

1 Preparation

1.1 Theoretical Background

1.1.1 γ -Radiation and Decay

There are different kinds of radiation. In the experiments there are EM-radiation, α -radiation, β^+ - and β^- -radiation as well as radiation of neutrons. Important for compton scattering is the γ -radiation, a part of the electro magnetic spectrum.

It is emitted when atoms decay and the new atom is in an excited state. Photons are emitted as the atom relaxes to the ground state.

The γ -radiation can be expanded in multipoles, i.e. a sum of spherical harmonics. The parity of the L-th multipole is $(-1)^L$. Induced by the decay the electric and magnetic part has the parity $(-1)^L$ resp. $(-1)^{L+1}$. Hence, the order of the magnetic part is higher by 1 than the electric, so that mainly the electric part is observed.

The standard rule to add two angular momentums in QM gives for the initial state J_i and the final state J_f of the relaxing atom:

$$|J_f - J_i| \leq L \leq |J_f + J_i|$$

1.1.2 Interaction of Photons with Matter

There are different effects that play a role when photons interact with matter. Which effect is dominant depends on the energy of the photon as well as the atomic number Z. All effects have in common that the photon is either absorbed or scattered, so that there are less photons after the interaction with matter in the initial direction. The intensity follows an exponential law:

$$I(d) = I(0) \cdot e^{-\mu \cdot d}$$

Compton scattering dominantes starting at about 100 keV up to 10 MeV. Photons with the energy E are scattered inelastically with a charged particle with the energy E_0 . The photon loses some energy and changes its direction, the electron's kinetic energy increases. Conservation of 4-momentum gives the energy of the scattered photon as a function of the initial energy E and the polar angle Θ :

$$E' = \frac{E}{1 + \frac{E}{E_0} \cdot (1 - \cos \Theta)}$$

1.1.3 Cross Section

The cross section σ gives the probability of a particle to interact with a target. Referring to some statics it is defined as:

$$\sigma = \frac{N_{int} \cdot F}{N_{all} \cdot N_{targets}}$$

with the number of interacting particles N_{int} , the cross section the beam F , the total number of particles in the beam N_{all} and the number of targets $N_{targets}$. It has the same unit as a plane and is often visualized as such. In fact the cross section of two colliding billiard balls is just there geometric cross section. In other cases the cross section decreases with the distance of the two objects, e.g. Rutherford Scattering which depends on the electromagnetic interaction. The differential cross section with respect to the solid angle Ω is often used and it is assumed to be independent of the azimuthal angle Φ .

There is a theoretical formula by Klein and Nishina for the cross section of compton scattering.

1.1.4 Experimental Setup

Monoenergetic γ -radiation is emitted by a ^{137}Cs source. The target is a Al-Cylinder. The detector is made of NaJ and can be moved on a circle around the target.

We will measure the energy and the number of the photons after the scattering as a function of the polar angle Θ . This is because we assume rotational symmetry.

1.1.5 Assignment 1

We measure the differential cross section with respect to the solid angle. For this the detector measures the number of incoming photons. For evaluating we use following formula:

$$\frac{d\sigma}{d\Omega} = \frac{R(\Delta\Omega)}{\Delta\Omega} \cdot \frac{1}{\Phi_0 n} \cdot \frac{1}{\epsilon}$$

In this formula $\frac{R(\Delta\Omega)}{\Delta\Omega}$ is a approximation of detector surface because it is not a small dot. $R(\Delta\Omega)$ is the photon stream, the number of detected photons per time interval. We also devide by Φ_0 the incoming photon stream. $\frac{1}{\epsilon}$ is a given number to desribe the not detected electrons and finally n is the number of electrons in the target, which follows from the cylindric geometry, the desity and the atomic number:

$$n = \frac{L}{A} \cdot Z\rho\pi\left(\frac{d}{2}\right)^2 \cdot l$$

1.1.6 Assignment 2

In assignment 2 we measure the energy of the photons after the scattering as a function of the angle Θ and the initial energy.

First, we must calibrate the system with known sources. Then we make the measurement and use the formula above in a slightly different way. A linear regression gives us the invariant mass of electrons.

$$\frac{1}{E'} = \frac{1}{E} + \frac{1}{m_0 \cdot c^2} \cdot (1 - \cos \Theta)$$

1.1.7 Assignment 3

The above formula gives the differential cross section for a single electron. For the whole atom it should be proportional to the atomic number, given that the photon energy is large with respect to the binding energy. With this assumption and the above equations from assignment we can write

$$\left(\frac{d\sigma}{d\Omega}\right)_e = const. \cdot R \frac{A}{\rho Z}$$

2 Evaluation

2.1 Calibration

First we have to calibrate the system. We only measure the number of entries in different channels, but we know at which energies the spectrum of ^{22}Na , ^{60}Co and ^{137}Cs have peaks due to radiation. With these values we can make a linear regression and map the channels onto energies.

Fig.2.1 shows the entries in each channel for ^{22}Na as an example. We fitted the peaks with gaussian functions to get better estimated values.

In Fig.2.2 you can see the channels of each peak and the corresponding energies.

Fig.2.3 finally shows the linear regression with the following parameters:

$$y - \text{intercept} = (-3.02810 \pm 1.48205) \cdot 10^2 \text{ keV}$$

$$\text{slope} = (3.64232 \pm 0.513385) \text{ keV}$$

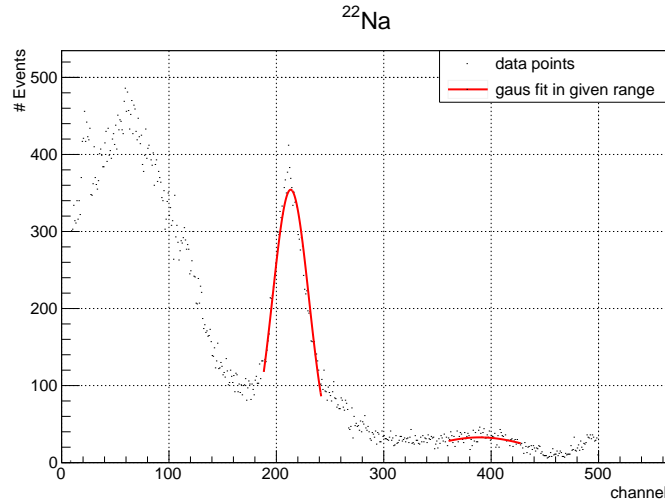


Figure 2.1: Measured channels for ^{22}Na . At estimated ranges we fitted gaussian distributions to get better estimates of the peaks.

Figure 2.2: Measured channels and corresponding energies. Data for a linear fit.

	mean	sigma	energy in keV
^{22}Na	213.33	16.88	511.0
^{22}Na	389.31	52.35	1274.6
^{60}Co	362.87	68.07	1173.2
^{60}Co	463.79	32.98	1332.6
^{137}Cs	278.01	16.78	661.7

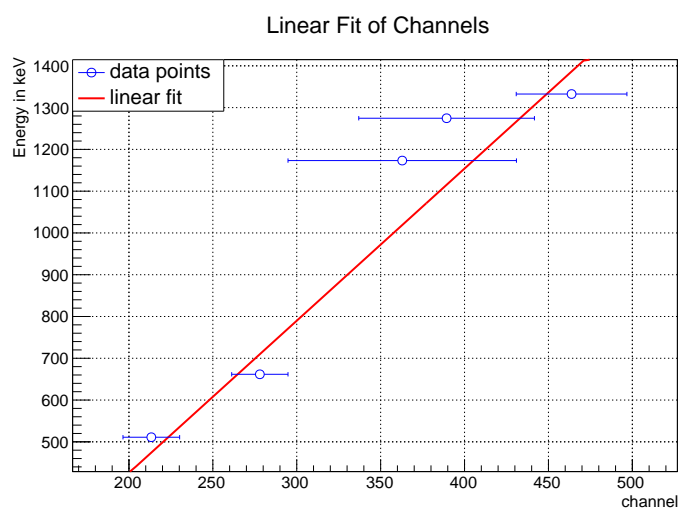


Figure 2.3: Linear fit of channels and corresponding energies. The slope and intersection make it possible so that we can map the channels on the energies.

2.2 Differential Cross Section

Next we measure the differential cross section with respect to the solid angle Ω . We assume that it has a rotation symmetry, i.e. it is independent of the azimuth ϕ .

We change the angle in 10° steps ranging from 20° to 100° and measure the background without target and the events with target. The sum of the difference is the total number scattered R .

To calculate the cross section we use the formular given in corresponding part of the preparation. The values used are in the appendix. To mention is the decay of the source, so we had to correct it by a factor $2^{-t/t_{1/2}}$ with $t_{1/2} = 30a$ and $t = 2016 - 1971 = 45a$. We also multiplied this value by the time we measured the events $t_{measurement} = 300s$

Fig.2.4 shows the polar angle Θ and the measured cross section with its errors.

Figure 2.4: Angles Θ and the measured differential cross section in cm^{-2} .

Θ in $^\circ$	$\frac{d\sigma}{d\Omega}$ in cm^{-2}	error of the cross section
20	0.684	0.041
30	0.558	0.033
40	0.442	0.026
50	0.348	0.021
60	0.282	0.017
70	0.240	0.014
80	0.212	0.013
90	0.204	0.012
100	0.200	0.012

2.3 Energy Shift and Invariant Mass of Electrons

The energy of a fixed angle is determined by a gaussian fit, an example is shown in Fig.2.5 for $\Theta = 50^\circ$. The procedure is the same as in the calibration subsection above. Using the measured linear fit from above we can acquire the energies given the channels.

As described in the preparation we plot the inverse energy $\frac{1}{E}$ over $1 - \cos(\Theta)$. The inverse of the slope gives the invariant mass of the electron in keV . The resulting plot is given in Fig.2.6 with following parameters for the fit:

$$y - \text{intercept} = (1.20237 \pm 0.553517) \cdot 10^{-4} \text{ keV}$$

$$\text{slope} = (4.55878 \pm 3.02294) \cdot 10^{-3} \text{ keV}$$

As a result we can calculate the electron mass to be:

$$(219.357 \pm 145.456) \text{ keV}$$

The uncertainty for $\frac{1}{E}$ is calculated by

$$\sigma_{energy}^2 = \left(\frac{1}{\text{slope}}\right)^2 \cdot (\sigma_{channel}^2 + \sigma_{intercept}^2 + (\text{channel} - \text{intercept})^2 \cdot \left(\frac{\sigma_{slope}}{\text{slope}}\right)^2)$$

Since $\sigma_{intercept}$ is a big number originating from the calibration and it is present even for small values of E , it dominates the uncertainty in this regime. The result is a big absolute uncertainty for big values of the inverse energy $\frac{1}{E}$, which is the reason for the appearance of the graph below.

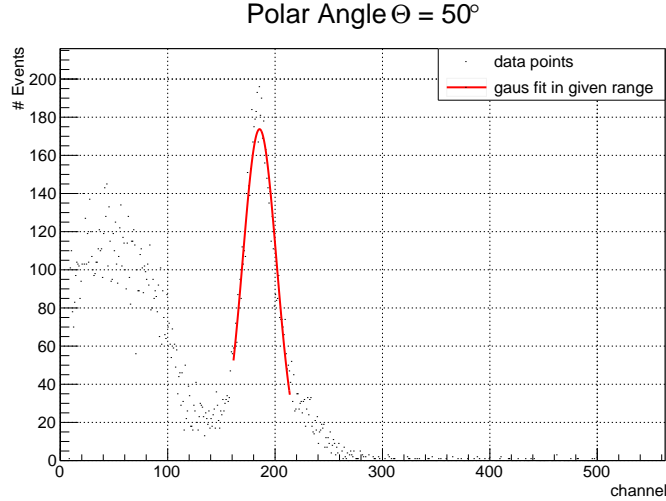


Figure 2.5: Fit to determine the energy at a fixed angle of $\Theta = 50^\circ$.

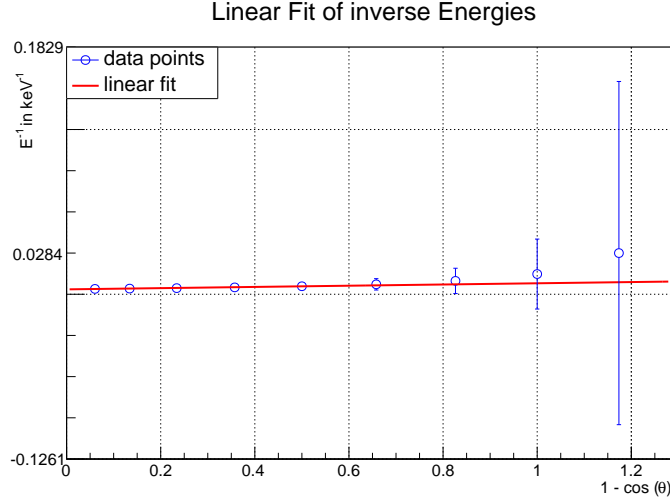


Figure 2.6: Fit to determine the invariant mass of electrons. The uncertainty is big for high values of $\frac{1}{E}$ due to the big uncertainty of the y-intercept in the calibration fit.

2.4 Cross Section for Different Materials

As described in the preparation the following formular should be true given that the binding energy is much smaller than the energy of the photons:

$$\left(\frac{d\sigma}{d\Omega}\right)_e = \text{const.} \cdot R \frac{A}{\rho Z}$$

We measured the total number of events for a fixed angle $\Theta = 20^\circ$ for different targets and subtracted the background $R = R_{\text{target}} - R_{\text{background}}$. The materials are aluminium, copper, iron and lead. Each target has the same size and shape.

To check the formular above we plot $\frac{R \cdot A}{\rho Z}$ over Z . This should be a horizontal line. Here A is the atomic mass, ρ is the density of the material and Z is the atomic number. The used values are given in Fig.2.7.

The resulting graph is shown in Fig.2.8. All data points except the one for lead are on a horizontal line. The reason is that the binding energy of lead is higher than for the others. Therefore the equation above doesn't hold true anymore because the electrons can't be simplified to be free anymore. It seems that with increasing atomic number, the error gets bigger.

Figure 2.7: Values used for the different materials in the equation above.

	aluminium	copper	iron	lead
ρ in $\frac{g}{cm^3}$	2.70	8.92	7.87	11.34
A in u	26.98	63.55	55.85	207.2
Z	13	29	26	82
R	19619	58964	55038	50573
$\sigma(R)$	1265.54	3688.28	3455.20	3477.25

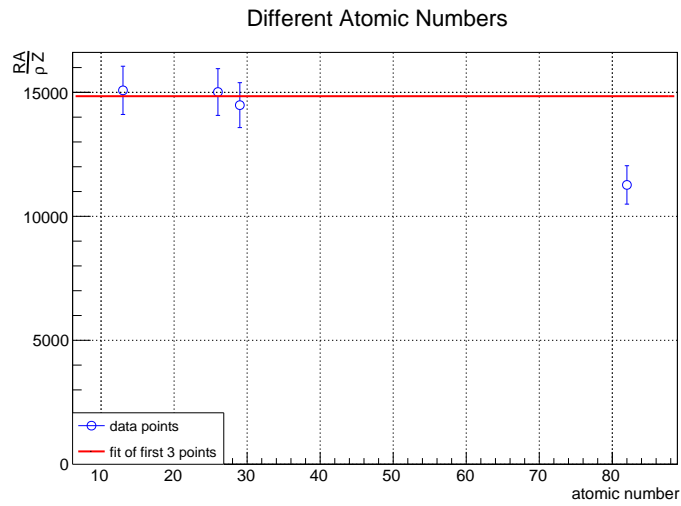


Figure 2.8: Plot of the quantity mentioned in the text over the atomic number Z with a fit of the first three data points. Lead doesn't align with the other materials. The reason is the binding energy of the electrons.