Angewandte Mathimatik Sheet 8 **Maximilian von Sternberg**

1. (a)

$$
\begin{aligned}
f'(x) =& \frac{2e^{2x} \cdot (e^{2x}+1) - (e^{2x}-1) \cdot 2e^{2x}}{(e^{2x}+1)^2} \\
f'(x) =& \frac{2e^{2x} \cdot (e^{2x}+1-e^{2x}+1)}{(e^{2x}+1)^2} \\
f'(x) =& \frac{4e^{2x}}{(e^{2x}+1)^2} \\
f'(x) =& \frac{4e^{2x}}{e^{4x}+2e^{2x}+1}
\end{aligned}
$$

   (b) Julia
   (c) Julia
   (d) Julia
   (e) It is more likely, that the mean value of a secant is between the values, used in the calculation. Espacially when the functions second differential is a small value, meaning that the rate of change is less significant.

2. (a)

$$
\begin{aligned}
[\log(\cosh(x)) + C]_{-1}^{5} =& \log(\cosh(5)) - \log(\cosh(-1)) \\
=& 3.87
\end{aligned}
$$

   (b)

$$
\begin{aligned}
f(x) =& \frac{\sinh(x)}{\cosh(x)} \\
=& \tanh(x)
\end{aligned}
$$

   (c) Julia
   (d) Julia
   (e) Julia
   (f) The Simpson Method performes best, because it was optimized to waste as little space as possible.

3. (a)

$$
\begin{aligned}
\frac{\partial}{\partial t} u(x_i, t_n) =& \frac{u(x_i, t_{n+1}) - u(x_i, t_n)}{\tau} \\
\frac{\partial^2}{\partial x^2} u(x_i, t_n) =& \frac{u(x_i, t_{n+1}) - u(x_i, t_n)}{\tau} \\
\frac{u(x_{i+1}, t_n) - 2u(x_i, t_n) + u(x_{i-1}, t_n)}{h^2} =& \frac{u(x_i, t_{n+1}) - u(x_i, t_n)}{\tau} \quad | \cdot \tau \\
\Leftrightarrow \tau \frac{u(x_{i+1}, t_n) - 2u(x_i, t_n) + u(x_{i-1}, t_n)}{h^2} =& u(x_i, t_{n+1}) - u(x_i, t_n) \quad | + u(x_i, t_n) \\
\Leftrightarrow \tau \frac{u(x_{i+1}, t_n) - 2u(x_i, t_n) + u(x_{i-1}, t_n)}{h^2} + u(x_i, t_n) =& u(x_i, t_{n+1})
\end{aligned}
$$

4. (a)

$$
\begin{aligned}
2x^6 - 5x^5 + x^2 - 6x + 1 =& (2x) \cdot x^5 - 5x^5 + x^2 - 6x + 1 \\
=& ((2x) - 5) \cdot x^5 + x^2 - 6x + 1 \\
=& (((2x) - 5) \cdot x^3 + 1) \cdot x^2 - 6x + 1 \\
=& (((2x) - 5) \cdot x^3 + 1) \cdot x - 6) \cdot x + 1
\end{aligned}
$$

(b)

$$
\begin{aligned}
& 12x^7 + 2x^4y^6 + x^3y^4 - 3x^2 - 7x^2y^4 - 2 + 9y + 2y^5 \\
=& (12x^7 - 3x^2) + (2y^5 + 9y) + (2x^4y^6 + x^3y^4 - 7x^2y^4) - 2 \\
=& ((12x^5 - 3) \cdot x^2) + ((2y^4 + 9) \cdot y) + (((2xy^2 + 1) \cdot x - 7) \cdot x^2y^4) - 2
\end{aligned}
$$

(c) Julia