Equations

Schrödinger:

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = H|\psi t\rangle \qquad \mathscr{L} = i\psi^*\dot{\psi} - \frac{1}{2m}\vec{\nabla}\psi^* \cdot \vec{\nabla}\psi$$

Klein-Gordon (Real Scalar Field):

$$(\partial \cdot \partial + m^2)\phi = 0 \qquad \mathscr{L} = \frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right] = \frac{1}{2}\left[\dot{\phi}^2 - \left(\vec{\nabla}\phi\right)^2 - m^2\phi^2\right]$$

Dirac (Complex Scalar Field):

$$(i\slashed{\partial} - m)\varphi = 0 \qquad \overline{\varphi}(i\slashed{\partial} + m) = 0 \qquad \mathscr{L} = \partial_\mu\varphi\partial^\mu\varphi^* - m^2\varphi\varphi^*$$

γ-Matrices

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad \gamma_5 = +i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma_0\gamma_1\gamma_2\gamma_3 = -\frac{i}{4!}\epsilon^{\alpha\beta\gamma\delta}\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \implies [\gamma^\mu, \gamma^\nu] = -2i\sigma^{\mu\nu} \qquad \{\gamma_5, \gamma^\mu\} = 0 \gamma_5\gamma_5 = +\mathbb{1}$$

$$\left(\gamma^0\right)^\dagger = \gamma^0 \quad \left(\gamma^0\right)^2 = \mathbb{1} \quad \left(\gamma^k\right)^\dagger = -\gamma^k \quad \left(\gamma^k\right)^2 = -\mathbb{1} \quad \left(\gamma^5\right)^\dagger = \gamma^5 \quad \left(\gamma^5\right)^2 = \mathbb{1}$$

	$\mathbb{1}$	γ_5	γ^μ	$\gamma_5\gamma^\mu$	$\sigma^{\mu\nu}$
γ_5	γ_5	$\mathbb{1}$	γ^μ	$\gamma_5\gamma^\mu$	$\frac{i}{2}\epsilon^{\mu\nu\pi\rho}\sigma_{\pi\rho}$
γ^α	$-\gamma_5\gamma^\alpha$	$+\gamma_5\gamma^\alpha$	$g^{\alpha\mu} - i\sigma^{\alpha\mu}$	$-\frac{1}{2}(2g^{\alpha\mu}\gamma_5 + \epsilon^{\alpha\mu}\sigma_{\pi\rho})$	$\epsilon^{\alpha\mu\nu\lambda}\gamma_5\gamma_\lambda + ig^{\alpha\mu}\gamma^\nu - ig^{\alpha\nu}\gamma^\mu$
$\gamma_5\gamma^\alpha$	$-\gamma^\alpha$	$-\gamma^\alpha$	$-(g^{\alpha\mu} - i\sigma^{\alpha\mu})$	$\epsilon^{\alpha\mu\lambda}\gamma_\lambda + ig^{\alpha\mu}\gamma_5\gamma^\nu - ig^{\alpha\nu}\gamma_5\gamma^\mu$
$\sigma^{\alpha\beta}$	$\sigma^{\alpha\beta}$	$\epsilon^{\alpha\beta\mu\lambda}\gamma_5\gamma_\lambda + ig^{\beta\mu}\gamma^\alpha - ig^{\alpha\mu}\gamma^\beta$	$\epsilon^{\alpha\beta\mu\lambda}\gamma_\lambda + ig^{\beta\mu}\gamma_5\gamma^\alpha - ig^{\alpha\mu}\gamma_5\gamma^\beta$	$i\epsilon^{\alpha\beta\mu\nu}\gamma_5 + g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu} + i\left[g^{\alpha\mu}\sigma^{\beta\nu} + g^{\beta\nu}\sigma^{\alpha\mu} - g^{\alpha\nu}\sigma^{\beta\nu} - g^{\beta\nu}\sigma^{\alpha\mu}\right]$

$$\slashed{A} = A^\mu\gamma_\mu \qquad \gamma^\mu\gamma_\mu = 4 \qquad \gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4g^{\nu\rho}$$

$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\pi\gamma_\mu = 2[\gamma^\pi\gamma^\nu\gamma^\rho\gamma^\sigma + \gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\pi]$$

Helicity & Chirality

$$\gamma_L = \frac{1-\gamma_5}{2} \qquad \gamma_R = \frac{1+\gamma_5}{2} \qquad \gamma_{L,R}^2 = \gamma_{L,R} \qquad \gamma_{L,R}\gamma_{R,L} = 0 \qquad \gamma_L + \gamma_R = \mathbb{1}$$

$$\gamma^\mu\gamma_{L,R} = \gamma_{R,L}\gamma^\mu \qquad \gamma_5\gamma_L = \gamma_L\gamma_5 = -\gamma_L \qquad \gamma_5\gamma_R = \gamma_R\gamma_5 = \gamma_R \qquad \overline{\gamma_{L,R}} = \gamma_0\gamma_{L,R}^\dagger\gamma_0 = \gamma_{R,L}$$

$P_L u_\downarrow = u_\downarrow$	$P_L u_\uparrow = 0$	$P_R u_\downarrow = 0$	$P_R u_\uparrow = u_\uparrow$	$\overline{u_\downarrow}P_L = 0$	$\overline{u_\uparrow}P_L = u_\uparrow$	$\overline{u_\downarrow}P_R = u_\downarrow$	$\overline{u_\uparrow}P_R = 0$
$\gamma_L u_L = u_L$	$\gamma_L u_R = 0$	$\gamma_R u_L = 0$	$\gamma_R u_R = u_R$	$\overline{u_L}\gamma_L = 0$	$\overline{u_R}\gamma_L = u_R$	$\overline{u_L}\gamma_R = u_L$	$\overline{u_R}\gamma_R = 0$
$P_L v_\downarrow = 0$	$P_L v_\uparrow = v_\uparrow$	$P_R v_\downarrow = v_\downarrow$	$P_R v_\uparrow = 0$	$\overline{v_\downarrow}P_L = v_\downarrow$	$\overline{v_\uparrow}P_L = 0$	$\overline{v_\downarrow}P_R = 0$	$\overline{v_\uparrow}P_R = v_\uparrow$
$\gamma_L v_L = v_L$	$\gamma_L v_R = 0$	$\gamma_R v_L = 0$	$\gamma_R v_R = v_R$	$\overline{v_L}\gamma_L = v_L$	$\overline{v_R}\gamma_L = v_R$	$\overline{v_L}\gamma_R = v_L$	$\overline{v_R}\gamma_R = v_R$

Traces

$$\mathrm{Tr}[\mathbb{1}] = 4 \qquad \mathrm{Tr}[\underbrace{\gamma^\mu\gamma^\nu...\gamma^\rho}_{\text{Odd Number}}] = 0 \qquad \mathrm{Tr}[\gamma^\mu\gamma^\nu] = 4g^{\mu\nu}$$

$$\mathrm{Tr}[\gamma_5] = 0 \qquad \mathrm{Tr}[\gamma_5\underbrace{\gamma^\mu\gamma^\nu...\gamma^\rho}_{\text{Odd Number}}] = 0 \qquad \mathrm{Tr}[\gamma_5\gamma^\mu\gamma^\nu] = 0$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) \qquad \mathrm{Tr}[\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = -4i\epsilon^{\mu\nu\rho\sigma}$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_L] = 2(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} + i\epsilon^{\mu\nu\rho\sigma})$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_R] = 2(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - i\epsilon^{\mu\nu\rho\sigma})$$

$$\mathrm{Tr}[(\slashed{p}_a + m_a)\gamma^\mu(\slashed{p}_b + m_b)\gamma^\nu] = 4\left[p_a^\mu p_b^\nu + p_b^\mu p_a^\nu + (m_a m_b - p_a \cdot p_b)g^{\mu\nu}\right]$$

Fermions

$$:a(x)b(x'): = a(x)b(x') - b(x')a(x)$$

$$\mathrm{T}[a(x)b(x')] = \theta(t-'t)a(x)b(x') - \theta(t'-t)b(x')a(x)$$

$$\{b^\dagger(p,s), b(p',s')\} = \{d^\dagger(p,s), d(p',s')\} = \tilde{\delta}(p-p')\delta_{ss'}$$

$$\psi(x) = \psi^+(x) + \psi^-(x) = \int \mathrm{d}\tilde{p}\sum_s \left[b(p,s)u(p,s)e^{-ip\cdot x} + d^\dagger(p,s)v(p,s)e^{+ip\cdot x}\right]$$

$$\overline{\psi}(x) = \overline{\psi}^+(x) + \overline{\psi}^-(x) = \int \mathrm{d}\tilde{p}\sum_s \left[d(p,s)\overline{v}(p,s)e^{-ip\cdot x} + b^\dagger(p,s)\overline{u}(p,s)e^{+ip\cdot x}\right]$$

$\psi^+(b)$ destroys e^- $\quad \psi^-(d^\dagger)$ creates e^+ $\quad \overline{\psi}^+(d)$ destroy e^+ $\quad \overline{\psi}^-(b^\dagger)$ creates e^-

Spinors (Fermions)

$$\psi = ue^{+i(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}} - E\cdot t)} = ve^{-i(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}} - E\cdot t)} \qquad (\slashed{p} - m)u = \overline{u}(\slashed{p} - m) = 0 \qquad (\slashed{p} + m)v = \overline{v}(\slashed{p} + m) = 0$$

$$\overline{u}^r(p)u^s(p) = +2m\delta^{rs} \qquad \overline{u}^r(p)v^s(p) = 0 \qquad u^{r\dagger}(p)u^s(p) = 2E\delta^{rs} \qquad \sum_{s=1,2} u^s(p)\overline{u}^s(p) = \slashed{p} + m$$

$$\overline{v}^r(p)v^s(p) = -2m\delta^{rs} \qquad \overline{v}^r(p)u^s(p) = 0 \qquad v^{r\dagger}(p)u^s(p) = 2E\delta^{rs} \qquad \sum_{s=1,2} v^s(p)\overline{v}^s(p) = \slashed{p} - m$$

Bosons

$$:a(x)b(x'): = a(x)b(x') + b(x')a(x)$$

$$\mathrm{T}[a(x)b(x')] = \theta(t-'t)a(x)b(x') + \theta(t'-t)b(x')a(x)$$

$$[\phi(\vec{\mathbf{x}},t), \phi(\vec{\mathbf{y}},t)] = [\Pi(\vec{\mathbf{x}},t), \Pi(\vec{\mathbf{y}},t)] = 0 \qquad [\phi(\vec{\mathbf{x}},t), \Pi(\vec{\mathbf{y}},t)] = i\delta^3(\vec{\mathbf{x}} - \vec{\mathbf{y}})$$

$$[a^\dagger(k,\lambda), a(k',\lambda')] = g^{\lambda\lambda'}\tilde{\delta}(k-k')$$

$$A_\mu(x) = A_\mu^+(x) + A_\mu^-(x) = \int \mathrm{d}\tilde{k}\sum_{\lambda=0}^3 \left[a(k,\lambda)\epsilon_\mu(k,\lambda)e^{-ik\cdot x} + a^\dagger(k,\lambda)\epsilon_\mu^*(k,\lambda)e^{+ik\cdot x}\right]$$

$A^+(a)$ destroys γ $\quad A^-(a^\dagger)$ creates γ

Polarization Vectors (Bosons)

External massless & Massive:

$$\sum_{\lambda=1}^2 \epsilon^\mu(k,\lambda) \cdot \epsilon^{*\nu}(k,\lambda) = -g^{\mu\nu} \qquad \sum_{\lambda=1}^3 \epsilon_\mu(q,\lambda)\epsilon_\nu^*(q,\lambda) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2}$$

Virtual massless:

$$\epsilon(k,\lambda) \cdot \epsilon^*(k,\lambda') = g^{\lambda\lambda'} \qquad \sum_\lambda g^{\lambda\lambda}\epsilon^\mu(k,\lambda) \cdot \epsilon^{*\nu}(k,\lambda) = g^{\mu\nu}$$

Decay : Decay Rates

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{32\pi^2}\frac{|\vec{\mathbf{p}}_{\text{CM}}|}{m_0^2}\overline{|\mathscr{M}|^2} \qquad |\vec{\mathbf{p}}_{\text{CM}}| = \frac{1}{2m_0}\sqrt{[m_0^2 - (m_1+m_2)^2][m_0^2 - (m_1-m_2)^2]}$$
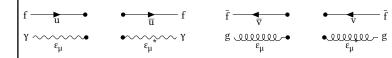
Scattering : Cross Sections

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{\mathbf{p}}_3|}{|\vec{\mathbf{p}}_1|}\overline{|\mathscr{M}|^2} \qquad |\vec{\mathbf{p}}| = \frac{1}{2\sqrt{s}}\sqrt{[s - (m_a+m_b)^2][s - (m_a-m_b)^2]}$$

$$s = (p_1+p_2)^2 = (p_3+p_4)^2 \approx +2p_1\cdot p_2 \approx +2p_3\cdot p_4 \qquad \sqrt{s} = E_{CM}$$

$$t = (p_1-p_3)^2 = (p_2-p_4)^2 \approx -2p_1\cdot p_3 \approx -2p_2\cdot p_4 \qquad s+t+u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

$$u = (p_1-p_4)^2 = (p_2-p_3)^2 \approx -2p_1\cdot p_4 \approx -2p_2\cdot p_3$$

$$\mathrm{f} + \bar{\mathrm{f}} \longrightarrow \mathrm{g} + \bar{\mathrm{g}}:$$

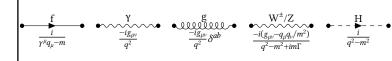$$p_1 = \frac{\sqrt{s}}{2}(1, 0, 0, +\beta_f) \qquad p_2 = \frac{\sqrt{s}}{2}(1, 0, 0, -\beta_f)$$

$$p_3 = \frac{\sqrt{s}}{2}(1, +\beta_g\sin\theta, 0, +\beta_g\cos\theta) \qquad p_4 = \frac{\sqrt{s}}{2}(1, -\beta_g\sin\theta, 0, -\beta_g\cos\theta)$$

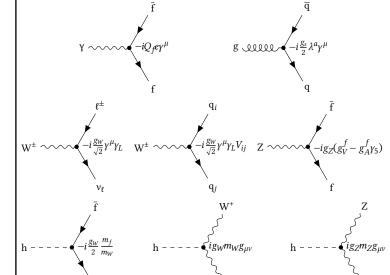$$m \approx 0: \qquad t = -\frac{s}{2}(1 - \cos\theta) \qquad u = -\frac{s}{2}(1 + \cos\theta)$$

Feynman Rules for $i\mathscr{M}$

Goes in opposite way of arrows with the first one being adjoint, $\overline{\psi} = \psi^\dagger\gamma^0$:

$$\frac{i}{\gamma^\mu q_\mu - m} \qquad \frac{-ig_{\mu\nu}}{q^2} \qquad \frac{-ig_{\mu\nu}}{q^2}\delta^{ab} \qquad \frac{-i(g_{\mu\nu} - q_\mu q_\nu/m^2)}{q^2 - m^2 + im\Gamma} \qquad \frac{i}{q^2 - m^2}$$

$$-iQ_f e\gamma^\mu \qquad -i\frac{g_s}{2}\lambda^a\gamma^\mu$$

$$-i\frac{g_W}{\sqrt{2}}\gamma^\mu\gamma_L \qquad -i\frac{g_W}{\sqrt{2}}\gamma^\mu\gamma_L V_{ij} \qquad -ig_Z(g_V^f - g_A^f\gamma_5)$$

$$-i\frac{g_W}{2}\frac{m_f}{m_W} \qquad ig_W m_W g_{\mu\nu} \qquad ig_Z m_Z g_{\mu\nu}$$

Constants

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} = \frac{g_W^2}{8M_Z^2\cos^2(\theta_W)} = \frac{1}{2v^2} \qquad g_W = g_Z\cos\theta_W = \frac{e}{\sin\theta_W}$$

$$g_V^f = \frac{1}{2}T_f^3 - Q_f\sin^2(\theta_W) \qquad g_A^f = \frac{1}{2}T_f^3 \qquad M_W = M_Z\cos\theta_W = \frac{1}{2}g_W v \qquad m_H = \sqrt{2\lambda}v$$

Fermions	Q_f	$I_W^{(3)}$	Y_L	Y_R	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	+1/2	−1	0	+1/2	0	+1/2	+1/2
e^-, μ^-, τ^-	−1	−1/2	−1	−2	−0.27	+0.23	−0.04	−1/2
u, c, t	+2/3	+1/2	+1/3	+4/3	+0.35	−0.15	+0.19	+1/2
d, s, b	−1/3	−1/2	+1/3	−2/3	−0.42	+0.08	−0.35	−1/2

Relativity

$$\beta = \frac{p}{E}$$

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{If } (\mu\nu\rho\sigma) \text{ is an even permutation of } (0123) \\ -1 & \text{If } (\mu\nu\rho\sigma) \text{ is an odd permutation of } (0123) \\ 0 & \text{Otherwise} \end{cases}$$

$$\epsilon_{\alpha\beta_1\gamma_1\delta_1}\epsilon^{\alpha\beta_2\gamma_2\delta_2} = +g_{\beta_1}^{\beta_2}g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\gamma_2} - g_{\beta_1}^{\beta_2}g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\delta_2} + g_{\beta_1}^{\gamma_2}g_{\gamma_1}^{\beta_2}g_{\delta_1}^{\delta_2} - g_{\beta_1}^{\gamma_2}g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\beta_2}$$
$$+ g_{\beta_1}^{\delta_2}g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\beta_2} - g_{\beta_1}^{\delta_2}g_{\gamma_1}^{\beta_2}g_{\delta_1}^{\gamma_2}$$

$$\epsilon_{\alpha\beta\gamma_1\delta_1}\epsilon^{\alpha\beta\gamma_2\delta_2} = -2\left(g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\delta_2} - g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\gamma_2}\right) \qquad \epsilon_{\alpha\beta\gamma\delta_1}\epsilon^{\alpha\beta\gamma\delta_2} = -6g_{\delta_1}^{\delta_2}$$