1) Submodular equation

$$P(\beta,\gamma)+P(\alpha,\delta)-P(\beta,\delta)-P(\alpha,\gamma)\geq 0$$

1.1 $\beta>\alpha$ $\delta>\gamma$.

$$P(\omega_m,\omega_n)=c(\omega_m-\omega_n)^2$$

$$\Rightarrow (\beta^2+\gamma^2-2\beta\gamma+\alpha^2+\delta^2-2\alpha\delta$$

$$-\beta^2-\gamma^2+2\beta\delta-\alpha^2-\delta^2+2\alpha\gamma\geq 0$$

$$\Rightarrow -2\beta\gamma-2\alpha\delta+2\beta\delta+2\alpha\gamma$$

$$\Rightarrow \beta\delta+\alpha\gamma-\beta\gamma-\alpha\delta$$

$$\therefore \beta\delta\geq\beta\gamma \;\&\; \alpha\gamma\geq\alpha\delta\geq 0.$$

$\therefore$ Submodular

1.2 Potts Model.

$$P(\omega_m,\omega_n)=c(1-\delta(\omega_m-\omega_n))$$

Assume $\delta>\gamma=\beta>\alpha$

$$=c(0+1-1-1)=-c<0$$

Not submodular

2 a b c d e f = β β γ α α δ

Source