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Conceptual Study and Analysis of Neutron Diffusion and Moderation in Nuclear Reactor

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Abstract

Conceptual study on diffusion and moderation of neutron in nonmultiplying and multiplying medium in nuclear reactor have been conducted by using mathematical explanation, experimental data and graphical representation. The corresponding governing equations have been explained. A brief discussion on neutron and criticality of a reactor is carried on. Necessity of a moderator medium in slow reactors has been discussed. Comparative analysis of usefulness and drawbacks as moderator for different medium (Graphite, Beryllium, light water, heavy water and Lithium Fluoride) have been studied in aspects of different features. Mathematical example of calculating numbers of collision required for thermalization process is shown. A primary starter source is introduced in the reactor to start the initial fission reaction. Once critical condition is reached chain reaction is established. Neutron is diffused from high to low concentration area according to Fick's law of diffusion. While going through the moderator medium fast neutron are slowed down. It is found that Fermi age and thermal diffusion length of neutron is smaller in light and heavy water than graphite or beryllium. Thus light and heavy water are effective moderator and are used conveniently in nuclear reactors.

Keywords: Diffusion, Moderation, Nonmultiplying Medium, Multiplying Medium, Fermi Age Equation.

1. Introduction

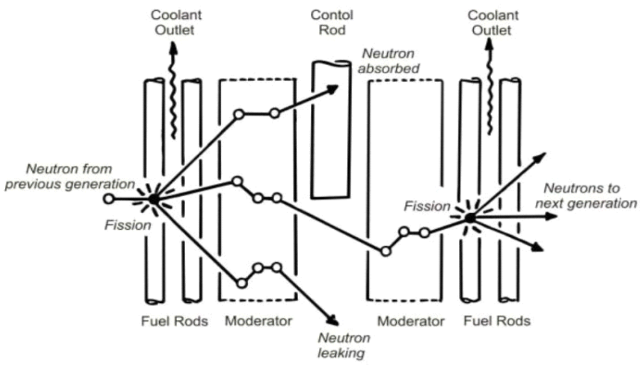

Fig. 1. Neutron life cycle [5]

Neutron generation is the prerequisite condition for a nuclear energy production. Different generation processes are used for different type of power plant. Relative advantages and disadvantages are needed to be analyzed to make sure the continuity of neutron generation. In a nuclear reactor fission of uranium-235 is induced by the absorption of neutrons by the uranium nucleus. Each nucleus may be divided into two new nucleus and on an average 2 or 3 neutrons are released per incident neutron. The neutrons strikes uranium and kinetic energy converts into heat energy. If one of the prompt neutron again strikes another U-235 nucleus a nuclear chain reaction will be established. In a controlled chain reaction in nuclear reactor, after achieving criticality multiple parallel fission reactions go on.

Neutrons produced from fission process have high energy and consequently high velocity. These are called fast neutrons. To increase the possibility of absorption of these neutrons in U-235 nucleus a moderation system is a must. Because fast neutrons do not affect the U-235, meanwhile it strikes the non-fissile U-238 with inelastic collision. The moderation process consists a suitable medium where many scattering collision with nucleus will reduce their energy to a state of thermal equilibrium within the medium. Commonly used moderator include

light water, beryllium, graphite, heavy water etc. But different type of moderation process have different degrees of advantages in various physical properties such as neutron age, slowing down length etc. Meanwhile cost is another crucial phenomena.
After getting thermally balanced, neutrons are called thermally balanced neutrons. At this lower velocity there is much more high probability of being absorbed by the fuel (U-235) to cause another fission. In design of the reactor leakage neutron must be considered. Neutron reducing phenomena can be controlled by employing a refractor around the reactor. This allows the neutron to diffuse in all direction.

2. Neutron diffusion

The net passage of neutrons from regions of higher neutron densities is called neutron diffusion. However diffusion of neutrons is a consequence of non-uniform density in the reactor assembly. Unfortunately determining the neutron distribution is a difficult problem. An approximation can be arrived at by solving the diffusion equation. This differential equation describes how high concentration solute diffuses to regions of lower concentration. This is similar to how neutrons behave in a reactor. Diffusion equation can be found out by the combination of two laws of physics:
1. Fick's law
2. Equation of continuity
Diffusion equation can be written as:

(1)

Where,
D = the diffusion co-efficient (in units of meter or centimeter)
Φ = neutron flux (in units of $cm^{-2}sec^{-1}$)
n - The number density of neutron at any point
s - The rate at which neutrons are emitted from sources per cm^3 in V
Σ_a - Macroscopic absorption cross-section
In the case of time independent problem the equation may be considered as:

(2)

This steady state diffusion equation is applicable to both multiplying i.e. system containing fissile materials that can produce additional neutrons as a result of neutron absorption and nonmultiplying systems, in which neutrons are introduced from an independent source(s=0).
In a real power reactor, there is no independent or extraneous source. In this case a production of neutron by fission is equal to neutron lost by absorption or by escape in different energy range in a given time. One of the basic properties to explain such case in a nuclear reactor is infinite multiplication factor, K_{eff} which is the ratio of rate of neutron production to rate of neutron absorption plus leakage.
If the value of K_{eff} is equal to 1; the power reactor is said to be critical which is desired; if less than 1 than said to be sub-critical and said to be supercritical if it is greater than 1. In the case of supercriticality, the condition may be out of control and accident may happen. On that case control rods which has a large absorption cross-section.
K_{eff} can be mathematically explained by 6 factor formula that is discussed below:

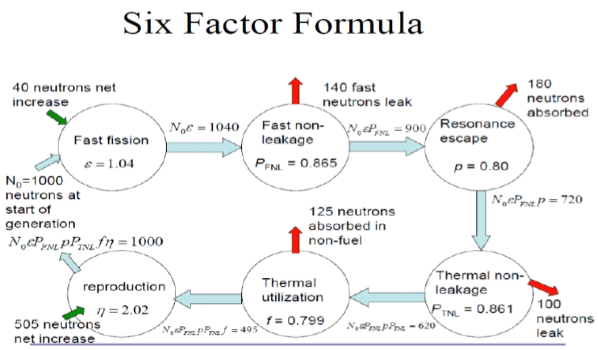

Fig. 2. Six factor formula

3. Neutron Moderation

The importance of a moderator:

The problem lies in the fact that whenever a thermal neutron causes fission it also leads to the release of fast neutrons. Now these fast neutrons have to be slowed down and brought to lower energy levels if they have to cause successful fission in turn. It is here that the concept of a moderator comes in the picture.

With high kinetic energy of fast neutron fission of U-235 nucleus is not likely to happen. A moderator is a medium which is used to absorb a portion of the kinetic energy of fast neutrons so that they come in the category of thermal neutrons which help to sustain a controlled chain reaction. To moderate a neutron kinetic energy is needed to be reduced from 2MeV to 0.025eV. Although moderators are necessary in most nuclear reactors this does not mean to say that all reactors require moderators. There is a special class of reactors known as fast reactors which do not use moderators [12] but depend on the use of fast moving neutrons for causing fission. Even otherwise it must be remembered that fast moving neutrons have lesser probability of getting absorbed and causing fission but it does not mean that they are incapable of causing the fission reaction. A fast moving neutron travels with a speed which is nearly in the region of 10% of the speed of light, while a thermal neutron travels with a speed which is typically of the order of a few kilometers per second. The loss of kinetic energy and slowing down of neutron can be expressed by Fermi age equation.

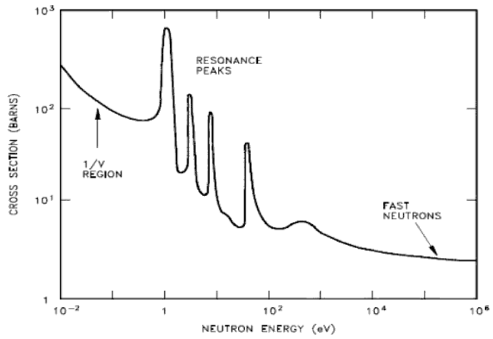

Fig. 3. [1]Fission cross section (in unit of [2]barns) of U-235 with different energy of neutron

1. Fission cross section is the probability of an incident neutron to cause fission in the target nucleus.
2. 1 barns = 10^{-24} cm^2.

Fermi age equation:

The complete slowing down diffusion equation is known as Fermi age equation. A detailed reactor theory combine Fermi age theory for non-thermal neutrons with diffusion theory for thermal neutrons. During the slowing down stage, neutrons cannot be treated like a collection of monoenergetic neutrons. Because they undergo considerable energy changes while diffusing. For Fermi age equation, in order to simplify calculations, it is assumed:

1. A continuous loss of energy for a slowing-down neutron.
2. Free path between collisions, λ_a considered constant and varies slightly with neutron energy.

Fermi age equation represents the spatial distribution of the slowing down density in the moderator. After some certain mathematical calculation with assumption, the Fermi age equation can be written as:

(3)

Where,

q - Slowing down density, the number of neutrons whose energy drops below a given energy E per second in a unit volume element of the medium.

τ - Fermi age or neutron age, slowing down area for neutrons calculated from Fermi age theory. Slowing down area is one sixth of the mean square distance from the source of a neutron in an infinite, homogeneous medium to the point at which the neutron reaches a given energy. It should be remembered that the Fermi age equation does not contain the time variable explicitly and that is therefore a time-independent or steady-state equation. Fermi age equation contains a complete description of neutron density distribution in both energy and space co-ordinates for neutrons undergoing moderation. In equation (3) the new variable τ is introduced as [8]:

(4)

If λ_s and λ_{tr} can be taken as constant over the slowing-down energy range or if they are replaced by suitable average values over the energy range integration of equation (4) in the process of having its energy reduced from E_0 to E is given by:

(5)

where,

(6)

Here, is the mean logarithmic reduction of neutron energy per collision. In this expression $C\lambda_s$ represents the total zigzag length of a neutron between the moment of its creation or the beginning of its slowing down and the moment of its arrival at energy E. If $\Lambda_s = C\lambda_s$; where, Λ_s is quite analogous to λ_a and the equation (5) may be written as:

(7)

Where L is called thermal diffusion length or slowing down length. Physically, it is a measure of the distance a fission neutron has traveled away from its point of creation by the time it reaches thermal energies.
Here,

(8); (9); (10)

Consider a mathematical analysis based on neutron age, τ_0 and slowing down length, L for fission neutrons of 2 MeV average energy to different thermal energy in graphite, beryllium, heavy water and light water by plotting graph. To solve this problem equations (6), (8), (9), (10) are substituted in equation (5) to get the following equation.

(11)

Table 1. Atomic or molecular weight (A) and Macroscopic cross-section (Σ_s) of some elements [4]

	Beryllium (Be)	Graphite (C)
A	9.013	12.011
Σ_s (cm^{-1})	0.865	0.385

Using these data in equation (11) a graph of Fermi age for varying energy of neutron for these four type of moderators may be produced. Now using equation , another graph of neutron energy versus slowing down length can be produced

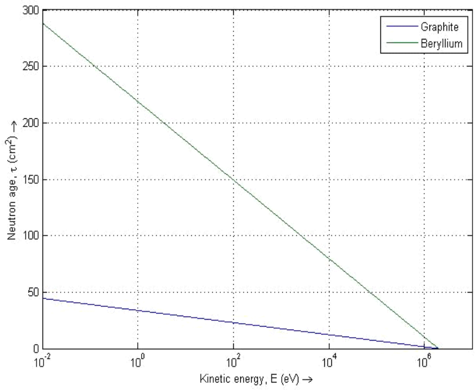

Fig. 4. Fermi age vs. Kinetic energy graph

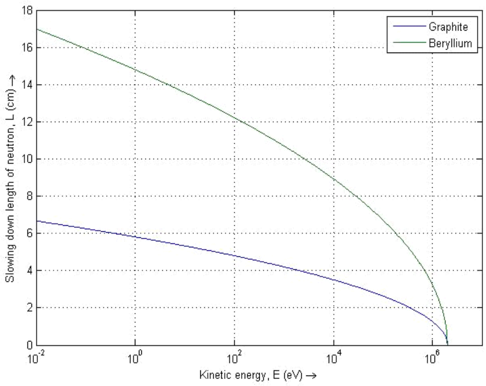

Fig. 5. Slowing down length vs. Kinetic energy

In moderation process, fast neutrons are collided with other particles in a suitable moderator medium. As more collision happens, kinetic energy is passed from fast neutron to other particles. There are some factors to denote a moderator whether it is good or bad. A good moderator should not absorb neutrons itself. The probability of absorbing neutron from flux is called absorption cross section of the moderator, denoted by Σ_a. The probability of neutron being scattered by moderator nucleus is denoted by scattering cross section, Σ_s. So, less the absorption cross section and larger the scattering cross section is better moderator. The moderator should slow down the neutron efficiently. A good moderator should thermalize the neutron in a few collisions. The least collision needed is the best. The first couple of collisions with a nucleus will be inelastic. It will transform some of the kinetic energy of neutron to potential energy of the moderator nucleus. The nucleus will be in an excited state. As kinetic energy of neutron is removed further collisions will be elastic. As in gas molecules' model, nucleus of closer mass will remove more energy in collision. So, nucleus of closer mass to neutron is more favored. Also a head-on collision will take more energy than a glancing collision. Both the energy reduction in a head-on and glancing collisions should be considered. Reduction in kinetic energy of fast neutron due to collision is expressed by mean logarithmic reduction of neutron energy per collision, and is given by:

(12)

where,
E_0 and E - kinetic energy of the neutron before and after one collision.
A - atomic mass of the moderator nucleus.
For a moderating medium consisting of more than one atom it is needed to consider the moderating and absorbing effect of all the atoms to calculate . The n umber of collisions to thermalize a neutron from the fission energy E_0 to the thermal energy E_{th} can be calculated from:

(13)

For example the number of collision needed to thermalize fast neutron with hydrogen and graphite moderator is calculated.
Assume, for fast neutron E_0 = 2 MeV; for thermal neutron E_{th} = 0.04 eV.
For hydrogen moderator:
A = 1, , collisions.
For graphite or carbon moderator:
A = 12, , n collisions.
It becomes apparent that a hydrogenous material is a better neutron moderator than graphite.The efficiency of a moderator is expressed as moderating ratio given by:

(14)

Table 2: Number of Collisions, on Average, to Moderate a Neutron from 2 MeV to 1 eV [3]

Moderator	ξ	Number of Collisions	$\xi\Sigma_s/\Sigma_a$
H	1.0	14	–
D	0.725	20	–
H_2O	0.920	16	71
D_2O	0.509	29	5670
He	0.425	43	83
Be	0.209	69	143
C	0.158	91	192
Na	0.084	171	1134
Fe	0.035	411	35
^{238}U	0.008	1730	0.0092

The ideal moderator is of low mass, high scattering cross section, and low absorption cross section. More neutron is absorbed with more collision. So a moderator which can reduce more kinetic energy in less collision is preferred. With a large density of moderator nucleus probability of collision is higher. In gas moderator medium density of gas nucleus is lower and so number of collisions is not enough to eliminate enough kinetic energy from the fast neutron. Thus, gas cannot be used as moderator.

4. Summary and Closing Remarks

Understanding the neutron distribution in space and energy space is necessary to design a nuclear reactor properly. Therefore, study of neutron generation, diffusion and moderation is important. For nuclear reactor fast neutrons can be moderated or thermalized using any of the described moderator medium. But the choice of most suitable moderator depends on type, construction, working principle of that specific nuclear reactor; cost, availability, pros and cons of different moderators.
1. Moderator medium must be solid or liquid. Gaseous medium must not be used as moderator.

2. Better moderation occurs with the moderators that is associated with smaller values of Fermi age and thermal diffusion length. So beryllium is better moderator than graphite according to Fermi age equation.
3. Moderating ratio of heavy water is more than light water. This makes heavy water a more efficient moderating medium than light water. But light water is much cheaper and easily available than heavy water. Light water is used in most reactors.
4. In the early age solid, pure graphite was generally used as moderator medium. But with presence of a small amount of boron poison or due to the unstable Wigner energy graphite is not reliable as a good moderator. Hence recently the usage of graphite as moderator medium is substantially reduced.
5. Beryllium has a good moderating effect in nuclear reactor. But it is a metal, expensive and toxic. Also it has a risk of sudden structural failure due to brittleness.
6. Lithium fluoride salt, typically in conjunction with beryllium fluoride salt is commonly used in molten salt reactors.
7. In fast reactors, where U-238 is used as fuel, no moderator medium is required.
8. The safety of a nuclear reactor is greatly dependent on the right choice of suitable moderator medium. A slight mistake or impurity in moderator can lead to devastating result or failure of operation due to not self-sustaining chain reaction.

5. References

[1] Bell, G. I., & Glasstone, S. (1974). Nuclear reactor theory. 1970. *Van Nostran Reinhold*, New York.
[2] Lamarsh, J. R. (1975). Introduction to nuclear engineering.
[3] Stacey, W. M. (2007). Nuclear reactor physics, *John Wiley & Sons*.
[4] Garg, S., Ahmed, F., & Kothari, L. S. (1986). Physics of nuclear reactors. *Tata McGraw-Hill*.
[5] Chaplin, R.A. (2012). Nuclear Reactor Design. Nuclear Energy Materials and Reactors, *UNESCO-EOLSS*, Vol. I
[6] Carpenter, J. M. (2004). Neutron production, moderation, and characterization of sources. *Argonne National Laboratory*.
[7] Knoll, G. F. (2010). Radiation detection and measurement. *John Wiley & Sons*.
[8] Dokhane, A. (2008). Chapter 5, Neutron Diffusion. Lecture note of the course Phys487 in King Saud University, Saudi Arabia. [9] Retrieved from: http://faculty.ksu.edu.sa/adokhane/Documents/Courses/PHYS487/Phys487_Lecture5.pdf [Cited on 2015 February 25].
[10] Ragheb, M. (11/19/2014). Neutron Diffusion in Non-multiplying Media. Lecture note of the course NPRE-402 in University of Illinois at Urbana-Champaign, USA. Spring 2015. Retrieved from: http://mragheb.com/NPRE 402 ME 405 Nuclear Power Engineering/Neutron HYPERLINK "http://mragheb.com/NPRE%20402%20ME%20405%20Nuclear%20Power%20Engineering/Neutron%20Diffusion%20in%20Nonmultiplying%20Media.pdf" HYPERLINK "http://mragheb.com/NPRE%20402%20ME%20405%20Nuclear%20Power%20Engineering/Neutron%20Diffusion%20in%20Nonmultiplying%20Media.pdf"Diffusion in HYPERLINK "http://mragheb.com/NPRE%20402%20ME%20405%20Nuclear%20Power%20Engineering/Neutron%20Diffusion%20in%20Nonmultiplying%20Media.pdf"Nonmultiplying HYPERLINK "http://mragheb.com/NPRE%20402%20ME%20405%20Nuclear%20Power%20Engineering/Neutron%20Diffusion%20in%20Nonmultiplying%20Media.pdf" HYPERLINK "http://mragheb.com/NPRE%20402%20ME%20405%20Nuclear%20Power%20Engineering/Neutron%20Diffusion%20in%20Nonmultiplying%20Media.pdf"Media.pdf [Cited on 2015 February 27].
[11] Neutron moderator. [Internet]. [Cited on 2015 March 7]. Wikipedia, the free encyclopedia. Retrieved from: http://en.wikipedia.org/wiki/Neutron_moderator [Cited on 2015 March 3].
[12] Neutron cross section. [Internet]. [Cited on 2015 March 7]. Wikipedia, the free encyclopedia. Retrieved from: http://en.wikipedia.org/wiki/Neutron_cross_section
[13] What is moderation. [Internet]. [Cited on 2015 March 10]. What is nuclear. Retrieved from: http://www.whatisnuclear.com/physics/moderation.html
[14] Moderators. [Internet]. [Cited on 2015 March 11]. DoITPoMS, University of Cambridge. Retrieved from: http://www.doitpoms.ac.uk/tlplib/nuclear_materials/moderators.php
[15] Muslim, A.G. (2013) Study the neutron Fermi age for selected reactor shielding and radioactive waste materials (M.Sc. Thesis).
Turner, J. E. (2007). Atoms, radiation, and radiation protection: James E. Turner.