Paper ID: MS-177

Thermal Resistance Approach to Analyze Temperature Distribution in Hollow Cylinders Made of Functionally Graded Material (FGM): Under Dirichlet Boundary Condition

S M Shayak Ibna Faruqui[1], Abul Al Arabi[2], M. S Parvej[3]

[1,3]Bangladesh University of Engineering and Technology
[2]Independant University, Bangladesh
E-mail: shayakbuet08@gmail.com

Abstract

In this paper a temperature distribution under dirichlet boundary condition was obtained for cylinders made of Functionally Graded Material (FGM) using Thermal Resistance Approach. Material property was assumed to vary along radial direction of the cylinder following Power Function. Thermal conductivity of the FGM cylinder is assumed to be indifferent to the temperature change and only a function of independent variable, radial distance. So the governing ODE stayed linear which allowed us to determine solutions analytically. In Thermal Resistance Approach the FGM cylinder was divided into homogeneous elements across the thickness having constant material property throughout each element and the radial variation of thermal conductivity of the whole body was assumed to be a piece-wise function of radial distance. Then a temperature profile was generated considering each element's thermal resistance. Results were compared with other studies and with obtained results from ANSYS solver.

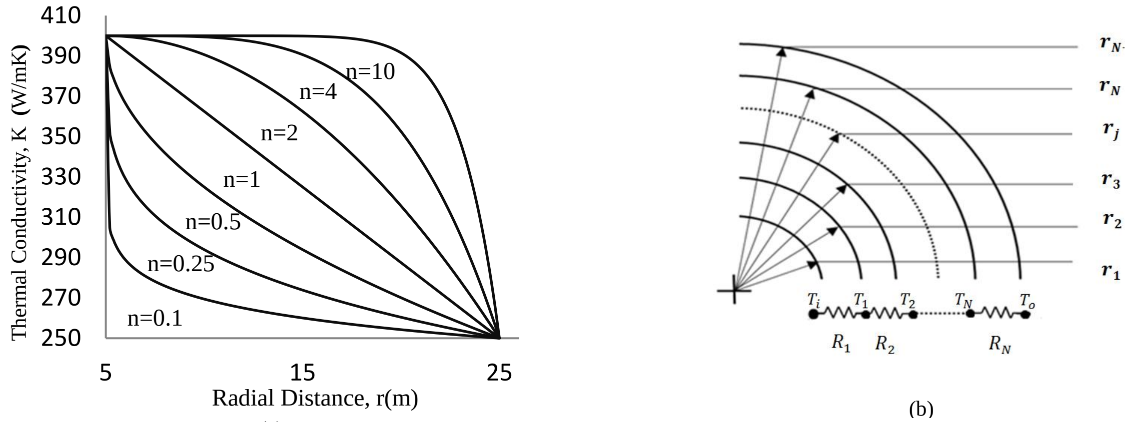
Keywords: Functionally graded material, Thermal resistance, Temperature distribution

1. Introduction

Functionally Graded Materials (FGMs) belong to a class of superior Composite Material in which material properties vary along any specific direction. The difference between traditional composites and FGMs is that, traditional composites are layered structures where the variation of mechanical properties is piecewise along a certain direction but in FGM, the variation is smooth and continuous. The thermal as well as structural behavior of FGMs has attracted many researchers in the past few years from different engineering sectors. FGMs are replacing traditional materials to meet the extreme performance requirements under complex working conditions because of their superiority over various metal alloys and composites. Recently In their work, S. Karampour et al. [2] solved the heat conduction equation for obtaining temperature distribution analytically using Legendre polynomials and Euler differential equations system. Celebi et al. [3] used Complimentary Function Method (CFM) to convert the boundary value problem into an initial value problem and solved the temperature problem. M. Jabbari et al. [4], [5], [6], [7] & [8] obtained results for temperature distribution directly by exact method and as he assumed the radial distribution of material property to follow an exponential function, getting exact solutions posed no complexity. X.L. Peng et al. [9] also converted the boundary value problem associated with the thermoelastic problem into a Fredholm integral equation and by numerically solving the resulting equation, the distribution of the temperature was obtained. Shao Z.S [10] presented a study about mechanical and thermal stresses in FGM hollow cylinder of finite length and Zimmerman et al [11] in uniformly heated FGM cylinder. In this study, temperature distribution was obtained for dirichlet boundary condition by Thermal Resistance Approach and obtained results were compared with [4] & [3] and then with the results from the ANSYS solver.

2. Formulation of Problem

A hollow cylinder is considered with an inner radius r_i and outer radius r_o where k_i and k_o are the value of thermal conductivity at inner and outer surfaces respectively. Thermal conductivity is assumed to vary from k_i to k_o along radial direction following a Power Function (1) where "n" is the power law index.



(a) (b)

Fig. 1. (a) Continuous distribution of thermal conductivity in FGM cylinder and (b) Thermal series resistances formed by cylindrical elements.

$$K_{fgm}(r) = k_i + (k_o - k_i)\left(\frac{r - r_i}{r_o - r_i}\right)^n \tag{1}$$

Governing equation and analytical (exact) solution for n=1

Assuming axial symmetry the temperature only varies radially for the cylinder. In cylindrical coordinate for steady state conduction heat transfer with no internal heat generation, the governing equation forms as (2).

$$\frac{d}{dr}\left(K_{fgm}(r).r\frac{dT}{dr}\right) = 0 \tag{2}$$

A solution of this ODE can be achieved for dirichlet boundary condition when temperature values at both inner and outer surfaces of the cylinder are known by simple double integration method. Considering *n=1* from (1), where $T(r_i) = T_i \; and \; T(r_o) = T_o$, for $k_i > k_o$,

$$T(r) = \frac{(T_i - T_o)}{ln\left(\frac{r_i k_o}{r_o k_i}\right)} ln\left(\frac{k_i(r_o - r_i)r}{(k_i(r_o - r) + k_o(r - r_i))r_i}\right) + T_i \tag{3}$$

Thermal resistance approach (TRA)

For dirichlet boundary conditions when both inner and outer surface temperature of the hollow cylindrical body is known, solutions can be obtained by dividing the cylindrical body along the radial direction into small cylindrical elements. Every element is assumed to be homogenous ensuring material property remains the same throughout the element. Each element's thermal conductivity can be assigned to it from (1) by taking average of the values at each two inner and outer boundaries. The radial variation of thermal conductivity of the whole body is then assumed to be a piece-wise function of radial distance, r. If the body is divided into *'N'* number of elements, each element will have a thermal conductivity according to this variation. After getting *'N'* number of different thermal conductivities for *'N'* different elements, thermal resistance can be computed for each of them. Thermal resistance across the j^{th} element can be determined using (4).

$$R_j = \frac{ln\left(\frac{r_{j+1}}{r_j}\right)}{2\pi L K_j} \tag{4}$$

Where "L" is the length of the cylindrical body, k_j is the thermal conductivity of j^{th} element, $r_j \; and \; r_{j+1}$ are inner and outer radius respectively. Then the total thermal resistance across the inner and outer surface of the whole body can be calculated from (5) where R_1, R_2, R_3, etc. are the thermal resistances of different elements.

$$R_{total} = R_1 + R_2 + \cdots + R_j + \cdots + R_N \tag{5}$$

However, in this approach it is assumed that there is no heat loss or additional thermal contact resistance at the contact surface of two adjoin elements. The temperature values are obtained at each of the contact surfaces of constituent elements. Temperature at contact surface of the j^{th} and $(j+1)^{th}$ element at $r = r_{j+1}$ is computed from (6). When $T(r_i) = T_i$ and $T(r_o) = T_o$,

$$T_{j+1} = \frac{(R_1+R_2+R_3+\cdots+R_{j-1}+R_j)T_o+(R_{j+1}+R_{j+2}+R_{j+3}+\cdots+R_{N-1}+R_N)T_i}{R_{total}} = \frac{\left(\sum_1^j R\right)T_o+\left(\sum_{j+1}^N R\right)T_i}{R_{total}} \tag{6}$$

3. Results, Validation and Discussion

Setting up $k_i = 400\, W/mk$ and $k_o = 250\ W/mk$, $T_i = 900k$ and $T_o = 400k$ solutions can be obtained and analyzed. k_i and k_o are thermal conductivities of Copper and Aluminum respectively and are assumed to be temperature independent. Figure 2(a) shows temperature distribution of homogenous material and FGM (for power law index, n=1) under same boundary and physical conditions where temperature distribution of FGM cylinder is also the comparison between results obtained by analytical (exact) method and thermal resistance approach. Figure 2(b) is obtained radial temperature distribution for different power law indexes (n=0, 2, 4, 10). Figure 2(c) & (d) shows radial distribution of temperature (left/primary vertical axis) with the corresponding radial distribution of thermal conductivity (right/secondary vertical axis). Effect of the distribution slope is prominent in the results for n=0.1 in fig.2(c) & (d). In figure 2(e) the effect of K_{fgm}'s distribution on the resulting distribution of thermal resistance across the thickness is observed. Figure 2(f) is the comparison among results obtained by M. Jabbari [4] and Celebi [3] with present work for inhomogeneity parameter *β=2* and results obtained from this work are in good agreement with them. Table.1. presents the variation in results when FGM cylinder is divided into different numbers of homogenous elements across the thickness denoted by "*N*". As the value of "*N*" increases the results converge. When the cylinder is divided into 2 homogenous elements, 1 temperature value at contact surface of the two elements is obtained taken at *r=15*. For *N=3* two results and for *N=6* five temperature values at 5 contact regions are obtained. For *N=40* the results are more refined.

Table 1. Radial distribution of temperature for different values of *N*

No. of Homogenous Elements, *N*	Temperature Distribution, *T(r)* (℃) — Radial distance, *r(m)*: 5	8	11	12	14	15	17	18	20	21	23	25
2	900					585.56						400
3	900			655.72				524.86				400
6	900	779.55		668.7		577.32		519.08		466.01		400
40	900	773.06	681.71	655.65	607.98	585.9	544.45	524.82	487.25	469.2	434.1	400

Table 2. Radial temperature distribution from TRA, Jabbari [4] and Celebi [3].

r/r_i	$T/T(r_i)$: $\beta = 0$ This Work	$\beta = 0$ Jabbari	$\beta = 0$ Celebi(CFM)	$\beta = -2$ This Work	$\beta = -2$ Jabbari	$\beta = -2$ Celebi(CFM)
1	1	1	1	1	1	1
1.04	0.784881646	0.784882	0.784882	0.816793017	0.814545	0.814545
1.08	0.577882931	0.577883	0.577883	0.625818359	0.621818	0.621818
1.12	0.378413133	0.378413	0.378413	0.426462605	0.421818	0.421818
1.16	0.185943737	0.185944	0.185944	0.218082217	0.214545	0.214545
1.2	0	0	0	0	0	0

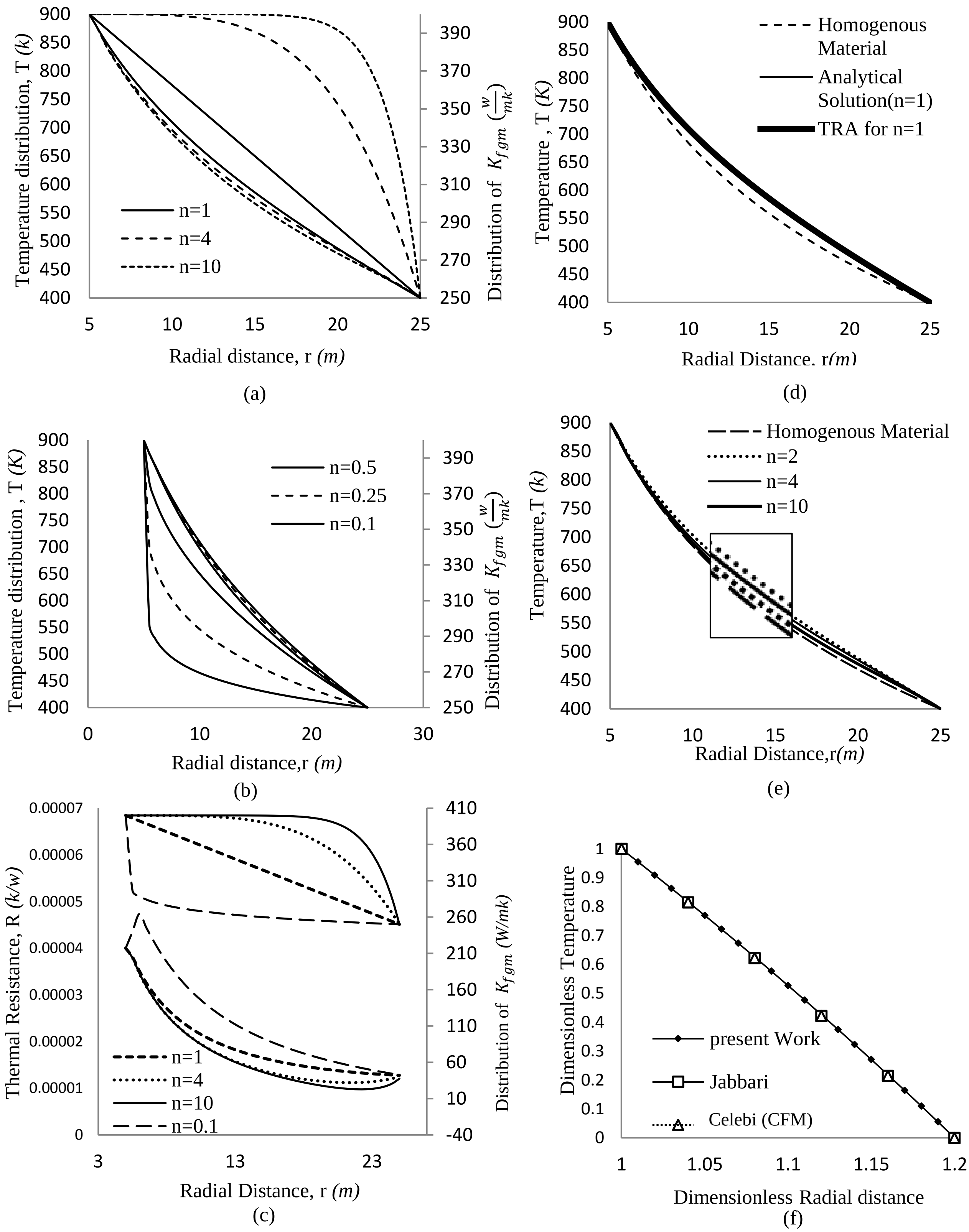


Fig. 2. (a)&(b) Radial distribution of temperature with corresponding distribution of k_{fgm} for various values of *n*, (c) Radial distribution of thermal resistance with corresponding radial distribution of k_{fgm}, (d)&(e) Radial distribution of temperature for various values of *n*, (f) Comparison of obtained results with Celebi[3] and Jabbari[4].

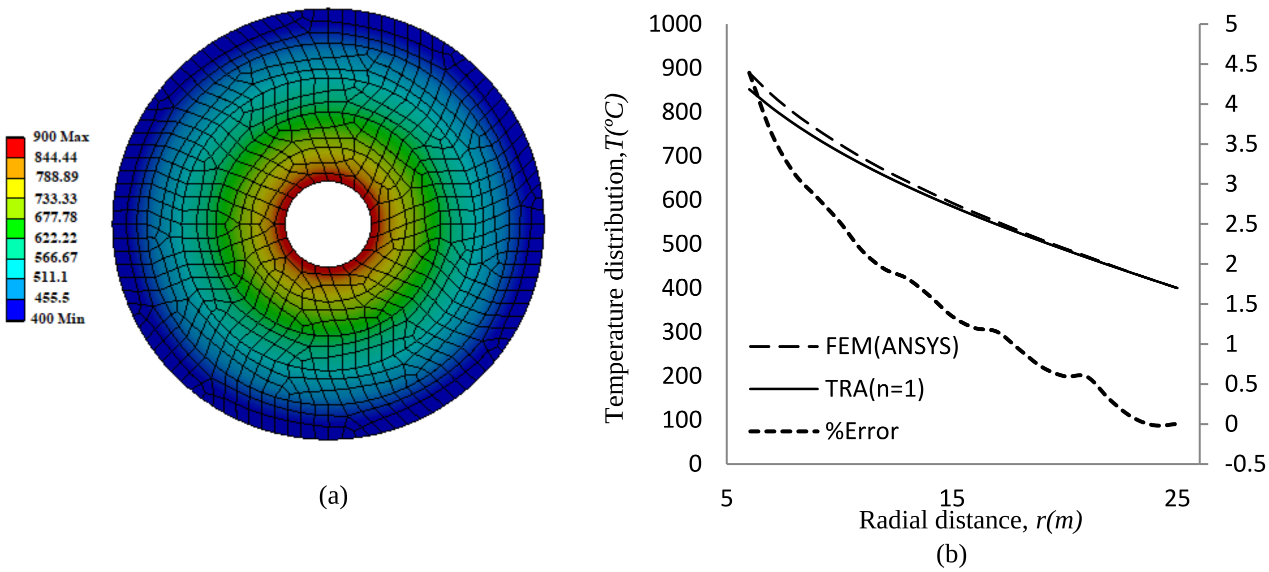


Fig. 3 (a) Temperature distribution, (b) Comparison of TRA results with FEM (ANSYS)

Table.2. shows comparison among results from this work with [3] and [4] for *β=0,-2*. Celebi [3] and Jabbari [4], [5] & [7] used a different power law where material property distribution follows an exponential function,

$$k(r) = K_c e^{\beta r} \tag{7}$$

Where K_c is a material constant and β is the inhomogeneity parameter. To obtain results a thick hollow cylinder was considered with $r_i = 1\, meter$ and $r_o = 1.2\, meters$ and boundary conditions were $T_i = 10^0 C\ and\ T_o = 0^o C$. From the results illustrated in Table-2 it can be observed that results obtained by thermal resistance approach is well supported by Jabbari [4] who obtained direct solutions and Celebi [3] who used Complimentary Functions Method or CFM. Figure 3(a) is the meshing region and temperature distribution obtained from ANSYS solver. The geometry specimen was divided into 5 elements considering each element homogenous and thermal conductivity of each element was obtained by taking average of the values at each two boundaries from (1). Boundary condition was same as thermal resistance approach. Figure 3(b) shows us the comparison of results obtained from ANSYS solver and the thermal resistance approach. It can be observed that moving from inner radius to the outer, results obtained by the ANSYS solver converges to TRA results and percentage of error drops from slight less than 4% at the innermost surface to 0% at the outer most surface.

5. Conclusion

This study presents solutions to the temperature distribution problem of FGM hollow cylinders by thermal resistance approach. Material gradient is assumed to follow a power law function and solutions are presented for different values of power law index. An analytical solution has also been obtained for n=1 and results have been compared with each other. For another case, the obtained results have been compared with some well-established data in the literature. In every case, the results well complied with the others. Results have also been obtained under same physical conditions using ANSYS solver which converged to obtained solutions from TRA. In conclusion, though Thermal Resistance Approach gives a discontinuous solution, this approach is undoubtedly more flexible than other methods present in the literature. Dividing the geometry into more constituent layers provides a closer solution towards continuity. The advantage of this approach over other methods is that, it allows us to get solution for any value of power law index (n) when exact or analytical solution is very complex due to non-linearity.

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