Beamforming

October 13, 2025

1 Beamforming Project

1.1 Problem Statement

Goal: Implement, compare, and analyze several classical and advanced acoustic beamforming algorithms on a controlled synthetic scene.

1.2 Scene Setup (Free-field, no flow)

- **Array Geometries**:
    1. **Uniform Circular Array (UCA):** 32 microphones, radius = 0.5 m
    2. **Spiral (aperiodic):** 48 microphones, radius 0.6 m
- **Sources (at z = 1.5 m):**
    - **S1:** (0.20, 0.00) m, tonal @ 2 kHz
    - **S2:** (0.26, 0.00) m, tonal @ 2 kHz, **coherent with S1** ($\rightarrow$ hard resolution test)
    - **S3:** (−0.35, 0.25) m, broadband (1–4 kHz), **incoherent**

NB: A tonal source is one whose sound is concentrated at one(or a few) discrete frequencies, while a broadband source has its energy spread over a wide band.

- **Frequencies Evaluated:** 1, 2, 4, 8 kHz
    - (8 kHz chosen to highlight **spatial aliasing** for the UCA)
- **Snapshots (Time Samples):**
  Default: (K = 200)
  Test with (K $\in$ {25, 50, 100, 200})
- **Noise:** Spatially white, **SNR = 10 dB (default)**

1.3 Beamforming Algorithms to Implement

1. **Conventional Beamforming (CB)** – Delay and Sum
2. **MVDR (Minimum Variance Distortionless Response)** – with diagonal loading

3. **MUSIC** – eigenvalue decomposition method
4. **Functional Beamforming (FB)** – parameter (m)
5. **CLEAN-SC (simplified)** – deconvolution method
6. **DAMAS** – Gauss–Seidel iterative solver, nonnegative constraint

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1.4 Tasks

1. **Implement** all six algorithms.
2. **Produce source maps** on a 2D scan grid.
   - Compare across methods, arrays, frequencies, and snapshots.
3. **Report performance metrics**:
   - Localization error
   - Resolution (FWHM of mainlobe)
   - MSR (Mainlobe-to-Sidelobe Ratio)
   - False positives
   - Runtime (computational cost)
   - Robustness vs snapshots
4. **Discussion**:
   - Effects of **spatial aliasing** (especially at 8 kHz)
   - **Coherent source behavior** (S1 & S2 separation)
   - **Array layout trade-offs** (UCA vs spiral)
5. **(Bonus)**: Inject a **steering-vector mismatch** (e.g., +2% speed-of-sound error).
   - Show which algorithms are more robust vs. sensitive.

1.5 Expected Deliverables

- Acoustic source maps (per method, frequency, snapshot, array type)
- Quantitative performance metrics (tables/plots)

- Comparative discussion on results
- (Optional) Bonus analysis with steering-vector mismatch

2 1. Conventional Beamforming: Delay-and-Sum (DAS)

1. Define the scene (speed of sound, sources, arrays, imaging grid).
2. Define the steering-vector function $g(\mathbf{x})$.
3. Define a frequency-domain cross-spectral matrix (CSM) for the sources + noise.
4. Implement DAS: $B(\mathbf{x}) = w^H C w$ with $w = g/||g||$.
5. Plot and interpret results.
6. Performance metrics: resolution, localization error, MSR.
7. Compare arrays & frequencies.

[2]:
```python
import numpy as np
import matplotlib.pyplot as plt

# Make plots a bit sharper
plt.rcParams["figure.dpi"] = 140

# Physics
c = 343.0           # speed of sound [m/s]
z_plane = 1.5       # imaging plane z [m]

# Reproducibility for any randomness (noise, etc.)
rng = np.random.default_rng(42)
```

2.1 Define the synthetic scene

- Two coherent tonal sources at 2 kHz (hard-resolution test).
- One broadband incoherent source (1–4 kHz).
- Two array geometries: UCA (circular) and aperiodic spiral.

[3]:
```python
# Step 1: Scene definition

# Sources (x, y) located on the imaging plane z = z_plane
sources = [
    {"name": "S1", "xy": ( 0.20,  0.00), "type": "tonal",     "f0": 2000.0,    ␣
 ↪"power": 1.0},
    {"name": "S2", "xy": ( 0.26,  0.00), "type": "tonal",     "f0": 2000.0,    ␣
 ↪"power": 1.0},
    {"name": "S3", "xy": (-0.35,  0.25), "type": "broadband", "band": (1e3,␣
 ↪4e3),"power": 0.7},
]
rho_S1S2 = 1.0  # coherence between S1 and S2 (1.0 = fully coherent)

# Arrays
```

```
def array_uca(num_mics=32, radius=0.5):
    """Uniform Circular Array on z=0."""
    ang = np.linspace(0, 2*np.pi, num_mics, endpoint=False)
    x = radius*np.cos(ang)
    y = radius*np.sin(ang)
    z = np.zeros_like(x)
    return np.vstack([x, y, z]).T

def array_spiral(num_mics=48, max_radius=0.6):
    """Aperiodic (Fermat) spiral to suppress sidelobes."""
    ga = np.pi * (3 - np.sqrt(5))  # golden angle
    n = np.arange(num_mics)
    r = max_radius * np.sqrt((n + 0.5)/num_mics)
    th = n * ga
    x = r*np.cos(th); y = r*np.sin(th); z = np.zeros_like(x)
    return np.vstack([x, y, z]).T

uca    = array_uca(32, 0.5)
spiral = array_spiral(48, 0.6)

# Imaging grid (x-y plane at z=z_plane)
grid_x = np.linspace(-0.7, 0.7, 141)
grid_y = np.linspace(-0.7, 0.7, 141)

# Frequencies to evaluate
freqs = [1000, 2000, 4000, 8000]  # Hz

# Noise level (spatially white)
SNR_dB_default = 10.0
```

Why these choices?

- The coherent pair (S1,S2) at 2 kHz stresses resolution limits.
- 8 kHz stresses spatial aliasing, especially on the circular array, since mic spacing becomes comparable to half-wavelength.

3 Define the steering vector g (Green's function)

We assume spherical spreading and a phase delay:

$$g_m(\mathbf{x}) = \frac{e^{-jkr_m}}{4\pi r_m}, \quad r_m = \|\mathbf{x} - \mathbf{x}_m\|, \quad k = 2\pi f/c.$$

[13]:
```
# Step 2: Steering vector / Green's function

def steering_vector(freq, mic_xyz, grid_xy, z_plane=1.5, c=343.0, decay=True):
    """
    Free-field monopole Green's function from a candidate source point␣
↪(grid_xy, z_plane)
    to each microphone.
    Returns g in shape (M,), M = number of microphones.
    """
    k = 2*np.pi*freq / c
    xg, yg = grid_xy
    grid_xyz = np.array([xg, yg, z_plane])
    r = np.linalg.norm(mic_xyz - grid_xyz, axis=1)  # distances mic <-> grid␣
↪point
    phase = np.exp(-1j * k * r)
    # The decay variable is used to switch the steering vector depending on␣
↪whether we are in the near-field or far-field case
    if decay:
        g = phase / (4*np.pi * np.maximum(r, 1e-6))  # avoid 1/0 # Spherical␣
↪Green's function. Best for near-field cases or when the methods expect the␣
↪physical Green's function, e.g., DAMAS/CLEAN-PSF.
    else:
        g = phase # "Phase-only" Green's function. This approximates a␣
↪far-field case where amplitude is approximately constant across all␣
↪microphones.
    return g
```

3.1 Build the CSM C at a single frequency

The CSM at a single frequncy f is defined as:

$$\mathbf{C} = \sum_k P_k \, \mathbf{g}_k \mathbf{g}_k^{\mathrm{H}} \; + \; (\text{cross-terms for coherence}) \; + \; \sigma_n^2 \mathbf{I}.$$

where: * the cross-terms = 0 for mutually uncorrelated sources. * P_k represents the autopower which is a measure of how strong source k is at this frequency f. * g_k represents the steering vector from source k to the M microphones at f. * $\mathbf{g}_k^{\mathrm{H}}$ represents the conjugate transpose of g_k. * σ_n^2 represents the noise variance per microphone. * The identity matrix $\mathbf{I} \in \mathbb{R}^{M \times M}$ is used to inject spatially white (uncorrelated) noise of variance σ_n^2 to each microphone (diagonal only). This sets the noise target SNR and provides diagonal loading which improves conditioning of the CSM and guarantees positive definiteness for methods that invert or eigendecompose C to keep the beamformer output numerically stable. The target SNR we chose is relative to the total sigal power to ensure that noise is independent of array size, and in general to provide realistic and numerically stable test data. In numerical analysis, we say that we want a well-defined problem.

Remark: 1. If two sources are **coherent** at the same frequency (e.g., S1/S2 at 2 kHz with fixed phase relation), the CSM also includes **cross-terms**:

$$P_{12} \, \mathbf{g}_1 \mathbf{g}_2^{\mathrm{H}} + P_{21} \, \mathbf{g}_2 \mathbf{g}_1^{\mathrm{H}}.$$

to model coherence. For **incoherent** sources, these cross-terms' expectations are zero and are omitted.

2. Since we are considering a single-frequency CSM at f, the following holds:

   - If $f \notin [1, 4]$ kHz, then S3 contributes nothing (we assume S3 emits no energy at that frequency).
   - If $f \in [1, 4]$ kHz, then S3 contributes a source term:

     $$P_3(f)\mathbf{g}_3\mathbf{g}_3^H$$

     Since S3 is incoherent with S1/S2, there are no cross-terms like $\mathbf{g}_3\mathbf{g}_1^H$

[15]:
```
# Step 3: Synthetic CSM (per frequency)

def synth_csm(freq, mic_xyz, sources, SNR_dB=10.0, rho12=1.0, z_plane=1.5,␣
↪c=343.0):
    """
    Construct a model CSM at 'freq' for the given array and source set.
    - S1/S2 (tonal): includes cross-terms controlled by rho12 to emulate␣
↪coherence.
    - Broadband sources: contribute only if 'freq' lies in their band.
    - Noise: spatially white, level chosen to achieve SNR_dB wrt signal power.
    """
    M = mic_xyz.shape[0]
    C = np.zeros((M, M), dtype=complex)

    # Tonal steering vectors for S1 and S2 (only if freq == f0)
    s1 = next(s for s in sources if s["name"] == "S1")
    s2 = next(s for s in sources if s["name"] == "S2")

    # Only build the steering vector g for S1/S2 at thier tone; otherwise set␣
↪to None (not active at this frequency)
    gS1 = steering_vector(freq, mic_xyz, s1["xy"], z_plane, c, True) if␣
↪(s1["type"]=="tonal" and np.isclose(freq, s1["f0"])) else None
    gS2 = steering_vector(freq, mic_xyz, s2["xy"], z_plane, c, True) if␣
↪(s2["type"]=="tonal" and np.isclose(freq, s2["f0"])) else None

    # Add the autopower contribution of each source to the CSM
    if gS1 is not None: C += s1["power"] * np.outer(gS1, gS1.conj()) # RHS reps␣
↪the auto-power contribution of S1 to the CSM at the given freq
    if gS2 is not None: C += s2["power"] * np.outer(gS2, gS2.conj()) # RHS reps␣
↪the auto-power contribution of S2 to the CSM at the given freq

    # Coherent cross-terms
    if (gS1 is not None) and (gS2 is not None) and (rho12 is not None):
        cross = rho12 * np.sqrt(s1["power"]*s2["power"]) # Cross-covariance of␣
↪S1 and S2
        C += cross * np.outer(gS1, gS2.conj()) # C_12 = rho12␣
↪\sqrt(P_1P_2)g_1g_2^H + rho_{12}^* \sqrt(P_1P_2)g_2g_1^H
        C += cross.conjugate() * np.outer(gS2, gS1.conj())
    # Broadband sources
    for s in sources:
        if s["type"] == "broadband":
            fmin, fmax = s["band"]
            if fmin <= freq <= fmax:
                gb = steering_vector(freq, mic_xyz, s["xy"], z_plane, c, True)
                C += s["power"] * np.outer(gb, gb.conj())

    # White noise level to hit target SNR wrt average signal power per mic
    sig_pow = float(np.real(np.trace(C)) / M) if np.real(np.trace(C)) > 0 else␣
↪1e-12
    SNR_lin = 10.0**(SNR_dB/10.0)
    noise_pow = sig_pow / SNR_lin
    C += noise_pow * np.eye(M)

    return C
```

3.2 DAS beamforming map

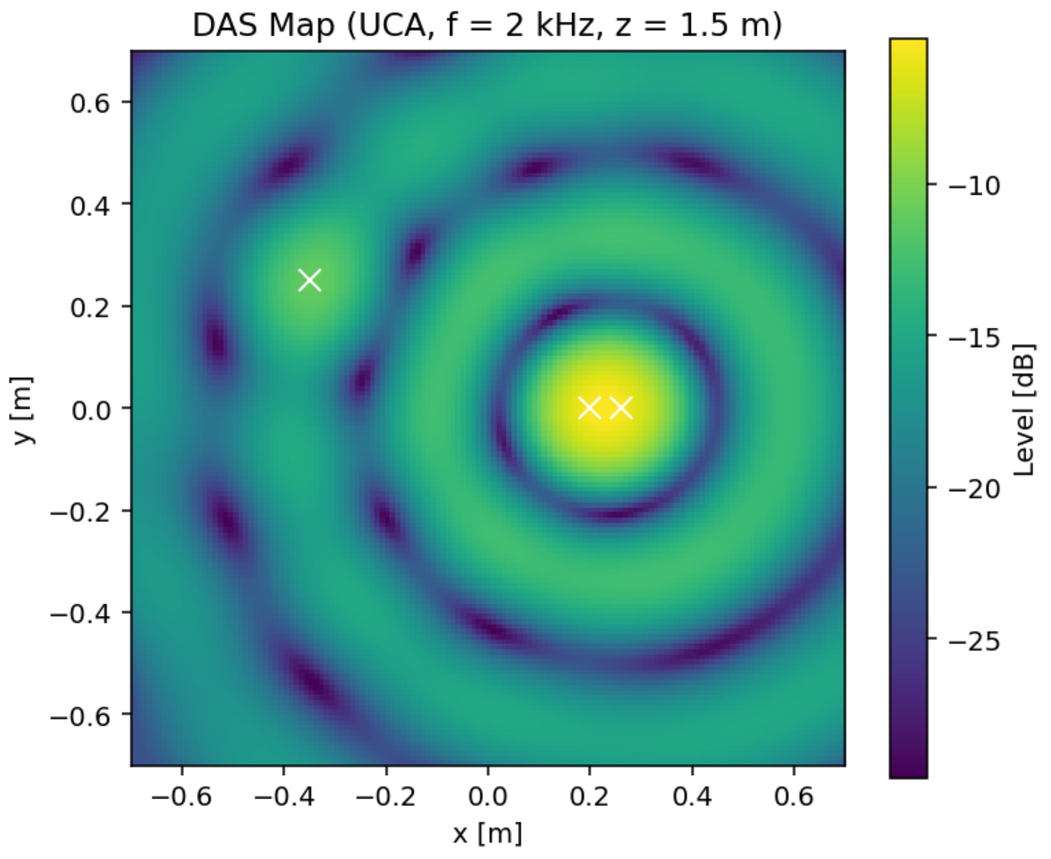
For each grid point:

1. Build $\mathbf{g}$,
2. Normalize $\mathbf{w} = \mathbf{g}/\|\mathbf{g}\|$ (prevents bias to nearer points),
3. Compute power $B = \mathbf{w}^{\mathrm{H}}\mathbf{C}\mathbf{w}$.

[6]:
```
# Step 4: DAS map computation

def das_map(freq, mic_xyz, C, grid_x, grid_y, z_plane=1.5, c=343.0):
    """
    DAS (Conventional) map: B(x) = w^H C w with w = g / ||g||.
    Returns a 2D array (ny, nx) aligned with (grid_y, grid_x).
    """
    nx, ny = len(grid_x), len(grid_y)
    B = np.zeros((ny, nx), dtype=float)
    for iy, gy in enumerate(grid_y):
        for ix, gx in enumerate(grid_x):
            g = steering_vector(freq, mic_xyz, (gx, gy), z_plane, c, True)
            norm = np.linalg.norm(g)
            w = g if norm < 1e-12 else g / norm
            B[iy, ix] = np.real((w.conj().T @ (C @ w)))
    return B
```

3.3 Visualize a first map (UCA, 2 kHz)

[7]:
```
# Step 5: Run a demo (UCA, 2 kHz)
freq = 2000
C_uca = synth_csm(freq, uca, sources, SNR_dB=SNR_dB_default, rho12=rho_S1S2,␣
 ↪z_plane=z_plane, c=c)
B_uca = das_map(freq, uca, C_uca, grid_x, grid_y, z_plane=z_plane, c=c)

# Plot in dB
plt.figure(figsize=(6,5))
plt.imshow(10*np.log10(np.maximum(B_uca, 1e-16)),
           extent=[grid_x.min(), grid_x.max(), grid_y.min(), grid_y.max()],
           origin="lower", aspect="equal")
plt.colorbar(label="Level [dB]")
plt.title("DAS Map (UCA, f = 2 kHz, z = 1.5 m)")
plt.xlabel("x [m]"); plt.ylabel("y [m]")

# Mark true sources
for s in sources:
    plt.plot(s["xy"][0], s["xy"][1], "wx", ms=8)
plt.show()
```

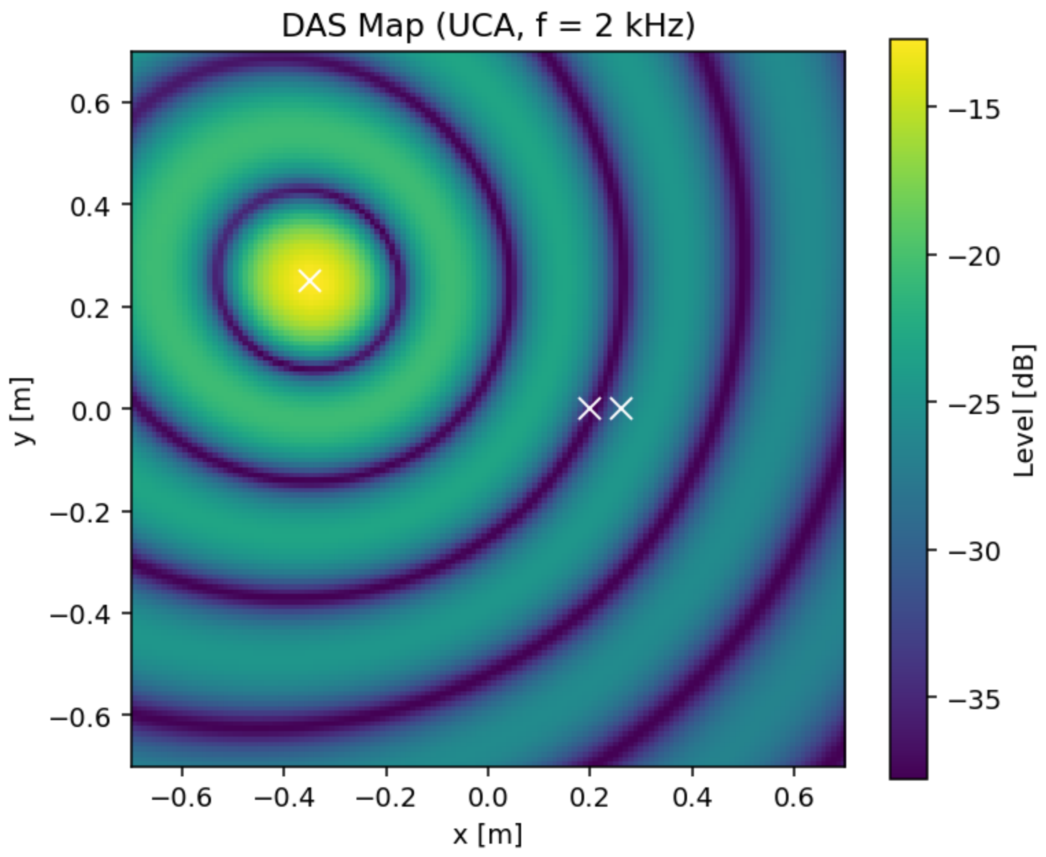


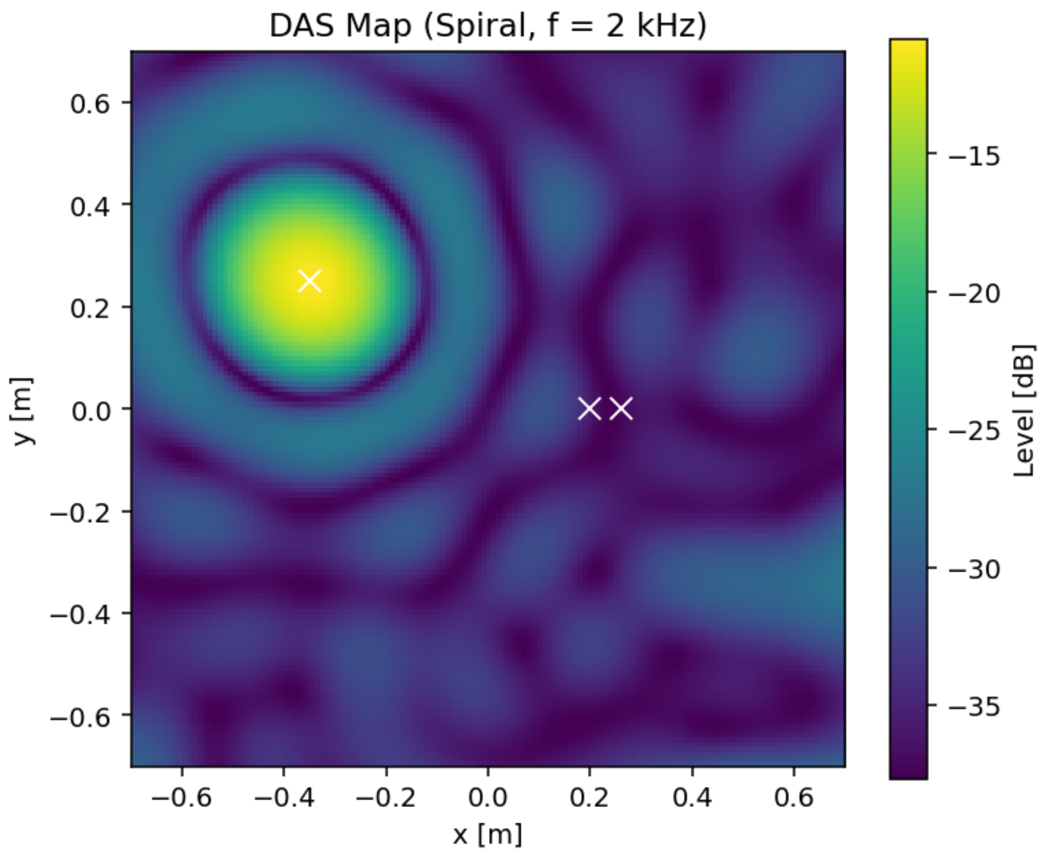
Interpretation:

- we see a strong mainlobe near $x \approx 0.2$ to 0.26 (coherent pair can blur/merge).
- we also see S3 (broadband) around $(-0.35, 0.25)$ at 2 kHz (within its 1–4 kHz band).
- Ringing around lobes are sidelobes (PSF of the array).

3.4 Quick comparison: Circular & spiral array

[8]:
```
freq_hi = 2500
C_uca_hi = synth_csm(freq_hi, uca, sources, SNR_dB=SNR_dB_default,␣
 ↪rho12=rho_S1S2, z_plane=z_plane, c=c)
B_uca_hi = das_map(freq_hi, uca, C_uca_hi, grid_x, grid_y, z_plane=z_plane, c=c)

plt.figure(figsize=(6,5))
plt.imshow(10*np.log10(np.maximum(B_uca_hi, 1e-16)),
           extent=[grid_x.min(), grid_x.max(), grid_y.min(), grid_y.max()],
           origin="lower", aspect="equal")
plt.colorbar(label="Level [dB]")
plt.title("DAS Map (UCA, f = 2 kHz)")
plt.xlabel("x [m]"); plt.ylabel("y [m]")
for s in sources:
    plt.plot(s["xy"][0], s["xy"][1], "wx", ms=8)
plt.show()

# Spiral at 2 kHz (generally nicer sidelobes)
C_sp_2k = synth_csm(freq_hi, spiral, sources, SNR_dB=SNR_dB_default,␣
 ↪rho12=rho_S1S2, z_plane=z_plane, c=c)
B_sp_2k = das_map(freq_hi, spiral, C_sp_2k, grid_x, grid_y, z_plane=z_plane,␣
 ↪c=c)

plt.figure(figsize=(6,5))
plt.imshow(10*np.log10(np.maximum(B_sp_2k, 1e-16)),
           extent=[grid_x.min(), grid_x.max(), grid_y.min(), grid_y.max()],
           origin="lower", aspect="equal")
plt.colorbar(label="Level [dB]")
plt.title("DAS Map (Spiral, f = 2 kHz)")
plt.xlabel("x [m]"); plt.ylabel("y [m]")
for s in sources:
    plt.plot(s["xy"][0], s["xy"][1], "wx", ms=8)
plt.show()
```


DAS Map (UCA, f = 2 kHz)
y [m]
x [m]
Level [dB]
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−15
−20
−25
−30
−35



3.5 Spiral array reduces regular grating lobes → cleaner maps than UCA with same aperture.

3.6 Useful metrics (peak, localization error, FWHM, MSR)

These help quantify performance beyond just eyeballing maps.

[9]:
```
# Step 6: Metrics utilities

def find_map_peak(B, grid_x, grid_y):
    """Return peak value and its (x,y) coordinate."""
    iy, ix = np.unravel_index(np.argmax(B), B.shape)
    return float(B[iy, ix]), (grid_x[ix], grid_y[iy])

def nearest_true_sources_at_freq(freq, sources):
    """Return list of ground-truth source positions active at this freq."""
    xy_list = []
    for s in sources:
        if s["type"] == "tonal" and np.isclose(freq, s["f0"]):
            xy_list.append(s["xy"])
        if s["type"] == "broadband":
            fmin, fmax = s["band"]
            if fmin <= freq <= fmax:
                xy_list.append(s["xy"])
    return xy_list

def localization_error(peak_xy, freq, sources):
    """Euclidean distance to the nearest *active* true source at this frequency.
↪"""
    actives = nearest_true_sources_at_freq(freq, sources)
    if not actives:
        return np.nan
    dists = [np.hypot(peak_xy[0]-x, peak_xy[1]-y) for (x,y) in actives]
    return float(np.min(dists))

def estimate_fwhm(B, grid_x, grid_y, peak_xy):
    """
    Rough FWHM estimate: width at half-max on a horizontal and vertical
    cut through the peak; return average of the two.
    """
    # Locate nearest grid indices
    ix = int(np.argmin(np.abs(grid_x - peak_xy[0])))
    iy = int(np.argmin(np.abs(grid_y - peak_xy[1])))
    peak = B[iy, ix]
    half = 0.5*peak

    # Horizontal cut (y fixed)
    row = B[iy, :]
    left  = ix - np.argmax(row[:ix][::-1] < half) if np.any(row[:ix] < half)␣
↪else 0
    right = ix + np.argmax(row[ix:]     < half)   if np.any(row[ix:]  < half)␣
↪else len(grid_x)-1
    fwhm_x = grid_x[right] - grid_x[left]

    # Vertical cut (x fixed)
    col = B[:, ix]
    down = iy - np.argmax(col[:iy][::-1] < half) if np.any(col[:iy] < half)␣
↪else 0
    up   = iy + np.argmax(col[iy:]       < half) if np.any(col[iy:] < half)␣
↪else len(grid_y)-1
    fwhm_y = grid_y[up] - grid_y[down]

    return float((abs(fwhm_x)+abs(fwhm_y))/2)

def mainlobe_sidelobe_ratio(B, peak_xy, grid_x, grid_y, exclude_radius=0.08):
    """
    MSR (in dB): 10*log10(P_main / P_max_sidelobe).
    We exclude a small disk around the peak to search for the highest sidelobe.
    """
    # Build coordinate grids
    X, Y = np.meshgrid(grid_x, grid_y)
    # Mask mainlobe neighborhood
    mask = ( (X-peak_xy[0])**2 + (Y-peak_xy[1])**2 ) <= exclude_radius**2
    Bl = B.copy()
    Bl[mask] = -np.inf  # remove mainlobe neighborhood
    side_max = np.max(Bl)
    peak_val = B[np.argmin(np.abs(grid_y-peak_xy[1])), np.argmin(np.
 ↪abs(grid_x-peak_xy[0]))]
    return 10*np.log10(peak_val / max(side_max, 1e-16))
```

3.6.1 Use them on a map:

[10]:
```
# Example on the UCA 2 kHz map
peak_val, peak_xy = find_map_peak(B_uca, grid_x, grid_y)
err = localization_error(peak_xy, 2000, sources)
fwhm = estimate_fwhm(B_uca, grid_x, grid_y, peak_xy)
msr_db = mainlobe_sidelobe_ratio(B_uca, peak_xy, grid_x, grid_y,␣
 ↪exclude_radius=0.08)

print(f"Peak at {peak_xy}, value={peak_val:.3e}")
print(f"Localization error to nearest active source: {err:.3f} m")
print(f"FWHM (avg of x/y cuts): {fwhm:.3f} m")
print(f"Mainlobe-to-Sidelobe Ratio (MSR): {msr_db:.1f} dB")
```

```
Peak at (0.23000000000000001, 1.1102230246251565e-16), value=3.013e-01
Localization error to nearest active source: 0.030 m
FWHM (avg of x/y cuts): 0.200 m
Mainlobe-to-Sidelobe Ratio (MSR): 1.8 dB
```

3.7 Batch runner (compare arrays & frequencies)

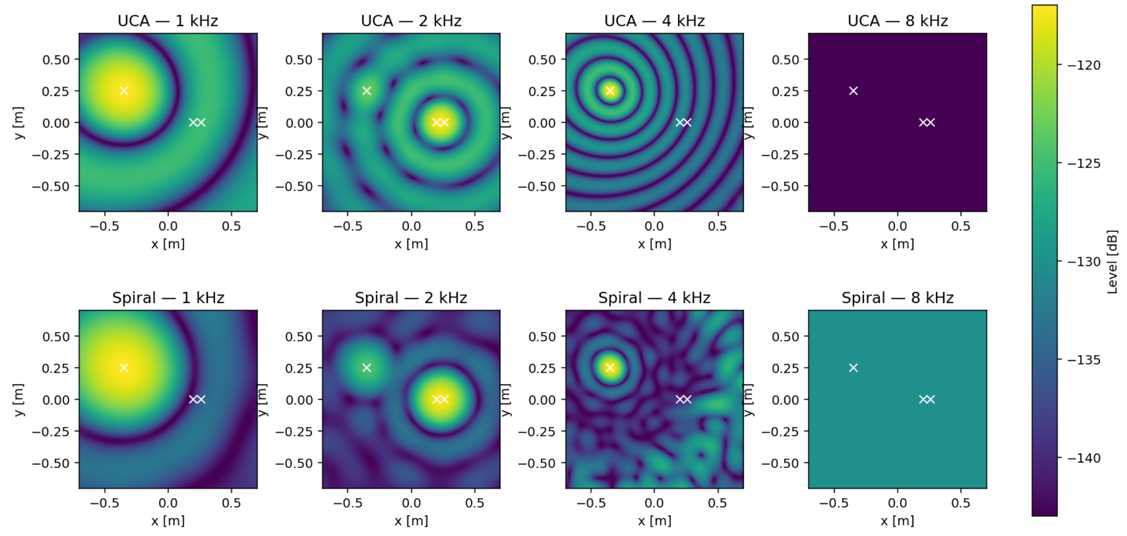
[11]:
```
# Step 7: Batch evaluation helper

def run_das_once(freq, mic_xyz, label):
    C = synth_csm(freq, mic_xyz, sources, SNR_dB=SNR_dB_default,␣
 ↪rho12=rho_S1S2, z_plane=z_plane, c=c)
    B = das_map(freq, mic_xyz, C, grid_x, grid_y, z_plane=z_plane, c=c)
    peak_val, peak_xy = find_map_peak(B, grid_x, grid_y)
    metrics = {
        "array": label,
        "freq": freq,
        "peak_xy": peak_xy,
        "loc_error_m": localization_error(peak_xy, freq, sources),
        "fwhm_m": estimate_fwhm(B, grid_x, grid_y, peak_xy),
        "msr_db": mainlobe_sidelobe_ratio(B, peak_xy, grid_x, grid_y,␣
 ↪exclude_radius=0.08),
    }
    return B, metrics

results = []
maps = []

for arr_label, arr in [("UCA", uca), ("Spiral", spiral)]:
    for f in freqs:
        B, m = run_das_once(f, arr, arr_label)
        maps.append((arr_label, f, B))
        results.append(m)

# Show a small summary
import pandas as pd
df = pd.DataFrame(results)
df
```

[11]:

```
    array  freq                                    peak_xy   loc_error_m  \
0     UCA  1000      (-0.3499999999999999, 0.2500000000000001)  1.241267e-16
1     UCA  2000  (0.2300000000000001, 1.1102230246251565e-16)  3.000000e-02
2     UCA  4000      (-0.3499999999999999, 0.2500000000000001)  1.241267e-16
3     UCA  8000    (-0.6399999999999999, -0.06999999999999995)           NaN
4  Spiral  1000      (-0.3499999999999999, 0.2500000000000001)  1.241267e-16
5  Spiral  2000  (0.2300000000000001, 1.1102230246251565e-16)  3.000000e-02
6  Spiral  4000      (-0.3499999999999999, 0.2500000000000001)  1.241267e-16
7  Spiral  8000                   (0.44000000000000017, -0.52)           NaN

   fwhm_m    msr_db
0   0.425  0.377386
1   0.200  1.839329
2   0.110  7.431672
3   1.400  0.000000
4   0.510  0.267663
5   0.235  1.287211
6   0.130  4.686734
7   1.400  0.000000
```

[12]:

```
# Visualize a grid of maps
ncols = len(freqs)
nrows = 2
fig, axes = plt.subplots(nrows, ncols, figsize=(3*ncols, 3*nrows),␣
 ↪constrained_layout=True)
for (arr_label, f, B), ax in zip(maps, axes.ravel()):
    im = ax.imshow(10*np.log10(np.maximum(B, 1e-16)),
                       extent=[grid_x.min(), grid_x.max(), grid_y.min(), grid_y.
↪max()],
                       origin="lower", aspect="equal")
    ax.set_title(f"{arr_label} - {f/1000:g} kHz")
    ax.set_xlabel("x [m]"); ax.set_ylabel("y [m]")
    for s in sources: ax.plot(s["xy"][0], s["xy"][1], "wx", ms=6)
fig.colorbar(im, ax=axes.ravel().tolist(), shrink=0.95, label="Level [dB]")
plt.show()
```



3.8 Key Points:

- **Steering vectors** model propagation; normalizing them avoids bias.
- The **CSM** embeds both power and **coherence** structure between microphones; DAS simply "looks" in a direction (or point) via **w**.
- **Resolution** (FWHM) improves with **aperture** and **frequency**, but high frequency risks **aliasing** if mic spacing is coarse—spiral arrays help here.
- **Coherent sources** S1/S2 are hard for DAS: it often merges them; advanced methods (MVDR, MUSIC, deconvolution) do better.

[ ]: