Paper ID: FM-02

Free Convection Heat & Mass Transfer on Steady MHD Boundary Layer Flow over a Hot Stretching Plate with Hall Current and Heat Generation

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Abstract

The present problem is focused on steady two-dimensional magnetohydrodynamic (MHD) free convection, heat and mass transfer flow of an incompressible electrically conducting fluid over a hot stretching plate under the influence of an applied uniform magnetic field with Hall current and heat generation. The numerical results concerned with the primary velocity, secondary velocity, temperature and concentration profiles effects of various parameters on the flow fields are investigated and presented graphically for air with a Prandtl number of 0.71. The results presented graphically illustrate that primary velocity field decrease due to increase of magnetic parameter but increase for the values of heat source parameter while there is no effect for Hall parameter. Again the secondary velocity is increased for the increasing values of magnetic and Hall parameter but negligible decreasing effect for heat generation parameter. Temperature field increases in the presence of heat generation parameter whereas it is decreased for the increasing values of magnetic parameter and Prandtl number. Again, the concentration is decreased for the increasing values of Schmidt number and magnetic parameter but negligible increasing effect for heat generation parameter. To verify the validity and accuracy of the present numerical results for the skin friction coefficient $f''(0)$ *and the local Nusselt number* $[-\theta'(0)]$ *are compared with results of Ishak et al. [7] in absence of Hall current and heat generation.*

Keywords: MHD, Hall current, Heat generation, Stretching sheet.

1. Introduction

MHD laminar boundary layer flow problems has become of its important applications in industrial manufacturing processes like plasma studies, petroleum industries, Magneto-hydrodynamics power generator, cooling of Nuclear reactors, boundary layer control in aerodynamics. Also , MHD laminar boundary layer behavior over a stretching surface is a significant type of flow having considerable practical applications in paper production, hot rolling, wire drawing, drawing of plastic films, metal and polymer extrusion, metal spinning and polymer processing. In this regard, various authors has been done a lot of works related to this field such as Venkatesulu and Rao [1] analyzed the effect of Hall Currents and Thermo-diffusion on convective heat and mass transfer flow of a viscous, chemically reacting rotating fluid through a porous medium past a vertical porous plate, Mathew *et al*. [2] studied Hall effects on heat and mass transfer through a porous medium in a rotating channel with radiation, Kumar and Singh [3] have been studied mathematical modeling of Soret and hall effects on oscillatory MHD free convective flow of radiating fluid in a rotating vertical porous channel filled with porous medium, Ruzicka [4] have also studied heat and mass transfer past a vertical flat porous plate through a porous medium with variable thermal conductivity, Chauhan [5] considered the effect of Hall current on MHD slip flow and heat transfer through a porous medium over an accelerated plate in a rotating system, further Hayat *et al.* [6] studied the slip effects on peristaltic transport in an inclined channel with mass transfer and chemical reaction. In view of the importance, the present work is focused on steady MHD free convection, heat and mass transfer flow of an incompressible electrically conducting fluid over a hot stretching sheet with Hall current and heat generation under the influence of an applied uniform magnetic field. The study extends the work of Ishak *et al*. [7] by considering the Hall and heat generation effects. Finally, the numerical values of the skin friction, wall temperature gradient and concentration gradient are also shown in a tabular form.

2. Governing Equations of the Present Problem and Similarity Analysis

Let us consider steady two dimensional MHD free convection heat and mass transfer in an incompressible electrically conducting fluid flow over a hot stretching sheet in a rotating system under the influence of an applied uniform magnetic field. The flow is subjected to a transverse magnetic field of strength B_0 which is assumed to be applied in the positive *y* –direction normal to the surface. By taking the magnetic Reynolds number is so small that the induced magnetic field is negligible. The pressure gradient, body force, viscous dissipation and joule heating effects are neglected because their values are generally small compared with the effects of internal heat generation. Under these above assumptions along with Boussinesq and usual boundary layer approximation, the dimensional governing equations of continuity, momentum, concentration and energy under the influence of externally imposed magnetic field with the presence of Hall current are [7] and [2]:

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma B_0^2}{\rho(1+m^2)}(u+mW) \tag{2}$$

$$u\frac{\partial W}{\partial x}+v\frac{\partial W}{\partial y}=v\frac{\partial^2 W}{\partial y^2}+\frac{\sigma B_0^2}{\rho(1+m^2)}(mu\text{-}W) \tag{3}$$

Energy Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}(T-T_\infty) \tag{4}$$

Concentration Equation: [1]

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{5}$$

Boundary conditions are:

$$\begin{aligned}&u=a,\ v=0, W=0, T=T_w=T_\infty+bx, C=C_w\ \ at\ \ y=0\\&u=0, v=0, T=T_\infty, C=C_\infty\ \ as\ \ y\to\infty\end{aligned}$$

where u ,v and W are the velocity components along x , y and z directions, T , T_w and T_∞ are the fluid temperature, the stretching sheet temperature and the free stream temperature respectively while C , C_w and C_∞ are the corresponding concentrations, k is the thermal conductivity, Q_0 is the heat generation constant, m is the Hall parameter, a is the constant, b is arbitrary constant, C_p specific heat with constant pressure, μ is the coefficient of viscosity, v is the kinematic viscosity , σ is the electrical conductivity, ρ is the fluid density, β is the thermal expansion coefficient, β^* is the concentration expansion coefficient, B_0 is the magnetic field intensity, g is the acceleration due to gravity, D is the coefficient of mass diffusivity respectively. We introduce the steam function $\psi(x,y)$ as defined by $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$.

To convert the governing equations into a set of similarity equations, we introduce the following similarity transformation:

$$W=axg, \eta=y\sqrt{\frac{a}{v}},\ \psi=x\sqrt{av}\, f, \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

From the above transformations, the non-dimensional, nonlinear and coupled ordinary differential equations are obtained as

$$f''' + ff'' + Gr\,\theta + Gm\,\varphi - \frac{M}{1+m^2} f' + \frac{Mm}{1+m^2} g = 0 \tag{6}$$

$$g'' - f'g + g' - \frac{M}{1+m^2} g + \frac{Mm}{1+m^2} f' = 0 \tag{7}$$

$$\theta'' + \Pr f\theta' - \Pr f'\theta + Q\theta = 0 \tag{8}$$

$$\varphi'' + \mathrm{Scf}\varphi' = 0 \tag{9}$$

The transform boundary conditions:

$$f = 0,\ f' = 1,\ g = 0,\ \theta = \varphi = 1\ at\ \eta = 0 \text{ and } f' = g = \theta = \varphi \to 0\ as\ \eta \to \infty$$

Where f', g, θ and φ are the dimensionless primary velocity, secondary velocity, temperature and concentration profiles respectively, η is the similarity variable, the prime denotes differentiation with respect to η . Also the non-dimensional parameters

$$Gr = \frac{g\beta(T_w - T_\infty)}{a^2 x}, \Pr = \frac{\nu}{\alpha}, M = \frac{\sigma B_0^2}{\rho a}, Q = \frac{Q_0}{a\rho c_p}, Gm = \frac{g\beta^*(C_w - C_\infty)}{a^2 x}, Sc = \frac{\nu}{D}$$

are the Grashof number, Prandtl number, magnetic parameter, heat generation parameter, modified Grashof number and Schmidt number respectively.

3. Results and Discussion

Numerical calculation for distribution of the primary velocity, secondary velocity, temperature and concentration profiles across the boundary layer are displayed in Fig. l- Fig.12 for different values of magnetic parameter *M*, Hall parameter *m*, heat source *Q*, Prandtl number *Pr* and Schmidt number *Sc* and for fixed values of *Gr* and *Gm*. Throughout the calculations, the bouncy parameter *Gr* = -5.0 and *Gm* = -5.0 are taken which correspond to a hot plate and other parameters are chosen arbitrary.

The effects of various parameters on primary velocity profile are shown in Fig. 1- Fig. 3. In Fig. 1 it is observed that the velocity decreases with an increase in the *M*. The magnetic parameter is found to retard the velocity at all points of the flow field. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Reverse effect arises for the increasing values of heat generation parameter which are shown in Fig.3 cause increasing heat generation would increase bouncy force as a result velocity increases. From Fig.2 it is observed that there is no effect on primary velocity profile for Hall parameter.

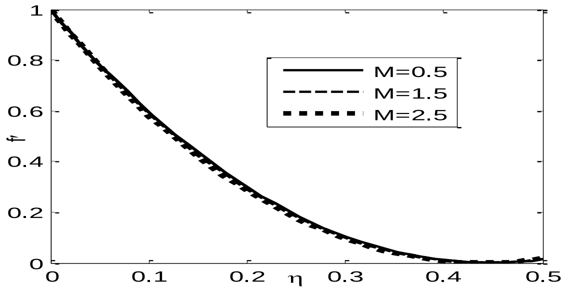

Fig.1. Primary velocity profile for various values of *M*

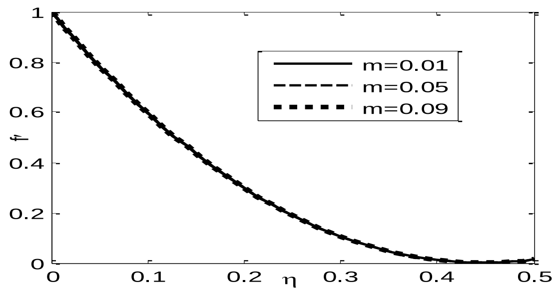

Fig.2. Primary velocity profile for various values of *m*

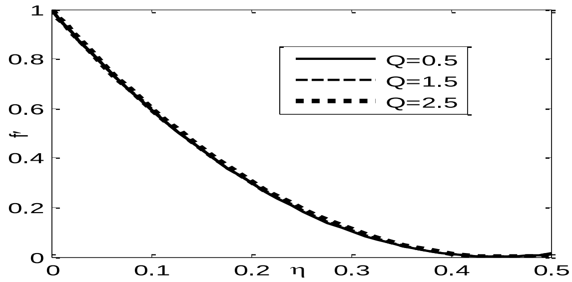

Fig.3. Primary velocity profile for various values of Q

Fig.4- Fig.6 display the effect of various entering parameters on secondary velocity profile. From these figures it is seen that the secondary velocity starts from minimum value at the plate and increases until it attains the maximum value within the boundary layer and then starts decreasing until it reaches the free stream area satisfying the far field boundary condition. The noticeable increasing effect of magnetic and Hall parameter on secondary velocity profile are observed which are shown in Fig.4 and Fig.5. Again, it is interesting to note that the other mentioned parameter has negligible decreasing effect on secondary velocity profile which are shown in Fig.6.

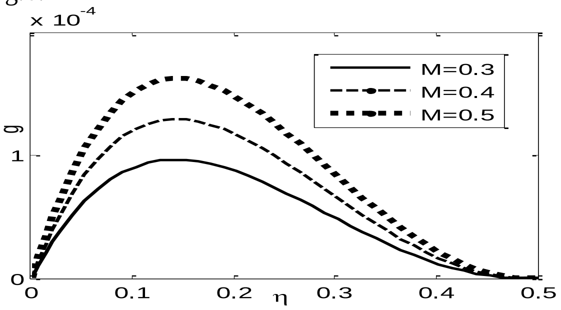

Fig.4. Secondary velocity profile for various values of M

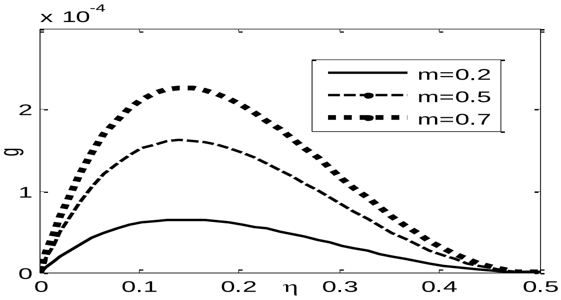

Fig.5. Secondary velocity profile for various values of m

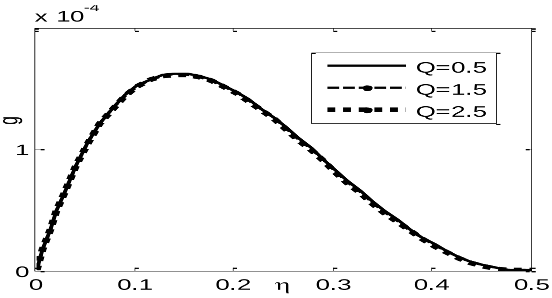

Fig.6. Secondary velocity profile for various values of Q

The effect of various parameters on temperature profile are shown in Fig.7-Fig.9. From these figures we see that, the temperature profile is starting at the initial point of the plate surface and increases until it attains the maximum value within the boundary layer and then starts decreasing until it reaches to zero far away from the plate satisfying the boundary condition. Fig.7 shows the temperature distribution for different values of the magnetic parameter M and observed that the thermal boundary layer decreases as M increases adjacent to the surface of the plate and the effect is not significant far away from the plate. Fig. 8 which illustrate the effect of Prandtl number Pr on the temperature profile. From this figure it is observed that the temperature decreases with an increase in the Prandtl number, which implies viscous boundary layer is thicker than the thermal boundary layer. From these plots it is evident that large values of Prandtl number result in thinning of the thermal boundary layer. In this case temperature asymptotically approaches to zero in free stream region. Fig.9 indicates the variation of heat generation parameter and it is noticed that the thermal boundary layer increased up to certain values of eta and then decreased far from the plate.

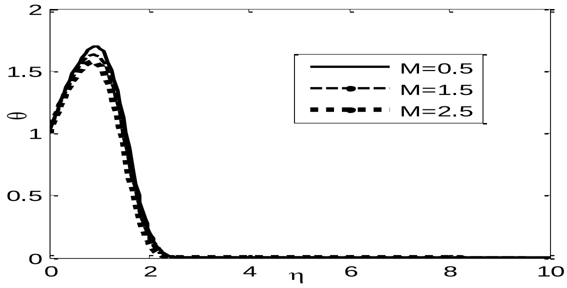

Fig.7. Temperature profile for various values of M

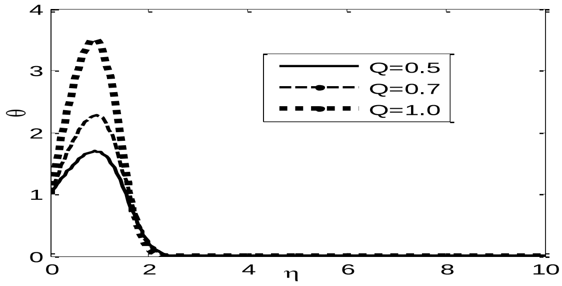

Fig.9. Temperature profile for various values of Q

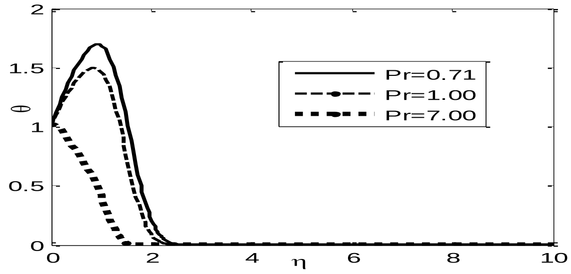

Fig.8. Temperature profile for various values of *Pr*

Fig. 10- Fig. 12 shows the concentration profiles obtained by the numerical simulation for various values of entering non-dimensional parameters. In Fig. 12 the effect of *Sc* is found to decrease the concentration because increasing in *Sc* decreases molecular diffusivity which result a decrease of the boundary layer. Hence the concentration of the species in lower for large values of *Sc*. From the Fig. 10 and Fig.11 it is observed that, the negligible decreasing and increasing effect on concentration profiles for increasing values of *M* and *Q*.

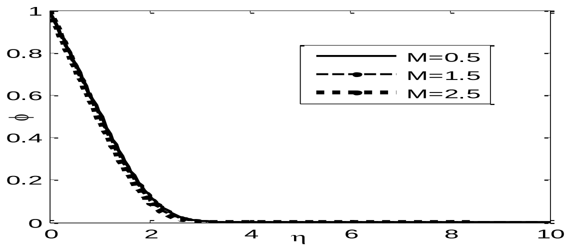

Fig.10. Concentration profile for various values of *M*

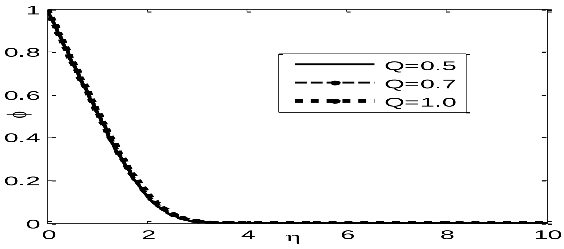

Fig.11. Concentration profile for various values of *Q*

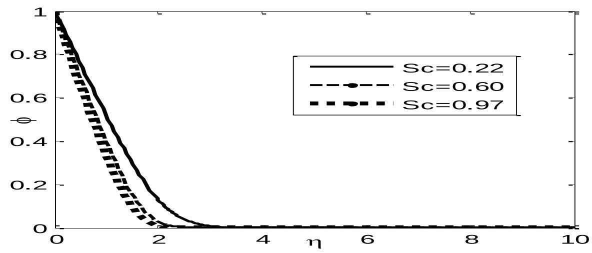

Fig.12. Concentration profile for various values of *Sc*

Again, from Table 1 it is observed that the skin friction is decreased for magnetic parameter and increased for heat generation parameter & Prandtl number. The rate of heat transfer is increased for magnetic parameter and Prandtl number as a result the thermal boundary layer is decreased but the reverse result arises for heat generation parameter. Again, the rate of concentration is increased for magnetic parameter and Schmidt number as a result the concentration boundary layer is decreased but reverse case arises for heat generation parameter. Table 2 show the compared result and found to be in good agreement between the numerical results of skin friction coeffient and local Nusselt number by Runge-Kutta sixth order shooting iteration technique and the results via Keller-Box method of Ishak et al. [7].

Table 1. The skin friction $f''(0)$, rate of heat transfer $-\theta'(0)$ and rate of concentration $-\varphi'(0)$ for different values of M, Q, Pr and

Table 2. The values of $f''(0)$ and rate of heat transfer $-\theta'(0)$ for different values of M when Pr =1.0, Gr = Gm = 0.5, Sc = 0.22 and

Sc are respectively.

M	Q	Pr	Sc	$f'(0)$	$-\theta'(0)$	$\varphi'(0)$
0.5	0.5	0.71	0.22	-4.5590	1.260	0.4890
1.5	0.5	0.71	0.22	-4.6670	1.196	0.5030
2.5	0.5	0.71	0.22	-4.7720	1.140	0.5180
0.5	**0.7**	0.71	0.22	-4.5406	2.190	0.4840
0.5	**1.0**	0.71	0.22	-4.5220	3.120	0.4808
0.5	0.5	**1.00**	0.22	-4.6721	1.070	-
0.5	0.5	**7.00**	0.22	-4.1203	0.440	-
0.5	0.5	0.71	**0.60**	-	-	0.6290
0.5	0.5	0.71	**0.97**	-	-	0.7100

Q = m = 0 respectively.

	Ishak *et. al*.[7]		Present results	
M	$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$
0.0	0.5607	1.0873	-0.56513	1.0945
0.1	0.5658	1.0863	-0.57035	1.0933
0.2	0.5810	1.0833	-0.58421	1.0785
0.5	0.6830	1.0630	-0.68538	1.0553
1.0	1.0000	1.0000	-1.0043	1.0021
2.0	1.8968	0.8311	-1.9004	0.9421
5.0	4.9155	0.4702	-4.9206	0.4811

4. Conclusions

The results are presented to display the flow characteristic like velocity, temperature and concentration. Following are the conclusions made from above analysis:

- The magnitude of primary velocity profile decreases with increasing magnetic parameter but reverse result rises in case of heat generation parameter. The noticeable increasing effect of magnetic and Hall parameter on secondary velocity profile are observed primary velocity profile. The thermal boundary layer decreases for *M* and *Pr* adjacent to the surface of the plate and the effect is not significant far away from the plate. The effect of *Sc* is found to decrease the concentration profile because increasing in *Sc* decreases molecular diffusivity which result a decrease of the boundary layer. Hence the concentration of the species in lower for large values of *Sc*.

5. References

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