PHY 3650: Quantum Information Processing

Spring 2025

Homework Set #1 - Due on January 20, 2025

The problems in this set are a bit verbose but not hard. Some rely on elementary physics. It is always good to start early. #5 is probably the hardest one, so leave it for last. The maximum points for each item are indicated in bold face.

1. Planck's spectral density for the radation emitted by black body as a function of the frequency is given by[1]

$$u(f) = \frac{8\pi h f^3}{c^3} \frac{1}{e^{hf/k_B T} - 1}.$$

   This expression can be used to measure the temperature of a hard-to-reach source of radiation, such as a star or hot furnace, when the temperature and spectral density of a reference source are known. Optical pyrometers are based on this principle.

   a) **(5)** Call T_r the temperature of a reference (i.e., known) radiation source and T_u the unknown temperature of another source whose spectral density can be measured. Find an expression relating the ratio of the spectral densities of these two sources to their temperatures.

   b) **(10)** Using a similar technique, about 70 years ago, Dicke, Penzias, and Wilson found out that the Earth is surrounded uniformly in all directions by electromagnetic radiation as if the universe itself were a black body, which they determined to be at a temperature of 3 K. Their finding gave strong support to the big-bang theory for the origin of the universe. Find the predominant wavelength of the radiation of a black body at 3 K (*Hint*: find at which frequency Planck's formula has a maximum and then convert it to wavelength). Does this wavelength give you a clue for the technique they used in their experimental investigation?

2. **(5)** In photolectric effect experiments, one observes that electrons are ejected with different velocities, even when the light is monchromatic (i.e., has a well-defined frequency). Provide a plausible explanation of this phenomenon using Einstein's theory.

3. Light with a wavelength of 2000 Å hits an aluminum surface. For aluminum, 4.2 eV are necessary to remove an electron.

   a) **(5)** What is the kinetic energy of the fastest photoelectron (i.e., the electron ejected by the light)?

   b) **(5)** What is the kinetic energy of the slowest photoelectron?

   c) **(5)** What is the cutoff wavelength for aluminum?

   d) **(5)** If the light intensity is 2.0 W/m^2, what is the number of photons per unit of time and area that hit the aluminum surface? *Hint*: intensity is the total amount of energy per unit of time and unit of area; start by finding out how much energy a single photon carries.

4. **(10)** Why we do not experience wave-like phenomena such as interference and diffraction in our daily lives? You may illustrate your answers by considering an object of mass 40 g moving at 1000 m/s. What is its de Broglie wavelength?

5. Starting from Bohr's quantization of the angular momentum ($L = n\hbar$) and assuming that the proton (mass m_p and charge e) stays stationary while the electron (mass m_e, charge $-e$) moves in a circular orbit around it, use elementary Newton's mechanics and Coulomb's law to:

   a) **(10)** Find an expression for the quantized radius of the orbit, r_n, of an electron in the hydrogen atom.

1. This expression gives the amount of energy emitted by the black-body radiation per unit of frequency and unit of volume. It differs with respect to other forms of the radiation distribution, such as when the density of emitted energy is defined per unit of wavelength. The two forms are related by the differential relation $u(f)\, df = u(\lambda)\, d\lambda$, recalling that $f = c/\lambda$.

b) **(10)** Find an expression for the quantized total energy of the electron E_n in the hydrogen atom.

c) **(10)** Find an expression relating the energy of emitted or absorbed photon when the electron changes its orbit from n to n'.

Hints: start by relating the radius of the orbit to the electron mass and its velocity; then use the angular momentum quantization to find the quantization of the radius; finally, relate the total energy (kinetic plus potential) to the radius.

Useful quantities for calculations you may have do:

$h = 6.63 \times 10^{-34}$ J·s
$k_B = 1.38 \times 10^{-23}$ J/K
$c = 3.00 \times 10^{8}$ m/s
$1\,\text{eV} = 1.602 \times 10^{-19}$ J
$1\,\text{Å} = 10^{-10}$ m.