Angewandte Mathematik in der Informatik Sheet 5 **Maximilian von Sternberg**

1. (a) Yes, it is a valid solution:

$$
\begin{aligned}
&y'' + 2y' = 3y \quad | - 2y' \\
\Leftrightarrow\ & y'' = 3y - 2y' \\
&= (e^{-3x})'' = 3 \cdot e^{-3x} - 2(e^{-3x})' \\
&= 9 \cdot e^{-3x} = 3 \cdot e^{-3x} - 2(e^{-3x})' \\
&= 9 \cdot e^{-3x} = 3 \cdot e^{-3x} + 6 \cdot e^{-3x} \\
&= 9 \cdot e^{-3x} = 9 \cdot e^{-3x}
\end{aligned}
$$

(b)

$$
\begin{aligned}
&x^2y'' - xy' + 2y = 0 \quad | - x^2y''; \cdot -1 \\
\Leftrightarrow\ & x^2y'' = xy' - 2y \\
&= x^2(x\cos(ln|x|))'' = x(x\cos(ln|x|))' - 2(x\cos(ln|x|)) \\
&= -\frac{x^2 \cdot \sin(\ln(|x|) + \cos(\ln(|x|))}{x}) = x(x\cos(ln|x|))' - 2(x\cos(ln|x|)) \\
&= x\sin(\ln|x|) + x\cos(\ln|x|) = x(x\cos(ln|x|))' - 2(x\cos(ln|x|)) \\
&= -x\sin(\ln|x|) - x\cos(\ln|x|) = -x\sin(ln|x|) + x\cos(ln|x| - 2(x\cos(ln|x|)) \\
&= -x\sin(\ln|x|) - x\cos(\ln|x|) = -x\sin(ln|x|) - x\cos(ln|x|)
\end{aligned}
$$

(c)

$$
\begin{aligned}
&xy' - 3y = x^3 \quad | + 3y; \frac{1}{x} \\
\Leftrightarrow\ & (x^3(C + ln|x|))' = x^2 + \frac{1}{x}3(x^3(C + ln|x|)) \\
&= x^2 + 3x^2(C + ln|x|) = x^2 + \frac{1}{x}3(x^3(C + ln|x|)) \\
&= x^2 + 3x^2(C + ln|x|) = x^2 + \frac{3x^3(C + ln|x|)}{x} \\
&= x^2 + 3x^2(C + ln|x|) = x^2 + 3x^2(C + ln|x|)
\end{aligned}
$$

$$
\begin{aligned}
17 &= 1^3 \cdot (C + ln1) = C \\
&\Rightarrow x^3 \cdot (17 + ln|x|)
\end{aligned}
$$

2.

$$
\begin{aligned}
y' &= \frac{x + e^{2x}}{y} \\
\frac{dy}{dx} &= \frac{x + e^{2x}}{y} \quad | \cdot y; \cdot dx \\
y \cdot dy &= x + e^{2x} \cdot dx \quad | \int \\
\frac{1}{2}y^2 &= \frac{1}{2}x^2 + \frac{1}{2} \cdot e^{2x} \quad | \cdot 2; \sqrt{} \\
y &= \sqrt{x^2 + e^{2x}}
\end{aligned}
$$

3. (a)

$$
\begin{aligned}
\dot{y}_0 &= 2\sqrt{2}\cos(\frac{\pi}{4}) = 2 \\
\dot{x}_0 &= 2\sqrt{2}\sin(\frac{\pi}{4}) = 2 \\
y_0 &= 1 \\
x_0 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\dot{y}_1 &= -g \cdot h + \dot{y}_0 = -\frac{5}{2} + 2 = -0,5 \\
\dot{x}_1 &= \dot{x}_0 = 2 \\
\dot{y}_2 &= -\frac{g}{4} + \dot{y}_1 = -\frac{5}{2} - 0,5 = -3 \\
\dot{x}_2 &= \dot{x}_1 = 2
\end{aligned}
$$

$$
\begin{aligned}
y_1 &= h \cdot \dot{y_0} + y_0 = \frac{1}{2} + 1 = \frac{3}{2} \\
x_1 &= h \cdot \dot{x_0} + x_0 = \frac{1}{2} \\
y_2 &= h \cdot \dot{y_1} + y_1 = -\frac{1}{8} + \frac{3}{2} = \frac{11}{8} \\
x_2 &= h \cdot \dot{x_1} + x_1 = 1
\end{aligned}
$$

(b) Julia

(c) Julia

(d) The bigger h gets, the steeper the error curve gets