

Equations

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle \quad (\partial \cdot \partial + m^2)\phi = 0 \quad (i\partial - m)\varphi = 0 \quad \bar{\varphi}(i\partial + m) = 0$$

Pauli and Dirac \gamma-Matrices Matrices

$$\sigma^1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \tilde{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

$$\{\sigma^i, \sigma^j\} = 2\delta^{ij} \quad [\sigma^i, \sigma^j] = 2i\epsilon^{ijk}\sigma^k \quad \sigma^i\sigma^j = \delta^{ij} + i\epsilon^{ijk}\sigma^k \quad \sigma^2\sigma^i\sigma^2 = -(\sigma^i)^*$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad \text{<- Chiral | Pauli-Dirac ->} \quad \gamma^0 = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} \quad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad \gamma_5 = +i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma_0\gamma_1\gamma_2\gamma_3 = -\frac{i}{4!}\epsilon^{\alpha\beta\gamma\delta}\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta \quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

$$\not{A} = A^\mu\gamma_\mu \quad \gamma^\mu\gamma_\mu = 4 \quad \gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu \quad \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4g^{\nu\rho} \quad \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu$$

$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\pi\gamma_\mu = 2[\gamma^\pi\gamma^\nu\gamma^\rho\gamma^\sigma + \gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\pi] \quad \{\gamma_5, \gamma^\mu\} = 0 \quad \gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu$$

$$(\gamma^0)^\dagger = \gamma^0 \quad (\gamma^0)^2 = \mathbb{1} \quad (\gamma^k)^\dagger = -\gamma^k \quad (\gamma^k)^2 = -\mathbb{1} \quad (\gamma^5)^\dagger = \gamma^5 \quad (\gamma^5)^2 = 1$$

$$\bar{\psi} = \psi^\dagger\gamma^0 \quad \bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0 \quad \bar{\gamma}_5 = -\gamma_5 \quad \bar{\gamma}^\mu = \gamma^\mu \quad \bar{\gamma}^\mu\bar{\gamma}_5 = \gamma^\mu\gamma_5 \quad \bar{\sigma}^{\mu\nu} = \sigma^{\mu\nu}$$

Spin, Helicity and Chirality

$$\vec{S} = \frac{1}{2}\vec{\Sigma} \quad h = \frac{\vec{S} \cdot \vec{p}}{|\vec{p}|} \quad P_L = \frac{1-h}{2} \quad P_R = \frac{1+h}{2} \quad Y_L = \frac{1-\gamma_5}{2} \quad Y_R = \frac{1+\gamma_5}{2}$$

$$Y_{L,R}^2 = Y_{L,R} \quad Y_{L,R}Y_{R,L} = 0 \quad Y_L + Y_R = \mathbb{1} \quad \gamma^\mu Y_{L,R} = Y_{R,L}\gamma^\mu \quad \gamma_5 Y_L = -Y_L \quad \gamma_5 Y_R = Y_R$$

$$\gamma_{L,R}^\dagger = Y_{L,R} \quad \overline{\gamma^\mu Y_{L,R}} = \gamma^\mu Y_{L,R} \quad \overline{Y_5 Y_L} = -Y_R \quad \overline{Y_5 Y_R} = Y_L \quad \overline{Y_{L,R}} = \gamma_0 Y_{L,R}^\dagger \gamma_0 = Y_{R,L}$$

$$u_L = u_\downarrow \quad u_R = u_\uparrow \quad v_L = v_\uparrow \quad v_R = v_\downarrow \quad \vec{p} = p(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$P_L u_L = u_\downarrow \quad P_L u_\uparrow = 0 \quad P_R u_\downarrow = 0 \quad P_R u_\uparrow = u_\uparrow \quad \overline{u}_L P_L = 0 \quad \overline{u}_\uparrow P_L = u_\uparrow \quad \overline{u}_L P_R = u_\downarrow \quad \overline{u}_\uparrow P_R = 0$$

$$Y_L u_L = u_L \quad Y_L u_R = 0 \quad Y_R u_L = 0 \quad Y_R u_R = u_R \quad \overline{u}_L Y_L = 0 \quad \overline{u}_R Y_L = u_R \quad \overline{u}_L Y_R = u_L \quad \overline{u}_R Y_R = 0$$

$$P_L v_L = 0 \quad P_L v_\uparrow = v_\uparrow \quad P_R v_\downarrow = v_\downarrow \quad P_R v_\uparrow = 0 \quad \overline{v}_L P_L = v_\downarrow \quad \overline{v}_\uparrow P_L = 0 \quad \overline{v}_L P_R = 0 \quad \overline{v}_\uparrow P_R = v_\uparrow$$

$$Y_L v_L = v_L \quad Y_L v_R = 0 \quad Y_R v_L = 0 \quad Y_R v_R = v_R \quad \overline{v}_L Y_L = 0 \quad \overline{v}_R Y_L = v_R \quad \overline{v}_L Y_R = v_L \quad \overline{v}_R Y_R = 0$$

$$u_\uparrow^T = N \begin{pmatrix} c & se^{i\phi} & kc & kse^{i\phi} \end{pmatrix} \approx N \begin{pmatrix} c & se^{i\phi} & c & se^{i\phi} \end{pmatrix} \quad N = \sqrt{E+m} \approx \sqrt{E}$$

$$u_\downarrow^T = N \begin{pmatrix} -s & ce^{i\phi} & ks & -kce^{i\phi} \end{pmatrix} \approx N \begin{pmatrix} -s & ce^{i\phi} & s & -ce^{i\phi} \end{pmatrix} \quad k = \frac{p}{E+m} \approx 1$$

$$v_\uparrow^T = N \begin{pmatrix} ks & -kce^{i\phi} & -s & ce^{i\phi} \end{pmatrix} \approx N \begin{pmatrix} s & -ce^{i\phi} & -s & ce^{i\phi} \end{pmatrix} \quad s = \sin\left(\frac{\theta}{2}\right)$$

$$v_\downarrow^T = N \begin{pmatrix} kc & kse^{i\phi} & c & se^{i\phi} \end{pmatrix} \approx N \begin{pmatrix} c & se^{i\phi} & c & se^{i\phi} \end{pmatrix} \quad c = \cos\left(\frac{\theta}{2}\right)$$

Spinors (Fermions) & Polarization Vectors (Bosons)

$$\psi = ue^{+i(\vec{p}\cdot\vec{x}-Et)} = ve^{-i(\vec{p}\cdot\vec{x}-Et)} \quad (\not{p}-m)u = \not{u}(\not{p}-m) = 0 = (\not{p}+m)v = \not{v}(\not{p}+m)$$

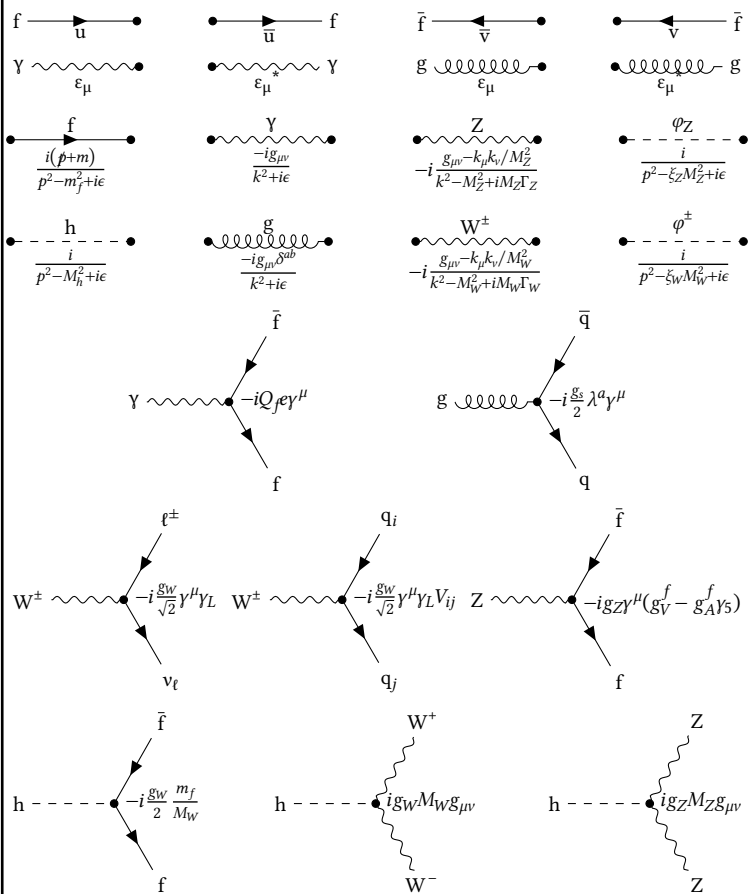
$$\not{u}^r(p)u^s(p) = +2m\delta^{rs} \quad \not{u}^r(p)v^s(p) = 0 \quad u^{r\dagger}(p)u^s(p) = 2E\delta^{rs} \quad \sum_{s=1,2} u^s(p)\bar{u}^s(p) = \not{p} + m$$

$$\not{v}^r(p)v^s(p) = -2m\delta^{rs} \quad \not{v}^r(p)u^s(p) = 0 \quad v^{r\dagger}(p)v^s(p) = 2E\delta^{rs} \quad \sum_{s=1,2} v^s(p)\bar{v}^s(p) = \not{p} - m$$

$$m = 0 : \sum_{\lambda=1}^2 \epsilon^\mu(k, \lambda) \cdot \epsilon^{*\nu}(k, \lambda) = -g^{\mu\nu} \quad m \neq 0 : \sum_{\lambda=1}^3 \epsilon_\mu(q, \lambda) \epsilon_\nu^*(q, \lambda) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2}$$

Feynman Rules for i\mathcal{M}

Goes in opposite way of arrows with the first one being adjoint, $\bar{\psi} = \psi^\dagger\gamma^0$:



Decay : Decay Rates

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2} \frac{|\vec{p}_{\text{CM}}|}{m_0^2} |\overline{\mathcal{M}}|^2 \quad |\vec{p}_{\text{CM}}| = \frac{1}{2m_0} \sqrt{[m_0^2 - (m_1 + m_2)^2][m_0^2 - (m_1 - m_2)^2]}$$

$$E_1 = \frac{m_0^2 + m_1^2 - m_2^2}{2m_0} \quad E_2 = \frac{m_0^2 + m_2^2 - m_1^2}{2m_0}$$

Scattering : Cross Sections

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_3|}{|\vec{p}_1|} |\overline{\mathcal{M}}|^2 \quad |\vec{p}_a| = |\vec{p}_b| = \frac{1}{2\sqrt{s}} \sqrt{[s - (m_a + m_b)^2][s - (m_a - m_b)^2]}$$

$$E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}} \quad E_2 = \frac{s + m_2^2 - m_1^2}{2\sqrt{s}} \quad E_3 = \frac{s + m_3^2 - m_4^2}{2\sqrt{s}} \quad E_4 = \frac{s + m_4^2 - m_3^2}{2\sqrt{s}}$$

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \approx +2p_1 \cdot p_2 \approx +2p_3 \cdot p_4 \quad \sqrt{s} = E_{\text{CM}}$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \approx -2p_1 \cdot p_3 \approx -2p_2 \cdot p_4 \quad s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \approx -2p_1 \cdot p_4 \approx -2p_2 \cdot p_3$$

$$m = 0 : s = 2|\vec{p}_{\text{CM}}|^2 = 2(p_{\text{CM}}^0)^2 \quad t = -\frac{s}{2}(1 - \cos\theta) \quad u = -\frac{s}{2}(1 + \cos\theta)$$

Constants

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} = \frac{g_W^2}{8M_Z^2 \cos^2(\theta_W)} = \frac{1}{2v^2} \quad M_W = M_Z \cos\theta_W = \frac{1}{2}g_W v \quad M_H = \sqrt{2}\lambda v$$

$$g_W = g_Z \cos\theta_W = \frac{e}{\sin\theta_W} \quad g_V^f = \frac{1}{2}T_f^3 - Q_f \sin^2(\theta_W) \quad g_A^f = \frac{1}{2}T_f^3 \quad g_{A,V} = \frac{1}{2}c_{A,V}$$

$$\alpha = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c} \quad c_L^f = g_V^f + g_A^f \quad c_R^f = g_V^f - g_A^f \quad g_V^f = \frac{c_L^f + c_R^f}{2} \quad g_A^f = \frac{c_L^f - c_R^f}{2}$$

Fermions	Q_f	$I_W^{(3)}$	Y_L	Y_R	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	+1/2	-1	0	+1/2	0	+1/2	+1/2
e^-, μ^-, τ^-	-1	-1/2	-1	-2	-0.27	+0.23	-0.04	-1/2
u, c, t	+2/3	+1/2	+1/3	+4/3	+0.35	-0.15	+0.19	+1/2
d, s, b	-1/3	-1/2	+1/3	-2/3	-0.42	+0.08	-0.35	-1/2

Particle	W^\pm	Z^0	H^0	e^\pm	μ^\pm	τ	u	d	c	s	t	b
Mass (MeV)	80 379	91 188	125 100	0.511	105.7	1777	2.16	4.67	1270	93	172 760	4180

$$1 \text{ kg} = 5.61 \times 10^{26} \text{ GeV} \quad 1 \text{ m} = 55.07 \times 10^{15} \text{ GeV}^{-1} \quad 1 \text{ s} = 51.52 \times 10^{24} \text{ GeV}^{-1}$$

Relativity & Quantum Mechanics

$$p^\mu \rightarrow i\delta^\mu$$

$$\beta = \frac{v}{c} = \frac{p}{E} = \tanh(\eta) \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{E}{m} = \cosh(\eta) \quad \gamma\beta = \frac{p}{m} = \sinh(\eta)$$

$$E^2 = m^2 + |p|^2 \quad \frac{d^3\vec{p}'}{E'} = \frac{d^3\vec{p}}{E} \quad \widetilde{dp} \equiv \frac{d^3\vec{p}}{(2\pi)^3 2E_p} \quad \delta(p-q) \equiv (2\pi)^3 2E_p \delta^3(\vec{p}-\vec{q})$$

$$x^\mu = (t, \vec{x}) \quad p^\mu = (E, \vec{p}) \quad \partial^\mu = (\partial_t, -\vec{\nabla}) \quad A^\mu = (\phi, \vec{A}) \quad J^\mu = (\rho, \vec{J})$$

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu \quad \mathcal{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} \quad \partial_\mu F^{\mu\nu} = J^\nu \quad \partial_\mu \mathcal{F}^{\mu\nu} = 0$$

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{If } (\mu\nu\rho\sigma) \text{ is an even permutation of } (0123) \\ -1 & \text{If } (\mu\nu\rho\sigma) \text{ is an odd permutation of } (0123) \\ 0 & \text{Otherwise} \end{cases}$$

$$\epsilon_{\alpha\beta\gamma_1\delta_1}\epsilon^{\alpha\beta_2\gamma_2\delta_2} = +g_{\beta_1\beta_2}^{\beta_1}\delta_{\gamma_1\gamma_2}^{\gamma_2} - g_{\beta_1\beta_2}^{\beta_2}\delta_{\gamma_1\gamma_2}^{\gamma_1} + g_{\beta_1\beta_2}^{\beta_2}\delta_{\gamma_1\gamma_2}^{\gamma_1} - g_{\beta_1\beta_2}^{\beta_1}\delta_{\gamma_1\gamma_2}^{\gamma_2} + g_{\beta_1\beta_2}^{\beta_2}\delta_{\gamma_1\gamma_2}^{\gamma_1} - g_{\beta_1\beta_2}^{\beta_1}\delta_{\gamma_1\gamma_2}^{\gamma_2} + g_{\beta_1\beta_2}^{\beta_2}\delta_{\gamma_1\gamma_2}^{\gamma_1} - g_{\beta_1\beta_2}^{\beta_1}\delta_{\gamma_1\gamma_2}^{\gamma_2}$$

$$\epsilon_{\alpha\beta\gamma_1\delta_1}\epsilon^{\alpha\beta_2\gamma_2\delta_2} = -2\left(g_{\gamma_1\gamma_2}^{\gamma_2}\delta_{\delta_1\delta_2}^{\delta_2} - g_{\gamma_1\gamma_2}^{\gamma_1}\delta_{\delta_1\delta_2}^{\delta_1}\right) \quad \epsilon_{\alpha\beta\gamma\delta_1}\epsilon^{\alpha\beta\gamma\delta_2} = -6g_{\delta_1\delta_2}^{\delta_2}$$