Equations

Schrödinger, Klein-Gordon (Real Scalar Field), Dirac (Complex Scalar Field):

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle \qquad \mathscr{L} = i\psi^*\dot{\psi} - \frac{1}{2m}\vec{\nabla}\psi^*\cdot\vec{\nabla}\psi$$

$$(\partial\cdot\partial + m^2)\phi = 0 \qquad \mathscr{L} = \frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right] = \frac{1}{2}\left[\dot{\phi}^2 - \left(\vec{\nabla}\phi\right)^2 - m^2\phi^2\right]$$

$$(i\not{\partial} - m)\varphi = 0 \qquad \overline{\varphi}(i\not{\partial} + m) = 0 \qquad \mathscr{L} = \partial_\mu\varphi\partial^\mu\varphi^* - m^2\varphi\varphi^*$$

Pauli Matrices

$$\sigma^1 = \sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad \sigma^2 = \sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \quad \sigma^3 = \sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \quad \vec{\Sigma} = \begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix}$$

$$\left\{\sigma^i, \sigma^j\right\} = 2\delta^{ij} \qquad \left[\sigma^i, \sigma^j\right] = 2i\epsilon^{ijk}\sigma^k \qquad \sigma^i\sigma^j = \delta^{ij} + i\epsilon^{ijk}\sigma^k \qquad \sigma^2\sigma^i\sigma^2 = -\left(\sigma^i\right)^*$$

$$\sigma^\mu = (\mathbb{1}, \vec{\sigma}) \qquad \overline{\sigma}^\mu = (\mathbb{1}, -\vec{\sigma}) \qquad (p\cdot\sigma)(p\cdot\overline{\sigma}) = p^2 \qquad \left(\vec{\sigma}\cdot\vec{\mathbf{p}}\right)^2 = |\vec{\mathbf{p}}|^2$$

$$p\cdot\overline{\sigma} = \begin{pmatrix}p^0 + p^3 & p^1 - ip^2\\ p^1 + ip^2 & p^0 - p^3\end{pmatrix} \qquad p\cdot\sigma = \begin{pmatrix}p^0 - p^3 & -(p^1 - ip^2)\\ -(p^1 + ip^2) & p^0 + p^3\end{pmatrix}$$

$$\sqrt{p\cdot\sigma} = \frac{E + m - \vec{\sigma}\cdot\vec{\mathbf{p}}}{\sqrt{2(E+m)}} \qquad \sqrt{p\cdot\overline{\sigma}} = \frac{E + m + \vec{\sigma}\cdot\vec{\mathbf{p}}}{\sqrt{2(E+m)}} \qquad \vec{\sigma}\cdot\vec{\mathbf{p}} = \begin{pmatrix}p_z & p_x - ip_y\\ p_x + ip_y & -p_z\end{pmatrix}$$

Dirac γ-Matrices

$$\gamma^\mu = \begin{pmatrix}0 & \sigma^\mu\\ \overline{\sigma}^\mu & 0\end{pmatrix} \quad \text{<- Chiral | Pauli-Dirac ->} \quad \gamma^0 = \begin{pmatrix}+\mathbb{1} & 0\\ 0 & -\mathbb{1}\end{pmatrix} \quad \gamma^k = \begin{pmatrix}0 & \sigma^k\\ -\sigma^k & 0\end{pmatrix}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \qquad \gamma_5 = +i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma_0\gamma_1\gamma_2\gamma_3 = -\frac{i}{4!}\epsilon^{\alpha\beta\gamma\delta}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \implies [\gamma^\mu, \gamma^\nu] = -2i\sigma^{\mu\nu} \qquad \{\gamma_5, \gamma^\mu\} = 0 \qquad \gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu$$

$$\left(\gamma^0\right)^\dagger = \gamma^0 \quad \left(\gamma^0\right)^2 = \mathbb{1} \qquad \left(\gamma^k\right)^\dagger = -\gamma^k \quad \left(\gamma^k\right)^2 = -\mathbb{1} \qquad \left(\gamma^5\right)^\dagger = \gamma^5 \quad \left(\gamma^5\right)^2 = \mathbb{1}$$

$$\overline{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0 \qquad \overline{\gamma_5} = -\gamma_5 \qquad \overline{\gamma^\mu} = \gamma^\mu \qquad \overline{\gamma^\mu\gamma_5} = \gamma^\mu\gamma_5 \qquad \overline{\sigma^{\mu\nu}} = \sigma^{\mu\nu}$$

$\mathbb{1}$	γ_5	γ^μ	$\gamma_5\gamma^\mu$	$\sigma^{\mu\nu}$
γ_5	$\mathbb{1}$	$+\gamma_5\gamma^\mu$	$+\gamma^\mu$	$\frac{i}{2}\epsilon^{\mu\nu\pi\rho}\sigma_{\pi\rho}$
γ^α	$-\gamma_5\gamma^\alpha$	$g^{\alpha\mu} - i\sigma^{\alpha\mu}$	$-\frac{1}{2}\left(2g^{\alpha\mu}\gamma_5 + \epsilon^{\alpha\mu\pi\rho}\sigma_{\pi\rho}\right)$	$\epsilon^{\alpha\mu\nu\lambda}\gamma_5\gamma_\lambda + ig^{\alpha\mu}\gamma^\nu - ig^{\alpha\nu}\gamma^\mu$
$\gamma_5\gamma^\alpha$	$-\gamma^\alpha$	$-\gamma^\alpha$	$-(g^{\alpha\mu} - i\sigma^{\alpha\mu})$	$\epsilon^{\alpha\mu\nu\lambda}\gamma_\lambda + ig^{\alpha\mu}\gamma_5\gamma^\nu - ig^{\alpha\nu}\gamma_5\gamma^\mu$
$\sigma^{\alpha\beta}$	$\sigma^{\alpha\beta}$	$\epsilon^{\alpha\beta\mu\lambda}\gamma_5\gamma_\lambda + ig^{\beta\mu}\gamma^\alpha - ig^{\alpha\mu}\gamma^\beta$	$\epsilon^{\alpha\beta\mu\lambda}\gamma_\lambda + ig^{\beta\mu}\gamma_5\gamma^\alpha - ig^{\alpha\mu}\gamma_5\gamma^\beta$	$i\epsilon^{\alpha\beta\mu\nu}\gamma_5 + g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu} + i\left[g^{\alpha\nu}\sigma^{\beta\mu} + g^{\beta\mu}\sigma^{\alpha\nu} - g^{\alpha\mu}\sigma^{\beta\nu} - g^{\beta\nu}\sigma^{\alpha\mu}\right]$

$$\not{A} = A^\mu\gamma_\mu \qquad \gamma^\mu\gamma_\mu = 4 \qquad \gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4g^{\nu\rho}$$

$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\pi\gamma_\mu = 2[\gamma^\pi\gamma^\nu\gamma^\rho\gamma^\sigma + \gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\pi]$$

$$\gamma^\mu\gamma^\nu\gamma^\rho = i\epsilon^{\mu\nu\rho\lambda}\gamma_\lambda\gamma_5 + g^{\mu\nu}\gamma^\rho - g^{\mu\rho}\gamma^\nu + g^{\nu\rho}\gamma^\mu$$

Spin, Helicity and Chirality

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\boldsymbol{\Sigma}} \qquad \tilde{h} = \frac{\vec{\mathbf{S}}\cdot\vec{\mathbf{p}}}{|\vec{\mathbf{p}}|} = \frac{\vec{\boldsymbol{\Sigma}}\cdot\vec{\mathbf{p}}}{2|\vec{\mathbf{p}}|} \qquad h = 2\tilde{h} = \frac{\vec{\boldsymbol{\Sigma}}\cdot\vec{\mathbf{p}}}{|\vec{\mathbf{p}}|} \qquad P_L = \frac{1-h}{2} \qquad P_R = \frac{1+h}{2}$$

$$\gamma_L = \frac{1-\gamma_5}{2} \qquad \gamma_R = \frac{1+\gamma_5}{2} \qquad \gamma_{L,R}^2 = \gamma_{L,R} \qquad \gamma_{L,R}\gamma_{R,L} = 0 \qquad \gamma_L + \gamma_R = \mathbb{1}$$

$$\gamma^\mu\gamma_{L,R} = \gamma_{R,L}\gamma^\mu \qquad \gamma_5\gamma_L = \gamma_L\gamma_5 = -\gamma_L \qquad \gamma_5\gamma_R = \gamma_R\gamma_5 = \gamma_R \qquad \gamma_{L,R}^\dagger = \gamma_{L,R}$$

$$\overline{\gamma^\mu\gamma_{L,R}} = \gamma^\mu\gamma_{L,R} \qquad \overline{\gamma_5\gamma_L} = -\gamma_R \qquad \overline{\gamma_5\gamma_R} = \gamma_L \qquad \overline{\gamma_{L,R}} = \gamma_0\gamma_{L,R}^\dagger\gamma_0 = \gamma_{R,L}$$

$$u_L = u_\downarrow \qquad u_R = u_\uparrow \qquad v_L = v_\uparrow \qquad v_R = v_\downarrow$$

$$\begin{array}{llllllll}
P_L u_\downarrow = u_\downarrow & P_L u_\uparrow = 0 & P_R u_\downarrow = 0 & P_R u_\uparrow = u_\uparrow & \overline{u_\downarrow}P_L = 0 & \overline{u_\uparrow}P_L = u_\uparrow & \overline{u_\downarrow}P_R = u_\downarrow & \overline{u_\uparrow}P_R = 0\\
\gamma_L u_L = u_L & \gamma_L u_R = 0 & \gamma_R u_L = 0 & \gamma_R u_R = u_R & \overline{u_L}\gamma_L = 0 & \overline{u_R}\gamma_L = u_R & \overline{u_L}\gamma_R = u_L & \overline{u_R}\gamma_R = 0\\
P_L v_\downarrow = 0 & P_L v_\uparrow = v_\uparrow & P_R v_\downarrow = v_\downarrow & P_R v_\uparrow = 0 & \overline{v_\downarrow}P_L = v_\downarrow & \overline{v_\uparrow}P_L = 0 & \overline{v_\downarrow}P_R = 0 & \overline{v_\uparrow}P_R = v_\uparrow\\
\gamma_L v_L = v_L & \gamma_L v_R = 0 & \gamma_R v_L = 0 & \gamma_R v_R = v_R & \overline{v_L}\gamma_L = 0 & \overline{v_R}\gamma_L = v_R & \overline{v_L}\gamma_R = v_L & \overline{v_R}\gamma_R = 0
\end{array}$$

$$\begin{array}{ll}
u_\uparrow^T = N\left(c \quad se^{i\phi} \quad kc \quad kse^{i\phi}\right) \approx N\left(c \quad se^{i\phi} \quad c \quad se^{i\phi}\right) & N = \sqrt{E+m} \approx \sqrt{E}\\
u_\downarrow^T = N\left(-s \quad ce^{i\phi} \quad ks \quad -kce^{i\phi}\right) \approx N\left(-s \quad ce^{i\phi} \quad s \quad -ce^{i\phi}\right) & k = \dfrac{p}{E+m} \approx 1\\
v_\uparrow^T = N\left(ks \quad -kce^{i\phi} \quad -s \quad ce^{i\phi}\right) \approx N\left(s \quad -ce^{i\phi} \quad -s \quad ce^{i\phi}\right) & s = \sin\left(\dfrac{\theta}{2}\right)\\
v_\downarrow^T = N\left(kc \quad kse^{i\phi} \quad c \quad se^{i\phi}\right) \approx N\left(c \quad se^{i\phi} \quad c \quad se^{i\phi}\right) & c = \cos\left(\dfrac{\theta}{2}\right)
\end{array}$$

$$\vec{\mathbf{p}} = p(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

Traces

Traces with an odd number of γ matrices are 0.

$$\mathrm{Tr}[\mathbb{1}] = 4 \qquad \mathrm{Tr}[\gamma^\mu\gamma^\nu] = 4g^{\mu\nu} \qquad \mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})$$

$$\mathrm{Tr}[\gamma_5] = 0 \qquad \mathrm{Tr}[\gamma_5\gamma^\mu\gamma^\nu] = 0 \qquad \mathrm{Tr}[\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = -4i\epsilon^{\mu\nu\rho\sigma}$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_L] = 2(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} + i\epsilon^{\mu\nu\rho\sigma})$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_R] = 2(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - i\epsilon^{\mu\nu\rho\sigma})$$

$$\mathrm{Tr}\left[\not{p}_1\gamma^\mu\gamma_L\not{p}_2\gamma^\nu(g_V - g_A\gamma_5)\not{p}_4\gamma_\mu\gamma_L\not{p}_3\gamma_\nu\gamma_L\right] = -16(g_V + g_A)(p_1\cdot p_4)(p_2\cdot p_3)$$

$$\mathrm{Tr}\left[(\not{p}_a + m_a)\gamma^\mu(\not{p}_b + m_b)\gamma^\nu\right] = 4\left[p_a^\mu p_b^\nu + p_b^\mu p_a^\nu + (m_a m_b - p_a\cdot p_b)g^{\mu\nu}\right]$$

$$\mathrm{Tr}\left[\not{a}\gamma^\mu\not{b}\gamma^\nu\right]\mathrm{Tr}\left[\not{c}\gamma_\mu\not{d}\gamma_\nu\gamma_5\right] = 0 \qquad \mathrm{Tr}\left[\not{a}\gamma^\mu\not{b}\gamma^\nu\gamma_L\right]\mathrm{Tr}\left[\not{c}\gamma_\mu\not{d}\gamma_\nu\gamma_R\right] = 16(a\cdot d)(b\cdot c)$$

$$\mathrm{Tr}\left[\not{a}\gamma^\mu\not{b}\gamma^\nu\gamma_L\right]\mathrm{Tr}\left[\not{c}\gamma_\mu\not{d}\gamma_\nu\gamma_L\right] = \mathrm{Tr}\left[\not{a}\gamma^\mu\not{b}\gamma^\nu\gamma_L\right]\mathrm{Tr}\left[\not{c}\gamma_\mu\not{d}\gamma_\nu\gamma_L\right] = 16(a\cdot c)(b\cdot d)$$

Spinors (Fermions)

$$\psi = ue^{+i(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}} - E\cdot t)} = ve^{-i(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}} - E\cdot t)} \qquad \overline{\psi} = \psi^\dagger\gamma^0$$

$$(\not{p} - m)u = \overline{u}(\not{p} - m) = 0 \qquad (\not{p} + m)v = \overline{v}(\not{p} + m) = 0$$

$$\overline{u}^r(p)u^s(p) = +2m\delta^{rs} \qquad \overline{u}^r(p)v^s(p) = 0 \qquad u^{r\dagger}(p)u^s(p) = 2E\delta^{rs} \qquad \sum_{s=1,2}u^s(p)\overline{u}^s(p) = \not{p} + m$$

$$\overline{v}^r(p)v^s(p) = -2m\delta^{rs} \qquad \overline{v}^r(p)u^s(p) = 0 \qquad v^{r\dagger}(p)v^s(p) = 2E\delta^{rs} \qquad \sum_{s=1,2}v^s(p)\overline{v}^s(p) = \not{p} - m$$

Polarization Vectors (Bosons)

External massless & Massive:

$$\sum_{\lambda=1}^{2}\epsilon^\mu(k,\lambda)\cdot\epsilon^{*\nu}(k,\lambda) = -g^{\mu\nu} \qquad \sum_{\lambda=1}^{3}\epsilon_\mu(q,\lambda)\epsilon_\nu^*(q,\lambda) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2}$$

Quantization : Real Scalar Field (Bosons)

$$:\frac{1}{2}\left[a^\dagger(k)a(k) + a(k)a^\dagger(k)\right]: = a^\dagger(k)a(k) \qquad :a(x)b(x'): = a(x)b(x') + b(x')a(x)$$

$$\mathrm{T}(a(x)b(x')) = \theta(t - t')a(x)b(x') + \theta(t' - t)b(x')a(x)$$

The "+" corresponds to positive frequency plane waves $e^{-ik\cdot x}$:

$$\mathscr{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi\phi \qquad \pi = \dot{\phi} \qquad \left[a^\dagger(k,\lambda), a(k',\lambda')\right] = g^{\lambda\lambda'}\tilde{\delta}(k - k')$$

$$\left[\phi(\vec{\mathbf{x}},t), \phi(\vec{\mathbf{y}},t)\right] = \left[\Pi(\vec{\mathbf{x}},t), \Pi(\vec{\mathbf{y}},t)\right] = 0 \qquad \left[\phi(\vec{\mathbf{x}},t), \Pi(\vec{\mathbf{y}},t)\right] = i\delta^3(\vec{\mathbf{x}} - \vec{\mathbf{y}})$$

$$\phi(x) = \int\widetilde{\mathrm{d}k}\left[a(k)e^{-ik\cdot x} + a^\dagger(k)e^{+ik\cdot x}\right]$$

$$|p\rangle = a^\dagger(p)|0\rangle \qquad |p_1, p_2\rangle = a^\dagger(p_2)a^\dagger(p_1)|0\rangle = |p_2, p_1\rangle$$

Quantization : Complex Scalar Field (Bosons)

$$\mathscr{L} = :\partial^\mu\varphi^\dagger\partial_\mu\varphi - m^2\varphi^\dagger\varphi: \qquad \pi = \dot{\varphi}^\dagger \quad \pi^\dagger = \dot{\varphi} \qquad \left[a_\pm(k), a_\pm^\dagger(k')\right] = \tilde{\delta}(k - k')$$

$$\left[\varphi(\vec{\mathbf{x}},t), \pi(\vec{\mathbf{y}},t)\right] = \left[\varphi^\dagger(\vec{\mathbf{x}},t), \pi^\dagger(\vec{\mathbf{y}},t)\right] = i\delta^3(\vec{\mathbf{x}} - \vec{\mathbf{y}})$$

$$\varphi(x) = \varphi^+(x) + \varphi^-(x) = \int\widetilde{\mathrm{d}k}\left[a_+(k)e^{-ik\cdot x} + a_-^\dagger(k)e^{+ik\cdot x}\right]$$

$$\varphi^\dagger(x) = \varphi^{+\dagger}(x) + \varphi^{-\dagger}(x) = \int\widetilde{\mathrm{d}k}\left[a_-(k)e^{-ik\cdot x} + a_+^\dagger(k)e^{+ik\cdot x}\right]$$

$$\left|p^+\right\rangle = a_+^\dagger(p)|0\rangle \qquad \left|p^-\right\rangle = a_-^\dagger(p)|0\rangle \qquad \left|p_1^+, p_2^-\right\rangle = a_-^\dagger(p_2)a_+^\dagger(p_1)|0\rangle$$

Quantization : Dirac Field (Fermions)

$$:a(x)b(x'): = a(x)b(x') - b(x')a(x)$$

$$\mathrm{T}(a(x)b(x')) = \theta(t -' t)a(x)b(x') - \theta(t' - t)b(x')a(x)$$

$$\mathscr{L} = :i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi: \qquad \pi_\alpha = \frac{\partial\mathscr{L}}{\partial\dot{\psi}_\alpha} = i\psi_\alpha^\dagger \qquad \pi_\alpha^\dagger = \frac{\partial\mathscr{L}}{\partial\dot{\psi}_\alpha^\dagger} = 0$$

$$\left\{b^\dagger(p,s), b(p',s')\right\} = \left\{d^\dagger(p,s), d(p',s')\right\} = \tilde{\delta}(p - p')\delta_{ss'}$$

$$\psi(x) = \psi^+(x) + \psi^-(x) = \int\widetilde{\mathrm{d}p}\sum_s\left[b(p,s)u(p,s)e^{-ip\cdot x} + d^\dagger(p,s)v(p,s)e^{+ip\cdot x}\right]$$

$$\overline{\psi}(x) = \overline{\psi}^+(x) + \overline{\psi}^-(x) = \int\widetilde{\mathrm{d}p}\sum_s\left[d(p,s)\overline{v}(p,s)e^{-ip\cdot x} + b^\dagger(p,s)\overline{u}(p,s)e^{+ip\cdot x}\right]$$

$$\left|e^-(p_1,s_1), e^-(p_2,s_2)\right\rangle = b^\dagger(p_2,s_2)b^\dagger(p_1,s_1)|0\rangle = -\left|e^-(p_2,s_2), e^-(p_1,s_1)\right\rangle$$

$$\psi^+(x)|p\rangle = \psi^+(x)b^\dagger(p)|0\rangle = |0\rangle u(p)e^{-ip\cdot x} \qquad \psi^+(b) \text{ destroys e}^-$$

$$\overline{\psi}^+(x)|p\rangle = \overline{\psi}^+(x)d^\dagger(p)|0\rangle = |0\rangle\overline{v}(p)e^{-ip\cdot x} \qquad \overline{\psi}^+(d) \text{ destroy e}^+$$

$$\langle p|\psi^-(x) = \langle 0|d(p)\psi^-(x) = v(p)e^{+ip\cdot x}\langle 0| \qquad \psi^-(d^\dagger) \text{ creates e}^+$$

$$\langle p|\overline{\psi}^-(x) = \langle 0|b(p)\overline{\psi}^-(x) = \overline{u}(p)e^{+ip\cdot x}\langle 0| \qquad \overline{\psi}^-(b^\dagger) \text{ creates e}^-$$

Quantization : Electromagnetic Field

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial\cdot A)^2 \qquad \pi^\mu = F^{\mu 0} - \frac{g^{\mu 0}}{\xi}(\partial\cdot A) \qquad \pi^0 = -\frac{1}{\xi}(\partial\cdot A) \qquad \pi^k = E^k$$

$$\left[A_\mu(\vec{\mathbf{x}},t), A_\nu(\vec{\mathbf{y}},t)\right] = \left[\dot{A}_\mu(\vec{\mathbf{x}},t), \dot{A}_\nu(\vec{\mathbf{y}},t)\right] = 0 \qquad \left[\dot{A}_\mu(\vec{\mathbf{y}},t), A_\nu(\vec{\mathbf{x}},t)\right] = ig_{\mu\nu}\delta^3(\vec{\mathbf{x}} - \vec{\mathbf{y}})$$

$$A_\mu(x) = A_\mu^+(x) + A_\mu^-(x) = \int\widetilde{\mathrm{d}k}\sum_{\lambda=0}^{3}\left[a(k,\lambda)\epsilon_\mu(k,\lambda)e^{-ik\cdot x} + a^\dagger(k,\lambda)\epsilon_\mu^*(k,\lambda)e^{+ik\cdot x}\right]$$

$$A_\mu^+(x)|k\rangle = A_\mu^+(x)a^\dagger(k)|0\rangle = |0\rangle\epsilon_\mu(k)e^{-ik\cdot x} \qquad \left[\dot{a}(k,\lambda), a^\dagger(k',\lambda')\right] = -g^{\lambda\lambda'}\tilde{\delta}(k - k')$$

$$\langle k|A_\mu^-(x) = \langle 0|a(k)A_\mu^-(x) = \epsilon_\mu^*(k)e^{+ik\cdot x}\langle 0| \qquad A^+(a) \text{ destroys } \gamma \quad A^-(a^\dagger) \text{ creates } \gamma$$

Classical Field Theory

$$S = \int_{t_i}^{t_f} L\,\mathrm{d}t = \int \mathrm{d}^4x\,\mathscr{L} \qquad L = \int \mathrm{d}^3\vec{\mathbf{x}}\,\mathscr{L}(\phi, \partial_\mu\phi) \qquad \frac{\partial\mathscr{L}}{\partial\phi_i} - \partial_\mu\frac{\partial\mathscr{L}}{\partial(\partial_\mu\phi_i)} = 0$$

$$H = \int \mathrm{d}^3\vec{\mathbf{x}}\,\mathscr{H} \qquad \mathscr{H} = \sum_i \Pi_i(x)\partial_0\phi_i(x) - \mathscr{L} \qquad \Pi_i(x) = \frac{\partial\mathscr{L}}{\partial(\partial_0\phi_i(x))}$$

$$\dot{\psi} = \frac{\partial\mathscr{H}}{\partial\Pi} - \vec{\nabla}\cdot\frac{\partial\mathscr{H}}{\partial(\vec{\nabla}\Pi)} \qquad \dot{\Pi} = -\frac{\partial\mathscr{H}}{\partial\psi} + \vec{\nabla}\cdot\frac{\partial\mathscr{H}}{\partial(\vec{\nabla}\psi)}$$

$$\delta\mathscr{L} = \mathscr{L}' - \mathscr{L} = \partial_\mu C^\mu \qquad J^\mu = C^\mu - \frac{\partial\mathscr{L}}{\partial\partial_\mu\phi}\delta\phi \qquad \partial_\mu J^\mu = \frac{\partial J^0}{\partial t} + \vec{\nabla}\cdot\vec{\mathbf{J}} = 0$$

$$Q = \int \mathrm{d}^3\vec{\mathbf{x}}\,J^0 \qquad \frac{\mathrm{d}Q}{\mathrm{d}t} = 0 \qquad A^\mu \to A^\mu + \partial^\mu\lambda$$

Ward identity: $k_\mu\mathscr{M}^\mu = 0$, where $\mathscr{M} = \epsilon_\mu\mathscr{M}^\mu$. Photon polarizations ϵ parallel to its direction of propagation don't contribute to the scattering amplitude.

Dyson Expansion, Propagators and Wick's Theorem

$$S = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\ldots\mathrm{d}^4x_n\,\mathrm{T}(\mathscr{H}_{\mathrm{int}}(x_1)\,\mathscr{H}_{\mathrm{int}}(x_2)\ldots\mathscr{H}_{\mathrm{int}}(x_n)) \qquad \sum_f \left|S_{fi}\right|^2 = 1$$

$$\Delta_F(x_1 - x_2) = \phi(x_1)\phi(x_2) = \langle 0|\mathrm{T}(\phi(x_1)\phi(x_2))|0\rangle \qquad D_F^{\mu\nu} = A^\mu(x_1)A^\nu(x_2)$$

$$\Delta_F(x_1 - x_2) = \varphi(x_1)\varphi^\dagger(x_2) = \varphi^\dagger(x_2)\varphi(x_1) \qquad S_{F\alpha\beta} = \psi_\alpha(x_1)\overline{\psi}_\beta(x_2) = -\overline{\psi}_\beta(x_2)\psi_\alpha(x_1)$$

$$\mathrm{T}(ABCD\ldots WXYZ) = :ABCD\ldots WXYZ: + :ABCD\ldots WXYZ: + :ABCD\ldots WXYZ: +$$
$$+ \cdots + :ABCD\ldots WXYZ: + :ABCD\ldots WXYZ: + \cdots + :ABCD\ldots WXYZ: + \ldots$$

Decay : Decay Rates

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{32\pi^2}\frac{|\vec{\mathbf{p}}_{\mathrm{CM}}|}{m_0^2}\overline{|\mathscr{M}|^2} \qquad |\vec{\mathbf{p}}_{\mathrm{CM}}| = \frac{1}{2m_0}\sqrt{\left[m_0^2 - (m_1 + m_2)^2\right]\left[m_0^2 - (m_1 - m_2)^2\right]}$$

$$E_1 = \frac{m_0^2 + m_1^2 - m_2^2}{2m_0} \qquad E_2 = \frac{m_0^2 + m_2^2 - m_1^2}{2m_0}$$

Scattering : Cross Sections

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{\mathbf{p}}_3|}{|\vec{\mathbf{p}}_1|}\overline{|\mathscr{M}|^2} \qquad |\vec{\mathbf{p}}_a| = |\vec{\mathbf{p}}_b| = \frac{1}{2\sqrt{s}}\sqrt{\left[s - (m_a + m_b)^2\right]\left[s - (m_a - m_b)^2\right]}$$

$$E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}} \qquad E_2 = \frac{s + m_2^2 - m_1^2}{2\sqrt{s}} \qquad E_3 = \frac{s + m_3^2 - m_4^2}{2\sqrt{s}} \qquad E_4 = \frac{s + m_4^2 - m_3^2}{2\sqrt{s}}$$

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \approx +2p_1\cdot p_2 \approx +2p_3\cdot p_4 \qquad \sqrt{s} = E_{CM}$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \approx -2p_1\cdot p_3 \approx -2p_2\cdot p_4 \qquad s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \approx -2p_1\cdot p_4 \approx -2p_2\cdot p_3$$

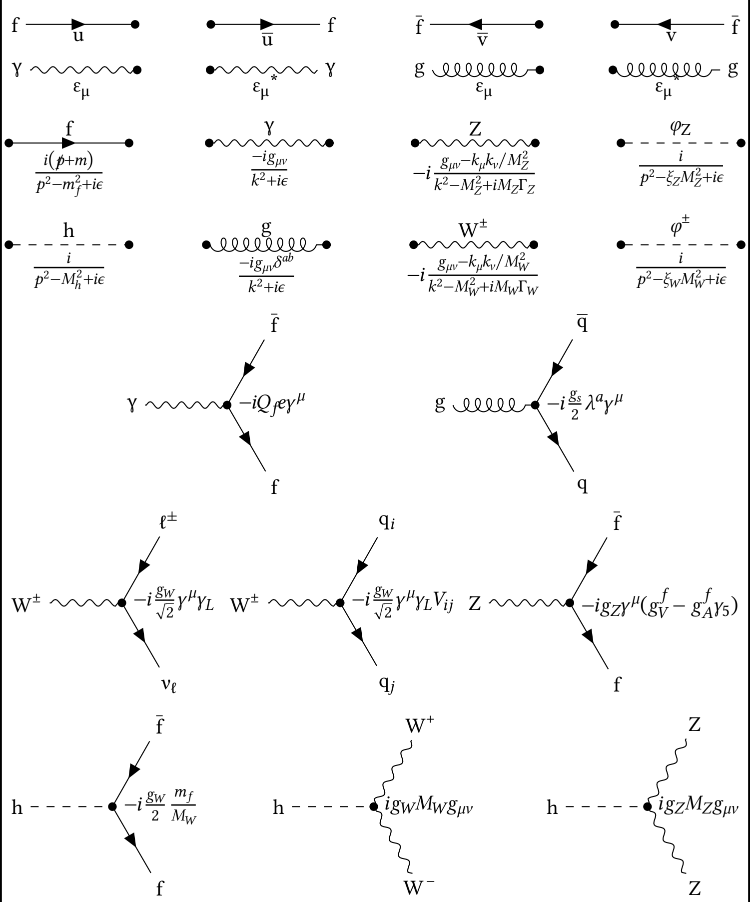
$$\mathrm{f} + \bar{\mathrm{f}} \longrightarrow \mathrm{g} + \bar{\mathrm{g}}: \qquad p_1 = \frac{\sqrt{s}}{2}\left(1, 0, 0, +\beta_f\right) \qquad p_2 = \frac{\sqrt{s}}{2}\left(1, 0, 0, -\beta_f\right)$$

$$p_3 = \frac{\sqrt{s}}{2}\left(1, +\beta_g\sin\theta, 0, +\beta_g\cos\theta\right) \qquad p_4 = \frac{\sqrt{s}}{2}\left(1, -\beta_g\sin\theta, 0, -\beta_g\cos\theta\right)$$

$$m = 0: \qquad s = 2|\vec{\mathbf{p}}_{\mathrm{CM}}|^2 = 2\left(p_{\mathrm{CM}}^0\right)^2 \qquad t = -\frac{s}{2}(1 - \cos\theta) \qquad u = -\frac{s}{2}(1 + \cos\theta)$$

Feynman Rules for $i\mathscr{M}$

Goes in opposite way of arrows with the first one being adjoint, $\overline{\psi} = \psi^\dagger\gamma^0$:



Constants

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} = \frac{g_W^2}{8M_Z^2\cos^2(\theta_W)} = \frac{1}{2v^2} \qquad M_W = M_Z\cos\theta_W = \frac{1}{2}g_W v \qquad M_H = \sqrt{2\lambda}v$$

$$g_W = g_Z\cos\theta_W = \frac{e}{\sin\theta_W} \qquad g_V^f = \frac{1}{2}T_f^3 - Q_f\sin^2(\theta_W) \qquad g_A^f = \frac{1}{2}T_f^3 \qquad g_{A,V} = \frac{1}{2}c_{A,V}$$

$$\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c} \qquad c_L^f = g_V^f + g_A^f \qquad c_R^f = g_V^f - g_A^f \qquad g_V^f = \frac{c_L^f + c_R^f}{2} \qquad g_A^f = \frac{c_L^f - c_R^f}{2}$$

Fermions	Q_f	$I_W^{(3)}$	Y_L	Y_R	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	+1/2	−1	0	+1/2	0	+1/2	+1/2
e^-, μ^-, τ^-	−1	−1/2	−1	−2	−0.27	+0.23	−0.04	−1/2
u, c, t	+2/3	+1/2	+1/3	+4/3	+0.35	−0.15	+0.19	+1/2
d, s, b	−1/3	−1/2	+1/3	−2/3	−0.42	+0.08	−0.35	−1/2

Particle	$W^\pm$	Z^0	H^0	$e^\pm$	$\mu^\pm$	τ	u	d	c	s	t	b
Mass (MeV)	80 379	91 188	125 100	0.511	105.7	1777	2.16	4.67	1270	93	172 760	4180

$$1\,\mathrm{kg} = 5.61\times10^{26}\,\mathrm{GeV} \qquad 1\,\mathrm{m} = 55.07\times10^{15}\,\mathrm{GeV}^{-1} \qquad 1\,\mathrm{s} = 51.52\times10^{24}\,\mathrm{GeV}^{-1}$$

Relativity & Quantum Mechanics

$$\beta = \frac{v}{c} = \frac{p}{E} = \tanh(\eta) \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh(\eta) \qquad \gamma\beta = \sin(\eta)$$

$$\cosh\left(\frac{\eta}{2}\right) = \sqrt{\frac{E+m}{2m}} \qquad \sinh\left(\frac{\eta}{2}\right) = \frac{|\vec{\mathbf{p}}|}{\sqrt{2m(E+m)}} \qquad p^\mu \to i\delta^\mu$$

$$E^2 = m^2 + |p|^2 \qquad \frac{\mathrm{d}^3\vec{\mathbf{p}}'}{E'} = \frac{\mathrm{d}^3\vec{\mathbf{p}}}{E} \qquad \widetilde{\mathrm{d}p} \equiv \frac{\mathrm{d}^3\vec{\mathbf{p}}}{(2\pi)^3 2E_p} \qquad \tilde{\delta}(p-q) \equiv (2\pi)^3 2E_p\,\delta^3(\vec{\mathbf{p}} - \vec{\mathbf{q}})$$

$$x^\mu = (t, \vec{\mathbf{x}}) \qquad p^\mu = (E, \vec{\mathbf{p}}) \qquad \partial^\mu = \left(\partial_t, -\vec{\nabla}\right) \qquad A^\mu = \left(\phi, \vec{\mathbf{A}}\right) \qquad J^\mu = \left(\rho, \vec{\mathbf{J}}\right)$$

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu \qquad \mathscr{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} \qquad \partial_\mu F^{\mu\nu} = J^\nu \qquad \partial_\mu\mathscr{F}^{\mu\nu} = 0$$

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{If } (\mu\nu\rho\sigma) \text{ is an even permutation of } (0123) \\ -1 & \text{If } (\mu\nu\rho\sigma) \text{ is an odd permutation of } (0123) \\ 0 & \text{Otherwise} \end{cases}$$

$$\epsilon_{\alpha\beta_1\gamma_1\delta_1}\epsilon^{\alpha\beta_2\gamma_2\delta_2} = +g_{\beta_1}^{\beta_2}g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\gamma_2} - g_{\beta_1}^{\beta_2}g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\delta_2} + g_{\beta_1}^{\gamma_2}g_{\gamma_1}^{\beta_2}g_{\delta_1}^{\delta_2} - g_{\beta_1}^{\gamma_2}g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\beta_2} + g_{\beta_1}^{\delta_2}g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\beta_2} - g_{\beta_1}^{\delta_2}g_{\gamma_1}^{\beta_2}g_{\delta_1}^{\gamma_2}$$

$$\epsilon_{\alpha\beta\gamma_1\delta_1}\epsilon^{\alpha\beta\gamma_2\delta_2} = -2\left(g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\delta_2} - g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\gamma_2}\right) \qquad \epsilon_{\alpha\beta\gamma\delta_1}\epsilon^{\alpha\beta\gamma\delta_2} = -6g_{\delta_1}^{\delta_2}$$

$$g_{\mu\nu}\Lambda^\mu_{\cdot\alpha}\Lambda^\nu_{\cdot\beta} = g_{\alpha\beta} \qquad \Lambda^\mu_{\cdot\nu} = g^\mu_\nu + \omega^\mu_\nu = \delta^\mu_\nu + \omega^\mu_{\cdot\nu} \qquad \omega_{\beta\alpha} = -\omega_{\alpha\beta} \qquad \omega_{kl} = -\epsilon^{klm}\theta^m \qquad \omega^{0k} = \eta^k$$

$$\delta V^\alpha \sim \omega^\alpha_{\cdot\beta}V^\beta = -\frac{i}{2}\omega_{\mu\nu}(J^{\mu\nu})^\alpha_{\cdot\beta}V^\beta \qquad (J^{\mu\nu})^\alpha_{\cdot\beta} = i(g^{\mu\alpha}g\beta^\nu - g^{\nu\alpha}g\beta^\mu)$$

$$[J^{\mu\nu}, J^{\rho\sigma}] = i(g^{\nu\rho}J^{\mu\sigma} - g^{\nu\sigma}J^{\mu\rho} - g^{\mu\rho}J^{\nu\sigma} + g^{\mu\sigma}J^{\nu\rho}) \qquad \vec{\mathbf{J}} = \left(J^{23}, J^{31}, J^{12}\right)$$

$$J^k = \frac{1}{2}\epsilon^{klm}J^{lm} \qquad K^k = J^{k0} \qquad J\pm^k = \frac{1}{2}\left(J^k \pm iK^k\right) \qquad \vec{\mathbf{K}} = \left(J^{10}, J^{20}, J^{30}\right)$$

$$\left[J^i, J^j\right] = i\epsilon^{ijk}J^k \qquad \left[K^i, K^j\right] = -i\epsilon^{ijk}J^k \qquad \left[K^i, J^j\right] = i\epsilon^{ijk}K^k$$

$$\left[J\pm^i, J\pm^j\right] = i\epsilon^{ijk}J\pm^k \qquad \left[J_+^i, J_-^j\right] = 0$$

$$(j_-, j_+) = (1/2, 0) \qquad \vec{\mathbf{S}}_- = \vec{\boldsymbol{\sigma}}/2 \qquad \vec{\mathbf{S}}_+ = 0 \qquad \chi_L \to \Lambda_L\chi_L = \exp\left\{\left(-i\vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\eta}}\right)\cdot\vec{\boldsymbol{\sigma}}/2\right\}\chi_L$$

$$(j_-, j_+) = (0, 1/2) \qquad \vec{\mathbf{S}}_+ = \vec{\boldsymbol{\sigma}}/2 \qquad \vec{\mathbf{S}}_- = 0 \qquad \chi_R \to \Lambda_R\chi_R = \exp\left\{\left(-i\vec{\boldsymbol{\theta}} + \vec{\boldsymbol{\eta}}\right)\cdot\vec{\boldsymbol{\sigma}}/2\right\}\chi_R$$

$$\Lambda_L^\dagger\Lambda_R = \Lambda_R^\dagger\Lambda_L = \mathbb{1} \qquad \sigma^2\Lambda_L^*\sigma^2 = \Lambda_R \qquad \sigma^2\chi_L^2 \to \Lambda_R(\sigma^2\chi_L^*)$$

Mathematics

Dirac Delta:

$$\delta^n(x' - x) = i\int\frac{\mathrm{d}^n p}{(2\pi)^n}e^{-ip(x'-x)} \qquad f(x_i) = 0 \implies \delta[f(x)] = \sum_i\left|\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x_i}^{-1}\cdot\delta(x - x_i)$$

Triangle Function:

$$\lambda(a, b, c) = \left[a - \left(\sqrt{b} + \sqrt{c}\right)^2\right]\left[a - \left(\sqrt{b} - \sqrt{c}\right)^2\right]$$

Residue Theorem:

$$\oint_\Gamma \mathrm{d}z\,f(z) = \pm2\pi i\sum_{k=1}^n \mathrm{Res}[f(a_k)] \qquad \mathrm{Res}[f(a_k)] = \lim_{z\to a_k}(z - a_k)f(z)$$

The "-" ("+") is used when Γ is oriented clockwise (counterclockwise).

$$f(z) = \frac{h(z)e^{-iz(t'-t)}}{(z - a_1)(z - a_2)\ldots} \overset{t'-t<0}{\Longrightarrow} \int_{\Re(z)=-\infty}^{\Re(z)=+\infty} f(z)\,\mathrm{d}z = +2\pi i\sum_{a_k\text{ in UHP}}\mathrm{Res}[f(a_k)]$$

$$\overset{t'-t>0}{\Longrightarrow} \int_{\Re(z)=-\infty}^{\Re(z)=+\infty} f(z)\,\mathrm{d}z = -2\pi i\sum_{a_k\text{ in LHP}}\mathrm{Res}[f(a_k)]$$