Knockoff - Construction - addendum

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We can separate $\tilde{X}$ to the part in the span of X (A) and the part orthogonal to X (B). Let's denote them $\tilde{X} = A + B$

Now we want that $X^T\tilde{X} = \Sigma - S \Rightarrow X^T(A+B) = \Sigma - S$
$\Rightarrow X^TA + X^TB = \Sigma - S \Rightarrow X^TA = \Sigma - S$
Since B is orthogonal to X.
$X^TA = \Sigma(\mathrm{I} - \Sigma^{-1}S) = X^TX(I - \Sigma^{-1}S)$
$\Rightarrow A = X(I - \Sigma^{-1}S)$

We also want that $\tilde{X}^T\tilde{X} = \Sigma$
$(A+B)^T(A+B) = A^TA + A^TB + B^TA + B^TB = \Sigma$
$\Rightarrow (X^TX - X^TX\Sigma^{-1}S + X^TB - S\Sigma^{-1}X^TX + S\Sigma^{-1}X^TX\Sigma^{-1}S - S\Sigma^{-1}X^TB + B^TX$
$- B^TX\Sigma^{-1}S + B^TB) = \Sigma$
Everything that has X^TB in it must be 0, since B is orthogonal to X.
We get that $B^TB = 2S - S\Sigma^{-1}S$

But this doesn't gurantee that $B^TX = 0$, for this we need to set $B = \tilde{U}C$
Where $C^TC = 2S - S\Sigma^{-1}S$.