Angewandte Mathematik in der Informatik    Sheet 6 **Maximilian von Sternberg**

1. (a)

$$
\begin{aligned}
\int_0^{\pi} \sqrt{r^2 + (\frac{dr}{d\theta})^2}\, d\theta &= \int_0^{\pi} \sqrt{(1-\cos\theta)^2 + (\sin\theta)^2}\, d\theta \\
&= \int_0^{\pi} \sqrt{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}\, d\theta \\
&= \int_0^{\pi} \sqrt{-2\cos\theta + 2}\, d\theta \\
&= \int_0^{\pi} \sqrt{-2(2\cos^2\frac{\theta}{2} - 1) + 2}\, d\theta \\
&= \int_0^{\pi} \sqrt{-4\cos^2\frac{\theta}{2} + 4}\, d\theta \\
&= \int_0^{\pi} \sqrt{4(1 + -\cos^2\frac{\theta}{2})}\, d\theta \\
&= \int_0^{\pi} \sqrt{4\sin^2\frac{\theta}{2}}\, d\theta \\
&= \int_0^{\pi} 2\sin\frac{\theta}{2}\, d\theta \\
&= [-4\cos\frac{\theta}{2}]_0^{\pi} \\
&= -4\cos\frac{\pi}{2} + 4\cos 0 \\
&= 4
\end{aligned}
$$

(b)

$$\begin{aligned}
\int_0^{2\pi} \sqrt{\sum_{i=1}^{n} (\gamma_i'(\theta))^2} &= \int_0^{2\pi} \sqrt{(2\cos\theta + 2)^2 + (-2\sin\theta)^2}\, d\theta \\
&= \int_0^{2\pi} \sqrt{4\cos^2\theta + 8\cos\theta + 4 + 4\sin^2\theta}\, d\theta \\
&= \int_0^{2\pi} \sqrt{8\cos\theta + 8}\, d\theta \\
&= \int_0^{2\pi} \sqrt{8(\cos\theta + 1)}\, d\theta \\
&= \int_0^{2\pi} \sqrt{8 \cdot 2\cos^2\frac{\theta}{2}}\, d\theta \\
&= 4 \cdot \int_0^{2\pi} |\cos\frac{\theta}{2}|\, d\theta \\
&= 4 \cdot \int_0^{\pi} |\cos u|\, \frac{du}{du'} \\
&= 8 \cdot \int_0^{\pi} |\cos u|\, du \\
&= 8 \cdot \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos u\, du \\
&= 8 \cdot [\sin u]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= 16
\end{aligned}$$

(c)

$$\begin{aligned}
\int_0^{8} \sqrt{\sum_{i=1}^{n} (\gamma_i'(\theta))^2} &= \int_0^{8} \sqrt{\frac{9}{4}t + 9 - \frac{9}{4}t} \\
&= \int_0^{8} \sqrt{9} \\
&= \int_0^{8} 3 \\
&= [3t]_0^8 \\
&= 24
\end{aligned}$$

2. (a)

$$\begin{aligned}
\begin{pmatrix} a & b \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} &= \begin{pmatrix} a \cdot \frac{1}{2} + b \cdot \frac{\sqrt{3}}{2} & -a \cdot \frac{\sqrt{3}}{2} + b \cdot \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} \\
&= a^2 \cdot \frac{1}{2} - ab \cdot \frac{\sqrt{3}}{2} + b^2 \cdot \frac{1}{2} + ab \cdot \frac{\sqrt{3}}{2} \\
&= a^2 \cdot \frac{1}{2} + b^2 \cdot \frac{1}{2} \\
&\Rightarrow \text{The Matrix is positivly definite}
\end{aligned}$$

(b)

$$
\begin{aligned}
0 &= det(\begin{pmatrix} t-2 & x \\ 0 & t-2 \end{pmatrix}) \\
&= (t-2)^2 \quad |\sqrt{} \\
&\Leftrightarrow 0 = t-2 \quad | + 2 \\
&\Leftrightarrow 2 = t \\
&\Rightarrow \text{The Matrix is positivly definite for every value of x}
\end{aligned}
$$

(c)

$$
\begin{aligned}
0 &= det(\begin{pmatrix} t-0,5 & 0 \\ 0 & t-x \end{pmatrix}) \\
&= (t-0,5) \cdot (t-x) \\
\Rightarrow t_1 &= 0,5 \\
t_2 &= x \\
&\Rightarrow \text{For } x \in \mathbb{R}^+/\{0\} \text{ the matrix is positivly define,} \\
&\text{for } x = 0 \text{ it is positivly semi definite and for } x \in \mathbb{R}^-/\{0\} \text{ it is undefinite}
\end{aligned}
$$

3. (a) Explicit

   Implicit: $0 = \begin{pmatrix} 4 \\ 10 \\ -2 \end{pmatrix}^T \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} - 15$

   Parametric: $\begin{pmatrix} 0 \\ 0 \\ -7,5 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \cdot t + \begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix} \cdot r$

   (b) Implicit

   Explicit: $z = -(x+y) - \sqrt{75}$

   Parametric: $\begin{pmatrix} 0 \\ 0 \\ -\sqrt{75} \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \cdot t + \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \cdot r$

   (c) Implicit

   Explicit: $z = \sqrt{25 - (x-6)^2 - (y-3)^2}$

   Parametric: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (25 \sin\phi \cos\theta) + 6 \\ (25 \sin\phi \sin\theta) + 3 \\ (25 \cos\phi) \end{pmatrix} for 0 \leq \phi, \theta \leq 2\pi$

4. (a) $\begin{pmatrix} 2x_1 & 0 \\ 0 & 2x_2 \end{pmatrix}$

   (b) $\begin{pmatrix} 3x_1^2 x_2^2 - x_2^3 & x_1^3 2x_2 - x_1 3x_2^2 \\ 2x_1 - x_2^3 & -x_1 3x_2^2 \end{pmatrix}$

   (c) $\begin{pmatrix} 4x_1 & -x_3 \sin(x_2 x_3) & -x_2 \sin(x_2 x_3) \\ 8x_1 & 1260 x_3^2 & 4 \\ -x_2 e^{-x_1 x_2} & -x_1 e^{-x_1 x_2} & 20 \end{pmatrix}$