

Equations

Schrödinger:

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = H |\psi t\rangle \qquad \mathcal{L} = i\psi^* \dot{\psi} - \frac{1}{2m} \vec{\nabla} \psi^* \cdot \vec{\nabla} \psi$$

Klein-Gordon (Real Scalar Field):

$$(\partial \cdot \partial + m^2)\phi = 0 \qquad \mathcal{L} = \frac{1}{2} \big[ \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 \big] = \frac{1}{2} \Big[ \dot{\phi}^2 - \big( \vec{\nabla} \phi \big)^2 - m^2 \phi^2 \Big]$$

Dirac (Complex Scalar Field):

$$(i \not{\partial} - m)\varphi = 0 \qquad \overline{\varphi}(i \not{\partial} + m) = 0 \qquad \mathcal{L} = \partial_\mu \varphi \partial^\mu \varphi^* - m^2 \varphi \varphi^*$$

$\gamma$ -Matrices

$$\{ \gamma^\mu, \gamma^\nu \} = 2 g^{\mu\nu} \quad \gamma_5 = + i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = - i \gamma_0 \gamma_1 \gamma_2 \gamma_3 = - \frac{i}{4!} \epsilon^{\alpha\beta\gamma\delta} \gamma_\alpha \gamma_\beta \gamma_\gamma \gamma_\delta$$
$$\sigma^{\mu\nu} = \frac{i}{2} [ \gamma^\mu, \gamma^\nu ] \implies [ \gamma^\mu, \gamma^\nu ] = -2i \sigma^{\mu\nu} \qquad \{ \gamma_5, \gamma^\mu \} = 0 \qquad \gamma^0 (\gamma^\mu)^\dagger \gamma^0 = \gamma^\mu$$
$$(\gamma^0)^\dagger = \gamma^0 \quad (\gamma^0)^2 = \mathbb{1} \qquad (\gamma^k)^\dagger = -\gamma^k \quad (\gamma^k)^2 = -\mathbb{1} \qquad (\gamma^5)^\dagger = \gamma^5 \quad (\gamma^5)^2 = \mathbb{1}$$
$$\overline{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0 \qquad \overline{\gamma_5} = -\gamma_5 \qquad \overline{\gamma^\mu} = \gamma^\mu \qquad \overline{\gamma^\mu \gamma_5} = \gamma^\mu \gamma_5 \qquad \overline{\sigma^{\mu\nu}} = \sigma^{\mu\nu}$$

$\mathbb{1}$	$\gamma_5$	$\gamma^\mu$	$\gamma_5 \gamma^\mu$	$\sigma^{\mu\nu}$
$\gamma_5$	$\mathbb{1}$	$+ \gamma_5 \gamma^\mu$	$+ \gamma^\mu$	$\frac{i}{2} \epsilon^{\mu\nu\sigma\rho} \sigma_{\sigma\rho}$
$\gamma^\alpha$	$-\gamma_5 \gamma^\alpha$	$g^{\alpha\mu} - i \sigma^{\alpha\mu}$	$-\frac{i}{2} (2 g^{\alpha\mu} \gamma_5 + \epsilon^{\alpha\mu\sigma\rho} \sigma_{\sigma\rho})$	$\epsilon^{\alpha\mu\nu\lambda} \gamma_5 \gamma_\lambda + i g^{\alpha\mu} \gamma^\nu - i g^{\alpha\nu} \gamma^\mu$
$\gamma_5 \gamma^\alpha$	$-\gamma^\alpha$	$-\gamma^\alpha$	$-(g^{\alpha\mu} - i \sigma^{\alpha\mu})$	$\epsilon^{\alpha\mu\nu\lambda} \gamma_\lambda + i g^{\alpha\mu} \gamma_5 \gamma^\nu - i g^{\alpha\nu} \gamma_5 \gamma^\mu$
$\sigma^{\alpha\beta}$	$\sigma^{\alpha\beta}$	$\epsilon^{\alpha\beta\mu\lambda} \gamma_5 \gamma_\lambda + i g^{\beta\mu} \gamma^\alpha - i g^{\alpha\mu} \gamma^\beta$	$\epsilon^{\alpha\beta\mu\lambda} \gamma_\lambda + i g^{\beta\mu} \gamma_5 \gamma^\alpha - i g^{\alpha\mu} \gamma_5 \gamma^\beta$	$i \epsilon^{\alpha\beta\mu\nu} \gamma_5 + g^{\alpha\mu} g^{\beta\nu} - g^{\alpha\nu} g^{\beta\mu} + \frac{i}{2} [ g^{\alpha\sigma} \sigma^{\beta\mu} + g^{\beta\mu} \sigma^{\alpha\nu} - g^{\alpha\mu} \sigma^{\beta\nu} - g^{\beta\nu} \sigma^{\alpha\mu} ]$

$$\not{A} = A^\mu \gamma_\mu \qquad \gamma^\mu \gamma_\mu = 4 \qquad \gamma^\mu \gamma^\nu \gamma_\mu = -2 \gamma^\nu \qquad \gamma^\mu \gamma^\nu \gamma^\rho \gamma_\mu = 4 g^{\mu\nu}$$
$$\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_\mu = -2 \gamma^\nu \gamma^\rho \gamma^\sigma \qquad \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\pi \gamma_\mu = 2 [ \gamma^\pi \gamma^\nu \gamma^\rho \gamma^\sigma + \gamma^\sigma \gamma^\rho \gamma^\nu \gamma^\pi ]$$

Spin, Helicity and Chirality

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \vec{\Sigma} = \frac{1}{2} \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$
$$\vec{S} = \frac{1}{2} \vec{\Sigma} \qquad \vec{h} = \frac{\vec{S} \cdot \vec{p}}{|\vec{p}|} = \frac{\vec{\Sigma} \cdot \vec{p}}{2|\vec{p}|} \qquad \vec{h} = 2h = \frac{\vec{\Sigma} \cdot \vec{p}}{|\vec{p}|} \qquad P_L = \frac{1-h}{2} \qquad P_R = \frac{1+h}{2}$$
$$\gamma_L = \frac{1-\gamma_5}{2} \qquad \gamma_R = \frac{1+\gamma_5}{2} \qquad \gamma_{L,R}^2 = \gamma_{L,R} \qquad \gamma_{L,R} \gamma_{R,L} = 0 \qquad \gamma_L + \gamma_R = \mathbb{1}$$
$$\gamma^\mu \gamma_{L,R} = \gamma_{R,L} \gamma^\mu \qquad \gamma_5 \gamma_L = \gamma_L \gamma_5 = -\gamma_L \qquad \gamma_5 \gamma_R = \gamma_R \gamma_5 = \gamma_R \qquad \gamma_{L,R}^\dagger = \gamma_{L,R}$$
$$\overline{\gamma^\mu \gamma_{L,R}} = \gamma^\mu \gamma_{L,R} \qquad \overline{\gamma_5 \gamma_L} = -\gamma_R \qquad \overline{\gamma_5 \gamma_R} = \gamma_L \qquad \overline{\gamma_{L,R}} = \gamma_0 \gamma_{L,R} \gamma_0^\dagger = \gamma_{R,L}$$

$$u_L = u_\downarrow \qquad u_R = u_\uparrow \qquad v_L = v_\uparrow \qquad v_R = v_\downarrow$$
$$P_L u_\downarrow = u_\downarrow \quad P_L u_\uparrow = 0 \quad P_R u_\downarrow = 0 \quad P_R u_\uparrow = u_\uparrow \quad \overline{u}_\downarrow P_L = 0 \quad \overline{u}_\uparrow P_L = u_\uparrow \quad \overline{u}_\downarrow P_R = u_\downarrow \quad \overline{u}_\uparrow P_R = 0$$
$$\gamma_L u_L = u_L \quad \gamma_L u_R = 0 \quad \gamma_R u_L = 0 \quad \gamma_R u_R = u_R \quad \overline{u}_L \gamma_L = 0 \quad \overline{u}_R \gamma_L = u_R \quad \overline{u}_L \gamma_R = u_L \quad \overline{u}_R \gamma_R = 0$$
$$P_L v_\downarrow = 0 \quad P_L v_\uparrow = v_\uparrow \quad P_R v_\downarrow = v_\downarrow \quad P_R v_\uparrow = 0 \quad \overline{v}_\downarrow P_L = v_\downarrow \quad \overline{v}_\uparrow P_L = 0 \quad \overline{v}_\downarrow P_R = 0 \quad \overline{v}_\uparrow P_R = v_\uparrow$$
$$\gamma_L v_L = v_L \quad \gamma_L v_R = 0 \quad \gamma_R v_L = 0 \quad \gamma_R v_R = v_R \quad \overline{v}_L \gamma_L = 0 \quad \overline{v}_R \gamma_L = v_R \quad \overline{v}_L \gamma_R = v_L \quad \overline{v}_R \gamma_R = 0$$

$$u_\uparrow = N \begin{pmatrix} c \\ s e^{i\phi} \\ kc \\ k s e^{i\phi} \end{pmatrix} \approx N \begin{pmatrix} c \\ s e^{i\phi} \\ c \\ s e^{i\phi} \end{pmatrix} \qquad u_\downarrow = N \begin{pmatrix} -s \\ c e^{i\phi} \\ ks \\ -k c e^{i\phi} \end{pmatrix} \approx N \begin{pmatrix} -s \\ c e^{i\phi} \\ s \\ -c e^{i\phi} \end{pmatrix}$$
$$v_\uparrow = N \begin{pmatrix} ks \\ -k c e^{i\phi} \\ -s \\ c e^{i\phi} \end{pmatrix} \approx N \begin{pmatrix} s \\ -c e^{i\phi} \\ -s \\ c e^{i\phi} \end{pmatrix} \qquad v_\downarrow = N \begin{pmatrix} kc \\ k s e^{i\phi} \\ c \\ s e^{i\phi} \end{pmatrix} \approx N \begin{pmatrix} c \\ c e^{i\phi} \\ c \\ s e^{i\phi} \end{pmatrix}$$
$$\vec{p} = p(\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$$
$$N = \sqrt{E+m} \approx \sqrt{E} \qquad k = \frac{p}{E+m} \approx 1 \qquad s = \sin\left(\frac{\theta}{2}\right) \qquad c = \cos\left(\frac{\theta}{2}\right)$$

Traces

$$\text{Tr}[\mathbb{1}] = 4 \qquad \text{Tr}[\underbrace{\gamma^\mu \gamma^\nu \dots \gamma^\rho}_{\text{Odd Number}}] = 0 \qquad \text{Tr}[\gamma^\mu \gamma^\nu] = 4 g^{\mu\nu}$$
$$\text{Tr}[\gamma_5] = 0 \qquad \text{Tr}[\gamma_5 \underbrace{\gamma^\mu \gamma^\nu \dots \gamma^\rho}_{\text{Odd Number}}] = 0 \qquad \text{Tr}[\gamma_5 \gamma^\mu \gamma^\nu] = 0$$
$$\text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] = 4(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) \qquad \text{Tr}[\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] = -4 i \epsilon^{\mu\nu\rho\sigma}$$
$$\text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_L] = 2(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho} + i \epsilon^{\mu\nu\rho\sigma})$$
$$\text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_R] = 2(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho} - i \epsilon^{\mu\nu\rho\sigma})$$

$$\text{Tr}[(\not{a} + m_a) \gamma^\mu (\not{b} + m_b) \gamma^\nu] = 4 \big[ p_a^\mu p_b^\nu + p_b^\mu p_a^\nu + (m_a m_b - p_a \cdot p_b) g^{\mu\nu} \big]$$

Fermions

$$:a(x)b(x'):= a(x)b(x') - b(x')a(x)$$
$$\text{T}[a(x)b(x')] = \theta(t - \text{'} t) a(x)b(x') - \theta(t' - t) b(x')a(x)$$
$$\{ b^\dagger(p,s), b(p',s') \} = \{ d^\dagger(p,s), d(p',s') \} = \tilde{\delta}(p - p') \delta_{ss'}$$
$$\psi(x) = \psi^+(x) + \psi^-(x) = \int d\vec{p} \sum_s \big[ b(p,s) u(p,s) e^{-ip \cdot x} + d^\dagger(p,s) v(p,s) e^{+ip \cdot x} \big]$$
$$\overline{\psi}(x) = \overline{\psi}^+(x) + \overline{\psi}^-(x) = \int d\vec{p} \sum_s \big[ d(p,s) \overline{v}(p,s) e^{-ip \cdot x} + b^\dagger(p,s) \overline{u}(p,s) e^{+ip \cdot x} \big]$$

$\psi^+(b)$  destroys  $e^-$      $\psi^-(d^\dagger)$  creates  $e^+$      $\overline{\psi}^+(d)$  destroy  $e^+$      $\overline{\psi}^-(b^\dagger)$  creates  $e^-$

Spinors (Fermions)

$$\psi = u e^{+i(\vec{p} \cdot \vec{x} - Et)} = v e^{-i(\vec{p} \cdot \vec{x} - Et)} \qquad \overline{\psi} = \psi^\dagger \gamma^0$$
$$(\not{p} - m) u = \overline{u} (\not{p} - m) = 0 \qquad (\not{p} + m) v = \overline{v} (\not{p} + m) = 0$$
$$\overline{u}(p) u^s(p) = +2m \delta^{rs} \qquad \overline{u}(p) v^s(p) = 0 \qquad u^\dagger(p) u^s(p) = 2E \delta^{rs} \qquad \sum_{s=1,2} u^s(p) \overline{u}^s(p) = \not{p} + m$$
$$\overline{v}(p) v^s(p) = -2m \delta^{rs} \qquad \overline{v}(p) u^s(p) = 0 \qquad u^\dagger(p) u^s(p) = 2E \delta^{rs} \qquad \sum_{s=1,2} v^s(p) \overline{v}^s(p) = \not{p} - m$$

Bosons

$$:a(x)b(x'):= a(x)b(x') + b(x')a(x)$$
$$\text{T}[a(x)b(x')] = \theta(t - \text{'} t) a(x)b(x') + \theta(t' - t) b(x')a(x)$$
$$[\phi(\vec{x},t), \phi(\vec{y},t)] = [\Pi(\vec{x},t), \Pi(\vec{y},t)] = 0 \qquad [\phi(\vec{x},t), \Pi(\vec{y},t)] = i \delta^3(\vec{x} - \vec{y})$$
$$\big[ a^\dagger(k,\lambda), a(k',\lambda') \big] = g^{\lambda\lambda'} \tilde{\delta}(k - k')$$
$$A_\mu(x) = A_\mu^+(x) + A_\mu^-(x) = \int d\vec{k} \sum_{\lambda=0}^3 \big[ a(k,\lambda) \epsilon_\mu(k,\lambda) e^{-ik \cdot x} + a^\dagger(k,\lambda) \epsilon_\mu^*(k,\lambda) e^{+ik \cdot x} \big]$$

$A^+(a)$  destroys  $\gamma$      $A^-(a^\dagger)$  creates  $\gamma$

Polarization Vectors (Bosons)

External massless & Massive:

$$\sum_{\lambda=1}^2 \epsilon^\mu(k,\lambda) \cdot \epsilon^{*\nu}(k,\lambda) = -g^{\mu\nu} \qquad \sum_{\lambda=1}^3 \epsilon_\mu(q,\lambda) \epsilon_\nu^*(q,\lambda) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2}$$

Virtual massless:

$$\epsilon(k,\lambda) \cdot \epsilon^*(k,\lambda') = g^{\lambda\lambda'} \qquad \sum_\lambda g^{\lambda\lambda} \epsilon^\mu(k,\lambda) \cdot \epsilon^{*\nu}(k,\lambda) = g^{\mu\nu}$$

Decay : Decay Rates

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2} \frac{|\vec{\mathbf{p}}_{\text{CM}}|}{m_0^2} |\overline{\mathcal{M}}|^2 \qquad |\vec{\mathbf{p}}_{\text{CM}}| = \frac{1}{2m_0} \sqrt{[m_0^2 - (m_1 + m_2)^2] [m_0^2 - (m_1 - m_2)^2]}$$

Scattering : Cross Sections

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{\mathbf{p}}_3|}{|\vec{\mathbf{p}}_1|} |\overline{\mathcal{M}}|^2 \qquad |\vec{\mathbf{p}}| = \frac{1}{2\sqrt{s}} \sqrt{[s - (m_a + m_b)^2] [s - (m_a - m_b)^2]}$$

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \approx +2p_1 \cdot p_2 \approx +2p_3 \cdot p_4 \qquad \sqrt{s} = E_{\text{CM}}$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \approx -2p_1 \cdot p_3 \approx -2p_2 \cdot p_4 \qquad s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \approx -2p_1 \cdot p_4 \approx -2p_2 \cdot p_3$$

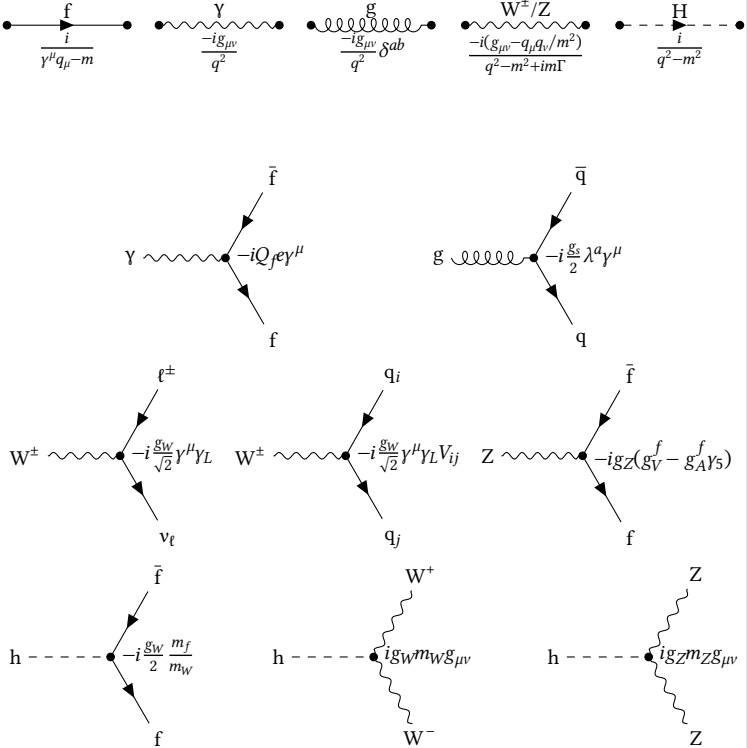
$$f + \bar{f} \longrightarrow g + \bar{g}:$$

$$p_1 = \frac{\sqrt{s}}{2} (1, 0, 0, +\beta_f) \qquad p_2 = \frac{\sqrt{s}}{2} (1, 0, 0, -\beta_f)$$
$$p_3 = \frac{\sqrt{s}}{2} (1, +\beta_g \sin\theta, 0, +\beta_g \cos\theta) \qquad p_4 = \frac{\sqrt{s}}{2} (1, -\beta_g \sin\theta, 0, -\beta_g \cos\theta)$$

$m = 0$  :     $s = 2 |\vec{\mathbf{p}}_{\text{CM}}|^2 = 2 (p_{\text{CM}}^0)^2 \qquad t = -\frac{s}{2} (1 - \cos\theta) \qquad u = -\frac{s}{2} (1 + \cos\theta)$

## Constants

Figure 1 displays Feynman diagrams for the production of a photon and a gluon. The top row shows the production of a photon ( $\gamma$ ) via quark-antiquark annihilation ( $q\bar{q}$ ) into a photon ( $\gamma$ ) and a gluon ( $g$ ). The bottom row shows the production of a gluon ( $g$ ) via quark-antiquark annihilation ( $q\bar{q}$ ) into a gluon ( $g$ ) and a photon ( $\gamma$ ). The diagrams are labeled with the corresponding particles and momenta.


$$f(x_i) = 0 \implies \delta[f(x)] = \sum_i \left| \frac{df}{dx} \right|_{x_i}^{-1} \cdot \delta(x - x_i)$$

$$\beta = \frac{v}{c} = \frac{p}{E} = \tanh(\eta) \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh(\eta) \quad \gamma\beta = \sinh(\eta)$$

$$E^2 = m^2 + |p|^2 \quad \frac{d^3\vec{p}'}{E'} = \frac{d^3\vec{p}}{E}$$

$$x^\mu = (t, \vec{x}) \quad p^\mu = (E, \vec{p}) \quad \partial^\mu = (\partial_t, -\vec{\nabla}) \quad A^\mu = (\phi, \vec{A}) \quad J^\mu = (\rho, \vec{J})$$

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu \quad \mathcal{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \quad \partial_\mu F^{\mu\nu} = J^\nu \quad \partial_\mu \mathcal{F}^{\mu\nu} = 0$$

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{If } (\mu\nu\rho\sigma) \text{ is an even permutation of } (0123) \\ -1 & \text{If } (\mu\nu\rho\sigma) \text{ is an odd permutation of } (0123) \\ 0 & \text{Otherwise} \end{cases}$$

$$\begin{aligned} \epsilon_{\alpha\beta\gamma_1\delta_1}\epsilon^{\alpha\beta_2\gamma_2\delta_2} &= +g_{\beta_1\gamma_1}^{ \beta_2\delta_2}g_{\delta_1}^{\gamma_2} - g_{\beta_1\gamma_1}^{\beta_2\gamma_2}g_{\delta_1}^{\delta_2} + g_{\beta_1\gamma_1}^{\gamma_2\delta_2}g_{\delta_1}^{\beta_2} - g_{\beta_1\gamma_1}^{\gamma_2\delta_2}g_{\delta_1}^{\beta_2} \\ &\quad + g_{\beta_1\gamma_1}^{\delta_2\beta_2}g_{\delta_1}^{\gamma_2} - g_{\beta_1\gamma_1}^{\delta_2\beta_2}g_{\delta_1}^{\gamma_2} \\ \epsilon_{\alpha\beta\gamma_1\delta_1}\epsilon^{\alpha\beta\gamma_2\delta_2} &= -2\left(g_{\gamma_1\delta_1}^{\gamma_2\delta_2} - g_{\gamma_1\delta_1}^{\delta_2\gamma_2}\right) \\ \epsilon_{\alpha\beta\gamma\delta}\epsilon^{\alpha\beta\gamma\delta} &= -6g_{\delta_1}^{\delta_2} \end{aligned}$$

## Mathematics

$$E \rightarrow i \frac{\partial}{\partial t} \quad \vec{\mathbf{p}} \rightarrow -i \vec{\nabla} \cdot \implies p^\mu \rightarrow i \delta^\mu$$

$$f(x_i) = 0 \implies \delta[f(x)] = \sum_i \left| \frac{df}{dx} \right|_{x_i}^{-1} \cdot \delta(x - x_i)$$

$$\lambda(a, b, c) = \left[ a - (\sqrt{b} + \sqrt{c})^2 \right] \left[ a - (\sqrt{b} - \sqrt{c})^2 \right]$$

$$\oint_{\Gamma} dz f(z) = 2\pi i \sum \text{Res}(f, x_k)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} = \frac{g_W^2}{8M_Z^2 \cos^2(\theta_W)} = \frac{1}{2v^2} \quad g_W = g_Z \cos \theta_W = \frac{e}{\sin \theta_W}$$

$$g_V^f = \frac{1}{2} T_f^3 - Q_f \sin^2(\theta_W) \quad g_A^f = \frac{1}{2} T_f^3 \quad M_W = M_Z \cos \theta_W = \frac{1}{2} g_W v \quad m_H = \sqrt{2} \lambda v$$