

1. (a)

$$\begin{aligned}
 & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 1 & 2 & 5 \\ 3 & 13 & 7 & 8 & 17 \\ 1 & 7 & 21 & 8 & 15 \\ 2 & 8 & 8 & 7 & 16 \\ 5 & 17 & 15 & 16 & 40 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 5 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 1 & 2 & 5 \\ 0 & 4 & 4 & 2 & 2 \\ 0 & 4 & 20 & 6 & 10 \\ 0 & 2 & 6 & 3 & 6 \\ 0 & 2 & 10 & 6 & 15 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 2 & \frac{1}{2} & 0 & 1 & 0 \\ 5 & \frac{1}{2} & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 1 & 2 & 5 \\ 0 & 4 & 4 & 2 & 2 \\ 0 & 0 & 16 & 4 & 8 \\ 0 & 0 & 4 & 2 & 5 \\ 0 & 0 & 8 & 5 & 14 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 2 & \frac{1}{2} & \frac{1}{2} & 1 & 0 \\ 5 & \frac{1}{2} & \frac{1}{2} & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 1 & 2 & 5 \\ 0 & 4 & 4 & 2 & 2 \\ 0 & 0 & 16 & 4 & 8 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 3 & 10 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 2 & \frac{1}{2} & \frac{1}{2} & 1 & 0 \\ 5 & \frac{1}{2} & \frac{1}{2} & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 1 & 2 & 5 \\ 0 & 4 & 4 & 2 & 2 \\ 0 & 0 & 16 & 4 & 8 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}
 \end{aligned}$$

(b) The matrix is symmetric and regular

(c)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 & 0 \\ 1 & 2 & 4 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 5 & 1 & 2 & 3 & 1 \end{pmatrix}$$

(d)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 & 0 \\ 1 & 2 & 4 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 5 & 1 & 2 & 3 & 1 \end{pmatrix} \cdot y = (11111)^T$$

$$y = \begin{pmatrix} 1 \\ -1 \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{5}{2} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 1 & 2 & 5 \\ 0 & 2 & 2 & 1 & 1 \\ 0 & 0 & 4 & 1 & 2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot x = \begin{pmatrix} 1 \\ -1 \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{5}{2} \end{pmatrix}$$

$$x = \begin{pmatrix} 7 \\ -\frac{19}{8} \\ -\frac{33}{8} \\ 7 \\ -\frac{5}{2} \end{pmatrix}$$

(e) If you directly wanted to calculate, you would have to use the Gauss algorithm, to transform the matrix into a unit matrix and perform these transformations on another unit matrix. The unit matrix will then transform into the inverse of the matrix.

2. (a)

$$x_1 = \frac{1}{1}(1 - (3 + 1 + 2 + 5)) = -10$$

$$x_2 = \frac{1}{13}(1 - (3 + 7 + 8 + 17)) = 2.26$$

$$x_3 = \frac{1}{21}(1 - (1 + 7 + 8 + 15)) = 1.43$$

$$x_4 = \frac{1}{7}(1 - (2 + 8 + 8 + 16)) = 3.29$$

$$x_5 = \frac{1}{40}(1 - (5 + 17 + 15 + 16)) = 1.3$$

(b) Julia

(c) Julia

(d)

3. (a)