CHAPTER 13

Nonparametric Statistics

Objectives

After completing this chapter, you should be able to

1. State the advantages and disadvantages of nonparametric methods.
2. Test hypotheses, using the sign test.
3. Test hypotheses, using the Wilcoxon rank sum test.
4. Test hypotheses, using the signed-rank test.
5. Test hypotheses, using the Kruskal-Wallis test.
6. Compute the Spearman rank correlation coefficient.
7. Test hypotheses, using the runs test.

Outline

Statistics Today

Too Much or Too Little?

Suppose a manufacturer of ketchup wishes to check the bottling machines to see if they are functioning properly. That is, are they dispensing the right amount of ketchup per bottle? A 40-ounce bottle is currently used. Because of the natural variation in the manufacturing process, the amount of ketchup in a bottle will not always be exactly 40 ounces. Some bottles will contain less than 40 ounces, and others will contain more than 40 ounces. To see if the variation is due to chance or to a malfunction in the manufacturing process, a runs test can be used. The runs test is a nonparametric statistical technique. See Statistics Today—Revisited at the end of this chapter. This chapter explains such techniques, which can be used to help the manufacturer determine the answer to the question.

Introduction

Statistical tests, such as the z, t, and F tests, are called parametric tests. **Parametric tests** are statistical tests for population parameters such as means, variances, and proportions that involve assumptions about the populations from which the samples were selected. One assumption is that these populations are normally distributed. But what if the population in a particular hypothesis-testing situation is *not* normally distributed? Statisticians have developed a branch of statistics known as **nonparametric statistics** or **distribution-free statistics** to use when the population from which the samples are selected is not normally distributed. Nonparametric statistics can also be used to test hypotheses that do not involve specific population parameters, such as μ, σ, or p.

For example, a sportswriter may wish to know whether there is a relationship between the rankings of two judges on the diving abilities of 10 Olympic swimmers. In another situation, a sociologist may wish to determine whether men and women enroll at random for a specific drug rehabilitation program. The statistical tests used in these situations are nonparametric or distribution-free tests. The term *nonparametric* is used for both situations.

The nonparametric tests explained in this chapter are the sign test, the Wilcoxon rank sum test, the Wilcoxon signed-rank test, the Kruskal-Wallis test, and the runs test.

In addition, the Spearman rank correlation coefficient, a statistic for determining the relationship between ranks, is explained.

13–1 Advantages and Disadvantages of Nonparametric Methods

As stated previously, nonparametric tests and statistics can be used in place of their parametric counterparts (z, t, and F) when the assumption of normality cannot be met. However, you should not assume that these statistics are a better alternative than the parametric statistics. There are both advantages and disadvantages in the use of nonparametric methods.

Objective 1

State the advantages and disadvantages of nonparametric methods.

Advantages

There are five advantages that nonparametric methods have over parametric methods:

1. They can be used to test population parameters when the variable is not normally distributed.
2. They can be used when the data are nominal or ordinal.
3. They can be used to test hypotheses that do not involve population parameters.
4. In some cases, the computations are easier than those for the parametric counterparts.
5. They are easy to understand.

Disadvantages

There are three disadvantages of nonparametric methods:

1. They are *less sensitive* than their parametric counterparts when the assumptions of the parametric methods are met. Therefore, larger differences are needed before the null hypothesis can be rejected.
2. They tend to use *less information* than the parametric tests. For example, the sign test requires the researcher to determine only whether the data values are above or below the median, not how much above or below the median each value is.
3. They are *less efficient* than their parametric counterparts when the assumptions of the parametric methods are met. That is, larger sample sizes are needed to overcome the loss of information. For example, the nonparametric sign test is about 60% as efficient as its parametric counterpart, the z test. Thus, a sample size of 100 is needed for use of the sign test, compared with a sample size of 60 for use of the z test to obtain the same results.

Since there are both advantages and disadvantages to the nonparametric methods, the researcher should use caution in selecting these methods. If the parametric assumptions can be met, the parametric methods are preferred. However, when parametric assumptions cannot be met, the nonparametric methods are a valuable tool for analyzing the data.

The basic assumption for nonparametric statistics is that the sample or samples are randomly obtained. When two or more samples are used, they must be independent of each other unless otherwise stated.

Interesting Fact

Older men have the biggest ears. James Heathcote, M.D., says, "On average, our ears seem to grow 0.22 millimeter a year. This is roughly a centimeter during the course of 50 years."

Ranking

Many nonparametric tests involve the **ranking** of data, that is, the positioning of a data value in a data array according to some rating scale. Ranking is an ordinal variable. For example, suppose a judge decides to rate five speakers on an ascending scale of 1 to 10, with 1 being the best and 10 being the worst, for categories such as voice, gestures, logical presentation, and platform personality. The ratings are shown in the chart.

Speaker	A	B	C	D	E
Rating	8	6	10	3	1

The rankings are shown next.

Speaker	E	D	B	A	C
Rating	1	3	6	8	10
Ranking	1	2	3	4	5

Since speaker E received the lowest score, 1 point, he or she is ranked first. Speaker D received the next-lower score, 3 points; he or she is ranked second; and so on.

What happens if two or more speakers receive the same number of points? Suppose the judge awards points as follows:

Speaker	A	B	C	D	E
Rating	8	6	10	6	3

The speakers are then ranked as follows:

Speaker	E	D	B	A	C
Rating	3	6	6	8	10
Ranking	1	Tie for 2nd and 3rd		4	5

When there is a tie for two or more places, the average of the ranks must be used. In this case, each would be ranked as

$$\frac{2+3}{2} = \frac{5}{2} = 2.5$$

Hence, the rankings are as follows:

Speaker	E	D	B	A	C
Rating	3	6	6	8	10
Ranking	1	2.5	2.5	4	5

Many times, the data are already ranked, so no additional computations must be done. For example, if the judge does not have to award points but can simply select the speakers who are best, second-best, third-best, and so on, then these ranks can be used directly.

P-values can also be found for nonparametric statistical tests, and the *P*-value method can be used to test hypotheses that use nonparametric tests. For this chapter, the *P*-value method will be limited to some of the nonparametric tests that use the standard normal distribution or the chi-square distribution.

Applying the Concepts 13–1

Ranking Data

The following table lists the percentages of patients who experienced side effects from a drug used to lower a person's cholesterol level.

Side effect	Percent
Chest pain	4.0
Rash	4.0
Nausea	7.0
Heartburn	5.4
Fatigue	3.8
Headache	7.3
Dizziness	10.0
Chills	7.0
Cough	2.6

Rank each value in the table.

See page 717 for the answer.

Exercises 13–1

1. What is meant by *nonparametric statistics?*
2. When should nonparametric statistics be used?
3. List the advantages and disadvantages of nonparametric statistics.

For Exercises 4 through 10, rank each set of data.

4. 3, 8, 6, 1, 4, 10, 7
5. 22, 66, 32, 43, 65, 43, 71, 34
6. 83, 460, 582, 177, 241
7. 19.4, 21.8, 3.2, 23.1, 5.9, 10.3, 11.1
8. 10.9, 20.2, 43.9, 9.5, 17.6, 5.6, 32.6, 0.85, 17.6
9. 28, 50, 52, 11, 71, 36, 47, 88, 41, 50, 71, 50
10. 90.6, 47.0, 82.2, 9.27, 327.0, 52.9, 18.0, 145.0, 34.5, 9.54

13–2 The Sign Test

Single-Sample Sign Test

Objective 2

Test hypotheses, using the sign test.

The simplest nonparametric test, the **sign test** for single samples, is used to test the value of a median for a specific sample. When using the sign test, the researcher hypothesizes the specific value for the median of a population; then he or she selects a sample of data and compares each value with the conjectured median. If the data value is above the conjectured median, it is assigned a plus sign. If it is below the conjectured median, it is assigned a minus sign. And if it is exactly the same as the conjectured median, it is assigned a 0. Then the numbers of plus and minus signs are compared. If the null hypothesis is true, the number of plus signs should be approximately equal to the number of minus signs. If the null hypothesis is not true, there will be a disproportionate number of plus or minus signs.

Test Value for the Sign Test

The test value is the smaller number of plus or minus signs.

For example, if there are 8 positive signs and 3 negative signs, the test value is 3. When the sample size is 25 or less, Table J in Appendix C is used to determine the critical value. For a specific α, if the test value is less than or equal to the critical value obtained from the table, the null hypothesis should be rejected. The values in Table J are obtained from the binomial distribution. The derivation is omitted here.

Example 13–1 Snow Cone Sales

A convenience store owner hypothesizes that the median number of snow cones she sells per day is 40. A random sample of 20 days yields the following data for the number of snow cones sold each day.

18	43	40	16	22
30	29	32	37	36
39	34	39	45	28
36	40	34	39	52

At $\alpha = 0.05$, test the owner's hypothesis.

Solution

Step 1 State the hypotheses and identify the claim.

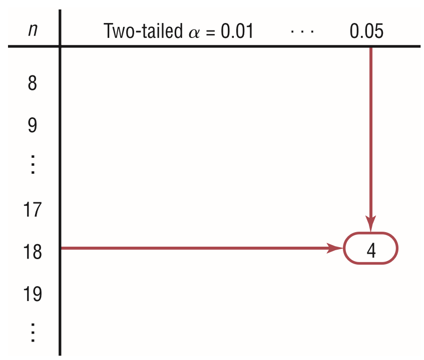
H_0: median = 40 (claim) and H_1: median $\neq$ 40

Step 2 Find the critical value. Compare each value of the data with the median. If the value is greater than the median, replace the value with a plus sign. If it is less than the median, replace it with a minus sign. And if it is equal to the median, replace it with a 0. The completed table follows.

−	+	0	−	−
−	−	−	−	−
−	−	−	+	−
−	0	−	−	+

Refer to Table J in Appendix C, using $n = 18$ (the total number of plus and minus signs; omit the zeros) and $\alpha = 0.05$ for a two-tailed test; the critical value is 4. See Figure 13–1.

Figure 13–1
Finding the Critical Value in Table J for Example 13–1



Step 3 Compute the test value. Count the number of plus and minus signs obtained in step 2, and use the smaller value as the test value. Since there are 3 plus signs and 15 minus signs, 3 is the test value.

Step 4 Make the decision. Compare the test value 3 with the critical value 4. If the test value is less than or equal to the critical value, the null hypothesis is rejected. In this case, the null hypothesis is rejected since $3 < 4$.

Step 5 Summarize the results. There is enough evidence to reject the claim that the median number of snow cones sold per day is 40.

When the sample size is 26 or more, the normal approximation can be used to find the test value. The formula is given. The critical value is found in Table E in Appendix C.

Formula for the *z* Test Value in the Sign Test When $n \geq 26$

$$z = \frac{(X + 0.5) - (n/2)}{\sqrt{n}/2}$$

where

X = smaller number of + or − signs
n = sample size

Example 13–2 Age of Foreign-Born Residents

Based on information from the U.S. Census Bureau, the median age of foreign-born U.S. residents is 36.4 years. A researcher selects a sample of 50 foreign-born U.S. residents in his area and finds that 21 are older than 36.4 years. At $\alpha = 0.05$, test the claim that the median age of the residents is at least 36.4 years.

Solution

Step 1 State the hypotheses and identify the claim.

H_0: MD = 36.4 (claim) and H_1: MD < 36.4

Step 2 Find the critical value. Since $\alpha = 0.05$ and $n = 50$, and since this is a left-tailed test, the critical value is -1.65, obtained from Table E.

Step 3 Compute the test value.

$$z = \frac{(X + 0.5) - (n/2)}{\sqrt{n}/2} = \frac{(21 + 0.5) - (50/2)}{\sqrt{50}/2} = \frac{-3.5}{3.5355} = -0.99$$

Step 4 Make the decision. Since the test value of -0.99 is greater than -1.65, the decision is to not reject the null hypothesis.

Step 5 Summarize the results. There is not enough evidence to reject the claim that the median age of the residents is at least 36.4.

In Example 13–2, the sample size was 50, and 21 residents are older than 36.4. So $50 - 21$, or 29, residents are not older than 36.4. The value of X corresponds to the smaller of the two numbers 21 and 29. In this case, $X = 21$ is used in the formula; since 21 is the smaller of the two numbers, the value of X is 21.

Suppose a researcher hypothesized that the median age of houses in a certain municipality was 40 years. In a random sample of 100 houses, 68 were older than 40 years. Then the value used for X in the formula would be $100 - 68$, or 32, since it is the smaller of the two numbers 68 and 32. When 40 is subtracted from the age of a house older than 40 years, the answer is positive. When 40 is subtracted from the age of a house that is less than 40 years old, the result is negative. There would be 68 positive signs and 32 negative signs (assuming that no house was exactly 40 years old). Hence, 32 would be used for X, since it is the smaller of the two values.

Paired-Sample Sign Test

The sign test can also be used to test sample means in a comparison of two dependent samples, such as a before-and-after test. Recall that when dependent samples are taken from normally distributed populations, the t test is used (Section 9–4). When the condition of normality cannot be met, the nonparametric sign test can be used, as shown in Example 13–3.

Example 13–3 Ear Infections in Swimmers

A medical researcher believed the number of ear infections in swimmers can be reduced if the swimmers use earplugs. A sample of 10 people was selected, and the number of infections for a four-month period was recorded. During the first two months, the swimmers did not use the earplugs; during the second two months, they did. At the beginning of the second two-month period, each swimmer was examined to make sure that no infections were present. The data are shown here. At $\alpha = 0.05$, can the researcher conclude that using earplugs reduced the number of ear infections?

Number of ear infections

Swimmer	Before, X_B	After, X_A
A	3	2
B	0	1
C	5	4
D	4	0
E	2	1
F	4	3
G	3	1
H	5	3
I	2	2
J	1	3

Interesting Fact

Room temperature is generally considered 72° since at this temperature a clothed person's body heat is allowed to escape at a rate that is most comfortable to him or her.

Solution

Step 1 State the hypotheses and identify the claim.

H_0: The number of ear infections will not be reduced.

H_1: The number of ear infections will be reduced (claim).

Step 2 Find the critical value. Subtract the after values X_A from the before values X_B and indicate the difference by a positive or negative sign or 0, according to the value, as shown in the table.

Swimmer	Before, X_B	After, X_A	Sign of difference
A	3	2	+
B	0	1	−
C	5	4	+
D	4	0	+
E	2	1	+
F	4	3	+
G	3	1	+
H	5	3	+
I	2	2	0
J	1	3	−

From Table J, with $n = 9$ (the total number of positive and negative signs; the 0 is not counted) and $\alpha = 0.05$ (one-tailed), at most 1 negative sign is needed to reject the null hypothesis because 1 is the smallest entry in the $\alpha = 0.05$ column of Table J.

Step 3 Compute the test value. Count the number of positive and negative signs found in step 2, and use the smaller value as the test value. There are 2 negative signs, so the test value is 2.

Step 4 Make the decision. There are 2 negative signs. The decision is to not reject the null hypothesis. The reason is that with $n = 9$, C.V. $= 1$ and $1 < 2$.

Step 5 Summarize the results. There is not enough evidence to support the claim that the use of earplugs reduced the number of ear infections.

When conducting a one-tailed sign test, the researcher must scrutinize the data to determine whether they support the null hypothesis. If the data support the null hypothesis, there is no need to conduct the test. In Example 13–3, the null hypothesis states that the number of ear infections will not be reduced. The data would support the null hypothesis if there were more negative signs than positive signs. The reason is that the before values X_B in most cases would be smaller than the after values X_A, and the $X_B - X_A$ values would be negative more often than positive. This would indicate that there is not enough evidence to reject the null hypothesis. The researcher would stop here, since there is no need to continue the procedure.

On the other hand, if the number of ear infections were reduced, the X_B values, for the most part, would be larger than the X_A values, and the $X_B - X_A$ values would most often be positive, as in Example 13–3. Hence, the researcher would continue the procedure. A word of caution is in order, and a little reasoning is required.

When the sample size is 26 or more, the normal approximation can be used in the same manner as in Example 13–2. The steps for conducting the sign test for single or paired samples are given in the Procedure Table.

Procedure Table

Sign Test for Single and Paired Samples

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s). For the single-sample test, compare each value with the conjectured median. If the value is larger than the conjectured median, replace it with a positive sign. If it is smaller than the conjectured median, replace it with a negative sign.

For the paired-sample sign test, subtract the after values from the before values, and indicate the difference with a positive or negative sign or 0, according to the value. Use Table J and $n =$ total number of positive and negative signs.

Check the data to see whether they support the null hypothesis. If they do, do not reject the null hypothesis. If not, continue with step 3.

Step 3 Compute the test value. Count the numbers of positive and negative signs found in step 2, and use the smaller value as the test value.

Step 4 Make the decision. Compare the test value with the critical value in Table J. If the test value is less than or equal to the critical value, reject the null hypothesis.

Step 5 Summarize the results.

Note: If the sample size n is 26 or more, use Table E and the following formula for the test value:

$$z = \frac{(X + 0.5) - (n/2)}{\sqrt{n}/2}$$

where

$X =$ smaller number of $+$ or $-$ signs

$n =$ sample size

Applying the Concepts 13–2

Clean Air

An environmentalist suggests that the median of the number of days per month that a large city failed to meet the EPA acceptable standards for clean air is 11 days per month. A random sample of 20 months shows the number of days per month that the air quality was below the EPA's standards.

15	14	1	9	0	3	3	1	10	8
6	16	21	22	3	19	16	5	23	13

1. What is the claim?
2. What test would you use to test the claim? Why?
3. What would the hypotheses be?
4. Select a value for α and find the corresponding critical value.
5. What is the test value?
6. What is your decision?
7. Summarize the results.
8. Could a parametric test be used?

See page 717 for the answers.

Exercises 13–2

1. Why is the sign test the simplest nonparametric test to use? The sign test uses only positive or negative signs.

2. What population parameter can be tested with the sign test? The median

3. In the sign test, what is used as the test value when $n < 26$? The smaller number of positive or negative signs

4. When $n \geq 26$, what is used in place of Table J for the sign test? The normal approximation

For Exercises 5 through 20, perform these steps.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

5. Ages When Married The median age at first marriage for men in the United States in 2008 was 27.6 years. Alumni officers at a large university contacted recent newlyweds to see if their median age was different. Their ages (in years) at marriage are listed below. At $\alpha = 0.05$ can it be concluded that the median age for these alumni is different?

31.8	39.9	34.1	22.9
29.2	33.9	34.0	36.9
33.8	36.2	26.1	35.1
23.1	25.2	32.6	26.3

6. Game Attendance An athletic director suggests the median number for the paid attendance at 20 local football games is 3000. The data for a sample are shown. At $\alpha = 0.05$, is there enough evidence to reject the claim? If you were printing the programs for the games, would you use this figure as a guide?

6210	3150	2700	3012	4875
3540	6127	2581	2642	2573
2792	2800	2500	3700	6030
5437	2758	3490	2851	2720

Source: *Pittsburgh Post Gazette.*

7. Cyber School Enrollment An educator hypothesizes that the median of the number of students enrolled in cyber schools in school districts in southwestern Pennsylvania is 25. At $\alpha = 0.05$, is there enough evidence to reject the educator's claim? The data are shown here. What benefit would this information provide to the school board of a local school district?

12	41	26	14	4
38	27	27	9	11
17	11	66	5	14
8	35	16	25	17

Source: *Pittsburgh Tribune-Review.*

8. Weekly Earnings of Women According to the Women's Bureau of the U.S. Department of Labor, the occupation with the highest median weekly earnings among women is pharmacist with median weekly earnings of \$1603. Based on the weekly earnings listed below from a sample of female pharmacists, can it be concluded that the median is less than \$1603? Use $\alpha = 0.05$.

1550	1355	1777
1430	1570	1701
2465	1655	1484
1429	1829	1812
1217	1501	1449

9. Natural Gas Costs For a specific year, the median price of natural gas was \$10.86 per 1000 cubic feet. A researcher wishes to see if there is enough evidence to reject the claim. Out of 42 households, 18 paid less than \$10.86 per 1000 cubic feet for natural gas. Test the claim at $\alpha = 0.05$. How could a prospective home buyer use this information?

Source: Based on information from the Energy Information Administration.

10. Family Income The median U.S. family income is believed to be \$63,211. In a survey of families in a particular neighborhood, it was found that out of 40 families surveyed, 10 had incomes below \$63,211. At the 0.05 level of significance is there sufficient evidence to conclude that the median income is not \$63,211?

11. Number of Faculty for Proprietary Schools An educational researcher believes that the median number of faculty for proprietary (for-profit) colleges and universities is 150. The data provided list the number of faculty at a selected number of proprietary colleges and universities. At the 0.05 level of significance, is there sufficient evidence to reject his claim?

372	111	165	95	191	83	136	149	37	119
142	136	137	171	122	133	133	342	126	64
61	100	225	127	92	140	140	75	108	96
138	318	179	243	109					

Source: *World Almanac.*

12. Television Viewers A researcher read that the median age for viewers of the Carson Daly show is 39. To test the claim, 75 viewers were surveyed, and 27 were under the age of 39. At $\alpha = 0.02$ test the claim. Give one reason why an advertiser might like to know the results of this study.

Source: Nielsen Media Research.

13. Students' Opinions on Lengthening the School Year One hundred students are asked if they favor increasing the school year by 20 days. The responses are 62 no, 36 yes, and 2 undecided. At $\alpha = 0.10$, test the hypothesis that 50% of the students are against extending the school year. Use the P-value method.

14. Deaths due to Severe Weather A meteorologist suggests that the median number of deaths per year from tornadoes in the United States is 60. The number of deaths for a sample of 11 years is shown. At $\alpha = 0.05$ is there enough evidence to reject the claim? If you took proper safety precautions during a tornado, would you feel relatively safe?

53	39	39	67	69	40
25	33	30	130	94	

Source: NOAA.

15. Diet Medication and Weight A study was conducted to see whether a certain diet medication had an effect on the weights (in pounds) of eight women. Their weights were taken before and six weeks after daily administration of the medication. The data are shown here. At $\alpha = 0.05$, can you conclude that the medication had an effect (increase or decrease) on the weights of the women?

Subject	A	B	C	D	E	F	G	H
Weight before	187	163	201	158	139	143	198	154
Weight after	178	162	188	156	133	150	175	150

16. Exam Scores A statistics professor wants to investigate the relationship between a student's midterm examination score and the score on the final. Eight students were selected, and their scores on the two examinations are noted below. At the 0.10 level of significance, is there sufficient evidence to conclude that there is a difference in scores?

Student	1	2	3	4	5	6	7	8
Midterm	75	92	68	85	65	80	75	80
Final	82	90	79	95	70	83	72	79

17. Increasing Supervisory Skills A large corporation sent several of its prospective supervisors to a two-day seminar in identifying and increasing supervisory skills. Participants were given a pretest at the start of the seminar and a posttest at the conclusion. Their scores are listed below. At $\alpha = 0.05$ can it be concluded that the training program was effective?

Employee	1	2	3	4	5	6	7	8
Pretest	70	65	73	72	80	77	69	68
Posttest	68	72	75	70	83	82	72	75

18. Effects of a Pill on Appetite A researcher wishes to test the effects of a pill on a person's appetite. Twelve subjects are allowed to eat a meal of their choice, and their caloric intake is measured. The next day, the same subjects take the pill and eat a meal of their choice. The caloric intake of the second meal is measured. The data are shown here. At $\alpha = 0.02$, can the researcher conclude that the pill had an effect on a person's appetite?

Subject	1	2	3	4	5	6	7
Meal 1	856	732	900	1321	843	642	738
Meal 2	843	721	872	1341	805	531	740

Subject	8	9	10	11	12
Meal 1	1005	888	756	911	998
Meal 2	900	805	695	878	914

19. Television Viewers A researcher wishes to determine if the number of viewers for 10 returning television shows has not changed since last year. The data are given in millions of viewers. At $\alpha = 0.01$, test the claim that the number of viewers has not changed. Depending on your answer, would a television executive plan to air these programs for another year?

Show	1	2	3	4	5	6
Last year	28.9	26.4	20.8	25.0	21.0	19.2
This year	26.6	20.5	20.2	19.1	18.9	17.8

Show	7	8	9	10
Last year	13.7	18.8	16.8	15.3
This year	16.8	16.7	16.0	15.8

Source: Based on information from Nielsen Media Research.

20. Routine Maintenance and Defective Parts A manufacturer believes that if routine maintenance (cleaning and oiling of machines) is increased to once a day rather than once a week, the number of defective parts produced by the machines will decrease. Nine machines are selected, and the number of defective parts produced over a 24-hour operating period is counted. Maintenance is then increased to once a day for a week, and the number of defective parts each machine produces is again counted over a 24-hour operating period. The data are shown here. At $\alpha = 0.01$, can the manufacturer conclude that increased maintenance reduces the number of defective parts manufactured by the machines?

Machine	1	2	3	4	5	6	7	8	9
Before	6	18	5	4	16	13	20	9	3
After	5	16	7	4	18	12	14	7	1

Extending the Concepts

Confidence Interval for the Median

The confidence interval for the median of a set of values less than or equal to 25 in number can be found by ordering the data from smallest to largest, finding the median, and using Table J. For example, to find the 95% confidence interval of the true median for 17, 19, 3, 8, 10, 15, 1, 23, 2, 12, order the data:

1, 2, 3, 8, 10, 12, 15, 17, 19, 23

From Table J, select $n = 10$ and $\alpha = 0.05$, and find the critical value. Use the two-tailed row. In this case, the critical value is 1. Add 1 to this value to get 2. In the ordered list, count from the left two numbers and from the right two numbers, and use these numbers to get the confidence interval, as shown:

1, 2, 3, 8, 10, 12, 15, 17, 19, 23

$2 \le \text{MD} \le 19$

Always add 1 to the number obtained from the table before counting. For example, if the critical value is 3, then count 4 values from the left and right.

For Exercises 21 through 25, find the confidence interval of the median, indicated in parentheses, for each set of data.

21. 3, 12, 15, 18, 16, 15, 22, 30, 25, 4, 6, 9 (95%)
$6 \le \text{median} \le 22$

22. 101, 115, 143, 106, 100, 142, 157, 163, 155, 141, 145, 153, 152, 147, 143, 115, 164, 160, 147, 150 (90%)
MD = 146; $141 \le \text{MD} \le 153$

23. 8.2, 7.1, 6.3, 5.2, 4.8, 9.3, 7.2, 9.3, 4.5, 9.6, 7.8, 5.6, 4.7, 4.2, 9.5, 5.1 (98%) $4.7 \le \text{median} \le 9.3$

24. 1, 8, 2, 6, 10, 15, 24, 33, 56, 41, 58, 54, 5, 3, 42, 31, 15, 65, 21 (99%) MD = 21; $5 \le \text{MD} \le 54$

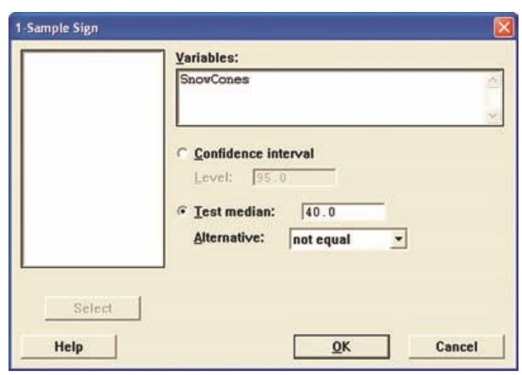
25. 12, 15, 18, 14, 17, 19, 25, 32, 16, 47, 14, 23, 27, 42, 33, 35, 39, 41, 21, 19 (95%) $17 \le \text{median} \le 33$

Technology *Step by Step*

MINITAB Step by Step

The Sign Test

1. Type the data for Example 13–1 into a column of MINITAB. Name the column SnowCones.
2. Select **Stat>Nonparametrics> 1-Sample Sign Test.**
3. Double-click SnowCones in the list box.
4. Click on Test median, then enter the hypothesized value of **40.**
5. Click [OK]. In the session window the *P*-value is 0.0075.



The Paired-Sample Sign Test

1. Enter the data for Example 13–3 into a worksheet; only the Before and After columns are necessary. Calculate a column with the differences to begin the process.
2. Select **Calc>Calculator.**

3. Type **D** in the box for Store result in variable.
4. Move to the Expression box, then click on Before, the subtraction sign, and After. The completed entry is shown.
5. Click [OK].

MINITAB will calculate the differences and store them in the first available column with the name "D." Use the instructions for the Sign Test on the differences *D* with a hypothesized value of zero.

Sign Test for Median: D

```
Sign test of median = 0.00000 versus not = 0.00000

    N  Below  Equal  Above      P       Median
D  10      2      1      7  0.1797      1.000
```

The *P*-value is 0.1797. Do not reject the null hypothesis.

Excel Step by Step

The Sign Test

Excel does not have a procedure to conduct the sign test. However, you may conduct this test by using the MegaStat Add-in available on your CD. If you have not installed this add-in, do so, following the instructions from the Chapter 1 Excel Step by Step.

1. Enter the data from Example 13–1 into column A of a new worksheet.
2. From the toolbar, select Add-Ins, **MegaStat>Nonparametric Tests>Sign Test.** *Note:* You may need to open MegaStat from the MegaStat.xls file on your computer's hard drive.
3. Type **A1:A20** for the Input range.
4. Type **40** for the Hypothesized value, and select the "not equal" Alternative.
5. Click [OK].

The *P*-value is 0.0075. Reject the null hypothesis.

13–3 The Wilcoxon Rank Sum Test

Objective 3

Test hypotheses, using the Wilcoxon rank sum test.

Interesting Fact

One in four married women now earns more than her husband.

The sign test does not consider the magnitude of the data. For example, whether a value is 1 point or 100 points below the median, it will receive a negative sign. And when you compare values in the pretest/posttest situation, the magnitude of the differences is not considered. The Wilcoxon tests consider differences in magnitudes by using ranks.

The two tests considered in this section and in Section 13–4 are the **Wilcoxon rank sum test,** which is used for independent samples, and the **Wilcoxon signed-rank test,** which is used for dependent samples. Both tests are used to compare distributions. The parametric equivalents are the z and t tests for independent samples (Sections 9–1 and 9–3) and the t test for dependent samples (Section 9–4). For the parametric tests, as stated previously, the samples must be selected from approximately normally distributed populations, but the only assumption for the Wilcoxon signed-rank tests is that the population of differences has a symmetric distribution.

In the Wilcoxon tests, the values of the data for both samples are combined and then ranked. If the null hypothesis is true—meaning that there is no difference in the population distributions—then the values in each sample should be ranked approximately the same. Therefore, when the ranks are summed for each sample, the sums should be approximately equal, and the null hypothesis will not be rejected. If there is a large difference in the sums of the ranks, then the distributions are not identical, and the null hypothesis will be rejected.

The first test to be considered is the Wilcoxon rank sum test for independent samples. For this test, both sample sizes must be greater than or equal to 10. The formulas needed for the test are given next.

Formula for the Wilcoxon Rank Sum Test When Samples Are Independent

$$z = \frac{R - \mu_R}{\sigma_R}$$

where

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2}$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1 + n_2 + 1)}{12}}$$

R = sum of ranks for smaller sample size (n_1)
n_1 = smaller of sample sizes
n_2 = larger of sample sizes
$n_1 \geq 10$ and $n_2 \geq 10$

Note that if both samples are the same size, either size can be used as n_1.

Example 13–4 illustrates the Wilcoxon rank sum test for independent samples.

Example 13–4 Times to Complete an Obstacle Course

Two independent samples of army and marine recruits are selected, and the time in minutes it takes each recruit to complete an obstacle course is recorded, as shown in the table. At $\alpha = 0.05$, is there a difference in the times it takes the recruits to complete the course?

Army	15	18	16	17	13	22	24	17	19	21	26	28	Mean = 19.67
Marines	14	9	16	19	10	12	11	8	15	18	25		Mean = 14.27

Solution

Step 1 State the hypotheses and identify the claim.

H_0: There is no difference in the times it takes the recruits to complete the obstacle course.

H_1: There is a difference in the times it takes the recruits to complete the obstacle course (claim).

Step 2 Find the critical value. Since $\alpha = 0.05$ and this test is a two-tailed test, use the z values of $+1.96$ and -1.96 from Table E.

Step 3 Compute the test value.

a. Combine the data from the two samples, arrange the combined data in order, and rank each value. Be sure to indicate the group.

Time	8	9	10	11	12	13	14	15	15	16	16	17
Group	M	M	M	M	M	A	M	A	M	A	M	A
Rank	1	2	3	4	5	6	7	8.5	8.5	10.5	10.5	12.5

Time	17	18	18	19	19	21	22	24	25	26	28
Group	A	M	A	A	M	A	A	A	M	A	A
Rank	12.5	14.5	14.5	16.5	16.5	18	19	20	21	22	23

b. Sum the ranks of the group with the smaller sample size. (*Note:* If both groups have the same sample size, either one can be used.) In this case, the sample size for the marines is smaller.

$$R = 1 + 2 + 3 + 4 + 5 + 7 + 8.5 + 10.5 + 14.5 + 16.5 + 21 = 93$$

c. Substitute in the formulas to find the test value.

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2} = \frac{(11)(11 + 12 + 1)}{2} = 132$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1 + n_2 + 1)}{12}} = \sqrt{\frac{(11)(12)(11 + 12 + 1)}{12}} = \sqrt{264} = 16.2$$

$$z = \frac{R - \mu_R}{\sigma_R} = \frac{93 - 132}{16.2} = -2.41$$

Step 4 Make the decision. The decision is to reject the null hypothesis, since $-2.41 < -1.96$.

Step 5 Summarize the results. There is enough evidence to support the claim that there is a difference in the times it takes the recruits to complete the course.

The steps for the Wilcoxon rank sum test are given in the Procedure Table.

Procedure Table

Wilcoxon Rank Sum Test

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s). Use Table E.

Step 3 Compute the test value.

a. Combine the data from the two samples, arrange the combined data in order, and rank each value.

b. Sum the ranks of the group with the smaller sample size. (*Note:* If both groups have the same sample size, either one can be used.)

c. Use these formulas to find the test value.

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2}$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1 + n_2 + 1)}{12}}$$

$$z = \frac{R - \mu_R}{\sigma_R}$$

where R is the sum of the ranks of the data in the smaller sample and n_1 and n_2 are each greater than or equal to 10.

Step 4 Make the decision.

Step 5 Summarize the results.

Applying the Concepts 13–3

School Lunch

A nutritionist decided to see if there was a difference in the number of calories served for lunch in elementary and secondary schools. She selected a random sample of eight elementary schools and another random sample of eight secondary schools in Pennsylvania. The data are shown.

Elementary	Secondary
648	694
589	730
625	750
595	810
789	860
727	702
702	657
564	761

1. Are the samples independent or dependent?
2. What are the hypotheses?
3. What nonparametric test would you use to test the claim?
4. What critical value would you use?
5. What is the test value?
6. What is your decision?
7. What is the corresponding parametric test?
8. What assumption would you need to meet to use the parametric test?
9. If this assumption were not met, would the parametric test yield the same results?

See page 717 for the answers.

Exercises 13–3

1. What are the minimum sample sizes for the Wilcoxon rank sum test? n_1 and n_2 are each greater than or equal to 10.

2. What are the parametric equivalent tests for the Wilcoxon rank sum tests? The t test for independent samples

3. What distribution is used for the Wilcoxon rank sum test? The standard normal distribution

For Exercises 4 through 11, use the Wilcoxon rank sum test. Assume that the samples are independent. Also perform each of these steps.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

4. Lengths of Prison Sentences A random sample of men and women in prison was asked to give the length of sentence each received for a certain type of crime. At $\alpha = 0.05$, test the claim that there is no difference in the sentence received by each gender. The data (in months) are shown here.

Males	8	12	6	14	22	27	32	24	26
Females	7	5	2	3	21	26	30	9	4

Males	19	15	13		
Females	17	23	12	11	16

5. Technology Proficiency Test The following are scores from a technology proficiency test required of all new incoming students at a particular college. Use the Wilcoxon rank sum test to see if there is a difference in scores between freshmen and transfer students at the 0.05 level of significance.

Freshmen	40	32	40	32	47	39	38	39	29	35	30
Transfers	38	43	35	45	37	36	36	33	46	44	41

6. Lifetimes of Handheld Video Games To test the claim that there is no difference in the lifetimes of two brands of handheld video games, a researcher selects a sample of 11 video games of each brand. The lifetimes (in months) of each brand are shown here. At $\alpha = 0.01$, can the researcher conclude that there is a difference in the distributions of lifetimes for the two brands?

Brand A	42	34	39	42	22	47	51	34	41	39	28
Brand B	29	39	38	43	45	49	53	38	44	43	32

7. Stopping Distances of Automobiles A researcher wishes to see if the stopping distance for midsize automobiles is different from the stopping distance for compact automobiles at a speed of 70 miles per hour. The data are shown. At $\alpha = 0.10$, test the claim that the stopping distances are the same. If one of your safety concerns is stopping distance, would it make a difference which type of automobile you purchase?

Automobile	1	2	3	4	5	6	7	8	9	10
Midsize	188	190	195	192	186	194	188	187	214	203
Compact	200	211	206	297	198	204	218	212	196	193

Source: Based on information from the National Highway Traffic Safety Administration.

8. Winning Baseball Games For the years 1970–1993 the National League (NL) and the American League (AL) (major league baseball) were each divided into two divisions: East and West. Below is a sample of the number of games won by each league's Eastern Division. At $\alpha = 0.05$, is there sufficient evidence to conclude a difference in the number of wins?

NL	89	96	88	101	90	91	92	96	108	100	95	
AL	108	86	91	97	100	102	95	104	95	89	88	101

Source: *World Almanac.*

9. Hunting Accidents A game commissioner wishes to see if the number of hunting accidents in counties in western Pennsylvania is different from the number of hunting accidents in counties in eastern Pennsylvania. A sample of counties from the two regions is selected, and the numbers of hunting accidents are shown. At $\alpha = 0.05$, is there a difference in the number of accidents in the two areas? If so, give a possible reason for the difference.

Western Pa.	10	21	11	11	9	17	13	8	15	17
Eastern Pa.	14	3	7	13	11	2	8	5	5	6

Source: Pennsylvania Game Commission.

10. Medical School Enrollments Samples of enrollments from medical schools that specialize in research and in primary care are listed below. At $\alpha = 0.05$, can it be concluded that there is a difference?

Research	474	577	605	663	813	443	565	696	692	217	
Primary care	783	546	442	662	605	474	587	555	427	320	293

Source: *U.S. News & World Report Best Graduate Schools.*

11. Speed of Pain Relievers Volunteers were randomly assigned to one of two groups to test the speed with which a pain reliever brought relief. One group took the standard dose of extra-strength acetaminophen (group A) while the other group (group N) took a newly approved pain-relieving drug. The number of minutes until symptoms abated is listed for each member of each group. At $\alpha = 0.05$ can it be concluded that there is a difference in time until pain is relieved?

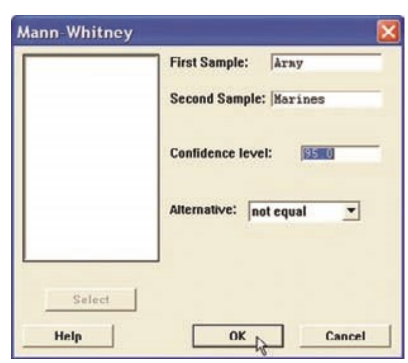
Group A	15	20	12	20	17	14	15	17	18	11
Group N	7	14	13	11	10	16	12	9	10	9

Technology *Step by Step*

MINITAB Step by Step

Wilcoxon Rank Sum Test (Mann-Whitney)

1. Enter the data for Example 13–4 into two columns of a worksheet.
2. Name the columns **Army** and **Marines.**
3. Select **Stat>Nonparametric>Mann-Whitney.**
4. Double-click Army for the First Sample.
5. Double-click Marines for the Second Sample.
6. Click [OK].



Mann-Whitney Test and CI: Army, Marines

```
             N  Median
   Army     12  18.500
   Marines  11  14.000

Point estimate for ETA1-ETA2 is 6.000
95.5 Percent CI for ETA1-ETA2 is (1.003, 9.998)
W = 183.0
Test of ETA1 = ETA2 vs ETA1 not = ETA2 is significant at 0.0178
The test is significant at 0.0177 (adjusted for ties)
```

The *P*-value for the test is 0.0177. Reject the null hypothesis. There is a significant difference in the times it takes the recruits to complete the course.

Excel
Step by Step

The Wilcoxon Mann-Whitney Test

Excel does not have a procedure to conduct the Mann-Whitney rank sum test. However, you may conduct this test by using the MegaStat Add-in available on your CD. If you have not installed this add-in, do so, following the instructions from the Chapter 1 Excel Step by Step.

1. Enter the data from Example 13–4 into columns A and B of a new worksheet.
2. From the toolbar, select Add-Ins, **MegaStat>Nonparametric Tests>Wilcoxon-Mann/Whitney Test.** *Note:* You may need to open MegaStat from the MegaStat.xls file on your computer's hard drive.
3. Type **A1:A12** in the box for Group 1.
4. Type **B1:B11** in the box for Group 2.
5. Check the option labeled Correct for ties, and select the "not equal" Alternative.
6. Click [OK].

Wilcoxon Mann-Whitney Test

n	Sum of ranks	
12	183	Group 1
11	93	Group 2
23	276	Total

```
144.00 Expected value
 16.23 Standard deviation
  2.37 z, corrected for ties
0.0177 P-value (two-tailed)
```

The *P*-value is 0.0177. Reject the null hypothesis.

13–4 The Wilcoxon Signed-Rank Test

Objective 4

Test hypotheses, using the signed-rank test.

When the samples are dependent, as they would be in a before-and-after test using the same subjects, the Wilcoxon signed-rank test can be used in place of the *t* test for dependent samples. Again, this test does not require the condition of normality. Table K is used to find the critical values.

The procedure for this test is shown in Example 13–5.

Example 13–5

Shoplifting Incidents

In a large department store, the owner wishes to see whether the number of shoplifting incidents per day will change if the number of uniformed security officers is doubled. A sample of 7 days before security is increased and 7 days after the increase shows the number of shoplifting incidents.

	Number of shoplifting incidents	
Day	**Before**	**After**
Monday	7	5
Tuesday	2	3
Wednesday	3	4
Thursday	6	3
Friday	5	1
Saturday	8	6
Sunday	12	4

Is there enough evidence to support the claim, at $\alpha = 0.05$, that there is a difference in the number of shoplifting incidents before and after the increase in security?

Solution

Step 1 State the hypotheses and identify the claim.

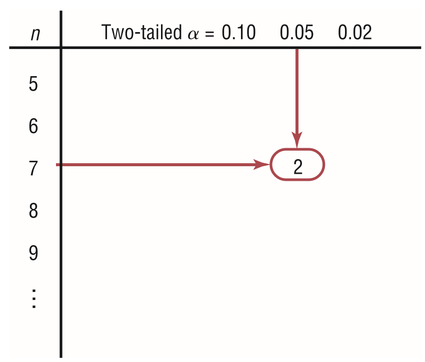
H_0: There is no difference in the number of shoplifting incidents before and after the increase in security.

H_1: There is a difference in the number of shoplifting incidents before and after the increase in security (claim).

Step 2 Find the critical value from Table K. Since $n = 7$ and $\alpha = 0.05$ for this two-tailed test, the critical value is 2. See Figure 13–2.

Figure 13–2

Finding the Critical Value in Table K for Example 13–5



Step 3 Find the test value.

a. Make a table as shown here.

Day	**Before, X_B**	**After, X_A**	**Difference $D = X_B - X_A$**	**Absolute value $\|D\|$**	**Rank**	**Signed rank**
Mon.	7	5				
Tues.	2	3				
Wed.	3	4				
Thurs.	6	3				
Fri.	5	1				
Sat.	8	6				
Sun.	12	4				

b. Find the differences (before minus after), and place the values in the Difference column.

$7 - 5 = 2$ $\quad$ $6 - 3 = 3$ $\quad$ $8 - 6 = 2$

$2 - 3 = -1$ $\quad$ $5 - 1 = 4$ $\quad$ $12 - 4 = 8$

$3 - 4 = -1$

c. Find the absolute value of each difference, and place the results in the Absolute value column. (*Note:* The absolute value of any number except 0 is the positive value of the number. Any differences of 0 should be ignored.)

$\|2\| = 2$	$\|3\| = 3$	$\|2\| = 2$
$\|-1\| = 1$	$\|4\| = 4$	$\|8\| = 8$
$\|-1\| = 1$		

d. Rank each absolute value from lowest to highest, and place the rankings in the Rank column. In the case of a tie, assign the values that rank plus 0.5.

Value	2	1	1	3	4	2	8
Rank	3.5	1.5	1.5	5	6	3.5	7

e. Give each rank a plus or minus sign, according to the sign in the Difference column. The completed table is shown here.

Day	**Before, X_B**	**After, X_A**	**Difference $D = X_B - X_A$**	**Absolute value $\|D\|$**	**Rank**	**Signed rank**
Mon.	7	5	2	2	3.5	+3.5
Tues.	2	3	−1	1	1.5	−1.5
Wed.	3	4	−1	1	1.5	−1.5
Thurs.	6	3	3	3	5	+5
Fri.	5	1	4	4	6	+6
Sat.	8	6	2	2	3.5	+3.5
Sun.	12	4	8	8	7	+7

Interesting Fact

Nearly one in three unmarried adults lives with a parent today.

f. Find the sum of the positive ranks and the sum of the negative ranks separately.

Positive rank sum $(+3.5) + (+5) + (+6) + (+3.5) + (+7) = +25$

Negative rank sum $(-1.5) + (-1.5) = -3$

g. Select the smaller of the absolute values of the sums ($|-3|$), and use this absolute value as the test value w_s. In this case, $w_s = |-3| = 3$.

Step 4 Make the decision. Reject the null hypothesis if the test value is less than or equal to the critical value. In this case, $3 > 2$; hence, the decision is not to reject the null hypothesis.

Step 5 Summarize the results. There is not enough evidence to support the claim that there is a difference in the number of shoplifting incidents. Hence, the security increase probably made no difference in the number of shoplifting incidents.

The rationale behind the signed-rank test can be explained by a diet example. If the diet is working, then the majority of the postweights will be smaller than the preweights. When the postweights are subtracted from the preweights, the majority of the signs will be positive, and the absolute value of the sum of the negative ranks will be small. This sum will probably be smaller than the critical value obtained from Table K, and the null hypothesis will be rejected. On the other hand, if the diet does not work, some people will gain weight, other people will lose weight, and still other people will remain about the same weight. In this case, the sum of the positive ranks and the absolute value of the sum of the negative ranks will be approximately equal and will be about one-half of the sum of the absolute value of all the ranks. In this case, the smaller of the absolute values of the two sums will still be larger than the critical value obtained from Table K, and the null hypothesis will not be rejected.

When $n \geq 30$, the normal distribution can be used to approximate the Wilcoxon distribution. The same critical values from Table E used for the z test for specific α values are used. The formula is

$$z = \frac{w_s - \dfrac{n(n+1)}{4}}{\sqrt{\dfrac{n(n+1)(2n+1)}{24}}}$$

where

n = number of pairs where difference is not 0
w_s = smaller sum in absolute value of signed ranks

The steps for the Wilcoxon signed-rank test are given in the Procedure Table.

Procedure Table

Wilcoxon Signed-Rank Test

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value from Table K.

Step 3 Compute the test value.

a. Make a table, as shown.

Before, X_B	After, X_A	Difference $D = X_B - X_A$	Absolute value $\lvert D \rvert$	Rank	Signed rank

b. Find the differences (before − after), and place the values in the Difference column.

c. Find the absolute value of each difference, and place the results in the Absolute value column.

d. Rank each absolute value from lowest to highest, and place the rankings in the Rank column.

e. Give each rank a positive or negative sign, according to the sign in the Difference column.

f. Find the sum of the positive ranks and the sum of the negative ranks separately.

g. Select the smaller of the absolute values of the sums, and use this absolute value as the test value w_s.

Step 4 Make the decision. Reject the null hypothesis if the test value is less than or equal to the critical value.

Step 5 Summarize the results.

Note: When $n \geq 30$, use Table E and the test value

$$z = \frac{w_s - \dfrac{n(n+1)}{4}}{\sqrt{\dfrac{n(n+1)(2n+1)}{24}}}$$

where

n = number of pairs where difference is not 0
w_s = smaller sum in absolute value of signed ranks

Applying the Concepts 13–4

Pain Medication

A researcher decides to see how effective a pain medication is. Eight subjects were asked to determine the severity of their pain by using a scale of 1 to 10, with 1 being very minor and 10 being very severe. Then each was given the medication, and after 1 hour, they were asked to rate the severity of their pain, using the same scale.

Subject	1	2	3	4	5	6	7	8
Before	8	6	2	3	4	6	2	7
After	6	5	3	1	2	6	1	6

1. What is the purpose of the study?
2. Are the samples independent or dependent?
3. What are the hypotheses?
4. What nonparametric test could be used to test the claim?
5. What significance level would you use?
6. What is your decision?
7. What parametric test could you use?
8. Would the results be the same?

See page 717 for the answers.

Exercises 13–4

1. What is the parametric equivalent test for the Wilcoxon signed-rank test? The *t* test for dependent samples

For Exercises 2 and 3, find the sum of the signed ranks. Assume that the samples are dependent. State which sum is used as the test value.

2.

Pretest	65	103	79	92	72	91	76	95
Posttest	72	105	64	95	78	92	76	93

3.

Pretest	108	97	115	162	156	105	153
Posttest	110	97	103	168	143	112	141

For Exercises 4 through 8, use Table K to determine whether the null hypothesis should be rejected.

4. $w_s = 62$, $n = 21$, $\alpha = 0.05$, two-tailed test
C.V. = 59; do not reject

5. $w_s = 18$, $n = 15$, $\alpha = 0.02$, two-tailed test
C.V. = 20; reject

6. $w_s = 53$, $n = 20$, $\alpha = 0.05$, two-tailed test
C.V. = 52; do not reject

7. $w_s = 102$, $n = 28$, $\alpha = 0.01$, one-tailed test
C.V. = 102; reject

8. $w_s = 33$, $n = 18$, $\alpha = 0.01$, two-tailed test
C.V. = 28; do not reject

9. Drug Prices Eight drugs were selected, and the prices for the human doses and the animal doses for the same amounts were compared. At $\alpha = 0.05$, can it be concluded that the prices for the animal doses are significantly less than the prices for the human doses? If the null hypothesis is rejected, give one reason why animal doses might cost less than human doses.

Human dose	0.67	0.64	1.20	0.51	0.87	0.74	0.50	1.22
Animal dose	0.13	0.18	0.42	0.25	0.57	0.57	0.49	1.28

Source: House Committee on Government Reform.

10. Property Assessments Use the sign test to test the hypothesis that the assessed value has changed between 2006 and 2010. Use $\alpha = 0.05$. Do you think land values in a large city would be normally distributed?

Ward	A	B	C	D	E	F	G	H	I	J	K
2006	184	414	22	99	116	49	24	50	282	25	141
2010	161	382	22	190	120	52	28	50	297	40	148

11. Weight Loss Through Diet Eight subjects were weighed before and after a new three-week "healthy" diet. At the 0.05 level of significance, can it be concluded that a difference in weight resulted? (Weights are in pounds.)

Subject	A	B	C	D	E	F	G	H
Before	150	195	188	197	204	175	160	180
After	152	190	185	191	200	170	162	179

12. Legal Costs for School Districts A sample of legal costs (in thousands of dollars) for school districts for two recent consecutive years is shown. At $\alpha = 0.05$, is there a difference in the costs?

Year 1	108	36	65	108	87	94	10	40
Year 2	138	28	67	181	97	126	18	67

Source: *Pittsburgh Tribune-Review.*

13. Drug Prices A researcher wishes to compare the prices for prescription drugs in the United States with those in Canada. The same drugs and dosages were compared in each country. At $\alpha = 0.05$, can it be concluded that the drugs in Canada are cheaper?

Drug	1	2	3	4	5	6
United States	3.31	2.27	2.54	3.13	23.40	3.16
Canada	1.47	1.07	1.34	1.34	21.44	1.47

Drug	7	8	9	10
United States	1.98	5.27	1.96	1.11
Canada	1.07	3.39	2.22	1.13

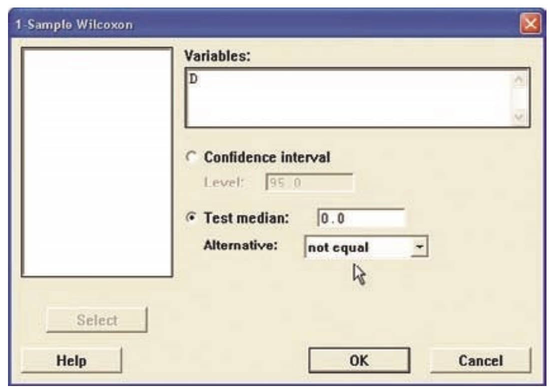
Source: IMS Health and other sources.

Technology *Step by Step*

MINITAB Step by Step

Wilcoxon Signed-Rank Test

Test the median value for the differences of two dependent samples. Use Example 13–5.



1. Enter the data into two columns of a worksheet. Name the columns **Before** and **After.**
2. Calculate the differences, using **Calc>Calculator.**
3. Type **D** in the box for Store result in variable.
4. In the expression box, type **Before − After.**
5. Click [OK].
6. Select **Stat>Nonparametric> 1-Sample Wilcoxon.**
7. Select C3 for the Variable.
8. Click on Test median. The value should be 0.
9. Click [OK].

Wilcoxon Signed-Rank Test: D

```
Test of median = 0.000000 versus median not = 0.000000
         N
        for   Wilcoxon          Estimated
   N   Test  Statistic      P      Median
D  7      7       25.0  0.076       2.250
```

The *P*-value of the test is 0.076. Do not reject the null hypothesis.

13–5 The Kruskal-Wallis Test

Objective 5

Test hypotheses, using the Kruskal-Wallis test.

The analysis of variance uses the F test to compare the means of three or more populations. The assumptions for the ANOVA test are that the populations are normally distributed and that the population variances are equal. When these assumptions cannot be met, the nonparametric **Kruskal-Wallis test,** sometimes called the *H test,* can be used to compare three or more means.

In this test, each sample size must be 5 or more. In these situations, the distribution can be approximated by the chi-square distribution with $k - 1$ degrees of freedom, where $k =$ number of groups. This test also uses ranks. The formula for the test is given next.

In the Kruskal-Wallis test, you consider all the data values as a group and then rank them. Next, the ranks are separated and the H formula is computed. This formula approximates the variance of the ranks. If the samples are from different populations, the sums of the ranks will be different and the H value will be large; hence, the null hypothesis will be rejected if the H value is large enough. If the samples are from the same population, the sums of the ranks will be approximately the same and the H value will be small; therefore, the null hypothesis will not be rejected. This test is always a right-tailed test. The chi-square table, Table G, with d.f. $= k - 1$, should be used for critical values.

Formula for the Kruskal-Wallis Test

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \cdots + \frac{R_k^2}{n_k}\right) - 3(N+1)$$

where

$R_1 =$ sum of ranks of sample 1
$n_1 =$ size of sample 1
$R_2 =$ sum of ranks of sample 2
$n_2 =$ size of sample 2
⋮
$R_k =$ sum of ranks of sample k
$n_k =$ size of sample k
$N = n_1 + n_2 + \cdots + n_k$
$k =$ number of samples

Example 13–6 illustrates the procedure for conducting the Kruskal-Wallis test.

Example 13–6 Hospital Infections

A researcher wishes to see if the total number of infections that occurred in three groups of hospitals is the same. The data are shown in the table. At $\alpha = 0.05$ is there enough evidence to reject the claim that the number of infections in the three groups of hospitals is the same?

Group A	Group B	Group C
557	476	105
315	232	110
920	80	167
178	116	155

Source: Pennsylvania Health Care Cost Containment Council.

Solution

Step 1 State the hypotheses and identify the claim.

H_0: There is no difference in the number of infections in the three groups of hospitals (claim).

H_1: There is a difference in the number of infections in the three groups of hospitals.

Step 2 Find the critical value. Use the chi-square table (Table G) with d.f. $= k - 1$, where $k =$ the number of groups. With $\alpha = 0.05$ and d.f. $= 3 - 1 = 2$, the critical value is 5.991.

Step 3 Compute the test value.

a. Arrange all the data from the lowest value to the highest value and rank each value.

Amount	Group	Rank
80	B	1
105	C	2
110	C	3
116	B	4
155	C	5
167	C	6
178	A	7
232	B	8
315	A	9
476	B	10
557	A	11
920	A	12

b. Find the sum of the ranks for each group.

Group A $7 + 9 + 11 + 12 = 39$

Group B $1 + 4 + 8 + 10 = 23$

Group C $2 + 3 + 5 + 6 = 16$

c. Substitute in the formula.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3}\right) - 3(N+1)$$

where

$N = 12 \qquad R_1 = 39 \qquad R_2 = 23 \qquad R_3 = 16$

$n_1 = n_2 = n_3 = 4$

Therefore,

$$H = \frac{12}{12(12+1)}\left(\frac{39^2}{4} + \frac{23^2}{4} + \frac{16^2}{4}\right) - 3(12+1)$$

$$= 5.346$$

Step 4 Make the decision. Since 5.346 is less than the critical value of 5.991, the decision is to not reject the null hypothesis.

Step 5 Summarize the results. There is not enough evidence to reject the claim that there is no difference in the number of infections in the groups of hospitals. Hence the differences are not significant at $\alpha = 0.05$.

The steps for the Kruskal-Wallis test are given in the Procedure Table.

Procedure Table

Kruskal-Wallis Test

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value. Use the chi-square table, Table G, with d.f. $= k - 1$ (k = number of groups).

Step 3 Compute the test value.

a. Arrange the data from lowest to highest and rank each value.

b. Find the sum of the ranks of each group.

c. Substitute in the formula

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \cdots + \frac{R_k^2}{n_k}\right) - 3(N+1)$$

where

$N = n_1 + n_2 + \cdots + n_k$
R_k = sum of ranks for kth group
k = number of groups

Step 4 Make the decision.

Step 5 Summarize the results.

Applying the Concepts 13–5

Heights of Waterfalls

You are doing research for an article on the waterfalls on our planet. You want to make a statement about the heights of waterfalls on three continents. Three samples of waterfall heights (in feet) are shown.

North America	Africa	Asia
600	406	330
1200	508	830
182	630	614
620	726	1100
1170	480	885
442	2014	330

1. What questions are you trying to answer?
2. What nonparametric test would you use to find the answer?
3. What are the hypotheses?
4. Select a significance level and run the test. What is the H value?
5. What is your conclusion?
6. What is the corresponding parametric test?
7. What assumptions would you need to make to conduct this test?

See page 718 for the answers.

Exercises 13–5

For Exercises 1 through 11, perform these steps.

a. State the hypotheses and identify the claim.
b. Find the critical value.
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

1. Calories in Cereals Samples of four different cereals show the following numbers of calories for the suggested servings of each brand. At $\alpha = 0.05$, is there a difference in the number of calories for the different brands?

Brand A	Brand B	Brand C	Brand D
112	110	109	106
120	118	116	122
135	123	125	130
125	128	130	117
108	102	128	116
121	101	132	114

2. Mathematics Literacy Scores Through the Organization for Economic Cooperation and Development (OECD), 15-year-olds are tested in member countries in mathematics, reading, and science literacy. Below are listed total mathematics literacy scores (i.e., both genders) for selected countries in different parts of the world. Test, using the Kruskal-Wallis test, to see if there is a difference in means at $\alpha = 0.05$.

Western Hemisphere	Europe	Eastern Asia
527	520	523
406	510	547
474	513	547
381	548	391
411	496	549

Source: www.nces.ed.gov

3. Lawnmower Costs A researcher wishes to compare the prices of three types of lawnmowers. At $\alpha = 0.10$, can it be concluded that there is a difference in the prices? Based on your answer, do you feel that the cost should be a factor in determining which type of lawnmower a person would purchase?

Gas-powered self-propelled	Gas-powered push	Electric
290	320	188
325	360	245
210	200	470
300	229	395
330	160	

4. Sodium Content of Microwave Dinners Three brands of microwave dinners were advertised as low in sodium. Samples of the three different brands show the following milligrams of sodium. At $\alpha = 0.05$, is there a difference in the amount of sodium among the brands?

Brand A	Brand B	Brand C
810	917	893
702	912	790
853	952	603
703	958	744
892	893	623
732		743
713		609
613		

5. Unemployment Benefits In Chapter 12 we did this exercise assuming that the populations were normally distributed and that the population variances were equal. Assume that this is not the case. Using the Kruskal-Wallis test, is the outcome affected? Do you think unemployment benefits are normally distributed? Test for a difference in means at $\alpha = 0.05$.

Florida	Pennsylvania	Maine
200	300	250
187	350	195
192	295	275
235	362	260
260	280	220
175	340	290

6. Job Offers for Chemical Engineers A recent study recorded the number of job offers received by newly graduated chemical engineers at three colleges. The data are shown here. At $\alpha = 0.05$, is there a difference in the average number of job offers received by the graduates at the three colleges?

College A	College B	College C
6	2	10
8	1	12
7	0	9
5	3	13
6	6	4

7. Expenditures for Pupils The expenditures in dollars per pupil for states in three sections of the country are listed below. At $\alpha = 0.05$, can it be concluded that there is a difference in spending between regions?

Eastern third	Middle third	Western third
6701	9854	7584
6708	8414	5474
9186	7279	6622
6786	7311	9673
9261	6947	7353

Source: *New York Times Almanac.*

8. Printer Costs An electronics store manager wishes to compare the costs (in dollars) of three types of computer printers. The data are shown. At $\alpha = 0.05$, can it be concluded that there is a difference in the prices? Based on your answer, do you think that a certain type of printer generally costs more than the other types?

Inkjet printers	Multifunction printers	Laser printers
149	98	192
199	119	159
249	149	198
239	249	198
99	99	229
79	199	

9. Number of Crimes per Week In a large city, the number of crimes per week in five precincts is recorded for five weeks. The data are shown here. At $\alpha = 0.01$, is there a difference in the number of crimes?

Precinct 1	Precinct 2	Precinct 3	Precinct 4	Precinct 5
105	87	74	56	103
108	86	83	43	98
99	91	78	52	94
97	93	74	58	89
92	82	60	62	88

10. Amounts of Caffeine in Beverages The amounts of caffeine in a regular (small) serving of assorted beverages are listed below. If someone wants to limit caffeine intake, does it really matter which beverage she or he chooses? Is there a difference in caffeine content at $\alpha = 0.05$?

Teas	Coffees	Colas
70	120	35
40	80	48
30	160	55
25	90	43
40	140	42

Source: *Doctor's Pocket Calorie, Fat & Carbohydrate Counter.*

11. Maximum Speeds of Animals A human is said to be able to reach a maximum speed of 27.89 miles per hour. The maximum speeds of various types of other animals are listed below. Based on these particular groupings is there evidence of a difference in speeds? Use the 0.05 level of significance.

Predatory mammals	Deerlike animals	Domestic animals
70	50	47.5
50	35	39.35
43	32	35
42	30	30
40	61	11

Technology *Step by Step*

MINITAB Step by Step

Kruskal-Wallis Test

Example: Milliequivalents of Potassium in Breakfast Drinks

A researcher tests three different brands of breakfast drinks to see how many milliequivalents of potassium per quart each contains. These data are obtained.

Brand A	Brand B	Brand C
4.7	5.3	6.3
3.2	6.4	8.2
5.1	7.3	6.2
5.2	6.8	7.1
5.0	7.2	6.6

At $\alpha = 0.05$, is there enough evidence to reject the hypothesis that all brands contain the same amount of potassium?

The data for this test must be "stacked." All the numeric data must be in one column, and the second column identifies the brand.

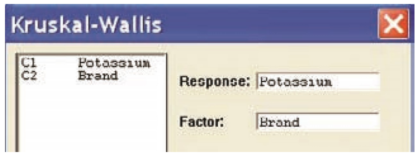
1. Stack the data for the example into two columns of a worksheet.
   a) First, enter all the potassium amounts into one column.
   b) Name this column **Potassium.**
   c) Enter code **A, B,** or **C** for the brand into the next column.
   d) Name this column **Brand.**

↓	C1	C2-T
	Potassium	Brand
1	4.7	A
2	3.2	A
3	5.1	A
4	5.2	A
5	5.0	A
6	5.3	B
7	6.4	B
8	7.3	B
9	6.8	B
10	7.2	B
11	6.3	C
12	8.2	C
13	6.2	C
14	7.1	C
15	6.6	C

The worksheet is shown.

2. Select **Stat>Nonparametric>Kruskal-Wallis.**
3. Double-click C1 Potassium to select it for Response.

This variable must be quantitative so the column for Brand will not be available in the list until the cursor is in the Factor text box.



4. Select C2 Brand for Factor.
5. Click [OK].

Kruskal-Wallis Test: Potassium versus Brand

```
Kruskal-Wallis Test on Potassium
Brand      N  Median  Ave Rank      Z
A          5   5.000       3.0  -3.06
B          5   6.800      10.6   1.59
C          5   6.600      10.4   1.47
Overall   15               8.0
H = 9.38  DF = 2  P = 0.009
```

The value $H = 9.38$ has a P-value of 0.009. Reject the null hypothesis.

Excel
Step by Step

The Kruskal-Wallis Test

Excel does not have a procedure to conduct the Kruskal-Wallis test. However, you may conduct this test by using the MegaStat Add-in available on your CD. If you have not installed this add-in, do so, following the instructions from the Chapter 1 Excel Step by Step.

1. Enter the data from previous example into columns A, B, and C of a new worksheet.
2. From the toolbar, select Add-Ins, **MegaStat>Nonparametric Tests>Kruskal-Wallis Test.** *Note:* You may need to open MegaStat from the MegaStat.xls file on your computer's hard drive.
3. Type **A1:C5** in the box for Input range.
4. Check the option labeled Correct for ties, and select the "not equal" Alternative.
5. Click [OK].

Kruskal-Wallis Test

Median	n	Avg. rank	
5.00	5	3.00	Group 1
6.80	5	10.60	Group 2
6.60	5	10.40	Group 3
6.30	15		Total
		9.380	H
		2	d.f.
		0.0092	P-value

Multiple comparison values for avg. ranks
6.77(0.05) 8.30(0.01)

The P-value is 0.0092. Reject the null hypothesis.

13–6

The Spearman Rank Correlation Coefficient and the Runs Test

Historical Note

Charles Spearman, who was a student of Karl Pearson, developed the Spearman rank correlation in the early 1900s. Other nonparametric statistical methods were also devised around this time.

The techniques of regression and correlation were explained in Chapter 10. To determine whether two variables are linearly related, you use the Pearson product moment correlation coefficient. Its values range from $+1$ to -1. One assumption for testing the hypothesis that $\rho = 0$ for the Pearson coefficient is that the populations from which the samples are obtained are normally distributed. If this requirement cannot be met, the nonparametric equivalent, called the **Spearman rank correlation coefficient** (denoted by r_s), can be used when the data are ranked.

Rank Correlation Coefficient

The computations for the rank correlation coefficient are simpler than those for the Pearson coefficient and involve ranking each set of data. The difference in ranks is found, and r_s is computed by using these differences. If both sets of data have the same ranks, r_s will be $+1$. If the sets of data are ranked in exactly the opposite way, r_s will be -1. If there is no relationship between the rankings, r_s will be near 0.

Objective 6

Compute the Spearman rank correlation coefficient.

Formula for Computing the Spearman Rank Correlation Coefficient

$$r_s = 1 - \frac{6\,\Sigma d^2}{n(n^2 - 1)}$$

where
d = difference in ranks
n = number of data pairs

This formula is algebraically equivalent to the formula for r given in Chapter 10, except that ranks are used instead of raw data.

The computational procedure is shown in Example 13–7. For a test of the significance of r_s, Table L is used for values of n up to 30. For larger values, the normal distribution can be used. (See Exercises 24 through 28 in the exercise section.)

Example 13–7

Bank Branches and Deposits

A researcher wishes to see if there is a relationship between the number of branches a bank has and the total number of deposits (in billions of dollars) the bank receives. A sample of eight regional banks is selected, and the number of branches and the amount of deposits are shown in the table. At $\alpha = 0.05$ is there a significant linear correlation between the number of branches and the amount of the deposits?

Bank	Number of branches	Deposits (in billions)
A	209	$23
B	353	31
C	19	7
D	201	12
E	344	26
F	132	5
G	401	24
H	126	5

Source: SNL Financial.

Solution

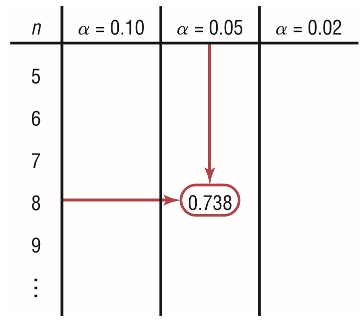
Step 1 State the hypotheses.

H_0: $\rho = 0$ and H_1: $\rho \neq 0$

Step 2 Find the critical value. Use Table L to find the value for $n = 8$ and $\alpha = 0.05$. It is 0.738. See Figure 13–3.

Figure 13–3

Finding the Critical Value in Table L for Example 13–7



Step 3 Find the test value.

a. Rank each data set as shown in the table.

Bank	Branches	Rank	Deposits	Rank
A	209	4	23	4
B	353	2	31	1
C	19	8	7	6
D	201	5	12	5
E	344	3	26	2
F	132	6	5	7
G	401	1	24	3
H	126	7	4	8

b. Let X_1 be the rank of the branches and X_2 be the rank of the deposits.

c. Subtract the ranking ($X_1 - X_2$).

$4 - 4 = 0$ $\quad 2 - 1 = 1$ $\quad 8 - 6 = 2$ $\quad$ etc.

d. Square the differences.

$0^2 = 0$ $\quad 1^2 = 1$ $\quad 2^2 = 4$ $\quad$ etc.

e. Find the sum of the squares

$0 + 1 + 4 + 0 + 1 + 1 + 4 + 1 = 12$

The results can be summarized in a table as shown.

X_1	X_2	$d = X_1 - X_2$	d^2
4	4	0	0
2	1	1	1
8	6	2	4
5	5	0	0
3	2	1	1
6	7	−1	1
1	3	−2	4
7	8	−1	1
			$\Sigma d^2 = 12$

Unusual Stat

You are almost twice as likely to be killed while walking with your back to traffic as you are when facing traffic, according to the National Safety Council.

f. Substitute in the formula for r_s.

$$r_s = 1 - \frac{6\,\Sigma d^2}{n(n^2-1)} \quad \text{where } n = \text{number of pairs}$$

$$r_s = 1 - \frac{6 \cdot 12}{6(6^2-1)} = 1 - \frac{72}{210} = 0.657$$

Step 4 Make the decision. Do not reject the null hypothesis since $r_s = 0.657$, which is less than the critical value of 0.738.

Step 5 Summarize the results. There is not enough evidence to say that there is a linear relationship between the number of branches a bank has and the deposits of the bank.

The steps for finding and testing the Spearman rank correlation coefficient are given in the Procedure Table.

Procedure Table

Finding and Testing the Spearman Rank Correlation Coefficient

Step 1 State the hypotheses.

Step 2 Rank each data set.

Step 3 Subtract the rankings $(X_1 - X_2)$.

Step 4 Square the differences.

Step 5 Find the sum of the squares.

Step 6 Substitute in the formula.

$$r_s = 1 - \frac{6\,\Sigma d^2}{n(n^2-1)}$$

where
d = difference in ranks
n = number of pairs of data

Step 7 Find the critical value.

Step 8 Make the decision.

Step 9 Summarize the results.

Objective 7

Test hypotheses, using the runs test.

The Runs Test

When samples are selected, you assume that they are selected at random. How do you know if the data obtained from a sample are truly random? Before the answer to this question is given, consider the following situations for a researcher interviewing 20 people for a survey. Let their gender be denoted by M for male and F for female. Suppose the participants were chosen as follows:

Situation 1 M M M M M M M M M M F F F F F F F F F F

It does not look as if the people in this sample were selected at random, since 10 males were selected first, followed by 10 females.

Consider a different selection:

Situation 2 F M F M F M F M F M F M F M F M F M F M

In this case, it seems as if the researcher selected a female, then a male, etc. This selection is probably not random either.

Finally, consider the following selection:

Situation 3 F F F M M F M F M M F F M M F F M M M F

This selection of data looks as if it may be random, since there is a mix of males and females and no apparent pattern to their selection.

Rather than try to guess whether the data of a sample have been selected at random, statisticians have devised a nonparametric test to determine randomness. This test is called the **runs test.**

A **run** is a succession of identical letters preceded or followed by a different letter or no letter at all, such as the beginning or end of the succession.

For example, the first situation presented has two runs:

Run 1: M M M M M M M M M M

Run 2: F F F F F F F F F F

The second situation has 20 runs. (Each letter constitutes one run.) The third situation has 11 runs.

Run 1:	F F F	Run 5:	F	Run 9:	F F
Run 2:	M M	Run 6:	M M	Run 10:	M M M
Run 3:	F	Run 7:	F F	Run 11:	F
Run 4:	M	Run 8:	M M		

Example 13–8

Determine the number of runs in each sequence.

a. M M F F F M F F

b. H T H H H

c. A B A A A B B A B B B

Solution

a. There are four runs, as shown.

$$\underbrace{\text{M M}}_{1} \quad \underbrace{\text{F F F}}_{2} \quad \underbrace{\text{M}}_{3} \quad \underbrace{\text{F F}}_{4}$$

b. There are three runs, as shown.

$$\underbrace{\text{H}}_{1} \quad \underbrace{\text{T}}_{2} \quad \underbrace{\text{H H H}}_{3}$$

c. There are six runs, as shown.

$$\underbrace{\text{A}}_{1} \quad \underbrace{\text{B}}_{2} \quad \underbrace{\text{A A A}}_{3} \quad \underbrace{\text{B B}}_{4} \quad \underbrace{\text{A}}_{5} \quad \underbrace{\text{B B B}}_{6}$$

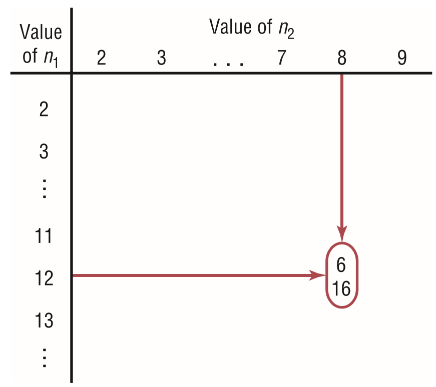
The test for randomness considers the number of runs rather than the frequency of the letters. For example, for data to be selected at random, there should not be too few or too many runs, as in situations 1 and 2. The runs test does not consider the questions of how many males or females were selected or how many of each are in a specific run.

To determine whether the number of runs is within the random range, use Table M in Appendix C. The values are for a two-tailed test with $\alpha = 0.05$. For a sample of 12 males and 8 females, the table values shown in Figure 13–4 mean that any number of runs from 7 to 15 would be considered random. If the number of runs is 6 or less or 16 or more, the sample is probably not random, and the null hypothesis should be rejected.

Example 13–9 shows the procedure for conducting the runs test by using letters as data. Example 13–10 shows how the runs test can be used for numerical data.

Figure 13–4

Finding the Critical Value in Table M



Example 13–9 Gender of Train Passengers

On a commuter train, the conductor wishes to see whether the passengers enter the train at random. He observes the first 25 people, with the following sequence of males (M) and females (F).

F F F M M F F F F M F M M M F F F F M M F F F M M

Test for randomness at $\alpha = 0.05$.

Solution

Step 1 State the hypotheses and identify the claim.

H_0: The passengers board the train at random, according to gender (claim).

H_1: The null hypothesis is not true.

Step 2 Find the number of runs. Arrange the letters according to runs of males and females, as shown.

Run	Gender
1	F F F
2	M M
3	F F F F
4	M
5	F
6	M M M
7	F F F F
8	M M
9	F F F
10	M M

There are 15 females (n_1) and 10 males (n_2).

Step 3 Find the critical value. Find the number of runs in Table M for $n_1 = 15$, $n_2 = 10$, and $\alpha = 0.05$. The values are 7 and 18. *Note:* In this situation the critical value is found after the number of runs is determined.

Step 4 Make the decision. Compare these critical values with the number of runs. Since the number of runs is 10 and 10 is between 7 and 18, do not reject the null hypothesis.

Step 5 Summarize the results. There is not enough evidence to reject the hypothesis that the passengers board the train at random according to gender.

Example 13–10 Ages of Drug Program Participants

Twenty people enrolled in a drug abuse program. Test the claim that the ages of the people, according to the order in which they enroll, occur at random, at $\alpha = 0.05$. The data are 18, 36, 19, 22, 25, 44, 23, 27, 27, 35, 19, 43, 37, 32, 28, 43, 46, 19, 20, 22.

Solution

Step 1 State the hypotheses and identify the claim.

H_0: The ages of the people, according to the order in which they enroll in a drug program, occur at random (claim).

H_1: The null hypothesis is not true.

Step 2 Find the number of runs.

a. Find the median of the data. Arrange the data in ascending order.

18 19 19 19 20 22 22 23 25 27 27

28 32 35 36 37 43 43 44 46

The median is 27.

b. Replace each number in the original sequence with an A if it is above the median and with a B if it is below the median. Eliminate any numbers that are equal to the median.

B A B B B A B A B A A A A A A B B B

c. Arrange the letters according to runs.

Run	Letters
1	B
2	A
3	B B B
4	A
5	B
6	A
7	B
8	A A A A A A
9	B B B

Step 3 Find the critical value. Table M shows that with $n_1 = 9$, $n_2 = 9$, and $\alpha = 0.05$, the number of runs should be between 5 and 15.

Step 4 Make the decision. Since there are 9 runs and 9 falls between 5 and 15, the null hypothesis is not rejected.

Step 5 Summarize the results. There is not enough evidence to reject the hypothesis that the ages of the people who enroll occur at random.

The steps for the runs test are given in the Procedure Table.

Procedure Table

The Runs Test

Step 1 State the hypotheses and identify the claim.

Step 2 Find the number of runs.

Note: When the data are numerical, find the median. Then compare each data value with the median and classify it as above or below the median. Other methods such as odd-even can also be used. (Discard any value that is equal to the median.)

Step 3 Find the critical value. Use Table M.

Step 4 Make the decision. Compare the actual number of runs with the critical value.

Step 5 Summarize the results.

Applying the Concepts 13–6

Tall Trees

As a biologist, you wish to see if there is a relationship between the heights of tall trees and their diameters. You find the following data for the diameter (in inches) of the tree at 4.5 feet from the ground and the corresponding heights (in feet).

Diameter (in.)	Height (ft)
1024	261
950	321
451	219
505	281
761	159
644	83
707	191
586	141
442	232
546	108

Source: *The World Almanac and Book of Facts.*

1. What question are you trying to answer?
2. What type of nonparametric analysis could be used to answer the question?
3. What would be the corresponding parametric test that could be used?
4. Which test do you think would be better?
5. Perform both tests and write a short statement comparing the results.

See page 718 for the answer.

Exercises 13–6

For Exercises 1 through 4, find the critical value from Table L for the rank correlation coefficient, given sample size n and α. Assume that the test is two-tailed.

1. $n = 14, \alpha = 0.01$ 0.716

2. $n = 28, \alpha = 0.02$ 0.488

3. $n = 10, \alpha = 0.05$ 0.648

4. $n = 9, \alpha = 0.01$ 0.833

For Exercises 5 through 14, perform these steps.

a. Find the Spearman rank correlation coefficient.

b. State the hypotheses.

c. Find the critical value. Use $\alpha = 0.05$.

d. Make the decision.

e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

5. Mathematics Achievement Test Scores The National Assessment of Educational Progress (U.S. Department of Education) tests mathematics, reading, and science achievement in grades 4 and 8. A random sample of states is selected, and their mathematics achievement scores are noted for fourth- and eighth-graders. At $\alpha = 0.05$ can a linear relationship be concluded between the data?

Grade 4	89	84	80	89	88	77	80
Grade 8	81	75	66	76	80	59	74

Source: *World Almanac.*

6. Subway and Commuter Rail Passengers Six cities are selected, and the number of daily passenger trips (in thousands) for subways and commuter rail service is obtained. At $\alpha = 0.05$, is there a relationship between the variables? Suggest one reason why the transportation authority might use the results of this study.

City	1	2	3	4	5	6
Subway	845	494	425	313	108	41
Rail	39	291	142	103	33	39

Source: American Public Transportation Association.

7. Motion Picture Releases and Gross Revenue In Chapter 10 it was demonstrated that there was a significant linear relationship between the numbers of releases that a motion picture studio put out and its gross receipts for the year. Is there a relationship between the two at the 0.05 level of significance?

No. of releases	361	270	306	22	35	10	8	12	21
Receipts	2844	1967	1371	1064	667	241	188	154	125

Source: www.showbizdata.com

8. Hospitals and Nursing Homes Find the Spearman rank correlation coefficient for the following data, which represent the number of hospitals and nursing homes in each of seven randomly selected states. At the 0.05 level of significance, is there enough evidence to conclude that there is a correlation between the two?

Hospitals	107	61	202	133	145	117	108
Nursing homes	230	134	704	376	431	538	373

Source: *World Almanac.*

9. Calories and Cholesterol in Fast-Food Sandwiches Use the Spearman rank correlation coefficient to see if there is a linear relationship between these two sets of data, representing the number of calories and the amount of cholesterol in fast-food sandwiches.

Calories	580	580	270	470	420	415	330	430
Cholesterol (mg)	205	225	285	270	185	215	185	220

Source: www.fatcalories.com

10. Book Publishing The data below show the number of books published in six different subject areas for the years 1980 and 2004. Use $\alpha = 0.05$ to see if there is a relationship between the two data sets. Do you think the same relationship will hold true 20 years from now? (In case you're curious, the subjects represented are agriculture, home economics, literature, music, science, and sports and recreation.)

1980	461	879	1686	357	3109	971
2004	1065	3639	4671	2764	8509	4806

Source: *New York Times Almanac.*

11. Gasoline Costs Shown is a comparison between the average gasoline prices charged by a gasoline station and a car rental company for 10 cities in the United States before the recent surge in gasoline prices. At $\alpha = 0.05$, is there a relationship between the prices? How might a person who travels a lot and rents an automobile use the information obtained from this study?

Car rental agency price	5.12	5.27	5.29	5.18	5.59
Gas station price	2.09	1.96	2.29	1.94	2.20
Car rental agency price	5.30	5.83	5.46	5.12	5.15
Gas station price	2.20	2.40	2.12	2.15	2.11

Source: AAA Oil Price Information Service and car rental agencies.

12. Motor Vehicle Thefts and Burglaries Is there a relationship between the number of motor vehicle (MV) thefts and the number of burglaries (per 100,000 population) for different metropolitan areas? Use $\alpha = 0.05$.

MV theft	220.5	499.4	285.6	159.2	104.3	444
Burglary	913.6	909.2	803.6	520.9	477.8	993.7

Source: *New York Times Almanac.*

13. Cyber School Enrollments Shown are the number of students enrolled in cyber school for five randomly selected school districts and the per-pupil costs for the cyber school education. At $\alpha = 0.10$, is there a relationship between the two variables? How might this information be useful to school administrators?

Number of students	10	6	17	8	11
Per-pupil cost	7200	9393	7385	4500	8203

Source: *Pittsburgh Tribune-Review.*

14. Drug Prices Shown are the price for a human dose of several prescription drugs and the price for an equivalent dose for animals. At $\alpha = 0.10$, is there a relationship between the variables?

Humans	0.67	0.64	1.20	0.51	0.87	0.74	0.50	1.22
Animals	0.13	0.18	0.42	0.25	0.57	0.57	0.49	1.28

Source: House Committee on Government Reform.

15. A school dentist wanted to test the claim, at $\alpha = 0.05$, that the number of cavities in fourth-grade students is random. Forty students were checked, and the number of cavities each had is shown here. Test for randomness of the values above or below the median.

0	4	6	0	6	2	5	3	1	5	1
2	2	1	3	7	3	6	0	2	6	0
2	3	1	5	2	1	3	0	2	3	7
3	1	5	1	1	2	2				

16. Daily Lottery Numbers Listed below are the daily numbers (daytime drawing) for the Pennsylvania State Lottery for February 2007. Using O for odd and E for even, test for randomness at $\alpha = 0.05$.

270	054	373	204	908	121	121
804	116	467	357	926	626	247
783	554	406	272	508	764	890
441	964	606	568	039	370	583

Source: www.palottery.com

17. Cola Orders Many eating facilities serve one brand of soft drinks only, but the College Corner Café serves two different brands. On a Friday night here are the orders for cola. Test for randomness at the 0.05 level of significance.

P	P	C	C	C	P	C	P	P	C	P
P	P	C	P	C	P	C	C	C	C	P
C	C	P	P	P	P	C				

18. Random Numbers Random? A calculator generated these integers randomly. Apply the runs test to see if you can reject the hypothesis that the numbers are truly random. Use $\alpha = 0.05$.

1	1	1	1	1	1	2	1	1	1	1
2	2	1	2	1	2	2	1	2	1	1
2	1	1								

19. Concert Seating As students, faculty, friends, and family arrived for the Spring Wind Ensemble Concert at Shafer Auditorium, they were asked whether they were going to sit in the balcony (B) or on the ground floor (G). Use the responses listed below and test for randomness at $\alpha = 0.05$.

B B G G B B G B B B B B B G B B
G G B B B B G G G G B G B B B G G

20. Twenty shoppers are in a checkout line at a grocery store. At $\alpha = 0.05$, test for randomness of their gender: male (M) or female (F). The data are shown here.

F M M F F M F M M F

F M M M F F F F F M

21. Employee Absences A supervisor records the number of employees absent over a 30-day period. Test for randomness, at $\alpha = 0.05$.

27	6	19	24	18	12	15	17	18	20
0	9	4	12	3	2	7	7	0	5
32	16	38	31	27	15	5	9	4	10

22. Skiing Conditions A ski lodge manager observes the weather for the month of February. If his customers are able to ski, he records S; if weather conditions do not permit skiing, he records N. Test for randomness, at $\alpha = 0.05$.

S S S S S N N N N N N N N N
N S S S N N N S S S S S S S S

23. Tossing a Coin Toss a coin 30 times and record the outcomes (H or T). Test the results for randomness at $\alpha = 0.05$. Repeat the experiment a few times and compare your results. Answers will vary.

Extending the Concepts

When $n \geq 30$, the formula $r = \dfrac{\pm z}{\sqrt{n-1}}$ can be used to find the critical values for the rank correlation coefficient. For example, if $n = 40$ and $\alpha = 0.05$ for a two-tailed test,

$$r = \frac{\pm 1.96}{\sqrt{40-1}} = \pm 0.314$$

Hence, any r_s greater than or equal to $+0.314$ or less than or equal to -0.314 is significant.

For Exercises 24 through 28, find the critical r value for each (assume that the test is two-tailed).

24. $n = 50$, $\alpha = 0.05$ ± 0.28

25. $n = 30$, $\alpha = 0.01$ ± 0.479

26. $n = 35$, $\alpha = 0.02$ ± 0.400

27. $n = 60$, $\alpha = 0.10$ ± 0.215

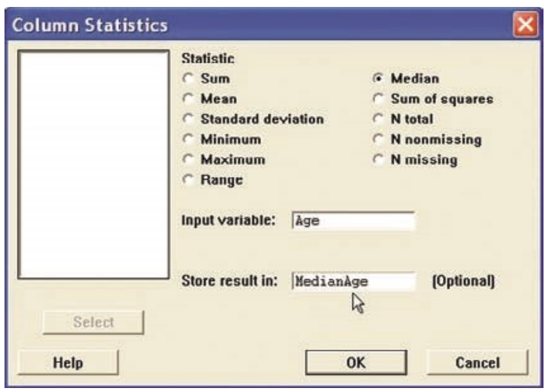
28. $n = 40$, $\alpha = 0.01$ ± 0.413

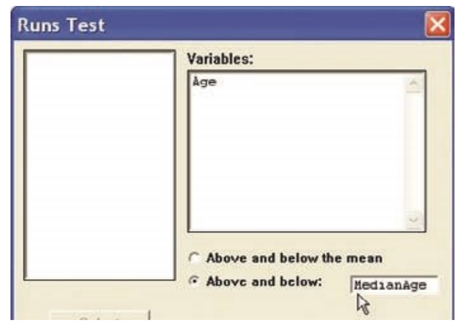
Technology *Step by Step*

MINITAB
Step by Step

Runs Test for Randomness

1. Sequence is important! Enter the data down C1 in the same order they were collected. Do not sort them! Use the data from Example 13–10.
2. Calculate the median and store it as a constant.
   a) Select **Calc>Column Statistics.**
   b) Check the option for Median.
   c) Use C1 Age for the Input Variable.
   d) Type the name of the constant MedianAge in the Store result in text box.
   e) Click [OK].





3. Select **Stat>Nonparametric>Runs Test.**
4. Select C1 Age as the variable.
5. Click the button for Above and below, then select MedianAge in the text box.
6. Click [OK]. The results will be displayed in the session window.

Runs Test: Age

```
Runs test for Age
Runs above and below K = 27

The observed number of runs = 9
The expected number of runs = 10.9
9 observations above K, 11 below
* N is small, so the following approximation may be invalid.
P-value = 0.378
```

The *P*-value is 0.378. Do not reject the null hypothesis.

Excel
Step by Step

Spearman Rank Correlation Coefficient

Example: Textbook Ratings

Two students were asked to rate eight different textbooks for a specific course on an ascending scale from 0 to 20 points. Points were assigned for each of several categories, such as reading level, use of illustrations, and use of color. At $\alpha = 0.05$, test the hypothesis that there is a significant linear correlation between the two students' ratings. The data are shown in the following table.

Textbook	Student 1's rating	Student 2's rating
A	4	4
B	10	6
C	18	20
D	20	14
E	12	16
F	2	8
G	5	11
H	9	7

Excel does not have a procedure to compute the Spearman rank correlation coefficient. However, you may compute this statistic by using the MegaStat Add-in available on your CD. If you have not installed this add-in, do so, following the instructions from the Chapter 1 Excel Step by Step.

1. Enter the rating scores from the example into columns A and B of a new worksheet.
2. From the toolbar, select Add-Ins, **MegaStat>Nonparametric Tests>Spearman Coefficient of Rank Correlation.** *Note:* You may need to open MegaStat from the MegaStat.xls file on your computer's hard drive.
3. Type **A1:B8** in the box for Input range.
4. Check the Correct for ties option.
5. Click [OK].

Spearman Coefficient of Rank Correlation

	#1	#2
#1	1.000	
#2	.643	1.000

```
        8 sample size

±0.707 critical value .05 (two-tail)
±0.834 critical value .01 (two-tail)
```

Since the correlation coefficient 0.643 is less than the critical value, there is not enough evidence to reject the null hypothesis of a nonzero correlation between the variables.

Summary

- In many research situations, the assumptions (particularly that of normality) for the use of parametric statistics cannot be met. Also, some statistical studies do not involve parameters such as means, variances, and proportions. For both situations, statisticians have developed nonparametric statistical methods, also called *distribution-free methods.* (13–1)
- There are several advantages to the use of nonparametric methods. The most important one is that no knowledge of the population distributions is required. Other advantages include ease of computation and understanding. The major disadvantage is that they are less efficient than their parametric counterparts when the assumptions for the parametric methods are met. In other words, larger sample sizes are needed to get results as accurate as those given by their parametric counterparts. (13–1)
- This list gives the nonparametric statistical tests presented in this chapter, along with their parametric counterparts.

Nonparametric test	Parametric test	Condition
Single-sample sign test (13–2)	z or t test	One sample
Paired-sample sign test (13–2)	z or t test	Two dependent samples
Wilcoxon rank sum test (13–3)	z or t test	Two independent samples
Wilcoxon signed-rank test (13–4)	t test	Two dependent samples
Kruskal-Wallis test (13–5)	ANOVA	Three or more independent samples
Spearman rank correlation coefficient (13–6)	Pearson's correlation coefficient	Relationships between variables
Runs test (13–6)	None	Randomness

- When the assumptions of the parametric tests can be met, the parametric tests should be used instead of their nonparametric counterparts.

Important Terms

distribution-free statistics 672

Kruskal-Wallis test 693

nonparametric statistics 672

parametric tests 672

ranking 673

run 703

runs test 703

sign test 675

Spearman rank correlation coefficient 700

Wilcoxon rank sum test 683

Wilcoxon signed-rank test 683

Important Formulas

Formula for the z test value in the sign test:

$$z = \frac{(X + 0.5) - (n/2)}{\sqrt{n}/2}$$

where

n = sample size (greater than or equal to 26)

X = smaller number of positive or negative signs

Formula for the Wilcoxon rank sum test:

$$z = \frac{R - \mu_R}{\sigma_R}$$

where

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2}$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1 + n_2 + 1)}{12}}$$

R = sum of ranks for smaller sample size (n_1)

n_1 = smaller of sample sizes

n_2 = larger of sample sizes

$n_1 \geq 10$ and $n_2 \geq 10$

Formula for the Wilcoxon signed-rank test:

$$z = \frac{w_s - \dfrac{n(n+1)}{4}}{\sqrt{\dfrac{n(n+1)(2n+1)}{24}}}$$

where

n = number of pairs where difference is not 0 and $n \geq 30$

w_s = smaller sum in absolute value of signed ranks

Formula for the Kruskal-Wallis test:

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \cdots + \frac{R_k^2}{n_k}\right) - 3(N+1)$$

where

R_1 = sum of ranks of sample 1

n_1 = size of sample 1

R_2 = sum of ranks of sample 2

n_2 = size of sample 2

$\vdots$

R_k = sum of ranks of sample k

n_k = size of sample k

$N = n_1 + n_2 + \cdots + n_k$

k = number of samples

Formula for the Spearman rank correlation coefficient:

$$r_s = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

where

d = difference in ranks

n = number of data pairs

Review Exercises

For Exercises 1 through 13, follow this procedure:

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

1. **Ages of City Residents** The median age for the total population of the state of Maine is 41.2, the highest in the nation. The mayor of a particular city believes that his population is considerably "younger" and that the median age there is 36 years. At $\alpha = 0.05$, is there sufficient evidence to reject his claim? The data here represent a random selection of persons from the household population of the city.

40	56	42	72	12	22
25	43	39	48	50	37
18	35	15	30	52	45
10	24	25	39	29	19
30	60	38	42	41	61

Source: www.factfinder.census.gov

2. **Lifetime of Truck Tires** A tire manufacturer claims that the median lifetime of a certain brand of truck tires is 40,000 miles. A sample of 30 tires shows that 12 lasted longer than 40,000 miles. Is there enough evidence to reject the claim at $\alpha = 0.05$? Use the sign test.

3. **Grocery Store Repricing** A grocery store chain has decided to help customers save money by instituting "temporary repricing" to help cut costs. Nine products from the sale flyer are featured below with their regular price and their "temporary" new price. Using the paired-sample sign test and $\alpha = 0.05$, is there evidence of a difference in price? Comment on your results.

Old	2.59	0.69	1.29	3.10	1.89	2.05	1.58	2.75	1.99
New	2.09	0.70	1.18	2.95	1.59	1.75	1.32	2.19	1.99

4. **Record High Temperatures** Shown here are the record high temperatures for Dawson Creek in British Columbia, Canada, and for Whitehorse in Yukon, Canada, for 12 months. Using the Wilcoxon rank sum test at $\alpha = 0.05$, do you find a difference in the record high temperatures? Use the *P*-value method.

Dawson Creek	52	60	57	71	86	89	94	93	88	80	66	52
Whitehorse	47	50	51	69	86	89	91	86	80	66	51	47

Source: Jack Williams, *The USA TODAY Weather Almanac.*

5. **Hours Worked by Student Employees** Student employees are a major part of most college campus employment venues. Two major departments that participate in student hiring are listed below with the number of hours worked by students for a month. At the 0.10 level of significance, is there sufficient evidence to conclude a difference? Is the conclusion the same for the 0.05 level of significance?

Athletics	20	24	17	12	18	22	25	30	15	19	
Library	35	28	24	20	25	18	22	26	31	21	19

6. **Fuel Efficiency of Automobiles** Twelve automobiles were tested to see how many miles per gallon each one obtained. Under similar driving conditions, they were tested again, using a special additive. The data are shown here. At $\alpha = 0.05$, did the additive improve gas mileage? Use the Wilcoxon signed-rank test.

Before		**After**	
13.6	18.3	22.6	23.7
18.2	19.5	21.9	20.8
16.1	18.2	25.3	25.3
15.3	16.7	28.6	27.2
19.2	21.3	15.2	17.2
18.8	17.2	16.3	18.5

7. **Lunch Costs** Full-time employees in a large city were asked how much they spent on a typical weekday lunch and how much they spent on the weekend. The amounts are listed below. At $\alpha = 0.05$, is there sufficient evidence to conclude a difference in the amounts spent?

Weekday	7.00	5.50	4.50	10.00	6.75	5.00	6.00
Weekend	6.00	10.00	7.00	12.00	8.50	7.00	8.00

8. **Breaking Strengths of Ropes** Samples of three types of ropes are tested for breaking strength. The data (in pounds) are shown here. At $\alpha = 0.05$, is there a difference in the breaking strength of the ropes? Use the Kruskal-Wallis test.

Cotton	**Nylon**	**Hemp**
230	356	506
432	303	527
505	361	581
487	405	497
451	432	459
380	378	507
462	361	562
531	399	571
366	372	499
372	363	475
453	306	505
488	304	561
462	318	532
467	322	501

Statistics Today

Too Much or Too Little?—Revisited

In this case, the manufacturer would select a sequence of bottles and see how many bottles contained more than 40 ounces, denoted by plus, and how many bottles contained less than 40 ounces, denoted by minus. The sequence could then be analyzed according to the number of runs, as explained in Section 13–6. If the sequence were not random, then the machine would need to be checked to see if it was malfunctioning. Another method that can be used to see if machines are functioning properly is *statistical quality control.* This method is beyond the scope of this book.

9. Beach Temperatures for July The National Oceanographic Data Center provides useful data for vacation planning. Below are listed beach temperatures in the month of July for various U.S. coastal areas. Using the 0.05 level of significance, can it be concluded that there is a difference in temperatures? Omit the Southern Pacific temperatures and repeat the procedure. Is the conclusion the same?

Southern Pacific	Western Gulf	Eastern Gulf	Southern Atlantic
67	86	87	76
68	86	87	81
66	84	86	82
69	85	86	84
63	79	85	80
62	85	84	86
		85	87

Source: www.nodc.noaa.gov

10. Homework Exercises and Exam Scores A statistics instructor wishes to see whether there is a relationship between the number of homework exercises a student completes and her or his exam score. The data are shown here. Using the Spearman rank correlation coefficient, test the hypothesis that there is no relationship at $\alpha = 0.05$.

Homework problems	63	55	58	87	89	52	46	75	105
Exam score	85	71	75	98	93	63	72	89	100

11. Shown below is the average number of viewers for 10 television shows for two consecutive years. At $\alpha = 0.05$, is there a relationship between the number of viewers?

Last year	28.9	26.4	20.8	25.0	21.0	19.2
This year	26.6	20.5	20.2	19.1	18.9	17.8

Last year	13.7	18.8	16.8	15.3
This year	16.8	16.7	16.0	15.8

12. Book Arrangements A bookstore has a display of sale books arranged on shelves in the store window. A combination of hardbacks (H) and paperbacks (P) is arranged as follows. Test for randomness at $\alpha = 0.05$.

H H H P P P P H P H P H H H H P P P P P
H H P P P H P P P P P P

13. Exam Scores An instructor wishes to see whether grades of students who finish an exam occur at random. Shown here are the grades of 30 students in the order that they finished an exam. (Read from left to right across each row, and then proceed to the next row.) Test for randomness, at $\alpha = 0.05$.

87	93	82	77	64	98
100	93	88	65	72	73
56	63	85	92	95	91
88	63	72	79	55	53
65	68	54	71	73	72

Data Analysis

The Data Bank is found in Appendix D, or on the World Wide Web by following links from www.mhhe.com/math/stat/bluman

1. From the Data Bank, choose a sample and use the sign test to test one of the following hypotheses.
   a. For serum cholesterol, test H_0: median = 220 milligram percent (mg%).
   b. For systolic pressure, test H_0: median = 120 millimeters of mercury (mm Hg).
   c. For IQ, test H_0: median = 100.
   d. For sodium level, test H_0: median = 140 mEq/l.
2. From the Data Bank, select a sample of subjects. Use the Kruskal-Wallis test to see if the sodium levels of smokers and nonsmokers are equal.
3. From the Data Bank select a sample of 50 subjects. Use the Wilcoxon rank sum test to see if the means of the sodium levels of the males differ from those of the females.

Chapter Quiz

Determine whether each statement is true or false. If the statement is false, explain why.

1. Nonparametric statistics cannot be used to test the difference between two means. False
2. Nonparametric statistics are more sensitive than their parametric counterparts. False
3. Nonparametric statistics can be used to test hypotheses about parameters other than means, proportions, and standard deviations. True
4. Parametric tests are preferred over their nonparametric counterparts, if the assumptions can be met. True

Select the best answer.

5. The ________ test is used to test means when samples are dependent and the normality assumption cannot be met.

   a. Wilcoxon signed-rank
   b. Wilcoxon rank sum
   c. Sign
   d. Kruskal-Wallis

6. The Kruskal-Wallis test uses the ________ distribution.

   a. z
   b. t
   c. Chi-square
   d. F

7. The nonparametric counterpart of ANOVA is the ________.

   a. Wilcoxon signed-rank test
   b. Sign test
   c. Runs test
   d. None of the above

8. To see if two rankings are related, you can use the ________.

   a. Runs test
   b. Spearman correlation coefficient
   c. Sign test
   d. Kruskal-Wallis test

Complete the following statements with the best answer.

9. When the assumption of normality cannot be met, you can use ________ tests. Nonparametric
10. When data are ________ or ________ in nature, nonparametric methods are used. Nominal, ordinal
11. To test to see whether a median was equal to a specific value, you would use the ________ test. Sign
12. Nonparametric tests are less ________ than their parametric counterparts. Sensitive

For the following exercises, use the traditional method of hypothesis testing unless otherwise specified.

13. **Home Prices** The median price for an existing home in 2009 was \$177,500. A random sample of homes for sale listed by a local realtor indicated homes available for the following prices. Test the claim that the median is not \$177,500. Use $\alpha = 0.05$.

184,500	174,900	155,000	210,000	235,500	399,900
355,900	182,500	229,900	199,900	169,900	219,900

Source: *World Almanac.*

14. **Lifetimes of Batteries** A battery manufacturer claims that the median lifetime of a certain brand of heavy-duty battery is 1200 hours. A sample of 25 batteries shows that 15 lasted longer than 1200 hours. Test the claim at $\alpha = 0.05$. Use the sign test.

15. **Weights of Turkeys** A special diet is fed to adult turkeys to see if they will gain weight. The before and after weights (in pounds) are given here. Use the paired-sample sign test at $\alpha = 0.05$ to see if there is weight gain.

Before	28	24	29	30	32	33	25	26	28
After	30	29	31	32	32	35	29	25	31

16. **Charity Donations** Two teams of 10 members each solicited donations for their participation in a charity walk for blood cancer research. The teams received the following amounts. At $\alpha = 0.05$ can it be concluded that there is a difference in amounts?

Team A	100	50	65	50	60	75	100	150	108	120
Team B	135	90	80	140	155	60	200	58	70	72

17. **Textbook Costs** Samples of students majoring in law and nursing are selected, and the amount each spent on textbooks for the spring semester is recorded here, in dollars. Using the Wilcoxon rank sum test at $\alpha = 0.10$, is there a difference in the amount spent by each group?

Law	167	158	162	106	98	206	112	121
Nursing	98	198	209	168	157	126	104	122

Law	133	145	151	199
Nursing	111	138	116	201

18. **Student Grade Point Averages** The grade point average of a group of students was recorded for one month. During the next nine-week grading period, the students attended a workshop on study skills. Their GPAs were recorded at the end of the grading period, and the data appear here. Using the Wilcoxon signed-rank test at $\alpha = 0.05$, can it be concluded that the GPA increased?

Before	3.0	2.9	2.7	2.5	2.1	2.6	1.9	2.0
After	3.2	3.4	2.9	2.5	3.0	3.1	2.4	2.8

19. **Sodium Content of Fast-Food Sandwiches** Sometimes calories and cholesterol are not the only considerations in healthy eating. Below are listed the sodium contents (in mg) for sandwiches from three popular fast-food restaurants. Use $\alpha = 0.05$.

No. 1	No. 2	No. 3
2940	2010	1130
3720	1850	1190
3180	1980	1220
2260	1640	1640
2780	1440	1240

Source: www.fatcalories.com

20. **Medication and Reaction Times** Three different groups of monkeys were fed three different medications for one month to see if the medication has any effect on reaction time. Each monkey was then taught to repeat a series of steps to receive a reward. The number of trials it took each to receive the reward is shown here. At $\alpha = 0.05$, does the medication have an effect on reaction time? Use the Kruskal-Wallis test. Use the *P*-value method.

Med. 1	8	7	11	14	8	6	5
Med. 2	3	4	6	7	9	3	4
Med. 3	8	14	13	7	5	9	12

21. **Drug Prices** Is there a relationship between the prescription drug prices in Canada and Great Britain? Use $\alpha = 0.10$.

Canada	1.47	1.07	1.34	1.34	1.47	1.07	3.39	1.11	1.13
Great Britain	1.67	1.08	1.67	0.82	1.73	0.95	2.86	0.41	1.70

Source: *USA TODAY.*

22. **Funding and Enrollment for Head Start Students** Is there a relationship between the amount of money (in millions of dollars) spent on the Head Start Program by the states and the number of students enrolled (in thousands)? Use $\alpha = 0.10$.

Funding	100	50	22	88	49	219
Enrollment	16	7	3	14	8	31

Source: Gannet News Service.

23. **Birth Registry** At the state registry of vital statistics, the birth certificates issued for females (F) and males (M) were tallied. At $\alpha = 0.05$, test for randomness. The data are shown here.

M M F F F F F F F F M M M M F F
M F M F M M M F F F

24. **Output of Motors** The output in revolutions per minute (rpm) of 10 motors was obtained. The motors were tested again under similar conditions after they had been reconditioned. The data are shown here. At $\alpha = 0.05$, did the reconditioning improve the motors' performance? Use the Wilcoxon signed-rank test.

Before	413	701	397	602	405	512	450	487	388	351
After	433	712	406	650	450	550	450	500	402	415

25. **State Lottery Numbers** A statistician wishes to determine if a state's lottery numbers are selected at random. The winning numbers selected for the month of February are shown here. Test for randomness at $\alpha = 0.05$.

321	909	715	700	487	808	509	606	943	761
200	123	367	012	444	576	409	128	567	908
103	407	890	193	672	867	003	578		

Critical Thinking Challenges

1. **Tolls for Bridge** Two commuters ride to work together in one car. To decide who pays the toll for a bridge on the way to work, they flip a coin and the loser pays. Explain why over a period of one year, one person might have to pay the toll 5 days in a row. There is no toll on the return trip. (*Hint:* You may want to use random numbers.)

2. **Olympic Medals** Shown in the next column are the type and number of medals each country won in the 2000 Summer Olympic Games. You are to rank the countries from highest to lowest. Gold medals are highest, followed by silver, followed by bronze. There are many different ways to rank objects and events. Here are several suggestions.

a. Rank the countries according to the total medals won.
b. List some advantages and disadvantages of this method.
c. Rank each country separately for the number of gold medals won, then for the number of silver medals won, and then for the number of bronze medals won. Then rank the countries according to the sum of the *ranks* for the categories.
d. Are the rankings of the countries the same as those in step *a*? Explain any differences.
e. List some advantages and disadvantages of this method of ranking.
f. A third way to rank the countries is to assign a weight to each medal. In this case, assign 3 points

for each gold medal, 2 points for each silver medal, and 1 point for each bronze medal the country won. Multiply the number of medals by the weights for each medal and find the sum. For example, since Austria won 2 gold medals, 1 silver medal, and 0 bronze medals, its rank sum is $(2 \times 3) + (1 \times 2) + (0 \times 1) = 8$. Rank the countries according to this method.

g. Compare the ranks using this method with those using the other two methods. Are the rankings the same or different? Explain.

h. List some advantages and disadvantages of this method.

i. Select two of the rankings, and run the Spearman rank correlation test to see if they differ significantly.

Summer Olympic Games 2000 Final Medal Standings

Country	Gold	Silver	Bronze
Austria	2	1	0
Canada	3	3	8
Germany	14	17	26
Italy	13	8	13
Norway	4	3	3
Russia	32	28	28
Switzerland	1	6	2
United States	40	24	33

Source: Reprinted with permission from the *World Almanac and Book of Facts.* World Almanac Education Group Inc.

Data Projects

Use a significance level of 0.05 for all tests below.

1. **Business and Finance** Monitor the price of a stock over a five-week period. Note the amount of gain or loss per day. Test the claim that the median is 0. Perform a runs test to see if the distribution of gains and losses is random.
2. **Sports and Leisure** Watch a basketball game, baseball game, or football game. For baseball, monitor an inning's pitches for balls and strikes (all fouls and balls in play also count as strikes). For football monitor a series of plays for runs versus passing plays. For basketball monitor one team's shots for misses versus made shots. For the collected data, conduct a runs test to see if the distribution is random.
3. **Technology** Use the data collected in data project 3 of Chapter 2 regarding song lengths. Consider only three genres. For example, use rock, alternative, and hip hop/rap. Conduct a Kruskal-Wallis test to determine if the mean song lengths for the genres are the same.
4. **Health and Wellness** Have everyone in class take her or his pulse during the first minute of class. Have everyone take his or her pulse again 30 minutes into class. Conduct a paired-sample sign test to determine if there is a difference in pulse rates.
5. **Politics and Economics** Find the ranking for each state for its mean SAT Mathematics scores, its mean SAT English score, and its mean for income. Conduct a rank correlation analysis using Math and English, Math and income, and English and income. Which pair has the strongest relationship?
6. **Your Class** Have everyone in class take his or her temperature on a healthy day. Test the claim that the median body temperature is 98.6°F.

Hypothesis-Testing Summary 3*

15. Test to see whether the median of a sample is a specific value when $n \geq 26$.

Example: H_0: median = 100

Use the sign test:

$$z = \frac{(X + 0.5) - (n/2)}{\sqrt{n}/2}$$

16. Test to see whether two independent samples are obtained from populations that have identical distributions.

Example: H_0: There is no difference in the ages of the subjects.

Use the Wilcoxon rank sum test:

$$z = \frac{R - \mu_R}{\sigma_R}$$

where

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2}$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1 + n_2 + 1)}{12}}$$

*This summary is a continuation of Hypothesis-Testing Summary 2 at the end of Chapter 12.

17. Test to see whether two dependent samples have identical distributions.

Example: H_0: There is no difference in the effects of a tranquilizer on the number of hours a person sleeps at night.

Use the Wilcoxon signed-rank test:

$$z = \frac{w_s - \dfrac{n(n+1)}{4}}{\sqrt{\dfrac{n(n+1)(2n+1)}{24}}}$$

when $n \geq 30$.

18. Test to see whether three or more samples come from identical populations.

Example: H_0: There is no difference in the weights of the three groups.

Use the Kruskal-Wallis test:

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \cdots + \frac{R_k^2}{n_k}\right) - 3(N+1)$$

19. Rank correlation coefficient.

$$r_s = 1 - \frac{6\,\Sigma d^2}{n(n^2-1)}$$

20. Test for randomness: Use the runs test.

Answers to Applying the Concepts

Section 13–1 Ranking Data

Percent	2.6	3.8	4.0	4.0	5.4	7.0	7.0	7.3	10.0
Rank	1	2	3.5	3.5	5	6.5	6.5	8	9

Section 13–2 Clean Air

1. The claim is that the median number of days that a large city failed to meet EPA standards is 11 days per month.
2. We will use the sign test, since we do not know anything about the distribution of the variable and we are testing the median.
3. H_0: median = 11 and H_1: median > 11.
4. If $\alpha = 0.05$, then the critical value is 5.
5. The test value is 9.
6. Since 9 > 5, do not reject the null hypothesis.
7. There is not enough evidence to conclude that the median is not 11 days per month.
8. We cannot use a parametric test in this situation.

Section 13–3 School Lunch

1. The samples are independent since two different random samples were selected.
2. H_0: There is no difference in the number of calories served for lunch in elementary and secondary schools.
   H_1: There is a difference in the number of calories served for lunch in elementary and secondary schools.
3. We will use the Wilcoxon rank sum test.
4. The critical value is ± 1.96 if we use $\alpha = 0.05$.
5. The test statistic is $z = -2.15$.
6. Since $-2.15 < -1.96$, we reject the null hypothesis and conclude that there is a difference in the number of calories served for lunch in elementary and secondary schools.
7. The corresponding parametric test is the two-sample t test.
8. We would need to know that the samples were normally distributed to use the parametric test.
9. Since t tests are robust against variations from normality, the parametric test would yield the same results.

Section 13–4 Pain Medication

1. The purpose of the study is to see how effective a pain medication is.
2. These are dependent samples, since we have before and after readings on the same subjects.
3. H_0: The severity of pain after is the same as the severity of pain before the medication was administered.
   H_1: The severity of pain after is less than the severity of pain before the medication was administered.
4. We will use the Wilcoxon signed-rank test.
5. We will choose to use a significance level of 0.05.
6. The test statistic is $w_s = 2.5$. The critical value is 4. Since 2.5 < 4, we reject the null hypothesis. There is enough evidence to conclude that the severity of pain after is less than the severity of pain before the medication was administered.
7. The parametric test that could be used is the t test for small dependent samples.
8. The results for the parametric test would be the same.

Section 13–5 Heights of Waterfalls

1. We are investigating the heights of waterfalls on three continents.
2. We will use the Kruskal-Wallis test.
3. H_0: There is no difference in the heights of waterfalls on the three continents.
   H_1: There is a difference in the heights of waterfalls on the three continents.
4. We will use the 0.05 significance level. The critical value is 5.991. Our test statistic is $H = 0.01$.
5. Since $0.01 < 5.991$, we fail to reject the null hypothesis. There is not enough evidence to conclude that there is a difference in the heights of waterfalls on the three continents.
6. The corresponding parametric test is analysis of variance (ANOVA).
7. To perform an ANOVA, the population must be normally distributed, the samples must be independent of each other, and the variances of the samples must be equal.

Section 13–6 Tall Trees

1. The biologist is trying to see if there is a relationship between the heights and diameters of tall trees.
2. We will use a Spearman rank correlation analysis.
3. The corresponding parametric test is the Pearson product moment correlation analysis.
4. Answers will vary.
5. The Pearson correlation coefficient is $r = 0.329$. The associated P-value is 0.353. We would fail to reject the null hypothesis that the correlation is zero. The Spearman's rank correlation coefficient is $r_s = 0.115$. We would reject the null hypothesis, at the 0.05 significance level, if $r_s > 0.648$. Since $0.115 < 0.648$, we fail to reject the null hypothesis that the correlation is zero. Both the parametric and nonparametric tests find that the correlation is not statistically significantly different from zero—it appears that no linear relationship exists between the heights and diameters of tall trees.