AM160 HW 1 Part 1

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\begin{aligned}
A_{n\times p}\theta_{p\times 1} &= y_{n\times 1}, \qquad rank(A) = n \implies AA^T \in R^{n\times n} \text{ is invertible} \\
\theta^* &= A^T(AA^T)^{-1}y \\
A\theta^* &= y \\
&\text{consider any other solution } \theta \text{ of } A\theta = y \\
&\langle \theta - \theta^*, \theta^* \rangle \\
&= \langle \theta - \theta^*, A^T(AA^T)^{-1}y \rangle \\
&= \left\langle \underbrace{A(\theta - \theta^*)}_{=0}, (AA^T)^{-1}y \right\rangle \\
&\therefore (\theta - \theta^*) \perp \theta^* \\
\|\theta\|^2 &= \langle \theta, \theta \rangle \\
&= \langle \theta - \theta^* + \theta^*,\ \theta - \theta^* + \theta^* \rangle \\
&= \langle \theta - \theta^*,\ \theta - \theta^* \rangle + \langle \theta^*, \theta^* \rangle + 2\langle \theta - \theta^*, \theta^* \rangle \\
&= \|\underbrace{\theta - \theta^*}_{\geq 0}\|^2 + \|\theta^*\|^2 \geq \|\theta^*\|^2 \\
&\therefore \text{ for any } \theta \neq \theta^*,\ \|\theta\|^2 > \|\theta^*\|^2 \text{ meaning } \|\theta^*\| \text{ must be the minimum solution}
\end{aligned}
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