Transient Heat Conduction in Nuclear Fuel Pin

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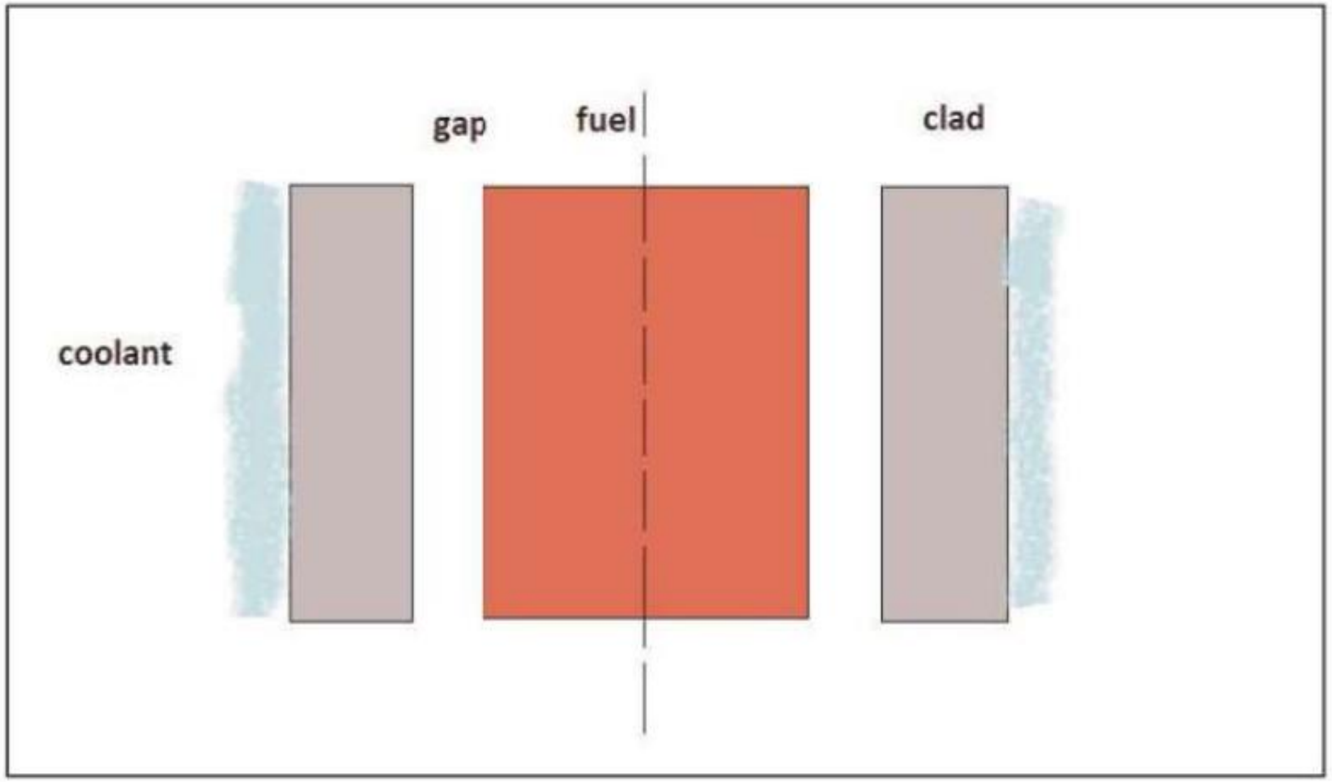
Radial Heat conduction in nuclear fuel elements

Reactor fuel pin is a cylindrical fuel element that contain fuel pellet, gap and cladding as shown in the Figure. Coolant can be in single phase or two phase. Fuel and clad properties are available as a function of temperature. The state of the coolant is obtained from convection equations. Thus, the information is provided by these to transient conduction solver. As the equation is non-linear due to complex property and boundary conditions, numerical solutions are called for.



Basic governing equation

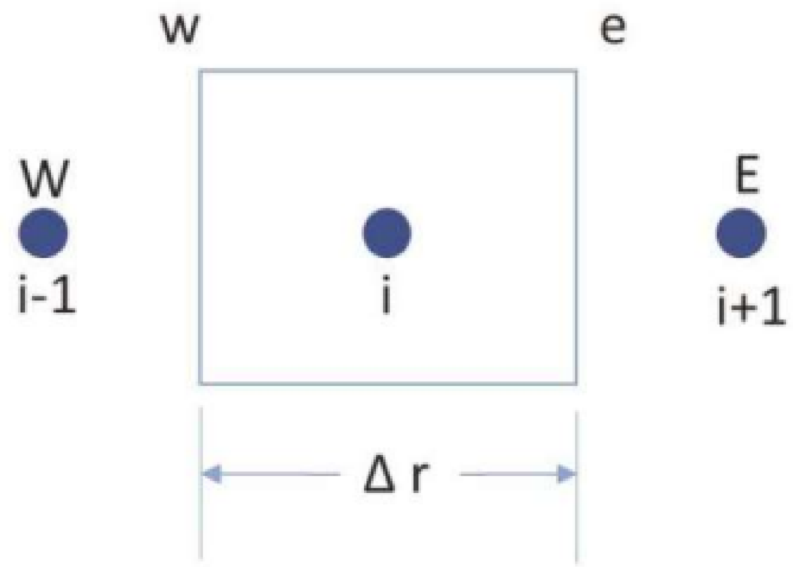
(after Over assumption of azimuthal symmetry and neglecting axial conduction affects)

$$\frac{\partial}{\partial t}\left(\rho c_p T\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + q'''$$

Discretized equations for fuel pin geometry

(using finite volume approach)

In this section, the governing equation (shown in the previous slide) is integrated over volume and time at $node_i$ to obtain the discretized equations.



Transient Term

$$\int_{w}^{e}\int_{t}^{t+\Delta t}\frac{\partial}{\partial t}\left(\rho c_p T\right) r dr dt = \Delta r_i r_i\left[\left(\rho c_p T_i^{n+1}\right) - \left(\rho c_p T_i^{n}\right)\right]$$

T_i^{n+1}, T_{i+1}^{n} represents average values in the control volume at n+1th and nth timesteps at ith node location.

Conduction Term

$$\int_{w}^{e}\int_{t}^{t+\Delta t}(1/r)\frac{\partial}{\partial t}\left(rk\frac{\partial}{\partial r}(T)\right)rdrdt = \left(\left(\frac{r_{i+1}^{n+1}k_{i+1}^{n+1}}{\Delta r_{i+1}^{n+1}}\left(T_{i+1}^{n+1} - T_{i}^{n+1}\right)\right) - \left(\frac{r_{i-1}^{n+1}k_{i-1}^{n+1}}{\Delta r_{i-1}^{n+1}}\left(T_{i}^{n+1} - T_{i-1}^{n+1}\right)\right)\right)$$

T_i^{n+1}, T_{i+1}^{n} represents average values in the control volume at $n+1^{th}$ and n^{th} timesteps at i^{th} node location.

Heat Generation Term

$$\int_w^e \int_t^{t+\Delta t} q''' r dr dt = (q_i''')^{n+1} \Delta t \left(\frac{r_{i+1}^2 - r_{i-1}^2}{2} \right) = (q_i''')^{n+1} r_i \Delta r_i \Delta t$$

T_i^{n+1}, T_{i+1}^n represents average values in the control volume at n+1th and nth timesteps at ith node location.

Substituting the discretization in the Governing Equation:

$$\int_w^e \int_t^{t+\Delta t} \frac{\partial}{\partial t}(\rho c_p T) r dr dt = \Psi\left(\int_w^e \int_t^{t+\Delta t} (1/r)\frac{\partial}{\partial t}\left(rk\frac{\partial}{\partial r}(T)\right) r dr dt - \int_w^e \int_t^{t+\Delta t} q''' r dr dt\right)$$

$$\left(\frac{\Delta r_i r_i \rho C_p}{\Delta t}\right)^{n+1} (T_i^{n+1}) - \left(\frac{\Delta r_i r_i \rho C_p}{\Delta t}\right)^{n} (T_i^{n})$$
$$= \Psi\left(\frac{r_{i+1}^{n+1} k_{i+1}^{n+1}}{\Delta r_{i+1}^{n+1}} T_{i+1}^{n+1}\right) + \Psi\left(\frac{r_{i-1}^{n+1} k_{i-1}^{n+1}}{\Delta r_{i-1}^{n+1}} T_{i-1}^{n+1}\right) - \Psi\left(\left(\frac{r_{i+1}^{n+1} k_{i+1}^{n+1}}{\Delta r_{i+1}^{n+1}} + \frac{r_{i-1}^{n+1} k_{i-1}^{n+1}}{\Delta r_{i-1}^{n+1}}\right) T_i^{n+1}\right) + \Psi(q_i''')^{n+1} r_i \Delta r_i$$

$$\left(\left(\Psi\frac{r_{i+1}^{n+1} k_{i+1}^{n+1}}{\Delta r_{i+1}^{n+1}} + \Psi\frac{r_{i-1}^{n+1} k_{i-1}^{n+1}}{\Delta r_{i-1}^{n+1}} + \left(\frac{\Delta r_i r_i \rho C_p}{\Delta t}\right)^{n+1}\right) T_i^{n+1}\right)$$
$$= \Psi\left(\frac{r_{i+1}^{n+1} k_{i+1}^{n+1}}{\Delta r_{i+1}^{n+1}} T_{i+1}^{n+1}\right) + \Psi\left(\frac{r_{i-1}^{n+1} k_{i-1}^{n+1}}{\Delta r_{i-1}^{n+1}} T_{i-1}^{n+1}\right) + \Psi(q_i''')^{n+1} r_i \Delta r_i + \left(\frac{\Delta r_i r_i \rho C_p}{\Delta t}\right)^{n} (T_i^{n})$$

Final Equation:

$$[CAP^{n+1}]T_i^{n+1} = [CAW^{n+1}]T_{i+1}^{n+1} + [CAE^{n+1}]T_{i-1}^{n+1} + S^{n+1}$$

Final Equation:

$$[CAP^{n+1}]T_i^{n+1} = [CAW^{n+1}]T_{i+1}^{n+1} + [CAE^{n+1}]T_{i-1}^{n+1} + S^{n+1}$$

$$AW^{n+1} = \frac{r_{i-1}^{n+1} k_{i-1}^{n+1}}{\Delta r_{i-1}^{n+1}}$$

$$CAW^{n+1} = \Psi AW^{n+1}$$

$$\Psi = \mathit{Implicit\ factor}$$

$$AE^{n+1} = \frac{r_{i+1}^{n+1} k_{i+1}^{n+1}}{\Delta r_{i+1}^{n+1}}$$

$$CAE^{n+1} = \Psi AE^{n+1}$$

$$AQ^{n+1} = (q_i''')^{n+1} r_i \Delta r_i$$

$$CAQ^{n+1} = \Psi AQ^{n+1}$$

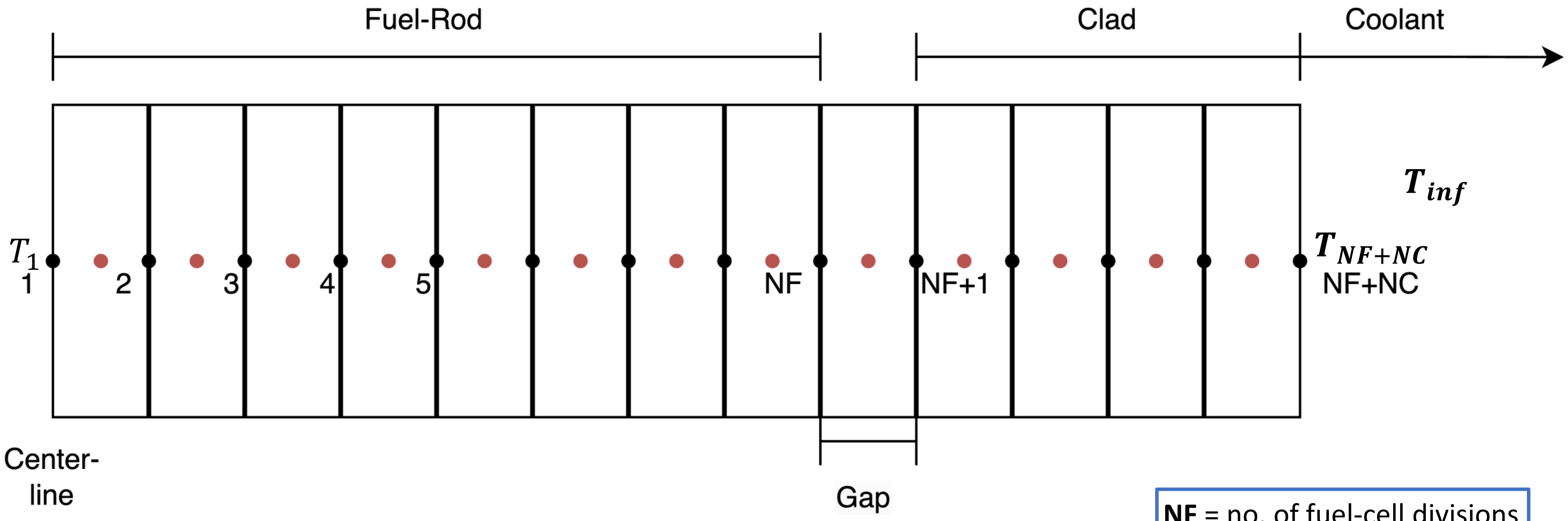
$$CAP^{n+1} = CAW^{n+1} + CAE^{n+1} + AT$$

$$S^{n+1} = CAQ^{n+1} + ATO \cdot T^n$$

$$AT = \left(\frac{\Delta r_i r_i \rho C_p}{\Delta t}\right)^{n+1}$$

$$ATO = \left(\frac{\Delta r_i r_i \rho C_p}{\Delta t}\right)^{n}$$

Discretization of Fuel Cell and Clad



NF = no. of fuel-cell divisions
NC = no. of clad-divisions
NT = NF + NC

Boundary Conditions

At Centre-line in Fuel Rod (Neumann Left Boundary Condition)

$$CAW_1^{n+1} = 0$$

$$S_1^{n+1} = CAQ_1^{n+1} + ATO_1 \cdot T_1^n + \Psi \cdot r_1 \cdot q_{flux}$$

q_{flux} = Heat flux

At Fuel-Gap Surface (Robin's Right Boundary Condition)

$$CAE_{NF}^{n+1} = \Psi AE_{NF}^{n+1} + \Psi \cdot r_{NF} \cdot HTC$$

HTC = Heat-Transfer Coefficient for fuel-gap-clad

$$CAP_{NF}^{n+1} = CAW_{NF}^{n+1} + CAE_{NF}^{n+1} + AT_{NF} + \Psi \cdot r_{NF} \cdot HTC$$

$$S_{NF}^{n+1} = CAQ_{NF}^{n+1} + ATO_{NF} \cdot T_{NF}^n + \Psi \cdot r_{NF} \cdot HTC \cdot \left(\frac{T_{NF}^n + T_{NF+1}^n}{2} \right)$$

At Gap-Clad Surface (Robin's Left Boundary Condition)

$$CAW_{NF+1}^{n+1} = \Psi AW_{NF+1}^{n+1} + \Psi \cdot r_{NF+1} \cdot HTC$$

$$CAP_{NF+1}^{n+1} = CAW_{NF+1}^{n+1} + CAE_{NF+1}^{n+1} + AT_{NF+1} + \Psi \cdot r_{NF+1} \cdot HTC$$

$$S_{NF+1}^{n+1} = CAQ_{NF+1}^{n+1} + ATO_{NF+1} \cdot T_{NF+1}^{n} + \Psi \cdot r_{NF+1} \cdot HTC \cdot \left(\frac{T_{NF}^{n} + T_{NF+1}^{n}}{2}\right)$$

At Clad-Coolant Surface (Robin's Right Boundary Condition)

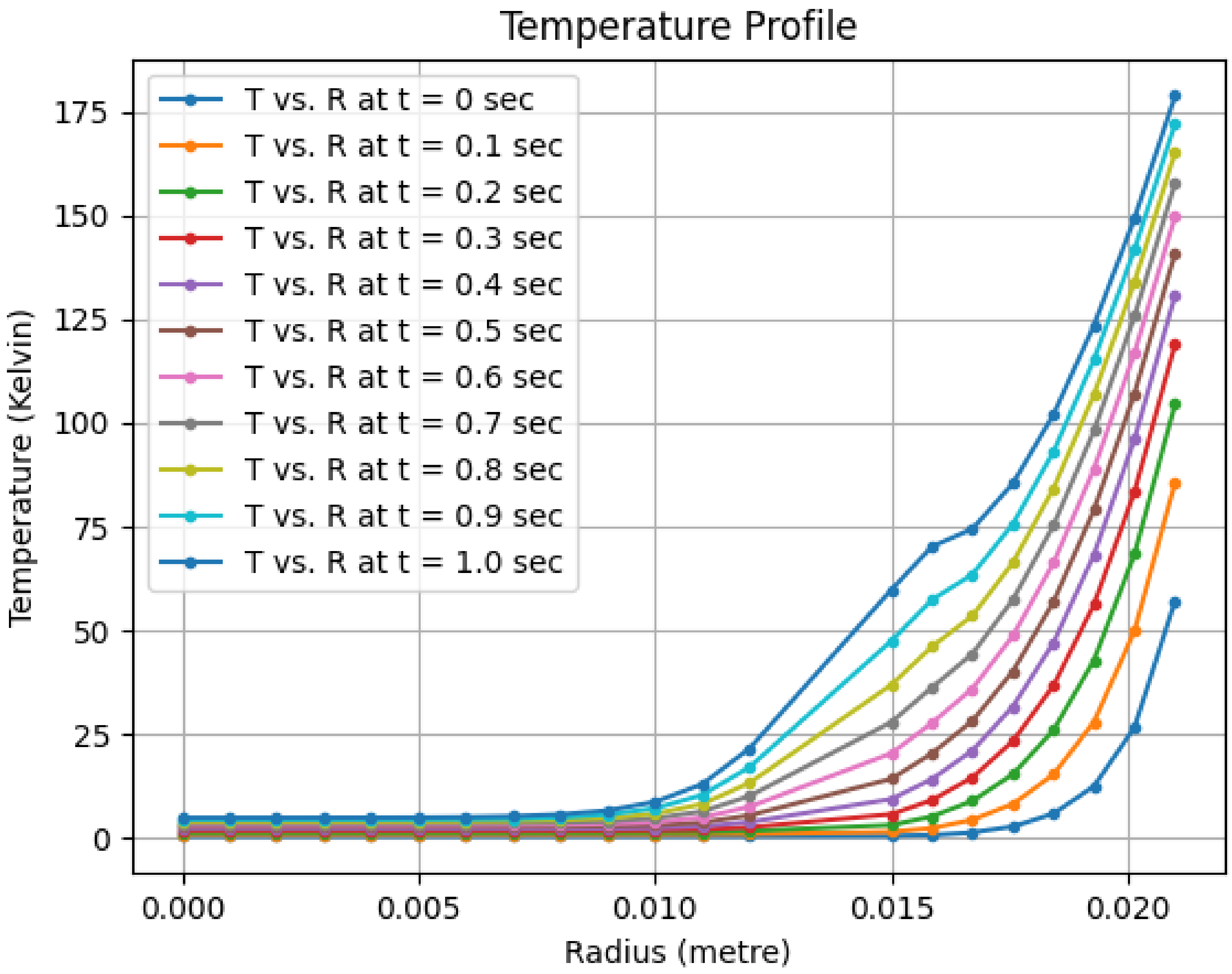
HTCC = Heat-Transfer Coefficient for clad-coolant
T_{inf} = Temperature of Coolant

$$CAE_{NF+NC}^{n+1} = 0$$

$$CAP_{NF+NC}^{n+1} = CAW_{NF+NC}^{n+1} + CAE_{NF+NC}^{n+1} + AT_{NF+NC} + \Psi \cdot r_{NF} \cdot HTCC$$

$$S_{NF+NC}^{n+1} = CAQ_{NF+NC}^{n+1} + ATO_{NF+NC} \cdot T_{NF+NC}^{n} + \Psi \cdot r_{NF+NC} \cdot HTCC \cdot T_{inf}$$

Transient State Temperature Profile



Steady State Temperature Profile