Reasoning about Identifier Spaces: How to Make Chord Correct

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***Abstract*—The Chord distributed hash table (DHT) is well-known and often used to implement peer-to-peer systems. Chord peers find other peers, and access their data, through a ring-shaped pointer structure in a large identifier space. Despite claims of proven correctness, i.e., eventual reachability, previous work has shown that the Chord ring-maintenance protocol is not correct under its original operating assumptions. Previous work has not, however, discovered whether Chord could be made correct under the same assumptions. The contribution of this paper is to provide the first specification of correct operations and initialization for Chord, an inductive invariant that is necessary and sufficient to support a proof of correctness, and two independent proofs of correctness. One proof is informal and intuitive, and applies to networks of any size. The other proof is based on a formal model in Alloy, and uses fully automated analysis to prove the assertions for networks of bounded size. The two proofs complement each other in several important ways.**

I. Introduction

Peer-to-peer systems are distributed systems featuring decentralized control, self-organization of similar nodes, fault-tolerance, and scalability. The best known peer-to-peer system is Chord, which was first presented in a 2001 SIGCOMM paper [1]. This paper was the fourth-most-cited paper in computer science for several years (according to Citeseer), and won the 2011 SIGCOMM Test-of-Time Award.

The Chord protocol maintains a network of nodes that can reach each other despite the fact that autonomous nodes can join the network, leave the network, or fail at any time. The nodes of a Chord network have identifiers in an m-bit identifier space, and reach each other through pointers in this identifier space. Because the network structure is based on adjacency in the identifier space, and $2^m - 1$ is adjacent to 0, the structure of a Chord network is a ring.

A Chord network is used to maintain a distributed hash table (DHT), which is a key-value store in which the keys are also identifiers in the same m-bit space. In turn, the hash table can be used to implement shared file storage, group directories, and many other purposes. Chord has been implemented many times, and used to build large-scale applications such as BitTorrent. And the continuing influence of Chord is easy to trace in more recent systems such as Dynamo [2].

The basic correctness property for Chord is eventual reachability: given ample time and no further joins, departures, or failures, the protocol can repair all defects in the ring structure. If the protocol is not correct in this sense, then some nodes of a Chord network will become permanently unreachable from other nodes. The introductions of the original Chord papers [1], [3] say, "Three features that distinguish Chord from many other peer-to-peer lookup protocols are its simplicity, provable correctness, and provable performance." An accompanying PODC paper [4] lists invariants of the ring-maintenance protocol.

The claims of simplicity and performance are certainly true. The Chord algorithms are far simpler and more completely specified than those of other DHTs, such as Pastry [5], Tapestry [6], CAN [7], and Kademlia [8]. Operations are fast because there are no atomic operations requiring locking of multiple nodes, and even queries are minimized.

Unfortunately, the claim of correctness is not true. The original specification with its original operating assumptions does not have eventual reachability, and *not one* of the seven properties claimed to be invariants in [4] is actually an invariant [9]. This was revealed by modeling the protocol in the Alloy language and checking its properties with the Alloy Analyzer [10].

The principal contribution of this paper is to provide the first specification of a version of Chord that is as efficient as the original, correct under reasonable operating assumptions, and actually proved correct. The new version corrects all the flaws that were revealed in [9], as well as some additional ones. The proof provides a great deal of insight into how rings in identifier spaces work, and is backed up by a formal, analyzable model.

Although other researchers have found problems with Chord implementations [11], [12], [13], they have not discovered any problems with the specification of Chord. Although other researchers have verified properties of DHTs [14], [15], they have not considered failures, which are by far the most difficult part of the problem. Other work on verifiable ring maintenance operations [16] uses multi-node atomic operations, which are avoided by Chord.

Some motivations and possible benefits of this work are presented below. They are categorized according to the audience or constituency that would benefit.

For those who implement Chord or rely on a Chord implementation: It seems obvious that they should have a precise and correct specification to follow. They should also know the invariant for Chord, as dynamic checking of the invariant is a design principle for enhancing DHT security [17].

Critics of this work have claimed that all the flaws in original Chord are either obvious and fixed by all implementers, or extremely unlikely to cause trouble during Chord execution. It is a fact that some implementations retain original flaws, citing [18] not because it is a bad implementation, but simply because the code is published and readable. Concerning whether the flaws cause real trouble or not, Chord implementations are certainly reported to have been unreliable. It is in the nature of distributed systems that failures are difficult to diagnose, and no one knows (or at least tells) what is really going on. Any means to increasing the reliability of distributed systems, especially without sacrificing efficiency, is an unmixed blessing.

For those interested in building more robust or more functional peer-to-peer systems based on Chord: Due to its simplicity and efficiency, it is an attractive idea to extend original Chord with stronger guarantees and additional properties. Work has already been done on protection against malicious peers [19], [20], [21], key consistency and data consistency [22], range queries [23], and atomic access to replicated data [24], [25].

For those who build on Chord, and reason about Chord behavior, their reasoning should have a sound foundation. Previous research on augmenting and strengthening Chord, as referenced above, relies on ambiguous descriptions of Chord and unsubstantiated claims about its behavior. These circumstances can lead to misunderstandings about how Chord works, as well as to unsound reasoning. For example, the performance analysis in [26] makes the assumption that every operation of a particular kind makes progress according to a particular measure, which is easily seen to be false [9].

For those interested in encouraging application of formal methods: This project has already had an impact, as developers at Amazon credit the discovery of Chord flaws [9] with convincing them that formal methods can be applied productively to real distributed systems [27].

The proof of correctness is also turning out to be an important case study. In this paper there are two independent proofs, one informal and one by model checking. The informal proof applies to networks of any size, and provides deep insight into how and why the protocol works. The Alloy model with its automated checking applies only to networks of bounded size, and offers limited insight, but it is an indispensable backup to the informal proof because it guards against human error. Also, it was an indispensable precursor to finding the general proof, because it indicated which theorems were likely to be true.

For those interested in formal proofs, the Alloy-only proof in [28] has been used as a test case for the Ivy proof system [29], and the new proof given here is being used as a test case for the Verdi proof system [30].

Finally, there are other possible uses for ring-shaped pointer structures in large identifier spaces (*e.g.,* [31], [7]). The reasoning about identifier spaces used in this paper may also be relevant to other work of this kind.

The paper begins with an overview of Chord using the revised, correct ring-maintenance operations (Section II), and a specification of these new operations (Section III). Although the specification is pseudocode for immediate accessibility, it is a paraphrase of the formal model in Alloy.

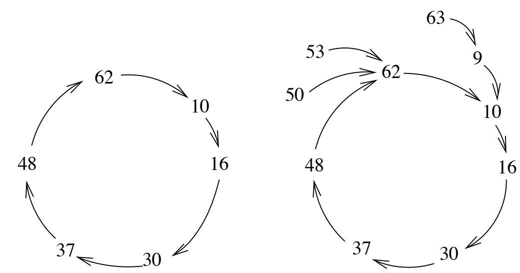

Fig. 1. Ideal (left) and valid (right) networks. Members are represented by their identifiers. Solid arrows are successor pointers.

Correct operations are necessary but not sufficient. It is also necessary to initialize a network correctly. Original Chord is initialized with a network of one node, which is not correct, and Section IV shows why. This section also introduces the inductive invariant for the proof, because a Chord network can safely be initialized in any state that satisfies the invariant.

Summarizing the previous two sections, Section V compares the revised Chord protocol with the original version, explaining how they differ. Together Sections IV and V present most of the problems with original Chord reported in [9] (as well as previously unreported ones). The problems are not presented first because they make more sense when explained along with their underlying nature and how to remove them.

The proof of correctness is largely based on reasoning about ring structures in identifier spaces. Section VI presents some useful theorems about these spaces and shows how they apply to Chord. The actual proof in Section VII follows a fairly conventional outline. Section VIII discusses the formal model and model-checked version of the proof.

II. Overview of correct Chord

Every member of a Chord network has an identifier (assumed unique) that is an m-bit hash of its IP address. Every member has a *successor list* of pointers to other members. The first element of this list is the *successor*, and is always shown as a solid arrow in the figures. Figure 1 shows two Chord networks with $m = 6$, one in the ideal state of a ring ordered by identifiers, and the other in the valid state of an ordered ring with appendages. In the networks of Figure 1, key-value pairs with keys from 31 through 37 are stored in member 37. While running the ring-maintenance protocol, a member also acquires and updates a *predecessor* pointer, which is always shown as a dotted arrow in the figures.

The ring-maintenance protocol is specified in terms of four operations, each of which is executed by a member and changes only the state of that member. In executing an operation, the member often queries another member or sequence of members, then updates its own pointers if necessary. The specification of Chord assumes that inter-node communication is bidirectional and reliable, so we are not concerned with Chord behavior when inter-node communication fails.

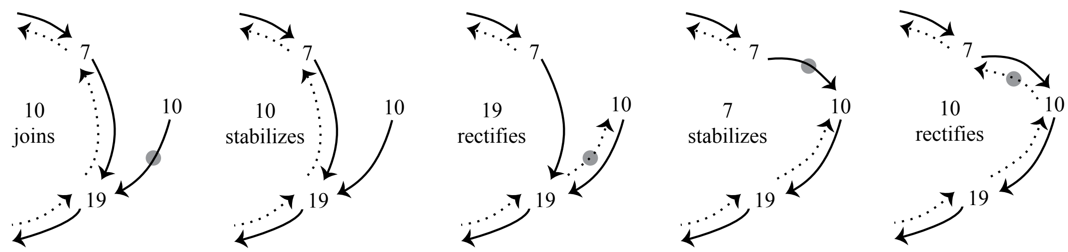

Fig. 2. A new node becomes part of the ring. A gray circle marks the pointer updated by an operation, if any. Dotted arrows are predecessors.

A node becomes a member in a *join* operation. A member is also referred to as a *live node*. When a member joins, it contacts some existing member to look up a member that is near to it in identifier space, and gets a successor list from that nearby member. The first stage of Figure 2 shows successor and predecessor pointers in a section of a network where 10 has just joined.

When a member *stabilizes*, it learns its successor's predecessor. It adopts the predecessor as its new successor, provided that the predecessor is closer in identifier order than its current successor. Because a member must query its successor to stabilize, this is also an opportunity for it to update its successor list with information from the successor. Members schedule their own stabilize operations, which should be periodic.

Between the first and second stages of Figure 2, 10 stabilizes. Because its successor's predecessor is 7, which is not a better successor for 10 than its current 19, this operation does not change the successor of 10.

After stabilizing (regardless of the result), a node notifies its successor of its identity. This causes the notified member to execute a *rectify* operation. The rectifying member adopts the notifying member as its new predecessor if the notifying member is closer in identifier order than its current predecessor, or if its current predecessor is dead. In the third stage of Figure 2, 10 has notified 19, and 19 has adopted 10 as its new predecessor.

In the fourth stage of Figure 2, 7 stabilizes, which causes it to adopt 10 as its new successor. In the last stage 7 notifies and 10 rectifies, so the predecessor of 10 becomes 7. Now the new member 10 is completely incorporated into the ring, and all the pointers shown are correct.

The protocol requires that a member or live node always responds to queries in a timely fashion. A node ceases to become a member in a *fail* operation, which can represent failure of the machine, or the node's silently leaving the network. A member that has failed is also referred to as a *dead node*. The protocol also requires that, after a member fails, it no longer responds to queries from other members. With this behavior, members can detect the failure of other members perfectly by observing whether they respond to a query before a timeout occurs.

Failures can produce gaps in the ring, which are repaired during stabilization. As a member attempts to query its successor for stabilization, it may find that its successor is dead. In this case it attempts to query the next member in its successor list and make this its new successor, continuing through the list until it finds a live successor.

There is an important operating assumption that successor lists are long enough, and failures are infrequent enough, so that a member is never left with no live successor in its list. Put another way, this is a fairness assumption about the relative rates of failures (which create dead entries in successor lists) and stabilizations (which replace dead entries with live ones).

As in the original Chord papers [1], [3], we wish to define a correctness property of eventual reachability: given ample time and no further disruptions, the ring-maintenance protocol can repair defects so that every member of a Chord network is reachable from every other member. Note that a network with appendages (nodes 50, 53, 63, 9 on the right side of Figure 1) cannot have full reachability, because an appendage cannot be reached by a member that is not in the same appendage.

A network is *ideal* when each pointer is globally correct. For example, on the right of Figure 1, the globally correct successor of 48 is 50 because it is the nearest member in identifier order. Because the ring-maintenance protocol is supposed to repair all imperfections, and because it is given ample time to do all the repairs, the correctness criterion can be strengthened slightly, to: *In any execution state, if there are no subsequent join or fail operations, then eventually the network will become ideal and remain ideal.*

Defining a member's *best successor* as its first successor pointing to a live node (member), a *ring member* is a member that can reach itself by following the chain of best successors. An *appendage member* is a member that is not a ring member. Of the seven invariants presented in [4] (and all violated by original Chord), the following four are necessary for correctness.

- There must be a ring, which means that there must be a non-empty set of ring members (*AtLeastOneRing*).
- There must be no more than one ring, which means that from each ring member, every other ring member is reachable by following the chain of best successors (*AtMostOneRing*).
- On the unique ring, the nodes must be in identifier order (*OrderedRing*).
- From each appendage member, the ring must be reachable by following the chain of best successors (*ConnectedAppendages*).

If any of these properties is violated, there is a defect in the structure that the ring-maintenance protocol cannot repair, and some members will be permanently unreachable from some other members. It follows that any inductive invariant must imply these properties.

The Chord papers define the lookup protocol, which is used to find the member primarily responsible for a key, namely the ring member with the smallest identifier greater than or equal to the key. The lookup protocol is not discussed further here. Chord papers also define the maintenance and use of finger tables, which greatly improve lookup speed by providing pointers that cross the ring like chords of a circle. Because finger tables are an optimization and they are built from successor lists, correctness does not depend on them.

III. Specification of ring-maintenance operations

A. Identifiers and node state

There is a type *Identifier* which is a string of *m* bits. Implicitly, whenever a member transmits the identifier of a member, it also transmits its IP address so that the recipient can reach the identified member. The pair is self-authenticating, as the identifier must be the hash of the IP address according to a chosen function.

The Boolean function *between* is used to test the order of identifiers. Because identifier order wraps around at zero, it is meaningless to test the order of two identifiers—each precedes and succeeds the other. This is why *between* has three arguments:

```
Boolean function between (n1,nb,n2: Identifier)
{  if (n1 < n3) return ( n1 < nb && nb < n2 )
   else         return ( n1 < nb || nb < n2 )
}
```

For *nb* to be *between n1* and *n2*, it must be equal to neither. Further properties of identifier spaces are presented in Section VI.

Each node that is a member of a Chord network has the following state variables:

```
myIdent: Identifier;
prdc: Identifier;
succList: list Identifier;      // length is r
```

where *myIdent* is the hash of its IP address, and *prdc* is the node's predecessor. *succList* is the node's entire successor list; the head of this list is its *first successor* or simply its *successor*. The parameter *r* is the fixed length of all successor lists.

B. Maintaining a shared-state abstraction

Reasoning about Chord requires reasoning about the global state, so the protocol must maintain the abstraction of a shared, global state. The algorithmic steps of the protocol must behave as if atomic and interleaved. In each algorithmic step, a node reads the state of at most one other node, and modifies only its own state.

In an implementation, a node reads the state of another node by querying it. If the node does not respond within a time parameter *t*, then it is presumed dead. If the node does respond, then the atomic step associated with the query is deemed to occur at the instant that the queried node responds with information about its own state.

To maintain the shared-state abstraction, the querying node must obey the following rules:

- The querying node does not know the instant that its query is answered; it only knows that the response was sent some time after it sent the query. So the querying node must treat its own state, between the time it sends the query and the time it finishes the step by updating its own state, as undefined. The querying node cannot respond to queries about its state from other nodes during this time.
- If the querying node is delaying response to a query because it is waiting for a response to its own query, it must return interim "response pending" messages so that it is not presumed dead.
- If a querying node is waiting for a response, and is queried by another node just to find out if it is alive or dead, it can respond immediately. This is possible because the response does not contain any information about its state.

This covers all possibilities except that of a deadlock due to circular waiting for query responses. Freedom from deadlock is covered in the proof of correctness in Section VII.

C. Join and fail operations

When a node is not a member of a Chord network, it has no Chord state variables, and does not respond to queries from Chord members. To join a Chord network, a node must first calculate its own Chord identifier *myIdent*. It must also know some member of the network—it does not even matter whether it is a ring member or appendage—and must ask the member to use the lookup protocol to find a member *newPrdc* such that *between (newPrdc, myIdent, head(newPrdc.succList))*.

Provided with this information, the node joins in a single atomic step, by executing the following pseudocode:

```
// Join step

// newPrdc has value from previous lookup
newPrdc: Identifier;

query newPrdc for newPrdc.succList;
if (query returns before timeout) {
   succList = newPrdc.succList;
   prdc = newPrdc;
}
else abort;
```

If the query fails then *newPrdc* has died, and the node has no choice but to try joining again later.

A fail operation is also a single atomic step. When a member node fails or leaves a Chord network, it deletes its Chord state variables and ceases to respond to queries. Fortunately, the proof of correctness shows that a node can re-join safely even if other nodes still have pointers to it from its former episode of membership.

D. Stabilize and rectify operations

A stabilize operation may require a sequence of steps. First, the stabilizing node executes a *StabilizeFromSuccessor* step:

```
// StabilizeFromSuccessor step

// newSucc not initialized
newSucc: Identifier;

query head(succList) for
      head(succList).prdc and
      head(succList).succList;
if (query returns before timeout) {
   // successor live, adopt its list as mine
   succList =
      append (
         head(succList),
         butLast(head(succList).succList)
      );
   newSucc = head(succList).prdc;
   if (between(myIdent,newSucc,head(succList)))
      // predecessor may be a better successor
      next step is StabilizeFromPredecessor;
      // else stabilization is complete
}
// successor is dead, remove from succList
else
   succList =
      append(tail(succList),last(succList)+1);
   next step is StabilizeFromSuccessor again;
```

First the node queries its successor for its successor's predecessor and successor list. If this query times out, then the node's successor is presumed dead. The node removes the dead successor from its successor list and does another *StabilizeFromSuccessor* step.[1] We know that eventually it will find a live successor in its list, because of the operating assumption (from Section II) that successor lists are long enough so that each list contains at least one live node.

Once the node has contacted a live successor, it adopts its successor list (all but the last entry) as its own second and later successors. It then tests the successor's predecessor to see if it might be a better first successor. If so, the node then executes a *StabilizeFromPredecessor* step. If not, the stabilization operation is complete.

The *StabilizeFromPredecessor* step is simple. The node queries its potential new successor for its successor list. If the new successor is live, the node adopts it and its successor list. If not, nothing changes. Either way, the stabilization operation is complete.

```
// StabilizeFromPredecessor step

// newSucc value came from previous step
newSucc: Identifier;

query newSucc for newSucc.succList;
if (query returns before timeout)
   // new successor is live, adopt it
   succList =
      append(newSucc,butLast(newSucc.succList));
   // else new successor is dead, no change
```

[1]The empty place in the successor list is filled with an artificial entry at the end, created by adding one to the last real entry. The reason for this entry will be made clear by the proof.

At the completion of each stabilization operation, regardless of the result, the stabilizing node sends a message to its successor notifying the successor of its presence as a predecessor. On receiving this notification, a node executes a single-step rectify operation, which may allow it to improve its predecessor pointer.

```
// Rectify step

// newPrdc value came from notification
newPrdc: Identifier;

if (between (prdc, newPrdc, myIdent))
   // newPrdc presumed live
   prdc = newPrdc;
else {
   query prdc to see if live;
   if (query returns before timeout)
      no change;
   // live newPrdc better than dead old one
   else prdc = newPrdc;
};
```

IV. Initialization and invariant

An *inductive invariant* is an invariant with the property that if the system satisfies the invariant before any event, then the system can be proved to satisfy the invariant after the event. By induction, if the system's initial state satisfies the invariant, then all system states satisfy the invariant.

Original Chord initializes a network with a single member that is its own successor, *i.e.,* the initial network is a ring of size 1. This is not correct, as shown in Figure 3 with successor lists of length 2. Appendage nodes 62 and 37 start with both list entries equal to 48. Then 48 fails, leaving members 62 and 37 with insufficient information to find each other.

Clearly the spirit of the operating assumption in Section II is that the chosen length of successor lists should provide enough redundancy to ensure safe operation. But we can hardly expect the successor lists to work if the redundancy is thrown away by filling them with duplicate entries. This is the problem with Figure 3—that 62 and 37 have no real redundancy in their successor lists, so one failure disconnects them from the network.

For members of a network with successor list length r to enjoy full redundancy, each member must have r distinct entries in its successor list. For this to be possible, the network must have at least $r+1$ members, and the inductive invariant must imply that this is so.

The inductive invariant for Chord is the result of a very long and arduous search, some of which is described in [28]. As one indication of the difficulty, the invariant must imply that the network has a minimum size, yet all operations are local, and no member knows how many other members there are.

As another indication of the difficulty, consider Figure 4, which is a counterexample to a trial invariant consisting of the conjunction of *AtLeastOneRing, AtMostOneRing, OrderedRing, ConnectedAppendages, NoDuplicates,* and *Or-*

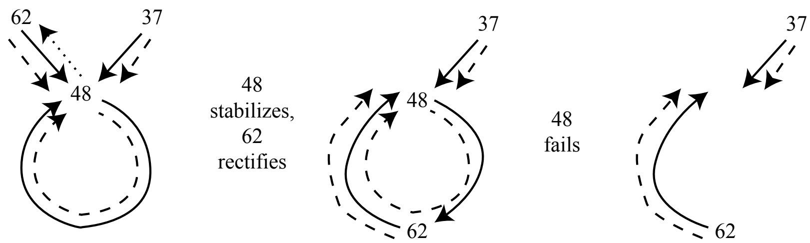

Fig. 3. Why the ring cannot be initialized at size 1. Dashed arrows are second-successor pointers. Predecessor pointers are not shown in the last two stages, as they are irrelevant. This problem was not reported in [9].

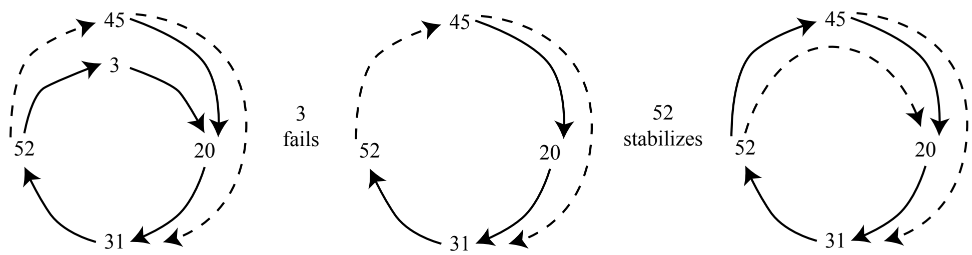

Fig. 4. A counterexample to a trial invariant. Only the relevant pointers are drawn.

deredSuccessorLists. Again $r = 2$. Let an *extended successor list* be the concatenation of a node with its successor list. *NoDuplicates* has the obvious meaning that the entries in any extended successor list are distinct. *OrderedSuccessorLists* says that for any ordered sublist *[x, y, z]* drawn from a node's extended successor list, whether the sublist is contiguous or not, *between [x, y, z]* holds.

In Figure 4, the first stage satisfies the trial invariant, having duplicate-free and ordered extended successor lists such as *[52, 3, 45]* and *[45, 20, 31]*. The appendage node 45 does not merge into the ring at the correct place, but that is part of normal Chord operation (see [9]). The second successor of ring node 52 points outside the ring, but that is also part of normal Chord operation. It is also part of normal Chord operation that 45 changes from being an appendage node to a ring member just because 3 fails. In the case shown in the figure, the result of all these quirks combined is that the ring becomes disordered.

The final invariant is much simpler than the earlier invariant used in [28]. It also has the major advantage of not requiring an extra operating assumption that is difficult to implement. It was discovered in the process of finding a general informal proof of the assertions verified automatically for networks of bounded size.

To explain the real invariant, we must introduce the concept of a *principal node*. A principal node is a member that is not skipped by any member's extended successor list. For example, if 30 is a principal node, then *[30, 34, 39]* and *[27, 30, 34]* and *[21, 27, 29]* can all be extended successor lists, but *[27, 29, 34]* cannot be, because 30 is between 29 and 34, and would therefore be skipped.

The real invariant is the conjunction of only two properties, *OneLiveSuccessor* and *SufficientPrincipals*. *OneLiveSuccessor* simply says that every successor list has at least one live entry. *SufficientPrincipals* says that the number of principal members is greater than or equal to $r + 1$, where r is the length of successor lists.

The proofs in Section VI will show that this deceptively simple invariant implies all of *AtLeastOneRing, AtMostOneRing, OrderedRing, ConnectedAppendages, NoDuplicates,* and *OrderedSuccessorLists*. Needless to say, it also implies that the network has a minimum size. (Note that the first stage of Figure 4 has no principal members, so the figure is not a counterexample to the real invariant.)

A typical Chord network has r from 2 to 5, so the set of principals need only have 3 to 6 nodes. Nevertheless, the existence of these few nodes protects the correctness of a network with millions of members. They wield great and mysterious powers!

V. Comparison of the versions

The *join, stabilize,* and *notified* operations of the original protocol are defined as pseudocode in [1] and [3]. These papers do not provide details about failure recovery. The only published paper with pseudocode for failure recovery is [4], where failure recovery is performed by the *reconcile, update,* and *flush* operations. The following table shows how operations of the two versions correspond. Although *rectify* in the new version is similar to *notified* in the old version, it seems more consistent to use an active verb form for its name.

old	**new**
join + reconcile	join
stabilize + reconcile + update	stabilize
notified + flush	rectify

In both old and new versions of Chord, members schedule their own maintenance operations except for *notified* and *rectify*, which occur when a member is notified by a predecessor. Although the operations are loosely expected to be periodic, scheduling is not formally constrained. As can be seen from the table, multiple smaller operations from the old version are assembled into larger new operations. This ensures that the successor lists of members are always fully populated with r entries, rather than having missing entries to be filled in by later operations. An incompletely populated successor list might lose (to failure) its last live successor. If the successor list belongs to an appendage member, this would mean that the appendage can no longer reach the ring, which is a violation of *ConnectedAppendages* [9].

Another systematic change from the old version to the new is that, before incorporating a pointer to a node into its state, a member checks that it is live. This prevents cases where a member replaces a pointer to a live node with a pointer to a dead one. A bad replacement can also cause a successor list to have no live successor. If the successor list belongs to a ring member, this will cause a break in the ring, and a violation of *AtLeastOneRing*. Together these two systematic changes also prevent scenarios in which the ring becomes disordered or breaks into two rings of equal size (violating *OrderedRing* or *AtMostOneRing*, respectively [9]).

A third systematic change was necessary because the old version does not say anything precise about communication between nodes, and does not say anything at all about atomic steps and maintaining a shared-state abstraction. The new operations are specified in terms of atomic steps, and the rules for maintaining a shared-state abstraction are stated explicitly.

The other major difference is the initialization, as discussed in Section IV.

In addition to these systematic changes, a number of small changes were made. Some were due to problems detected by Alloy modeling and analysis of the original version. Others were required to ensure that, after each atomic step of a stabilize operation, the global state satisfies the invariant.

These differences do not change the efficiency of Chord operations in any significant way. Checking some pointers to make sure they point to live nodes (new version) requires more queries than in the old version. On the other hand, in the old version stabilize, reconcile, and update operations are all separate, and can all entail queries. In this respect the old version requires more queries than the new version.

There is an additional bonus in the new version for implementers. Consider what happens when a member node fails, recovers, and wishes to rejoin, all of which could occur within a short period of time. It was previously thought necessary for the node to wait until all previous references to its identifier had been cleared away (with high probability), because obsolete pointers could make the state incorrect. This wait was included in the first Chord implementation [32]. Yet the wait is unnecessary, as Chord is provably correct even with obsolete pointers.

In the spirit of [17], it is a good security practice to monitor that invariants are satisfied. Both the conjuncts of the inductive invariant are global, and thus unsuitable for local monitoring. The right properties to monitor are *NoDuplicates* and *OrderedSuccessorLists*, which can be checked on individual successor lists. These are properties that must be true for Chord networks of any size.

Although the new initialization with $r+1$ principal nodes may not be inefficient, it is certainly more difficult to implement than initialization of original Chord. An alternative approach might be to start the network with a single node, and monitor the network as a whole until it has $r+1$ principal nodes. For example, all nodes might send their successor lists (whenever there is a change) around the ring, to be collected and checked by the single original node. Once the original node sees a sufficient set of principal nodes, it could send a signal around the ring that monitoring is no longer necessary. This scheme is discussed further in Section VII-B.

VI. Reasoning about ring structures in identifier spaces

A. Theorems about identifier spaces

An identifier space is built from a finite totally-ordered set by adding the concept that its greatest element is less than its smallest element. This makes identifier order circular, so the identifier space itself can be thought of as a ring.

The viewpoint of this paper is that identifier spaces have less structure than algebraic rings. Algebraic rings are generalizations of integer arithmetic, with operators such as sum and product that combine quantities. In Chord identifiers are not quantities, and it makes no sense to add or multiply them. This is in contrast to the formalization of Pastry [15], where distance in the identifier space is assumed to be meaningful and is used in the protocol.

In this section definitions and theorems about identifier spaces are presented in the Alloy syntax. In the Alloy model the concepts of identifier and node (potential network member) are conflated, so that `Node` is declared as a totally ordered set upon which an identifier space is built. Details about the Alloy model and bounded verification can be found in Section VIII. These theorems have been proven for unbounded identifier spaces using merely substitution and simplification.

Section III-A already introduced the Boolean function *between,* defined in Alloy as:

```
pred between[n1,nb,n2: Node] {
   lt[n1,n2] =>   ( lt[n1,nb] && lt[nb,n2] )
              else ( lt[n1,nb] || lt[nb,n2] ) }
```

where `lt`, `&&`, `||` are the notations for less than (in the total ordering), logical and, and logical or, respectively. The definition has the form of an if-then-else expression.

Here is a simple theorem in Alloy syntax:

```
assert AnyBetweenAny {
   all disj n1,n2: Node | between[n1,n2,n1] }
```

AnyBetweenAny says that for any distinct (disjoint) *n1* and *n2,* *n2* is between *n1* and *n1*.

For proofs, we also need a different predicate *includedIn*, which is like *between* except that the included identifier can be equal to either of the boundary identifiers:

```
pred includedIn[n1,nb,n2: Node] {
  lt[n1,n2] =>   ( lte[n1,nb] && lte[nb,n2] )
            else ( lte[n1,nb] || lte[nb,n2] ) }
```

In the *AnyIncludedInAny* theorem, the two arguments need not be disjoint:

```
assert AnyIncludedInAny {
   all n1,n2: Node | includedIn[n1,n2,n1] }
```

A very useful theorem allows us to reason about the fact or assumption that *between* does *not* hold.

```
assert IncludedReversesBetween {
   all disj n1,n2: Node, nb: Node |
       ! between[n1,nb,n2]
   <=> includedIn[n2,nb,n1]       }
```

Provided that the boundaries of an interval are distinct, if an identifier *nb* cannot be found in the portion of the identifier space from *n1* to *n2* (exclusive), then it must be found in the portion of the identifier space from *n2* to *n1* (inclusive).

B. Theorems about successor lists

This section introduces definitions and theorems about a second kind of ring. Successor lists whose entries are identifiers (in the first kind of ring) are used to create ring-shaped networks (the second kind of ring). A number of terms concerning successor lists in network states were introduced briefly in Section IV. For clarity, they will be redefined here.

Definition: An *extended successor list* (ESL) is a successor list with the node that owns it prepended to the list. The length of an ESL is $r + 1$.

Definition: A *principal node* is a member that is not skipped by any ESL. That is, for all principal nodes *p*, there is no contiguous pair *[x, y]* in any ESL such that *between [x, p, y]*.

Definition: The property *OneLiveSuccessor* holds in a state if every member has at least one live successor.

Definition: The property *SufficientPrincipals* holds in a state if the number of principal nodes is greater than or equal to $r + 1$.

Definition: The property *Invariant* is the conjunction of *OneLiveSuccessor* and *SufficientPrincipals*.

Definition: The property *NoDuplicates* holds in a state if no ESL has multiple copies of the same entry.

Definition: The property *OrderedSuccessorLists* holds in a state if for all sublists *[x, y, z]* of ESLs, whether contiguous sublists or not, *between [x, y, z]*.

The remainder of this section proves that *Invariant* implies the successor-list properties *NoDuplicates* and *OrderedSuccessorLists*.

Theorem: In any ring structure whose state is maintained in successor lists, *Invariant* implies *NoDuplicates*.

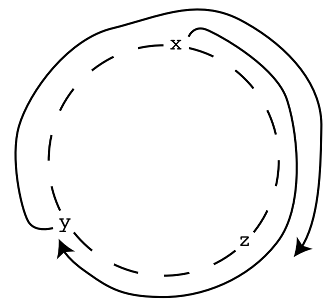

Fig. 5. The dashed line depicts the identifier space. The solid arrows show the path around the identifier space of a segment of an ESL *[x, ..., y, ..., z]*.

Proof:

Contrary to the theorem, assume that there is a network state for which *Invariant* is true and *NoDuplicates* is false. Then some node has an extended successor list with the form *[..., x, ..., x, ...]* for some identifier *x*.

From *AnyBetweenAny*, for all principal nodes *p* distinct from *x*, *between [x, p, x]*. Because of the definition of principal nodes, all of the principal nodes distinct from *x* must be listed in the ellipsis between the two occurrences of *x* in the successor list.

From *SufficientPrincipals*, the portion of the extended successor list *[x, ..., x]* must have length at least $r + 2$, because there are at least r principal nodes distinct from *x*. But the length of the entire extended successor list is $r + 1$, which yields a contradiction. □

If we visualize an identifier space as a ring ordered clockwise, then an ESL is a path that touches the ring wherever the ESL has an entry (as in Figure 5). This proof shows that the existence of a minimum number of nodes that cannot be skipped by ESLs prevents paths from wrapping around the identifier space, which is a major cause of trouble.

Theorem: In any ring structure whose state is maintained in successor lists, *Invariant* implies *OrderedSuccessorLists*.

Proof:

Contrary to the theorem, assume that there is a network state for which *Invariant* is true and *OrderedSuccessorLists* is false. Then some node has an ESL with the form *[..., x, ..., y, ..., z, ...]* where *! between [x, y, z]*. From the previous theorem, *x, y,* and *z* are all distinct.

From *IncludedReversesBetween*, *includedIn [z, y, x]*. So the disordered ESL segment *[x, ..., y, ..., z]* wraps around the identifier ring (see Figure 5), touching the identifier space first at *x*, passing by *z*, touching at *y*, passing by *x* again, then finally touching at *z*.

The maximum length of the disordered ESL segment is $r + 1$. From *SufficientPrincipals*, every entry in it must be a principal node, as there are at least $r + 1$ principals, and every principal must be included. From this and *NoDuplicates*, no entry can be duplicated within this segment.

So z is a principal node, but z is skipped between x and y, which is a contradiction. □

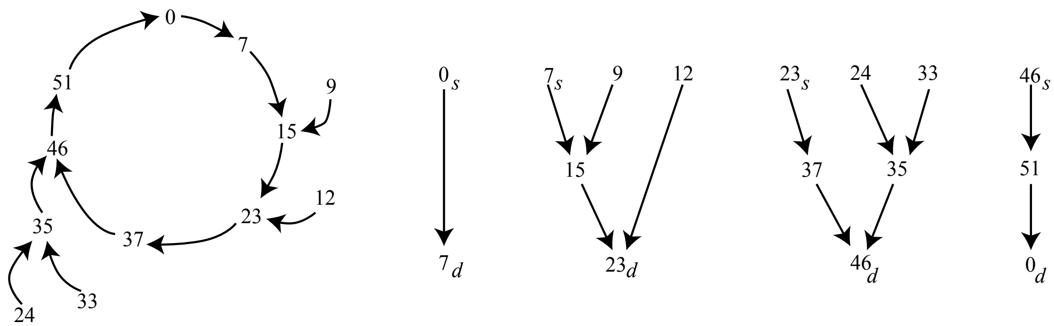

Fig. 6. For a network, the *bestSucc* relation is pictured on the left, and the *splitBestSucc* relation is pictured on the right. Although it cannot be seen from best successors only, the principal nodes are 0, 7, 23, and 46.

This proof continues the theme that ESLs must not wrap around the identifier ring, showing that it causes disorder in a successor list. If a disordered successor list became part of the network structure, then the network ring would be disordered, violating *OrderedRing* from Section II.

C. Theorem about networks built on successor lists

This section is concerned with proving that *Invariant* implies the four necessary properties introduced in Section II.

Definition: A network member's *best successor* or is the first live node in its successor list.

Definition: A *ring member* is a network member that can be reached by following the chain of best successors beginning at itself.

Definition: An *appendage member* is a network member that is not a ring member.

Definition: The property *AtLeastOneRing* holds in a state if there is at least one ring member.

Definition: The property *AtMostOneRing* holds in a state if, from every ring member, it is possible to reach every other ring member by following the chain of best successors beginning at itself.

Definition: The property *OrderedRing* holds in a state if on the unique ring, the nodes are in identifier order. That is, if nodes *n1* and *n2* are ring members, and *n2* is the best successor of *n1*, then there is no other ring member *nb* such that *between [n1, nb, n2]*.

Definition: The property *ConnectedAppendages* holds in a state if, from every appendage member, a ring member can be reached by following the chain of best successors beginning at itself.

Theorem: In any ring structure whose state is maintained in successor lists, *Invariant* implies *AtLeastOneRing*, *AtMostOneRing*, *OrderedRing*, and *ConnectedAppendages*.

Proof:

The best-successor relation *bestSucc* is a binary relation on network members. We define from it a relation *splitBestSucc* that is the same except that every principal node *p* is replaced by two nodes p_s and p_d, where p_s (*s* for source) is in the domain of the relation but not the range, and p_d (*d* for destination) is in the range of the relation but not the domain. Figure 6 displays as graphs the *bestSucc* and *splitBestSucc* relations for the same network. It is possible to deduce many properties of the *splitBestSucc* graph, as follows:

(1) From *Invariant*, every member has a best successor. So the only nodes with no outgoing edges are p_d nodes representing principal members only as *being* best successors.

(2) p_s nodes have no incoming edges, as they represent principal nodes only as *having* best successors. There can be other nodes with no incoming edges, because there can be members that are no member's successor.

Note: The next few points concern maximal paths in the *splitBestSucc* graph. These are paths beginning at nodes with no incoming edges. By definition, they can only end at p_d nodes, and can have no internal nodes representing principal nodes.

(3) Just as a successor list does not skip principal nodes, a maximal path of best successors does not skip principal nodes. That is because an adjacent pair *[x, y]* in a chain of best successors is taken from the successor list of *x*, and the only possible entries between *x* and *y* in the successor list are dead entries.

(4) A maximal path is acyclic. Contrary to this statement, assume that the path contains a cycle *x leads to x*. Then this path skips all principal nodes, which is a contradiction of the fact that paths have no internal nodes representing principal nodes.

From (1-4), we know that the graph of *splitBestSucc* is an inverted forest (a "biological" forest, with roots on the bottom and leaves on the top). Each tree is rooted at a p_d node.

(5) A maximal path is ordered by identifiers. Contrary to this statement, let the path contain *[x, ..., y, ..., z]* where not *between [x, y, z]*. Because the path is acyclic, *x, y,* and *z* are all distinct.

From *IncludedReversesBetween*, *includedIn [z, y, x]*. So the disordered path segment *[x, ..., y, ..., z]* wraps around the identifier ring exactly as depicted in Figure 5. Note that a path in a *splitBestSucc* graph is not the same as an ESL, which was the original subject of Figure 5, but shares some properties with it.

From the figure, we can see that *x* cannot be a principal node p_s, because it is skipped by the path segment from *y* to *z*. Also *z* cannot be a principal node p_d, because it is skipped by the path segment from *x* to *y*. Also *y* cannot be a principal node, by definition, because it is interior to a *splitBestSucc* path. So this disordered path segment skips *every* principal node in identifier space, which contradicts (3).

Note: The next few points concern the *splitBestSucc* relation restricted to p_s and p_d nodes.

(6) Every p_s node is a leaf of exactly one tree. It must be a leaf of some tree, because it begins a path of best successors that must end at a p_d node. It cannot be a leaf of more than one tree, because no node has more than one best successor.

(7) Every tree rooted at a p_d node has exactly one leaf that is a p_s node. It cannot have two such leaves $p1_s$ and $p2_s$, because the source principal closest to the destination principal would be skipped by the path of the furthest source principal.

It must have at least one such leaf p_s. Contrary to this statement, imagine that it does not. Then the principal node pc_s closest to p_d in reverse identifier order begins a path that leads to some other principal destination, skipping p_d in identifier order, which is a contradiction.

Summary:

(6) and (7) show that the *bestSucc* relation restricted to principal nodes is a bijection. In terms of *splitBestSucc*, the ring is formed by the concatenation of the unique maximal paths, one from each tree in the forest, starting at p_s nodes. This proves *AtLeastOneRing* and *AtMostOneRing*. From (5) we know that the ring is ordered by identifiers, so *OrderedRing* holds. All the nodes not on these unique maximal paths are appendage members, and each has a path to a principal node on the ring, so *ConnectedAppendages* holds. □

VII. Proof of Chord correctness

This section presents the proof of the theorem given in Section II:

Theorem: In any execution state, if there are no subsequent join or fail operations, then eventually the network will become ideal and remain ideal.

The most important part of this theorem is knowing that *Invariant* holds in all states, because this property and the properties it implies are the ones that all Chord users can count on at all times. We do not expect churn (joins and failures) to ever stop long enough for a network to become ideal. Rather, this part of the theorem simply tells us that the repair algorithm always makes progress, and cannot get into unproductive loops.

A. Establishing the invariant

First it is necessary to prove that *Invariant,* which is true of any initial state, is preserved by every atomic step of the protocol.

We begin with a failure step, because it requires a constraint based on the operating assumption in Section II: a member cannot fail if it would leave another member with no live successor. In other words, failure steps preserve the property of *OneLiveSuccessor* by operating assumption. No other kind of step can violate *OneLiveSuccessor*.

The other conjunct of *Invariant* is *SufficientPrincipals*, which says that the number of principal nodes must be at least $r + 1$. Rectify operations cannot violate this property, as they do not affect successor lists. In this section we will show that failure steps of non-principal nodes, join steps, *StabilizeFromSuccessor* steps, and *StabilizeFromPredecessor* steps do not cause principal nodes to become skipped in successor lists. This is the only way that they could violate *SufficientPrincipals.* The remaining case, that of failures of principal nodes, will be discussed in the next section.

Failure of a non-principal member *m* causes the disappearance of *m*'s successor list. But only being skipped in a successor list can make a node non-principal, so the disappearance of *m*'s successor list cannot make another node non-principal.

In a successful join, the new ESL created is *[myIdent, newPrdc.succList]*. We know that there is no principal node between *myIdent* and *head (newPrdc.succList)*, because at the time of the query there is no principal node between *newPrdc* and *head (newPrdc.succList)*, and *myIdent* is between those two. We also know that *newPrdc.succList* cannot skip a principal node, by definition.

There are two cases in a *StabilizeFromSuccessor* step where a successor list is altered. In the first case the new ESL is a concatenation of pieces of the ESLs of the stabilizing node and its first successor, joined where they overlap at the first successor. Since neither of the original ESLs can skip a principal node, their overlap cannot, either.

In the second case a dead entry is removed from the stabilizing node's list, which cannot cause it to skip a principal. This leaves an empty space at the end which is temporarily padded with the last real entry plus one. This is the only value choice that preserves the invariant by guaranteeing that no principal node is skipped by accident. It does not matter whether the artificial entry points to a real node or not, as it will be gone by the time that the stabilization operation is complete.

There is only one case in a *StabilizeFromPredecessor* step where a successor list is altered. The new ESL created is *[myIdent, newSucc, butLast (newSucc.succList)]*. In the previous *StabilizeFromSuccessor* step, this node tested that *between [myIdent, newSucc, head (succList)]*. This node cannot make any other changes to its successor list between that step and this *StabilizeFromPredecessor* step, so it is still true. Therefore we know that there is no principal node between *myIdent* and *newSucc*, because there is no principal node between *myIdent* and *head (succList)*, and *newSucc* is between those two. We also know that *[newSucc, butLast (newSucc.succList)]* cannot skip a principal node, because it is part of the ESL of *newSucc*.

B. Failure of principal nodes

It is very clear from Section VI that potential problems in a Chord network would be caused by disordered successor lists and paths of best successors, and that disorder is equivalent to wrapping around the identifier space. It is equally clear that principal nodes are anchor points that prevent disorder, and that there must be at least $r + 1$ of them to make sure that no successor list wraps around the identifier space. This is why a Chord network must be initialized to have $r + 1$ principal nodes.

Apart from initialization, a member of a Chord network becomes a principal node when it has been a member long enough so that every node that should know about it does know about it. More specifically, it should appear in the successor lists of its r predecessors, which will happen after a sequence of r stabilizations in which each predecessor learns about the node from its successor.

It is extremely important that Section VII-A showed that none of the operations or steps of operations discussed there can demote a node from principal to non-principal. In other words, the *only* action that can reduce the size of the set of principal nodes is failure of a principal node itself.

As a Chord network grows and matures, a significant fraction of its nodes will be members long enough to become principals. This means that the number of principal nodes is proportional to the size of the network; once the network is large enough there is no possibility that *SufficientPrincipals* will be violated. Section V presented the idea of global monitoring of small Chord networks as a way to implement initialization with $r+1$ principal nodes. It is a simple change to continue monitoring until the number of principal nodes has reached some multiple of r, after which the network is safe.

C. Queries have no circular waits

Section III-B explained how inter-node queries must be organized to maintain a shared-state abstraction. Sometimes a node must delay answering a query because it is waiting for the answer to its own query, which raises the specter of deadlock due to circular waiting.

Note that a rectify step only queries to see if a node is still alive, and does not read any of the node's state. Queries like these can always be answered immediately, so cannot cause waiting.

Note also that a join step requires a query, but no other node can be querying a node that has not joined yet. So the joining node, also, cannot be part of a circular wait.

This leaves queries due to the two stabilization steps, which are always directed to first successors or potential first successors. This means that, if there is a circular wait due to queries, it must encompass the entire ring. This possibility is sufficiently remote to ignore.

D. Proving progress

This section shows that in a network satisfying *Invariant*, if there are no join or fail operations, then eventually the network will become ideal—meaning that all its pointers are globally correct—and remain ideal.

Progress proceeds in a sequence of phases. In the first phase, all leading dead entries are removed from successor lists, so that every member's first successor is its best successor. Every time a member with a leading dead entry begins stabilization, it first executes a *StabilizeFromSuccessor* step, which will remove the leading dead entry. It will continue executing *StabilizeFromSuccessor* steps until all the leading dead entries are removed. Eventually all members will stabilize (this is an operating assumption), after which all leading dead entries will be removed from all successor lists.

Needless to say, these effective *StabilizeFromSuccessor* steps can be interleaved with other stabilize and rectify operations. However, rectify operations do not change successor lists. Even if a stabilization operation causes a node to change its successor, the steps are carefully designed so that the node will not change its successor to a dead entry. So, in the absence of failures, eventually all first successors will be best successors, and will remain so.

In the second phase, which can proceed concurrently with or subsequent to the first phase, all first successors and predecessors become correct. Let s be the current size of the network (number of members). This number is only changed by join and fail operations, and not by repair operations, so it remains the same throughout a repair-only phase as hypothesized by the theorem. The error of a first successor or of a predecessor is defined as 0 if it points to the globally correct member (in the sense of identifier order), 1 if it points to the next-most-correct member, . . . $s - 1$ if it points to the least globally correct member, and s if it points to a dead node.

Whenever there is a merge in the *bestSucc* or *splitBestSucc* graph (see Figure 6), there are two nodes *n1* and *n2* with successors merging at *n3*, and for some choice of symbolic names, *between [n1, n2, n3]*. There are three cases: (1) *n3.prdc* (the current predecessor of *n3*) is better (has a smaller error) than *n2*, meaning that *between [n2, n3.prdc, n3]*; (2) *n3.prdc* is *n2*; (3) *n3.prdc* is worse (has a larger error) than *n2*, meaning that *between [n3.prdc, n2, n3]*. In each of these three cases there is a sequence of enabled operations that will reduce the cumulative error in the network, as follows:

Case 1: Either *n1* or *n2* stabilizes, adopting *n3.prdc* as its successor and reducing the error of its successor. When the stabilizing node notifies *n3.prdc* and *n3.prdc* rectifies, it will change its predecessor pointer if and only if the change reduces error.

Case 2: n1 stabilizes, adopting *n2* as its successor and reducing the error of its successor. When *n1* notifies *n2* and *n2* rectifies, it will change its predecessor pointer if and only if the change reduces error.

Case 3: n2 stabilizes, which will not change its successor, but will have the effect of notifying *n3*. When *n3* rectifies, it will reduce the error of its predecessor by changing it to *n2*.

These cases show that, as long as there is a merge in the *bestSucc* graph, some operation or operations are enabled that will reduce the cumulative error of successor and predecessor pointers. Equally important, all operations are carefully designed so that a change never increases the error. At the same time, some of these operations will reduce the number of merges. For example, in Figure 6, let the merge of 24 and 33 at 35 be an example of Case 2. When 24 changes its successor to 33, which is not currently the successor of any node, the total number of merges is reduced.

As the network is finite, eventually there will be no merges in the *bestSucc* graph, which means that every node is a ring member. Because the ring is always ordered, the errors of all successors will be 0. The errors of all predecessors will also be 0, because whenever a successor pointer reaches its final value by stabilization, it notifies its successor. That node will update its predecessor pointer, and will never again change it, because no other candidate value can be superior. This is the completion of the second phase.

In the third and final phase, after all first successors are correct, the tails of all successor lists become correct (if they are not already). Let the error of a successor list of length r be defined as the length of its suffix beginning at the first entry that is not globally correct. At the beginning of this phase the maximum error of every successor list is $r - 1$, as the first entry is guaranteed correct.

Let *n2* be the successor of *n1*, and let the error of *n2*'s successor list be e. When *n1* stabilizes, the error of its successor list becomes $max(e - 1, 0)$, as it is adopting *n2*'s successor list, after first prepending a correct entry (*n2*) and dropping an entry at the end. Thus improvements to successor lists propagate backward in identifier order. In the worst case, after a backward chain of $r - 1$ stabilizations, the successor list of the last node of the chain will be globally correct. The correct list will continue propagating backward, leaving correctness in its wake. □

VIII. The Alloy model and bounded verification

As introduced in Section I, there is an Alloy model including specification of the operations, correctness properties, and assertions of the proof.[2] The reasons for using Alloy for this purpose can be found in [33].

The Alloy proof is direct rather than insightful. For example, there are assertions of all the theorems in Section VI. The Alloy Analyzer uses exhaustive enumeration to verify automatically that the theorems are true for all model instances up to some size bounds (see below). But unlike Section VI, this verification gives no insight into why the theorems are true.

The Alloy proof treats progress somewhat differently from Section VII-D. The model defines enabling predicates for all operation cases, where an enabling predicate is true if and only if a step or sequence of steps is enabled and will change the state of the network if it occurs. An assertion states that if a network is not ideal, some operation is enabled that will change the state. Another assertion states that if a network is ideal, no operation will change the state. Thus the model does not include the metric part of Section VII-D, which shows that every state change reduces error.

The model is and has been an indispensable part of this research, for two reasons: First, it protects against human error in the long informal proof. Second, it was a necessary tool for getting to the proof. Without long periods of model exploration, it would not have been possible to discover that the obvious invariants are not sufficient, nor to discover an invariant that is. Without the formal model and automated verification, one wastes too much time trying to prove assertions that are not true.

The model is analyzed for all instances with $r \leq 3$ and $n \leq 9$, where n is the size of the identifier/node space. For the largest instances, the possible number of nodes is more than twice the sufficient number of principal nodes.

It is worth noting what experimenting with models and bounds is like. With $r = 2$, many new counterexamples (to the current draft model) were found by increasing the number of nodes from 5 to 6, and no new counterexamples were ever found by increasing the number of nodes from 6 to 7 or more. No new counterexamples were ever found by increasing r from 2 to 3. This makes $r = 3$ and $n = 9$ seem more than adequate.

IX. Conclusion

The Chord ring-maintenance protocol is interesting in several ways. The design is extraordinary in its achievement of consistency and fault-tolerance with such simplicity, so little synchronization overhead, and such weak assumptions of fairness. Unlike most protocols, which work according to self-evident principles, it is quite difficult to understand how and why Chord works.

Now that our understanding of the protocol has a firm foundation, it should be possible to exploit this knowledge to improve peer-to-peer networks further. If these efficient networks become more robust, they may find a whole new generation of applications. For example, it may be possible to weaken the rather strong assumptions about failure detection. It is certainly possible to enhance security just by checking local invariants, and it may be possible to improve or verify more rigorously enhancements such as protection against malicious peers [19], [20], [21], key consistency and data consistency [22], range queries [23], and atomic access to replicated data [24], [25].

As a case study in practical verification, the Chord project illustrates the value of a variety of techniques. Simple analysis for bug-finding [9], fully automated verification through bounded model-checking [28], and informal mathematical proof, all had important roles to play.

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[2]http://www.research.att.com/~pamela > How to Make Chord Correct.

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