Relativity

$$\beta = \frac{p}{E}$$

Equations

Schrödinger:

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi t\rangle \qquad \mathscr{L} = i\psi^*\dot{\psi} - \frac{1}{2m}\vec{\nabla}\psi^*\cdot\vec{\nabla}\psi$$

Klein-Gordon (Real Scalar Field):

$$(\partial\cdot\partial + m^2)\phi = 0 \qquad \mathscr{L} = \frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right] = \frac{1}{2}\left[\dot{\phi}^2 - \left(\vec{\nabla}\phi\right)^2 - m^2\phi^2\right]$$

Dirac (Complex Scalar Field):

$$(i\partial\!\!\!/ - m)\varphi = 0 \qquad \overline{\varphi}(i\partial\!\!\!/ + m) = 0 \qquad \mathscr{L} = \partial_\mu\varphi\partial^\mu\varphi^* - m^2\varphi\varphi^*$$

γ-Matrices

$$\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu} \quad \gamma_5 = +i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma_0\gamma_1\gamma_2\gamma_3 = -\frac{i}{4!}\epsilon^{\alpha\beta\gamma\delta}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu] \implies [\gamma^\mu,\gamma^\nu] = -2i\sigma^{\mu\nu} \qquad \{\gamma_5,\gamma^\mu\} = 0\gamma_5\gamma_5 = +\mathbb{1}$$

$$\left(\gamma^0\right)^\dagger = \gamma^0 \quad \left(\gamma^0\right)^2 = \mathbb{1} \qquad \left(\gamma^k\right)^\dagger = -\gamma^k \quad \left(\gamma^k\right)^2 = -\mathbb{1} \qquad \left(\gamma^5\right)^\dagger = \gamma^5 \quad \left(\gamma^5\right)^2 = \mathbb{1}$$

$\mathbb{1}$	γ_5	γ^μ	$\gamma_5\gamma^\mu$	$\sigma^{\mu\nu}$
γ_5	$\mathbb{1}$	$+\gamma_5\gamma^\mu$	$+\gamma^\mu$	$\frac{i}{2}\epsilon^{\mu\nu\pi\rho}\sigma_{\pi\rho}$
γ^α	$-\gamma_5\gamma^\alpha$	$g^{\alpha\mu} - i\sigma^{\alpha\mu}$	$-\frac{1}{2}\left(2g^{\alpha\mu}\gamma_5 + \epsilon^{\alpha\mu\pi\rho}\sigma_{\pi\rho}\right)$	$\epsilon^{\alpha\mu\nu\lambda}\gamma_5\gamma_\lambda + ig^{\alpha\mu}\gamma^\nu - ig^{\alpha\nu}\gamma^\mu$
$\gamma_5\gamma^\alpha$	$-\gamma^\alpha$	$-\gamma^\alpha$	$-(g^{\alpha\mu} - i\sigma^{\alpha\mu})$	$\epsilon^{\alpha\mu\nu\lambda}\gamma_\lambda + ig^{\alpha\mu}\gamma_5\gamma^\nu - ig^{\alpha\nu}\gamma_5\gamma^\mu$
$\sigma^{\alpha\beta}$	$\sigma^{\alpha\beta}$	$\epsilon^{\alpha\beta\mu\lambda}\gamma_5\gamma_\lambda + ig^{\beta\mu}\gamma^\alpha - ig^{\alpha\mu}\gamma^\beta$	$\epsilon^{\alpha\beta\mu\lambda}\gamma_\lambda + ig^{\beta\mu}\gamma_5\gamma^\alpha - ig^{\alpha\mu}\gamma_5\gamma^\beta$	$i\epsilon^{\alpha\beta\mu\nu}\gamma_5 + g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu} + i\left[g^{\alpha\nu}\sigma^{\beta\mu} + g^{\beta\mu}\sigma^{\alpha\nu} - g^{\alpha\mu}\sigma^{\beta\nu} - g^{\beta\nu}\sigma^{\alpha\mu}\right]$

$$A\!\!\!/ = A^\mu\gamma_\mu \qquad \gamma^\mu\gamma_\mu = 4 \qquad \gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4g^{\mu\nu}$$

$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\nu\gamma^\rho\gamma^\sigma \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\pi\gamma_\mu = 2[\gamma^\pi\gamma^\nu\gamma^\rho\gamma^\sigma + \gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\pi]$$

Traces

$$\mathrm{Tr}[\mathbb{1}] = 4 \qquad \mathrm{Tr}[\underbrace{\gamma^\mu\gamma^\nu\ldots\gamma^\rho}_{\text{Odd Number}}] = 0 \qquad \mathrm{Tr}[\gamma^\mu\gamma^\nu] = 4g^{\mu\nu}$$

$$\mathrm{Tr}[\gamma_5] = 0 \qquad \mathrm{Tr}[\gamma_5\underbrace{\gamma^\mu\gamma^\nu\ldots\gamma^\rho}_{\text{Odd Number}}] = 0 \qquad \mathrm{Tr}[\gamma_5\gamma^\mu\gamma^\nu] = 0$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) \qquad \mathrm{Tr}[\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = -4i\epsilon^{\mu\nu\rho\sigma}$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_L] = 2(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} + i\epsilon^{\mu\nu\rho\sigma})$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_R] = 2(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - i\epsilon^{\mu\nu\rho\sigma})$$

$$\mathrm{Tr}\left[(p\!\!\!/_a + m_a)\gamma^\mu(p\!\!\!/_b + m_b)\gamma^\nu\right] = 4\left[p_a^\mu p_b^\nu + p_b^\mu p_a^\nu + (m_a m_b - p_a\cdot p_b)g^{\mu\nu}\right]$$

Spinors (Fermions)

$$\psi = ue^{i\left(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}} - E\cdot t\right)} = ve^{-i\left(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}} - E\cdot t\right)} \quad \overline{u}(p\!\!\!/ - m) = (p\!\!\!/ - m)u = 0 \quad \overline{v}(p\!\!\!/ + m) = (p\!\!\!/ + m)v = 0$$

$$\overline{u}^r(p)u^s(p) = +2m\delta^{rs} \quad \overline{u}^r(p)v^s(p) = 0 \quad u^{r\dagger}(p)u^s(p) = 2E\delta^{rs} \quad \sum_{s=1,2} u^s(p)\overline{u}^s(p) = p\!\!\!/ + m$$

$$\overline{v}^r(p)v^s(p) = -2m\delta^{rs} \quad \overline{v}^r(p)u^s(p) = 0 \quad u^{r\dagger}(p)u^s(p) = 2E\delta^{rs} \quad \sum_{s=1,2} v^s(p)\overline{v}^s(p) = p\!\!\!/ - m$$

Polarization Vectors (Bosons)

External massless & Massive:

$$\sum_{\lambda=1}^{2}\epsilon^\mu(k,\lambda)\cdot\epsilon^{*\nu}(k,\lambda) = -g^{\mu\nu} \qquad \sum_{\lambda=1}^{3}\epsilon_\mu(q,\lambda)\epsilon_\nu^*(q,\lambda) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2}$$

Virtual massless:

$$\epsilon(k,\lambda)\cdot\epsilon^*(k,\lambda') = g^{\lambda\lambda'} \qquad \sum_\lambda g^{\lambda\lambda}\epsilon^\mu(k,\lambda)\cdot\epsilon^{*\nu}(k,\lambda) = g^{\mu\nu}$$

Levi-Civita Symbol

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{If } (\mu\nu\rho\sigma) \text{ is an even permutation of } (0123) \\ -1 & \text{If } (\mu\nu\rho\sigma) \text{ is an odd permutation of } (0123) \\ 0 & \text{Otherwise} \end{cases}$$

$$\begin{aligned}\epsilon_{\alpha\beta_1\gamma_1\delta_1}\epsilon^{\alpha\beta_2\gamma_2\delta_2} = &+g_{\beta_1}^{\beta_2}g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\gamma_2} - g_{\beta_1}^{\beta_2}g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\delta_2} + g_{\beta_1}^{\gamma_2}g_{\gamma_1}^{\beta_2}g_{\delta_1}^{\delta_2} - g_{\beta_1}^{\gamma_2}g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\beta_2} \\ &+ g_{\beta_1}^{\delta_2}g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\beta_2} - g_{\beta_1}^{\delta_2}g_{\gamma_1}^{\beta_2}g_{\delta_1}^{\gamma_2}\end{aligned}$$

$$\epsilon_{\alpha\beta\gamma_1\delta_1}\epsilon^{\alpha\beta\gamma_2\delta_2} = -2\left(g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\delta_2} - g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\gamma_2}\right) \qquad \epsilon_{\alpha\beta\gamma\delta_1}\epsilon^{\alpha\beta\gamma\delta_2} = -6g_{\delta_1}^{\delta_2}$$

Fermions

$$:a(x)b(x'): = a(x)b(x') - b(x')a(x)$$

$$\mathrm{T}[a(x)b(x')] = \theta(t -' t)a(x)b(x') - \theta(t' - t)b(x')a(x)$$

$$\left\{b^\dagger(p,s), b(p',s')\right\} = \left\{d^\dagger(p,s), d(p',s')\right\} = \tilde{\delta}(p - p')\delta_{ss'}$$

$$\psi(x) = \psi^+(x) + \psi^-(x) = \int \mathrm{d}\tilde{p}\sum_s\left[b(p,s)u(p,s)e^{-ip\cdot x} + d^\dagger(p,s)v(p,s)e^{+ip\cdot x}\right]$$

$$\overline{\psi}(x) = \overline{\psi}^+(x) + \overline{\psi}^-(x) = \int \mathrm{d}\tilde{p}\sum_s\left[d(p,s)\overline{v}(p,s)e^{-ip\cdot x} + b^\dagger(p,s)\overline{u}(p,s)e^{+ip\cdot x}\right]$$

$\psi^+(b)$ destroys e^- $\quad$ $\psi^-(d^\dagger)$ creates e^+ $\quad$ $\overline{\psi}^+(d)$ destroy e^+ $\quad$ $\overline{\psi}^-(b^\dagger)$ creates e^-

Bosons

$$:a(x)b(x'): = a(x)b(x') + b(x')a(x)$$

$$\mathrm{T}[a(x)b(x')] = \theta(t -' t)a(x)b(x') + \theta(t' - t)b(x')a(x)$$

$$\left[\phi(\vec{\mathbf{x}},t), \phi(\vec{\mathbf{y}},t)\right] = \left[\Pi(\vec{\mathbf{x}},t), \Pi(\vec{\mathbf{y}},t)\right] = 0 \qquad \left[\phi(\vec{\mathbf{x}},t), \Pi(\vec{\mathbf{y}},t)\right] = i\delta^3(\vec{\mathbf{x}} - \vec{\mathbf{y}})$$

$$\left[a^\dagger(k,\lambda), a(k',\lambda')\right] = g^{\lambda\lambda'}\tilde{\delta}(k - k')$$

$$A_\mu(x) = A_\mu^+(x) + A_\mu^-(x) = \int \mathrm{d}\tilde{k}\sum_{\lambda=0}^{3}\left[a(k,\lambda)\epsilon_\mu(k,\lambda)e^{-ik\cdot x} + a^\dagger(k,\lambda)\epsilon_\mu^*(k,\lambda)e^{+ik\cdot x}\right]$$

$A^+(a)$ destroys γ $\quad$ $A^-(a^\dagger)$ creates γ

Helicity & Chirality

$$\gamma_L = \frac{1-\gamma_5}{2} \qquad \gamma_R = \frac{1+\gamma_5}{2} \qquad \gamma_{L,R}^2 = \gamma_{L,R} \qquad \gamma_{L,R}\gamma_{R,L} = 0 \qquad \gamma_L + \gamma_R = \mathbb{1}$$

$$\gamma^\mu\gamma_{L,R} = \gamma_{R,L}\gamma^\mu \qquad \gamma_5\gamma_L = \gamma_L\gamma_5 = -\gamma_L \qquad \gamma_5\gamma_R = \gamma_R\gamma_5 = \gamma_R \qquad \overline{\gamma_{L,R}} = \gamma_0\gamma_{L,R}^\dagger\gamma_0 = \gamma_{R,L}$$

$$P_L u_\downarrow = u_\downarrow \quad P_L u_\uparrow = 0 \qquad P_R u_\downarrow = 0 \quad P_R u_\uparrow = u_\uparrow \qquad \overline{u_\downarrow}P_L = 0 \quad \overline{u_\uparrow}P_L = u_\uparrow \qquad \overline{u_\downarrow}P_R = u_\downarrow \quad \overline{u_\uparrow}P_R = 0$$

$$\gamma_L u_L = u_L \quad \gamma_L u_R = 0 \qquad \gamma_R u_L = 0 \quad \gamma_R u_R = u_R \qquad \overline{u_L}\gamma_L = 0 \quad \overline{u_R}\gamma_L = u_R \qquad \overline{u_L}\gamma_R = u_L \quad \overline{u_R}\gamma_R = 0$$

$$P_L v_\downarrow = 0 \quad P_L v_\uparrow = v_\uparrow \qquad P_R v_\downarrow = v_\downarrow \quad P_R v_\uparrow = 0 \qquad \overline{v_\downarrow}P_L = v_\downarrow \quad \overline{v_\uparrow}P_L = 0 \qquad \overline{v_\downarrow}P_R = 0 \quad \overline{v_\uparrow}P_R = v_\uparrow$$

$$\gamma_L v_L = v_L \quad \gamma_L v_R = 0 \qquad \gamma_R v_L = 0 \quad \gamma_R v_R = v_R \qquad \overline{v_L}\gamma_L = 0 \quad \overline{v_R}\gamma_L = v_R \qquad \overline{v_L}\gamma_R = v_L \quad \overline{v_R}\gamma_R = 0$$

Decay : Decay Rates

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{32\pi^2}\frac{\left|\vec{\mathbf{p}}_{\mathrm{CM}}\right|}{m_0^2}\overline{|\mathscr{M}|^2} \qquad \left|\vec{\mathbf{p}}_{\mathrm{CM}}\right| = \frac{1}{2m_0}\sqrt{\left[m_0^2 - (m_1+m_2)^2\right]\left[m_0^2 - (m_1-m_2)^2\right]}$$

Scattering : Cross Sections

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}\frac{\left|\vec{\mathbf{p}}_3\right|}{\left|\vec{\mathbf{p}}_1\right|}\overline{|\mathscr{M}|^2} \qquad \left|\vec{\mathbf{p}}\right| = \frac{1}{2\sqrt{s}}\sqrt{\left[s - (m_a+m_b)^2\right]\left[s - (m_a-m_b)^2\right]}$$

$$s = (p_1+p_2)^2 = (p_3+p_4)^2 \approx +2p_1\cdot p_2 \approx +2p_3\cdot p_4 \qquad \sqrt{s} = E_{CM}$$

$$t = (p_1-p_3)^2 = (p_2-p_4)^2 \approx -2p_1\cdot p_3 \approx -2p_2\cdot p_4 \qquad s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

$$u = (p_1-p_4)^2 = (p_2-p_3)^2 \approx -2p_1\cdot p_4 \approx -2p_2\cdot p_3$$

$\mathrm{f} + \bar{\mathrm{f}} \longrightarrow \mathrm{g} + \bar{\mathrm{g}}$:

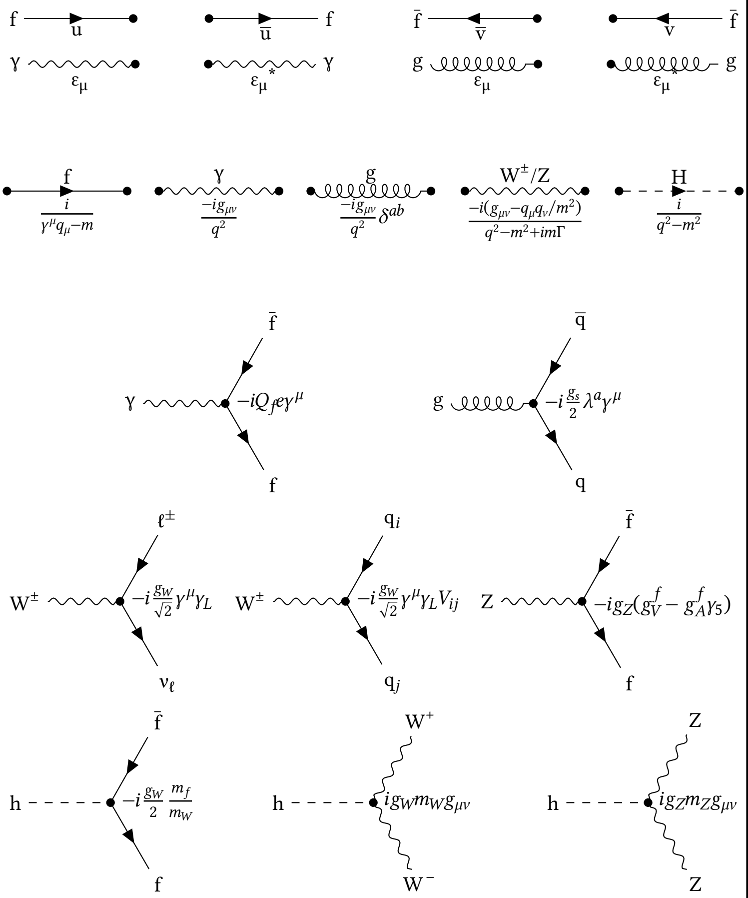
$$p_1 = \frac{\sqrt{s}}{2}\left(1,0,0,+\beta_f\right) \qquad p_2 = \frac{\sqrt{s}}{2}\left(1,0,0,-\beta_f\right)$$

$$p_3 = \frac{\sqrt{s}}{2}\left(1,+\beta_g\sin\theta,0,+\beta_g\cos\theta\right) \qquad p_4 = \frac{\sqrt{s}}{2}\left(1,-\beta_g\sin\theta,0,-\beta_g\cos\theta\right)$$

$$m \approx 0\ : \qquad t = -\frac{s}{2}(1-\cos\theta) \qquad u = -\frac{s}{2}(1+\cos\theta)$$

Feynman Rules for $i\mathscr{M}$

Goes in opposite way of arrows with the first one being adjoint, $\bar{\psi} = \psi^\dagger \gamma^0$:



Constants

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} = \frac{g_W^2}{8M_Z^2\cos^2(\theta_W)} = \frac{1}{2v^2} \qquad g_W = g_Z\cos\theta_W = \frac{e}{\sin\theta_W}$$

$$g_V^f = \frac{1}{2}T_f^3 - Q_f\sin^2(\theta_W) \qquad g_A^f = \frac{1}{2}T_f^3 \qquad M_W = M_Z\cos\theta_W = \frac{1}{2}g_W v \qquad m_H = \sqrt{2\lambda}v$$

Fermions	Q_f	$I_W^{(3)}$	Y_L	Y_R	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	+1/2	−1	0	+1/2	0	+1/2	+1/2
e^-, μ^-, τ^-	−1	−1/2	−1	−2	−0.27	+0.23	−0.04	−1/2
u, c, t	+2/3	+1/2	+1/3	+4/3	+0.35	−0.15	+0.19	+1/2
d, s, b	−1/3	−1/2	+1/3	−2/3	−0.42	+0.08	−0.35	−1/2