Hw06

Task 6a

- Verifying if the *7.112* equation holds true.

To complete this task, I created the *check_7112* function which firstly computed the R21 matrix by multiplying R2 and R1 and then RR21 using s21v21 and c21 (using the Maple script). After subtracting R21 from RR21 the result will be the zero matrix.

- Creating *R_from_theta_half_axis* function

Firstly, it computes the v_x matrix and then finds theta angle from theta_half. After computing sin and cos of theta, we can easily find R from Rodrigues' formula.

Task 6b

- Checking *7.116* equation

I found q1 from c1 and s1v1 and q2 from c2 and s2v2. After that I could also find q21 from c21 and s21v21. Using 7.116 equation, I computed qq21 by multiplying q2 with q1 as shown in the picture bellow.

$$\begin{bmatrix} q_1 & -q_2 & -q_3 & -q_4 \\ q_2 & q_1 & -q_4 & q_3 \\ q_3 & q_4 & q_1 & -q_2 \\ q_4 & -q_3 & q_2 & q_1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

Therefore, it can be verified that qq21-q21 gives us the zero vector.

- *R_from_q* function

This function has as input parameter a quaternion and I used the parameters from the vector to compute the R matrix using *7.66* formula as shown below.

$$\begin{bmatrix} 1 & -2\,q_1q_4 & 2\,q_1q_3 \\ 2\,q_1q_4 & 1 & -2\,q_1q_2 \\ -2\,q_1q_3 & 2\,q_1q_2 & 1 \end{bmatrix} + 2 \begin{bmatrix} -q_3^2 - q_4^2 & q_2q_3 & q_2q_4 \\ q_2q_3 & -q_2^2 - q_4^2 & q_3q_4 \\ q_2q_4 & q_3q_4 & -q_2^2 - q_3^2 \end{bmatrix}$$

- *q_from_R* function

For finding cos of theta, I used the following equation $\frac{trace(R)-1}{2}$. After finding the angle, I applied the following equation for the vector v.

$$\sin\theta\,\vec{v} = \frac{1}{2}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

Using these parameters, I could also compute the quaternion.

Task 6c

- Finding q1 and R1 for the $\frac{\pi}{2}$ rotation around x-axis.

The representation of the rotation matrix around the x-axis is the following,

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

so, we could use *q_from_R* function to find its correspondent quaternion.

- Finding q2 and R2 for the $\frac{\pi}{2}$ rotation around y-axis.

The representation of the rotation matrix around the y-axis is the following,

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

so, we could use *q_from_R* function to find its correspondent quaternion.

- q21

Using the composition of quaternions from *7.116* formula, I computed q21 from q2 and q1.

- R21

R21 was found by a simple matrix multiplication between R2 and R1.

- R21 from q21

Using *R_from_q* function I constructed the R21 matrix from the q21 quaternion which was the same as the previous R21 matrix.

- q21 from R21

Using *q_from_R* function I constructed the q21 quaternion from the R21 matrix which was the same as the previous q21 quaternion.