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Water Based Ferrofluid and Nanofluid Mixed Convective Flow through a Square Channel: Effect of Magnetic Induction

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Abstract

In this paper, a numerical investigation has been performed to investigate the effect of magnetic induction on the mixed convective flow of water based nanofluids through a square channel containing a heated block. The governing equations of the flow model are transformed into a dimensionless form and solved numerically employing the finite element method based on Galerkin's weighted residual technique. The obtained numerical results are presented in terms of streamlines and isotherms inside the channel. In addition, the variation of average Nusselt number and average temperature within the channel for three kinds of fluid are examined for the effect of magnetic induction. Results reveal that the magnetic field at a fixed solid volume fraction of Fe_3O_4/water and Al_2O_3/water nanofluids together with base fluid play a significant role on the flow field and temperature distribution. It is found that the heat transfer rate decreases significantly with an increase of magnetic field intensity.

Keywords: Magnetic induction; Mixed convection; Ferrofluid; Nanofluid; Finite element method.

1. Introduction

Buoyancy driven flow and heat transfer characteristics in vented channel is an interesting research area due to its numerous applications in engineering fields such as cooling of electronic devices, high performance building insulation, furnaces, lubrication technologies, chemical processing equipments, solar heat collectors, drying technologies etc. on the other hand, the presences of centered obstruction modify the above phenomena which extends its engineering and industrial applications. Based on such geometrical importance in engineering many researchers have recently studied the heat transfer problems.

Nasrin and Alim [1] carried out a control volume finite element solution for forced and natural convection flow in a vertical channel with a heat generating circular cylinder and found that the heat transfer rate strongly depend on the governing parameters. The problem of natural convection heat transfer and fluid flow in a vertical closed chamber filled with Al_2O_3/water nanofluid analyzed by Nasrin *et al.* [2]. Later on, the authors [3] also studied free convective flow and thermal behaviors of nanofluid inside a complicatedcavity consists of a centered heated diamond shaped hollow obstacle.

Magnetic field effects on the fluid flow and temperature fields have received much attention due to its technological importance in engineering such as coolers of nuclear reactors, micro-electronic devices and purification of molten metals. Rashidi *et al.* [4] studied numerically mixed convection heat transfer of nanofluid flow in a vertical channel with sinusoidal walls under the effect of magnetic field and indicated that the average Nusselt number and Poiseuille number increases by increasing the Hartman number. Kareem *et al.* [5] performed a numerical study of mixed convection heat transfer in a two-dimensional trapezoidal lid-driven enclosure filled with nanofluids and found that the Nusselt number increases due to the increasing volume fraction, but it decreases with the greater diameter of the nanoparticles. Javed *et al.* [6] analyzed the numerical results of heat transfer for mixed convection in an incompressible steady lid-driven fluid flow inside a trapezoidal cavity in the presence of a uniform magnetic field.

Ferrofluid is a liquid colloidal suspension of magnetite nanoparticles in base fluid that becomes strongly magnetized in the presence of a magnetic field. Sheikholeslami and Gorji-Bandpy [7] employed Lattice Boltzmann Method to obtain numerical solutions for the free convection flow of ferrofluid in a cavity heated from bottom wall in the presence of external magnetic field. The results of [7] indicated that the thermal boundary layer thickness and Nusselt number increased with the increase of volume fraction, Rayleigh number and heat source length respectively. Gul *et al*. [8] presented a numericall study of mixed convection flow of ferrofluid in a vertical channel and their results showed that the temperature and velocity of ferrofluid depend strongly on the thermal conductivity along with magnetic field. Jhumur and Saha [9] analyzed unsteady mixed convection flow in a T-shaped ventilated cavity filled with single phase ferrofluid under the influence of magnetic field.

To the best of authors' knowledge, no attention has been afforded to the problem of water based ferrofluid and nanofluid mixed convective flow through a square channel under the influence of magnetic field. The objective of this study is to examine numerically the flow and heat transfer behavior in a vented channel filled with ferrofluid and nanofluid. Steady two dimensional mixed convection has been induced by subjecting well thermal conditions. The numerical solution is obtained for inlet velocity, specified volume fraction with different Hartman number and Richardson number and detail discussion is presented in the following section.

2. Physical Model

We consider a two dimensional square channel with side length L and a square block of length $0.2L$ is placed at the centre of the enclosure. The bottom wall and the sides of the block are maintained at a uniform temperature, T_h, while the remaining walls are kept perfectly insulated. The directions of inflow and outflow of the channel are located at the bottom of the left wall and at the top of the right wall, respectively. The inflow state is defined by (u_i, T_i) while the outflow is preserved by zero diffusion flux. The gravitational force, g acts in the downward direction. A uniform magnetic field of strength B_0 is applied along the x-direction normal to the side wall. The thermo-physical properties of the considered base fluid and nanoparticles are specified in Table 1. Figure 1 shows a schematic diagram of the modeled enclosure.

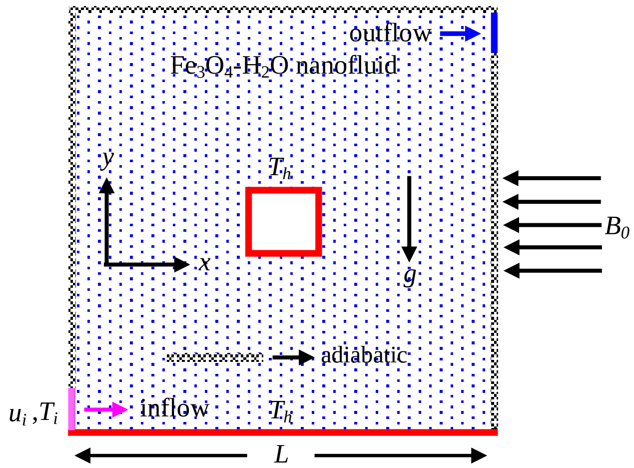

Fig.1. Schematic diagram of the physical model

3. Mathematical formulation

In the present problem, the flow in the channel is assumed to be steady two dimensional, laminar, incompressible and viscous dissipation effects is ignored. Under the usual Boussinesq approximation, the governing equation of the present problem can be defined as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{1}{\rho_{nf}}[g\,\beta_{nf}\,\rho_{nf}\,(T-T_c)]-\frac{1}{\rho_{nf}}[\sigma_{nf}B_0^2 v] \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Where $\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s$ [10] is the density, $(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s$ [10] is the heat capacitance, $\beta_{nf}=(1-\phi)\beta_f+\phi\beta_s$ [10] is the thermal expansion coefficient, $\alpha_{nf}=k_{nf}/(\rho C_p)_{nf}$ is the thermal diffusivity, $\sigma_{nf}=\sigma_f\,(1+((3((\sigma_s/\sigma_f)-1)\phi)/(((\sigma_s/\sigma_f)+2)-((\sigma_s/\sigma_f)-1)\phi))$ [10] is the electrical conductivity, $k_{nf}=k_f\,((k_s+2k_f-2\phi(k_f-k_s))/(k_s+2k_f+\phi(k_f-k_s)))$ [10] is the thermal conductivity and $\mu_{nf}=\mu_f\,(1-\phi)^{-2.5}$ [11] is the dynamic viscosity of nanofluid and for hybrid nanofluid [3], $\rho_{nf}=(1-\phi_1-\phi_2)\rho_f+\phi_1\rho_{s1}+\phi_2\rho_{s2}$ is the density, $(\rho C_p)_{nf}=(1-\phi_1-\phi_2)(\rho C_p)_f+\phi_1(\rho C_p)_{s1}+\phi_2(\rho C_p)_{s2}$ is the heat capacitance, $\beta_{nf}=(1-\phi_1-\phi_2)\beta_f+\phi_1\beta_{s1}+\phi_2\beta_{s2}$ is the thermal expansion coefficient, $\alpha_{nf}=k_{nf}/(\rho C_p)_{nf}$ is the thermal diffusivity, $\sigma_{nf}=\sigma_f\,(1+(\frac{(3((\sigma_{s1}/\sigma_f)-1)\phi_1+3((\sigma_{s2}/\sigma_f)-1)\phi_2)}{(((\sigma_{s1}+\sigma_{s1})/\sigma_f)+2)-((\sigma_{s1}/\sigma_f)-1)\phi_1)-((\sigma_{s2}/\sigma_f)-1)\phi_2)}))$ is the electrical conductivity, $k_{nf}=k_f\,(\frac{(k_{s1}+k_{s2}+2k_f-2\phi_1\,(k_f-k_{s1})-2\phi_2\,(k_f-k_{s2}))}{(k_{s1}+k_{s2}+2k_f+\phi_1\,(k_f-k_{s1})+\phi_2\,(k_f-k_{s2}))})$ is the thermal conductivity and $\mu_{nf}=\mu_f\,(1-\phi_1-\phi_2)^{-2.5}$ is the dynamic viscosity of nanofluid. Here nanofluid is calculated by seting ϕ_1 in place of ϕ and for ferrofliud, ϕ is replaced by ϕ_2.

Table1. Thermo physical properties of base fluid and nanoparticles [3 , 8]:

Physical properties	Base fluid	Al_2O_3	Fe_3O_4
C_p (J/kgK)	4179	765	670
ρ(kg/m^3)	997.1	3970	5180
k(W/mK)	0.613	40	9.7
β(1/K)	2.1×10^{-4}	0.085×10^{-4}	0.5×10^{-5}
σ (W/mK)	0.05	10^{-12}	0.112×10^{-6}

To obtain non-dimensional governing equations, we incorporate the following dimensionless dependent and independent variables:

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ U=\frac{u\,L}{\alpha_f},\ V=\frac{vL}{\alpha_f},\ P=\frac{p\,L^2}{\rho_{nf}\,\alpha_f^2}\ \text{and}\ \theta=\frac{T-T_i}{T_h-T_i} \tag{5}$$

Introducing the above relations into the equations (1)-(4), we obtained the non-dimensional governing equations are written as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{6}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{\nu_{nf}}{\alpha_f}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{7}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{\nu_{nf}}{\alpha_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\left(\frac{\beta_{nf}}{\beta_f}\right)Ri\,\theta-\left(\frac{\rho_f}{\rho_{nf}}\right)\left(\frac{\sigma_{nf}}{\sigma_f}\right)\frac{Ha^2}{\mathrm{Re}}V \tag{8}$$

$$U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f PrRe}\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right) \tag{9}$$

Where $Pr = \frac{\nu_f}{\alpha_f}$ be the Prandtl number, $Ha^2 = \frac{\sigma_f B_0^2 L^2}{\rho_f \nu_f}$ be the Hartmann number, $Re = \frac{U_0 L^3}{\nu}$ be the Reynolds number $Gr = \frac{g\beta_f \Delta T L^3}{\nu^2}$ be the Grashof number and $Ri = \frac{Gr}{Re^2}$ be the Richardson number.

The appropriate boundary conditions for the present problem can be written in dimensionless form as:
at the solid walls: $U = V = 0$, at the inlet: $U = 1, V = 0, \theta = 0$
at the outlet: Convective boundary condition and P = 0, at the bottom wall and the side walls of the block: $\theta = 1$
at the left, right and top walls: $\frac{\partial\theta}{\partial X} = \frac{\partial\theta}{\partial Y} = 0$
the average Nusselt number at the heated wall can be calculated by the following expression:

$$Nu = \frac{1}{L}\int_0^1 \overline{Nu}\, dX \tag{10}$$

and the average temperature of the fluid domain inside the cavity can be defined as follows:

$$\theta_{av} = \int (\theta/\overline{V})\, d\overline{V} \tag{11}$$

where $\overline{V}$ is the volume of the enclosure.

4. Grid refinement test

In order to get the appropriate mesh system, a grid independence test is performed for a square channel at respective values of Pr = 1.47, ϕ = 1% , Ri = 1 and Re = 50. Five different meshes were used to complete the grid refinement test where the mesh size of 13083 nodes and 25706 elements provided a satisfactory solution for the present physical problem. The comparison of the numerical values of average Nusselt number is shown in the Table 2 and Fig. 2 and the initial mesh mode for the present numerical simulation is presented in the Fig.3.

Table 2. Grid sensitivity test at Pr = 1.47, ϕ = 1% , Ri = 1 and Re = 50.

Nodes (Elements)	Ns: 2481 Es: 4550	Ns: 3215 Es: 6014	Ns: 5245 Es: 10058	Ns: 13083 Es: 25706	Ns: 15081 Es: 29658
Nu_{av}	5.82478	5.83029	5.84246	5.84347	5.86089

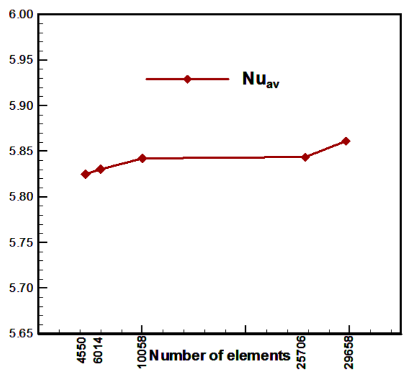

Fig.2. Grid refinement test.

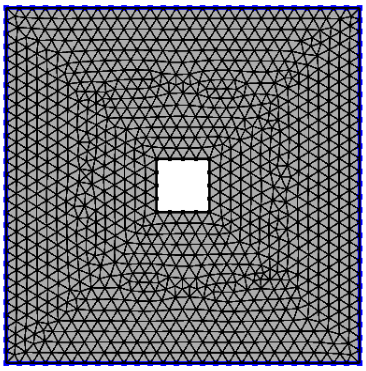

Fig. 3. Mesh generation of the square channel.

5. Code validation

The present numerical code has been verified by comparing the obtained numerical results with the published results of Pirmohammadi *et al.* [12], which is performed for the base fluid along with the finite elements scheme. The finding of this comparison is presented in **Table-3** and found to be a good agreement. Code validation of this study has been boosted up the confidence level in the numerical simulation.

Table 3. Comparison of average Nusselt number for Pr = 0.70 and various values of Ha.

Ra	*Ha*	Pirmohammadi *et al.* [12] (Nu_{av})	Present study (Nu_{av})	Error (%)
10^4	0	2.29	2.245	2.0
	10	1.97	1.928	2.2
	50	1.06	1.037	2.2
	100	1.02	1.003	1.7

6. Results and discussion

In this section, the influence of two controlling parameters obtained for mixed convection flow of nanofluid in a vented channel, namely, Hartmann number and Richardson number are analyzed through streamlines and isotherm contours. The range is considered as $0 \le Ha \le 100$ and $0.1 \le Ri \le 10$ while the remaining parameters are kept fixed at Pr = 1.47, Re = 50 and $\phi = \phi_1 = \phi_2 = 3\%$.

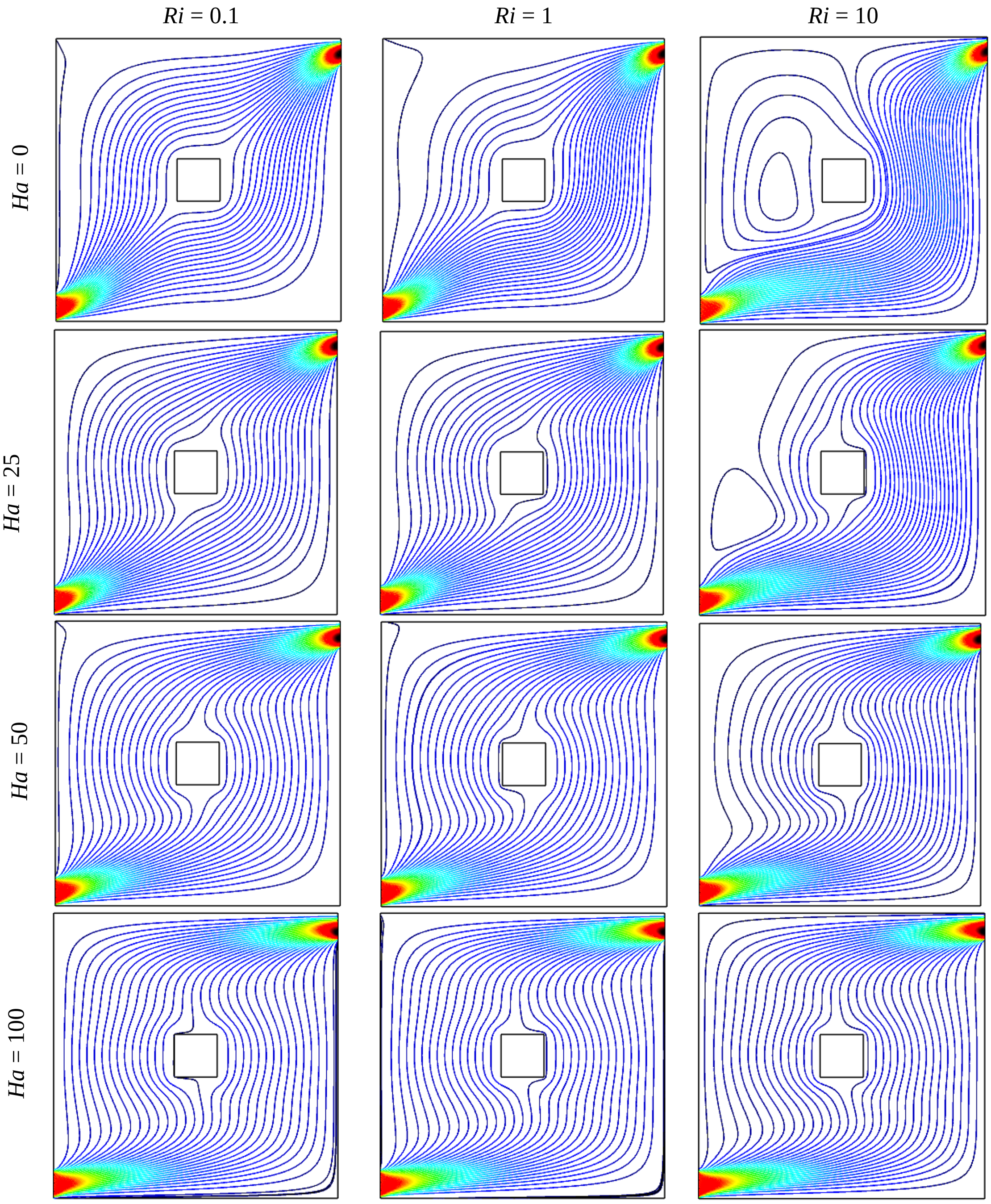

Fig. 4. Streamlines for the effect of magnetic field with different Richardson number.

Figure 4 shows the influence of magnetic field on streamlines at different Richardson number with certain values of controlling parameters. From these figures, it is observed that the flow pattern inside the channel appear as an onion shaped from entry to exit port of the enclosure and these shapes are elongated due to increasing strength of the magnetic field at Ri = 0.1occupying entire regions of the enclosure. In addition, the flow fields in pure mixed convection regime (Ri = 1) are almost similar to the force convection regime. It is interesting to note that the trend of flow patterns changed rapidly from mixed convection to free convection dominant region. A counter clockwise rotating circulation is developed occupying the centered block of the cavity and this vortex is reduced in size with increasing Ha which is disappearing for the greater effect of Ha. Moreover, an increasing in Ha leads to a decrease the strength of fluid current. This is because; applied magnetic field creates Lorentz forces that slow down the fluid motions within the enclosure. Thus, the flow fields strongly depend on the effect of magnetic field.

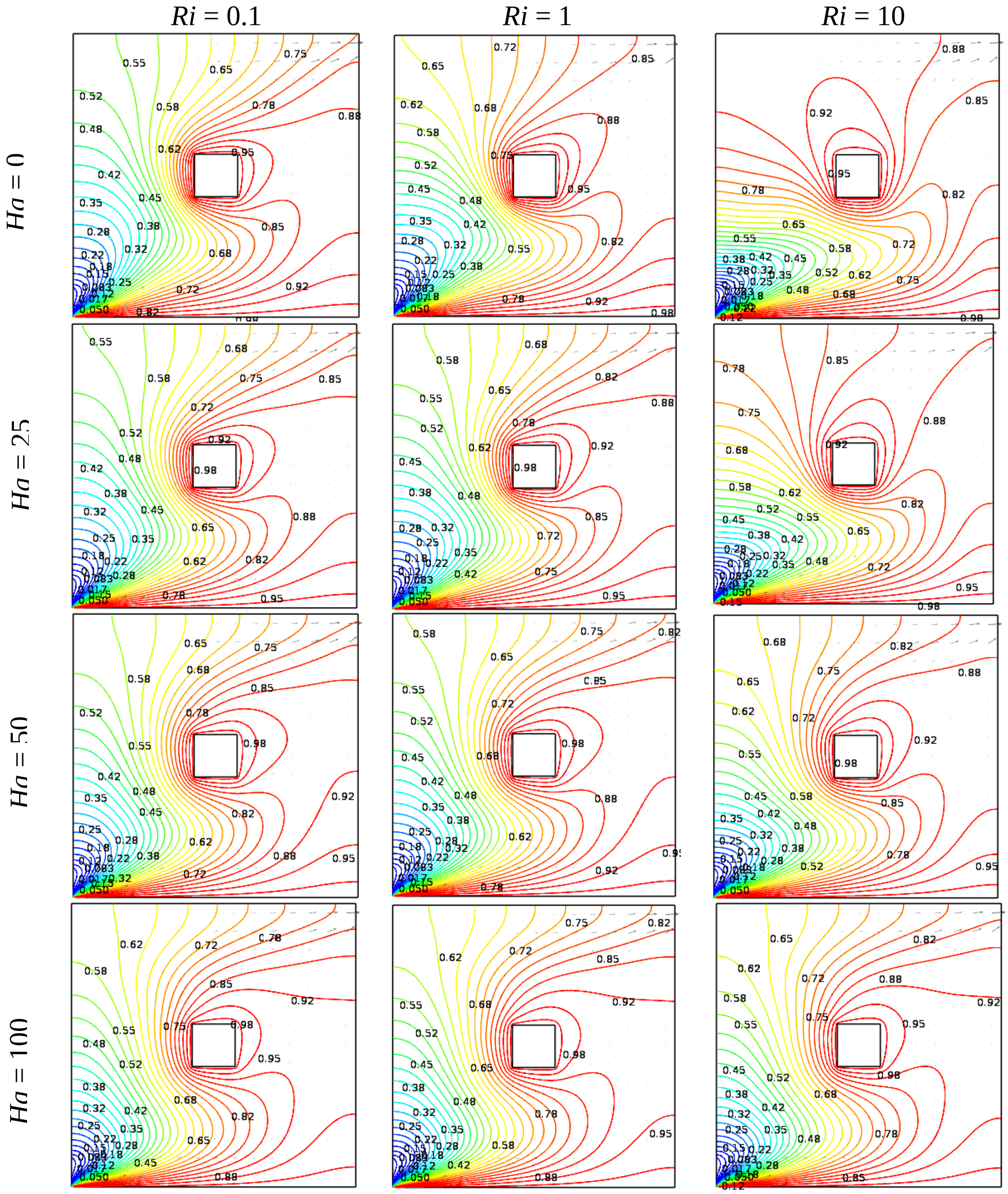

Fig. 5. Isotherms for the effect of magnetic field with different Richardson number.

Figure 5 presents the temperature contours of ferrofluid for the effect of magnetic parameter, *Ha* at various Richardson numbers. The isotherm contours are spread within the entire channel for the selected values of *Ha* and condensed close to the heated walls near to the inlet port of the enclosure. It is clear from these figures that with the increasing of *Ha*, the isotherms are less affected. Moreover, significant change is observed in the convection dominant region than mixed convection region because the buoyancy force due to greater *Ri* becomes more predominant than conduction at lower *Ri*.

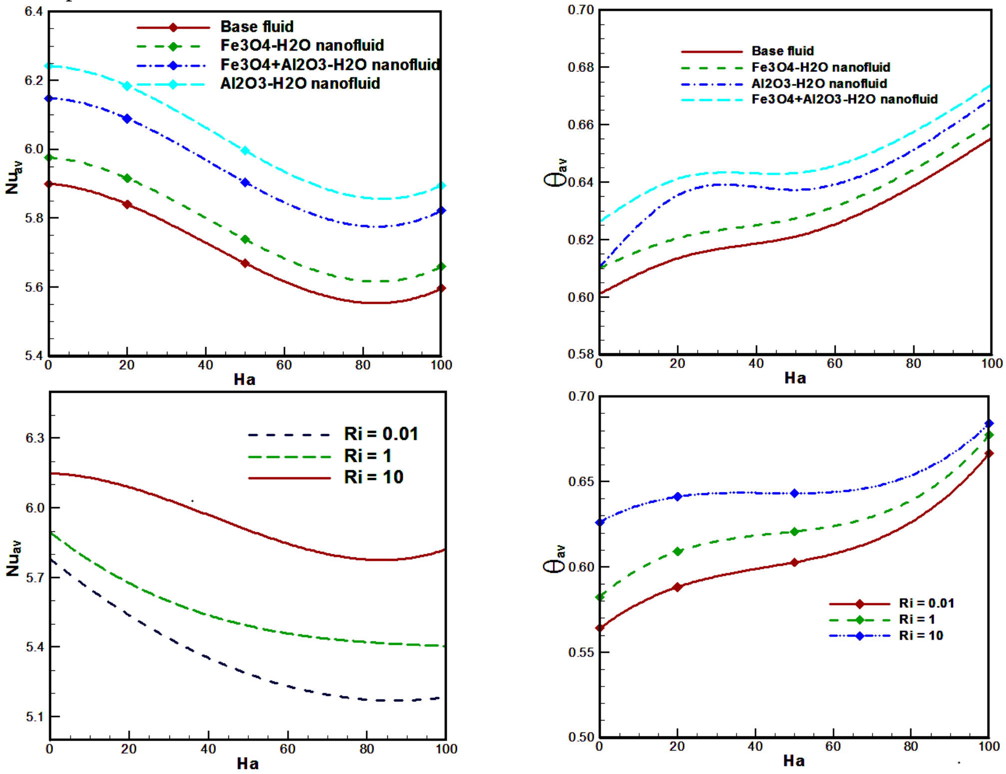

Fig. 6. (a) Average Nusselt number (left) and (b)average temperature (right) for various Hartmann number, nanofluids and Richardson number.

The variation of average Nusselt number on the heated bottom wall of the cavity for different *Ha*, nanofluids and *Ri* has been presented in the Fig. 6(a). From these figures, it is clear that the average Nusselt number is an increasing function of *Ri* and decreasing function of *Ha* in every case of base fluid, ferrofluid and nanofluid. This is because, greater *Ri* causes convection dominant region while *Ha* increases fluid temperature. In addition, the heat transfer rate in terms of the average Nusselt number for water based nanofluid containing magnetite nanoparticles (Fe_3O_4) and non-magnetite nanoparticles (Al_2O_3) is found to be more effective than the base fluid. On the other hand, Fig. 6(b) represents the variation of average temperature of nanofluid within the channel for the different values of *Ha* and *R*i. It can be observed from these figures that average temperature is increasing for both parameters, *Ha* and *Ri*. Moreover, mean temperature also increases for using magnetite nanofluid and non-magnetite nanofluid. It is well known that rising *Ha* increases fluid temperature. It is important to note that the effect of magnetic field is more noticeable while magnetite nanoparticles and non-magnetic nanoparticles suspended together in base fluid.

7. Conclusion

A numerical study is performed for mixed convection flow of water based nanofluid and ferrofluid in a square channel with central heated block. A detail parametric analysis is done for the streamlines, isotherms, average Nusselt number on the heated wall and average temperature of the nanofluid inside the vented enclosure to show the effects of selected governing parameters. From this discussion and the graphical presentation it can be concluded that the flow field is affected significantly for the effect of magnetic field parameter. The value of average Nusselt number is highest in the convection dominant region with a greater Richardson number in the presence of lower magnetic field effect. Moreover, the average temperature of nanofluid inside the channel increased due to the higher *Ha* and *Ri* with lower *Re*. In addition, overall heat transfer rate is more prominent in nanofluid and ferrofluid than base fluid.

8. References

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