Homework 3 - DSP - project

submitted by

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In [14]:
```
import numpy as np
import matplotlib.pyplot as plt
import dsp_hw1_py as hw1
import dsp_hw2_py as hw2
```

New Basic Blocks - BThresh (Binary Threshold) and NGauss (Normalized Gaussian)

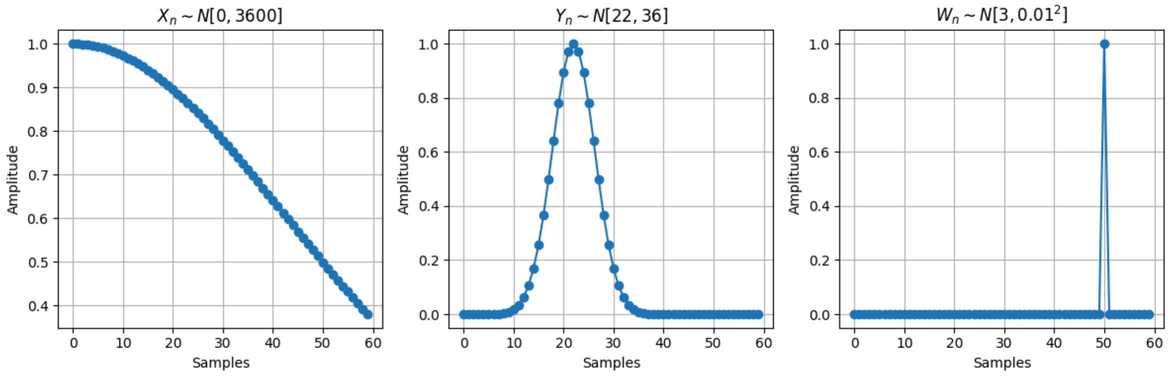
these are basic blocks that are created freely, the first returns a 1 if the threshold is exceeded and 0 otherwise,
the second returns a gaussian normalized such that its peak is at height 1.

In [15]:
```
def BThresh(signal, th):
    return hw1.Scalar(1,hw2.Threshold(signal,th)>0 )

print(BThresh([3,4,5,6],6))
print(BThresh(7,-5))
print(BThresh(2.67,3))
```

```
[0 0 0 1]
[1]
[0]
```

We can see that BThresh operates as expected via the three examples.

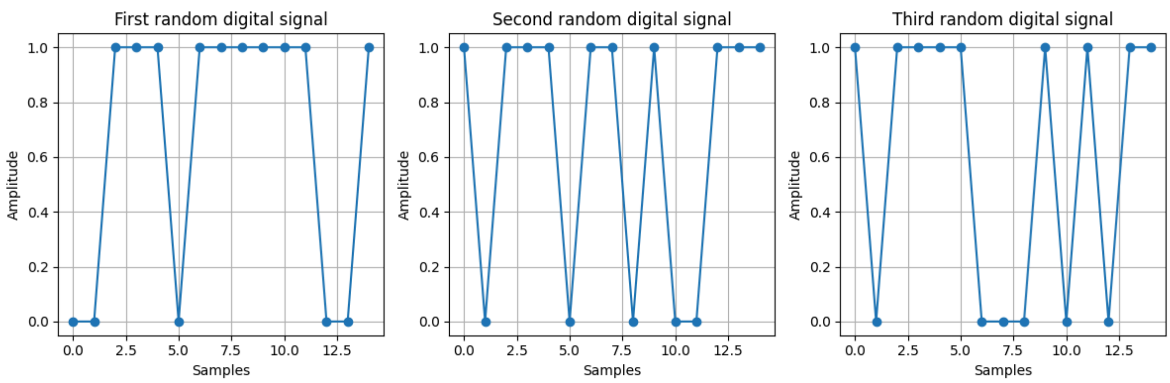
In [16]:
```
def GaussN(u: float, s: float, N: float) -> np.ndarray:
    n = np.arange(N)
    gauss_signal = np.exp(-((n - u)**2) / ((s+1e-6)**2) )  #find unnormalized ga
    gauss_signal = gauss_signal/( np.max(gauss_signal)+1e-6) # normalizing
    return gauss_signal

canonized_normal_dist = GaussN(0,60,60)
mean_shifted_high_variance = GaussN(22,6,60)
delta= GaussN(50,0.01,60)



hw1.plot_three_side_by_side(np.arange(60),canonized_normal_dist,np.arange(60),me
```



We can notice that really similar to Gauss in hw1 the examples maintain the expected characteristics, however their amplitude at the maximum is now 1.

New Composite Blocks (Blocks Built off of other blocks)

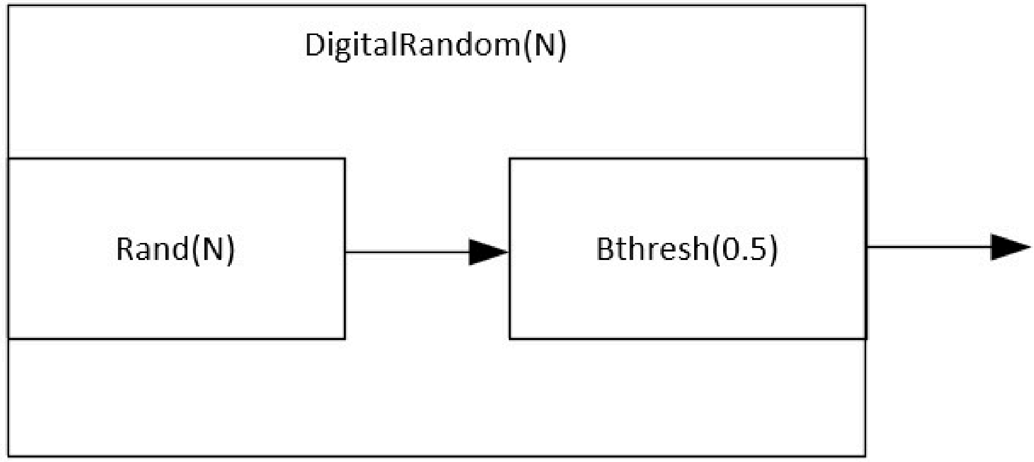
In [17]:
```
def DigitalRandom(N):
    x = BThresh(hw2.Rand(N),0.5)
    return x

x,y,z = DigitalRandom(15),DigitalRandom(15),DigitalRandom(15)

hw1.plot_three_side_by_side(np.arange(15),x,np.arange(15),y,np.arange(15),z,titl
```



DigitalRandom Explanation

- we notice how we created three vectors of length 15, we made a $\mathcal{U}[0,1]$ random vector and applied BThresh with a threshold of 1/2 to create them,
  the vectors are random but approximately 1/2 of the time they are 1 and the other half zero since $P(\mathcal{U}[0,1] > \frac{1}{2}) = 0.5$

DigitalRandom Block Diagram



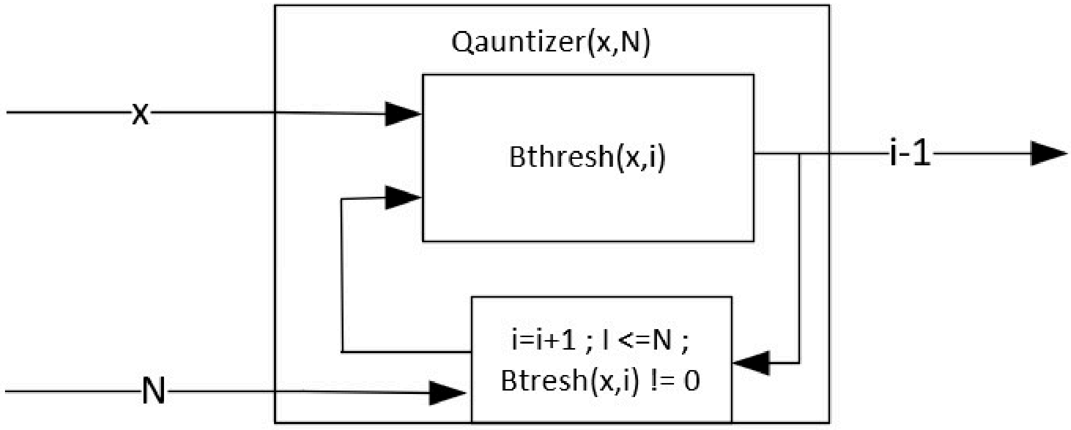
Quantizer Helper Block

- quantizes a random scalar float from 0 to N to an integer level by repeatedly applying Bthresh with a rising threshold, we essentially get a floor function
- Used just to roll a random natural number

In [18]:
```
def Quantizer(x,N):
    if x ==0:
        return 0
    for i in range(N+1):
        if BThresh(x,i) == 0:
            return i-1

## EXAMPLES

print(Quantizer(2.99, 7))
print(Quantizer(3.99, 4))
print(Quantizer(5.891263, 6))
```

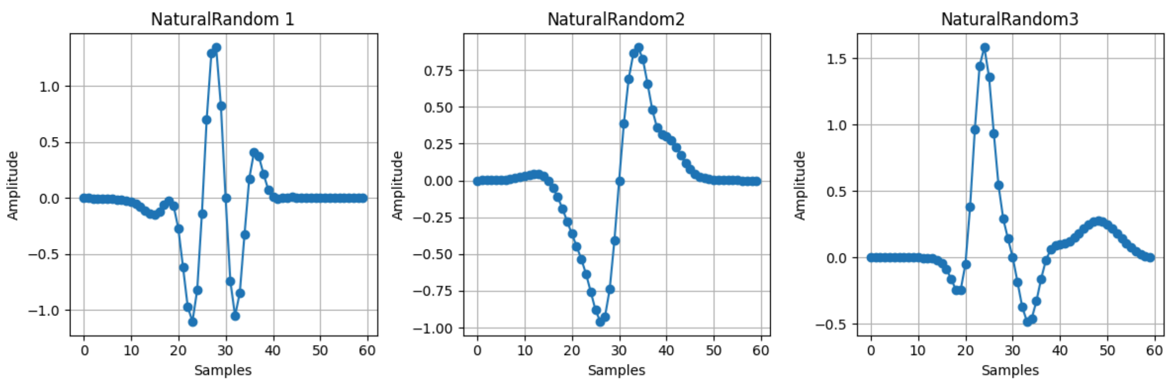
```
2
3
5
```

Quantizer Block Diagram



In [19]:

```
def NaturalRandom(N):
    x = np.zeros(N)
    for i in range(4):
        a_i = hw1.Scalar(hw1.Add(hw2.Rand(1),-0.5),4)
        f_i = hw1.Add(1, Quantizer(hw1.Prod(hw2.Rand(1),7),8))
        mi = hw1.WGN(1,N/2,N/10)
        si = hw1.WGN(1,N/10,3)
        g_i = GaussN(mi,si,N)
        x = hw1.Add(x,hw1.Prod(hw1.Scalar(hw1.Sine(f_i,N,0),a_i),g_i))
    return x

hw1.plot_three_side_by_side(np.arange(60),NaturalRandom(60),np.arange(60),Natura
```

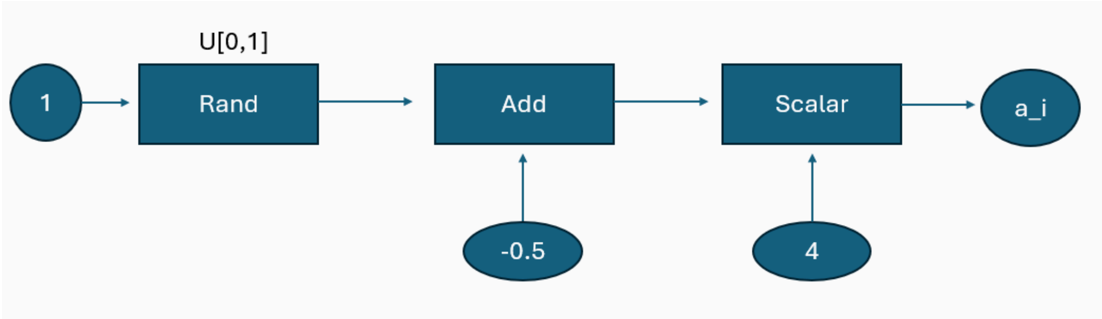


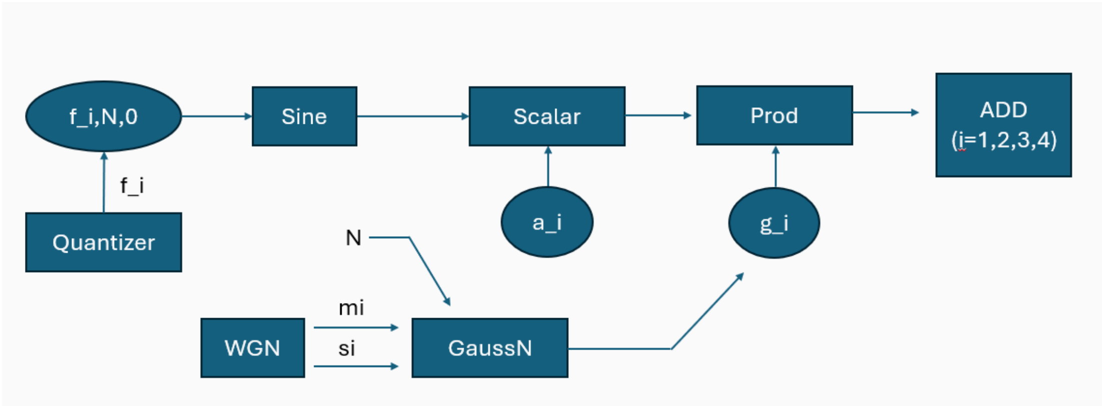
NaturalRandom Explanation

We realize the formula that adds four localized sines multiplied by normalized gaussians - the results exhibit sine-like behaviour localized in time,
we notice that the amplitudes are around between -2 and 2 which is as expected since those are the a_i coefficient ranges

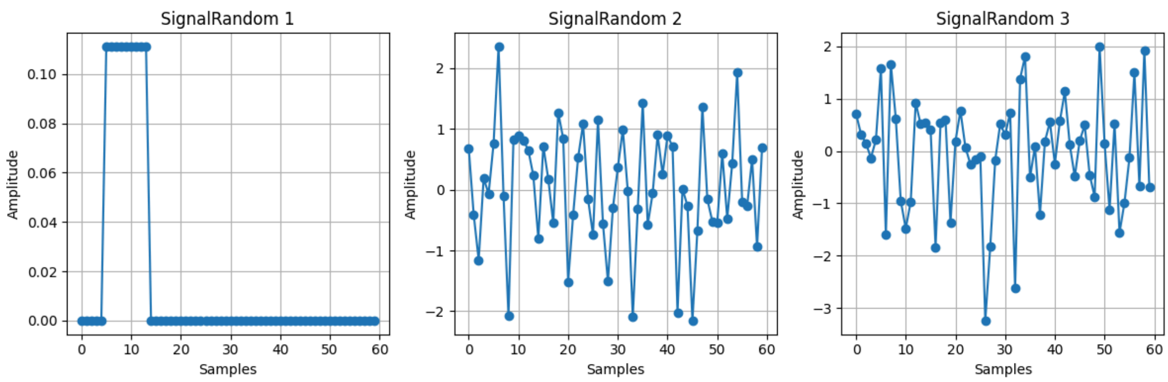
NaturalRandom Block Diagram

- Generating $a_i \sim \mathcal{U}[-2, 2]$ by shifting and scaling a uniform R.V





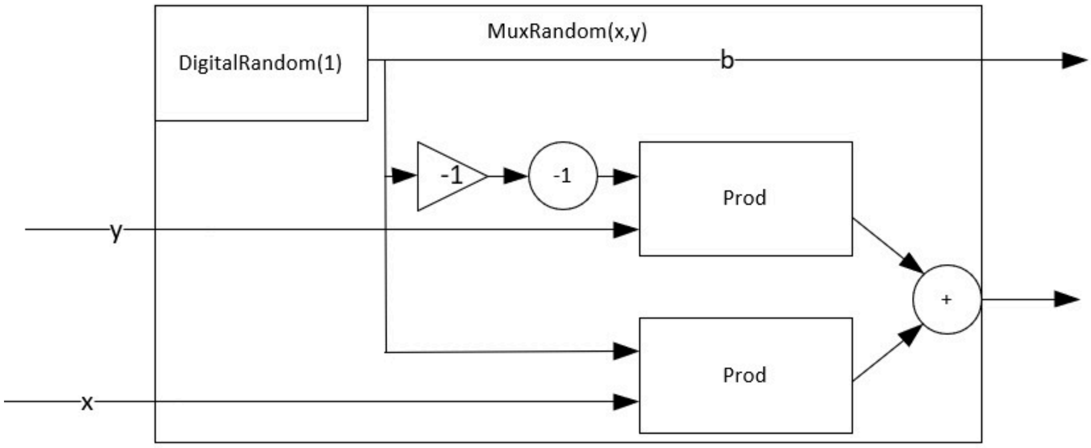
In [20]:
```
def MuxRandom(x,y):
    b = DigitalRandom(1)
    return hw1.Add(hw1.Prod(y,b),hw1.Prod(x,1-b)), b
#EXAMPLE
# print(MuxRandom(1,2))
# print(MuxRandom(1,2))
# print(MuxRandom(1,2))
x,c = MuxRandom(hw1.Sine(2,60,0),hw1.Rect(5,14,60))
y,d = MuxRandom(DigitalRandom(60),hw1.WGN(60,0,1))
z,f = MuxRandom(hw1.WGN(60,0,1),hw1.Rect(5,14,60))
hw1.plot_three_side_by_side(np.arange(60),x,np.arange(60),y,np.arange(60),z,titl


print(c,d,f)
```




```
[1] [1] [0]
```

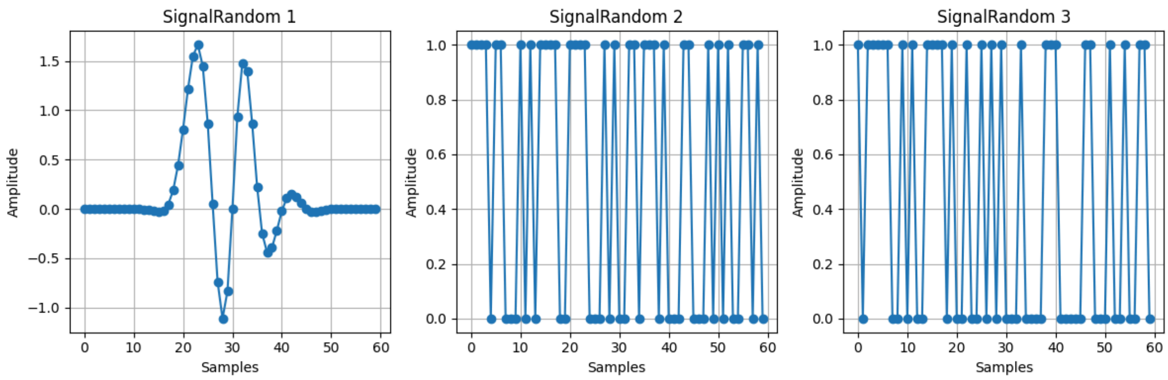
Mux Random Explanation

Using the bernoulli RV we generate via SignalRandom(1) and 1-that bernoulli R.V we can have a set of complimentary binary random variables,
multiplying our signals by them creates the Random Mux.



SignalRandom

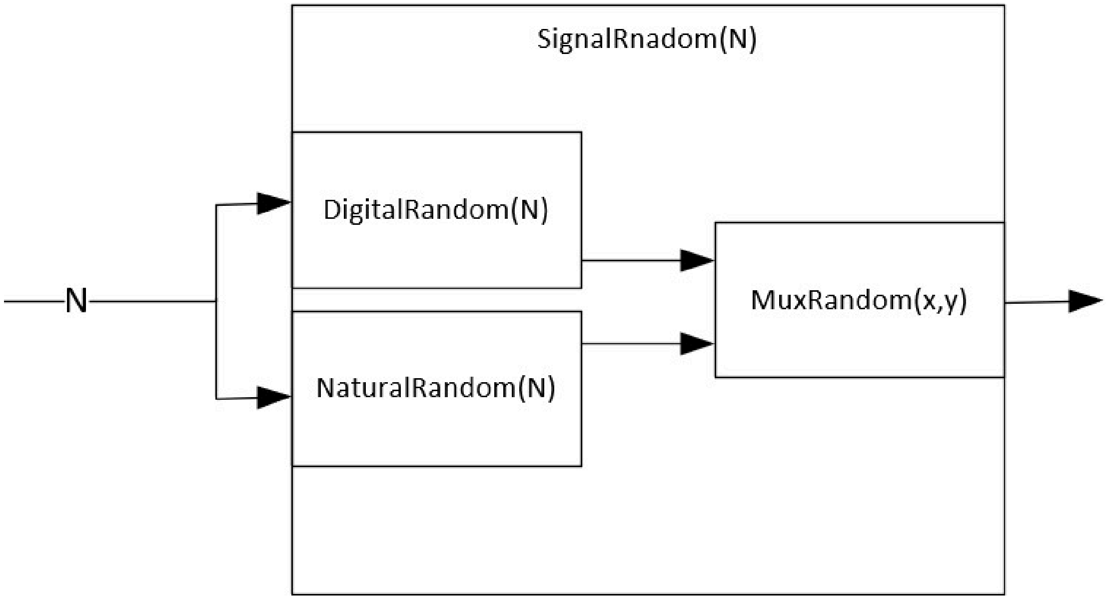
In [21]:
```
def SignalRandom(N):
    return MuxRandom(DigitalRandom(N),NaturalRandom(N))
x, c= SignalRandom(60)
y,d = SignalRandom(60)
z,e = SignalRandom(60)
hw1.plot_three_side_by_side(np.arange(60),x,np.arange(60),y,np.arange(60),z,titl


print(c,d,e)
```




```
[1] [0] [0]
```

SignalRandom Explanation

as expected, we generated at random either a digital signal or an analogue signal in either instance.
every slot upon re-running the program is either digial or analogue with probability 1/2 due to the signalMux block.
This is a private instance of the Mux Block

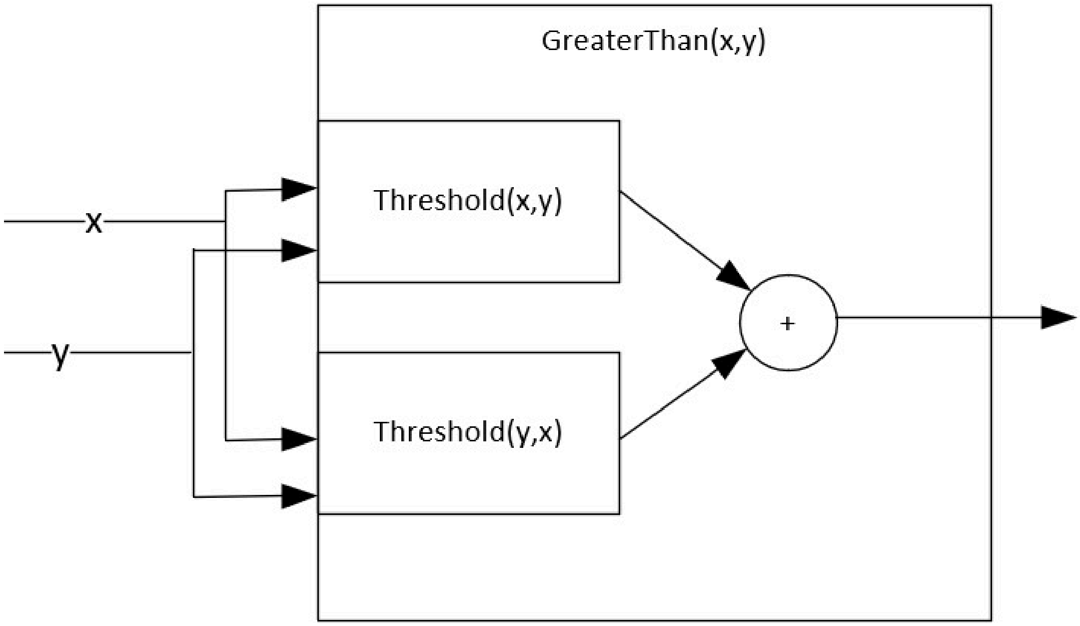


In [22]:
```
def GreaterThan(x,y):
    return hw1.Add(hw1.Prod(BThresh(x,y),x),hw1.Prod(BThresh(y,x),y))

### EXAMPLES
print(GreaterThan(5,6))
print(GreaterThan(-3,8.3285))
print(GreaterThan(0,-13))
```

```
[6]
[8.3285]
[0]
```

GreaterThan Explanation

we can see how we get in the outputs the bigger numbers as expected, our method of achieveing that is with a use of ThreshB and Prod.
ThreshB acts as a mask, such that only the greater number is multiplied by 1 (the one who isn't wiped out by the threshold), and the smaller number is zeroed out
we can see it works on whole, real and negative numbers



Part 2 & 3 - Binary Classifier using if Statements

Explanation for Binary Classifier

We can note that due to the characterization of the noise and the signal, our signals have different means on average - specifically, a natural signal
is centered around 0 whereas a digital signal's average appears to grow with the number of samples. for a 30-sample signal, we can classify based on the mean -
if the mean is greater than 0.2 we can assume the signal is most likely digital and otherwise natural. - so if the mean is 0.2 and above, we want c to be 0.5 and above. so let's call the return value r and make it the mean + 0.3 to make the 0.2 bound.

$$r = \bar{X} + 0.3$$

now lets bind r and return it:

$$\tilde{x} \triangleq \text{GreaterThan}(0, r)$$

(binding the mean from below) Afterwards we just need to reverse the prediction (if the output is greater than 1/2 we predict natural and not digital)

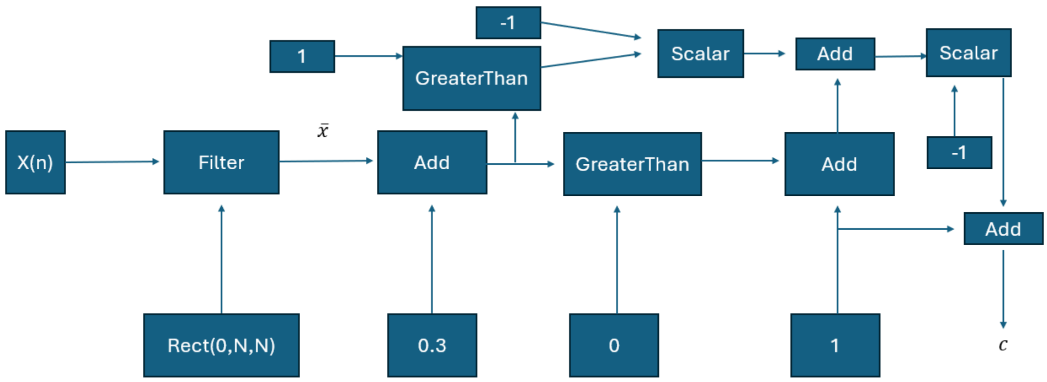
$$c \triangleq 1 - (\tilde{x} + 1 - \text{GreaterThan}(1, \tilde{x}))$$

In [243…

```
def BinaryClassifier(x: np.ndarray):
    N=len(x) # assume 30
    ave_sig = hw1.Filter([1], hw1.Rect(0,N, N),x)
```

```
    r = hw1.Add(ave_sig, 0.3)
    x_tilde = GreaterThan(r,0)
    c = hw1.Add(1,hw1.Scalar(-1,hw1.Add(hw1.Add(x_tilde,1),hw1.Scalar(-1, Greate
    return c
```

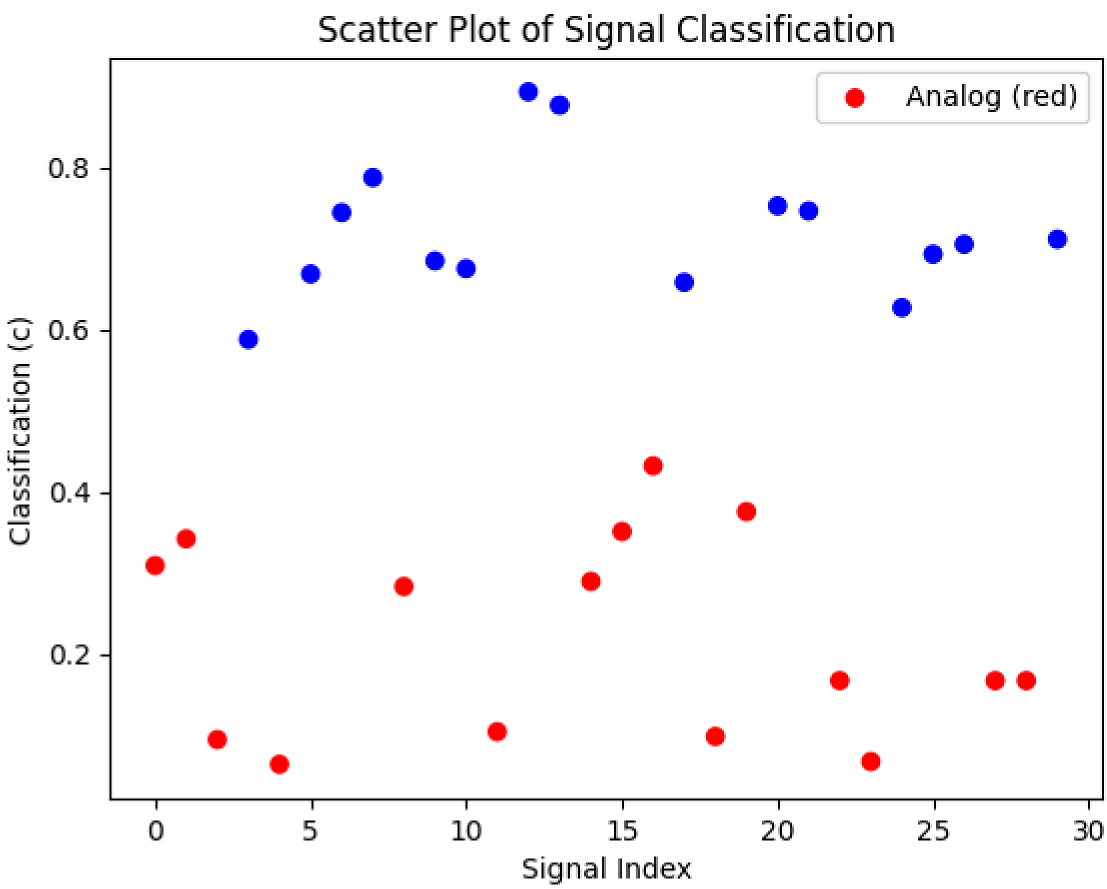
BinaryClassifier Block Diagram



In [245…

```
labels = []
classification = []
for i in range(30):
    signal,label =SignalRandom(30)
    labels.append(label)
    if i <=9:
        signal = hw1.Add(signal,hw1.Scalar(0.1,hw1.WGN(30,0,1)))
    elif i<=19 :
        signal = hw1.Add(signal,hw1.Scalar(0.5,hw1.WGN(30,0,1)))
    classification.append(BinaryClassifier(signal))

diff=0
for i in range(30):
    if labels[i]==1 and BThresh(classification[i],0.5) ==0 :
       diff+=1
    if labels[i]==0 and BThresh(classification[i],0.5) ==1:
       diff+=1

print(diff)

# Prepare data for scatter plot
x_values = np.arange(30)  # Index of signal
y_values = classification
# Create scatter plot
colors = ['blue' if label ==1  else 'red' for label in labels]

plt.scatter(x_values, y_values, c=colors, label="Classification Results")
plt.xlabel("Signal Index")
plt.ylabel("Classification (c)")
plt.title("Scatter Plot of Signal Classification")
plt.legend([ "Analog (red)","Digital (blue)"])
plt.show()
```

```
0
```



Illegal Part 2 & 3- Binary Classifier using if Statements

here because we're still proud of it

Helper Function: Absquared

In [24]:
```
def Absquared(z):
    return np.real(z*hw2.Conj(z)) #the real isn't necessary, it's just there to

print(Absquared([[3-1j,10-1j],[1j,0]]))
print(np.abs([[3-1j,1j],[1j,0]])**2)
```

```
[[ 10. 101.]
 [  1.   0.]]
[[10.  1.]
 [ 1.  0.]]
```

In [25]:
```
def illegal_BinaryClassifier(x: np.ndarray):
    N=len(x) # assume 30
    sum_sig = hw1.Filter([1], hw1.Scalar(hw1.Rect(0,N, N),N),x)
    if not (GreaterThan(sum_sig, 0)):#the signal is Natural, really low values
        return 1
    if GreaterThan(sum_sig, 16) == sum_sig:
        return 0 #digital, high values
    x_stft = Absquared(hw2.STFT(x,10))
```

```
    x_stft_rowsums=[]
    for i in range(10):
        x_stft_rowsums.append(hw1.Filter([1], hw1.Scalar(hw1.Rect(0,3,3),3),x_st
    high_freq = hw1.Filter([1],hw1.Scalar(hw1.Rect(0,5,10),5),x_stft_rowsums)
    freq_diff = hw1.Add(hw1.Filter([1],hw1.Scalar(hw1.Rect(0,10,10),10),x_stft_r
    if GreaterThan(freq_diff,35)==freq_diff: #digital
        return 0
    else:
        return 1
```

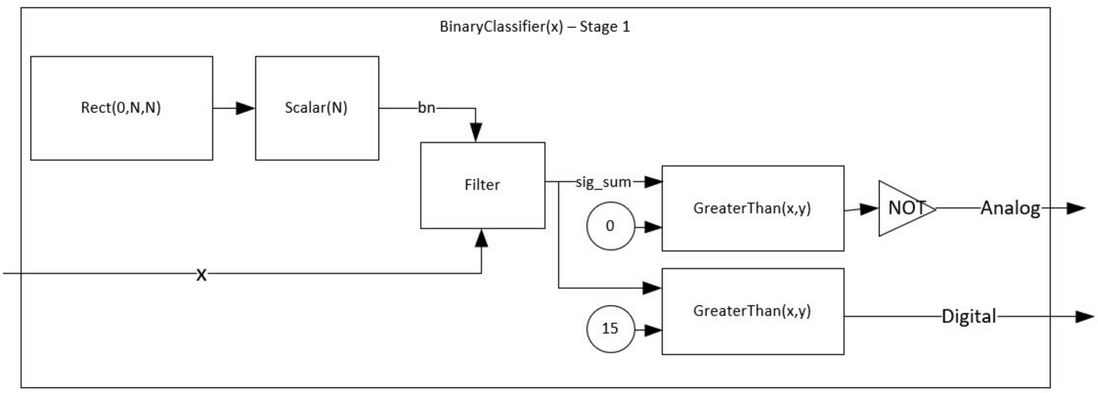
Explanation for Binary classifier

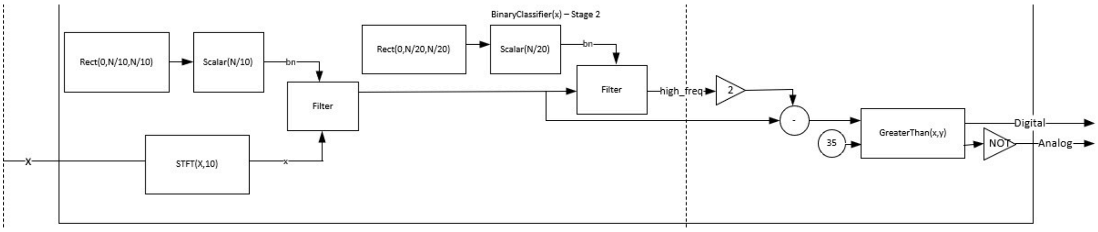
Using the code section commented out above we noticed the following:

- the average of the analogue signals samples tends to be centered around zero, the sum of samples is small, and is sometimes negative,
  whereas the digital signal is always nonzero- and is positive with every sample being a bernoulli RV with mean 1/2, this attribute carries over
  largely when we add the noise since with mean zero it tends to cancel itself when adding up the samples.
  We leveraged that so that our first classification barrier is whether the sum is negative (natural) or large (binary).

- if we didn't get a definitive conclusion, we looked at the squared absolute values of the STFT - we noticed that if the difference between the high and low frequency components is greater than 35 that's indicative of the signal being digital, otherwise its more likely to be natural.

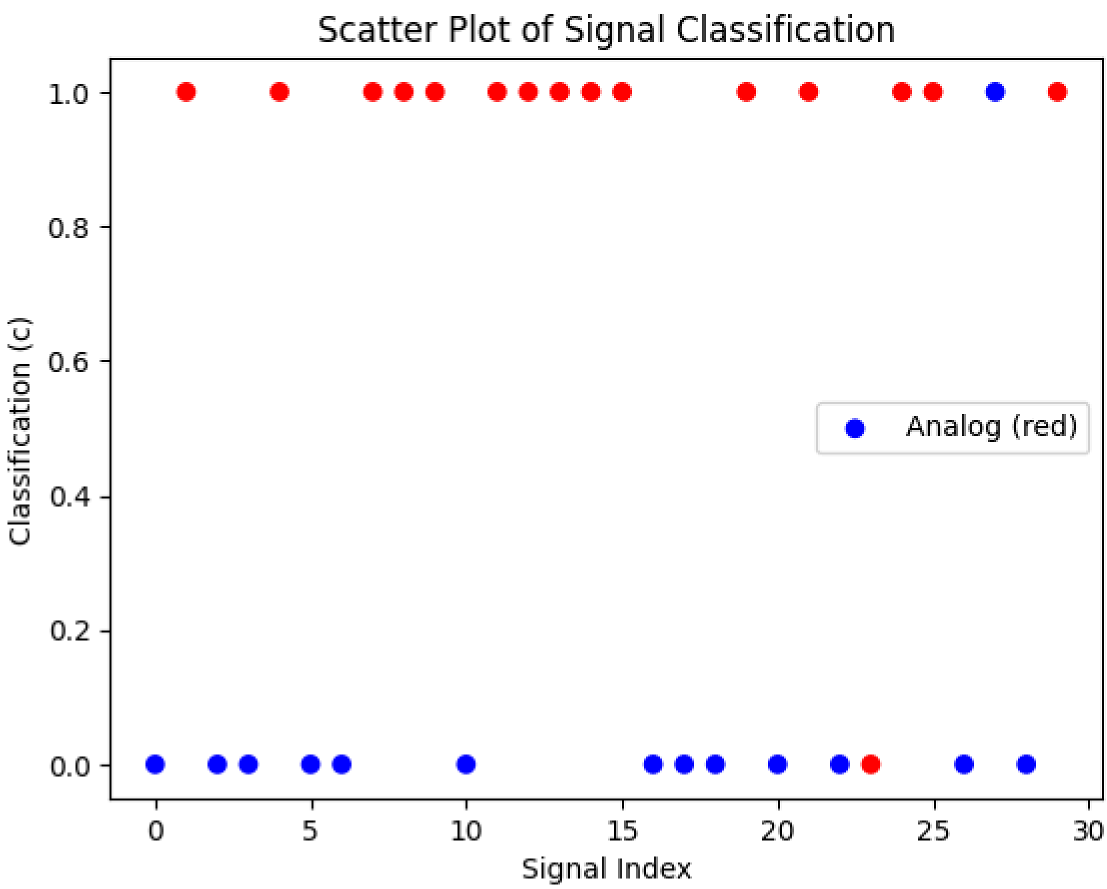
we realized this two-stage filtering function in blocks.





In [26]:
```python
labels = []
classification = []
for i in range(30):
    signal,label =SignalRandom(30)
    labels.append(label)
    if i <=9:
        signal = hw1.Add(signal,hw1.Scalar(0.1,hw1.WGN(30,0,1)))
    elif i<=19 :
        signal = hw1.Add(signal,hw1.Scalar(0.5,hw1.WGN(30,0,1)))
    classification.append(illegal_BinaryClassifier(signal))

diff=0
for i in range(30):
    diff = diff + sum(np.abs(labels[i]-classification[i]))
print(diff)

# Prepare data for scatter plot
x_values = np.arange(30)  # Index of signals
y_values = classification
# Create scatter plot
colors = ['blue' if label == 0 else 'red' for label in labels]

plt.scatter(x_values, y_values, c=colors, label="Classification Results")
plt.xlabel("Signal Index")
plt.ylabel("Classification (c)")
plt.title("Scatter Plot of Signal Classification")
plt.legend([ "Analog (red)","Digital (blue)"])
plt.show()
```

2


Scatter Plot of Signal Classification
Classification (c)
1.0
0.8
0.6
0.4
0.2
0.0
Analog (red)
0
5
10
15
20
25
30
Signal Index