

Am160HW1part2

January 25, 2025

```
[1]: import torch
import numpy as np
import matplotlib.pyplot as plt
import scipy.special
```

Import needed libraries

```
[2]: # Generate data
def generate_data(num_samples=100, noise=0.01):
    # generating function:  $y_n = 2x_n + \cos(25x_n) + N(0, 1)$ , where  $N$  is a Gaussian distribution
    x = torch.linspace(-1, 1, num_samples).unsqueeze(1)
    N = noise*torch.randn_like(x)
    y = 2*x + torch.cos(25*x) + N
    return x, y

# Split data into training and testing sets
num_train = 20
num_test = 100

x_train, y_train = generate_data(num_train)
x_test, y_test = generate_data(num_test)
```

generate training and testing data sets from the generating function $y_n = 2x_n + \cos(25x_n) + N(0, 1)$ where N is the gaussian distribution `torch.randn_like(x)`

```
[3]: # create random Fourier features
def create_random_fourier_features(x, degree):
    # Create random Fourier features
    # random Fourier feature (RFF) model, where the basis function  $\phi_i(x_n) = \cos(2\pi \omega_i x_n)$ 
    # Here,  $\omega_i$  is sampled from the Gaussian  $N(0, 1)$  and  $i \in [1, 2, 3, \dots, p]$ .
    # print('x', x.size())
    w = torch.randn(degree, 1) * 20 # vector of random weights (p x 1) sampled from  $N(0, 1)$ 
    phi = torch.cos(2*torch.pi*torch.mm(w, x.T)) # random features (n x p)
    # if degree == 3:
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#     plt.plot(x, phi.T)
#     plt.title('Random Fourier Features at degree {}'.format(degree))
#     plt.show()
return phi.T

# Solve using Moore-Penrose pseudoinverse with regularization
def solve_with_regularized_pseudoinverse(x_train_rff, y_train, x_test_rff,
    ↪ y_test, lambda_reg=1e-12):
    # Compute regularized pseudoinverse
    I = torch.eye(x_train_rff.size(1))
    gram_matrix = x_train_rff.T @ x_train_rff + lambda_reg * I

    # Check conditioning of the gram matrix

    condition_number = torch.linalg.cond(gram_matrix)
    if condition_number > 1e10:
        print(f"Warning: Ill-conditioned matrix at degree {x_train_rff.size(1)}
    ↪ 1}, condition number: {condition_number}")

    pseudoinverse = torch.linalg.pinv(gram_matrix) @ x_train_rff.T
    weights = pseudoinverse @ y_train
    # print('weights', weights.size())

    # Compute predictions
    y_train_pred = x_train_rff @ weights
    y_test_pred = x_test_rff @ weights

    # Compute errors
    train_error = ((y_train - y_train_pred)**2).mean().item()
    test_error = ((y_test - y_test_pred)**2).mean().item()

    return train_error, test_error

```

Then we make our Random Fourier Feature (RFF) model with the basis functions given by $\cos(2\pi\omega_i x_n)$ where ω_i is also sampled from the Gaussian Distribution. We then solve this model using the Regularized Pseudoinverse function found in the linear_models_bias_variance colab

```

[4]: # Train and evaluate models with varying degrees
def train_and_evaluate(max_degree, lambda_reg=1e-2):
    train_errors = []
    test_errors = []
    param_counts = []

    for degree in range(1, max_degree + 1, 1):
        print(f"Training model with Random Fourier Features degree {degree}...")

        # Prepare data with Legendre polynomial features

```

```

    # print('x_train', x_train.size())
    x_train_poly = create_random_fourier_features(x_train, degree)
    # print('x_train_poly', x_train_poly.size())
    # print('x_test', y_train.size())
    x_test_poly = create_random_fourier_features(x_test, degree)

    train_error, test_error = solve_with_regularized_pseudoinverse(
        x_train_poly, y_train, x_test_poly, y_test,
    ↪ lambda_reg=lambda_reg
    )
    # Store errors and parameter counts
    train_errors.append(train_error)
    test_errors.append(test_error)
    param_counts.append((degree + 1)) # Number of parameters = degree + 1

    print(f"Degree {degree} completed. Train Error: {train_error:.6f}, Test_
    ↪ Error: {test_error:.6f}\n")

    return param_counts, train_errors, test_errors

```

now we iterate through each degree to to generate training and testing sets and solving them with the regularized psuedoinverse

```

[5]: # Perform experiment
max_degree = 200 # Extend degree range to observe double descent
lambda_reg = 0 # Regularization strength
param_counts, train_errors, test_errors = train_and_evaluate(max_degree,
    ↪ lambda_reg=lambda_reg)

# Plot results
plt.figure(figsize=(10, 6))
plt.loglog(param_counts, train_errors, label="Training Error", marker='o')
plt.loglog(param_counts, test_errors, label="Test Error", marker='o')
# plt.ylim([1e-3,100])
plt.xscale('log')
plt.yscale('log')
plt.xlabel("Number of Parameters (Polynomial Degree + 1)")
plt.ylabel("Mean Squared Error")
plt.title("Double Descent Phenomenon with Random Fourier Features")
plt.legend()
plt.grid(True)
plt.show()

```

Training model with Random Fourier Features degree 1...

Degree 1 completed. Train Error: 1.980057, Test Error: 5.576274

Training model with Random Fourier Features degree 2...

Degree 2 completed. Train Error: 2.000684, Test Error: 1.890980

Training model with Random Fourier Features degree 3...
Degree 3 completed. Train Error: 1.997739, Test Error: 1.907450

Training model with Random Fourier Features degree 4...
Degree 4 completed. Train Error: 1.994866, Test Error: 1.897685

Training model with Random Fourier Features degree 5...
Degree 5 completed. Train Error: 1.482525, Test Error: 6.219657

Training model with Random Fourier Features degree 6...
Degree 6 completed. Train Error: 1.714273, Test Error: 2.189224

Training model with Random Fourier Features degree 7...
Degree 7 completed. Train Error: 1.948289, Test Error: 11.989459

Training model with Random Fourier Features degree 8...
Degree 8 completed. Train Error: 1.479652, Test Error: 2.678691

Training model with Random Fourier Features degree 9...
Degree 9 completed. Train Error: 1.678730, Test Error: 6294.702637

Training model with Random Fourier Features degree 10...
Degree 10 completed. Train Error: 1.478413, Test Error: 6.397077

Training model with Random Fourier Features degree 11...
Degree 11 completed. Train Error: 1.478427, Test Error: 2.164553

Training model with Random Fourier Features degree 12...
Degree 12 completed. Train Error: 1.487332, Test Error: 3.073854

Training model with Random Fourier Features degree 13...
Degree 13 completed. Train Error: 1.478411, Test Error: 26.869732

Training model with Random Fourier Features degree 14...
Degree 14 completed. Train Error: 1.478411, Test Error: 32.085892

Training model with Random Fourier Features degree 15...
Degree 15 completed. Train Error: 1.478411, Test Error: 2.624975

Training model with Random Fourier Features degree 16...
Degree 16 completed. Train Error: 1.478411, Test Error: 3669.324951

Training model with Random Fourier Features degree 17...
Degree 17 completed. Train Error: 1.478411, Test Error: 2.094013

Training model with Random Fourier Features degree 18...

Degree 18 completed. Train Error: 1.478411, Test Error: 6.300245

Training model with Random Fourier Features degree 19...

Degree 19 completed. Train Error: 1.478411, Test Error: 309.282745

Training model with Random Fourier Features degree 20...

Degree 20 completed. Train Error: 1.478411, Test Error: 2.364437

Training model with Random Fourier Features degree 21...

Degree 21 completed. Train Error: 1.478411, Test Error: 2.864808

Training model with Random Fourier Features degree 22...

Degree 22 completed. Train Error: 1.478411, Test Error: 3.152756

Training model with Random Fourier Features degree 23...

Degree 23 completed. Train Error: 1.478411, Test Error: 2.425458

Training model with Random Fourier Features degree 24...

Warning: Ill-conditioned matrix at degree 23, condition number: 24315310080.0

Degree 24 completed. Train Error: 1.478410, Test Error: 2.543692

Training model with Random Fourier Features degree 25...

Degree 25 completed. Train Error: 1.478410, Test Error: 2.199484

Training model with Random Fourier Features degree 26...

Degree 26 completed. Train Error: 1.478411, Test Error: 2.159898

Training model with Random Fourier Features degree 27...

Degree 27 completed. Train Error: 1.478411, Test Error: 2.175420

Training model with Random Fourier Features degree 28...

Degree 28 completed. Train Error: 1.478411, Test Error: 2.105335

Training model with Random Fourier Features degree 29...

Degree 29 completed. Train Error: 1.478411, Test Error: 2.776276

Training model with Random Fourier Features degree 30...

Degree 30 completed. Train Error: 1.478410, Test Error: 2.042315

Training model with Random Fourier Features degree 31...

Degree 31 completed. Train Error: 1.478411, Test Error: 6.601558

Training model with Random Fourier Features degree 32...

Degree 32 completed. Train Error: 1.478411, Test Error: 2.220818

Training model with Random Fourier Features degree 33...

Degree 33 completed. Train Error: 1.478410, Test Error: 2.005373

Training model with Random Fourier Features degree 34...
Degree 34 completed. Train Error: 1.478411, Test Error: 2.018739

Training model with Random Fourier Features degree 35...
Warning: Ill-conditioned matrix at degree 34, condition number: 13559853056.0
Degree 35 completed. Train Error: 1.478410, Test Error: 1.948067

Training model with Random Fourier Features degree 36...
Degree 36 completed. Train Error: 1.478411, Test Error: 1.958353

Training model with Random Fourier Features degree 37...
Degree 37 completed. Train Error: 1.478411, Test Error: 2.052426

Training model with Random Fourier Features degree 38...
Degree 38 completed. Train Error: 1.478410, Test Error: 2.284833

Training model with Random Fourier Features degree 39...
Degree 39 completed. Train Error: 1.478411, Test Error: 2.470566

Training model with Random Fourier Features degree 40...
Warning: Ill-conditioned matrix at degree 39, condition number: 49364475904.0
Degree 40 completed. Train Error: 1.478411, Test Error: 2.032817

Training model with Random Fourier Features degree 41...
Warning: Ill-conditioned matrix at degree 40, condition number: 12582745088.0
Degree 41 completed. Train Error: 1.478411, Test Error: 2.342231

Training model with Random Fourier Features degree 42...
Degree 42 completed. Train Error: 1.478410, Test Error: 2.069420

Training model with Random Fourier Features degree 43...
Degree 43 completed. Train Error: 1.478410, Test Error: 1.963001

Training model with Random Fourier Features degree 44...
Degree 44 completed. Train Error: 1.478411, Test Error: 1.926086

Training model with Random Fourier Features degree 45...
Degree 45 completed. Train Error: 1.478411, Test Error: 2.078733

Training model with Random Fourier Features degree 46...
Degree 46 completed. Train Error: 1.478411, Test Error: 2.047062

Training model with Random Fourier Features degree 47...
Degree 47 completed. Train Error: 1.478410, Test Error: 1.995301

Training model with Random Fourier Features degree 48...
Degree 48 completed. Train Error: 1.478410, Test Error: 2.066126

Training model with Random Fourier Features degree 49...
Degree 49 completed. Train Error: 1.478411, Test Error: 2.068995

Training model with Random Fourier Features degree 50...
Degree 50 completed. Train Error: 1.478411, Test Error: 2.030173

Training model with Random Fourier Features degree 51...
Degree 51 completed. Train Error: 1.478411, Test Error: 1.907946

Training model with Random Fourier Features degree 52...
Degree 52 completed. Train Error: 1.478411, Test Error: 2.028050

Training model with Random Fourier Features degree 53...
Warning: Ill-conditioned matrix at degree 52, condition number: 387159883776.0
Degree 53 completed. Train Error: 1.478411, Test Error: 1.884737

Training model with Random Fourier Features degree 54...
Warning: Ill-conditioned matrix at degree 53, condition number: 15428139008.0
Degree 54 completed. Train Error: 1.478411, Test Error: 1.977409

Training model with Random Fourier Features degree 55...
Degree 55 completed. Train Error: 1.478411, Test Error: 1.856093

Training model with Random Fourier Features degree 56...
Degree 56 completed. Train Error: 1.478411, Test Error: 1.886671

Training model with Random Fourier Features degree 57...
Warning: Ill-conditioned matrix at degree 56, condition number: 51428646912.0
Degree 57 completed. Train Error: 1.478411, Test Error: 1.963499

Training model with Random Fourier Features degree 58...
Degree 58 completed. Train Error: 1.478411, Test Error: 1.947759

Training model with Random Fourier Features degree 59...
Warning: Ill-conditioned matrix at degree 58, condition number: 4781411663872.0
Degree 59 completed. Train Error: 1.478411, Test Error: 1.886335

Training model with Random Fourier Features degree 60...
Degree 60 completed. Train Error: 1.478411, Test Error: 1.957199

Training model with Random Fourier Features degree 61...
Warning: Ill-conditioned matrix at degree 60, condition number: 14460365824.0
Degree 61 completed. Train Error: 1.478410, Test Error: 1.940134

Training model with Random Fourier Features degree 62...
Warning: Ill-conditioned matrix at degree 61, condition number: 17865154560.0
Degree 62 completed. Train Error: 1.478411, Test Error: 1.950321

Training model with Random Fourier Features degree 63...
Degree 63 completed. Train Error: 1.478411, Test Error: 2.065261

Training model with Random Fourier Features degree 64...
Degree 64 completed. Train Error: 1.478411, Test Error: 1.934534

Training model with Random Fourier Features degree 65...
Degree 65 completed. Train Error: 1.478411, Test Error: 1.917644

Training model with Random Fourier Features degree 66...
Degree 66 completed. Train Error: 1.478410, Test Error: 2.007437

Training model with Random Fourier Features degree 67...
Degree 67 completed. Train Error: 1.478411, Test Error: 1.911755

Training model with Random Fourier Features degree 68...
Degree 68 completed. Train Error: 1.478411, Test Error: 1.941412

Training model with Random Fourier Features degree 69...
Degree 69 completed. Train Error: 1.478411, Test Error: 1.977754

Training model with Random Fourier Features degree 70...
Warning: Ill-conditioned matrix at degree 69, condition number: 33115619328.0
Degree 70 completed. Train Error: 1.478411, Test Error: 2.007624

Training model with Random Fourier Features degree 71...
Degree 71 completed. Train Error: 1.478411, Test Error: 1.912352

Training model with Random Fourier Features degree 72...
Warning: Ill-conditioned matrix at degree 71, condition number: 11534538752.0
Degree 72 completed. Train Error: 1.478410, Test Error: 1.928352

Training model with Random Fourier Features degree 73...
Warning: Ill-conditioned matrix at degree 72, condition number: 28889886720.0
Degree 73 completed. Train Error: 1.478411, Test Error: 1.920961

Training model with Random Fourier Features degree 74...
Degree 74 completed. Train Error: 1.478411, Test Error: 1.953461

Training model with Random Fourier Features degree 75...
Degree 75 completed. Train Error: 1.478411, Test Error: 1.985156

Training model with Random Fourier Features degree 76...
Degree 76 completed. Train Error: 1.478411, Test Error: 1.939456

Training model with Random Fourier Features degree 77...
Warning: Ill-conditioned matrix at degree 76, condition number: 10828179456.0
Degree 77 completed. Train Error: 1.478411, Test Error: 1.936948

Training model with Random Fourier Features degree 78...
Degree 78 completed. Train Error: 1.478410, Test Error: 1.955943

Training model with Random Fourier Features degree 79...
Warning: Ill-conditioned matrix at degree 78, condition number: 21109794816.0
Degree 79 completed. Train Error: 1.478411, Test Error: 1.883391

Training model with Random Fourier Features degree 80...
Degree 80 completed. Train Error: 1.478411, Test Error: 2.098910

Training model with Random Fourier Features degree 81...
Warning: Ill-conditioned matrix at degree 80, condition number: 69225291776.0
Degree 81 completed. Train Error: 1.478411, Test Error: 1.961098

Training model with Random Fourier Features degree 82...
Degree 82 completed. Train Error: 1.478410, Test Error: 2.047185

Training model with Random Fourier Features degree 83...
Degree 83 completed. Train Error: 1.478411, Test Error: 1.922040

Training model with Random Fourier Features degree 84...
Warning: Ill-conditioned matrix at degree 83, condition number: 20061755392.0
Degree 84 completed. Train Error: 1.478411, Test Error: 1.936917

Training model with Random Fourier Features degree 85...
Warning: Ill-conditioned matrix at degree 84, condition number: 472292392960.0
Degree 85 completed. Train Error: 1.478411, Test Error: 1.942871

Training model with Random Fourier Features degree 86...
Degree 86 completed. Train Error: 1.478411, Test Error: 1.878894

Training model with Random Fourier Features degree 87...
Degree 87 completed. Train Error: 1.478411, Test Error: 1.882996

Training model with Random Fourier Features degree 88...
Degree 88 completed. Train Error: 1.478411, Test Error: 1.947202

Training model with Random Fourier Features degree 89...
Warning: Ill-conditioned matrix at degree 88, condition number: 34377940992.0
Degree 89 completed. Train Error: 1.478411, Test Error: 2.025529

Training model with Random Fourier Features degree 90...
Warning: Ill-conditioned matrix at degree 89, condition number: 18414493696.0
Degree 90 completed. Train Error: 1.478411, Test Error: 2.002899

Training model with Random Fourier Features degree 91...
Warning: Ill-conditioned matrix at degree 90, condition number: 965869109248.0

Degree 91 completed. Train Error: 1.478411, Test Error: 1.913563

Training model with Random Fourier Features degree 92...
Warning: Ill-conditioned matrix at degree 91, condition number: 21628706816.0
Degree 92 completed. Train Error: 1.478411, Test Error: 2.006163

Training model with Random Fourier Features degree 93...
Warning: Ill-conditioned matrix at degree 92, condition number: 11230727168.0
Degree 93 completed. Train Error: 1.478410, Test Error: 1.930813

Training model with Random Fourier Features degree 94...
Degree 94 completed. Train Error: 1.478411, Test Error: 1.912156

Training model with Random Fourier Features degree 95...
Degree 95 completed. Train Error: 1.478411, Test Error: 1.941357

Training model with Random Fourier Features degree 96...
Warning: Ill-conditioned matrix at degree 95, condition number: 19331567616.0
Degree 96 completed. Train Error: 1.478411, Test Error: 1.932097

Training model with Random Fourier Features degree 97...
Warning: Ill-conditioned matrix at degree 96, condition number: 23589201920.0
Degree 97 completed. Train Error: 1.478411, Test Error: 1.809477

Training model with Random Fourier Features degree 98...
Degree 98 completed. Train Error: 1.478411, Test Error: 1.918812

Training model with Random Fourier Features degree 99...
Degree 99 completed. Train Error: 1.478411, Test Error: 1.920285

Training model with Random Fourier Features degree 100...
Warning: Ill-conditioned matrix at degree 99, condition number: 189573906432.0
Degree 100 completed. Train Error: 1.478411, Test Error: 1.908042

Training model with Random Fourier Features degree 101...
Warning: Ill-conditioned matrix at degree 100, condition number: 276146290688.0
Degree 101 completed. Train Error: 1.478410, Test Error: 1.910715

Training model with Random Fourier Features degree 102...
Degree 102 completed. Train Error: 1.478411, Test Error: 1.911466

Training model with Random Fourier Features degree 103...
Warning: Ill-conditioned matrix at degree 102, condition number: 16272988160.0
Degree 103 completed. Train Error: 1.478411, Test Error: 1.850031

Training model with Random Fourier Features degree 104...
Warning: Ill-conditioned matrix at degree 103, condition number: 11101450240.0
Degree 104 completed. Train Error: 1.478411, Test Error: 1.892721

Training model with Random Fourier Features degree 105...
Warning: Ill-conditioned matrix at degree 104, condition number: 16454049792.0
Degree 105 completed. Train Error: 1.478411, Test Error: 2.060794

Training model with Random Fourier Features degree 106...
Warning: Ill-conditioned matrix at degree 105, condition number: 51583438848.0
Degree 106 completed. Train Error: 1.478411, Test Error: 1.809188

Training model with Random Fourier Features degree 107...
Warning: Ill-conditioned matrix at degree 106, condition number: 15794397184.0
Degree 107 completed. Train Error: 1.478411, Test Error: 1.861843

Training model with Random Fourier Features degree 108...
Degree 108 completed. Train Error: 1.478411, Test Error: 1.964188

Training model with Random Fourier Features degree 109...
Warning: Ill-conditioned matrix at degree 108, condition number: 154714849280.0
Degree 109 completed. Train Error: 1.478410, Test Error: 1.833931

Training model with Random Fourier Features degree 110...
Warning: Ill-conditioned matrix at degree 109, condition number: 10458247168.0
Degree 110 completed. Train Error: 1.478411, Test Error: 1.951233

Training model with Random Fourier Features degree 111...
Degree 111 completed. Train Error: 1.478411, Test Error: 1.901048

Training model with Random Fourier Features degree 112...
Warning: Ill-conditioned matrix at degree 111, condition number: 16609083392.0
Degree 112 completed. Train Error: 1.478411, Test Error: 1.912413

Training model with Random Fourier Features degree 113...
Warning: Ill-conditioned matrix at degree 112, condition number: 12035147776.0
Degree 113 completed. Train Error: 1.478411, Test Error: 1.893437

Training model with Random Fourier Features degree 114...
Warning: Ill-conditioned matrix at degree 113, condition number: 14875486208.0
Degree 114 completed. Train Error: 1.478411, Test Error: 1.908352

Training model with Random Fourier Features degree 115...
Degree 115 completed. Train Error: 1.478411, Test Error: 1.873511

Training model with Random Fourier Features degree 116...
Warning: Ill-conditioned matrix at degree 115, condition number: 321069318144.0
Degree 116 completed. Train Error: 1.478411, Test Error: 1.882043

Training model with Random Fourier Features degree 117...
Warning: Ill-conditioned matrix at degree 116, condition number: 32402178048.0

Degree 117 completed. Train Error: 1.478411, Test Error: 1.882173

Training model with Random Fourier Features degree 118...
Warning: Ill-conditioned matrix at degree 117, condition number: 199106445312.0
Degree 118 completed. Train Error: 1.478411, Test Error: 1.888897

Training model with Random Fourier Features degree 119...
Warning: Ill-conditioned matrix at degree 118, condition number: 13437878272.0
Degree 119 completed. Train Error: 1.478411, Test Error: 1.896384

Training model with Random Fourier Features degree 120...
Degree 120 completed. Train Error: 1.478411, Test Error: 1.997518

Training model with Random Fourier Features degree 121...
Warning: Ill-conditioned matrix at degree 120, condition number: 64461828096.0
Degree 121 completed. Train Error: 1.478411, Test Error: 1.856667

Training model with Random Fourier Features degree 122...
Warning: Ill-conditioned matrix at degree 121, condition number: 37989068800.0
Degree 122 completed. Train Error: 1.478411, Test Error: 1.894671

Training model with Random Fourier Features degree 123...
Warning: Ill-conditioned matrix at degree 122, condition number: 38336540672.0
Degree 123 completed. Train Error: 1.478411, Test Error: 1.988573

Training model with Random Fourier Features degree 124...
Warning: Ill-conditioned matrix at degree 123, condition number: 14570490880.0
Degree 124 completed. Train Error: 1.478410, Test Error: 1.902647

Training model with Random Fourier Features degree 125...
Warning: Ill-conditioned matrix at degree 124, condition number: 17310054400.0
Degree 125 completed. Train Error: 1.478411, Test Error: 1.781521

Training model with Random Fourier Features degree 126...
Warning: Ill-conditioned matrix at degree 125, condition number: 16390746112.0
Degree 126 completed. Train Error: 1.478411, Test Error: 1.898117

Training model with Random Fourier Features degree 127...
Degree 127 completed. Train Error: 1.478411, Test Error: 1.919022

Training model with Random Fourier Features degree 128...
Degree 128 completed. Train Error: 1.478411, Test Error: 1.924627

Training model with Random Fourier Features degree 129...
Warning: Ill-conditioned matrix at degree 128, condition number: 14791120896.0
Degree 129 completed. Train Error: 1.478411, Test Error: 1.921600

Training model with Random Fourier Features degree 130...

Warning: Ill-conditioned matrix at degree 129, condition number: 77279502336.0
Degree 130 completed. Train Error: 1.478411, Test Error: 1.983673

Training model with Random Fourier Features degree 131...
Warning: Ill-conditioned matrix at degree 130, condition number: 65077035008.0
Degree 131 completed. Train Error: 1.478411, Test Error: 1.906433

Training model with Random Fourier Features degree 132...
Degree 132 completed. Train Error: 1.478411, Test Error: 1.853019

Training model with Random Fourier Features degree 133...
Degree 133 completed. Train Error: 1.478411, Test Error: 2.094488

Training model with Random Fourier Features degree 134...
Warning: Ill-conditioned matrix at degree 133, condition number: 70463283200.0
Degree 134 completed. Train Error: 1.478411, Test Error: 1.948303

Training model with Random Fourier Features degree 135...
Warning: Ill-conditioned matrix at degree 134, condition number: 11520234496.0
Degree 135 completed. Train Error: 1.478410, Test Error: 1.867670

Training model with Random Fourier Features degree 136...
Warning: Ill-conditioned matrix at degree 135, condition number: 17415919616.0
Degree 136 completed. Train Error: 1.478411, Test Error: 1.858520

Training model with Random Fourier Features degree 137...
Warning: Ill-conditioned matrix at degree 136, condition number: 24085839872.0
Degree 137 completed. Train Error: 1.478411, Test Error: 1.926774

Training model with Random Fourier Features degree 138...
Warning: Ill-conditioned matrix at degree 137, condition number: 52320935936.0
Degree 138 completed. Train Error: 1.478411, Test Error: 1.933919

Training model with Random Fourier Features degree 139...
Degree 139 completed. Train Error: 1.478410, Test Error: 1.919104

Training model with Random Fourier Features degree 140...
Degree 140 completed. Train Error: 1.478411, Test Error: 1.899995

Training model with Random Fourier Features degree 141...
Degree 141 completed. Train Error: 1.478411, Test Error: 1.891930

Training model with Random Fourier Features degree 142...
Degree 142 completed. Train Error: 1.478411, Test Error: 1.832836

Training model with Random Fourier Features degree 143...
Warning: Ill-conditioned matrix at degree 142, condition number: 18335469568.0
Degree 143 completed. Train Error: 1.478411, Test Error: 1.903058

Training model with Random Fourier Features degree 144...
Warning: Ill-conditioned matrix at degree 143, condition number: 368631611392.0
Degree 144 completed. Train Error: 1.478411, Test Error: 1.961637

Training model with Random Fourier Features degree 145...
Warning: Ill-conditioned matrix at degree 144, condition number: 52866924544.0
Degree 145 completed. Train Error: 1.478411, Test Error: 1.910643

Training model with Random Fourier Features degree 146...
Warning: Ill-conditioned matrix at degree 145, condition number: 11919651840.0
Degree 146 completed. Train Error: 1.478411, Test Error: 1.938600

Training model with Random Fourier Features degree 147...
Degree 147 completed. Train Error: 1.478411, Test Error: 1.862435

Training model with Random Fourier Features degree 148...
Warning: Ill-conditioned matrix at degree 147, condition number: 10963120128.0
Degree 148 completed. Train Error: 1.478411, Test Error: 1.892520

Training model with Random Fourier Features degree 149...
Warning: Ill-conditioned matrix at degree 148, condition number: 20560437248.0
Degree 149 completed. Train Error: 1.478411, Test Error: 1.880728

Training model with Random Fourier Features degree 150...
Warning: Ill-conditioned matrix at degree 149, condition number: 47680688128.0
Degree 150 completed. Train Error: 1.478411, Test Error: 1.860042

Training model with Random Fourier Features degree 151...
Warning: Ill-conditioned matrix at degree 150, condition number: 26156173312.0
Degree 151 completed. Train Error: 1.478411, Test Error: 1.830525

Training model with Random Fourier Features degree 152...
Degree 152 completed. Train Error: 1.478411, Test Error: 1.898576

Training model with Random Fourier Features degree 153...
Warning: Ill-conditioned matrix at degree 152, condition number: 27976847360.0
Degree 153 completed. Train Error: 1.478411, Test Error: 1.921895

Training model with Random Fourier Features degree 154...
Degree 154 completed. Train Error: 1.478411, Test Error: 1.857267

Training model with Random Fourier Features degree 155...
Warning: Ill-conditioned matrix at degree 154, condition number: 66218033152.0
Degree 155 completed. Train Error: 1.478411, Test Error: 1.891784

Training model with Random Fourier Features degree 156...
Warning: Ill-conditioned matrix at degree 155, condition number: 45899341824.0

Degree 156 completed. Train Error: 1.478411, Test Error: 1.988897

Training model with Random Fourier Features degree 157...

Warning: Ill-conditioned matrix at degree 156, condition number: 68488523776.0

Degree 157 completed. Train Error: 1.478410, Test Error: 1.864939

Training model with Random Fourier Features degree 158...

Warning: Ill-conditioned matrix at degree 157, condition number: 30319177728.0

Degree 158 completed. Train Error: 1.478411, Test Error: 1.889879

Training model with Random Fourier Features degree 159...

Warning: Ill-conditioned matrix at degree 158, condition number: 42880319488.0

Degree 159 completed. Train Error: 1.478411, Test Error: 1.927906

Training model with Random Fourier Features degree 160...

Warning: Ill-conditioned matrix at degree 159, condition number: 132847845376.0

Degree 160 completed. Train Error: 1.478410, Test Error: 1.892171

Training model with Random Fourier Features degree 161...

Degree 161 completed. Train Error: 1.478410, Test Error: 1.896902

Training model with Random Fourier Features degree 162...

Degree 162 completed. Train Error: 1.478411, Test Error: 1.920150

Training model with Random Fourier Features degree 163...

Degree 163 completed. Train Error: 1.478411, Test Error: 1.940743

Training model with Random Fourier Features degree 164...

Degree 164 completed. Train Error: 1.478411, Test Error: 1.917095

Training model with Random Fourier Features degree 165...

Warning: Ill-conditioned matrix at degree 164, condition number: 158307205120.0

Degree 165 completed. Train Error: 1.478411, Test Error: 1.911195

Training model with Random Fourier Features degree 166...

Warning: Ill-conditioned matrix at degree 165, condition number: 20525600768.0

Degree 166 completed. Train Error: 1.478411, Test Error: 1.943841

Training model with Random Fourier Features degree 167...

Warning: Ill-conditioned matrix at degree 166, condition number: 12717872128.0

Degree 167 completed. Train Error: 1.478411, Test Error: 1.942510

Training model with Random Fourier Features degree 168...

Warning: Ill-conditioned matrix at degree 167, condition number: 167551270912.0

Degree 168 completed. Train Error: 1.478410, Test Error: 1.869054

Training model with Random Fourier Features degree 169...

Degree 169 completed. Train Error: 1.478411, Test Error: 1.885276

Training model with Random Fourier Features degree 170...
Warning: Ill-conditioned matrix at degree 169, condition number: 73891700736.0
Degree 170 completed. Train Error: 1.478411, Test Error: 1.878596

Training model with Random Fourier Features degree 171...
Warning: Ill-conditioned matrix at degree 170, condition number: 34740592640.0
Degree 171 completed. Train Error: 1.478410, Test Error: 1.868592

Training model with Random Fourier Features degree 172...
Warning: Ill-conditioned matrix at degree 171, condition number: 10911220736.0
Degree 172 completed. Train Error: 1.478410, Test Error: 1.914728

Training model with Random Fourier Features degree 173...
Degree 173 completed. Train Error: 1.478411, Test Error: 1.861178

Training model with Random Fourier Features degree 174...
Warning: Ill-conditioned matrix at degree 173, condition number: 19757969408.0
Degree 174 completed. Train Error: 1.478411, Test Error: 1.878526

Training model with Random Fourier Features degree 175...
Warning: Ill-conditioned matrix at degree 174, condition number: 394170466304.0
Degree 175 completed. Train Error: 1.478411, Test Error: 1.919481

Training model with Random Fourier Features degree 176...
Warning: Ill-conditioned matrix at degree 175, condition number: 46901800960.0
Degree 176 completed. Train Error: 1.478411, Test Error: 1.897353

Training model with Random Fourier Features degree 177...
Warning: Ill-conditioned matrix at degree 176, condition number: 80002121728.0
Degree 177 completed. Train Error: 1.478411, Test Error: 1.902413

Training model with Random Fourier Features degree 178...
Warning: Ill-conditioned matrix at degree 177, condition number: 447856115712.0
Degree 178 completed. Train Error: 1.478411, Test Error: 1.931403

Training model with Random Fourier Features degree 179...
Warning: Ill-conditioned matrix at degree 178, condition number: 49292029952.0
Degree 179 completed. Train Error: 1.478411, Test Error: 1.899580

Training model with Random Fourier Features degree 180...
Degree 180 completed. Train Error: 1.478410, Test Error: 1.914274

Training model with Random Fourier Features degree 181...
Warning: Ill-conditioned matrix at degree 180, condition number: 10880441344.0
Degree 181 completed. Train Error: 1.478410, Test Error: 1.891728

Training model with Random Fourier Features degree 182...

Warning: Ill-conditioned matrix at degree 181, condition number: 17129222144.0
Degree 182 completed. Train Error: 1.478411, Test Error: 1.916343

Training model with Random Fourier Features degree 183...
Degree 183 completed. Train Error: 1.478411, Test Error: 1.907884

Training model with Random Fourier Features degree 184...
Warning: Ill-conditioned matrix at degree 183, condition number: 1417713156096.0
Degree 184 completed. Train Error: 1.478411, Test Error: 1.896339

Training model with Random Fourier Features degree 185...
Warning: Ill-conditioned matrix at degree 184, condition number: 70104383488.0
Degree 185 completed. Train Error: 1.478411, Test Error: 1.917522

Training model with Random Fourier Features degree 186...
Warning: Ill-conditioned matrix at degree 185, condition number: 48846221312.0
Degree 186 completed. Train Error: 1.478411, Test Error: 1.869709

Training model with Random Fourier Features degree 187...
Warning: Ill-conditioned matrix at degree 186, condition number: 55031377920.0
Degree 187 completed. Train Error: 1.478411, Test Error: 1.864065

Training model with Random Fourier Features degree 188...
Degree 188 completed. Train Error: 1.478411, Test Error: 1.918343

Training model with Random Fourier Features degree 189...
Warning: Ill-conditioned matrix at degree 188, condition number: 299144314880.0
Degree 189 completed. Train Error: 1.478411, Test Error: 1.891260

Training model with Random Fourier Features degree 190...
Warning: Ill-conditioned matrix at degree 189, condition number: 627567493120.0
Degree 190 completed. Train Error: 1.478410, Test Error: 1.852828

Training model with Random Fourier Features degree 191...
Warning: Ill-conditioned matrix at degree 190, condition number: 56200876032.0
Degree 191 completed. Train Error: 1.478411, Test Error: 1.942399

Training model with Random Fourier Features degree 192...
Warning: Ill-conditioned matrix at degree 191, condition number: 18727561216.0
Degree 192 completed. Train Error: 1.478411, Test Error: 1.865220

Training model with Random Fourier Features degree 193...
Warning: Ill-conditioned matrix at degree 192, condition number: 128858021888.0
Degree 193 completed. Train Error: 1.478411, Test Error: 1.897249

Training model with Random Fourier Features degree 194...
Warning: Ill-conditioned matrix at degree 193, condition number: 44266700800.0
Degree 194 completed. Train Error: 1.478411, Test Error: 1.895789

Training model with Random Fourier Features degree 195...

Warning: Ill-conditioned matrix at degree 194, condition number: 12789522432.0

Degree 195 completed. Train Error: 1.478411, Test Error: 1.929968

Training model with Random Fourier Features degree 196...

Warning: Ill-conditioned matrix at degree 195, condition number:
18065123180544.0

Degree 196 completed. Train Error: 1.478411, Test Error: 1.891874

Training model with Random Fourier Features degree 197...

Warning: Ill-conditioned matrix at degree 196, condition number: 77189283840.0

Degree 197 completed. Train Error: 1.478411, Test Error: 1.865973

Training model with Random Fourier Features degree 198...

Warning: Ill-conditioned matrix at degree 197, condition number: 244517388288.0

Degree 198 completed. Train Error: 1.478411, Test Error: 1.925338

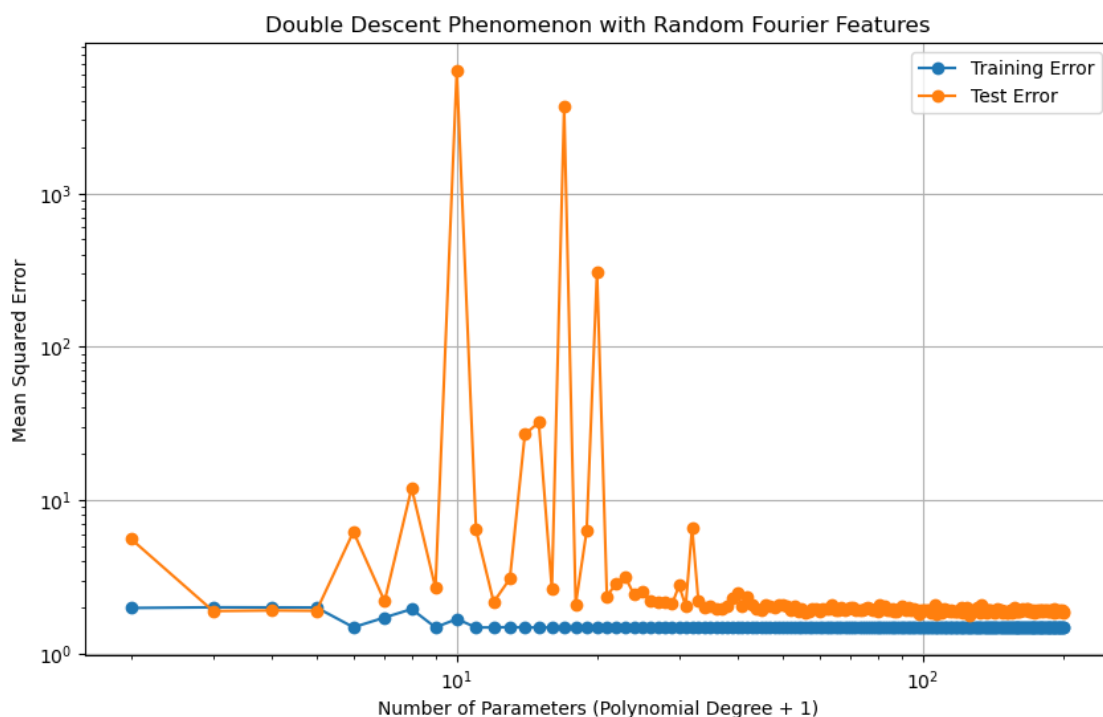
Training model with Random Fourier Features degree 199...

Warning: Ill-conditioned matrix at degree 198, condition number: 147242385408.0

Degree 199 completed. Train Error: 1.478411, Test Error: 1.846301

Training model with Random Fourier Features degree 200...

Degree 200 completed. Train Error: 1.478411, Test Error: 1.881204



Finally we run our model across 200 degrees and no stabilization to ensure we can see double descent and we plot our results in matplotlib

The Graph seems to show double or even more descents, however I was not able to get the training error very close to 0 since it flattend out around 1.5, the testing error however goes through it's descents before stablizing and trending towards the training error.