Homework 01

(due day in two weeks, 3/20)

Problem 1: (40 points)

Please determine whether the following functions belong to periodic function. If yes, please fine the period.

(1) $\sin\frac{n\pi x}{l}$ (2) $\cos\frac{n\pi x}{l}$ (3) $f(x)=a_0\cdot e^{-x}$

(4) $f(x)=a_0+a_1\cos\frac{\pi x}{l}+b_1\sin\frac{\pi x}{l}+a_2\cos\frac{2\pi x}{l}+b_2\sin\frac{2\pi x}{l}+\ldots+a_n\cos\frac{n\pi x}{l}+b_n\sin\frac{n\pi x}{l}+\ldots$

Problem 2: (10 points)

Set $f_1(-x)=-f_1(x)$, $f_2(-x)=f_2(x)$, $f_3(-x)=-f_3(x)$, $g(x)=f_1(x)\,f_2(x)$, and $h(x)=f_1(x)\,f_3(x)$. Please determine the result of $g(-x)$ and $h(-x)$

Problem 3: (10 points)

If $f(x+2\pi)=f(x)$ and $f(x)=\begin{cases}-1, -\pi\le x<0\\ 1, 0<x<\pi\end{cases}$, please find f(x)'s Fourier Series.

Problem 4: (40 points)

Given the function $f(x)=\begin{cases}0, -\pi < x < 0\\ \sin x, 0 \le x < \pi\end{cases}$

(1) Find its Fourier series

(2) Show that $\frac{1}{2}+\frac{1}{1\times 3}-\frac{1}{3\times 5}+\frac{1}{5\times 7}-\frac{1}{7\times 9}+\ldots=\frac{\pi}{4}$