Implementation

This program solves a Boundary Value problem using the spectral collocation method over with a variable amount of discretization and then measures the error of the estimation with the analytical solution, first we define the dft module which will let us perform a discrete fourier transform on our function and then we create problemsetup to define our forcing function. Then we define the bvp and and use the tools outlines in the dtf module to solve the bvp. We then compare our solution to the provided analytical one and determine the error

Questions

1.

in `bvp.f90` and `dft.f90` we do `use utility, only: fp, pi` which enables us to use `real (fp)` throughout these files

2.

`dft.f90` implements basic matrix multiplication and a discrete fourier transform so it isnt super specific to this boundary value problem and could be used for others. `bvp.f90` uses the functions provided by `dft.f90` and then uses them to solve the specific BVP setup in `problemsetup.f90`

3.

because the size of arrays that we use are dependent on the level discretization. The more steps we take the bigger our matrices are. They are allocated based on N in the `allocate_data` subroutine and then at the end dealoccated by `deallocate_data`

4.

elemental functions operate element wise on matrices allowing us to fill out our resulting array with the result at each point of discretization.

5.

Amount of discretization	Error
21	$9.817\,949\,242\,563\,184\,2 \times 10^{-3}$
41	$5.580\,084\,851\,963\,818\,0 \times 10^{-7}$
61	$1.090\,616\,486\,010\,276\,3 \times 10^{-11}$
81	$6.217\,248\,937\,900\,876\,6 \times 10^{-15}$
101	$5.551\,115\,123\,125\,782\,7 \times 10^{-15}$

the error starts to decrease much much slower after 81 this indicates that we've reached a level of discretization small enough that we have a suitable estimation

6.

with precision set to 6 decimal points

Amount of discretization	**Error**
21	$9.822\,607\,04 \times 10^{-3}$
41	$8.106\,231\,69 \times 10^{-6}$
61	$2.384\,185\,79 \times 10^{-6}$
81	$4.529\,953\,00 \times 10^{-6}$
101	$3.814\,697\,27 \times 10^{-6}$

the error reaches its minimum value much faster and it is less precise

7.

this assignment had us build the tools to perform a discrete fourier transform and then defines a forcing function and then we define a function that we solve a boundary value problem for using the dft tools we created earlier

8.

the real kind is 16 and the error is a much much longer number