

Equations

Schrödinger:

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = H |\psi(t)\rangle \quad \mathcal{L} = i\dot{\psi}^* \dot{\psi} - \frac{1}{2m} \vec{\nabla} \psi^* \cdot \vec{\nabla} \psi$$

Klein-Gordon (Real Scalar Field):

$$(\partial \cdot \partial + m^2)\phi = 0 \quad \mathcal{L} = \frac{1}{2} [\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2] = \frac{1}{2} [\dot{\phi}^2 - (\vec{\nabla} \phi)^2 - m^2 \phi^2]$$

Dirac (Complex Scalar Field):

$$(i\partial\!\!\!/ - m)\varphi = 0 \quad \overline{\varphi}(i\partial\!\!\!/ + m) = 0 \quad \mathcal{L} = \partial_\mu \varphi \partial^\mu \varphi^* - m^2 \varphi \varphi^*$$

γ-Matrices

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad \gamma_5 = +i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma_0\gamma_1\gamma_2\gamma_3 = -\frac{i}{4!}\epsilon^{\alpha\beta\gamma\delta}\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \implies [\gamma^\mu, \gamma^\nu] = -2i\sigma^{\mu\nu} \quad \{\gamma_5, \gamma^\mu\} = 0 \quad \gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu$$

$$(\gamma^0)^\dagger = \gamma^0 \quad (\gamma^0)^2 = 1 \quad (\gamma^k)^\dagger = -\gamma^k \quad (\gamma^k)^2 = -1 \quad (\gamma^5)^\dagger = \gamma^5 \quad (\gamma^5)^2 = 1$$

1	γ ₅	γ ^μ	γ ₅ γ ^μ	σ ^{μν}
γ ₅	1	+γ ₅ γ ^μ	+γ ^μ	$\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$
γ ^α	-γ ₅ γ ^α	g ^{αμ} - iσ ^{αμ}	- $\frac{1}{2}(2g^{\alpha\mu}\gamma_5 + \epsilon^{\alpha\mu\alpha'}\sigma_{\alpha'\beta})$	$\epsilon^{\alpha\mu\nu\lambda}\gamma_\lambda + i g^{\alpha\mu}\gamma^\nu - i g^{\alpha\nu}\gamma^\mu$
γ ₅ γ ^α	-γ ^α	-γ ^α	-(g ^{αμ} - iσ ^{αμ})	$\epsilon^{\alpha\mu\nu\lambda}\gamma_\lambda + i g^{\alpha\mu}\gamma^\nu - i g^{\alpha\nu}\gamma^\mu$
σ ^{αβ}	σ ^{αβ}	$\epsilon^{\alpha\beta\mu\lambda}\gamma_5\gamma_\lambda + i g^{\alpha\beta}\gamma^\mu - i g^{\mu\alpha}\gamma^\beta$	$\epsilon^{\alpha\beta\mu\lambda}\gamma_\lambda + i g^{\alpha\beta}\gamma_5\gamma^\mu - i g^{\alpha\mu}\gamma_5\gamma^\beta$	$i\epsilon^{\alpha\beta\mu\nu}\gamma_5 + g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu} + \frac{1}{4}[g^{\alpha\mu}g^{\beta\nu} + g^{\beta\mu}g^{\alpha\nu} - g^{\alpha\nu}g^{\beta\mu} - g^{\beta\nu}g^{\alpha\mu}]$

$$\begin{aligned} \not{A} &= A^\mu \gamma_\mu & \gamma^\mu \gamma_\mu &= 4 & \gamma^\mu \gamma^\nu \gamma_\mu &= -2\gamma^\nu & \gamma^\mu \gamma^\nu \gamma^\rho \gamma_\mu &= 4g^{\mu\nu} \\ \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_\mu &= -2\gamma^\nu \gamma^\rho \gamma^\sigma & \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\pi \gamma_\mu &= 2[\gamma^\pi \gamma^\nu \gamma^\rho \gamma^\sigma + \gamma^\sigma \gamma^\rho \gamma^\nu \gamma^\pi] \end{aligned}$$

Helicity & Chirality

$$Y_L = \frac{1 - \gamma_5}{2} \quad Y_R = \frac{1 + \gamma_5}{2} \quad Y_{L,R}^2 = Y_{L,R} \quad Y_L R Y_R L = 0 \quad Y_L + Y_R = 1$$

$$\gamma^\mu Y_{L,R} = Y_{R,L} \gamma^\mu \quad \gamma_5 Y_L = Y_L \gamma_5 = -Y_L \quad \gamma_5 Y_R = Y_R \gamma_5 = Y_R \quad \overline{Y_{L,R}} = Y_{0Y_{L,R}} \gamma_0 = Y_{R,L}$$

$$\begin{array}{llll} P_L u_L = u_L & P_L u_T = 0 & P_R u_L = 0 & P_R u_T = u_T \\ Y_L u_L = u_L & Y_L u_R = 0 & Y_R u_L = 0 & Y_R u_R = u_R \\ P_L v_L = 0 & P_L v_T = v_T & P_R v_L = v_L & P_R v_T = 0 \\ Y_L v_L = v_L & Y_L v_R = 0 & Y_R v_L = 0 & Y_R v_R = v_R \end{array} \quad \begin{array}{llll} \overline{u}_L P_L = 0 & \overline{u}_T P_L = u_T & \overline{u}_L P_R = u_L & \overline{u}_T P_R = 0 \\ \overline{u}_L Y_L = 0 & \overline{u}_R Y_L = u_R & \overline{u}_L Y_R = u_L & \overline{u}_R Y_R = 0 \\ \overline{v}_L P_L = v_L & \overline{v}_T P_L = 0 & \overline{v}_L P_R = 0 & \overline{v}_T P_R = v_T \\ \overline{v}_L Y_L = 0 & \overline{v}_R Y_L = v_R & \overline{v}_L Y_R = v_L & \overline{v}_R Y_R = 0 \end{array}$$

Traces

$$\text{Tr}[1] = 4 \quad \text{Tr}[\underbrace{\gamma^\mu \gamma^\nu \dots \gamma^\rho}_{\text{Odd Number}}] = 0 \quad \text{Tr}[\gamma^\mu \gamma^\nu] = 4g^{\mu\nu}$$

$$\text{Tr}[\gamma_5] = 0 \quad \text{Tr}[\gamma_5 \underbrace{\gamma^\mu \gamma^\nu \dots \gamma^\rho}_{\text{Odd Number}}] = 0 \quad \text{Tr}[\gamma_5 \gamma^\mu \gamma^\nu] = 0$$

$$\text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) \quad \text{Tr}[\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] = -4i\epsilon^{\mu\nu\rho\sigma}$$

$$\text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_L] = 2(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} + i\epsilon^{\mu\nu\rho\sigma})$$

$$\text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_R] = 2(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - i\epsilon^{\mu\nu\rho\sigma})$$

$$\text{Tr}[(\not{p}_a + m_a)\gamma^\mu(\not{p}_b + m_b)\gamma^\nu] = 4[p_a^\mu p_b^\nu + p_b^\mu p_a^\nu + (m_a m_b - p_a \cdot p_b)g^{\mu\nu}]$$

Fermions

$$:a(x)b(x'): = a(x)b(x') - b(x')a(x)$$

$$\mathbf{T}[a(x)b(x')] = \theta(t-t')a(x)b(x') - \theta(t'-t)b(x')a(x)$$

$$\{b^\dagger(p,s), b(p',s')\} = \{d^\dagger(p,s), d(p',s')\} = \tilde{\delta}(p-p')\delta_{ss'}$$

$$\psi(x) = \psi^+(x) + \psi^-(x) = \int d\tilde{p} \sum_s [b(p,s)u(p,s)e^{-ip \cdot x} + d^\dagger(p,s)v(p,s)e^{+ip \cdot x}]$$

$$\bar{\psi}(x) = \bar{\psi}^+(x) + \bar{\psi}^-(x) = \int d\tilde{p} \sum_s [d(p,s)\bar{v}(p,s)e^{-ip \cdot x} + b^\dagger(p,s)\bar{u}(p,s)e^{+ip \cdot x}]$$

$$\psi^+(b) \text{ destroys } e^- \quad \psi^-(d^\dagger) \text{ creates } e^+ \quad \bar{\psi}^+(d) \text{ destroy } e^+ \quad \bar{\psi}^-(b^\dagger) \text{ creates } e^-$$

Spinors (Fermions)

$$\psi = ue^{+i(\vec{p}\cdot\vec{x}-Et)} = v e^{-i(\vec{p}\cdot\vec{x}-Et)} \quad (\not{p}-m)u = \bar{u}(\not{p}-m) = 0 \quad (\not{p}+m)v = \bar{v}(\not{p}+m) = 0$$

$$\bar{u}^\dagger(p)u^s(p) = +2m\delta^{rs} \quad \bar{u}^\dagger(p)v^s(p) = 0 \quad u^{\dagger\dagger}(p)u^s(p) = 2E\delta^{rs} \quad \sum_{s=1,2} u^s(p)\bar{u}^s(p) = \not{p} + m$$

$$\bar{v}^\dagger(p)v^s(p) = -2m\delta^{rs} \quad \bar{v}^\dagger(p)u^s(p) = 0 \quad u^{\dagger\dagger}(p)u^s(p) = 2E\delta^{rs} \quad \sum_{s=1,2} v^s(p)\bar{v}^s(p) = \not{p} - m$$

Bosons

$$:a(x)b(x'): = a(x)b(x') + b(x')a(x)$$

$$\mathbf{T}[a(x)b(x')] = \theta(t-t')a(x)b(x') + \theta(t'-t)b(x')a(x)$$

$$[\phi(\vec{\mathbf{x}},t), \phi(\vec{\mathbf{y}},t)] = [\Pi(\vec{\mathbf{x}},t), \Pi(\vec{\mathbf{y}},t)] = 0 \quad [\phi(\vec{\mathbf{x}},t), \Pi(\vec{\mathbf{y}},t)] = i\delta^3(\vec{\mathbf{x}} - \vec{\mathbf{y}})$$

$$[a^\dagger(k,\lambda), a(k',\lambda')] = g^{\lambda\lambda'}\delta(k-k')$$

$$A_\mu(x) = A_\mu^+(x) + A_\mu^-(x) = \int d\vec{k} \sum_{\lambda=0}^3 [a(k,\lambda)\epsilon_\mu(k,\lambda)e^{-ik \cdot x} + a^\dagger(k,\lambda)\epsilon_\mu^*(k,\lambda)e^{+ik \cdot x}]$$

$$A^+(a) \text{ destroys } \gamma \quad A^-(a^\dagger) \text{ creates } \gamma$$

Polarization Vectors (Bosons)

External massless & Massive:

$$\sum_{\lambda=1}^2 \epsilon^\mu(k,\lambda) \cdot \epsilon^{*\nu}(k,\lambda) = -g^{\mu\nu} \quad \sum_{\lambda=1}^3 \epsilon_\mu(q,\lambda)\epsilon_\nu^*(q,\lambda) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2}$$

Virtual massless:

$$\epsilon(k,\lambda) \cdot \epsilon^*(k,\lambda') = g^{\lambda\lambda'} \quad \sum_\lambda g^{\lambda\lambda'} \epsilon^\mu(k,\lambda) \cdot \epsilon^{*\nu}(k,\lambda) = g^{\mu\nu}$$

Decay : Decay Rates

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2} \frac{|\vec{\mathbf{p}}_{\text{CM}}|}{m_0^2} |\mathcal{M}|^2 \quad |\vec{\mathbf{p}}_{\text{CM}}| = \frac{1}{2m_0} \sqrt{[m_0^2 - (m_1 + m_2)^2][m_0^2 - (m_1 - m_2)^2]}$$

Scattering : Cross Sections

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{\mathbf{p}}_3|}{|\vec{\mathbf{p}}_1|} |\overline{\mathcal{M}}|^2 \quad |\vec{\mathbf{p}}| = \frac{1}{2\sqrt{s}} \sqrt{[s - (m_a + m_b)^2][s - (m_a - m_b)^2]}$$

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \approx +2p_1 \cdot p_2 \approx +2p_3 \cdot p_4 \quad \sqrt{s} = E_{CM}$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \approx -2p_1 \cdot p_3 \approx -2p_2 \cdot p_4 \quad s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \approx -2p_1 \cdot p_4 \approx -2p_2 \cdot p_3$$

$$f + \bar{f} \longrightarrow g + \bar{g}:$$

$$p_1 = \frac{\sqrt{s}}{2}(1, 0, 0, +\beta_f) \quad p_2 = \frac{\sqrt{s}}{2}(1, 0, 0, -\beta_f)$$

$$p_3 = \frac{\sqrt{s}}{2}(1, +\beta_g \sin \theta, 0, +\beta_g \cos \theta) \quad p_4 = \frac{\sqrt{s}}{2}(1, -\beta_g \sin \theta, 0, -\beta_g \cos \theta)$$

$$m \approx 0 : \quad t = -\frac{s}{2}(1 - \cos \theta) \quad u = -\frac{s}{2}(1 + \cos \theta)$$

Feynman Rules for $i\mathcal{M}$

Goes in opposite way of arrows with the first one being adjoint, $\bar{\psi} = \psi^\dagger \gamma^0$:

$$\begin{array}{llll} f \longrightarrow \text{u} \bullet & \bullet \longrightarrow \text{u} & \bar{f} \longleftarrow \text{v} \bullet & \bullet \longleftarrow \text{v} \bar{f} \\ \gamma \text{ wavy } \epsilon_\mu & \bullet \text{ wavy } \epsilon_\mu & g \text{ curly } \epsilon_\mu & \bullet \text{ curly } \epsilon_\mu \end{array}$$

$$\begin{array}{llll} f \longrightarrow \frac{i}{\gamma^\mu q_\mu - m} \bullet & \bullet \text{ wavy } \frac{-ig_{\mu\nu}}{q^2} \bullet & \bullet \text{ curly } \frac{-ig_{\mu\nu}}{q^2} \delta^{ab} \bullet & \bullet \text{ wavy } \frac{-i(g_{\mu\nu} - q_\mu q_\nu / m^2)}{q^2 - m^2 + im\Gamma} \bullet \\ & & & \bullet \text{ dashed } \frac{i}{q^2 - m^2} \bullet \end{array}$$

$$\begin{array}{ll} \gamma \text{ wavy } \bullet \begin{array}{l} \nearrow \bar{f} \\ \searrow f \end{array} \text{ } -iQ_f e \gamma^\mu & g \text{ curly } \bullet \begin{array}{l} \nearrow \bar{q} \\ \searrow q \end{array} \text{ } -ig_s \frac{\gamma^a}{2} \lambda^a \gamma^\mu \end{array}$$

$$\begin{array}{lll} W^\pm \text{ wavy } \bullet \begin{array}{l} \nearrow t^\pm \\ \searrow \nu_\ell \end{array} \text{ } -i\frac{g_W}{\sqrt{2}} \gamma^\mu Y_L & W^\pm \text{ wavy } \bullet \begin{array}{l} \nearrow q_i \\ \searrow q_j \end{array} \text{ } -i\frac{g_W}{\sqrt{2}} \gamma^\mu Y_{ij} & Z \text{ wavy } \bullet \begin{array}{l} \nearrow \bar{f} \\ \searrow f \end{array} \text{ } -ig_Z (g_V^f - g_A^f \gamma_5) \end{array}$$

$$\begin{array}{lll} h \text{ dashed } \bullet \begin{array}{l} \nearrow \bar{f} \\ \searrow f \end{array} \text{ } -i\frac{g_W}{2} \frac{m_f}{m_W} & h \text{ dashed } \bullet \begin{array}{l} \nearrow W^+ \\ \searrow W^- \end{array} \text{ } i g_W m_W g_{\mu\nu} & h \text{ dashed } \bullet \begin{array}{l} \nearrow Z \\ \searrow Z \end{array} \text{ } i g_Z m_Z g_{\mu\nu} \end{array}$$

Constants

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} = \frac{g_W^2}{8M_Z^2 \cos^2(\theta_W)} = \frac{1}{2v^2} \qquad g_W = g_Z \cos \theta_W = \frac{e}{\sin \theta_W}$$
$$g_V^f = \frac{1}{2} T_f^3 - Q_f \sin^2(\theta_W) \qquad g_A^f = \frac{1}{2} T_f^3 \qquad M_W = M_Z \cos \theta_W = \frac{1}{2} g_W v \qquad m_H = \sqrt{2} \lambda v$$

Fermions	Q_f	$I_W^{(3)}$	Y_L	Y_R	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	+1/2	−1	0	+1/2	0	+1/2	+1/2
e^-, μ^-, τ^-	−1	−1/2	−1	−2	−0.27	+0.23	−0.04	−1/2
u, c, t	+2/3	+1/2	+1/3	+4/3	+0.35	−0.15	+0.19	+1/2
d, s, b	−1/3	−1/2	+1/3	−2/3	−0.42	+0.08	−0.35	−1/2

Relativity

$$E^2 = m^2 + |p|^2 \qquad \beta = \frac{p}{E}$$

$$\beta = \tanh(\eta) \qquad \gamma = \cosh(\eta) \qquad \gamma \beta = \sin(\eta)$$

$$x^\mu = (t, \vec{\mathbf{x}}) \qquad p^\mu = (E, \vec{\mathbf{p}}) \qquad \partial^\mu = \left(\partial_t, -\vec{\nabla}\right) \qquad A^\mu = \left(\phi, \vec{\mathbf{A}}\right) \qquad J^\mu = \left(\rho, \vec{\mathbf{J}}\right)$$
$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{If } (\mu\nu\rho\sigma) \text{ is an even permutation of } (0123) \\ -1 & \text{If } (\mu\nu\rho\sigma) \text{ is an odd permutation of } (0123) \\ 0 & \text{Otherwise} \end{cases}$$

$$\begin{aligned} \epsilon_{\alpha\beta_1\gamma_1\delta_1}\epsilon^{\alpha\beta_2\gamma_2\delta_2} &= +g_{\beta_1}^{\beta_2}g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\gamma_2} - g_{\beta_1}^{\beta_2}g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\delta_2} + g_{\beta_1}^{\gamma_2}g_{\gamma_1}^{\beta_2}g_{\delta_1}^{\delta_2} - g_{\beta_1}^{\gamma_2}g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\beta_2} \\ &\qquad + g_{\beta_1}^{\delta_2}g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\beta_2} - g_{\beta_1}^{\delta_2}g_{\gamma_1}^{\beta_2}g_{\delta_1}^{\gamma_2} \\ \epsilon_{\alpha\beta\gamma\delta_1}\epsilon^{\alpha\beta\gamma_2\delta_2} &= -2\Big(g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\delta_2} - g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\gamma_2}\Big) \qquad \epsilon_{\alpha\beta\gamma\delta_1}\epsilon^{\alpha\beta\gamma\delta_2} = -6g_{\delta_1}^{\delta_2} \end{aligned}$$

Quantum Mechanics

$$E \rightarrow i\frac{\partial}{\partial t} \qquad \vec{\mathbf{p}} \rightarrow -i\vec{\nabla}.$$