General Information: This assignment contains written and/or programming tasks. Combine all the answers to the written tasks in a single PDF document, named `{lastname}-written.pdf`. You can also scan or take pictures of (readable) handwritten papers. JPEG/PNG image files are accepted in this case and they should be named `{exercisenumber}-{lastname}-written.{jpeg/png}`. Make sure that we can follow the manual calculations. Do not combine too many small steps into one. The programming tasks have to be solved in *Julia* and the source code files have to be submitted using the following naming scheme: `{exercisenumber}-{lastname}.jl`.

(1) a) (0.5 points) Given the function

$$y(x) = e^{-3x},$$

determine if the given function is a solution to the following differential equation (detail your answer)

$$y'' + 2y' = 3y$$

b) (0.5 points) Show that the function

$$y = x\cos(\ln|x|)$$

is a solution to the differential equation

$$x^2y'' - xy' + 2y = 0$$

c) (1 point) Show that the function

$$y(x) = x^3(C + \ln|x|)$$

is a solution to the differential equation

$$xy' - 3y = x^3$$

also, find the particular solution for the given initial condition

$$y(1) = 17$$

.

(2) (2 points) The following differential equation is given:

$$y' = \frac{x + e^{2x}}{y}.$$

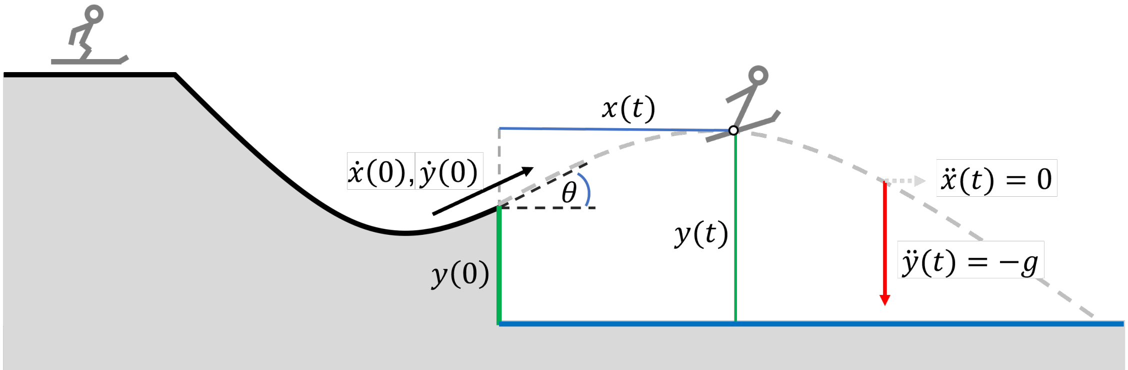
Compute the general solution $y(x)$ by separation of variables.

(3) (3 points) We consider a ski jumper who decides to take off at an angle θ with velocity v. If we neglect all external forces other than gravity ($g \approx 9.81$) we have the following ordinary differential equations describing the vertical and horizontal acceleration of the skier at time t:

$$\ddot{y}(t) = -g$$
$$\ddot{x}(t) = 0.$$

The initial values (i.e. at $t = 0$), are the vertical and horizontal velocity

$$\dot{y}(0) = v\sin\theta$$
$$\dot{x}(0) = v\cos\theta$$



and the height (and distance) of the skier at takeoff

$$y(0) = h_0$$
$$x(0) = 0.$$

In the following exercise you should solve these differential equations numerically with the *explicit euler* method. That is, we want to compute the distance x and height y of the skier for discrete times $t_0, ..., t_n$ where $t_n = t_0 + n \cdot h$. To this end you will need to apply the method twice for each dimension to solve for the values at t_{n+1}: First, to approximate the velocity $\dot{x}_{n+1}, \dot{y}_{n+1}$ from the acceleration $\ddot{x}(t_n), \ddot{y}(t_n)$ and second, to get the distance x_{n+1} and height y_{n+1} from the approximated velocities $\dot{x}_n, \dot{y}_n$.

a) (1 point) Calculate by hand the first two timesteps $t_1 = h, t_2 = 2h$, with $h = \frac{1}{4}$ of the explicit euler method.
Compute the initial values $\dot{x}_0, \dot{y}_0, x_0, y_0$ from the following parameters

$$h_0 = 1,\ \theta = \frac{\pi}{4},\ v = 2\sqrt{2}.$$

For simplicity you can assume that the gravity is $g = 10m/s^2$.

b) (1 point) Implement in Julia the explicit euler method to solve the aformentioned ordinary differential equations – see the template Julia code `ski_jump_euler.jl`.

c) (0.5 points) For each step compute the error between the true solution and the approximation, e.g. for the vertical component $|y(t_n) - y_n|$.

d) (0.5 points) Run the method with different stepsizes h.
What do you observe? Explain the change in behaviour geometrically.