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Finite Element Analysis of Bio-heat Transfer Problems with Different Heating Condition on Skin Surface

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Abstract:

Most of the therapy aims to raise the temperature of the cancerous tissue above a therapeutic value to kill or destroy it. One of the main concerns of these therapies is to minimize the damage of the unaffected cells. Therefore acknowledgement of temperature profile of living tissue is necessary before the treatment starts. This needs the heat transfer analysis of biological body. Pennes bio-heat equation represents the heat transfer model of the human body. In this paper, a one-dimensional finite element model has been developed to analyse the bio-heat transfer problems based on the Pennes equation. A C program has been coded to solve the developed model. Using the developed program, the temperature profile in the human body subject to constant and sinusoidal spatial heating on skin surface has been computed. The point and stochastic heating on the skin surface are also investigated. Numerical results are compared with the analytical result for the steady state case. The developed finite element code can be used to measure the temperature of human tissue with different heating and cooling conditions.

Keywords: Finite element, Bio-heat, Pennes equation, Spatial heating.

1. Introduction:

Heat transfer in living tissue has become a very interesting topic for scientists and engineers because of its broad application in bioengineering and designing of medical equipments. Many therapeutic applications need the proper thermal description of the human body. Again many medical operations rely on engineering methods to determine safety and risk level involved in surgeries. At present mathematical modelling of Bio heat transfer is widely used in treating tumours, cryosurgery, laser eye surgery and many other applications. The success of hyperthermia treatment greatly depends on the proper knowledge of heat transfer in blood perfused tissue [1].

Among the various model proposed to study the heat transfer in biological bodies, the most widely used ine is proposed by Pennes [2]. Though initially it was developed to predict heat transfer in human forearm later it was implemented in various biological research works due to its simplicity i.e. uniform thermal conductivity, perfusion rate and metabolic heating.

Numerous analytical and numerical solutions of the bio-heat transfer problem are found in the literature. Analytical solution using Green function is presented in [3]. In some cases time dependent surface heat flux, oscillatory heat flux and sometimes cooling of the skin [3] are considered as boundary conditions. In some numerical analysis sinusoidal spatial heating, point heating [4] was used as external heating.

In this paper a finite element model (FEM) [5] has been developed for the numerical solution of the 1D unsteady Pennes equations with different spatial heating and boundary conditions. The Crank-Nicolson method is used for the time discretization. A C code has been developed which can be used to measure the temperature distribution in the human tissue.

2. Bio Heat Transfer:

For the study of bio-heat transfer in human tissue the most useful one is Pennes equation which is:

$$c\rho \frac{\partial T}{\partial t} = \left\{ \frac{\partial}{\partial x}\left(K \frac{\partial T}{\partial x} \right) + \omega_b \rho_b C_b (T_a - T) + Q_m + Q_r \right\} \tag{1}$$

where ρ, c, K are respectively the density, the specific heat, and the thermal conductivity of the tissue; ρ_b, C_b denote density and specific heat of blood; ω_b the blood perfusion; T_a the known arterial temperature, and $T(x,t)$ is unknown tissue temperature; Q_m is the metabolic heat generation, and $Q_r(x,t)$ the heat source due to spatial heating with respect to time t.

For a one dimensional problem of length L, let $T(x,o) = T_0(x)$ is initial temperature, T_c is the body core temperature, h_0 is the heat convection coefficient between the skin surface and the surrounding air, T_f is the surrounding air temperature. Thus the boundary conditions

$$T = T_0(x) = T_c, \quad \text{at } x = L \tag{2(a)}$$

$$-k \frac{dT_0(x)}{dx} = h_0[T_f - T(x)], \quad \text{at } x = 0 \tag{2(b)}$$

In some cases boundary condition is time dependent .So time dependent boundary conditions can be introduced as

$$-k \frac{dT_0(x)}{dx} = f_1(t) \quad \text{at } x = 0 \tag{3}$$

Where $f_1(t)$ is the time dependent surface heat flux. Here, the skin surface is defined at $x = 0$ and the body core at $x = L$.

3. Finite element discretization:

The first step of the finite element discretization is to develop a weak form that is a weighted-integral statement and is equivalent to both the governing equation as well as certain type of boundary conditions. The simplest form of the equation (1) is

$$c\rho \frac{\partial T}{\partial t} - \left\{ \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) + CT + q \right\} = 0 \tag{4}$$

Where $C = w_b \rho_b C_b$ and $q = CT_a + Q_m + Q_r$

The weak form of the differential equation (Applying weighted residual method) is derived as

$$\int_{xa}^{xb} \left[Wc\rho \frac{dT}{dt} - k \frac{dW}{dx}\frac{dT}{dx} + CWT - Wq \right] + (WQ)x_a + (WQ)x_b = 0 \tag{5}$$

Where W is the weighted function and Q is the secondary variable.

In this paper a linear element is considered whose temperature function is given by

$$T_h^e(x) = \sum_{j=1}^{2} \varphi_j^e(x) T_j^e \tag{6}$$

Using the linear approximation function of equation as (6) finally a linear equation was derived of the following form

$$[C]\{\dot{T}\} + [K]\{T\} = \{q\} + \{Q\} \tag{7}$$

Where C is the capacitance matrix, K is heat conductive matrix and T is unknown temperature and others are known vectors.

4. Time Discretization Scheme

A simple time integration scheme to solve equation (7) was derived by assuming that C and K are constant. In such case, matrix differential equation can be discretized on time as:

$$C\frac{T^{n+1}-T^{n}}{\Delta T}+\alpha K T^{n+1}+(1-\alpha)KT^{n}=0 \tag{8}$$

Where T^{n+1} and T^{n} are the vectors of unknown nodal values at times $n\Delta T$ and $(n+1)\Delta T$ respectively. α is a weighting factor which must be chosen in between 0 and 1. In equation (8) the standard approximation for time derivative was used.

$$\dot{T}=\frac{T^{n+1}-T^{n}}{\Delta T}$$

When the value of α is considered 0.5, the process is called the popular Crank-Nicolson method. The discretized equation (8) can be written as:

$$\left(\frac{1}{\Delta T}C+\alpha K\right)T^{n+1}=\left(\frac{1}{\Delta T}C-(1-\alpha)K\right)T^{n}+Q+q \tag{9}$$

The equation (9) was solved using an iterative procedure. The initial temperature is known and then temperature of the next step can be calculated from the solution of equation (9) via the Gauss elimination technique.

5. Analysis and Numerical Results:

A FEM code has been developed using C language to solve the numerical solution of the finite element model described in the previous section. In this paper, a tissue of length 3mm from the skin surface is considered for the calculation. The tissue properties and parameters of the boundary conditions (Table 1) are applied as given in [3, 6].

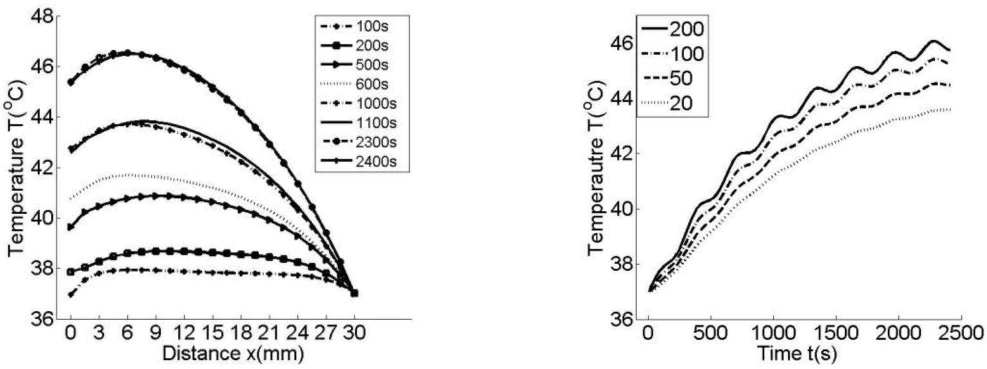
The initial temperature (at t=0) of all nodes is considered as constant (37°C). As the body core temperature remains constant at all-time thus the body core temperature is considered as 37^0 C.

Table 1.Thermal Properties

Parameters	**Symbol**	**value**
Thermal conductivity	K	0.5 w/m^2
Convection Coefficient	h_o	10 w/m^2
Forced convection coefficient	h_f	100 w/m^2
Environmental Temperature	T_f	25 °C
Temperature of the Artery	T_a	37 °C
Body core temperature	T_c	37 °C
Metabolic heat generation	Q_m	33800 w/m^2
Density of blood	ρ_b	1000 kg/m^3
Density of tissue	ρ	1000 kg/m^3
Specific heat of blood	C_b	4200 J/kg.°C
Specific heat of tissue	C	4200 J/kg.°C
Blood perfusion	ω_b	0.0005 ml/s/ml

Investigation-1: Surface adiabatic condition and sinusoidal spatial heating

In case of heating by laser, microwave, the expression for specific absorption rate can be simplified as $Q_r=\eta P_0(t)\exp(-\eta x)$ in which heat flux decays exponentially with respect to distance from the skin surface [6, 7]. Here $P_o(t)$ is the time dependent heating power on skin surface and η is the scattering coefficient.



(a) (b)

Fig. 1. Temperature distribution subjected to sinusoidal spatial heating of human tissue;

(P_o (t)=250+200cos(0.02t) W/m^2); (a) at different times; (b) at different scattering coefficient.

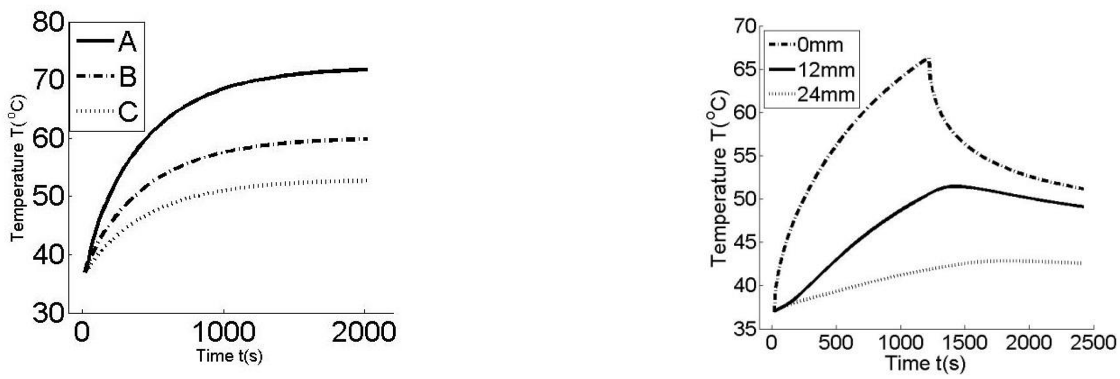
Fig. 1(a) shows the temperature distribution with respect to distance from the skin surface at different times. In this figure there is cross of temperature curve at different times which indicates the temperature oscillation inside tissue. Fig. 1(b) shows the effect of scattering coefficient (for four different values of scattering coefficient). Clearly this curve indicates the sinusoidal effect and also demonstrates that the higher frequency results in higher amplitude.

Investigation-2: Surface heating

Here, we analysed the thermal behaviour of living tissue subjected to time dependent surface heat flux. This is very beneficial in case of thermal injury analysis i.e. thermal burn analysis [8].The calculated tissue temperature for three different constant surfaces heating is shown in Fig. 2(a). Curves A, B, C indicates the temperature at f_1 = 1000 W/m^2, f_1 = 500 W/m^2, f_1 = 200 W/m^2 respectively (when P_o(t)=250). These curves indicate that the larger heat flux brings higher temperature. Moreover temperature increases as time increases.

Transient temperature at three positions subject to surface step heating (Considering Q_r=0) is shown in Fig 2(b).

Where $f_1(t)= \begin{cases} 1000 \ \frac{w}{m2}, t \leq 1200s \\ 0 \ \frac{w}{m2}, t > 1200s \end{cases}$. The practical application of these types of heating can be found in eye surgery or skin burn due to a flash fire, hot plate, liquid or hot gas for a short period [8].



(a) (b)

Fig. 2. Tissue temperature response on surface heat flux; (a) Constant surface heat flux; (b) Step heating

Fig. 2(a) indicates that larger heat flux brings higher temperature. Moreover temperature increases as time increases. Fig. 2(b) shows that as time increases temperature also increases but this becomes reverse after 1200s when $f_1(t) = 0$.

Investigation-3: Effect of heating frequency and blood perfusion

The calculated result for different heating frequency and blood perfusion is shown in Fig. 3 when simultaneously constant spatial heating (when $P_o(t)=250$) and sinusoidal surface heating is applied. The sinusoidal heating at the skin surface can be expressed as $f(t)=q_0+q_w\cos(\omega_1 t)$. Where q_0 and q_w are the constant term and oscillation amplitude of sinusoidal heat flux respectively and ω_1 represents the heating frequency.
Here curve A and B we use blood perfusion as 0.0005, where in curve C & D it is 0.004. When in curve A and C we uses a heating frequency of value 0.02 where in B & D it is 0.01.

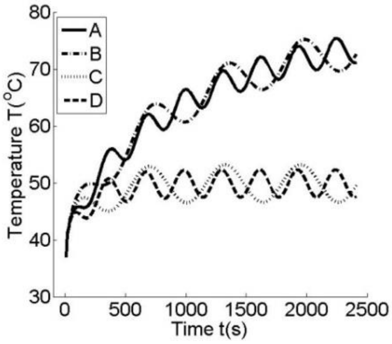


Fig. 3 Effect of heating frequency and blood perfusion on tissue temperature.

This figure indicates that higher blood perfusion leads to lower temperature. This curve also indicates that frequency of the temperature varies with that of the surface heating.

Investigation-4: Point and Stochastic heating

Point heating is adopted when a deep tumour is selected and treatment is applied to thermally kill it. In this heating the target region is heated up to a certain high temperature within a short period [9]. So acknowledgement of temperature distribution under such heating condition is necessary. In contrast to previous heating style where heat flux decays exponentially here the point heating term is expressed as $Q_r(x,t) = P_1(t)\delta(X - X_0)$. Where $P_1(t)$ is the point heating power and $\delta(x-x_0)$ is the Dirac delta function. $\delta(x-x_0)$ has a value 1 at $x = x_0$ and 0 elsewhere in x.

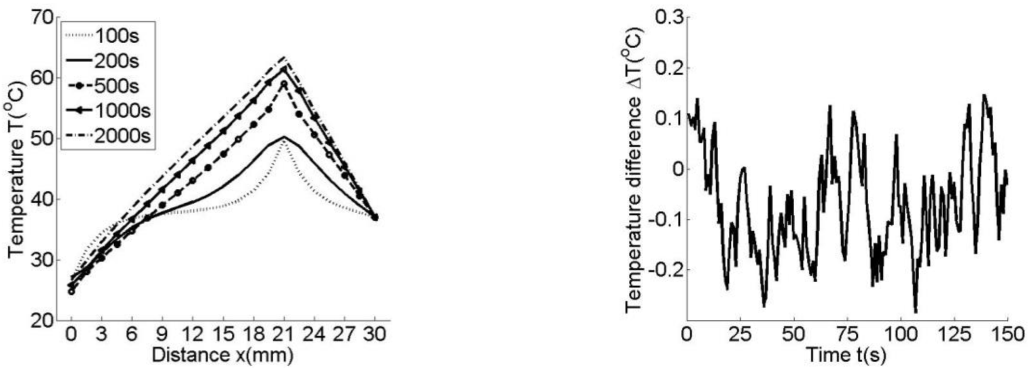


Fig. 4 Impact of point and stochastic heating on tissue temperature distribution;
(a) Point heating; (b) Stochastic heating;

In Fig. 4(a) temperature distribution at different times is shown where point heating with a point heat source of $P_1(t)$=2500 W/m^2 at a distance 21mm (x_0 = 21mm) below the skin surface is applied. In this figure at skin surface convection boundary condition is applied (h_f=100 W/m^2 and T_f=15 ºC).This figure demonstrates that due to point heat source the position of the maximum temperature remain constant at the site of the point source at different time. Here the maximum temperature is more than 60^0C thus it can be helpful in case of thermal ablation.

Previously we considered environmental temperature as a constant value but in practical cases it does not remain constant at all. It generally fluctuates over time so it is necessary to study the impact of such variance.

In Fig. 4(b) stochastic heating is shown where surrounding fluids temperature is usually a stochastic value and it is taken as $T_e = T_f + \lambda_t (0.05 - \sigma_i)$, where λ_t is a constant which is taken as 5^0C and σ_i the random number between 0 and 1. Clearly the figures demonstrate that due to irregular cooling medium temperature the tissue temperature fluctuates within a narrow range. This may be helpful in thermal comfort analysis as it indicates that the biological body trends to keep its temperature stable.

6. Conclusion:

A system for finite element analysis of bio-heat transfer has been developed. Various types of heating pattern i.e. spatial heating, surface heating, point heating, stochastic heating and their impacts are investigated and briefly discussed in this paper. The effect of heating frequency, blood perfusion and scattering coefficient are also shown. It is found that for destroying a target cell point heating is more suitable than other heating as it increases the temperature of the target region and it has relatively low impact on nearby unaffected cells. As heating apparatus such as laser or microwave may have different power and scattering coefficient. Results obtained in this paper can be used to select suitable apparatus. During treatment, fluctuation of environmental fluid temperature may be out of considered as its impact on tissue temperature is almost negligible as shown in stochastic heating. Results described in this paper could be useful to accurately predict the thermal behaviour of living tissues and can be extended to such application as parameter estimation, clinical treatment and medical protocols design and to optimize for surgical treatment.

7. Acknowledgement:

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