Script_Layer_thickness

June 6, 2024

1 Boundaries layer Thickness

The objectives of this script are to compare the different scaling, and theory to the measurement made one the Coriolis Plateform at LEGI for Spin up experiments.

Here we will define several Layer thicknesses describing at least 3 differents regimes:

- The classical boundary layer
- The Von karman layer (Rotating disk theory)
- The Turbulent Ekman layer

Briefly, we expect an initial growth of the Boundarie layer with two different behaviours depending on whether the water layer is stratified or not.

Then a limit of this growth determined by the diffusion of the momentum (Von Karman regime)

And finally the impact of the rotational effects on the flow (Turbulent Ekman regime)

The aim is to characterise the effects of stratification and rotation regimes on boundary layer dynamics and to validate our experimental system.

1.0.1 Outline of the script

1. A Definition of Boundary layer δ_1
2. Momentum thickness
3. Ekman Boundary layer
4. Ekman spirale

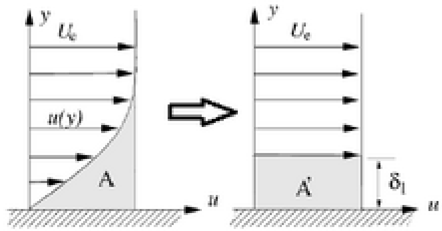
Designation	T_{ini}s	T_{end}s	ΔT °C	N	f	*Comment*	*Temporal resolution PIV*
EXP01	0	120	0		0	Fail in camera acquisition	
EXP02	0	120	0		0		
EXP03	120	60	0		0.104		
EXP04	100	54.5	0		0.125		Frequence btwn burts 1/s

Designation	T_{ini}s	T_{end}s	ΔT °C	N	f	*Comment*	*Temporal resolution PIV*
EXP05	80	48	0		0.157	Fail in camera acquisition	Frequence btwn burts 4/s
EXP06	80	48	0		0.157		Frequence btwn burts 4/s
EXP07	0	120	0		0		Frequence btwn burts 5/s
EXP08	0	120	9°C		0	optical refraction / mixing	Frequence btwn burts 5/s
EXP09	120	60	0		0.104		Frequence btwn burts 5/s
EXP10	120	60	12°C		0.104	optical refraction / mixing	Frequence btwn burts 5/s
EXP11	98	541	12°C		0.128	Spin down	Frequence btwn burts 5/s
EXP12	0	120	19°C	0.24	0	Test Stereo x Stratif	Frequence btwn burts 5/s
EXP13	0	120	0		0	Stereo	Frequence btwn burts 5/s
EXP14	0	120	0		0	Stereo	Frequence btwn burts 5/s
EXP15	0	120	0		0	Stereo + Colorant	Frequence btwn burts 5/s
EXP16	120	60	20°C		0.104	Stereo + Colorant	Frequence btwn burts 5/s
EXP17	100	54.5	5°C		0.125	Stereo + Colorant / Not enough particles to PIV	Frequence btwn burts 5/s

The sliding averages is done one a 3 second period to decompose the component due to the mean flow $\overline{u}$ and the turbulent fluctuating part u'

2 Boundary layer approximation

2.1 First definition : Displacement thickness



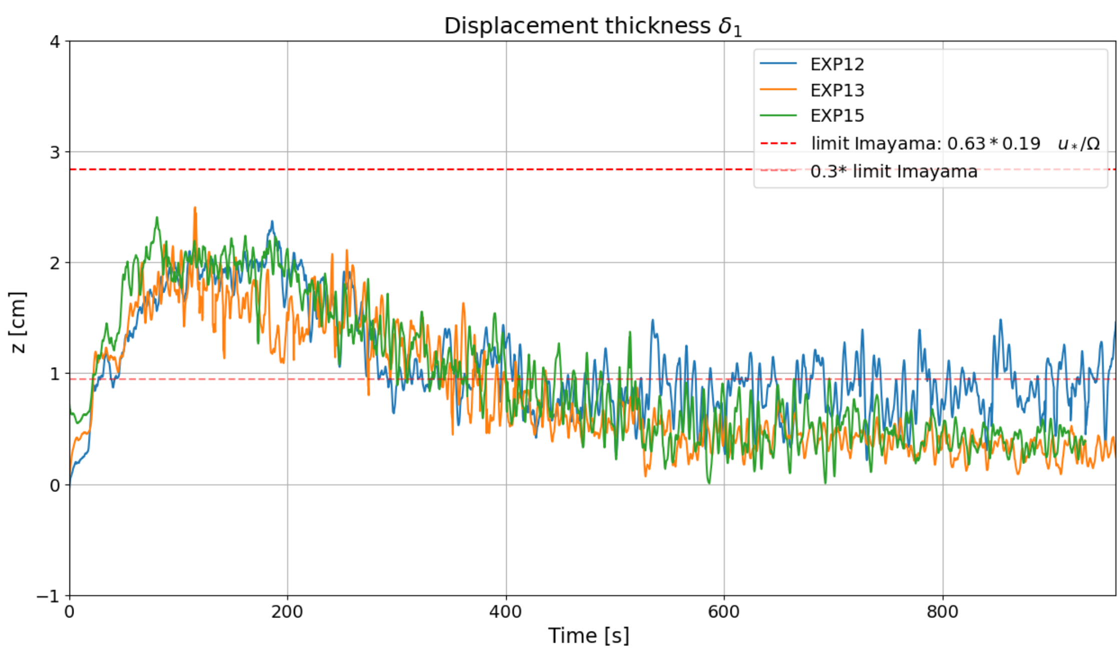
Source :https://fr.wikipedia.org/wiki/Couche_limite#/media/Fichier:Param%C3%A8tres_caract%C3%A9ristiqu

If we consider A and A’ to be surfaces such that : $A = *{0}^{∞} (U{∞}* - u) dy = A’ = *{1} U{∞}* $.

Assuming $A = A'$, the *displacement thickness* can be written as

$$\delta_1 = \frac{\int_0^\infty (U_\infty - u)}{U_\infty} dy$$

Reference : Course *“Turbulence, diffusion and transport: Master Environmental Fluid Mechanics”* **Joel Sommeria** p p90



We can see that for all the spin up experiments **without initial rotation** there is initially an increase in displacement thickness and then stabilisation.

The initial growth seems to follow a $\sqrt{t}$ law, (see the following log-log graph) and, as expected, is independant on the stratification. The conservation of momentum for a rectiligne mouvement would require a linear growth wich is not the case, that means the cylindrical shape of the plateform induce a convertion of the momentum. (**Maybe try to plot the sum of the displacement thikness for u and for v ?**)

This state of equilibrium seems to be more variable for Exp 12 and to a lesser extent EXP 15. These two experiments have a stratification (N(EXP12) $\gg$ N(EXP12)). We could therefore think of ossilation due to internal waves, **this remains to be confirmed**.

The red curves is the empirical limits of the growth of the displacement thikness above a rotating-disk

$$\delta_1 = 0.63 * 0.19 \quad u_*/\Omega$$

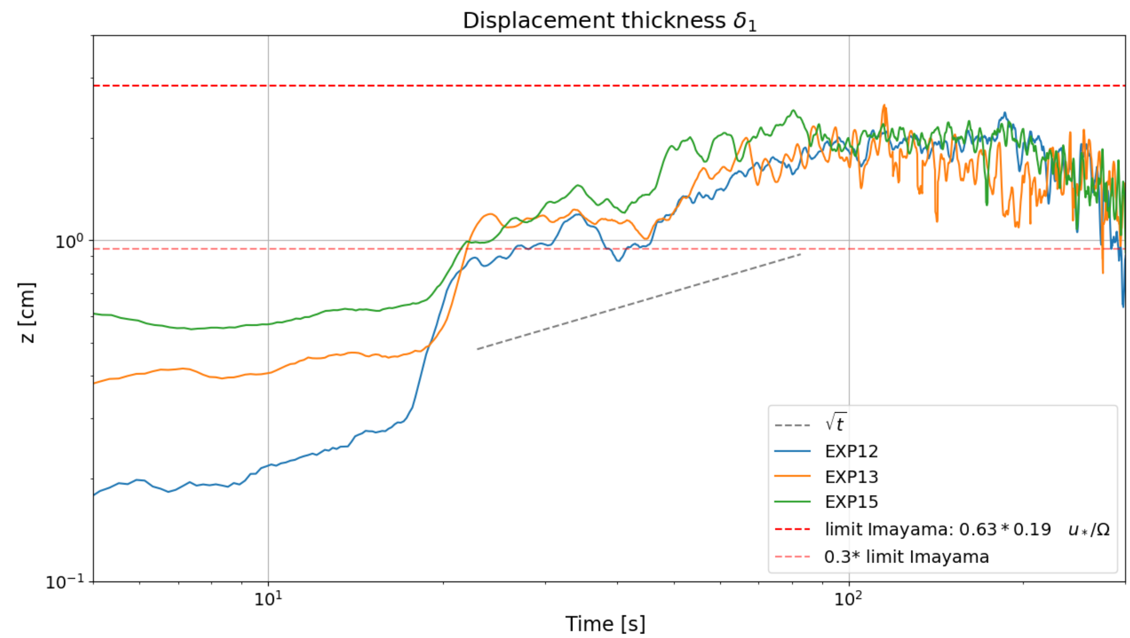
(**ref:** Table1 *Imayama et al., 2014*)

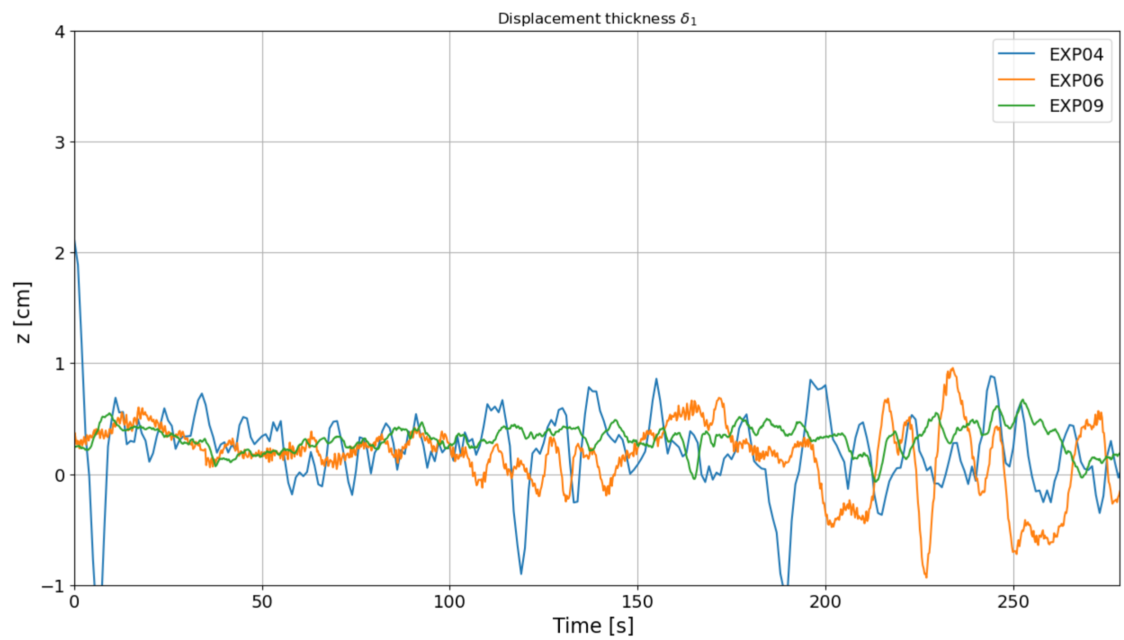
The lighter red curve is the same limit with a factor 1/3 which we also observe bellow when we compare the **Pollard's growth law** and the the initial growth of EXP12.

All three curves are stoped around 2cm by the diffusion of momentum, then decrease more linearly, until it reach a quasi thershold (the slight decay is certainly to the slow decay of u_*)

There is two ways to consider that:

1. We consider the normal Imayama limit then **the stabilisation of the turbulent boundary layer by the effect of rotation occurs earlier (is lower) in our case of a closed geometry** than in the case of a rotating disc and therefore with open edges. and an other regim would explaine what happend after 200s
2. We consider the 0.3 Imayama limit at long time and this bump can be explained by a somewhat *diffusion of the rotation effect*. By this I mean it taks time for the entire layer to "feel" rotation and thus there is an continuous adjustment (growth) of ω since we are reasoning in an integral way. And therefore of a progressive lowering of this limit to reach this final value





The interpretation of the spinning case is much more complicated because in the definition of the *thickness of displacement* we calculate the integral of the area A but in the case of a spin up with rotation we have a maximum speed which is located near the wall, (a bulge) which is faster than the speed at infinity. This is why we can find negative values.

However, when the values are positive, they are of the same order as those found in the graphs above.

2.2 Second definition : δ_{95}

Here we define the height limit δ_{95} such that $\overline{u}(z = \delta_{95}) = 95\% U_\infty$.
The shape of the velocity profile do not allow me to use the usual 99% criterion .

- For initial growth, in the stratified case we expect to find **Pollard's law** (1973):

$$h(t) = \frac{u_*}{\sqrt{Nf}} \left[4(1 - \cos(ft))\right]^{1/4} \quad t < \frac{T_f}{2}$$
$$h(t) = 1.7 \left(\frac{u_*}{\sqrt{Nf}}\right) \quad t > \frac{T_f}{2}$$

with $u_* = \sqrt{\frac{\tau}{\rho}}$ the friction velocity at the surface and we use the notation

$$L_{P73} = \frac{u_*}{\sqrt{Nf}}$$

as characteristic length from the P73's scaling law.

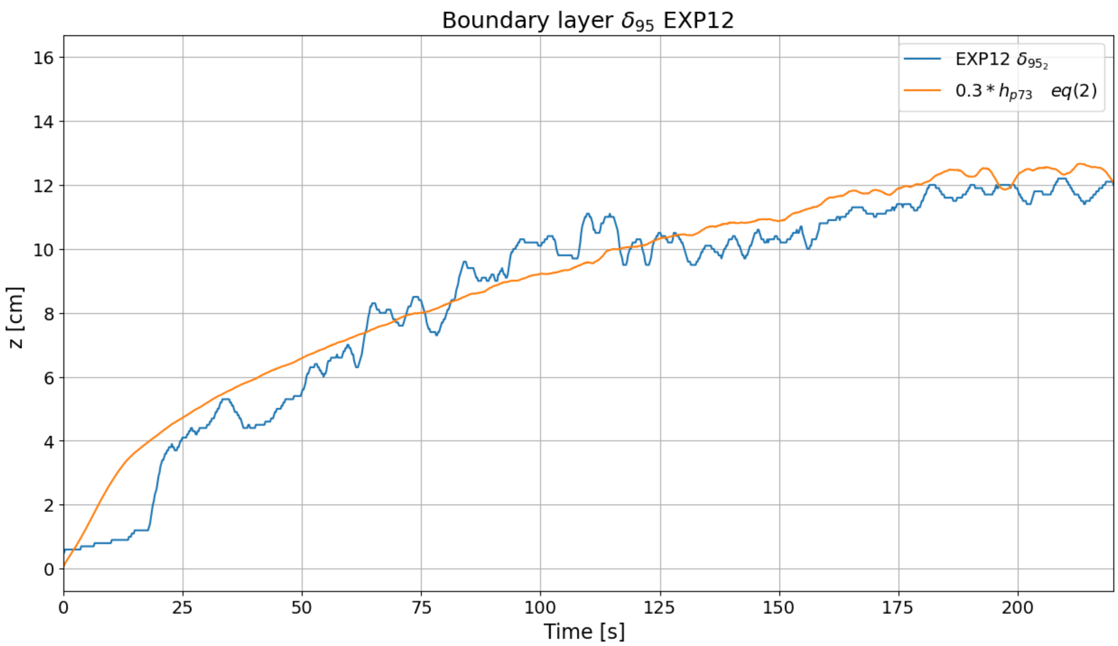
We also define from the equation (3.1) in the paper of P73 a characteristic velocity of the ML

$$U_{p73} = \frac{u_*^2}{f L_{p73}} = u_* \sqrt{\frac{N}{f}}.$$

P73 reduced to

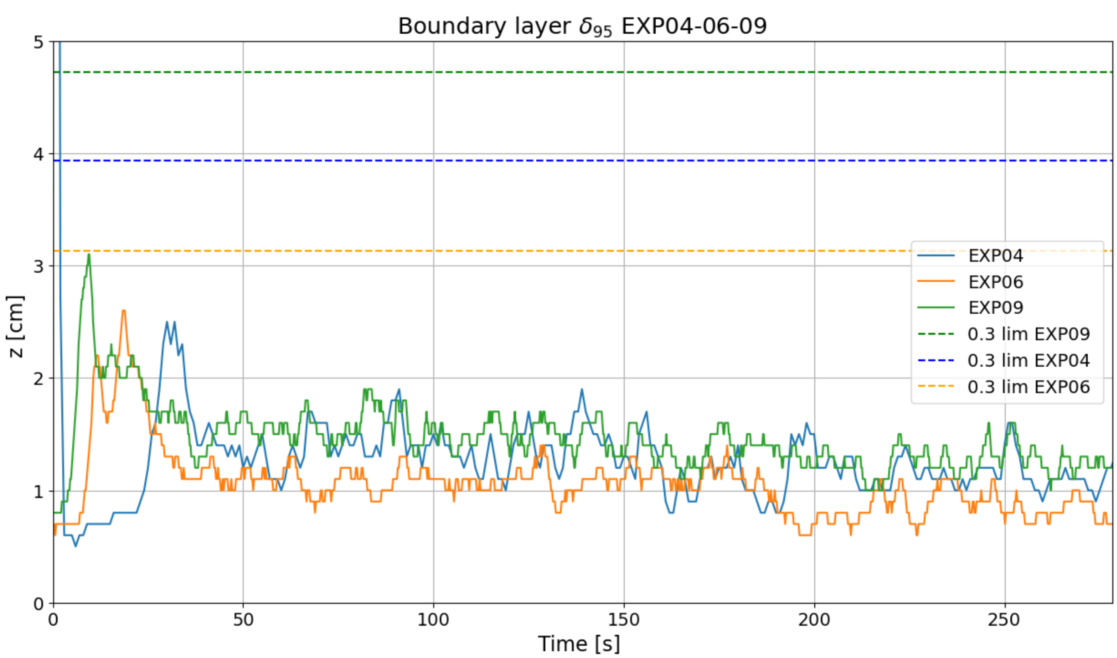
$$h(t) = u_* \left(2\right)^{1/4} \sqrt{\frac{t}{N}}$$

```
/Users/maxcoppin/anaconda3/envs/gotm/lib/python3.9/site-
packages/xarray/core/computation.py:808: RuntimeWarning: invalid value
encountered in sqrt
  result_data = func(*input_data)
```



The plot above shows the initial growth of the δ_{95} height for EXP12. This experiment was a spin up with no initial rotation and a stratification of the order of $N = 0.24$ over the first 30 cm. The orange curve represents the theoretical value obtained from equation (2) **with a 1/3 factor**. In spite of this factor, we still find a law in $t^{1/2}$.

I cannot continue the graph after 220 sec because the growth continue and it exeed the limit of the observed domain and thus the algorithm crash.



There, either my algorithm fail, either the Imuyama limits fail. I'd lean more towards the first option

3 Momentum thickness

We can define

Reference : Course *"Turbulence, diffusion and transport: Master Environmental Fluid Mechanics"* **Joel Sommeria** p p94

4 Ekman Boundary layer

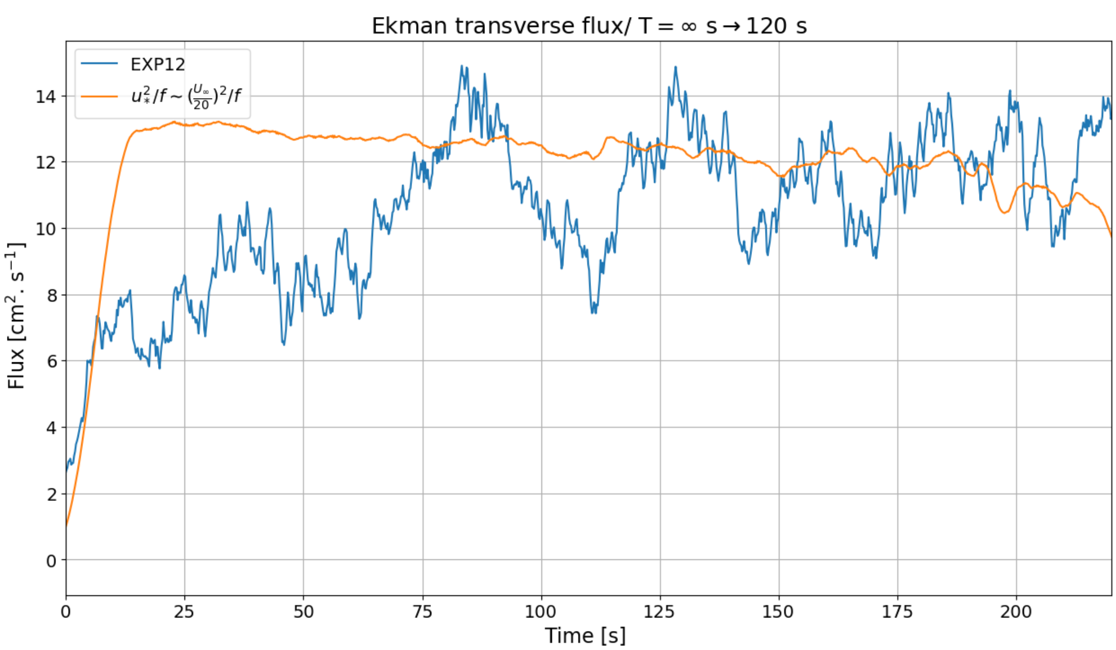
Reference : - *Sous et al., 2013*

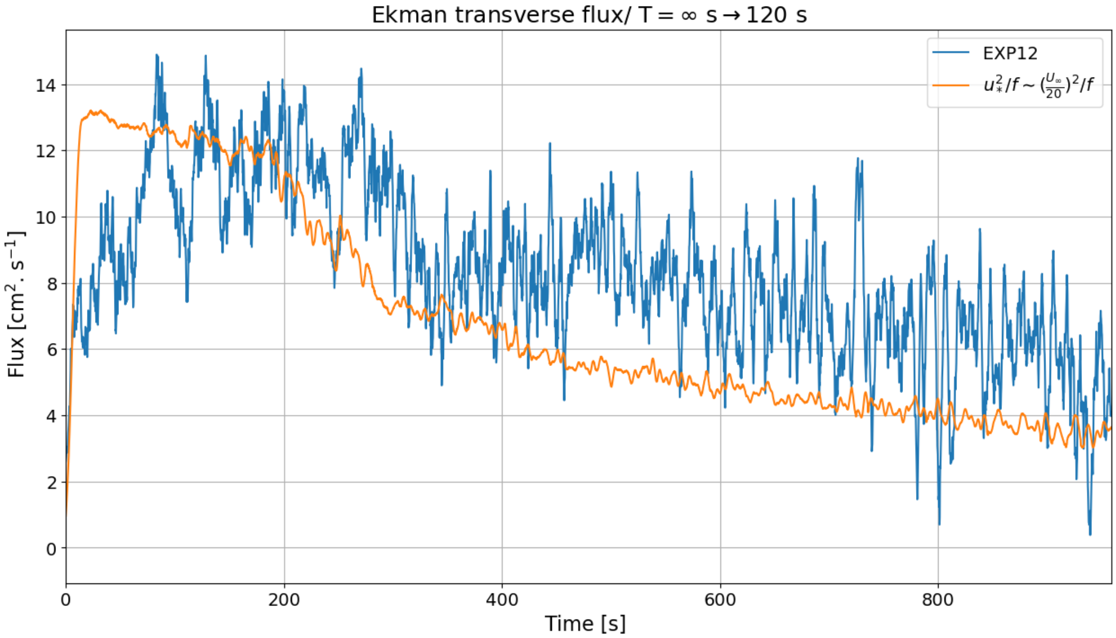
Ekman transverse flux :

$$\int_0^\infty u_r dz = -\frac{u_*^2}{f}$$

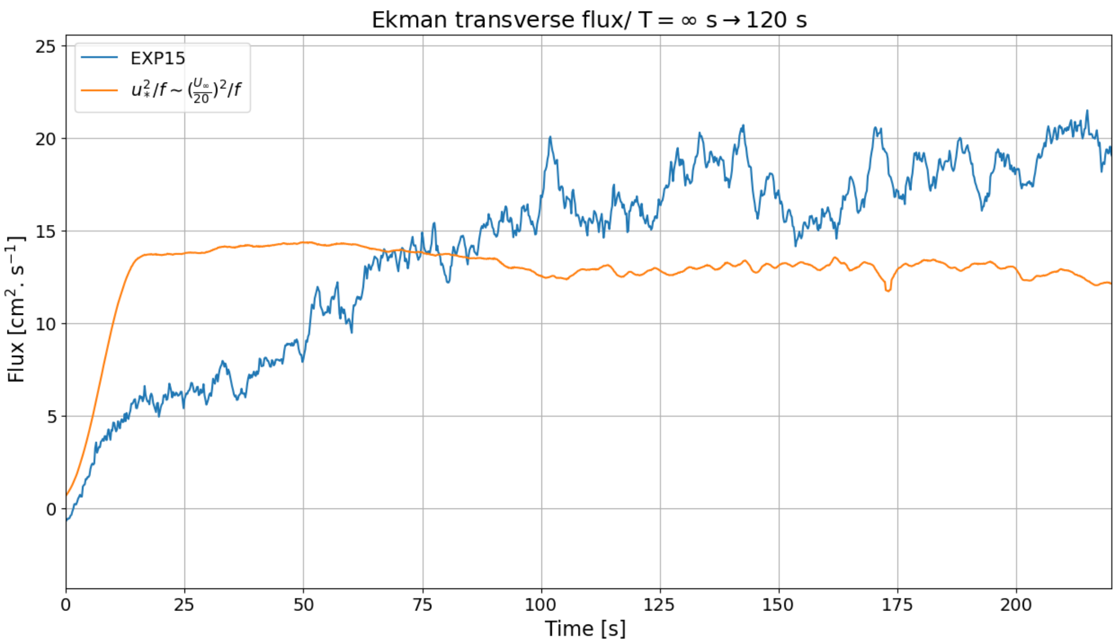
- Note that v is the cartesian component accross the laser sheet which is tangent to the disc of radius $R = 4.5m$. Thus this velocity component v is strictly equal to u_r the radial component only in the middle of the slice. However, given that $(2 R)/l_{laser sheet} 88 $ we can approximate $v \sim u_r$.

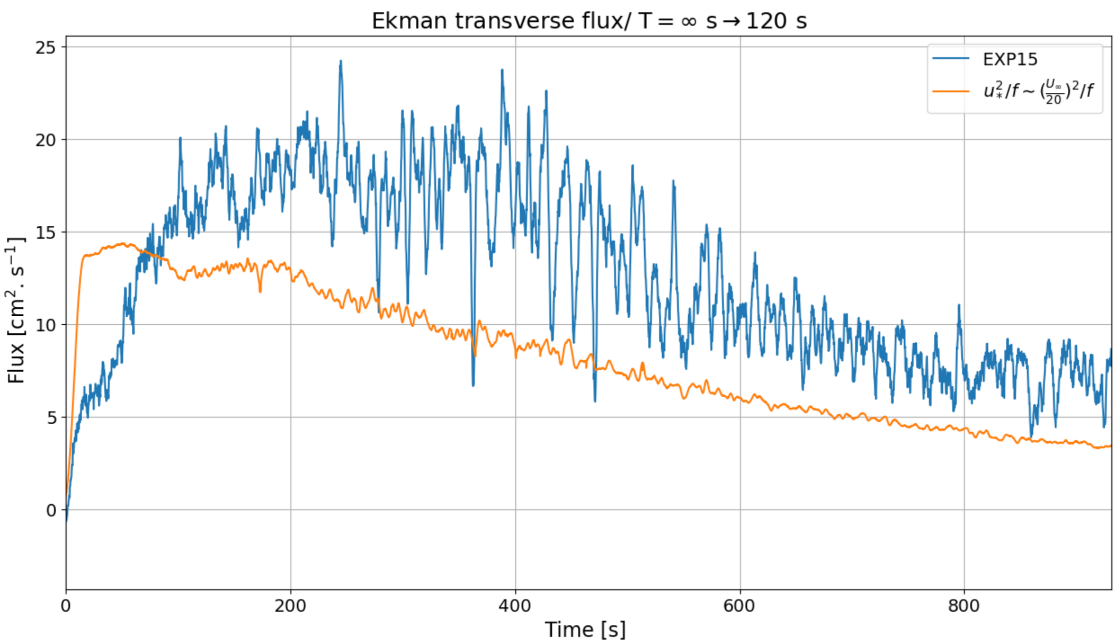
$l_{\text{laser sheet}} = 34$ cm for the stereo

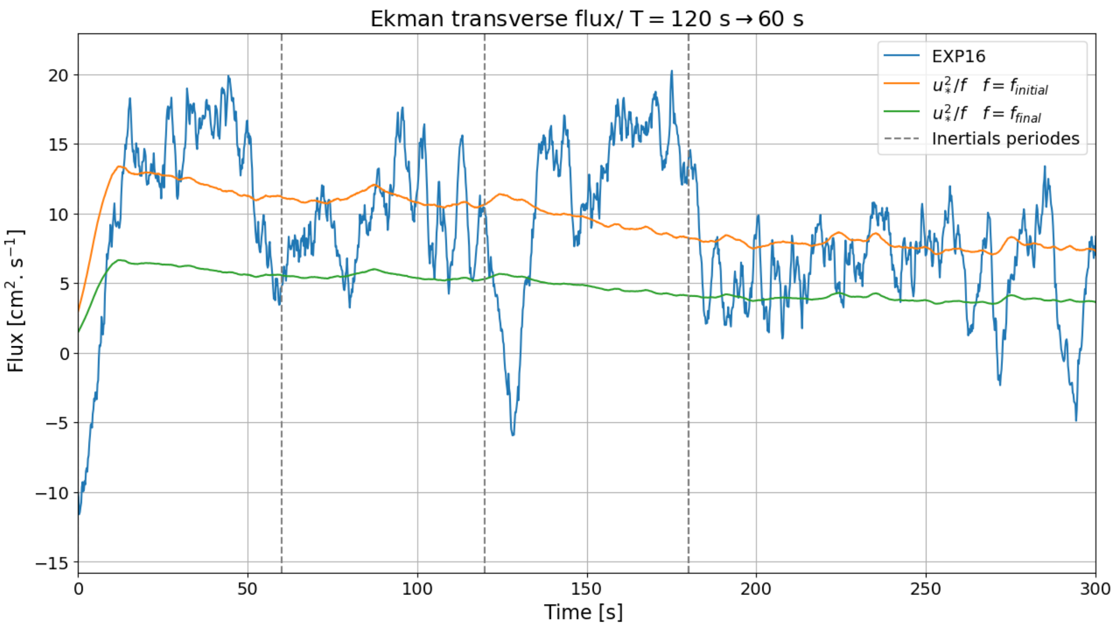


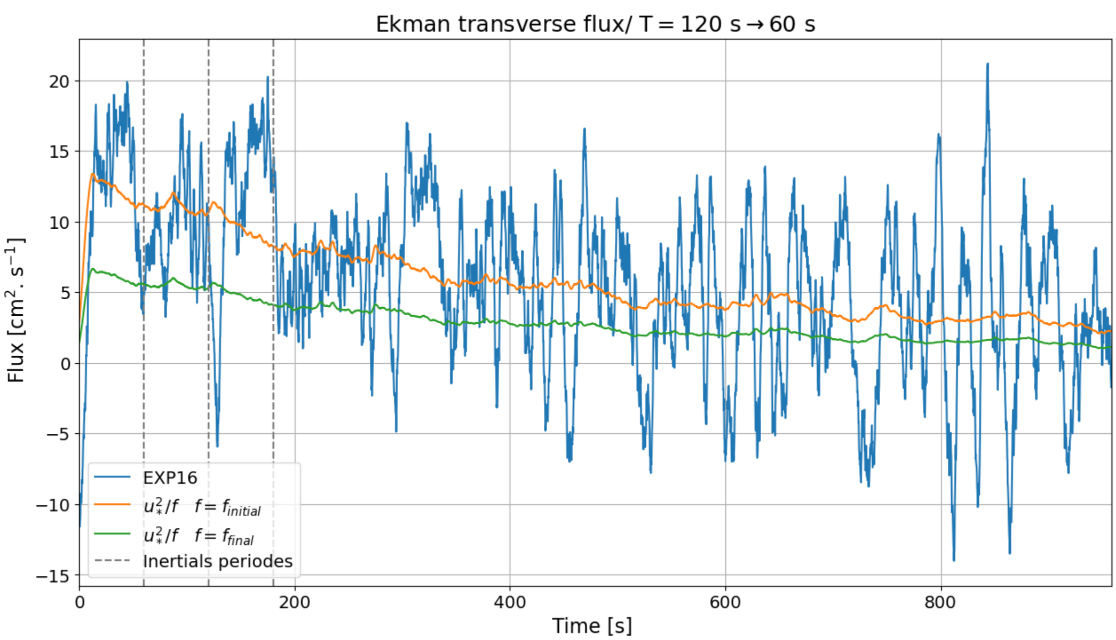


The figures aboves shows the time evolution of the transverse flux $\int_0^{z_{max}} \overline{v} dz$ in blue for the EXP12 case of a spin-up without initial rotation. And in orange the curve $u_*^2/f \sim (\frac{U_\infty}{20})^2/f$ which represents the radial transport in the Ekman theory for a parameter f corresponding to the final rotation. The Top panel is only a zoom over the 220 first secondes. What we can observe is that at the beginning, as expected, the transport is low because there is no transport due to Ekman pumping since there is no initial rotation. Then, after 100-200 seconds, the effects of rotation are felt and transport is approchin the value verified by Ekman pumping. the gap may be due to the underestimation of u_* which is taken $u_* \sim \frac{U_\infty}{20}$ The transport values at short times can be explained by the deflection of the free surface by centrifugal effects.


Ekman transverse flux/ T = ∞ s → 120 s
EXP15
$u_*^2/f \sim (\frac{U_\infty}{20})^2/f$
Flux [cm^2. s^{-1}]
Time [s]
25
20
15
10
5
0
0
25
50
75
100
125
150
175
200

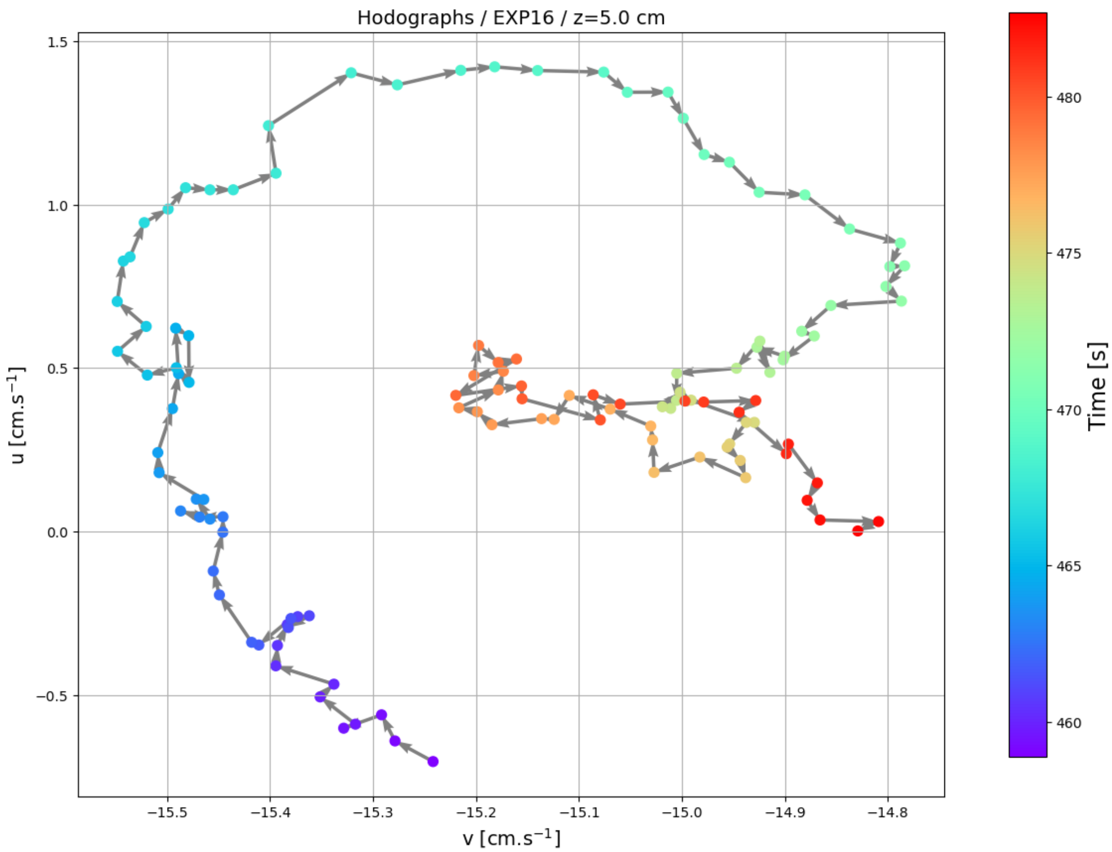

Ekman transverse flux/ T = ∞ s → 120 s
EXP15
$u_*^2/f \sim (\frac{U_\infty}{20})^2/f$
Flux [cm^2. s^{-1}]
Time [s]
25
20
15
10
5
0
0
200
400
600
800



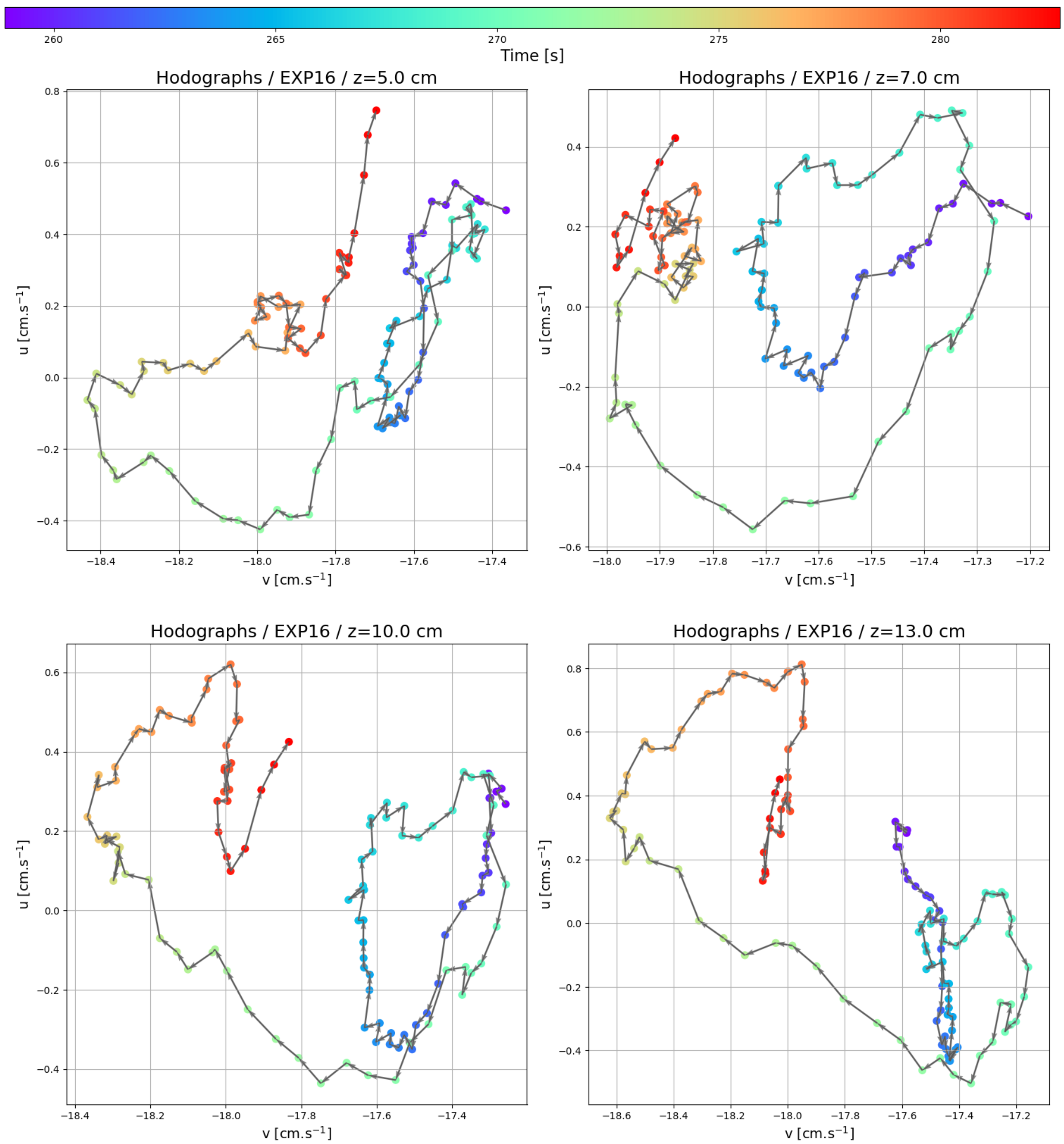


In this case of spin up with an initial rotation, in orange is the estimate of the ekman pumping for a coriolis parameter corresponding to the initial rotation and in green for the final rotation. It is quite difficult to draw any conclusions given the wide variations.

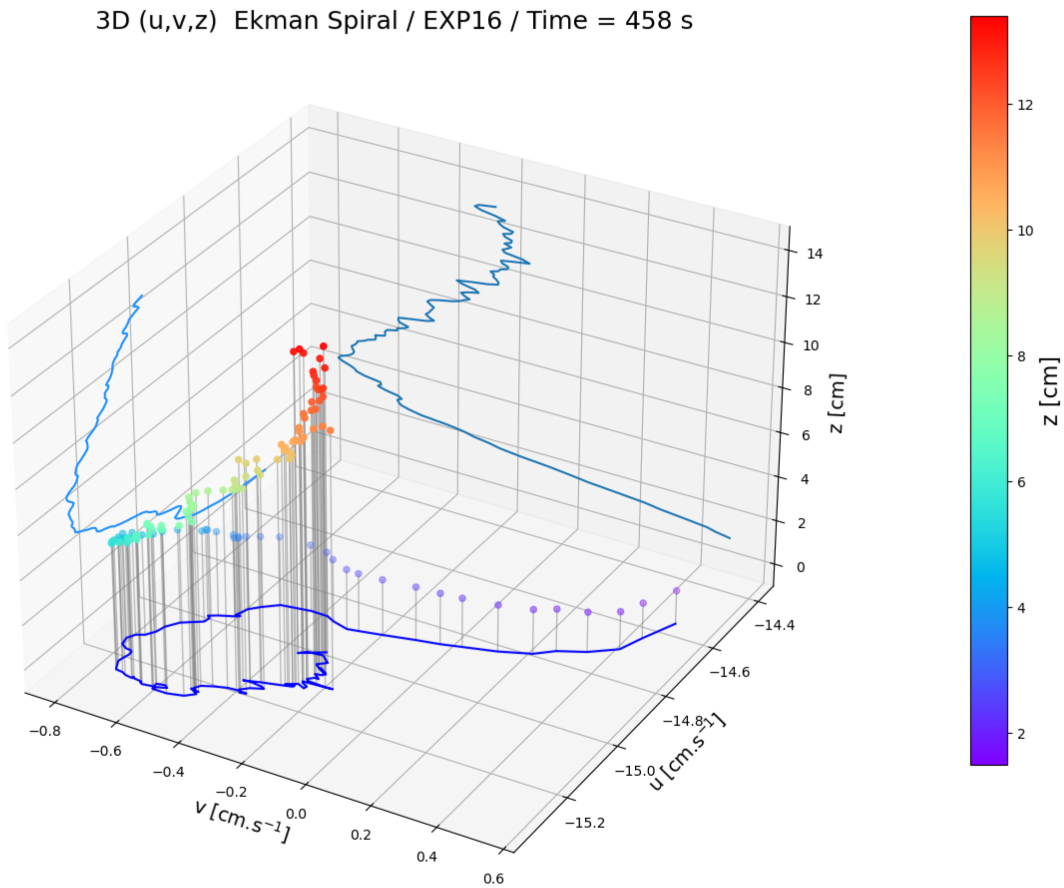
4.1 Ekman Spiral




```
/var/folders/gl/qr_d872d4sj6k_rm1k6tskj80000gq/T/ipykernel_12711/1918904863.py:3
6: UserWarning: This figure includes Axes that are not compatible with
tight_layout, so results might be incorrect.
  plt.tight_layout()
```



Time [s]
260
265
270
275
280
Hodographs / EXP16 / z=5.0 cm
Hodographs / EXP16 / z=7.0 cm
Hodographs / EXP16 / z=10.0 cm
Hodographs / EXP16 / z=13.0 cm
u [cm.s^{-1}]
v [cm.s^{-1}]


3D (u,v,z) Ekman Spiral / EXP16 / Time = 458 s
v [cm.s⁻¹]
u [cm.s⁻¹]
z [cm]
z [cm]