

1. (a) $F(x) = \frac{3}{2} \sqrt[3]{x^2} + -\frac{1}{4} \cdot \sin(7 - 4x) + c$

(b)

$$\begin{aligned} F(x) &= \int \frac{x^2 + 1}{x^3 + 3x} dx \\ &= \int \frac{x^2 + 1}{z \cdot z'} dz \\ &= \int \frac{x^2 + 1}{z \cdot 3(x^2 + 1)} dz \\ &= \int \frac{1}{z \cdot 3} dz \\ &= \frac{1}{3} \int \frac{1}{z} dz \\ &= \frac{1}{3} \ln(|z|) + c \\ &= \frac{\ln(|x^3 + 3x|)}{3} + c \end{aligned}$$

(c)

$$\begin{aligned} F(x) &= \int \frac{x - 1}{x^2 - 1} dx \\ &= \int \frac{x - 1}{(x + 1)(x - 1)} dx \\ &= \int \frac{1}{x + 1} dx \\ &= \ln(|x + 1|) + c \end{aligned}$$

(d)

$$\begin{aligned} F(x) &= \int \sin(x) \cdot x dx \\ &= -x \cdot \cos(x) - \int -\cos(x) dx \\ &= -x \cdot \cos(x) + \sin(x) + c \end{aligned}$$

2.

$$\begin{aligned}
 F(x) &= \int_0^8 \sqrt{1+x^2} dx \\
 &= \int_{\operatorname{asinh}(0)}^{\operatorname{asinh}(8)} \sqrt{1+(\sinh(u))^2} \cosh(u) du \\
 &= \int_{\operatorname{asinh}(0)}^{\operatorname{asinh}(8)} \sqrt{\cosh^2(u)} \cosh(u) du \\
 &= \int_{\operatorname{asinh}(0)}^{\operatorname{asinh}(8)} \cosh^2(u) du \\
 &= \left[\frac{1}{2} \cosh(u) \sinh(u) + \frac{1}{2} u \right]_{\operatorname{asinh}(0)}^{\operatorname{asinh}(8)} \\
 &= \frac{8}{2} \cosh(\operatorname{asinh}(8)) + \frac{\operatorname{asinh}(8)}{2} \\
 &= 33,637267134
 \end{aligned}$$

3.

$$\begin{aligned}
 x+2 &= x^3 - 2x + 2 & | - (x+2) \\
 0 &= x^3 - 3x \\
 0 &= x(x^2 - 3) \\
 x_1 &= 0 \\
 0 &= x^2 - 3 \\
 x_{2/3} &= \pm\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 A &= \left| \int_{-\sqrt{3}}^0 x^3 - 2x + 2 - (x+2) dx \right| + \left| \int_0^{-\sqrt{3}} x^3 - 2x + 2 + (x+2) dx \right| \\
 &= \left| \int_{-\sqrt{3}}^0 x^3 - 3x dx \right| + \left| \int_0^{-\sqrt{3}} x^3 - 3x dx \right| \\
 &= \left| \left[\frac{1}{4} x^4 - \frac{3}{2} x^2 \right]_{-\sqrt{3}}^0 \right| + \left| \left[\frac{1}{4} x^4 - \frac{3}{2} x^2 \right]_0^{-\sqrt{3}} \right| \\
 &= \frac{9}{2}
 \end{aligned}$$

4. (a) We have to find a c which corresponds to the appropriate value that the function stays continuous. As for my hand I have provided the functions in julia. $x(300) = 20550$
- (b) In beschleunigung.jl
- (c) The steps are only an arbitrary sample in the function. Therefore if we use a lower step size the accuracy of our guess will become worse.

5. (a)

$$\begin{aligned} V &= \int_0^8 \int_{-4}^4 -\sqrt{\frac{5}{4}} \cdot x^2 + 2y + 80 \, dy \, dx \\ &= \int_0^8 \left[\frac{-\sqrt{5} \cdot x^2 y}{2} + y^2 + 80y \right]_{-4}^4 \, dx \\ &= \int_0^8 640 - 4\sqrt{5} \cdot x^2 \, dx \\ &= \left[-\frac{4}{3}\sqrt{5} \cdot x^3 + 640x \right]_0^8 \\ &= 5120 - \frac{2048\sqrt{5}}{3} \end{aligned}$$

(b)

$$S = \int_0^8 \int_{-4}^4 |(1, 0, \partial_x f)^T \times (0, 1, \partial_y)^T| \, dy \, dx$$