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Study of Free Convection Heat and Mass Transfer about a Semi-Infinite Vertical Plate with Heat Absorption and Chemical Reaction in Presence of Magnetic Field

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Abstract

An analysis is performed to study the free convection heat and mass transfer flow of an electrically conducting incompressible viscous fluid about a semi-infinite vertical plate under the action of heat absorption, chemical reaction in presence of magnetic field with constant heat flux. The effects of magnetic parameter, Prandtl number, Schmidt number, heat absorption parameter and chemical reaction parameter on velocity, temperature and concentration profiles are discussed numerically and shown graphically. Therefore, the results of velocity field increases for increasing values of Prandtl number and chemical reaction parameter but other parameters decrease the velocity profile of the fluid flow. The temperature field decreases in the presence of magnetic parameter and Prandtl number but other parameters increase the temperature profile of the fluid flow. Also, the concentration profile decreases for increasing the values of magnetic parameter, reaction parameter and Schmidt number but there is no effect on concentration profile for heat absorption parameter. To verify the validity and accuracy of the present numerical results for the skin friction coefficient $f''(0)$ *and the local Sherwood number* $[-\varphi'(0)]$ *are compared with results of Ahmed A. Afify [11] in absence of heat absorption and found to be in very good agreement.*

Keywords: MHD, Heat absorption, Chemical reaction, Constant heat flux.

1. Introduction

The boundary layer flow of heat and mass transfer with chemical reaction has a great practical important to engineers and scientist because of its universal occurrence in many branches of science and engineering. Specially, it has a considerable importance in chemical and hydro mathematical industries. In many chemical processes, a chemical reaction occurs between a foreign mass and a fluid in which a plate is moving. These processes has numerous applications such as polymer production, manufacturing of ceramics or glassware and food processing. In recent years, MHD flow problems have become more important industrially. Indeed, MHD laminar boundary layer behavior over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. This problem has also an important bearing on metallurgy where magnetohydrodynamic (MHD) techniques have recently been used. In this regard, Sonth *et al*. [1] discussed heat and mass transfer in a visco-elastic fluid flow over an accelerating surface with heat source/sink and viscous dissipation, Tan *et al*. [2] studied heat and mass transfer over an impermeable stretching plate and Sing [3] studied the heat and mass transfer in MHD boundary layer flow past an inclined plate with viscous dissipation in porous medium. The effect of chemical reaction on free-convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet was investigated by Afify [4] in the presence of transverse magnetic field. Cortell [5] studied the magneto hydrodynamics flow of a power-law fluid over a stretching sheet. Abel and Mahesh [6] presented an analytical and numerical solution for heat transfer in a steady laminar flow of an incompressible viscoelastic fluid over a stretching sheet with power-law surface temperature, including the effects of variable thermal conductivity and non-uniform heat source and radiation. Iskak *et al*. [7] discussed heat transfer analysis for mixed convection flow through a vertical stretching sheet, Ali *et al*. [8] have analyzed the MHD viscos flow of heat transfer over a permeable shrinking sheet with surface heat flux, Iskak *et al*. [9] also analyzed the heat transfer of unsteady mixed convection flow over vertical stretching sheet, Rajesh [10] have studied the MHD effects on free convection and mass transform flow through a porous medium with variable temperature. Since the study of heat and mass transfer is important in many cases, in the present paper we studied the free convection heat and mass transfer about a semi-infinite vertical plate with heat absorption and chemical reaction in presence of magnetic field. The boundary layer equations are transformed by a similarity transformation into a system of coupled non-linear ordinary differential equations and which are solved numerically by shooting iteration technique along with Runge- Kutta sixth order method. The study extends the work of Ahmed A. Afify [11] by considering heat absorption effects. Finally, the numerical values of the skin friction and concentration gradient are also shown in a tabular form.

2. Governing Equations of the Present Problem

Consider a two dimensional steady laminar MHD viscous incompressible electrically conducting fluid along a vertical plate. A magnetic field of strength B_0 is introduced to the normal to the direction to the flow. The uniform plate temperature $T_w\ (>T\infty)$, where T_∞ is the temperature of the fluid far away from the plate. Let u and v be the velocity components along the x and y axis respectively in the boundary layer region. Under the above assumptions and usual boundary layer approximation, the dimensional governing equations of continuity, momentum, concentration and energy under the influence of externally imposed magnetic field are [2]:

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

Energy Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho \mathrm{c_p}}\frac{\partial^2 T}{\partial y^2}+Q^*(\mathrm{C-C_\infty}) \tag{3}$$

Concentration Equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-\lambda^*(C-C_\infty) \tag{4}$$

Boundary conditions are:

$$u=ax,\ v=\ 0,\frac{\partial T}{\partial y}=-\frac{q}{\kappa},C=C_w \quad at \ \ y=0 \ \ and \ u=0,T=T_\infty,C=C_\infty \ \ as \quad y\to\infty$$

where u and v are the velocity components along x and y directions, T , T_w and T_∞ are the fluid temperature, the plate temperature and the free stream temperature respectively while C, C_w and C_∞ are the corresponding concentrations, a is the constant, κ is the thermal conductivity, λ^* is the reaction rate constant, Q^* is the heat absorption rate constant, q is the heat flux, C_p specific heat with constant pressure, ν is the kinematic viscosity , σ is the electrical conductivity, ρ is the fluid density, β is the thermal expansion coefficient, $\beta*$ is the concentration expansion coefficient, B_0 is the magnetic field intensity, g is the acceleration due to gravity, D_m is the coefficient of mass diffusivity respectively. We introduce the steam function $\psi(x,y)$ as defined by $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$.

To convert the governing equations into a set of similarity equations, we introduce the following similarity transformation:

$$\eta = y\sqrt{\frac{U_0}{\upsilon x}},\ \psi = \sqrt{U_0 \upsilon x}\, f(\eta),\ \theta = \frac{T - T_\infty}{q}\kappa\sqrt{\frac{U_0}{\upsilon x}},\ \phi = \frac{C - C_\infty}{C_w - C_\infty}\sqrt{\frac{U_0}{\upsilon x}}$$

From the above transformations, the non-dimensional, nonlinear and coupled ordinary differential equations are obtained as

$$f''' + \frac{1}{2} ff'' - Mf' + Gr\theta + Gm\phi = 0 \tag{5}$$

$$\theta'' + \frac{1}{2}\Pr(f\,\theta' - f'\theta) + \Pr R_a \varphi = 0 \tag{6}$$

$$\varphi'' + \frac{1}{2}Sc(f\varphi' - f'\varphi) - Sc C_r \varphi = 0 \tag{7}$$

The transform boundary conditions:

$$f = 0,\ f' = 1,\ \theta' = -1, \varphi = 1 \text{ at } \eta = 0 \text{ and } f' = \theta = \varphi \to 0 \text{ as } \eta \to \infty$$

Where f', θ and φ are the dimensionless velocity, temperature and concentration profiles respectively, η is the similarity variable, the prime denotes differentiation with respect to η.
Also the non-dimensional parameters are

$$Gr = \frac{g\beta qx}{U_0^2\kappa}\sqrt{\frac{\nu x}{U_0}}, Gm = \frac{g\beta^*(C_w - C)x}{U_0^2}\sqrt{\frac{\nu x}{U_0}}, M = \frac{\sigma B_0 x}{\rho U_0}, \Pr = \frac{\nu}{\alpha}, R_a = \frac{\kappa m Q^* x}{qU_0 D_m}, C_r = \frac{\lambda^* x}{U_0}, Sc = \frac{\nu}{D_m}$$

are the Grashof number, modified Grashof number, magnetic parameter, Prandtl number, heat absorption parameter, chemical reaction parameter and Schmidt number respectively.

3. Results and Discussion

Numerical calculation for distribution of the velocity, temperature and concentration profiles across the boundary layer are displayed in Fig. l- Fig.12 for different values of magnetic parameter M, Prandtl number Pr, heat absorption parameter R_a, chemical reaction parameter C_r and Schmidt number Sc and for fixed values of Gr and Gm. The values of Prandtl number Pr are chosen for 1.00 which correspond to salt water and Schmidt number are taken 0.66 which correspond to water vapor. Throughout the calculations, the bouncy parameter Gr = -5.0 and Gm = -5.0 are taken which correspond to a hot plate and other parameters are chosen arbitrary. The effects of various parameters on velocity profile are shown in Fig. 1- Fig. 4. In Fig. 1 it is observed that the velocity decreases with an increase in the M. The magnetic parameter is found to retard the velocity at all points of the flow field. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Similar effect arises for the increasing values of heat absorption parameter but reverse trend arises for the increasing values of Prandtl number and chemical reaction parameter which are shown in Fig.4, Fig.2 and Fig.3.

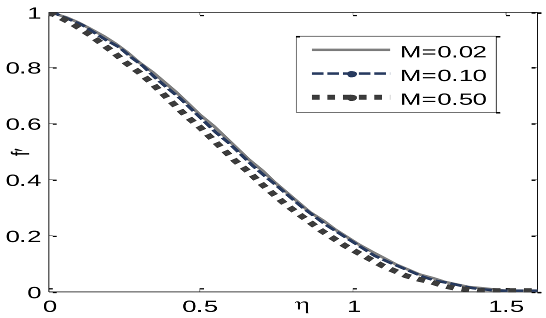

Fig.1. Velocity profile for various values of M

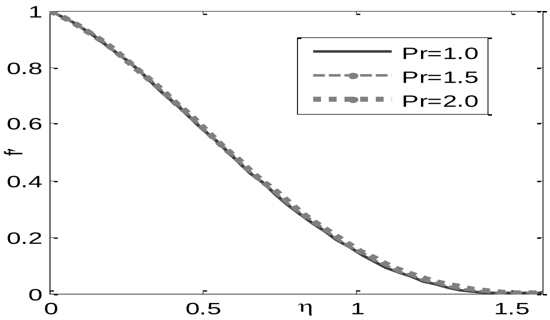

Fig.2. Velocity profile for various values of Pr

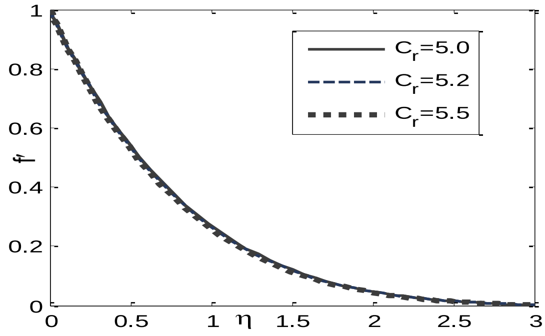

Fig.3. Velocity profile for various values of C_r

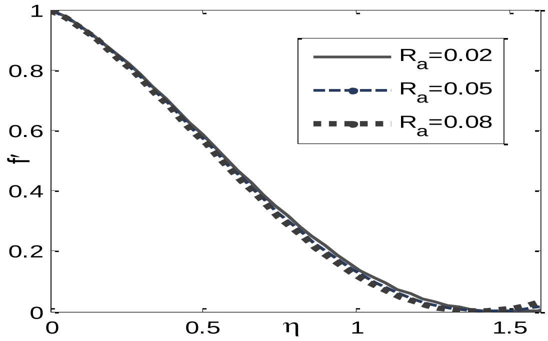

Fig.4. Velocity profile for various values of Ra

The effect of various parameters on temperature profile are shown in Fig.5 - Fig.8. From these figures we see that, the temperature profile is starting at the initial point of the plate surface and then decreasing until it reaches to zero far away from the plate satisfying the boundary condition. Fig.5 shows the temperature distribution for different values of the magnetic field parameter M and observed that the thermal boundary layer decreases as M increases adjacent to the surface of the plate and the effect is not significant far away from the plate. Fig. 6 which illustrate the effect of Prandtl number Pr on the temperature profile. From this figure it is observed that the temperature decreases with an increase in the Prandtl number, which implies viscous boundary layer is thicker than the thermal boundary layer. From these plots it is evident that large values of Prandtl number result in thinning of the thermal boundary layer. In this case temperature asymptotically approaches to zero in free stream region. Fig.7 and Fig.8 indicates the variation of heat absorption parameter and chemical reaction parameter. It is noticed that the thermal boundary layer is increasing.

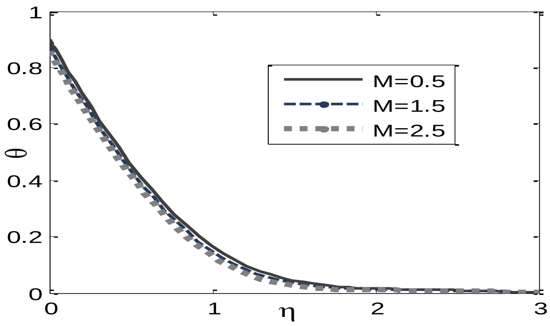

Fig.5. Temperature profile for various values of M

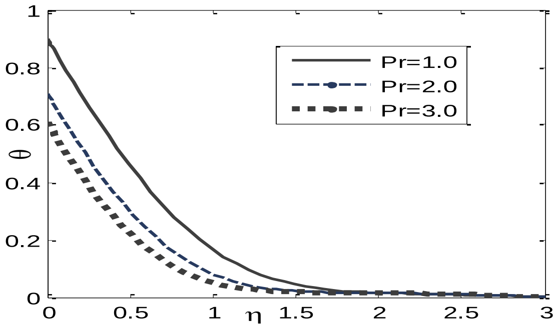

Fig.6. Temperature profile for various values of Pr

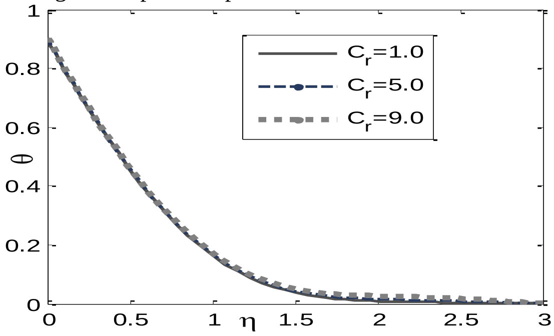

Fig.7. Temperature profile for various values of C_r

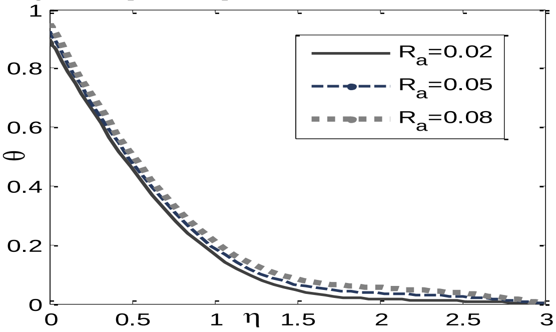

Fig.8. Temperature profile for various values of R_a

Fig. 9 - Fig. 12 shows the concentration profiles obtained by the numerical simulation for various values of entering non-dimensional parameters. In Fig. 12 the effect of Sc is found to decrease the concentration because increasing in Sc decreases molecular diffusivity which result a decrease of the boundary layer. Hence the concentration of the species in lower for large values of Sc. From the Fig. 9 and Fig.11 it is observed that, the negligible decreasing and increasing effect on concentration profiles for increasing values of M and R_a. The concentration profile is increased for increasing values of chemical reaction parameter which are shown in Fig.10.

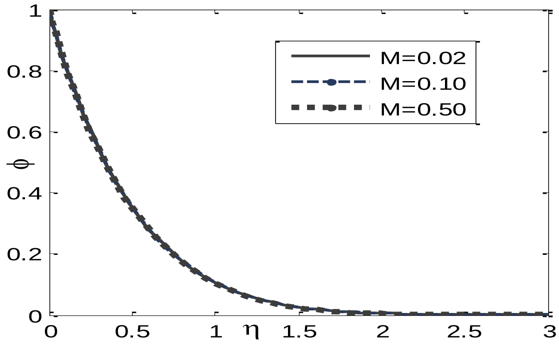

Fig.9. Concentration profile for various values of M

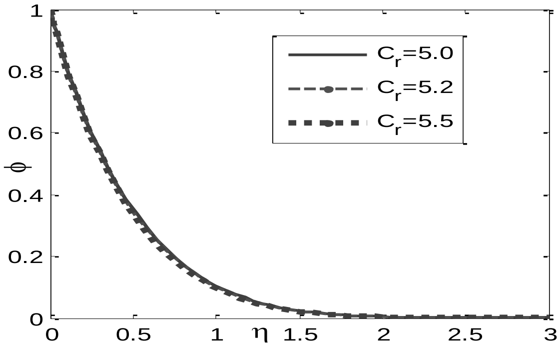

Fig.10. Concentration profile for various values of C_r

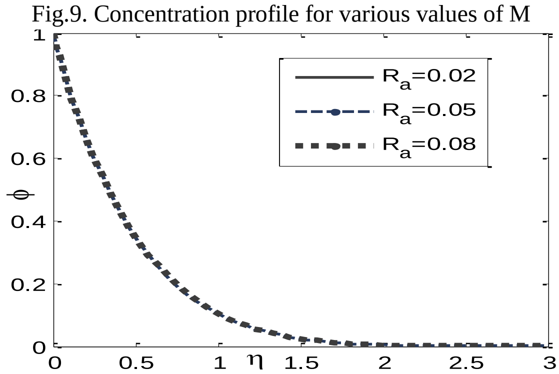

Fig.11. Concentration profile for various values of R_a

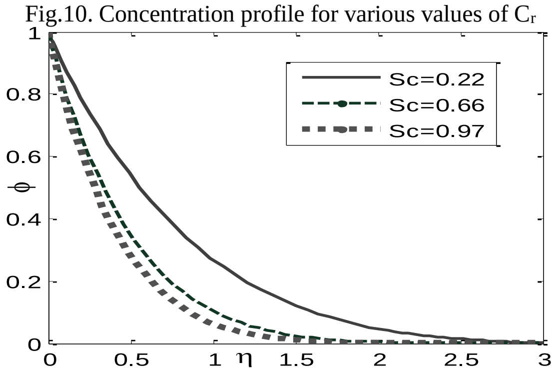

Fig.12. Concentration profile for various values of Sc

Again, from Table 1 it is observed that the skin friction is decreased for magnetic parameter and heat absorption parameter as a result the momentum boundary layer is decreased but reverse result arises for chemical reaction parameter & Prandtl number. Again, the rate of concentration is increased for magnetic parameter, chemical reaction parameter and Schmidt number as a result the concentration boundary layer is decreased but the rate is constant for heat absorption parameter. Also Table 2 depicts the comparison result for various values of M with Ahmed A. Afify[11] in absence of heat absorption parameter and found to be in good agreement.

Table 1. The skin friction $f'(0)$ and rate of concentration $-\varphi'(0)$ for different values of M, R_a, C_r, Pr and Sc are respectively by considering hot plate

M	R_a	C_r	Pr	Sc	$f'(0)$	$\varphi'(0)$
0.02	0.02	5.0	1.0	0.66	-0.2379	1.9799
0.10	0.02	5.0	1.0	0.66	-0.2743	1.9805
0.50	0.02	5.0	1.0	0.66	-0.4461	1.9836
0.5	**0.05**	5.0	1.0	0.66	-0.4675	1.98359
0.5	**0.08**	5.0	1.0	0.66	-0.4868	1.98359
0.5	0.02	**5.2**	1.0	0.66	-0.4258	2.0156
0.5	0.02	**5.5**	1.0	0.66	-0.3912	2.0627
0.5	0.02	5.0	**1.5**	0.66	-0.4423	-
0.5	0.02	5.0	**2.0**	0.66	-0.4378	-
0.5	0.02	5.0	1.0	**0.22**	-	1.1764
0.5	0.02	5.0	1.0	**0.66**	-	1.9836
0.5	0.02	5.0	1.0	**0.97**	-	2.3915

Table 2. The values of $f''(0)$ and rate of concentration transfer $-\varphi(0)$ for different values of M when Pr =0.71, Gr = Gm = 0.5, Sc = 00.1, C_r = 0.1 and R_a = 0 respectively.

	Ahmed A. Afify[11]		Present results	
M	$f'(0)$	$-\varphi'(0)$	$f'(0)$	$-\varphi'(0)$
0.1	0.39405	0.77174	0.40123	0.76892
0.5	0.94516	0.73770	0.93851	0.74532
1.0	1.45174	0.67701	1.46123	0.67823

4. Conclusions

In the present paper is an investigation of steady MHD free convection, heat and mass transfer flow of an incompressible electrically conducting fluid over a stretching sheet in a rotating system under the influence of an applied uniform magnetic field with Hall current. The leading equations are solved numerically by the shooting method along with Runge- Kutta fourth order integration scheme. The results are presented to display the flow characteristic like velocity, temperature and concentration. Following are the conclusions made from above analysis:

- The magnitude of velocity profile decreases with increasing magnetic parameter. The magnetic parameter is found to retard the velocity at all points of the flow field. As, M, increase, the Lorentz force, which opposes the flow, also increases and leads to enhanced deceleration of the flow.
- The thermal boundary layer decreases as M increases adjacent to the surface of the plate and the effect is not significant far away from the plate. The temperature decreases with an increase in the Prandtl number, which implies viscous boundary layer is thicker than the thermal boundary layer. From these plots it is evident that large values of Prandtl number result in thinning of the thermal boundary layer.
- The effect of Sc is found to decrease the concentration profile because increasing in Sc decreases molecular diffusivity which result a decrease of the boundary layer. Hence the concentration of the species in lower for large values of Sc.
- The skin friction is decreased for magnetic parameter and heat absorption parameter as a result the momentum boundary layer is decreased but reverse result arises for chemical reaction parameter & Prandtl number. Again, the rate of concentration is increased for magnetic parameter, chemical reaction parameter and Schmidt number as a result the concentration boundary layer is decreased but reverse case arises for heat absorption parameter

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