Equations

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle \qquad (\partial\cdot\partial + m^2)\phi = 0 \qquad (i\not\partial - m)\varphi = 0 \qquad \overline{\varphi}(i\not\partial + m) = 0$$

Pauli and Dirac γ-Matrices Matrices

$$\sigma^1 = \sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad \sigma^2 = \sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \quad \sigma^3 = \sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \quad \vec{\mathbf{\Sigma}} = \begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix}$$

$$\{\sigma^i, \sigma^j\} = 2\delta^{ij} \qquad [\sigma^i, \sigma^j] = 2i\epsilon^{ijk}\sigma^k \qquad \sigma^i\sigma^j = \delta^{ij} + i\epsilon^{ijk}\sigma^k \qquad \sigma^2\sigma^i\sigma^2 = -(\sigma^i)^*$$

$$\gamma^\mu = \begin{pmatrix}0 & \sigma^\mu\\ \overline{\sigma}^\mu & 0\end{pmatrix} \quad \text{<- Chiral | Pauli-Dirac ->} \quad \gamma^0 = \begin{pmatrix}+\mathbb{1} & 0\\ 0 & -\mathbb{1}\end{pmatrix} \quad \gamma^k = \begin{pmatrix}0 & \sigma^k\\ -\sigma^k & 0\end{pmatrix}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad \gamma_5 = +i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma_0\gamma_1\gamma_2\gamma_3 = -\frac{i}{4!}\epsilon^{\alpha\beta\gamma\delta}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta \quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

$$\not A = A^\mu\gamma_\mu \quad \gamma^\mu\gamma_\mu = 4 \quad \gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu \quad \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4g^{\nu\rho} \quad \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu$$

$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\pi\gamma_\mu = 2[\gamma^\pi\gamma^\nu\gamma^\rho\gamma^\sigma + \gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\pi] \qquad \{\gamma_5, \gamma^\mu\} = 0 \qquad \gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu$$

$$(\gamma^0)^\dagger = \gamma^0 \quad (\gamma^0)^2 = \mathbb{1} \quad (\gamma^k)^\dagger = -\gamma^k \quad (\gamma^k)^2 = -\mathbb{1} \quad (\gamma^5)^\dagger = \gamma^5 \quad (\gamma^5)^2 = \mathbb{1}$$

$$\overline{\psi} = \psi^\dagger\gamma^0 \quad \overline{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0 \quad \overline{\gamma_5} = -\gamma_5 \quad \overline{\gamma^\mu} = \gamma^\mu \quad \overline{\gamma^\mu\gamma_5} = \gamma^\mu\gamma_5 \quad \overline{\sigma^{\mu\nu}} = \sigma^{\mu\nu}$$

Spin, Helicity and Chirality

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{\Sigma}} \quad h = \frac{\vec{\mathbf{S}}\cdot\vec{\mathbf{p}}}{|\vec{\mathbf{p}}|} \quad P_L = \frac{1-h}{2} \quad P_R = \frac{1+h}{2} \quad \gamma_L = \frac{1-\gamma_5}{2} \quad \gamma_R = \frac{1+\gamma_5}{2}$$

$$\gamma_{L,R}^2 = \gamma_{L,R} \quad \gamma_{L,R}\gamma_{R,L} = 0 \quad \gamma_L + \gamma_R = \mathbb{1} \quad \gamma^\mu\gamma_{L,R} = \gamma_{R,L}\gamma^\mu \quad \gamma_5\gamma_L = -\gamma_L \quad \gamma_5\gamma_R = \gamma_R$$

$$\gamma_{L,R}^\dagger = \gamma_{L,R} \quad \overline{\gamma^\mu\gamma_{L,R}} = \gamma^\mu\gamma_{L,R} \quad \overline{\gamma_5\gamma_L} = -\gamma_R \quad \overline{\gamma_5\gamma_R} = \gamma_L \quad \overline{\gamma_{L,R}} = \gamma_0\gamma_{L,R}^\dagger\gamma_0 = \gamma_{R,L}$$

$$u_L = u_\downarrow \quad u_R = u_\uparrow \quad v_L = v_\uparrow \quad v_R = v_\downarrow \quad \vec{\mathbf{p}} = p(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$P_L u_\downarrow = u_\downarrow \quad P_L u_\uparrow = 0 \quad P_R u_\downarrow = 0 \quad P_R u_\uparrow = u_\uparrow \quad \overline{u_\downarrow}P_L = 0 \quad \overline{u_\uparrow}P_L = u_\uparrow \quad \overline{u_\downarrow}P_R = u_\downarrow \quad \overline{u_\uparrow}P_R = 0$$

$$\gamma_L u_L = u_L \quad \gamma_L u_R = 0 \quad \gamma_R u_L = 0 \quad \gamma_R u_R = u_R \quad \overline{u_L}\gamma_L = 0 \quad \overline{u_R}\gamma_L = u_R \quad \overline{u_L}\gamma_R = u_L \quad \overline{u_R}\gamma_R = 0$$

$$P_L v_\downarrow = 0 \quad P_L v_\uparrow = v_\uparrow \quad P_R v_\downarrow = v_\downarrow \quad P_R v_\uparrow = 0 \quad \overline{v_\downarrow}P_L = v_\downarrow \quad \overline{v_\uparrow}P_L = 0 \quad \overline{v_\downarrow}P_R = 0 \quad \overline{v_\uparrow}P_R = v_\uparrow$$

$$\gamma_L v_L = v_L \quad \gamma_L v_R = 0 \quad \gamma_R v_L = 0 \quad \gamma_R v_R = v_R \quad \overline{v_L}\gamma_L = 0 \quad \overline{v_R}\gamma_L = v_R \quad \overline{v_L}\gamma_R = v_L \quad \overline{v_R}\gamma_R = 0$$

$$u_\uparrow^T = N\begin{pmatrix}c & se^{i\phi} & kc & kse^{i\phi}\end{pmatrix} \approx N\begin{pmatrix}c & se^{i\phi} & c & se^{i\phi}\end{pmatrix} \qquad N = \sqrt{E+m} \approx \sqrt{E}$$

$$u_\downarrow^T = N\begin{pmatrix}-s & ce^{i\phi} & ks & -kce^{i\phi}\end{pmatrix} \approx N\begin{pmatrix}-s & ce^{i\phi} & s & -ce^{i\phi}\end{pmatrix} \qquad k = \frac{p}{E+m} \approx 1$$

$$v_\uparrow^T = N\begin{pmatrix}ks & -kce^{i\phi} & -s & ce^{i\phi}\end{pmatrix} \approx N\begin{pmatrix}s & -ce^{i\phi} & -s & ce^{i\phi}\end{pmatrix} \qquad s = \sin\left(\frac{\theta}{2}\right)$$

$$v_\downarrow^T = N\begin{pmatrix}kc & kse^{i\phi} & c & se^{i\phi}\end{pmatrix} \approx N\begin{pmatrix}c & se^{i\phi} & c & se^{i\phi}\end{pmatrix} \qquad c = \cos\left(\frac{\theta}{2}\right)$$

Spinors (Fermions) & Polarization Vectors (Bosons)

$$\psi = ue^{+i(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}} - E\cdot t)} = ve^{-i(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}} - E\cdot t)} \qquad (\not p - m)u = \overline{u}(\not p - m) = 0 = (\not p + m)v = \overline{v}(\not p + m)$$

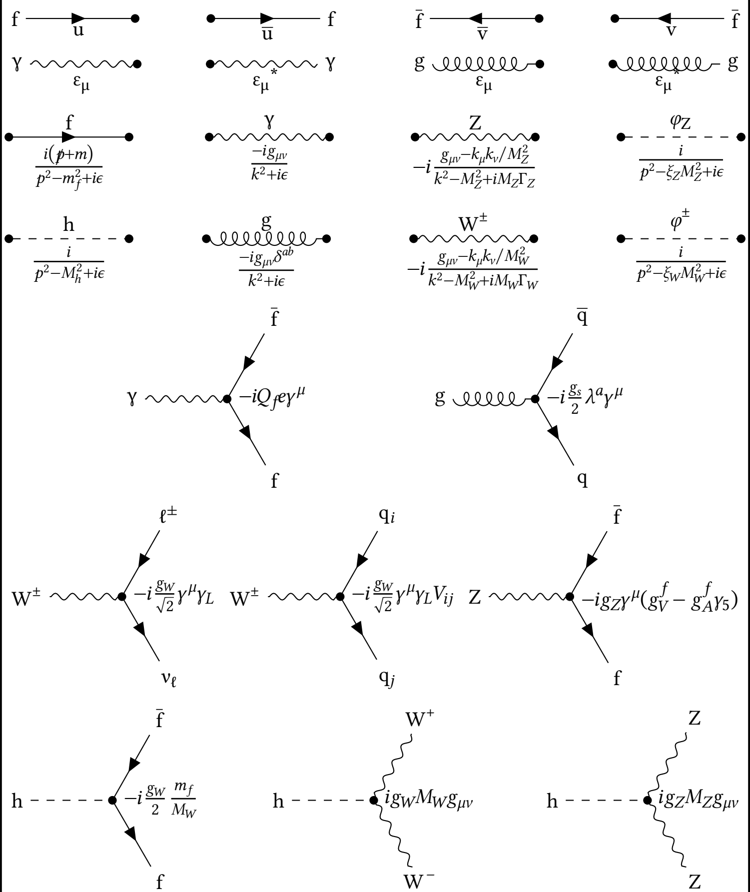
$$\overline{u}^r(p)u^s(p) = +2m\delta^{rs} \quad \overline{u}^r(p)v^s(p) = 0 \quad u^{r\dagger}(p)u^s(p) = 2E\delta^{rs} \quad \sum_{s=1,2} u^s(p)\overline{u}^s(p) = \not p + m$$

$$\overline{v}^r(p)v^s(p) = -2m\delta^{rs} \quad \overline{v}^r(p)u^s(p) = 0 \quad v^{r\dagger}(p)v^s(p) = 2E\delta^{rs} \quad \sum_{s=1,2} v^s(p)\overline{v}^s(p) = \not p - m$$

$$m = 0 : \sum_{\lambda=1}^{2}\epsilon^\mu(k,\lambda)\cdot\epsilon^{*\nu}(k,\lambda) = -g^{\mu\nu} \qquad m \neq 0 : \sum_{\lambda=1}^{3}\epsilon_\mu(q,\lambda)\epsilon_\nu^*(q,\lambda) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2}$$

Feynman Rules for $i\mathscr{M}$

Goes in opposite way of arrows with the first one being adjoint, $\overline{\psi} = \psi^\dagger\gamma^0$:



Decay : Decay Rates

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2}\frac{|\vec{\mathbf{p}}_{CM}|}{m_0^2}\overline{|\mathscr{M}|^2} \qquad |\vec{\mathbf{p}}_{CM}| = \frac{1}{2m_0}\sqrt{\left[m_0^2 - (m_1+m_2)^2\right]\left[m_0^2 - (m_1-m_2)^2\right]}$$

$$E_1 = \frac{m_0^2 + m_1^2 - m_2^2}{2m_0} \qquad E_2 = \frac{m_0^2 + m_2^2 - m_1^2}{2m_0}$$

Scattering : Cross Sections

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{\mathbf{p}}_3|}{|\vec{\mathbf{p}}_1|}\overline{|\mathscr{M}|^2} \qquad |\vec{\mathbf{p}}_a| = |\vec{\mathbf{p}}_b| = \frac{1}{2\sqrt{s}}\sqrt{\left[s - (m_a+m_b)^2\right]\left[s - (m_a-m_b)^2\right]}$$

$$E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}} \quad E_2 = \frac{s + m_2^2 - m_1^2}{2\sqrt{s}} \quad E_3 = \frac{s + m_3^2 - m_4^2}{2\sqrt{s}} \quad E_4 = \frac{s + m_4^2 - m_3^2}{2\sqrt{s}}$$

$$s = (p_1+p_2)^2 = (p_3+p_4)^2 \approx +2p_1\cdot p_2 \approx +2p_3\cdot p_4 \qquad \sqrt{s} = E_{CM}$$

$$t = (p_1-p_3)^2 = (p_2-p_4)^2 \approx -2p_1\cdot p_3 \approx -2p_2\cdot p_4 \qquad s+t+u = m_1^2+m_2^2+m_3^2+m_4^2$$

$$u = (p_1-p_4)^2 = (p_2-p_3)^2 \approx -2p_1\cdot p_4 \approx -2p_2\cdot p_3$$

$$m = 0 : s = 2|\vec{\mathbf{p}}_{CM}|^2 = 2(p_{CM}^0)^2 \qquad t = -\frac{s}{2}(1-\cos\theta) \qquad u = -\frac{s}{2}(1+\cos\theta)$$

Constants

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} = \frac{g_W^2}{8M_Z^2\cos^2(\theta_W)} = \frac{1}{2v^2} \qquad M_W = M_Z\cos\theta_W = \frac{1}{2}g_W v \qquad M_H = \sqrt{2\lambda}v$$

$$g_W = g_Z\cos\theta_W = \frac{e}{\sin\theta_W} \qquad g_V^f = \frac{1}{2}T_f^3 - Q_f\sin^2(\theta_W) \qquad g_A^f = \frac{1}{2}T_f^3 \qquad g_{A,V} = \frac{1}{2}c_{A,V}$$

$$\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c} \qquad c_L^f = g_V^f + g_A^f \qquad c_R^f = g_V^f - g_A^f \qquad g_V^f = \frac{c_L^f + c_R^f}{2} \qquad g_A^f = \frac{c_L^f - c_R^f}{2}$$

Fermions	Q_f	$I_W^{(3)}$	Y_L	Y_R	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	+1/2	−1	0	+1/2	0	+1/2	+1/2
e^-, μ^-, τ^-	−1	−1/2	−1	−2	−0.27	+0.23	−0.04	−1/2
u, c, t	+2/3	+1/2	+1/3	+4/3	+0.35	−0.15	+0.19	+1/2
d, s, b	−1/3	−1/2	+1/3	−2/3	−0.42	+0.08	−0.35	−1/2

Particle	$W^\pm$	Z^0	H^0	$e^\pm$	$\mu^\pm$	τ	u	d	c	s	t	b
Mass (MeV)	80 379	91 188	125 100	0.511	105.7	1777	2.16	4.67	1270	93	172 760	4180

$$1\,\text{kg} = 5.61\times10^{26}\,\text{GeV} \qquad 1\,\text{m} = 55.07\times10^{15}\,\text{GeV}^{-1} \qquad 1\,\text{s} = 51.52\times10^{24}\,\text{GeV}^{-1}$$

Relativity & Quantum Mechanics

$$\beta = \frac{v}{c} = \frac{p}{E} = \tanh(\eta) \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh(\eta) \qquad \gamma\beta = \sin(\eta) \qquad p^\mu \to i\delta^\mu$$

$$E^2 = m^2 + |p|^2 \qquad \frac{d^3\vec{\mathbf{p}}'}{E'} = \frac{d^3\vec{\mathbf{p}}}{E} \qquad \widetilde{dp} \equiv \frac{d^3\vec{\mathbf{p}}}{(2\pi)^3 2E_p} \qquad \tilde{\delta}(p-q) \equiv (2\pi)^3 2E_p\delta^3(\vec{\mathbf{p}} - \vec{\mathbf{q}})$$

$$x^\mu = (t, \vec{\mathbf{x}}) \qquad p^\mu = (E, \vec{\mathbf{p}}) \qquad \partial^\mu = (\partial_t, -\vec{\nabla}) \qquad A^\mu = (\phi, \vec{\mathbf{A}}) \qquad J^\mu = (\rho, \vec{\mathbf{J}})$$

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu \qquad \mathscr{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} \qquad \partial_\mu F^{\mu\nu} = J^\nu \qquad \partial_\mu\mathscr{F}^{\mu\nu} = 0$$

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{If } (\mu\nu\rho\sigma) \text{ is an even permutation of } (0123)\\ -1 & \text{If } (\mu\nu\rho\sigma) \text{ is an odd permutation of } (0123)\\ 0 & \text{Otherwise}\end{cases}$$

$$\epsilon_{\alpha\beta_1\gamma_1\delta_1}\epsilon^{\alpha\beta_2\gamma_2\delta_2} = +g_{\beta_1}^{\beta_2}g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\gamma_2} - g_{\beta_1}^{\beta_2}g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\delta_2} + g_{\beta_1}^{\gamma_2}g_{\gamma_1}^{\beta_2}g_{\delta_1}^{\delta_2} - g_{\beta_1}^{\gamma_2}g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\beta_2} + g_{\beta_1}^{\delta_2}g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\beta_2} - g_{\beta_1}^{\delta_2}g_{\gamma_1}^{\beta_2}g_{\delta_1}^{\gamma_2}$$

$$\epsilon_{\alpha\beta\gamma_1\delta_1}\epsilon^{\alpha\beta\gamma_2\delta_2} = -2\left(g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\delta_2} - g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\gamma_2}\right) \qquad \epsilon_{\alpha\beta\gamma\delta_1}\epsilon^{\alpha\beta\gamma\delta_2} = -6g_{\delta_1}^{\delta_2}$$