General Information: This assignment contains written and/or programming tasks. Combine all the answers to the written tasks in a single PDF document, named `{lastname}-written.pdf`. You can also scan or take pictures of (readable) handwritten papers. JPEG/PNG image files are accepted in this case and they should be named `{exercisenumber}-{lastname}-written.{jpeg/png}`. Make sure that we can follow the manual calculations. Do not combine too many small steps into one. The programming tasks have to be solved in *Julia* and the source code files have to be submitted using the following naming scheme: `{exercisenumber}-{lastname}.jl`.

(1) (0.25, 0.25, 0.5 Points) Compute the following integrals.

a)

$$\int \frac{1}{\sqrt[3]{x}} + \cos(7-4x)\mathrm{d}x$$

b)

$$\int \frac{x^2+1}{x^3+3x}\,\mathrm{d}x$$

c)

$$\int \frac{x-1}{x^2-1}\mathrm{d}x$$

d)

$$\int \sin(x)\cdot x\,\mathrm{d}x$$

Notes:
(1a) (1b) Use integral substitution

(2) (0.5, 0.25 Points) Compute the integral

$$\int_0^8 \sqrt{1+x^2}\,\mathrm{d}x\,. \qquad (\star)$$

Substitute $x = \sinh(u)$ with $\mathrm{d}x = \cosh(u)\,\mathrm{d}u$. Transform the borders of the definite integral and compute the integral over u. You can use the equation of identity and an additive theorem of the hyperbolic functions.

Check your solution using the *Julia* Package **QuadGK** (numerical integration).

Snippet:

```julia
# Definite integral of x^2 - 2*x for 0 < x < 8
using QuadGK
quadgk(x -> x^2 - 2*x, 0.0, 8.0)
```

`QuadGK.quadgk()` returns a numeric value computed by quadrature of the function and an estimated numerical error.

(3) Compute the area enclosed by the curve $y = x^3 - 2x + 2$ and the line $y = x + 2$. Make a sketch and do the calculations by hand.

a) (1 Points) Integration over x. Pay attention to what is happening at $x = \pm\sqrt{3}$ and $x = 0$.

(4) Measuring an acceleration $a(t)$ leads to the function

$$a(t) = \begin{cases} 1.5 & 0 \leq t \leq 60\,, \\ 0 & 60 < t \leq 150\,, \\ -\frac{t}{150} + 1 & 150 < t \leq 300\,, \end{cases}$$

which is plotted in Figure 1.

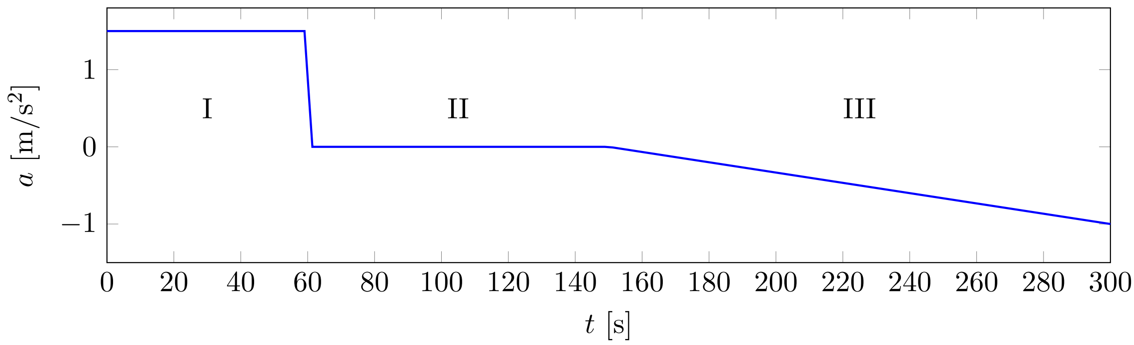

Figure 1: Measured acceleration $a(t)$.

a) (1.5 Points) Compute the velocity $v(t)$ and the position $x(t)$ over time by integrating $a(t)$ (multiple times). In order to do so solve the initial value problem

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}v(t) &= \dot{v}(t) = a(t)\,, \\ \frac{\mathrm{d}}{\mathrm{d}t}x(t) &= \dot{x}(t) = v(t)\,, \\ v(0) = v_0 &= x(0) = x_0 = 0\,. \end{aligned}$$

Plot your solutions $v(t)$ and $x(t)$ as in Figure 1. Compute $x(300)$.

Note: Pay special attention to the different integration limits. The velocity $v(t)$ for $200 < t \leq 300$, for example, depends on the velocity in the interval $0 \leq t \leq 200$.

b) (1.5 Points) Let

$$t_j = j \quad \text{for } j = 0, 1, 2, \ldots, 300$$

be the decomposition of the interval $[0, 300]$. Determine the approximations $\hat{v} \approx v$ and $\hat{x} \approx x$ by summing up the discrete values $a_j = a(t_j)$ and $v_j = v(t_j)$. Finally plot the absolute errors

$$|\hat{v}(t_j) - v(t_j)| \quad \text{and} \quad |\hat{x}(t_j) - x(t_j)|$$

per time step t_j using *Julia*.

c) (0.25 Punkte) What has to be taken into account after increasing the time steps $t_j = 5j$ for $j = 0, 1, \ldots, 60$? Create an explanatory sketch for this purpose.

(5) Consider the function

$$f(x, y) = -\sqrt{\frac{5}{4}x^2 + 2y + 80}$$

and the region $A = [0, 8] \times [-4, 4]$ in the xy-plain, cf. Figure 2.

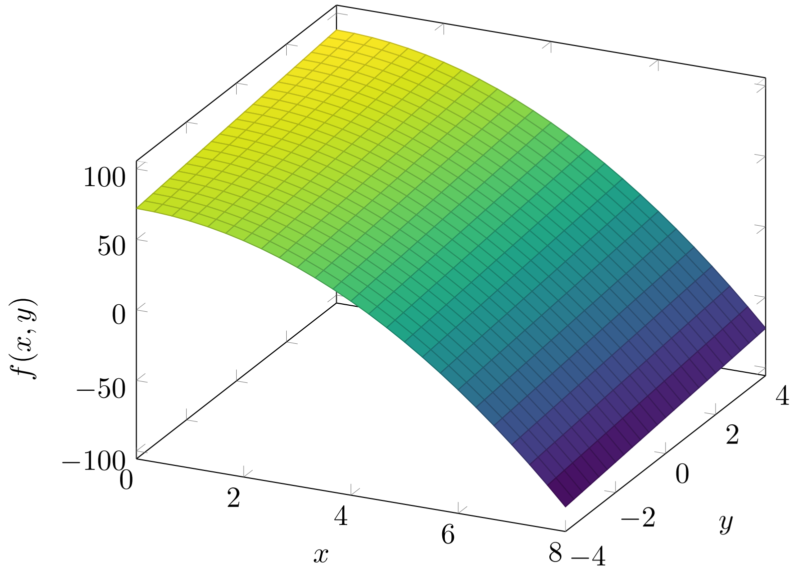

Figure 2: The function $f(x, y)$ of two variables.

a) (0.5 Points) Compute the volume between the xy-plain $z = 0$ and the function $f(x, y)$ in the region A

$$V = \int_0^8 \int_{-4}^4 f(x, y) \mathrm{d}y \, \mathrm{d}x \, .$$

b) (0.5 Points) Compute the surface area of $f(x, y)$ within A. It is computed by integrating the magnitude of the cross product of the tangent plane directional vectors (sum of the areas of the infinitesimal tangential planes)

$$S = \int_0^8 \int_{-4}^4 \left| (1, 0, \partial_x f)^T \times (0, 1, \partial_y f)^T \right| \mathrm{d}y \, \mathrm{d}x \, .$$

Note: Use the result of $(\star)$ for (5b).