

1. Underfitting is, when you don't represent a dataset adequately. It is only a loose representation of the data and the error is really big. For us you can see this in the linear fit.

Overfitting can be when the function is too specific to the dataset; this would make it impossible to make future predictions.

2. 2

3. (a)

$$\begin{pmatrix} -1 & 1 & -1 & 1 \\ 3 & -2 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \end{pmatrix} \cdot x = \begin{pmatrix} 0 \\ -5 \\ 10 \\ 20 \end{pmatrix}$$

$$\Leftrightarrow x = \begin{pmatrix} \frac{5}{4} \\ \frac{25}{4} \\ \frac{15}{4} \\ \frac{-5}{4} \end{pmatrix}$$

$$\Rightarrow \frac{5}{4}x^3 + \frac{25}{4}x^2 + \frac{15}{4}x + \frac{-5}{4}$$

(b)

$$l_0 = \frac{(x - 2.75)(x - 4)}{(-0.75)(-2)} = \frac{11 - 6.75x + x^2}{1.5} \quad l_0 = \frac{(x - 2.75)(x - 4)}{(-0.75)(-2)} = \frac{11 - 6.75x + x^2}{1.5}$$

(c)

Interpolation in x:  $g(x, y) = f(0, y) + (1 - x) \cdot f(1, y)$   
 $\Rightarrow$  Interpolation in y:  $f(x, y) = g(x, 0) + (1 - y) \cdot g(x, 1)$   
 $= f(0, 0) + (1 - x) \cdot f(1, 0) + (1 - y) \cdot (f(0, 1) + (1 - x) \cdot f(1, 1))$   
 $= f(x, y) = \begin{pmatrix} 1 - x \\ x \end{pmatrix}^T \begin{pmatrix} f(0, 0) & f(0, 1) \\ f(1, 0) & f(1, 1) \end{pmatrix} \begin{pmatrix} 1 - y \\ y \end{pmatrix}$   
 Interpolation in y:  $g(x, y) = f(x, 0) + (1 - y) \cdot f(x, 1)$   
 $\Rightarrow$  Interpolation in x:  $f(x, y) = g(0, y) + (1 - x) \cdot g(1, y)$   
 $= f(0, 0) + (1 - y) \cdot f(1, 0) + (1 - x) \cdot (f(0, 1) + (1 - y) \cdot f(1, 1))$   
 $= f(x, y) = \begin{pmatrix} 1 - x \\ x \end{pmatrix}^T \begin{pmatrix} f(0, 0) & f(0, 1) \\ f(1, 0) & f(1, 1) \end{pmatrix} \begin{pmatrix} 1 - y \\ y \end{pmatrix} \quad \square$