Mechanics

Part 1: Motion

To describe motion we need some key terms.

Key terms

- **Velocity**
  Velocity measures the rate of change of displacement. It is measure in ms^{-1}
  It has the symbol v.
  It is the vector quantity of speed. Speed is the magnetude of velocity.

$$V = \frac{\Delta S}{\Delta T}$$

- **Displacement**
  Displacement measures how far a point if from an origin. It is measured in metres and is a vector quantity . *it has the symbol,* s
- **Average velocity**
  Total distance covered divided by time elapsed
- **Instantaneous velocity**
  Rate of change of displacement at a particular instant
- **Acceleration**
  The rate of change of velocity the symal is a.
  The unit it ms^{-2}. It is a vector

$$Acceleration = \frac{\Delta V}{\Delta T}$$

$$V_{final} - V_{initial} = \Delta V$$

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Displacement / Time graph

- A **curved** line means that velocity is changing i.e accelerating
- A **steeper** line indicates a higher velocity
- A **negative** gradient means that the object is going in the opposite direction
- A **zero** gradient means that the object is stationary

Velocity / Time graph

- The gradient of a $\frac{V}{T}$ graph is $\frac{\Delta V}{\Delta T}$ which is the acceration.
- The area under a $v\ t$ graph gives us the total displacement
- A zero gradient means velocity is constant
- A negative gradient means either: It is decelerating in a $+$ direction or it is accelerating in a $-$ direction

Kinematic equations for uniformly accelerated motions

Using SUVAT equations, a stopwatch and a ball to find the accereration due to gravity

1. Measure the height (gives us displacement)
2. Drop ball and measure time taken to travel distance
3. We know the initial velocity (u) is zero
4. Calculate accereration (a) using SUVAT equation

Measured distance, $s = 4.53 \pm 0.05m \pm 0.011\%$

Measured time

```
0.84
0.85
0.88
1.07
1.05
1.01
0.93
0.95
0.85
1.00
0.86
0.95
```

$t = 0.937 \quad uncertainty = \frac{1.07 - 0.84}{2} = 0.115 \pm 0.11$
$t_{average} = 0.9 \pm 0.1s$

$a = \frac{2s}{t^2} = \frac{2 \times 4.53}{0.9^2} = 11.2ms^{-2}$
$a_{min} = \frac{2 \times 4.48}{1.0^2} = 8.96ms^{-2}$
$a_m ax = \frac{2 \times 4.58}{0.8^2} \qquad uncertainty = \frac{a_{max} a_{min}}{2} = 2.67 \approx 3$
$then\ a = 11 \pm 3ms^{-2}$

Using rule for propagating uncertainties

$$a = \frac{2 \times s}{t^2} \text{absolute uncertainty in 2 s} = 0.05 + 0.05$$

So real uncertainty in $2s = \frac{0.1}{9.06} = 0.011$

Total real uncertainty in $0.11 + 0.11 + 0.011 = 0.231$ Total absolute uncertainty is $0.231 \times (\text{best estimate of a}) = 0.231 \times 11.2 = 2.5872 \approx 3$

Finding acceleration of fan cart

Variables

- $u = 0$
- $t =$ measured
- $s = 1.0m$
- $a = ?$

What equation will we use?

$$S = ut + \frac{1}{2}at^2$$

What should we measure?

(slow)Time$=2.14\ 2.06\ 2.08$ Average $2.09s \pm 0.04$
(fast)Time$=1.81\ 1.86\ 1.76$ Average 1.81 ± 0.05

Rearrange & solve

Slow $a = 0.458ms^{-2}$ $\quad \frac{2\times1}{2.09^2}$ $\quad a_{max} = 0.0499$ $\quad \frac{2\times1.05}{2.05^2}$ $\quad a_{min} = 0.419$ $\quad \frac{2.095}{2.13^2}$

$$uncertainty(a) = \frac{0.499 - 0.419}{2} = 0.04$$

$$a = 0.46 \pm 0.04ms^{-2}$$

Fast

$a = \frac{2\times1}{1.81^2} = 0.61ms^{-2}$

$a_{min} = \frac{2\times1.05}{1.76^2} = 0.549ms^{-2}$

$a_{max} = \frac{2\times0.95}{1.81^2} = 0.677ms^{-2}$

$$\frac{a_{max} - a_{min}}{2} = 0.06$$

$$a = 0.61 \pm 0.06ms^2$$

Relative Motion 2D

$$V_{CrB} = V_{CrG} - V_{BrG}$$

1. **A plane** has an airpseed of $200ms^{-1}(V_{PrA})$ north (000).
   The wind has a relative velocity to the ground of $40ms^{-1}$ east (090)
   What is the planes velocity relative to the ground?

V_{PrA}
V_{ArG}
V_{PrG}

$$V_{PrA} = V_{PrG} - V_{ArP}$$

$$V_{PrG} = V_{PrA} + V_{ArG}$$

$$Magnitude\ of\ V_{PrG} = \sqrt{200^2 + 40^2} = 203.96$$

$$Bearing\ \theta = tan_1\left(\frac{40}{200}\right) = 11.3^{\circ}$$

Bearing in 011.3

Projectile Motion

An object is considered a projectile if the only force acting on it is the force due to gravity.
The key to working with projectiles is recognising that vertical motion can be considered separately from horizontal motion.

Vertically, projectiles accelerate at $-9.81ms^{-2}$ (due to the force due to gravity)

Horizontally, projectiles travel at a constant velocity (as there are NO horizontal forces).

e.g

Juliet has to throw the keys to the front door 15 metres horizontally. She wants to throw the straight out.

1. For how long will they be in the air?
2. Hence how fast will they need to be thrown to travel 15 m horizontally

3. Vertically $S = 20M$ $a = -9.8ms^{-2}$ $u = 0$ $t = ?$

$$ut + \frac{1}{2}at^2 t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times (-20)}{-9.8}} = 2.020s$$

4. $V = \frac{d}{t}$ when v is constant
   She wants keys to go $15m$ in $2.02s$
   so $v = \frac{15}{2.02} = 7.4ms^{-1}$

Solid Friction

This is a force that reists motion.
There are two types:

> Static friction: which is the force that prevents a stationary object from sliding across a surface.

> Dynamic/Kinetic friction: This is the force that tries to slow down an object sliding across as surface.

Friction is due to forces caused by the roughness of the two surfaces in contact
The friction force depends on two things, one of them is the nature of the surfaces.

Two factors affect the size of static friction force.

1. The normal reaction force F_N
2. The nature of the two surfaces

$$F_F \leq \mu_s F_n$$

This expression ifs the biggest friction force that these two surfaces can provide when pushed together by the force F_N.
If the the force pushing the object is smaller than $\mu_s F_N$, then the friction will be able to oppose it. If not the object will no longer be in equilibrium

F_F will oppose F_{push} provided that $F_{push} < \mu_s F_N$

When the object starts moving the equation for the friction force becomes

$$F_{friction} = \mu_D F_N$$

$\mu_d < \mu_s$ For any surface

Fores does not depend on surface area in contact.

####### 2.3 Work, Energy and Power

Energy

Kinetic Energy

energy due to motion

$$W = F \times s$$

A force is applied to a mass casuing it to accelerate. How much work is done?

$$W = ma \times \frac{1}{2}at^2$$

$$= \frac{1}{2} \times ma^2t^2$$

$$= \frac{1}{2}m\left(\frac{\Delta V}{\Delta t}\right)^2 t^2 = \frac{1}{2}mv_f^2$$

By the work-energy equivalence we see that energy of a body in motion at speed v mist be $K = \frac{1}{2}mv^2$

Gravitational potential Energy

Energy due to the objects position in a gravitational field.

How much work is need to move a mass, m, and displacement, s in a gravitational field of strength g?

$W = Fs$ Assume we moved it at a constant speed and force appllied to lift it was upwards.

$$W = mg\Delta h$$

$$\Delta E_p = mg\Delta h$$

Conservation of Energy

- In an isolated system the total amount of energy remains constant.
- Energy cannot be created or destroyed it can only be transformed from one object to another or be transformed from one type to another

Elastic potential Energy

Energy stored in stretched of deformed objects.

Hookes laws says that the extension of a spring is proportional to the force is applies and force is in opposite direction to the extension. i.e $F = -kx$
Where.

- $F =$ the tension force (N)
- $k =$ spring constant (Nm^{-1})
- $x =$ extension (m)
  To figure out how much energy is stored in a stretched spring we work out how much work is done to stretch it.
  The work done is the area under an $F - s$ graph
  Area of triangle is 1/2base x Height

$$W = \frac{1}{2} \times x \times kx = \frac{1}{2}kx^2$$

  Work = energy transferred

$$E_p = \frac{1}{2}kx^2$$

e.g An eraser is launched vertically by used a bent ruler.

$$F = 3.4N$$

$$x = -0.05m \quad F = -kx$$

$$k = \frac{F}{-x} = \frac{3.4}{-0.05} = 68Nm^{-1}$$

How high will it go if ruler tip is bent 30cm down?

$$E_{initial} = E_p = \frac{1}{2}kx^2 \quad = \frac{1}{2} \times 68 \times 0.03^2 \quad = 0.0306$$

$$E_f = E_{pg} = mg\Delta h \qquad M_{eraser} = 0.015kg$$

$$= mg(h + 0.03)$$

By conservation of energy $E_{PE}lost = E_{PG}gained$

$$0.0306 = 0.015 \times 9.81 \times (h + 0.03)$$

$$\frac{0.0306}{0.14715} - 0.03 = 0.17795$$

or about 18cm

Power

> Rate of energy transfer
>
> $$P = \frac{\Delta E}{\Delta t}$$

> Work done per unit time
>
> $$P = \frac{W}{\Delta t}$$

It is measured in watts (W) which is equivalent to joules per second (Js^{-1})

Using our second definition, we can substitute in the expression for work:
$W = Fcos\theta$

Thus the equation for power can be written as

$$P = \frac{Fscos\theta}{\Delta t}$$

If it is moving at a constant speed then we can replace $\frac{S}{\Delta t}$ with v
So:

$$P = Fvcos\theta$$

Efficiency

$$\frac{Useful\ Energy\ Output}{Total\ Energy\ Input} = e$$

$$\frac{Power\ out}{Power\ in} = e$$

A ball is launched vertially. Measure height and determine launch psed

GPE gained = KE lost

$$m \times g \times \Delta h = \frac{1}{2} \times v^2$$

$$g \times \Delta h = \frac{1}{2}v^2$$

$$2g \times \Delta h = v^2$$

$$v = \sqrt{2g\Delta h}$$

$$= \sqrt{2 \times 9.81 \times 0.89}$$

Mass is 17g , or 0.017kg - Use this to find the spring constant

$$E_{PE} lost = E_{PG}\ gained$$

$\frac{1}{2}kx^2 = mg\Delta h$ | X measured to be 0.073m

$$\frac{1}{2}k = \frac{mg\Delta g}{x^2}$$

$$k = \frac{2mg\Delta h}{x^2}$$

$$= \frac{2 \times 0.017 \times 9.81 \times 0.89}{0.073^2}$$

$$= 55.70474Nm^{-1} \approx 56Nm^{-1}$$

Momentum

Can be thought of “as hard it is to stop a thing that is moving”

It is defined as the product of the mass of the object and it’s velocity.

$$\vec{P} = m \times \vec{v}$$

Where

- P is momentum (vector)
- m is mass in kg
- v is velocity in ms^{-1}

Conservation of momentum

The total momentum of the system will remain constant unless an outside force acts on the system.

Explosions experiment (conservation of momentum)

$$P_{i1} = 0 \qquad P_{i2} = 0 \qquad P_{itotal} = 0$$

$$P_{f1} = m_1 \times v_1 P_{f2} = m_2 \times v_2$$

$$P_{ftotal} = P_{f1} + P_{f2}$$

$$P_{f1} = 0.50 \times 0.49 = 0.245 kgms^{-1}$$

$$P_{f2} = 0.50 \times 0.50 = 0.25 kgms^{-1}$$

$$P_{ftotal} \approx 0 kgms^{-1}$$

No external fores applied to two trolley system so momentum was conserved, i.e $P_{itotal} = P_{ftotal}$

Impulse

Impulse is another word for change in momentum.

> Impulse $= F\Delta t = \Delta p$

If we want to stop something that is moving, we need to produce an impulse by applying a force over an amount of time

Example

To stop a falling eg

1. An egg falls 2.5m to the ground in the first case and into Ben's hands in the second case.
2. In both cases the momentum form the same inital value to zero, so Δpm the impulse is that same. In the first case Δt is small, so F needs to be big and the eggs breaks due to the large acceleration

In the second the fall was cushioned so the force needed to achieve the same impulse is smaller

For these two cases, we can graph force vs time

We can use our definition of impulse to represent newton's second law more accurately.

$$\Delta p = F\Delta t$$

$$F = \frac{\Delta p}{\Delta t}$$

Topic Six

Describing Circular Motion

1. Speed
2. Angular Displacement
3. The radian
4. Angular and transitional motion
5. Acceleration in circular motion.

The circumference of the circlle, 2 πr is the distance covered per rotation. THe time to complete a rotation is the time period, T.
The average speed ,v, is given by $v = \frac{d}{t} = \frac{2\pi r}{T}$
A rotating rigid object will spen through a given angle at a given time, we can define the angular speed ω.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\frac{360^{\circ}}{T}$$

i.e the angle through which it spins is divided by the time.

Measuring angular displacement in radians

A radian is an angle where the arc length is equal to the radius

1 radian is angle for which arc-length is exactly equal to the radius

$$\theta = 2\pi \ radians$$

We can convert between arc length and angle as follows:
$d = \theta \times r$

We can also correct between linear speed,v, and angular speed ω in (radians^{-1})

$$V = \frac{\Delta d}{\Delta t} = \frac{\Delta \theta \times r}{\Delta t} = \omega \times r$$

Converting between radians & degrees.

- 2π radians = 360° degrees
- 1 radian = $\frac{360}{2\pi}$ degrees
- $\frac{2\pi}{360}$ radians = 1 degrees

Speed and acceleration in circular motion

The velocity of an object undergoing circular motion is tangent to the circle.

Because the velocity is always changing direction the object must be accelerating.

$$a = \frac{\Delta V}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

In this situation we can find Δv : $\Delta v = v_f - v_i$

We can see that Δv is towards the center of the circle so $\frac{\Delta v}{\Delta t}$ is towards the centre of the circle.

Centripetal acceleration

How can an object moving in a a circle be accelerating towards the centre and not get any closer to the centre?

If objects were not accelerating, it's velocity would not change, it would continue in the same direction at the same speed moving further and further away from the centre of the circle.

To keep the object on the circular path there needs to be acceleration at a right angle to the velocity (towards the centre of the circle) causing the direction of velocity to change.

Quantifying Centripetal acceleration & force

1. How does speed affect a_c(centripetal acceleration)

Increasing the speed , v of the object increases centripetal acceleration

2. how does radius affect a_c ?
Increasing the radius, r, of the obejcts path decreases the centripetal acceleration required for the object to remain on that path (keeping speed constant)s

a_c is a centripetal acceleration
v is speed
r is radians of path (m)

Equation for a_c

$$a_c = \frac{v^2}{r}$$

Note the v can be written as

$$v = \frac{2\pi r}{T}$$

where T is time for a complete revolution

Centripetal Force

Since objects in circular motion are accelerating there wont be a **net** force acting on them. We call this the centripetal force: F

- ω is rotaional velocity
- F_c is centripetal force
- a_c is centripetal acceleration
- r is the radius of the circle
- m is mass

$$F_c = ma_c = \frac{mv^2}{r} \quad = \omega r = m \quad m\omega^2 r$$

Using the apparatus we obtained centripetal force vs angular velocity

The linear relationship (with approximately zero y intercept) indicates that $Fproportional\omega^2 f = k\omega^2$\$

The gradient (k) seems to depend both on radius **and** mass. This is constistent with our equation:

$$F_c = m\omega^2 r$$

$$a_c = \frac{v^2}{r} = g \qquad 9.81ms^{-2}$$

Circular motion with two forces

e.g A real conical pendulum

- String length = 1.5m
- Radius or orbit, r = 0.5m
- Mass, m = 0.1kg

Using we can find theta:

$$\theta = sin^{-1} = 19.471^\circ$$

Now we know θ and adjacent side, we can find F_{net} (opposite side)

$$tan\theta = \frac{F_{net}}{0.981}$$

$$F_{net} = 0.981 \times tan(19.471) = 0.3468N$$

Is this the same as F_c for this motion?

$$F_c = ma_c = \frac{mv^2}{r} \quad r = 0.5 \quad v = \frac{2\pi r}{T}$$

$$ST_{measured} = 12.7\ 11.91\ 11.99\ 12.15\ 11.78$$

$$ST_{average} = 12s$$

$$T = 2.4s$$

$$V = \frac{2\pi \times 0.5}{2.4} = 1.3ms^{-1} \qquad F = \frac{mv^2}{r} = \frac{0.1 \times 1.3^3}{0.5} = 0.338N$$

=

Vertical Centripetal Motion

a_{top} is towards the centre (downwards) it is equal to $\frac{v^2}{r}$
a_{bottom} is towards the centre (upwards) it is equal to $\frac{v^2}{r^2}$

The net force at the bottom is equal to t the net force at the top is equal to $\frac{mv^2}{r}$

if $a_c = g$ then the object will just be able to safely complete the loop If we increase V. so that $a_c > g$ then the object will safely make it around the loop
If we decrease v, so that $A - c < g$ then the object will fall out of the loop.

So from * we see tht the minimum speed at the top of looop is given by $\frac{V_{min^2}}{r} = g$

Examples \ Question

What is the minimum accelerating the coaster must have so that it doesn’t fall of the top of the loop

$$a_c = \frac{v^2}{r} = g \qquad 9.81ms^{-2}$$

Calculate the minimum speed the coaster must have so it doesn't fall off the top of the loop

$$v = \sqrt{gr} = \sqrt{9.81 \times 7.5} = 8.6ms^{-1}$$

Calculate the minum height of the assuming the spseed is zero at the top of the rim

Gravity

> Gravity is the natural phenomenon by which physical bodes appear to attract each other with a force proportional to their masses

$$F = G\frac{m_1 m_2}{r^2}$$

Where : $G[\frac{Nm^2}{kg^2}]$ - gravity force
$m_1 \ and \ m_2$ are the masses of the objects
r is the distance between the enters of the objects m_1 and m_2

Gravitational constant = $6.67 \times 10^{-11} Nm^2kg^{-2}$

Example: Finding the mass of the earth
Radius of Earth: $637100 metres$
M_1:$1.0kg$
$F_g : 1.0 \times 9.81 = 9.81N$
M_z

Gravitational Fields

Gravitational field strength at a point is the force per unit mass expierenced by a small test mass placed at the point

$$g = \frac{F_g}{m} = \frac{GM}{r^2} = \frac{GM}{r^2}$$

Example

Find g at the surface of earth given
$M_E = 5.972 \times 10^{24} kg$
$r_E = 6.371 \times 10^6 m$
$G = 6.674 \times 10^{-11} Nm^2kg^{-2}$

$$g = \frac{GM}{r^2} = 9.814Nkg{-}1$$

Note - gravitational field strength at a point is the same thing as the acceleration due to gravity at that point.

The direction of g is the direction in which a mass placed at that point would experience a force.

In a field line representation the direction of the field is represented by arrows and the strength of the field is represented by arrows.The strength of field is represneted by how close together the field lines are

Orbital Motion of Satellites

The speed of a saterllite can be found if we know r and - [ ]

$$V = \frac{2\pi r}{T}$$

The centrpitel force is provided by gravity: $F_G = F_c$

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad = \quad \frac{GM}{r} = v^2 \qquad v = \sqrt{\frac{GM}{r}}$$

EE.g ISS is in orbit at altitude $4.00 \times 10^5 m$
Earth’s radius $6.37‘ \times 10^6 m$ Find its velocity (orbital)
Earth’s mass : $5.972 \times 10^{24} kg$