Equations

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle \qquad (\partial\cdot\partial+m^2)\phi=0 \qquad (i\not\partial - m)\varphi = 0 \qquad \overline{\varphi}(i\not\partial+m)=0$$

$$E = (\vec{\alpha}\cdot\vec{\mathbf{p}}+\beta m)\varphi \implies i\frac{\partial\varphi}{\partial t} = (-i\vec{\alpha}\cdot\vec{\nabla}+\beta m)\varphi \implies (i\gamma^\mu\partial_\mu - m)\varphi = (i\gamma^\mu p_\mu - m)\varphi = 0$$

Pauli Spin Matrices & Dirac γ-Matrices Matrices

$$\sigma^1=\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix} \qquad \sigma^2=\sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix} \qquad \sigma^3=\sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix} \qquad \vec{\mathbf{\Sigma}}=\begin{pmatrix}\vec{\sigma}&0\\0&\vec{\sigma}\end{pmatrix}$$

$$\{\sigma^i,\sigma^j\}=2\delta^{ij} \qquad [\sigma^i,\sigma^j]=2i\epsilon^{ijk}\sigma^k \qquad \sigma^i\sigma^j=\delta^{ij}+i\epsilon^{ijk}\sigma^k \qquad \sigma^2\sigma^i\sigma^2=-(\sigma^i)^*$$

$$\gamma^\mu=\begin{pmatrix}0&\sigma^\mu\\\overline{\sigma}^\mu&0\end{pmatrix} \qquad \beta=\gamma^0=\begin{pmatrix}+\mathbb{1}&0\\0&-\mathbb{1}\end{pmatrix} \qquad \alpha_i=\begin{pmatrix}0&\sigma_i\\\sigma_i&0\end{pmatrix} \qquad \gamma^k=\begin{pmatrix}0&\sigma^k\\-\sigma^k&0\end{pmatrix}$$

$$\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu} \qquad \gamma_5=+i\gamma^0\gamma^1\gamma^2\gamma^3=-i\gamma_0\gamma_1\gamma_2\gamma_3=-\frac{i}{4!}\epsilon^{\alpha\beta\gamma\delta}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta \qquad \sigma^{\mu\nu}=\frac{i}{2}[\gamma^\mu,\gamma^\nu]$$

$$\not A=A^\mu\gamma_\mu \qquad \gamma^\mu\gamma_\mu=4 \qquad \gamma^\mu\gamma^\nu\gamma_\mu=-2\gamma^\nu \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu=4g^{\nu\rho} \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu=-2\gamma^\sigma\gamma^\rho\gamma^\nu$$

$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\pi\gamma_\mu=2[\gamma^\pi\gamma^\nu\gamma^\rho\gamma^\sigma+\gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\pi] \qquad \{\gamma_5,\gamma^\mu\}=0 \qquad \gamma^0(\gamma^\mu)^\dagger\gamma^0=\gamma^\mu$$

$$(\gamma^0)^\dagger=\gamma^0 \qquad (\gamma^0)^2=\mathbb{1} \qquad (\gamma^k)^\dagger=-\gamma^k \qquad (\gamma^k)^2=-\mathbb{1} \qquad (\gamma^5)^\dagger=\gamma^5 \qquad (\gamma^5)^2=\mathbb{1}$$

$$\overline{\psi}=\psi^\dagger\gamma^0 \qquad \overline{\Gamma}=\gamma^0\Gamma^\dagger\gamma^0 \qquad \overline{\gamma_5}=-\gamma_5 \qquad \overline{\gamma^\mu}=\gamma^\mu \qquad \overline{\gamma^\mu\gamma_5}=\gamma^\mu\gamma_5 \qquad \overline{\sigma^{\mu\nu}}=\sigma^{\mu\nu}$$

Spin, Helicity and Chirality

$$\vec{\mathbf{S}}=\frac{1}{2}\vec{\mathbf{\Sigma}} \qquad h=\frac{\vec{\mathbf{S}}\cdot\vec{\mathbf{p}}}{|\vec{\mathbf{p}}|} \qquad P_L=\frac{1-h}{2} \qquad P_R=\frac{1+h}{2} \qquad \gamma_L=\frac{1-\gamma_5}{2} \qquad \gamma_R=\frac{1+\gamma_5}{2}$$

$$\gamma_{L,R}^2=\gamma_{L,R} \qquad \gamma_{L,R}\gamma_{R,L}=0 \qquad \gamma_L+\gamma_R=\mathbb{1} \qquad \gamma^\mu\gamma_{L,R}=\gamma_{R,L}\gamma^\mu \qquad \gamma_5\gamma_L=-\gamma_L \qquad \gamma_5\gamma_R=\gamma_R$$

$$\gamma_{L,R}^\dagger=\gamma_{L,R} \qquad \overline{\gamma^\mu\gamma_{L,R}}=\gamma^\mu\gamma_{L,R} \qquad \overline{\gamma_5\gamma_L}=-\gamma_R \qquad \overline{\gamma_5\gamma_R}=\gamma_L \qquad \overline{\gamma_{L,R}}=\gamma_0\gamma_{L,R}^\dagger\gamma_0=\gamma_{R,L}$$

$$u_L=u_\downarrow \qquad u_R=u_\uparrow \qquad v_L=v_\uparrow \qquad v_R=v_\downarrow \qquad \vec{\mathbf{p}}=p(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)$$

$$P_Lu_\downarrow=u_\downarrow \quad P_Lu_\uparrow=0 \quad P_Ru_\downarrow=0 \quad P_Ru_\uparrow=u_\uparrow \quad \overline{u_\downarrow}P_L=0 \quad \overline{u_\uparrow}P_L=u_\uparrow \quad \overline{u_\downarrow}P_R=u_\downarrow \quad \overline{u_\uparrow}P_R=0$$

$$\gamma_Lu_L=u_L \quad \gamma_Lu_R=0 \quad \gamma_Ru_L=0 \quad \gamma_Ru_R=u_R \quad \overline{u_L}\gamma_L=0 \quad \overline{u_R}\gamma_L=u_R \quad \overline{u_L}\gamma_R=u_L \quad \overline{u_R}\gamma_R=0$$

$$P_Lv_\downarrow=0 \quad P_Lv_\uparrow=v_\uparrow \quad P_Rv_\downarrow=v_\downarrow \quad P_Rv_\uparrow=0 \quad \overline{v_\downarrow}P_L=v_\downarrow \quad \overline{v_\uparrow}P_L=0 \quad \overline{v_\downarrow}P_R=0 \quad \overline{v_\uparrow}P_R=v_\uparrow$$

$$\gamma_Lv_L=v_L \quad \gamma_Lv_R=0 \quad \gamma_Rv_L=0 \quad \gamma_Rv_R=v_R \quad \overline{v_L}\gamma_L=0 \quad \overline{v_R}\gamma_L=v_R \quad \overline{v_L}\gamma_R=v_L \quad \overline{v_R}\gamma_R=0$$

$$u_\uparrow^T=N\left(c \quad se^{i\phi} \quad kc \quad kse^{i\phi}\right)\approx N\left(c \quad se^{i\phi} \quad c \quad se^{i\phi}\right) \qquad N=\sqrt{E+m}\approx\sqrt{E}$$

$$u_\downarrow^T=N\left(-s \quad ce^{i\phi} \quad ks \quad -kce^{i\phi}\right)\approx N\left(-s \quad ce^{i\phi} \quad s \quad -ce^{i\phi}\right) \qquad k=\frac{p}{E+m}\approx 1$$

$$v_\uparrow^T=N\left(ks \quad -kce^{i\phi} \quad -s \quad ce^{i\phi}\right)\approx N\left(s \quad -ce^{i\phi} \quad -s \quad ce^{i\phi}\right) \qquad s=\sin(\theta/2)$$

$$v_\downarrow^T=N\left(kc \quad kse^{i\phi} \quad c \quad se^{i\phi}\right)\approx N\left(c \quad se^{i\phi} \quad c \quad se^{i\phi}\right) \qquad c=\cos(\theta/2)$$

Spinors (Fermions) & Polarization Vectors (Bosons)

$$\psi=ue^{+i(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}}-Et)}=ve^{-i(\vec{\mathbf{p}}\cdot\vec{\mathbf{x}}-Et)} \qquad (\not p-m)u=\overline{u}(\not p-m)=0=(\not p+m)v=\overline{v}(\not p+m)$$

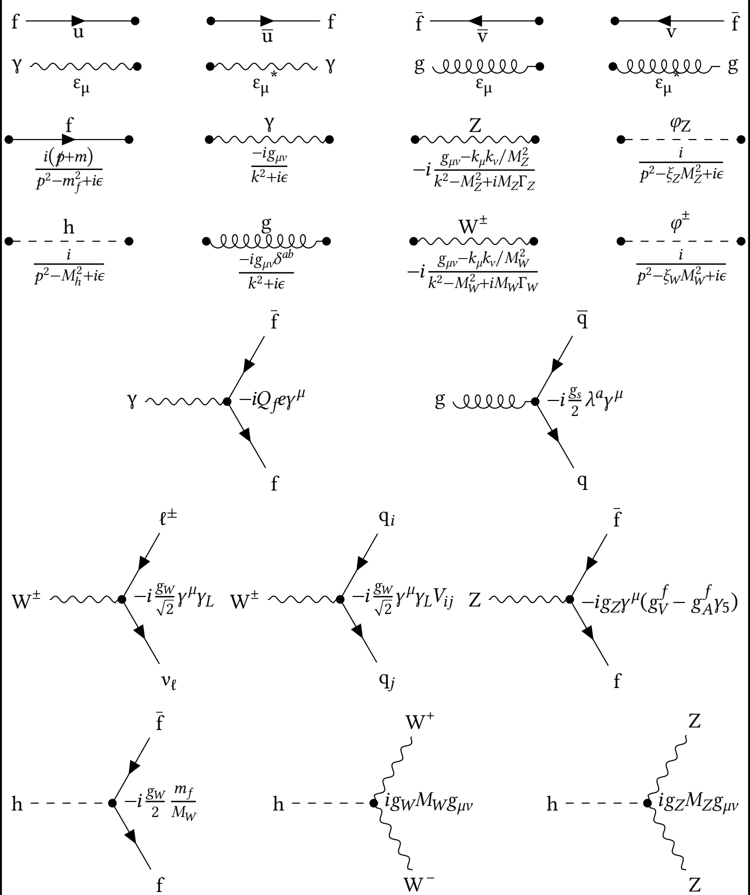
$$\overline{u}^r(p)u^s(p)=+2m\delta^{rs} \qquad \overline{u}^r(p)v^s(p)=0 \qquad u^{r\dagger}(p)u^s(p)=2E\delta^{rs} \qquad \sum_{s=1,2}u^s(p)\overline{u}^s(p)=\not p+m$$

$$\overline{v}^r(p)v^s(p)=-2m\delta^{rs} \qquad \overline{v}^r(p)u^s(p)=0 \qquad v^{r\dagger}(p)v^s(p)=2E\delta^{rs} \qquad \sum_{s=1,2}v^s(p)\overline{v}^s(p)=\not p-m$$

$$m=0: \sum_{\lambda=1}^{2}\epsilon^\mu(k,\lambda)\cdot\epsilon^{*\nu}(k,\lambda)=-g^{\mu\nu} \qquad m\neq 0: \sum_{\lambda=1}^{3}\epsilon_\mu(q,\lambda)\epsilon_\nu^*(q,\lambda)=-g_{\mu\nu}+\frac{q_\mu q_\nu}{M^2}$$

Feynman Rules for $i\mathscr{M}$

Goes in opposite way of arrows with the first one being adjoint, $\overline{\psi}=\psi^\dagger\gamma^0$:



Decay : Decay Rates

$$\frac{d\Gamma}{d\Omega}=\frac{1}{32\pi^2}\frac{|\vec{\mathbf{p}}_{CM}|}{m_0^2}\overline{|\mathscr{M}|^2} \qquad |\vec{\mathbf{p}}_{CM}|=\frac{1}{2m_0}\sqrt{\left[m_0^2-(m_1+m_2)^2\right]\left[m_0^2-(m_1-m_2)^2\right]}$$

$$E_1=\frac{m_0^2+m_1^2-m_2^2}{2m_0} \qquad E_2=\frac{m_0^2+m_2^2-m_1^2}{2m_0}$$

Scattering : Cross Sections

$$\frac{d\sigma}{d\Omega}=\frac{1}{64\pi^2 s}\frac{|\vec{\mathbf{p}}_3|}{|\vec{\mathbf{p}}_1|}\overline{|\mathscr{M}|^2} \qquad |\vec{\mathbf{p}}_a|=|\vec{\mathbf{p}}_b|=\frac{1}{2\sqrt{s}}\sqrt{\left[s-(m_a+m_b)^2\right]\left[s-(m_a-m_b)^2\right]}$$

$$E_1=\frac{s+m_1^2-m_2^2}{2\sqrt{s}} \qquad E_2=\frac{s+m_2^2-m_1^2}{2\sqrt{s}} \qquad E_3=\frac{s+m_3^2-m_4^2}{2\sqrt{s}} \qquad E_4=\frac{s+m_4^2-m_3^2}{2\sqrt{s}}$$

$$s=(p_1+p_2)^2=(p_3+p_4)^2\approx+2p_1\cdot p_2\approx+2p_3\cdot p_4 \qquad \sqrt{s}=E_{CM}$$

$$t=(p_1-p_3)^2=(p_2-p_4)^2\approx-2p_1\cdot p_3\approx-2p_2\cdot p_4 \qquad s+t+u=m_1^2+m_2^2+m_3^2+m_4^2$$

$$u=(p_1-p_4)^2=(p_2-p_3)^2\approx-2p_1\cdot p_4\approx-2p_2\cdot p_3$$

$$m=0: s=2|\vec{\mathbf{p}}_{CM}|^2=2(p_{CM}^0)^2 \qquad t=-\frac{s}{2}(1-\cos\theta) \qquad u=-\frac{s}{2}(1+\cos\theta)$$

Constants

$$\frac{G_F}{\sqrt{2}}=\frac{g_W^2}{8M_W^2}=\frac{g_W^2}{8M_Z^2\cos^2(\theta_W)}=\frac{1}{2v^2} \qquad M_W=M_Z\cos\theta_W=\frac{1}{2}g_Wv \qquad M_H=\sqrt{2\lambda}v$$

$$g_W=g_Z\cos\theta_W=\frac{e}{\sin\theta_W} \qquad g_V^f=\frac{1}{2}T_f^3-Q_f\sin^2(\theta_W) \qquad g_A^f=\frac{1}{2}T_f^3 \qquad g_{A,V}=\frac{1}{2}c_{A,V}$$

$$\alpha=\frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c} \qquad c_L^f=g_V^f+g_A^f \qquad c_R^f=g_V^f-g_A^f \qquad g_V^f=\frac{c_L^f+c_R^f}{2} \qquad g_A^f=\frac{c_L^f-c_R^f}{2}$$

Fermions	Q_f	$I_W^{(3)}$	Y_L	Y_R	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	+1/2	−1	0	+1/2	0	+1/2	+1/2
e^-, μ^-, τ^-	−1	−1/2	−1	−2	−0.27	+0.23	−0.04	−1/2
u, c, t	+2/3	+1/2	+1/3	+4/3	+0.35	−0.15	+0.19	+1/2
d, s, b	−1/3	−1/2	+1/3	−2/3	−0.42	+0.08	−0.35	−1/2

$$Q=I_3+\frac{1}{2}(A+S+C+B+T) \qquad A=\frac{1}{3}\left(n_q-n_{\overline{q}}\right) \qquad Y=A+S+C+B+T$$

$$C=n_c-n_{\overline{c}} \qquad S=-(n_s-n_{\overline{s}}) \qquad T=n_t-n_{\overline{t}} \qquad B=-(n_s-n_{\overline{s}})$$

Particle	$W^\pm$	Z^0	H^0	$e^\pm$	$\mu^\pm$	τ	u	d	c	s	t	b
Mass (MeV)	80 379	91 188	125 100	0.511	105.7	1777	2.16	4.67	1270	93	172 760	4180

$$1\,\text{kg}=5.61\times10^{26}\,\text{GeV} \qquad 1\,\text{m}=55.07\times10^{15}\,\text{GeV}^{-1} \qquad 1\,\text{s}=51.52\times10^{24}\,\text{GeV}^{-1}$$

Relativity & Quantum Mechanics

$$p^\mu\to i\delta^\mu$$

$$\beta=\frac{v}{c}=\frac{p}{E}=\tanh(\eta) \qquad \gamma=\frac{1}{\sqrt{1-\beta^2}}=\frac{E}{m}=\cosh(\eta) \qquad \gamma\beta=\frac{p}{m}=\sin(\eta)$$

$$E^2=m^2+|p|^2 \qquad \frac{d^3\vec{\mathbf{p}}'}{E'}=\frac{d^3\vec{\mathbf{p}}}{E} \qquad \widetilde{dp}\equiv\frac{d^3\vec{\mathbf{p}}}{(2\pi)^3 2E_p} \qquad \tilde{\delta}(p-q)\equiv(2\pi)^3 2E_p\delta^3(\vec{\mathbf{p}}-\vec{\mathbf{q}})$$

$$x^\mu=(t,\vec{\mathbf{x}}) \qquad p^\mu=(E,\vec{\mathbf{p}}) \qquad \partial^\mu=\left(\partial_t,-\vec{\nabla}\right) \qquad A^\mu=\left(\phi,\vec{\mathbf{A}}\right) \qquad J^\mu=\left(\rho,\vec{\mathbf{J}}\right)$$

$$F^{\mu\nu}\equiv\partial^\mu A^\nu-\partial^\nu A^\mu \qquad \mathscr{F}^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} \qquad \partial_\mu F^{\mu\nu}=J^\nu \qquad \partial_\mu\mathscr{F}^{\mu\nu}=0$$

$$\epsilon_{\alpha\beta_1\gamma_1\delta_1}\epsilon^{\alpha\beta_2\gamma_2\delta_2}=+g_{\beta_1}^{\beta_2}g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\gamma_2}-g_{\beta_1}^{\beta_2}g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\delta_2}+g_{\beta_1}^{\gamma_2}g_{\gamma_1}^{\beta_2}g_{\delta_1}^{\delta_2}-g_{\beta_1}^{\gamma_2}g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\beta_2}+g_{\beta_1}^{\delta_2}g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\beta_2}-g_{\beta_1}^{\delta_2}g_{\gamma_1}^{\beta_2}g_{\delta_1}^{\gamma_2}$$

$$\epsilon_{\alpha\beta\gamma_1\delta_1}\epsilon^{\alpha\beta\gamma_2\delta_2}=-2\left(g_{\gamma_1}^{\gamma_2}g_{\delta_1}^{\delta_2}-g_{\gamma_1}^{\delta_2}g_{\delta_1}^{\gamma_2}\right) \qquad \epsilon_{\alpha\beta\gamma\delta_1}\epsilon^{\alpha\beta\gamma\delta_2}=-6g_{\delta_1}^{\delta_2}$$

Other

$$\text{Luminosity}: L_{\text{int}}=\int\mathscr{L}(t)\,dt \qquad \mathscr{L}(t)=\frac{1}{\sigma}\frac{dN}{dt}=f\cdot\frac{n_1\cdot n_2}{4\pi\sigma_x\sigma_y} \qquad N=\sigma L_{\text{int}}$$

$$\text{Beam Current}: I_i=N_i\cdot e\cdot f\cdot b$$

$$\text{Bethe-Bloch}: \frac{dE}{dx}=-Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\log\left(\frac{2\beta^2\gamma^2m_ec^2}{I}\right)-\beta^2\right] \qquad I=10\,\text{eV}\cdot Z$$

$$\text{Bremsstrahlung }(\beta\gamma\sim1000): \frac{dE}{dx}=-\frac{4\alpha^3N_A}{m_e^2}\frac{EZ^2}{A}\ln\left(184.15Z^{-1/3}\right)$$

$$\text{Cherenkov}: \frac{\partial^2N}{\partial E\partial x}=\frac{\alpha^2z^2\sin^2(\theta_c)}{r_em_ec^2}=\frac{\alpha^2z^2}{r_em_ec^2}\left(1-\frac{1}{\beta^2n^2}\right)\approx370z^2\sin^2(\theta_c)(\text{eV cm})^{-1}$$

$$\text{Deviation of scattered angle from Coulomb}: \theta=\frac{13.6}{\beta p}\frac{q\Delta L}{X_0}\left[1+0.0038\ln\left(\frac{\Delta L}{X_0}\right)\right]$$