Equations

Schrödinger:

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi t\rangle \qquad \mathscr{L} = i\psi^*\dot{\psi} - \frac{1}{2m}\vec{\nabla}\psi^*\cdot\vec{\nabla}\psi$$

Klein-Gordon (Real Scalar Field):

$$(\partial\cdot\partial + m^2)\phi = 0 \qquad \mathscr{L} = \frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right] = \frac{1}{2}\left[\dot{\phi}^2 - \left(\vec{\nabla}\phi\right)^2 - m^2\phi^2\right]$$

Dirac (Complex Scalar Field):

$$(i\partial\!\!\!/ - m)\varphi = 0 \qquad \overline{\varphi}(i\partial\!\!\!/ + m) = 0 \qquad \mathscr{L} = \partial_\mu\varphi\partial^\mu\varphi^* - m^2\varphi\varphi^*$$

Pauli Matrices

$$\sigma^\mu = (\mathbb{1}, \vec{\sigma}) \qquad \overline{\sigma}^\mu = (\mathbb{1}, -\vec{\sigma})$$

$$\sigma^1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \vec{\boldsymbol{\Sigma}} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

$$\{\sigma^i, \sigma^j\} = 2\delta^{ij} \qquad [\sigma^i, \sigma^j] = 2i\epsilon^{ijk}\sigma^k \qquad \sigma^i\sigma^j = \delta^{ij} + i\epsilon^{ijk}\sigma^k \qquad \sigma^2\sigma^i\sigma^2 = -(\sigma^i)^*$$

$$\vec{\sigma}\cdot\vec{\mathbf{p}} = \begin{pmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{pmatrix} \qquad (\vec{\sigma}\cdot\vec{\mathbf{p}})^2 = |\vec{\mathbf{p}}|^2 \qquad (p\cdot\sigma)(p\cdot\overline{\sigma}) = p^2$$

$$p\cdot\overline{\sigma} = \begin{pmatrix} p^0 + p^3 & p^1 - ip^2 \\ p^1 + ip^2 & p^0 - p^3 \end{pmatrix} \qquad p\cdot\sigma = \begin{pmatrix} p^0 - p^3 & -(p^1 - ip^2) \\ -(p^1 + ip^2) & p^0 + p^3 \end{pmatrix}$$

$$\sqrt{p\cdot\sigma} = \frac{E + m - \vec{\sigma}\cdot\vec{\mathbf{p}}}{\sqrt{2(E+m)}} \qquad \sqrt{p\cdot\overline{\sigma}} = \frac{E + m + \vec{\sigma}\cdot\vec{\mathbf{p}}}{\sqrt{2(E+m)}}$$

Dirac γ-Matrices

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad \gamma_5 = +i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma_0\gamma_1\gamma_2\gamma_3 = -\frac{i}{4!}\epsilon^{\alpha\beta\gamma\delta}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \implies [\gamma^\mu, \gamma^\nu] = -2i\sigma^{\mu\nu} \qquad \{\gamma_5, \gamma^\mu\} = 0 \qquad \gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu$$

$$\left(\gamma^0\right)^\dagger = \gamma^0 \quad \left(\gamma^0\right)^2 = \mathbb{1} \qquad \left(\gamma^k\right)^\dagger = -\gamma^k \quad \left(\gamma^k\right)^2 = -\mathbb{1} \qquad \left(\gamma^5\right)^\dagger = \gamma^5 \quad \left(\gamma^5\right)^2 = \mathbb{1}$$

$$\overline{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0 \qquad \overline{\gamma_5} = -\gamma_5 \qquad \overline{\gamma^\mu} = \gamma^\mu \qquad \overline{\gamma^\mu\gamma_5} = \gamma^\mu\gamma_5 \qquad \overline{\sigma^{\mu\nu}} = \sigma^{\mu\nu}$$

$\mathbb{1}$	γ_5	γ^μ	$\gamma_5\gamma^\mu$	$\sigma^{\mu\nu}$
γ_5	$\mathbb{1}$	$+\gamma_5\gamma^\mu$	$+\gamma^\mu$	$\frac{i}{2}\epsilon^{\mu\nu\pi\rho}\sigma_{\pi\rho}$
γ^α	$-\gamma_5\gamma^\alpha$	$g^{\alpha\mu} - i\sigma^{\alpha\mu}$	$-\frac{1}{2}\left(2g^{\alpha\mu}\gamma_5 + \epsilon^{\alpha\mu\pi\rho}\sigma_{\pi\rho}\right)$	$\epsilon^{\alpha\mu\nu\lambda}\gamma_5\gamma_\lambda + ig^{\alpha\mu}\gamma^\nu - ig^{\alpha\nu}\gamma^\mu$
$\gamma_5\gamma^\alpha$	$-\gamma^\alpha$	$-\gamma^\alpha$	$-(g^{\alpha\mu} - i\sigma^{\alpha\mu})$	$\epsilon^{\alpha\mu\nu\lambda}\gamma_\lambda + ig^{\alpha\mu}\gamma_5\gamma^\nu - ig^{\alpha\nu}\gamma_5\gamma^\mu$
$\sigma^{\alpha\beta}$	$\sigma^{\alpha\beta}$	$\epsilon^{\alpha\beta\mu\lambda}\gamma_5\gamma_\lambda + ig^{\beta\mu}\gamma^\alpha - ig^{\alpha\mu}\gamma^\beta$	$\epsilon^{\alpha\beta\mu\lambda}\gamma_\lambda + ig^{\beta\mu}\gamma_5\gamma^\alpha - ig^{\alpha\mu}\gamma_5\gamma^\beta$	$i\epsilon^{\alpha\beta\mu\nu}\gamma_5 + g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu} + i\left[g^{\alpha\nu}\sigma^{\beta\mu} + g^{\beta\mu}\sigma^{\alpha\nu} - g^{\alpha\mu}\sigma^{\beta\nu} - g^{\beta\nu}\sigma^{\alpha\mu}\right]$

$$A\!\!\!/ = A^\mu\gamma_\mu \qquad \gamma^\mu\gamma_\mu = 4 \qquad \gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4g^{\nu\rho}$$

$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\pi\gamma_\mu = 2[\gamma^\pi\gamma^\nu\gamma^\rho\gamma^\sigma + \gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\pi]$$

$$\gamma^\mu\gamma^\nu\gamma^\rho = i\epsilon^{\mu\nu\rho\lambda}\gamma_\lambda\gamma_5 + g^{\mu\nu}\gamma^\rho - g^{\mu\rho}\gamma^\nu + g^{\nu\rho}\gamma^\mu$$

Spin, Helicity and Chirality

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\boldsymbol{\Sigma}} \qquad \tilde{h} = \frac{\vec{\mathbf{S}}\cdot\vec{\mathbf{p}}}{|\vec{\mathbf{p}}|} = \frac{\vec{\boldsymbol{\Sigma}}\cdot\vec{\mathbf{p}}}{2|\vec{\mathbf{p}}|} \qquad \hat{h} = 2h = \frac{\vec{\boldsymbol{\Sigma}}\cdot\vec{\mathbf{p}}}{|\vec{\mathbf{p}}|} \qquad P_L = \frac{1-h}{2} \qquad P_R = \frac{1+h}{2}$$

$$\gamma_L = \frac{1-\gamma_5}{2} \qquad \gamma_R = \frac{1+\gamma_5}{2} \qquad \gamma_{L,R}^2 = \gamma_{L,R} \qquad \gamma_{L,R}\gamma_{R,L} = 0 \qquad \gamma_L + \gamma_R = \mathbb{1}$$

$$\gamma^\mu\gamma_{L,R} = \gamma_{R,L}\gamma^\mu \qquad \gamma_5\gamma_L = \gamma_L\gamma_5 = -\gamma_L \qquad \gamma_5\gamma_R = \gamma_R\gamma_5 = \gamma_R \qquad \gamma_{L,R}^\dagger = \gamma_{L,R}$$

$$\overline{\gamma^\mu\gamma_{L,R}} = \gamma^\mu\gamma_{L,R} \qquad \overline{\gamma_5\gamma_L} = -\gamma_R \qquad \overline{\gamma_5\gamma_R} = \gamma_L \qquad \overline{\gamma_{L,R}} = \gamma_0\gamma_{L,R}^\dagger\gamma_0 = \gamma_{R,L}$$

$$u_L = u_\downarrow \qquad u_R = u_\uparrow \qquad v_L = v_\uparrow \qquad v_R = v_\downarrow$$

$$P_L u_\downarrow = u_\downarrow \quad P_L u_\uparrow = 0 \qquad P_R u_\downarrow = 0 \quad P_R u_\uparrow = u_\uparrow \qquad \overline{u_\downarrow}P_L = 0 \quad \overline{u_\uparrow}P_L = u_\uparrow \qquad \overline{u_\downarrow}P_R = u_\downarrow \quad \overline{u_\uparrow}P_R = 0$$

$$\gamma_L u_L = u_L \quad \gamma_L u_R = 0 \qquad \gamma_R u_L = 0 \quad \gamma_R u_R = u_R \qquad \overline{u_L}\gamma_L = 0 \quad \overline{u_R}\gamma_L = u_R \qquad \overline{u_L}\gamma_R = u_L \quad \overline{u_R}\gamma_R = 0$$

$$P_L v_\downarrow = 0 \quad P_L v_\uparrow = v_\uparrow \qquad P_R v_\downarrow = v_\downarrow \quad P_R v_\uparrow = 0 \qquad \overline{v_\downarrow}P_L = v_\downarrow \quad \overline{v_\uparrow}P_L = 0 \qquad \overline{v_\downarrow}P_R = 0 \quad \overline{v_\uparrow}P_R = v_\uparrow$$

$$\gamma_L v_L = v_L \quad \gamma_L v_R = 0 \qquad \gamma_R v_L = 0 \quad \gamma_R v_R = v_R \qquad \overline{v_L}\gamma_L = 0 \quad \overline{v_R}\gamma_L = v_R \qquad \overline{v_L}\gamma_R = v_L \quad \overline{v_R}\gamma_R = 0$$

$$u_\uparrow = N\begin{pmatrix} c \\ se^{i\phi} \\ kc \\ kse^{i\phi} \end{pmatrix} \approx N\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix} \qquad u_\downarrow = N\begin{pmatrix} -s \\ ce^{i\phi} \\ ks \\ -kce^{i\phi} \end{pmatrix} \approx N\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix}$$

$$v_\uparrow = N\begin{pmatrix} ks \\ -kce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix} \approx N\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix} \qquad v_\downarrow = N\begin{pmatrix} kc \\ kse^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix} \approx N\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}$$

$$\vec{\mathbf{p}} = p(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$N = \sqrt{E+m} \approx \sqrt{E} \qquad k = \frac{p}{E+m} \approx 1 \qquad s = \sin\left(\frac{\theta}{2}\right) \qquad c = \cos\left(\frac{\theta}{2}\right)$$

Traces

$$\mathrm{Tr}[\mathbb{1}] = 4 \qquad \mathrm{Tr}[\underbrace{\gamma^\mu\gamma^\nu\dots\gamma^\rho}_{\text{Odd Number}}] = 0 \qquad \mathrm{Tr}[\gamma^\mu\gamma^\nu] = 4g^{\mu\nu}$$

$$\mathrm{Tr}[\gamma_5] = 0 \qquad \mathrm{Tr}[\gamma_5\underbrace{\gamma^\mu\gamma^\nu\dots\gamma^\rho}_{\text{Odd Number}}] = 0 \qquad \mathrm{Tr}[\gamma_5\gamma^\mu\gamma^\nu] = 0$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) \qquad \mathrm{Tr}[\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = -4i\epsilon^{\mu\nu\rho\sigma}$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_L] = 2(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} + i\epsilon^{\mu\nu\rho\sigma})$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_R] = 2(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - i\epsilon^{\mu\nu\rho\sigma})$$

$$\mathrm{Tr}\left[(p\!\!\!/_a + m_a)\gamma^\mu(p\!\!\!/_b + m_b)\gamma^\nu\right] = 4\left[p_a^\mu p_b^\nu + p_b^\mu p_a^\nu + (m_a m_b - p_a\cdot p_b)g^{\mu\nu}\right]$$

$$\mathrm{Tr}\left[a\!\!\!/\gamma^\mu b\!\!\!/\gamma^\nu\right]\mathrm{Tr}\left[c\!\!\!/\gamma_\mu d\!\!\!/\gamma_\nu\gamma_5\right] = 0 \qquad \mathrm{Tr}\left[a\!\!\!/\gamma^\mu b\!\!\!/\gamma^\nu\gamma_L\right]\mathrm{Tr}\left[c\!\!\!/\gamma_\mu d\!\!\!/\gamma_\nu\gamma_R\right] = 16(a\cdot d)(b\cdot c)$$

$$\mathrm{Tr}\left[a\!\!\!/\gamma^\mu b\!\!\!/\gamma^\nu\gamma_L\right]\mathrm{Tr}\left[c\!\!\!/\gamma_\mu d\!\!\!/\gamma_\nu\gamma_L\right] = \mathrm{Tr}\left[a\!\!\!/\gamma^\mu b\!\!\!/\gamma^\nu\gamma_L\right]\mathrm{Tr}\left[c\!\!\!/\gamma_\mu d\!\!\!/\gamma_\nu\gamma_L\right] = 16(a\cdot c)(b\cdot d)$$

Spinors (Fermions)

$$\psi = ue^{+i(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}} - E\cdot t)} = ve^{-i(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}} - E\cdot t)} \qquad \overline{\psi} = \psi^\dagger\gamma^0$$

$$(p\!\!\!/ - m)u = \overline{u}(p\!\!\!/ - m) = 0 \qquad (p\!\!\!/ + m)v = \overline{v}(p\!\!\!/ + m) = 0$$

$$\overline{u}^r(p)u^s(p) = +2m\delta^{rs} \quad \overline{u}^r(p)v^s(p) = 0 \quad u^{r\dagger}(p)u^s(p) = 2E\delta^{rs} \quad \sum_{s=1,2} u^s(p)\overline{u}^s(p) = p\!\!\!/ + m$$

$$\overline{v}^r(p)v^s(p) = -2m\delta^{rs} \quad \overline{v}^r(p)u^s(p) = 0 \quad v^{r\dagger}(p)v^s(p) = 2E\delta^{rs} \quad \sum_{s=1,2} v^s(p)\overline{v}^s(p) = p\!\!\!/ - m$$

Polarization Vectors (Bosons)

External massless & Massive:

$$\sum_{\lambda=1}^{2}\epsilon^\mu(k,\lambda)\cdot\epsilon^{*\nu}(k,\lambda) = -g^{\mu\nu} \qquad \sum_{\lambda=1}^{3}\epsilon_\mu(q,\lambda)\epsilon_\nu^*(q,\lambda) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2}$$

Virtual massless:

$$\epsilon(k,\lambda)\cdot\epsilon^*(k,\lambda') = g^{\lambda\lambda'} \qquad \sum_\lambda g^{\lambda\lambda}\epsilon^\mu(k,\lambda)\cdot\epsilon^{*\nu}(k,\lambda) = g^{\mu\nu}$$

Quantization : Real Scalar Field (Bosons)

$$:\frac{1}{2}\left[a^\dagger(k)a(k) + a(k)a^\dagger(k)\right]: = a^\dagger(k)a(k) \qquad :a(x)b(x'): = a(x)b(x') + b(x')a(x)$$

$$\mathbf{T}(a(x)b(x')) = \theta(t-t')a(x)b(x') + \theta(t'-t)b(x')a(x)$$

The "+" corresponds to positive frequency plane waves $e^{-ik\cdot x}$:

$$\mathscr{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi\phi \qquad \pi = \dot{\phi} \qquad \left[a^\dagger(k,\lambda), a(k',\lambda')\right] = g^{\lambda\lambda'}\tilde{\delta}(k-k')$$

$$\left[\phi(\vec{\mathbf{x}},t), \phi(\vec{\mathbf{y}},t)\right] = \left[\Pi(\vec{\mathbf{x}},t), \Pi(\vec{\mathbf{y}},t)\right] = 0 \qquad \left[\phi(\vec{\mathbf{x}},t), \Pi(\vec{\mathbf{y}},t)\right] = i\delta^3(\vec{\mathbf{x}} - \vec{\mathbf{y}})$$

$$\phi(x) = \int \tilde{\mathrm{d}}k\left[a(k)e^{-ik\cdot x} + a^\dagger(k)e^{+ik\cdot x}\right]$$

$$|p\rangle = a^\dagger(p)\,|0\rangle \qquad |p_1, p_2\rangle = a^\dagger(p_2)a^\dagger(p_1)\,|0\rangle = |p_2, p_1\rangle$$

Quantization : Complex Scalar Field (Bosons)

$$\mathscr{L} = :\partial^\mu\varphi^\dagger\partial_\mu\varphi - m^2\varphi^\dagger\varphi: \qquad \pi = \dot{\varphi}^\dagger \quad \pi^\dagger = \dot{\varphi} \qquad \left[a_\pm(k), a_\pm^\dagger(k')\right] = \tilde{\delta}(k-k')$$

$$\left[\varphi(\vec{\mathbf{x}},t), \pi(\vec{\mathbf{y}},t)\right] = \left[\varphi^\dagger(\vec{\mathbf{x}},t), \pi^\dagger(\vec{\mathbf{y}},t)\right] = i\,\delta^3(\vec{\mathbf{x}} - \vec{\mathbf{y}})$$

$$\varphi(x) = \varphi^+(x) + \varphi^-(x) = \int \tilde{\mathrm{d}}k\left[a_+(k)e^{-ik\cdot x} + a_-^\dagger(k)e^{+ik\cdot x}\right]$$

$$\varphi^\dagger(x) = \varphi^{+\dagger}(x) + \varphi^{-\dagger}(x) = \int \tilde{\mathrm{d}}k\left[a_-(k)e^{-ik\cdot x} + a_+^\dagger(k)e^{+ik\cdot x}\right]$$

$$|p^+\rangle = a_+^\dagger(p)\,|0\rangle \qquad |p^-\rangle = a_-^\dagger(p)\,|0\rangle \qquad |p_1^+, p_2^-\rangle = a_-^\dagger(p_2)a_+^\dagger(p_1)\,|0\rangle$$

Quantization : Dirac Field (Fermions)

$$:a(x)b(x'): = a(x)b(x') - b(x')a(x)$$

$$\mathbf{T}(a(x)b(x')) = \theta(t -' t)a(x)b(x') - \theta(t'-t)b(x')a(x)$$

$$\mathscr{L} = :i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi: \qquad \pi_\alpha = \frac{\partial\mathscr{L}}{\partial\dot{\psi}_\alpha} = i\psi_\alpha^\dagger \qquad \pi_\alpha^\dagger = \frac{\partial\mathscr{L}}{\partial\dot{\psi}_\alpha^\dagger} = 0$$

$$\left\{b^\dagger(p,s), b(p',s')\right\} = \left\{d^\dagger(p,s), d(p',s')\right\} = \tilde{\delta}(p-p')\delta_{ss'}$$

$$\psi(x) = \psi^+(x) + \psi^-(x) = \int \widetilde{\mathrm{d}p}\sum_s\left[b(p,s)u(p,s)e^{-ip\cdot x} + d^\dagger(p,s)v(p,s)e^{+ip\cdot x}\right]$$

$$\overline{\psi}(x) = \overline{\psi}^+(x) + \overline{\psi}^-(x) = \int \widetilde{\mathrm{d}p}\sum_s\left[d(p,s)\overline{v}(p,s)e^{-ip\cdot x} + b^\dagger(p,s)\overline{u}(p,s)e^{+ip\cdot x}\right]$$

$$|e^-(p_1,s_1), e^-(p_2,s_2)\rangle = b^\dagger(p_2,s_2)b^\dagger(p_1,s_1)\,|0\rangle = -\,|e^-(p_2,s_2), e^-(p_1,s_1)\rangle$$

$$\psi^+(x)\,|p\rangle = \psi^+(x)b^\dagger(p)\,|0\rangle = |0\rangle\,u(p)e^{-ip\cdot x} \qquad \psi^+(b) \text{ destroys } e^-$$

$$\overline{\psi}^+(x)\,|p\rangle = \overline{\psi}^+(x)d^\dagger(p)\,|0\rangle = |0\rangle\,\overline{v}(p)e^{-ip\cdot x} \qquad \overline{\psi}^+(d) \text{ destroy } e^+$$

$$\langle p|\,\psi^-(x) = \langle 0|\,d(p)\psi^-(x) = v(p)e^{+ip\cdot x}\langle 0| \qquad \psi^-(d^\dagger) \text{ creates } e^+$$

$$\langle p|\,\overline{\psi}^-(x) = \langle 0|\,b(p)\overline{\psi}^-(x) = \overline{u}(p)e^{+ip\cdot x}\langle 0| \qquad \overline{\psi}^-(b^\dagger) \text{ creates } e^-$$

Quantization : Electromagnetic Field

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial\cdot A)^2 \qquad \pi^\mu = F^{\mu 0} - \frac{g^{\mu 0}}{\xi}(\partial\cdot A) \qquad \pi^0 = -\frac{1}{\xi}(\partial\cdot A) \qquad \pi^k = E^k$$

$$\left[A_\mu(\vec{\mathbf{x}},t), A_\nu(\vec{\mathbf{y}},t)\right] = \left[\dot{A}_\mu(\vec{\mathbf{x}},t), \dot{A}_\nu(\vec{\mathbf{y}},t)\right] = 0 \qquad \left[\dot{A}_\mu(\vec{\mathbf{y}},t), A_\nu(\vec{\mathbf{x}},t)\right] = i g_{\mu\nu}\delta^3(\vec{\mathbf{x}}-\vec{\mathbf{y}})$$

$$A_\mu(x) = A^+_\mu(x) + A^-_\mu(x) = \int \widetilde{\mathrm{d}k} \sum_{\lambda=0}^{3}\left[a(k,\lambda)\epsilon_\mu(k,\lambda)e^{-ik\cdot x} + a^\dagger(k,\lambda)\epsilon^*_\mu(k,\lambda)e^{+ik\cdot x}\right]$$

$$A^+_\mu(x)\left|k\right\rangle = A^+_\mu(x)a^\dagger(k)\left|0\right\rangle = \left|0\right\rangle \epsilon_\mu(k)e^{-ik\cdot x} \qquad \left[a(k,\lambda), a^\dagger(k',\lambda')\right] = -g^{\lambda\lambda'}\tilde{\delta}(k-k')$$

$$\left\langle k\right| A^-_\mu(x) = \left\langle 0\right| a(k)A^-_\mu(x) = \epsilon^*_\mu(k)e^{+ik\cdot x}\left\langle 0\right| \qquad A^+(a) \text{ destroys } \gamma \qquad A^-(a^\dagger) \text{ creates } \gamma$$

Dyson Expansion, Propagators and Wick's Theorem

$$S_{fi} = \langle f|\Phi(\infty)\rangle = \langle f|S|i\rangle \qquad \sum_f \left|S_{fi}\right|^2 = 1$$

$$S = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\dots\mathrm{d}^4x_n\,\mathbf{T}(\mathscr{H}_{\text{int}}(x_1)\,\mathscr{H}_{\text{int}}(x_2)\dots\mathscr{H}_{\text{int}}(x_n))$$

$$\Delta_F(x_1-x_2) = \overbracket{\phi(x_1)\phi}(x_2) = \langle 0|\mathbf{T}(\phi(x_1)\phi(x_2))|0\rangle \qquad D_F^{\mu\nu} = \overbracket{A^\mu(x_1)A^\nu}(x_2)$$

$$\Delta_F(x_1-x_2) = \overbracket{\varphi(x_1)\varphi^\dagger}(x_2) = \overbracket{\varphi^\dagger(x_2)\varphi}(x_1) \qquad S_{F\alpha\beta} = \overbracket{\psi_\alpha(x_1)\overline{\psi_\beta}}(x_2) = -\overbracket{\overline{\psi_\beta}(x_2)\psi_\alpha}(x_1)$$

$$\mathbf{T}(ABCD\dots WXYZ) = \,:ABCD\dots WXYZ:+$$
$$+ :\overbracket{AB}CD\dots WXYZ: + :\overbracket{ABC}D\dots WXYZ: + \dots + :ABCD\dots WX\overbracket{YZ}:+$$
$$+ :\overbracket{AB}\,\overbracket{CD}\dots WXYZ: + \dots + :ABCD\dots \overbracket{WX}\,\overbracket{YZ}: + \dots$$

Decay : Decay Rates

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{32\pi^2}\frac{|\vec{\mathbf{p}}_{\text{CM}}|}{m_0^2}\overline{|\mathscr{M}|^2} \qquad |\vec{\mathbf{p}}_{\text{CM}}| = \frac{1}{2m_0}\sqrt{\left[m_0^2-(m_1+m_2)^2\right]\left[m_0^2-(m_1-m_2)^2\right]}$$

Scattering : Cross Sections

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{\mathbf{p}}_3|}{|\vec{\mathbf{p}}_1|}\overline{|\mathscr{M}|^2} \qquad |\vec{\mathbf{p}}| = \frac{1}{2\sqrt{s}}\sqrt{\left[s-(m_a+m_b)^2\right]\left[s-(m_a-m_b)^2\right]}$$

$$s = (p_1+p_2)^2 = (p_3+p_4)^2 \approx +2p_1\cdot p_2 \approx +2p_3\cdot p_4 \qquad \sqrt{s} = E_{CM}$$

$$t = (p_1-p_3)^2 = (p_2-p_4)^2 \approx -2p_1\cdot p_3 \approx -2p_2\cdot p_4 \qquad s+t+u = m_1^2+m_2^2+m_3^2+m_4^2$$

$$u = (p_1-p_4)^2 = (p_2-p_3)^2 \approx -2p_1\cdot p_4 \approx -2p_2\cdot p_3$$

$$\mathrm{f} + \bar{\mathrm{f}} \longrightarrow \mathrm{g} + \bar{\mathrm{g}}:$$

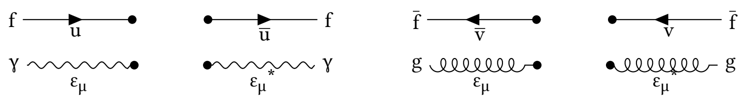
$$p_1 = \frac{\sqrt{s}}{2}\left(1,0,0,+\beta_f\right) \qquad p_2 = \frac{\sqrt{s}}{2}\left(1,0,0,-\beta_f\right)$$

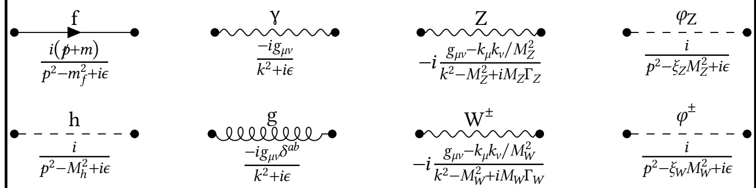
$$p_3 = \frac{\sqrt{s}}{2}\left(1,+\beta_g\sin\theta,0,+\beta_g\cos\theta\right) \qquad p_4 = \frac{\sqrt{s}}{2}\left(1,-\beta_g\sin\theta,0,-\beta_g\cos\theta\right)$$

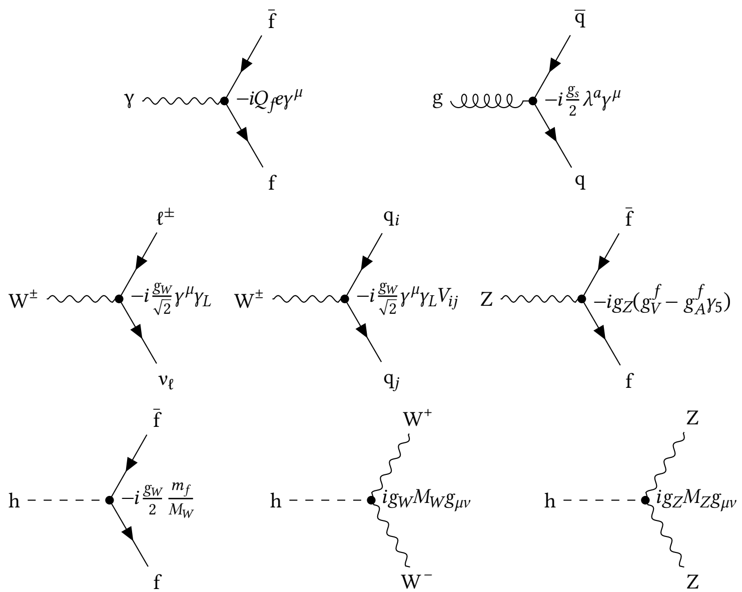
$$m=0: \qquad s = 2|\vec{\mathbf{p}}_{\text{CM}}|^2 = 2\left(p^0_{\text{CM}}\right)^2 \qquad t = -\frac{s}{2}(1-\cos\theta) \qquad u = -\frac{s}{2}(1+\cos\theta)$$

Feynman Rules for $i\mathscr{M}$

Goes in opposite way of arrows with the first one being adjoint, $\overline{\psi} = \psi^\dagger\gamma^0$:







Constants

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} = \frac{g_W^2}{8M_Z^2\cos^2(\theta_W)} = \frac{1}{2v^2} \qquad M_W = M_Z\cos\theta_W = \frac{1}{2}g_W v \qquad M_H = \sqrt{2\lambda}v$$

$$g_W = g_Z\cos\theta_W = \frac{e}{\sin\theta_W} \qquad g_V^f = \frac{1}{2}T_f^3 - Q_f\sin^2(\theta_W) \qquad g_A^f = \frac{1}{2}T_f^3 \qquad g_{A,V} = \frac{1}{2}c_{A,V}$$

$$c_L^f = \frac{g_V^f + g_A^f}{2} \qquad c_R^f = \frac{g_V^f - g_A^f}{2} \qquad g_V^f = c_L^f + c_R^f \qquad g_A^f = c_L^f - c_R^f$$

Fermions	Q_f	$I_W^{(3)}$	Y_L	Y_R	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	+1/2	−1	0	+1/2	0	+1/2	+1/2
e^-, μ^-, τ^-	−1	−1/2	−1	−2	−0.27	+0.23	−0.04	−1/2
u, c, t	+2/3	+1/2	+1/3	+4/3	+0.35	−0.15	+0.19	+1/2
d, s, b	−1/3	−1/2	+1/3	−2/3	−0.42	+0.08	−0.35	−1/2

Relativity

$$\beta = \frac{v}{c} = \frac{p}{E} = \tanh(\eta) \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh(\eta) \qquad \gamma\beta = \sin(\eta)$$

$$E^2 = m^2 + |p|^2 \qquad \frac{\mathrm{d}^3\vec{\mathbf{p}}'}{E'} = \frac{\mathrm{d}^3\vec{\mathbf{p}}}{E} \qquad \widetilde{\mathrm{d}p} \equiv \frac{\mathrm{d}^3\vec{\mathbf{p}}}{(2\pi)^3 2E_p} \qquad \tilde{\delta}(p-q) \equiv (2\pi)^3 2E_p\,\delta^3(\vec{\mathbf{p}}-\vec{\mathbf{q}})$$

$$x^\mu = (t,\vec{\mathbf{x}}) \qquad p^\mu = (E,\vec{\mathbf{p}}) \qquad \partial^\mu = \left(\partial_t, -\vec{\boldsymbol{\nabla}}\right) \qquad A^\mu = \left(\phi,\vec{\mathbf{A}}\right) \qquad J^\mu = \left(\rho,\vec{\mathbf{J}}\right)$$

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu \qquad \mathscr{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} \qquad \partial_\mu F^{\mu\nu} = J^\nu \qquad \partial_\mu\mathscr{F}^{\mu\nu} = 0$$

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{If } (\mu\nu\rho\sigma) \text{ is an even permutation of } (0123) \\ -1 & \text{If } (\mu\nu\rho\sigma) \text{ is an odd permutation of } (0123) \\ 0 & \text{Otherwise} \end{cases}$$

$$\epsilon_{\alpha\beta_1\gamma_1\delta_1}\epsilon^{\alpha\beta_2\gamma_2\delta_2} = +g^{\beta_2}_{\beta_1}g^{\delta_2}_{\gamma_1}g^{\gamma_2}_{\delta_1} - g^{\beta_2}_{\beta_1}g^{\gamma_2}_{\gamma_1}g^{\delta_2}_{\delta_1} + g^{\gamma_2}_{\beta_1}g^{\beta_2}_{\gamma_1}g^{\delta_2}_{\delta_1} - g^{\gamma_2}_{\beta_1}g^{\delta_2}_{\gamma_1}g^{\beta_2}_{\delta_1}$$
$$+ g^{\delta_2}_{\beta_1}g^{\gamma_2}_{\gamma_1}g^{\beta_2}_{\delta_1} - g^{\delta_2}_{\beta_1}g^{\beta_2}_{\gamma_1}g^{\gamma_2}_{\delta_1}$$

$$\epsilon_{\alpha\beta\gamma_1\delta_1}\epsilon^{\alpha\beta\gamma_2\delta_2} = -2\left(g^{\gamma_2}_{\gamma_1}g^{\delta_2}_{\delta_1} - g^{\delta_2}_{\gamma_1}g^{\gamma_2}_{\delta_1}\right) \qquad \epsilon_{\alpha\beta\gamma\delta_1}\epsilon^{\alpha\beta\gamma\delta_2} = -6g^{\delta_2}_{\delta_1}$$

Quantum Mechanics

$$E \to i\frac{\partial}{\partial t} \qquad \vec{\mathbf{p}} \to -i\vec{\boldsymbol{\nabla}}\cdot \implies p^\mu \to i\partial^\mu$$

Mathematics

Dirac Delta:

$$\delta^n(x'-x) = i\int\frac{\mathrm{d}^n p}{(2\pi)^n}e^{-ip(x'-x)} \qquad f(x_i) = 0 \implies \delta[f(x)] = \sum_i\left|\frac{\mathrm{d}f}{\mathrm{d}x}\right|^{-1}_{x_i}\cdot\delta(x-x_i)$$

Triangle Function:

$$\lambda(a,b,c) = \left[a - \left(\sqrt{b}+\sqrt{c}\right)^2\right]\left[a - \left(\sqrt{b}-\sqrt{c}\right)^2\right]$$

Residue Theorem:

$$\oint_\Gamma \mathrm{d}z\,f(z) = \pm 2\pi i\sum_{k=1}^{n}\mathrm{Res}[f(a_k)] \qquad \mathrm{Res}[f(a_k)] = \lim_{z\to a_k}(z-a_k)f(z)$$

The "-" ("+") is used when Γ is oriented clockwise (counterclockwise).

$$f(z) = \frac{h(z)e^{-iz(t'-t)}}{(z-a_1)(z-a_2)\dots} \overset{t'-t<0}{\Longrightarrow} \int_{\Re(z)=-\infty}^{\Re(z)=+\infty} f(z)\,\mathrm{d}z = +2\pi i\sum_{a_k \text{ in UHP}}\mathrm{Res}[f(a_k)]$$
$$\overset{t'-t>0}{\Longrightarrow} \int_{\Re(z)=-\infty}^{\Re(z)=+\infty} f(z)\,\mathrm{d}z = -2\pi i\sum_{a_k \text{ in LHP}}\mathrm{Res}[f(a_k)]$$