# Equations

Schrödinger:

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t}=H|\psi t\rangle \qquad \mathscr{L}=i\psi^*\dot{\psi}-\frac{1}{2m}\vec{\nabla}\psi^*\cdot\vec{\nabla}\psi$$

Klein-Gordon (Real Scalar Field):

$$(\partial\cdot\partial+m^2)\phi=0 \qquad \mathscr{L}=\frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi-m^2\phi^2\right]=\frac{1}{2}\left[\dot{\phi}^2-\left(\vec{\nabla}\phi\right)^2-m^2\phi^2\right]$$

Dirac (Complex Scalar Field):

$$(i\not{\partial}-m)\varphi=0 \qquad \overline{\varphi}(i\not{\partial}+m)=0 \qquad \mathscr{L}=\partial_\mu\varphi\partial^\mu\varphi^*-m^2\varphi\varphi^*$$

# Pauli Matrices

$$\sigma^\mu=(\mathbb{1},\vec{\sigma}) \qquad \overline{\sigma}^\mu=(\mathbb{1},-\vec{\sigma})$$

$$\sigma^1=\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix} \quad \sigma^2=\sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix} \quad \sigma^3=\sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix} \quad \vec{\mathbf{\Sigma}}=\begin{pmatrix}\vec{\sigma}&0\\0&\vec{\sigma}\end{pmatrix}$$

$$\left\{\sigma^i,\sigma^j\right\}=2\delta^{ij} \qquad \left[\sigma^i,\sigma^j\right]=2i\epsilon^{ijk}\sigma^k \qquad \sigma^i\sigma^j=\delta^{ij}+i\epsilon^{ijk}\sigma^k \qquad \sigma^2\sigma^i\sigma^2=-(\sigma^i)^*$$

$$\vec{\sigma}\cdot\vec{\mathbf{p}}=\begin{pmatrix}p_z&p_x-ip_y\\p_x+ip_y&-p_z\end{pmatrix} \qquad \left(\vec{\sigma}\cdot\vec{\mathbf{p}}\right)^2=\left|\vec{\mathbf{p}}\right|^2 \qquad (p\cdot\sigma)(p\cdot\overline{\sigma})=p^2$$

$$p\cdot\overline{\sigma}=\begin{pmatrix}p^0+p^3&p^1-ip^2\\p^1+ip^2&p^0-p^3\end{pmatrix} \qquad p\cdot\sigma=\begin{pmatrix}p^0-p^3&-(p^1-ip^2)\\-(p^1+ip^2)&p^0+p^3\end{pmatrix}$$

$$\sqrt{p\cdot\sigma}=\frac{E+m-\vec{\sigma}\cdot\vec{\mathbf{p}}}{\sqrt{2(E+m)}} \qquad \sqrt{p\cdot\overline{\sigma}}=\frac{E+m+\vec{\sigma}\cdot\vec{\mathbf{p}}}{\sqrt{2(E+m)}}$$

# Dirac $\gamma$-Matrices

$$\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu} \qquad \gamma_5=+i\gamma^0\gamma^1\gamma^2\gamma^3=-i\gamma_0\gamma_1\gamma_2\gamma_3=-\frac{i}{4!}\epsilon^{\alpha\beta\gamma\delta}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta$$

$$\sigma^{\mu\nu}=\frac{i}{2}[\gamma^\mu,\gamma^\nu]\implies[\gamma^\mu,\gamma^\nu]=-2i\sigma^{\mu\nu} \qquad \{\gamma_5,\gamma^\mu\}=0 \qquad \gamma^0(\gamma^\mu)^\dagger\gamma^0=\gamma^\mu$$

$$\left(\gamma^0\right)^\dagger=\gamma^0 \quad \left(\gamma^0\right)^2=\mathbb{1} \qquad \left(\gamma^k\right)^\dagger=-\gamma^k \quad \left(\gamma^k\right)^2=-\mathbb{1} \qquad \left(\gamma^5\right)^\dagger=\gamma^5 \quad \left(\gamma^5\right)^2=\mathbb{1}$$

$$\overline{\Gamma}=\gamma^0\Gamma^\dagger\gamma^0 \qquad \overline{\gamma_5}=-\gamma_5 \qquad \overline{\gamma^\mu}=\gamma^\mu \qquad \overline{\gamma^\mu\gamma_5}=\gamma^\mu\gamma_5 \qquad \overline{\sigma^{\mu\nu}}=\sigma^{\mu\nu}$$

| $\mathbb{1}$ | $\gamma_5$ | $\gamma^\mu$ | $\gamma_5\gamma^\mu$ | $\sigma^{\mu\nu}$ |
|---|---|---|---|---|
| $\gamma_5$ | $\mathbb{1}$ | $+\gamma_5\gamma^\mu$ | $+\gamma^\mu$ | $\frac{i}{2}\epsilon^{\mu\nu\pi\rho}\sigma_{\pi\rho}$ |
| $\gamma^\alpha$ | $-\gamma_5\gamma^\alpha$ | $g^{\alpha\mu}-i\sigma^{\alpha\mu}$ | $-\frac{1}{2}\left(2g^{\alpha\mu}\gamma_5+\epsilon^{\alpha\mu\pi\rho}\sigma_{\pi\rho}\right)$ | $\epsilon^{\alpha\mu\nu\lambda}\gamma_5\gamma_\lambda+ig^{\alpha\mu}\gamma^\nu-ig^{\alpha\nu}\gamma^\mu$ |
| $\gamma_5\gamma^\alpha$ | $-\gamma^\alpha$ | $-\gamma^\alpha$ | $-(g^{\alpha\mu}-i\sigma^{\alpha\mu})$ | $\epsilon^{\alpha\mu\nu\lambda}\gamma_\lambda+ig^{\alpha\mu}\gamma_5\gamma^\nu-ig^{\alpha\nu}\gamma_5\gamma^\mu$ |
| $\sigma^{\alpha\beta}$ | $\sigma^{\alpha\beta}$ | $\epsilon^{\alpha\beta\mu\lambda}\gamma_5\gamma_\lambda+ig^{\beta\mu}\gamma^\alpha-ig^{\alpha\mu}\gamma^\beta$ | $\epsilon^{\alpha\beta\mu\lambda}\gamma_\lambda+ig^{\beta\mu}\gamma_5\gamma^\alpha-ig^{\alpha\mu}\gamma_5\gamma^\beta$ | $i\epsilon^{\alpha\beta\mu\nu}\gamma_5+g^{\alpha\mu}g^{\beta\nu}-g^{\alpha\nu}g^{\beta\mu}+i\left[g^{\alpha\nu}\sigma^{\beta\mu}+g^{\beta\mu}\sigma^{\alpha\nu}-g^{\alpha\mu}\sigma^{\beta\nu}-g^{\beta\nu}\sigma^{\alpha\mu}\right]$ |

$$\not{A}=A^\mu\gamma_\mu \qquad \gamma^\mu\gamma_\mu=4 \qquad \gamma^\mu\gamma^\nu\gamma_\mu=-2\gamma^\nu \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu=4g^{\nu\rho}$$

$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu=-2\gamma^\sigma\gamma^\rho\gamma^\nu \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\pi\gamma_\mu=2[\gamma^\pi\gamma^\nu\gamma^\rho\gamma^\sigma+\gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\pi]$$

$$\gamma^\mu\gamma^\nu\gamma^\rho=i\epsilon^{\mu\nu\rho\lambda}\gamma_\lambda\gamma_5+g^{\mu\nu}\gamma^\rho-g^{\mu\rho}\gamma^\nu+g^{\nu\rho}\gamma^\mu$$

# Spin, Helicity and Chirality

$$\vec{\mathbf{S}}=\frac{1}{2}\vec{\mathbf{\Sigma}} \qquad \tilde{h}=\frac{\vec{\mathbf{S}}\cdot\vec{\mathbf{p}}}{|\vec{\mathbf{p}}|}=\frac{\vec{\mathbf{\Sigma}}\cdot\vec{\mathbf{p}}}{2|\vec{\mathbf{p}}|} \qquad \tilde{h}=2h=\frac{\vec{\mathbf{\Sigma}}\cdot\vec{\mathbf{p}}}{|\vec{\mathbf{p}}|} \qquad P_L=\frac{1-h}{2} \qquad P_R=\frac{1+h}{2}$$

$$\gamma_L=\frac{1-\gamma_5}{2} \qquad \gamma_R=\frac{1+\gamma_5}{2} \qquad \gamma_{L,R}^2=\gamma_{L,R} \qquad \gamma_{L,R}\gamma_{R,L}=0 \qquad \gamma_L+\gamma_R=\mathbb{1}$$

$$\gamma^\mu\gamma_{L,R}=\gamma_{R,L}\gamma^\mu \qquad \gamma_5\gamma_L=\gamma_L\gamma_5=-\gamma_L \qquad \gamma_5\gamma_R=\gamma_R\gamma_5=\gamma_R \qquad \gamma_{L,R}^\dagger=\gamma_{L,R}$$

$$\overline{\gamma^\mu\gamma_{L,R}}=\gamma^\mu\gamma_{L,R} \qquad \overline{\gamma_5\gamma_L}=-\gamma_R \qquad \overline{\gamma_5\gamma_R}=\gamma_L \qquad \overline{\gamma_{L,R}}=\gamma_0\gamma_{L,R}^\dagger\gamma_0=\gamma_{R,L}$$

$$u_L=u_\downarrow \qquad u_R=u_\uparrow \qquad v_L=v_\uparrow \qquad v_R=v_\downarrow$$

$$P_Lu_\downarrow=u_\downarrow \quad P_Lu_\uparrow=0 \quad P_Ru_\downarrow=0 \quad P_Ru_\uparrow=u_\uparrow \quad \overline{u_\downarrow}P_L=0 \quad \overline{u_\uparrow}P_L=u_\uparrow \quad \overline{u_\downarrow}P_R=u_\downarrow \quad \overline{u_\uparrow}P_R=0$$

$$\gamma_Lu_L=u_L \quad \gamma_Lu_R=0 \quad \gamma_Ru_L=0 \quad \gamma_Ru_R=u_R \quad \overline{u_L}\gamma_L=0 \quad \overline{u_R}\gamma_L=u_R \quad \overline{u_L}\gamma_R=u_L \quad \overline{u_R}\gamma_R=0$$

$$P_Lv_\downarrow=0 \quad P_Lv_\uparrow=v_\uparrow \quad P_Rv_\downarrow=v_\downarrow \quad P_Rv_\uparrow=0 \quad \overline{v_\uparrow}P_L=v_\downarrow \quad \overline{v_\uparrow}P_L=0 \quad \overline{v_\downarrow}P_R=0 \quad \overline{v_\uparrow}P_R=v_\uparrow$$

$$\gamma_Lv_L=v_L \quad \gamma_Lv_R=0 \quad \gamma_Rv_L=0 \quad \gamma_Rv_R=v_R \quad \overline{v_L}\gamma_L=0 \quad \overline{v_R}\gamma_L=v_R \quad \overline{v_L}\gamma_R=v_L \quad \overline{v_R}\gamma_R=0$$

$$u_\uparrow=N\begin{pmatrix}c\\se^{i\phi}\\kc\\kse^{i\phi}\end{pmatrix}\approx N\begin{pmatrix}c\\se^{i\phi}\\c\\se^{i\phi}\end{pmatrix} \qquad u_\downarrow=N\begin{pmatrix}-s\\ce^{i\phi}\\ks\\-kce^{i\phi}\end{pmatrix}\approx N\begin{pmatrix}-s\\ce^{i\phi}\\s\\-ce^{i\phi}\end{pmatrix}$$

$$v_\uparrow=N\begin{pmatrix}ks\\-kce^{i\phi}\\-s\\ce^{i\phi}\end{pmatrix}\approx N\begin{pmatrix}s\\-ce^{i\phi}\\-s\\ce^{i\phi}\end{pmatrix} \qquad v_\downarrow=N\begin{pmatrix}kc\\kse^{i\phi}\\c\\se^{i\phi}\end{pmatrix}\approx N\begin{pmatrix}c\\se^{i\phi}\\c\\se^{i\phi}\end{pmatrix}$$

$$\vec{\mathbf{p}}=p(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)$$

$$N=\sqrt{E+m}\approx\sqrt{E} \qquad k=\frac{p}{E+m}\approx 1 \qquad s=\sin\left(\frac{\theta}{2}\right) \qquad c=\cos\left(\frac{\theta}{2}\right)$$

# Traces

$$\mathrm{Tr}[\mathbb{1}]=4 \qquad \mathrm{Tr}[\underbrace{\gamma^\mu\gamma^\nu\dots\gamma^\rho}_{\text{Odd Number}}]=0 \qquad \mathrm{Tr}[\gamma^\mu\gamma^\nu]=4g^{\mu\nu}$$

$$\mathrm{Tr}[\gamma_5]=0 \qquad \mathrm{Tr}[\gamma_5\underbrace{\gamma^\mu\gamma^\nu\dots\gamma^\rho}_{\text{Odd Number}}]=0 \qquad \mathrm{Tr}[\gamma_5\gamma^\mu\gamma^\nu]=0$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma]=4(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}) \qquad \mathrm{Tr}[\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma]=-4i\epsilon^{\mu\nu\rho\sigma}$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_L]=2(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}+i\epsilon^{\mu\nu\rho\sigma})$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_R]=2(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}-i\epsilon^{\mu\nu\rho\sigma})$$

$$\mathrm{Tr}\left[(\not{p}_a+m_a)\gamma^\mu(\not{p}_b+m_b)\gamma^\nu\right]=4\left[p_a^\mu p_b^\nu+p_b^\mu p_a^\nu+(m_am_b-p_a\cdot p_b)g^{\mu\nu}\right]$$

$$\mathrm{Tr}\left[\not{a}\gamma^\mu\not{b}\gamma^\nu\right]\mathrm{Tr}\left[\not{c}\gamma_\mu\not{d}\gamma_\nu\gamma_5\right]=0 \qquad \mathrm{Tr}\left[\not{a}\gamma^\mu\not{b}\gamma^\nu\gamma_L\right]\mathrm{Tr}\left[\not{c}\gamma_\mu\not{d}\gamma_\nu\gamma_R\right]=16(a\cdot d)(b\cdot c)$$

$$\mathrm{Tr}\left[\not{a}\gamma^\mu\not{b}\gamma^\nu\gamma_L\right]\mathrm{Tr}\left[\not{c}\gamma_\mu\not{d}\gamma_\nu\gamma_L\right]=\mathrm{Tr}\left[\not{a}\gamma^\mu\not{b}\gamma^\nu\gamma_L\right]\mathrm{Tr}\left[\not{c}\gamma_\mu\not{d}\gamma_\nu\gamma_L\right]=16(a\cdot c)(b\cdot d)$$

# Spinors (Fermions)

$$\psi=ue^{+i(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}}-E\cdot t)}=ve^{-i(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}}-E\cdot t)} \qquad \overline{\psi}=\psi^\dagger\gamma^0$$

$$(\not{p}-m)u=\overline{u}(\not{p}-m)=0 \qquad (\not{p}+m)v=\overline{v}(\not{p}+m)=0$$

$$\overline{u}^r(p)u^s(p)=+2m\delta^{rs} \qquad \overline{u}^r(p)v^s(p)=0 \qquad u^{r\dagger}(p)u^s(p)=2E\delta^{rs} \qquad \sum_{s=1,2}u^s(p)\overline{u}^s(p)=\not{p}+m$$

$$\overline{v}^r(p)v^s(p)=-2m\delta^{rs} \qquad \overline{v}^r(p)u^s(p)=0 \qquad v^{r\dagger}(p)v^s(p)=2E\delta^{rs} \qquad \sum_{s=1,2}v^s(p)\overline{v}^s(p)=\not{p}-m$$

# Polarization Vectors (Bosons)

External massless & Massive:

$$\sum_{\lambda=1}^{2}\epsilon^\mu(k,\lambda)\cdot\epsilon^{*\nu}(k,\lambda)=-g^{\mu\nu} \qquad \sum_{\lambda=1}^{3}\epsilon_\mu(q,\lambda)\epsilon_\nu^*(q,\lambda)=-g_{\mu\nu}+\frac{q_\mu q_\nu}{M^2}$$

Virtual massless:

$$\epsilon(k,\lambda)\cdot\epsilon^*(k,\lambda')=g^{\lambda\lambda'} \qquad \sum_\lambda g^{\lambda\lambda}\epsilon^\mu(k,\lambda)\cdot\epsilon^{*\nu}(k,\lambda)=g^{\mu\nu}$$

# Quantization : Real Scalar Field (Bosons)

$$:\frac{1}{2}\left[a^\dagger(k)a(k)+a(k)a^\dagger(k)\right]:=a^\dagger(k)a(k) \qquad :a(x)b(x'):=a(x)b(x')+b(x')a(x)$$

$$\mathbf{T}(a(x)b(x'))=\theta(t-t')a(x)b(x')+\theta(t'-t)b(x')a(x)$$

The "+" corresponds to positive frequency plane waves $e^{-ik\cdot x}$:

$$\mathscr{L}=\frac{1}{2}\partial^\mu\phi\partial_\mu\phi-\frac{1}{2}m^2\phi\phi \qquad \pi=\dot{\phi} \qquad \left[a^\dagger(k,\lambda),a(k',\lambda')\right]=g^{\lambda\lambda'}\tilde{\delta}(k-k')$$

$$\left[\phi(\vec{\mathbf{x}},t),\phi(\vec{\mathbf{y}},t)\right]=\left[\Pi(\vec{\mathbf{x}},t),\Pi(\vec{\mathbf{y}},t)\right]=0 \qquad \left[\phi(\vec{\mathbf{x}},t),\Pi(\vec{\mathbf{y}},t)\right]=i\delta^3(\vec{\mathbf{x}}-\vec{\mathbf{y}})$$

$$\phi(x)=\int\widetilde{\mathrm{d}k}\left[a(k)e^{-ik\cdot x}+a^\dagger(k)e^{+ik\cdot x}\right]$$

$$|p\rangle=a^\dagger(p)\,|0\rangle \qquad |p_1,p_2\rangle=a^\dagger(p_2)a^\dagger(p_1)\,|0\rangle=|p_2,p_1\rangle$$

# Quantization : Complex Scalar Field (Bosons)

$$\mathscr{L}=:\partial^\mu\varphi^\dagger\partial_\mu\varphi-m^2\varphi^\dagger\varphi: \qquad \pi=\dot{\varphi}^\dagger \qquad \pi^\dagger=\dot{\varphi} \qquad \left[a_\pm(k),a_\pm^\dagger(k')\right]=\tilde{\delta}(k-k')$$

$$\left[\varphi(\vec{\mathbf{x}},t),\pi(\vec{\mathbf{y}},t)\right]=\left[\varphi^\dagger(\vec{\mathbf{x}},t),\pi^\dagger(\vec{\mathbf{y}},t)\right]=i\,\delta^3(\vec{\mathbf{x}}-\vec{\mathbf{y}})$$

$$\varphi(x)=\varphi^+(x)+\varphi^-(x)=\int\widetilde{\mathrm{d}k}\left[a_+(k)e^{-ik\cdot x}+a_-^\dagger(k)e^{+ik\cdot x}\right]$$

$$\varphi^\dagger(x)=\varphi^{+\dagger}(x)+\varphi^{-\dagger}(x)=\int\widetilde{\mathrm{d}k}\left[a_-(k)e^{-ik\cdot x}+a_+^\dagger(k)e^{+ik\cdot x}\right]$$

$$\left|p^+\right\rangle=a_+^\dagger(p)\,|0\rangle \qquad \left|p^-\right\rangle=a_-^\dagger(p)\,|0\rangle \qquad \left|p_1^+,p_2^-\right\rangle=a_-^\dagger(p_2)a_+^\dagger(p_1)\,|0\rangle$$

# Quantization : Dirac Field (Fermions)

$$:a(x)b(x'):=a(x)b(x')-b(x')a(x)$$

$$\mathbf{T}(a(x)b(x'))=\theta(t-'t)a(x)b(x')-\theta(t'-t)b(x')a(x)$$

$$\mathscr{L}=:i\overline{\psi}\gamma^\mu\partial_\mu\psi-m\overline{\psi}\psi: \qquad \pi_\alpha=\frac{\partial\mathscr{L}}{\partial\dot{\psi}_\alpha}=i\psi_\alpha^\dagger \qquad \pi_\alpha^\dagger=\frac{\partial\mathscr{L}}{\partial\dot{\psi}_\alpha^\dagger}=0$$

$$\left\{b^\dagger(p,s),b(p',s')\right\}=\left\{d^\dagger(p,s),d(p',s')\right\}=\tilde{\delta}(p-p')\delta_{ss'}$$

$$\psi(x)=\psi^+(x)+\psi^-(x)=\int\widetilde{\mathrm{d}p}\sum_s\left[b(p,s)u(p,s)e^{-ip\cdot x}+d^\dagger(p,s)v(p,s)e^{+ip\cdot x}\right]$$

$$\overline{\psi}(x)=\overline{\psi}^+(x)+\overline{\psi}^-(x)=\int\widetilde{\mathrm{d}p}\sum_s\left[d(p,s)\overline{v}(p,s)e^{-ip\cdot x}+b^\dagger(p,s)\overline{u}(p,s)e^{+ip\cdot x}\right]$$

$$\left|e^-(p_1,s_1),e^-(p_2,s_2)\right\rangle=b^\dagger(p_2,s_2)b^\dagger(p_1,s_1)\,|0\rangle=-\left|e^-(p_2,s_2),e^-(p_1,s_1)\right\rangle$$

$$\psi^+(x)\,|p\rangle=\psi^+(x)b^\dagger(p)\,|0\rangle=|0\rangle\,u(p)e^{-ip\cdot x} \qquad \psi^+(b)\text{ destroys }\mathrm{e}^-$$

$$\overline{\psi}^+(x)\,|p\rangle=\overline{\psi}^+(x)d^\dagger(p)\,|0\rangle=|0\rangle\,\overline{v}(p)e^{-ip\cdot x} \qquad \overline{\psi}^+(d)\text{ destroy }\mathrm{e}^+$$

$$\langle p|\,\psi^-(x)=\langle 0|\,d(p)\psi^-(x)=v(p)e^{+ip\cdot x}\,\langle 0| \qquad \psi^-(d^\dagger)\text{ creates }\mathrm{e}^+$$

$$\langle p|\,\overline{\psi}^-(x)=\langle 0|\,b(p)\overline{\psi}^-(x)=\overline{u}(p)e^{+ip\cdot x}\,\langle 0| \qquad \overline{\psi}^-(b^\dagger)\text{ creates }\mathrm{e}^-$$

# Quantization : Electromagnetic Field

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial\cdot A)^2 \qquad \pi^\mu = F^{\mu 0} - \frac{g^{\mu 0}}{\xi}(\partial\cdot A) \qquad \pi^0 = -\frac{1}{\xi}(\partial\cdot A) \qquad \pi^k = E^k$$

$$\left[A_\mu(\vec{\mathbf{x}},t), A_\nu(\vec{\mathbf{y}},t)\right] = \left[\dot{A}_\mu(\vec{\mathbf{x}},t), \dot{A}_\nu(\vec{\mathbf{y}},t)\right] = 0 \qquad \left[\dot{A}_\mu(\vec{\mathbf{y}},t), A_\nu(\vec{\mathbf{x}},t)\right] = i g_{\mu\nu}\delta^3(\vec{\mathbf{x}}-\vec{\mathbf{y}})$$

$$A_\mu(x) = A^+_\mu(x) + A^-_\mu(x) = \int \widetilde{\mathrm{d}k} \sum_{\lambda=0}^{3}\left[a(k,\lambda)\epsilon_\mu(k,\lambda)e^{-ik\cdot x} + a^\dagger(k,\lambda)\epsilon^*_\mu(k,\lambda)e^{+ik\cdot x}\right]$$

$$A^+_\mu(x)\,|k\rangle = A^+_\mu(x)a^\dagger(k)\,|0\rangle = |0\rangle\,\epsilon_\mu(k)e^{-ik\cdot x} \qquad \left[a(k,\lambda), a^\dagger(k',\lambda')\right] = -g^{\lambda\lambda'}\tilde{\delta}(k-k')$$

$$\langle k|\,A^-_\mu(x) = \langle 0|\,a(k)A^-_\mu(x) = \epsilon^*_\mu(k)e^{+ik\cdot x}\langle 0| \qquad A^+(a) \text{ destroys } \gamma \qquad A^-(a^\dagger) \text{ creates } \gamma$$

# Dyson Expansion, Propagators and Wick's Theorem

$$S_{fi} = \langle f|\Phi(\infty)\rangle = \langle f|S|i\rangle \qquad \sum_f \left|S_{fi}\right|^2 = 1$$

$$S = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\dots\mathrm{d}^4x_n\,\mathbf{T}(\mathscr{H}_{\text{int}}(x_1)\,\mathscr{H}_{\text{int}}(x_2)\dots\mathscr{H}_{\text{int}}(x_n))$$

$$\Delta_F(x_1-x_2) = \overbracket{\phi(x_1)\phi(x_2)} = \langle 0|\mathbf{T}(\phi(x_1)\phi(x_2))|0\rangle \qquad D_F^{\mu\nu} = \overbracket{A^\mu(x_1)A^\nu(x_2)}$$

$$\Delta_F(x_1-x_2) = \overbracket{\varphi(x_1)\varphi^\dagger(x_2)} = \overbracket{\varphi^\dagger(x_2)\varphi(x_1)} \qquad S_{F\alpha\beta} = \overbracket{\psi_\alpha(x_1)\overline{\psi}_\beta(x_2)} = -\overbracket{\overline{\psi}_\beta(x_2)\psi_\alpha(x_1)}$$

$$\mathbf{T}(ABCD\dots WXYZ) = \,:ABCD\dots WXYZ:+$$
$$+ :\overbracket{AB}CD\dots WXYZ: + :\overbracket{ABC}D\dots WXYZ: + \dots + :ABCD\dots WX\overbracket{YZ}:+$$
$$+ :\overbracket{AB}\,\overbracket{CD}\dots WXYZ: + \dots + :ABCD\dots \overbracket{WX}\,\overbracket{YZ}: + \dots$$

# Decay : Decay Rates

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{32\pi^2}\frac{|\vec{\mathbf{p}}_{\text{CM}}|}{m_0^2}|\mathscr{M}|^2 \qquad |\vec{\mathbf{p}}_{\text{CM}}| = \frac{1}{2m_0}\sqrt{\left[m_0^2-(m_1+m_2)^2\right]\left[m_0^2-(m_1-m_2)^2\right]}$$

# Scattering : Cross Sections

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{\mathbf{p}}_3|}{|\vec{\mathbf{p}}_1|}|\mathscr{M}|^2 \qquad |\vec{\mathbf{p}}| = \frac{1}{2\sqrt{s}}\sqrt{\left[s-(m_a+m_b)^2\right]\left[s-(m_a-m_b)^2\right]}$$

$$s = (p_1+p_2)^2 = (p_3+p_4)^2 \approx +2p_1\cdot p_2 \approx +2p_3\cdot p_4 \qquad \sqrt{s} = E_{CM}$$
$$t = (p_1-p_3)^2 = (p_2-p_4)^2 \approx -2p_1\cdot p_3 \approx -2p_2\cdot p_4 \qquad s+t+u = m_1^2+m_2^2+m_3^2+m_4^2$$
$$u = (p_1-p_4)^2 = (p_2-p_3)^2 \approx -2p_1\cdot p_4 \approx -2p_2\cdot p_3$$

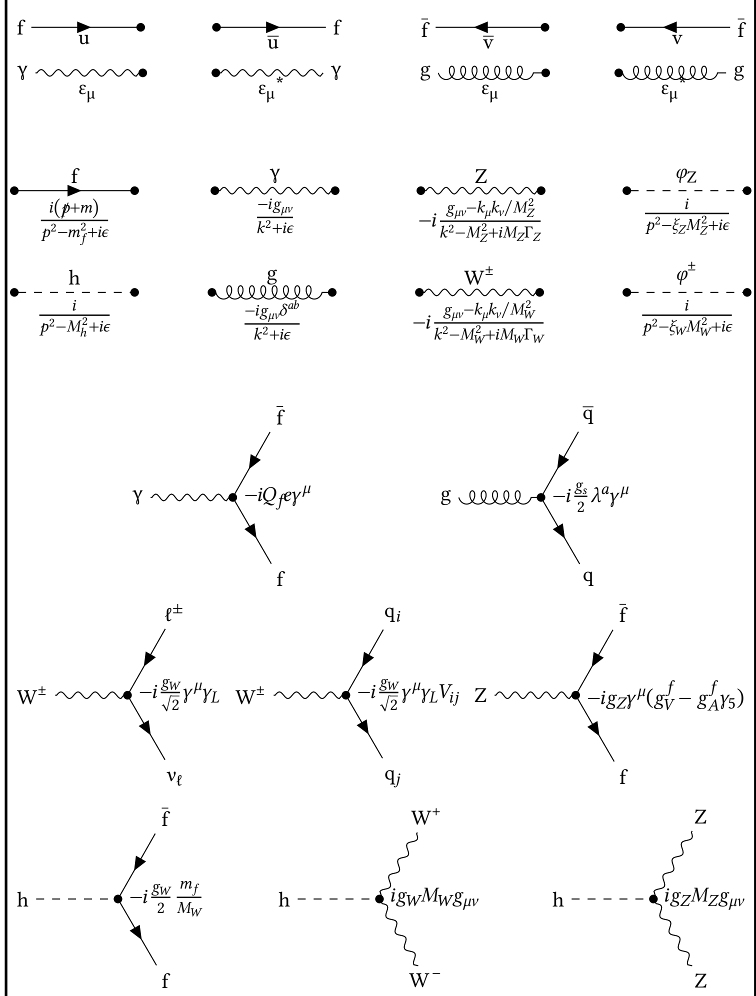
$\mathrm{f}+\bar{\mathrm{f}} \longrightarrow \mathrm{g}+\bar{\mathrm{g}}$:

$$p_1 = \frac{\sqrt{s}}{2}\left(1,0,0,+\beta_f\right) \qquad p_2 = \frac{\sqrt{s}}{2}\left(1,0,0,-\beta_f\right)$$
$$p_3 = \frac{\sqrt{s}}{2}\left(1,+\beta_g\sin\theta,0,+\beta_g\cos\theta\right) \qquad p_4 = \frac{\sqrt{s}}{2}\left(1,-\beta_g\sin\theta,0,-\beta_g\cos\theta\right)$$

$$m=0: \qquad s = 2|\vec{\mathbf{p}}_{\text{CM}}|^2 = 2\left(p^0_{\text{CM}}\right)^2 \qquad t = -\frac{s}{2}(1-\cos\theta) \qquad u = -\frac{s}{2}(1+\cos\theta)$$

# Feynman Rules for $i\mathscr{M}$

Goes in opposite way of arrows with the first one being adjoint, $\overline{\psi} = \psi^\dagger\gamma^0$:




# Constants

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} = \frac{g_W^2}{8M_Z^2\cos^2(\theta_W)} = \frac{1}{2v^2} \qquad M_W = M_Z\cos\theta_W = \frac{1}{2}g_W v \qquad M_H = \sqrt{2\lambda}v$$

$$g_W = g_Z\cos\theta_W = \frac{e}{\sin\theta_W} \qquad g_V^f = \frac{1}{2}T_f^3 - Q_f\sin^2(\theta_W) \qquad g_A^f = \frac{1}{2}T_f^3 \qquad g_{A,V} = \frac{1}{2}c_{A,V}$$

$$c_L^f = \frac{g_V^f+g_A^f}{2} \qquad c_R^f = \frac{g_V^f-g_A^f}{2} \qquad g_V^f = c_L^f + c_R^f \qquad g_A^f = c_L^f - c_R^f$$

| Fermions | $Q_f$ | $I_W^{(3)}$ | $Y_L$ | $Y_R$ | $c_L$ | $c_R$ | $c_V$ | $c_A$ |
|---|---|---|---|---|---|---|---|---|
| $\nu_e, \nu_\mu, \nu_\tau$ | 0 | +1/2 | −1 | 0 | +1/2 | 0 | +1/2 | +1/2 |
| $e^-, \mu^-, \tau^-$ | −1 | −1/2 | −1 | −2 | −0.27 | +0.23 | −0.04 | −1/2 |
| u, c, t | +2/3 | +1/2 | +1/3 | +4/3 | +0.35 | −0.15 | +0.19 | +1/2 |
| d, s, b | −1/3 | −1/2 | +1/3 | −2/3 | −0.42 | +0.08 | −0.35 | −1/2 |

# Relativity

$$\beta = \frac{v}{c} = \frac{p}{E} = \tanh(\eta) \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh(\eta) \qquad \gamma\beta = \sin(\eta)$$

$$E^2 = m^2 + |p|^2 \qquad \frac{\mathrm{d}^3\vec{\mathbf{p}}'}{E'} = \frac{\mathrm{d}^3\vec{\mathbf{p}}}{E} \qquad \widetilde{\mathrm{d}p} \equiv \frac{\mathrm{d}^3\vec{\mathbf{p}}}{(2\pi)^3 2E_p} \qquad \tilde{\delta}(p-q) \equiv (2\pi)^3 2E_p\delta^3(\vec{\mathbf{p}}-\vec{\mathbf{q}})$$

$$x^\mu = (t,\vec{\mathbf{x}}) \qquad p^\mu = (E,\vec{\mathbf{p}}) \qquad \partial^\mu = \left(\partial_t, -\vec{\nabla}\right) \qquad A^\mu = \left(\phi,\vec{\mathbf{A}}\right) \qquad J^\mu = \left(\rho,\vec{\mathbf{J}}\right)$$

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu \qquad \mathscr{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} \qquad \partial_\mu F^{\mu\nu} = J^\nu \qquad \partial_\mu\mathscr{F}^{\mu\nu} = 0$$

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{If } (\mu\nu\rho\sigma) \text{ is an even permutation of } (0123) \\ -1 & \text{If } (\mu\nu\rho\sigma) \text{ is an odd permutation of } (0123) \\ 0 & \text{Otherwise} \end{cases}$$

$$\begin{aligned}\epsilon_{\alpha\beta_1\gamma_1\delta_1}\epsilon^{\alpha\beta_2\gamma_2\delta_2} = &+g^{\beta_2}_{\beta_1}g^{\delta_2}_{\gamma_1}g^{\gamma_2}_{\delta_1} - g^{\beta_2}_{\beta_1}g^{\gamma_2}_{\gamma_1}g^{\delta_2}_{\delta_1} + g^{\gamma_2}_{\beta_1}g^{\beta_2}_{\gamma_1}g^{\delta_2}_{\delta_1} - g^{\gamma_2}_{\beta_1}g^{\delta_2}_{\gamma_1}g^{\beta_2}_{\delta_1} \\ &+ g^{\delta_2}_{\beta_1}g^{\gamma_2}_{\gamma_1}g^{\beta_2}_{\delta_1} - g^{\delta_2}_{\beta_1}g^{\beta_2}_{\gamma_1}g^{\gamma_2}_{\delta_1}\end{aligned}$$

$$\epsilon_{\alpha\beta\gamma_1\delta_1}\epsilon^{\alpha\beta\gamma_2\delta_2} = -2\left(g^{\gamma_2}_{\gamma_1}g^{\delta_2}_{\delta_1} - g^{\delta_2}_{\gamma_1}g^{\gamma_2}_{\delta_1}\right) \qquad \epsilon_{\alpha\beta\gamma\delta_1}\epsilon^{\alpha\beta\gamma\delta_2} = -6g^{\delta_2}_{\delta_1}$$

# Quantum Mechanics

$$E \to i\frac{\partial}{\partial t} \qquad \vec{\mathbf{p}} \to -i\vec{\nabla}\cdot \implies p^\mu \to i\delta^\mu$$

# Mathematics

Dirac Delta:

$$\delta^n(x'-x) = i\int\frac{\mathrm{d}^n p}{(2\pi)^n}e^{-ip(x'-x)} \qquad f(x_i) = 0 \implies \delta[f(x)] = \sum_i\left|\frac{\mathrm{d}f}{\mathrm{d}x}\right|^{-1}_{x_i}\cdot\delta(x-x_i)$$

Triangle Function:

$$\lambda(a,b,c) = \left[a-\left(\sqrt{b}+\sqrt{c}\right)^2\right]\left[a-\left(\sqrt{b}-\sqrt{c}\right)^2\right]$$

Residue Theorem:

$$\oint_\Gamma \mathrm{d}z\,f(z) = \pm 2\pi i\sum_{k=1}^{n}\mathrm{Res}[f(a_k)] \qquad \mathrm{Res}[f(a_k)] = \lim_{z\to a_k}(z-a_k)f(z)$$

The "-" ("+") is used when Γ is oriented clockwise (counterclockwise).

$$f(z) = \frac{h(z)e^{-iz(t'-t)}}{(z-a_1)(z-a_2)\dots} \overset{t'-t<0}{\implies} \int_{\Re(z)=-\infty}^{\Re(z)=+\infty} f(z)\,\mathrm{d}z = +2\pi i\sum_{a_k \text{ in UHP}}\mathrm{Res}[f(a_k)]$$
$$\overset{t'-t>0}{\implies} \int_{\Re(z)=-\infty}^{\Re(z)=+\infty} f(z)\,\mathrm{d}z = -2\pi i\sum_{a_k \text{ in LHP}}\mathrm{Res}[f(a_k)]$$