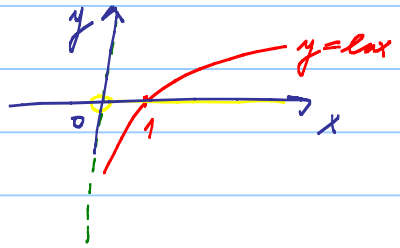
Note Title 16. 9. 2024

① $f(x) = \sqrt[3]{2 - \ln x}$ $\quad x > 0$

$D(f): (0, \infty)$



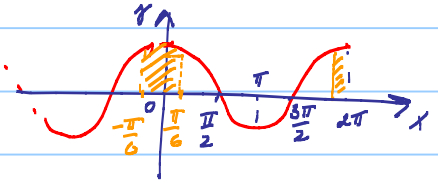
② $g(x) = \ln(2\cos x - \sqrt{3})$

$2\cos x - \sqrt{3} > 0$ $\quad \cos x = \frac{\sqrt{3}}{2} \ldots x = \frac{\pi}{6}$

$2\cos x > \sqrt{3}$

$\cos x > \frac{\sqrt{3}}{2}$

$D(g): \left\{ \left(-\frac{\pi}{6} + 2k\pi, \frac{\pi}{6} + 2k\pi \right); k \in \mathbb{Z} \right\}$



③ $h(x) = \sqrt{2 - \log_{\frac{1}{2}} x}$

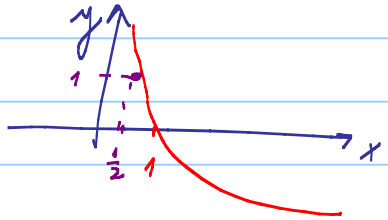
$2 - \log_{\frac{1}{2}} x \geq 0 \quad \wedge \quad x > 0$

$-\log_{\frac{1}{2}} x \geq -2$

$\log_{\frac{1}{2}} x \leq 2$ $\quad$!

$x \geq \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

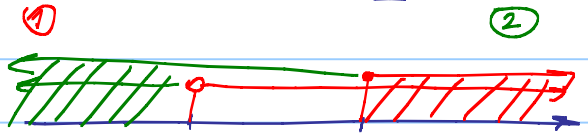
$D(h): \left\langle \frac{1}{4}, \infty \right)$



④ $i(x) = \sqrt{\frac{x-1}{x+1}}$ $\quad \frac{x-1}{x+1} \geq 0$

$(x - 1 \geq 0 \wedge x + 1 > 0) \cup (x - 1 \leq 0 \wedge x + 1 < 0)$

$(x \geq 1 \wedge x > -1) \cup (x \leq 1 \wedge x < -1)$



$D(i): (-\infty, -1) \cup \langle 1, \infty)$

⑤ $j(x) = \arcsin(2x+1)$

$-1 \leq 2x + 1 \leq 1$

$-1 \leq 2x + 1 \quad \wedge \quad 2x + 1 \leq 1$

$2x + 1 \geq -1$ $\quad 2x \leq 0$

$2x \geq -2$ $\quad x \leq 0$

$x \geq -1$

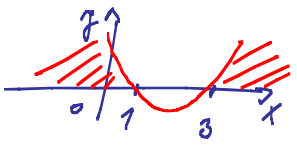
$D(j): \langle -1, 0 \rangle$

⑥ $k(x) = \operatorname{arctg} \frac{1}{\sqrt{x^2-4x+3}}$

$x^2-4x+3 > 0$

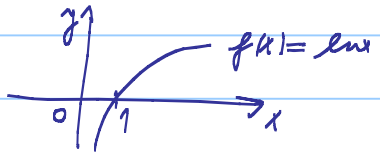
$(x-3)(x-1) > 0$

$3 \quad 1$

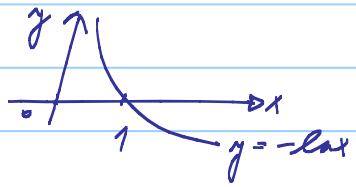


$D(k): (-\infty, 1) \cup (3, \infty)$

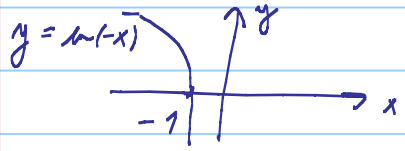
PR ① $f(x) = \ln x$



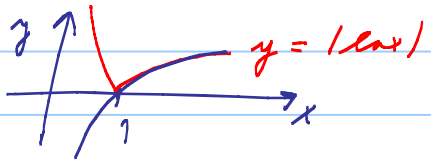
② $g(x) = -\ln x$



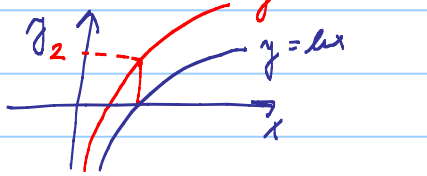
③ $a(x) = \ln(-x)$



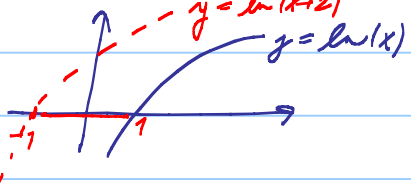
④ $i(x) = |\ln x|$



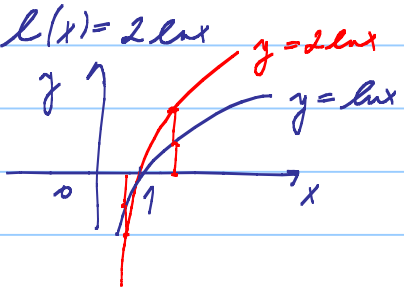
⑤ $j(x) = \ln(x) + 2$



⑥ $l(x) = \ln(x+2)$



⑦ $l(x) = 2\ln x$



PR $f(x) = \frac{x^3-3}{2+x^3}$ $\quad 2+x^3 \neq 0$ $\quad D(f): \mathbb{R} - \{\sqrt[3]{-2}\}$

$x^3 \neq -2$

$f(x_1) \stackrel{?}{=} f(x_2)$

$\frac{x_1^3-3}{2+x_1^3} = \frac{x_2^3-3}{2+x_2^3}$

$(x_1^3-3)(2+x_2^3) = (x_2^3-3)(2+x_1^3)$

$2x_1^3 + x_1^3x_2^3 - 6 - 3x_2^3 = 2x_2^3 + x_2^3x_1^3 - 6 - 3x_1^3$

$5x_1^3 = 5x_2^3$

FUKCIA $f(x)$ JE PROSTÁ

$x_1 = x_2 \Rightarrow$

$f^{-1}: x = \frac{y^3-3}{2+y^3} \Rightarrow 2x + xy^3 = y^3 - 3$

$xy^3 - y^3 = -2x - 3$

$y^3(x-1) = -2x-3$

$y^3 = \frac{-2x-3}{x-1} \qquad x \neq 1$

$y = \sqrt[3]{\frac{2x+3}{1-x}}$

(b) $g(x) = \frac{x^2-3}{2+x^2}$

$2 + x^2 \neq 0 \qquad D(g): \mathbb{R}$

$x^2 \neq -2$

$g(x_1) \overset{?}{=} g(x_2)$

$\frac{x_1^2-3}{2+x_1^2} = \frac{x_2^2-3}{2+x_2^2}$

$2x_1^2 + x_1^2x_2^2 - 6 - 3x_2^2 = 2x_2^2 - 6 + x_1^2x_2^2 - 3x_1^2$

$5x_1^2 = 5x_2^2$

$x_1^2 = x_2^2 \Rightarrow$ FUNKCIA NIE JE PROSTÁ

(c) $h(x) = 5 + \ln(x+1) \qquad D(h): (-1, \infty)$

$x + 1 > 0$

$x > -1$

$h(x_1) \overset{?}{=} h(x_2)$

$5 + \ln(x_1+1) = 5 + \ln(x_2+1)$

$x_1 + 1 = x_2 + 1$

$x_1 = x_2 \Rightarrow$ FUNKCIA h JE PROSTÁ

$h^{-1}: \quad x = 5 + \ln(y+1)$

$x - 5 = \ln(y+1)$

$e^{\ln(y+1)} = e^{(x-5)}$

$y + 1 = e^{x-5}$

$y = e^{x-5} - 1$