Exercise Question

1) $$\tilde{N}orm_x[a, A] = \frac{1}{\sqrt{2\pi A^2}} e^{-\frac{(x-a)^2}{2A^2}}$$

$$\tilde{N}orm_x[b, B] = \frac{1}{\sqrt{2\pi B^2}} e^{-\frac{(x-b)^2}{2B^2}}$$

$$\int \tilde{N}orm_x[a, A]\, \tilde{N}orm[b, B]$$

We can use the convolution theoram to integrate thus

$$F^{-1}[F(Norm_x(a, A)\, F(Norm(b, B)]$$

Where F is the fourier transform

$$F(Norm_x[a, A]) = \int_{-\infty}^{\infty} Norm(a, A) e^{-2\pi i k x}\, dx$$

And F^{-1} is the inverse

$$F^{-1}(F(Norm(a, A)) = \int_{-\infty}^{\infty} F(k) e^{2\pi i k x}\, dx$$

Replacing $x' = x - a$

Fourier transform can be written as

$$F(Norm(a, A) = \frac{1}{\sqrt{2\pi A^2}} \int_{-\infty}^{\infty} e^{-\frac{x'^2}{2A^2}} e^{-2\pi i k (x'+a)}\, dx'$$

$$\Rightarrow \frac{e^{-2\pi i k a}}{\sqrt{2\pi A^2}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2A^2}} [\cos(2\pi k x') - i \sin(2\pi k x')]\, dx'$$

Since x' is odd thus $\int \sin = 0$.

$$F(Norm(a, A) = \frac{e^{-2\pi i k a}}{\sqrt{2\pi} A} \int e^{-\frac{x'^2}{2A^2}} \cos(2\pi k x')\, dx'$$

Similarly

$$F(Norm(b, B) = \frac{e^{-2\pi i k b}}{\sqrt{2\pi} B} \int e^{-\frac{x''^2}{2B^2}} \cos(2\pi k x'')\, dx''$$

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$\therefore \left[f(Norm(a, A)) f(Norm(b, B)) \right] =$

$$\frac{1 \, e^{-2\pi i K(a+b)}}{(\sqrt{2\pi} A)(\sqrt{2\pi} B)} \int e^{\frac{-(x'^2 + x''^2)}{2A^2B^2}} \cos(2\pi K x') \cos(2\pi K x'') \, dx' dx''$$

$\downarrow$

$$\frac{Norm_a(b, A+B)}{\cancel{Norm}}$$

Since Fourier Transforms are invertible

$$\therefore Norm_a(b, A+B) \int Norm_x[E_x(A^{-1}a + B^{-1}b), E_x] \, dx$$