Assignment 3

Maximilian von Sternberg
Angewandte Mathematik

March 28, 2022

Exercise 1. Question a.

$$\begin{aligned}
f(x,y) &= -4x^2 + 2*(2x-y)^3 - (x+y)^3 + x + 4y^2 - y - 4 \\
&= 2*(2x-y)^3 - (x+y)^3 + x + 4y^2 - 4x^2 - y - 4 \\
&= 2*(8x^3 - y^3 - 12x^2y + 6xy^2) - (x^3 + 3x^2y + 3xy^2 + y^3) + x + 4y^2 - 4x^2 - y - 4 \\
&= 16x^3 - 2y^3 - 24x^2y + 12xy^2 - x^3 - 3x^2y - 3xy^2 - y^3 + x + 4y^2 - 4x^2 - y - 4 \\
&= 15x^3 - 3y^3 - 27x^2y + 9xy^2 - 4x^2 + 4y^2 + x - y - 4
\end{aligned}$$

$$\frac{\partial f(x,y)}{\partial x} = 45x^2 - 54yx + 9y^2 - 8x + 1$$

$$\frac{\partial f(x,y)}{\partial x} = -9y^2 - 27x^2 + 18xy + 8y - 1$$

Question b.

$$\begin{aligned}
0 = \frac{\partial f(0,y)}{\partial y} &= -9y^2 + 8y - 1 \\
\text{Quadratic-Formula} : y &= \frac{-8 \pm \sqrt{8^2 - 4(-9)(-1)}}{2(-9)} \\
&= \frac{-8 \pm \sqrt{64 - 36}}{-18} \\
y_1 = 0,738 & \\
y_2 = 0,15 &
\end{aligned}$$

$$
\begin{aligned}
0 = \frac{\partial f(x,-1/2)}{\partial x} &= 45x^2 + 35x + 3,25 \\
\text{Quadratic-Formula} : y &= \frac{-35 \pm \sqrt{35^2 - 4*43*3,25}}{2*45} \\
&= \frac{-35 \pm \sqrt{1225 - 559}}{90} \\
x_1 = -0,108 & \\
x_2 = -0,67 &
\end{aligned}
$$

Question c.

$$
\begin{aligned}
\nabla f(-\frac{1}{2},-\frac{1}{2}) &= (5,-9,5) \\
\nabla f(\frac{1}{2},\frac{1}{2}) &= (-3,-1,5) \\
\nabla f(-\frac{1}{2},\frac{1}{2}) &= (32,-10,5) \\
\nabla f(\frac{1}{2},-\frac{1}{2}) &= (24,-18,5) \\
\nabla f(-\frac{1}{2},-\frac{1}{2}) &= (3,25,-7,25)
\end{aligned}
$$

Exercise 2. The distance is $x(t) = v_x * t$. Now we need to maximize the time for which $y(t) > 0$ and where v_x is sufficiently big. We can combine these two properties by making v_x and v_y dependent on θ.

$$
\begin{pmatrix} v * cos\theta \\ v * sin\theta \end{pmatrix} \tag{1}
$$

the best t is now the non trivial solution for $0 = tv * sin\theta - \frac{gt^2}{2}$

$$
\begin{aligned}
0 &= tv_y * sin\theta - \frac{gt^2}{2} \quad | + \frac{gt^2}{2} \\
\Leftrightarrow \frac{gt^2}{2} &= tv_y * sin\theta \quad | * \frac{1}{vsin\theta} \\
\Leftrightarrow \frac{gt^2}{2v * sin\theta} &= t
\end{aligned}
$$

Now that we have the zero dependant on t we can combine the three functions to the length function

$$
f(\theta) = v * cos\theta * \frac{gt^2}{2v * sin\theta}
$$

To get the min of this function we just have to finde $0 = f'(\theta)$

Exercise 3. Question b. Because f'(x) = 0 you would divide by zero which is impossible

Question c. If you use f' for the function the roots will be the local minima of f