EXPLORING FRACTAL DIMENSION
DRP SPRING 2025

Maxwell Seay and Aaron Huntley
February 27, 2025

Contents

1. Preface

2. Sketch

2.1. Measure

2.2. Dimension

2.3. Examples of fractal/fractal-like sets

2.4. Claims/proofs

3. Cantor Set

4. Fractal Construction

5. Notions of Measure

6. Notions of Dimension

6.1. Netto's Theorem

References

To cite in line use the command [Fal85], if there is a paper you need the citation for go to https://zbmath.org and copy the bibtex into the citations file

1. Preface

2. Sketch

2.1. **Measure.**

- Hausdorff measure
- Lebesgue measure

2.2. **Dimension.**

- Hausdorff dimension
- Topological dimension

2.3. **Examples of fractal/fractal-like sets.**

- Kock curve
- Dragon curves
- Osgood curves
- Cantor set
- DeRham curve
- Space-filling curves

2.4. **Claims/proofs.**

- Lebesgue measure of cantor set is 0.
- Hausdorff dimension of the cantor set is $\frac{\ln(2)}{\ln(3)}$.
- All space filling curves have positive lebesgue measure.
- Netto's theorem.
- The Hausdorff dimension of a space filling curve is equal to the dimension of its codomain.
- Osgood curves are not space filling and don't have Hausdorff dimension 2.
- All Osgood curves have positive Lebesgue measure.

3. Cantor Set

The cantor set is a bounded set of real numbers which lies on the interval $[0, 1]$. It contains infinite elements.

4. Fractal Construction

5. Notions of Measure

6. Notions of Dimension

6.1. **Netto's Theorem.** If you have a continuous bijection it *can't* hop dimensions.

"If one relaxes the requirement of continuity, then all smooth manifolds of bounded dimension have equal cardinality, the cardinality of the continuum. Therefore, there exist discontinuous bijections between any two of them, as Georg Cantor showed in 1878."

You can use environments such as:

Theorem 6.1 (Netto's theorem)**.**

and

Definition 6.2 (Hausdorff measure)**.**

References

[Fal85] K. J. Falconer, *The geometry of fractal sets*, Camb. Tracts Math., vol. 85, Cambridge University Press, Cambridge, 1985 (English).