**General Information:** This assignment contains written and/or programming tasks. Combine all the answers to the written tasks in a single PDF document, named `{lastname}-written.pdf`. You can also scan or take pictures of (readable) handwritten papers. JPEG/PNG image files are accepted in this case and they should be named `{exercisenumber}-{lastname}-written.{jpeg/png}`. Make sure that we can follow the manual calculations. Do not combine too many small steps into one. The programming tasks have to be solved in *Julia* and the source code files have to be submitted using the following naming scheme: `{exercisenumber}-{lastname}.jl`.

(1) (1 Point) Compute the arc length of the following functions. **Note** that the arc length for polar coordinates is defined as $L = \int ds$ with $ds = \sqrt{r^2 + (\frac{dr}{d\theta})^2}\, d\theta$

a) $r(\theta) = 1 - cos\theta$ for $0 \leq \theta < \pi$ and sketch the plot of $r(\theta)$ for $0 \leq \theta < 2\pi$.
**Hint:** $cos(2a) = 2cos^2(a) - 1$.

b) $\gamma(\theta) = (2(\theta + sin\theta), 2(1 + cos\theta))$ for $0 \leq \theta < 2\pi$

c) $x(t) = t^{\frac{3}{2}}$, $y(t) = (4 - t)^{\frac{3}{2}}$ for $0 \leq t \leq 8$

(2) (1 Point) Definiteness of matrices: Check the following matrices on their definiteness with respect to free variables:

a) rotation matrix $\varphi = \frac{2\pi}{6}$

$$rot(\varphi) = \begin{bmatrix} cos(\varphi) & -sin(\varphi) \\ sin(\varphi) & cos(\varphi) \end{bmatrix}$$

b) shearing matrix, $x \in \mathbb{R}$

$$shear(x) = \begin{bmatrix} 2 & x \\ 0 & 2 \end{bmatrix}$$

c) scaling matrix, $x \in \mathbb{R}$

$$scale(x) = \begin{bmatrix} 0.5 & 0 \\ 0 & x \end{bmatrix}$$

d) (0.5 Points) Consider some arbitrary high-dimensional non-convex function. Are local minima/maxima encountered less frequently than saddle points? **Hint**: Think in terms of the eigenvalues of the Hessian matrix.

(3) (1.5 Points) For each of the following surfaces state whether they are given in implicit, explicit, or parametric form and compute the two missing representations.

a) $2z = 4x + 10y - 15$

b) $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})(x, y, z)^\intercal + 5 = 0$

c) $(x - 6)^2 + (y - 3)^2 + z^2 = 25$

(4) (2 Points) Consider the following functions $f, g\colon \mathbb{R}^2 \to \mathbb{R}^2$ and $h\colon \mathbb{R}^3 \to \mathbb{R}^3$ of several real variables

a) $f(x) = (x_1^2, x_2^2)^\intercal$,

b) $g(x) = (x_1^3 x_2^2 - x_1 x_2^3 - 1, x_1^2 - x_1 x_2^3 - 4)^\intercal$,

c) $h(x) = (2x_1^2 - \cos(x_2 x_3) - \frac{3}{2}, 4x_1^2 - 420x_2^3 + 4x_3 - 1, 20x_3 + \exp(-x_1 x_2) + 10)^\intercal$,

and compute the Jacobian matrices by hand.

Familiarize yourself with the backslash operator \ in *Julia*. Implement the multivariate Newton method and the three Jacobian matrices in the provided template `multivariate_newton.jl`. Do not explicitly invert the Jacobian matrices, cf. lecture notes. Test your code by running `runtests.jl` to see whether your solution is correct.

```
# Snippet for backslash operator.
A = [-4 -1; 2 2]  # 2x2 matrix
b = [-3; 0]  # right-hand side
x = A\b  # solving the system of linear equations
A*x == b
```

d) (0.5 Points) Which conditions need to be met in order to use Newton's Method? Could you use another optimization method that we discussed in the lecture? If so, describe the differences between both approaches.

(5) (1 Point) Given the vector field $F = [y, -x]^T$ compute the work $W$ done on a particle moving clockwise around $r(t) = (\cos(t) + 3, sin(t) - 3)^T$ for $0 \leq t < 2\pi$. What would happen if we change the orientation of the particle? Re-parameterize its curve and calculate the result.