Conservation of linear momentum

Consider two masses $m_1$and m_2,located at (x_1, y_1) and (x_2, y_2) respectively. At $t = 0$, the masses have velocities (v_{x1}, v_{y1}) and (v_{x2}, v_{y2}) respectively. The masses interact with each other via a gravitational attraction. At every instant the force on m_1 due to m_2 is given by $\vec{F}_{12} = (f_{x12}, f_{y12}) = (-\frac{Gm_1m_2}{d_{12}^3}(x_1 - x_2), -\frac{Gm_1m_2}{d_{12}^3}(y_1 - y_2))$. Similary the force on the mass $m_2$due to $m_1$is given by $\vec{F}_{21} = (f_{x21}, f_{y21}) = (-\frac{Gm_1m_2}{d_{12}^3}(x_2 - x_1), -\frac{Gm_1m_2}{d_{12}^3}(y_2 - y_1))$. The total external force and the total external torque acting on this system is zero, and hence it follows that the total linar momentum and total anuglar momentum is always zero.

1. First set up this system on python with trivial parameter values, $G = m_1 = m_2 = 1$. Let the masses be placed a distance of $d_{12} = 1$ unit apart. Let the initial velocities all be 0. Update the positions using the velocities, $(x_1, y_1) \to (x_1 + v_{x1}dt, y_1 + v_{y1}dt)$ and $(x_2, y_2) \to (x_2 + v_{x2}dt, y_2 + v_{y2}dt)$. Similarly update the velocities using the forces, $(v_{x1}, v_{y1}) \to (v_{x1} + f_{x12}dt, v_{y1} + f_{y12}dt)$ and $(v_{x2}, v_{y2}) \to (v_{x2} + f_{x21}dt, v_{y2} + f_{y21}dt)$. Make a movie or plot the trajectories of the particles. You should see them accelerate towards each other. They will eventually cross each other and move away from each other, and then come back again (No collision is included here, so they pass through each other)

2. Next let their initial postions be on the x axis, once again separated by 1 unit. Give them initial velocities along the y direction, but let $v_{y1} = -v_{y2}$. As you increase this intial velocity from 0, you will notice that the particles start orbiting each other. Calculate the momentum for each individual particle, $\vec{p}_1 = (p_{x1}, p_{y1}) = (m_1v_{x1}, m_1v_{y1})$ and similarly for m_2. Show graphically that though their individual momenta are not 0, the total momentum is 0, which is also the initial value of the mometum. As you increase the initial velocity, observe that orbits become more and more elliptical. For large values, the masses "escape" each other's gravitational field and do not have elliptic orbits any more. But the total momentum still remains 0.

3. A variant of the above is to use slightly unbalanced initial velocities, so $v_{y1} = -v_{y2} + \delta$. What is the initial total momentum? Find the trajectories of the masses, and show that the centre of mass of the system moves with this same initial momentum.

4. You could also use highly unbalanced masses and initial velocities, where $m_1 >> m_2$ and $v_{y1} = -m_2v_{y2}/m_2$

As an illustration of how the principle of conservation of linear momentum is used, use the values and procedures listed in problem 36, page 410, Chapter 10 of Matter and Interactions.

Conservation of Angular momentum (optional, if time permits)

To show the conservation of angular momentum, let one of the masses m_1, represent the sun, which is static. So, your code should be modified such that the positions and velocities of $m_1$are not updated and remain fixed. Place m_1 at the origin $(0, 0)$. For various initial conditions that give orbits or unbounded motion, find the angular momentum $\vec{r} \times \vec{p} = (0, 0, xp_y - yp_x)$ and show that it is constant in time.