Predictive Analytics Using statistics (UCS654)

PARAMETER ESTIMATION

1. $f(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$x_1, x_2, \ldots, x_n$ sample of size n.

$L(x_1, x_2, \cdots x_n) = f(x_1) \cdot f(x_2) \cdots f(x_n)$

$\Rightarrow \left(\dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right)\left(\dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right) \cdots$

taking ln on both sides

$$\ln(L) = -\frac{n}{2}\ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left| \frac{(x_i-\mu)^2}{2\sigma^2} \right| \longrightarrow (1)$$

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} -\left(2\frac{(x_i-\mu)^2}{2\sigma^2}\right) = 0$$

$$\sum_{i=1}^{n} (x_i-\mu) = 0$$

$$n\bar{x} - n\mu = 0$$

$$\boxed{\bar{x} = \mu}$$

$\boxed{\theta_1 = \bar{x}}$ is therefore sample mean

$$\frac{\partial \ln(L)}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2\sigma^2} = 0$$

$$n = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2} = 0$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\boxed{\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2}$$

② Binomial distribution $\rightarrow$ $\left({}^nC_{x_i}\,\theta^{x_i}(1-\theta)^{n-x_i}\right)$

$$L = \prod_{i=1}^{n} {}^nC_{x_i}\,\theta^{x_i}(1-\theta)^{n-x_i}$$

$$\log L = \sum_{i=1}^{n}\left(\log({}^nC_{x_i}) + \log\theta^{x_i} + \log(1-\theta)^{n-x_i}\right)$$

$$\log L = \sum_{i=1}^{n}\log({}^nC_{x_i}) + \log\theta\sum_{i=1}^{n}x_i + \log(1-\theta)\sum_{i=1}^{n}(n-x_i)$$

$$\frac{d\log(L)}{d\theta} = 0$$

$$\frac{1}{\theta}\sum x_i - \frac{1}{1-\theta}\sum(n-x_i) = 0$$

$$\frac{1}{\theta}\sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta}\sum x_i = 0$$

$$\frac{1}{\theta(1-\theta)}\sum x_i = \frac{n^2}{1-\theta}$$

$$\boxed{\theta = \frac{\sum x_i}{n^2}}$$