EXPLORING FRACTAL DIMENSION
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Maxwell Seay and Aaron Huntley

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- Lebesgue measure

2.2. Dimension.

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- Kock curve
- Dragon curves
- Osgood curves
- Cantor set
- DeRham curve
- Space-filling curves

2.4. Claims/proofs.

- Lebesgue measure of cantor set is 0.
- Hausdorff dimension of the cantor set is $\frac{\ln(2)}{\ln(3)}$.
- All space filling curves have positive lebesgue measure.
- Netto's theorem.
- The Hausdorff dimension of a space filling curve is equal to the dimension of its codomain.
- Osgood curves are not space filling and don't have Hausdorff dimension 2.
- All Osgood curves have positive Lebesgue measure.

3. Cantor Set

The cantor set is a set of real numbers. It lies on the interval $[0, 1]$. Its has an infinite number of elements.

4. Fractal Construction

5. Notions of Measure

6. Notions of Dimension

6.1. **Netto's Theorem.** If you have a continuous bijection it *can't* hop dimensions.

"If one relaxes the requirement of continuity, then all smooth manifolds of bounded dimension have equal cardinality, the cardinality of the continuum. Therefore, there exist discontinuous bijections between any two of them, as Georg Cantor showed in 1878."

You can use environments such as:

Theorem 6.1 (Netto's theorem)**.**

and

Definition 6.2 (Hausdorff measure)**.**

References

[Fal85] K. J. Falconer, *The geometry of fractal sets*, Camb. Tracts Math., vol. 85, Cambridge University Press, Cambridge, 1985 (English).