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Non-isothermal Heating and Magnetic Field Effect on Non-Newtonian Power Law Fluid in Triangular Channel

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Abstract

In this paper, effect of external magnetic field and non-isothermal heating in a triangular cavity filled with non-Newtonian power law fluid is analyzed numerically. Cavity has a non-isothermally heated bottom wall while other walls are kept cold. Simulations are carried out for wide variety of Rayleigh numbers (Ra = 10^3 ~ 10^6), Hartmann number (Ha = 0-60) and power law index (n = 0.5, 1, 1.5). Galerkin weighted residuals method of finite analysis is adopted in this study and grid independency test is performed to ensure numerical accuracy of the solution. Results are shown on the basis of streamlines, isotherms and average Nusselt number (Nu) plots. Result indicates that distortion of isotherm ensures convective dominance thus better heat transfer at lower Hartmann number. For any Hartmann number, heat transfer rate decreases with the increment of power law index and no significant changes are observed at higher Hartmann number.

Keywords: Rayleigh number, Magnetic field, Power-law index, Non-Newtonian fluid

1. Introduction

Natural convection fluid flow by buoyancy force has been a topic of interest for many researchers [1, 2] and has various applications of engineering such as electronic cooling systems [3, 4], nuclear reactors and heat exchangers [5] etc. The interaction of induced electric currents with the applied magnetic field results in a magneto hydrodynamic (MHD) forces which influences the flow of an electrically conducting fluid in the magnetic field. An externally applied magnetic field is a media for the process of manufacturing metal [6].

MHD natural convection simulation in cavities on different fluids and boundary conditions has been studied by various numerical methods widely recently [7-10]. The flow of non-Newtonian power law fluids has wide potential application in many process industries [11], in bio-medical Engineering [12] and also under MHD convection [13]. Researches with different geometric shaped models and numerical methods were done on natural convection for non-Newtonian power law fluids. Khezzar et al. [14] investigated natural convection of non-Newtonian power-law type fluids in two-dimensional rectangular tilted enclosures numerically and indicated that the increment and decrement of average Nusselt number for non-Newtonian power law fluids depended on Rayleigh number, Prandtl number, aspect ratio and power-law index. Examining two-dimensional steady state natural convection of power-law fluids numerically, Matin et al. [15] revealed that the non-Newtonian fluids were more efficient than Newtonian fluids for cooling and insulating purposes. Transient natural convection of non-Newtonian power law fluids in a square enclosure with differentially heated vertical side walls subjected to constant wall temperatures was studied by Kim et al. [16]. They showed that the mean Nusselt number increased for the decrement of power-law index for a given set of values of *Ra* and *Pr*.

The aim of present study is to simulate MHD natural convection of non-Newtonian power-law flow in a sinusoidally heated cavity with non-isothermal heating numerically. Moreover, it is intended to analyze various magnitude of magnetic field effect on the flow and on the thermal fields evidently with the alterations of different considered parameters (Rayleigh number, power-law index).

2. Problem specification

The details of the problem are presented in Figure 1. In the figure, right angled triangular-shape geometry of unit length (L) for bottom and vertical wall has been considered. The inclined wall has been modeled as the hypotenuse (√2L) of the right angle triangle. The entire cavity is filled with non-Newtonian power law fluid to transfer heat inside the cavity. Gravity is working along the negative Y axis. Both the inclined and vertical walls of the cavity are kept at low temperature ($T = T_c$), while the base is kept non-isothermally heated ($T = T_c + sin^2(\pi x)$). A uniform magnetic field with a constant magnitude B is applied in horizontal direction.

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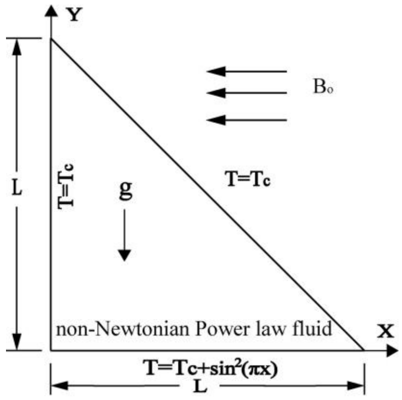

Fig. 1. Geometry of the present study

3. Mathematical formulation

A set of governing equations has been formed assuming that a two-dimensional right angled triangular enclosure filled with an incompressible non-Newtonian power-law fluid. The flow is considered to be incompressible and laminar. The density variation is approximated by the standard Boussinesq model. According to these assumptions, the governing equations like continuity, momentum and energy equation in non dimensional form can be written as follows-

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{1}$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+\frac{\Pr}{\sqrt{Ra}}\left[2\frac{\partial}{\partial x}\left(\frac{\mu_a}{K}\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\mu_a}{K}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)\right] \tag{2}$$

$$\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+\frac{\Pr}{\sqrt{Ra}}\left[2\frac{\partial}{\partial y}\left(\frac{\mu_a}{K}\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial x}\left(\frac{\mu_a}{K}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)\right]-\frac{\Pr \mathrm{Ha}^2}{\sqrt{Ra}}v \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\sqrt{Ra}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

In order to get the numerical solution of the system following scales are implemented to get the non-dimensional governing equations-

$$x=\frac{\bar{x}}{L},\ y=\frac{\bar{y}}{L},\ \mathrm{u}=\frac{\bar{u}}{\left(\frac{\alpha}{L}\right)Ra^{0.5}},\ \mathrm{v}=\frac{\bar{v}}{\left(\frac{\alpha}{L}\right)Ra^{0.5}},\ P=\frac{\bar{P}}{\rho\left(\frac{\alpha}{L}\right)^2 Ra},\ T=\frac{\bar{T}-T_C}{T_H-T_C}. \tag{5}$$

For a purely-viscous non-Newtonian fluid according to Ostwald–DeWaele power-law [17] model the shear stress tensor can be expressed as-

$$\tau_{ij}=2\mu_a D_{ij}=\mu_a\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right) \tag{6}$$

where D_{ij} indicates the rate-of-deformation tensor for the two dimensional Cartesian coordinate and μ_a is the apparent viscosity that is derived for the two-dimensional Cartesian coordinates as-

$$\bar{\mu}_a=K\left[2\left[\left(\frac{\partial \bar{\mu}}{\partial \bar{x}}\right)^2+\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2\right]+\left(\frac{\partial \bar{v}}{\partial \bar{x}}+\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2\right]^{\frac{(n-1)}{2}} \tag{7}$$

In the above equations $(\bar{u},\bar{v}),\bar{T}$ and $\bar{P}$ are the dimensional velocities, temperature and pressure respectively, ρ is the density, σ is the electrical conductivity, *B* is the uniform magnetic field, and *n* is the power law index. Therefore, the deviation of n from unity indicates the degree of deviation from Newtonian behavior. Dimensional form of the above equation is:

$$\mu_a = K\left[2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]^{\frac{(n-1)}{2}} \tag{8}$$

Boundary conditions used to solve the present problem are mentioned in Table 1.

Table 1.Boundary conditions in non- dimensional form.

Boundary wall	Flow Field	Thermal Field
Vertical wall	$U=0,V=0$	$T=0$
Inclined wall	$U=0,V=0$	$T=0$
Bottom wall	$U=0,V=0$	$T=T_c+\sin^2(\pi * x)$

The non-dimensional governing parameters used are Prandtl number (*Pr*), Rayleigh number (*Ra*) and Hartmann number (*Ha*) these are defined below:

$$Pr=\frac{\nu_f}{\alpha_f};Ha=\sqrt{\frac{\sigma_{nf}B^2L^2}{\nu_{nf}\rho_{nf}}};Ra=\frac{g\beta_f(T_h-T_c)L^3}{\upsilon_f\alpha_f}; \tag{9}$$

The characteristics of heat transfer are obtained by average Nusselt number and can be expressed as-

$$Nu_{av}=-\int_0^1\frac{\partial T}{\partial Y}dX \tag{10}$$

4. Numerical Procedure

Numerical methods

The Galerkin weighted residual finite element method (FEM) has been deployed to the present problem to obtain a numerical solution. From Boussinesq approximation a set of algebraic equations has been formulated and the iterative process is used to solve this algebraic equation set. Triangular mesh formulation has been used to discretize the entire domain into several elements. Converging nature of the numerical solution has been confirmed and the converging criteria used is $|\boldsymbol{\Gamma}^{n+1}-\boldsymbol{\Gamma}^{n}|\le 10^{-6}$ where *n* is the number of iteration and is general dependent variable.

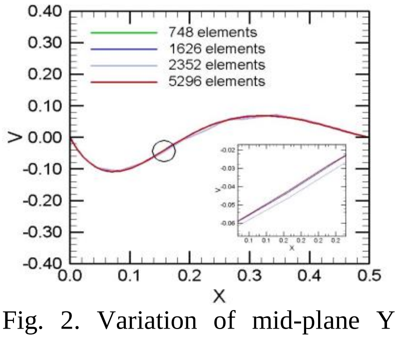

Fig. 2. Variation of mid-plane Y velocity (V) with X at *Ha* = 60, *Ra* = 10^5 and *Pr* = 10

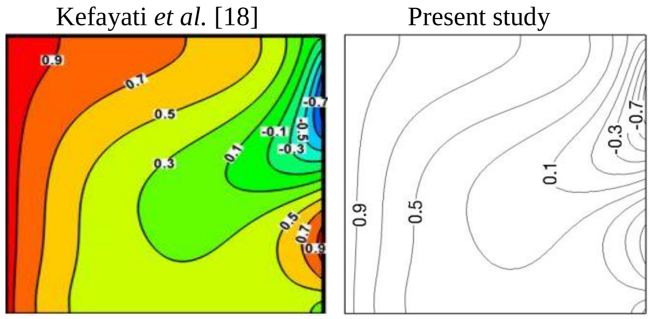

Fig. 3. Comparison of the isotherms for *Ra* = 10^5 and *Ha* = 60 between numerical results by Kefayati *et al.* [18] and the present results.

Grid independence

An extensive mesh testing procedure is conducted to guarantee a grid independent solution and its results have been presented in Figure 2. To check the accuracy of the numerical solution several mesh element numbers (748, 1626, 2352 and 5296) have been checked. From the figure, it is seen that mid-plane Y-velocity profiles for grid elements of 2352 and 5296 almost overlap each other thus making the solution grid independent. So, grid size of 5296 mesh elements is considered to be the optimum for the present study and other numerical simulation has been carried out taking this grid as independent.

Code validation

Code validation is done in light of isotherm and is validated with the consequences of Kefayati et al. [18] at *Ra* = 10^5 and *Ha* = 60 in Figure 3. From the figure, it is evident that present result is completely in par with the previous study. So, the present numerical code and solution procedure are completely reliable, and so is the numerical solution.

5. Results and discussion

Effect of power law index and Hartmann number on isotherm and streamline

Fig. 4 and 5 shows the comparative analysis of streamline and isotherm contours at various Hartmann number and power law index at Pr = 10 and Ra = 10^5. The streamline shows two vortices, one anticlockwise and another clockwise. In the given problem, the working fluid gets heated at the bottom wall and moves upward using Buoyancy force. Again, the particle in contact with cold wall moves downwards and thus creates the vortices. As triangular cavity is used, it is seen that the vortex displaying clockwise rotation of the fluid is squeezed. It is considered as secondary vortex and has less strength comparable to primary vortex in the cavity. The isotherm has closely packed lines near bottom wall which indicates better convection.

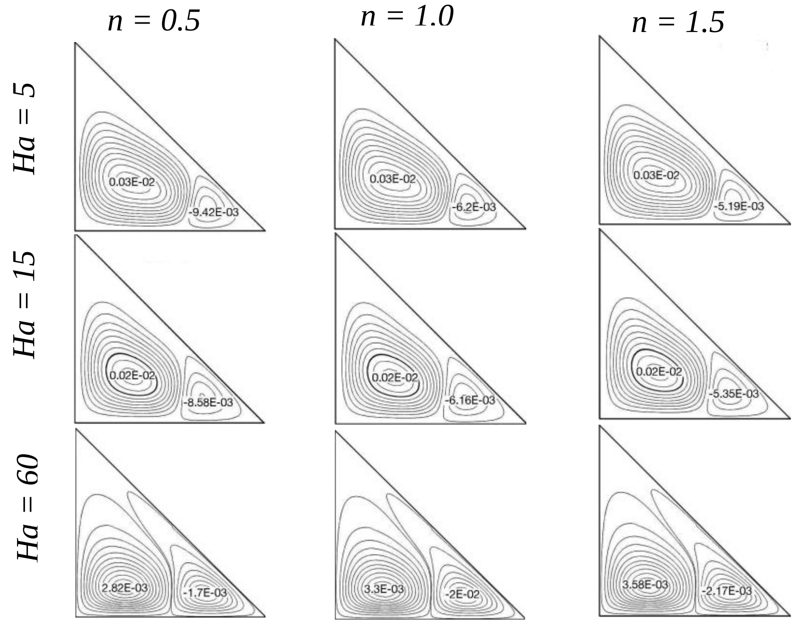

Fig. 4. Comparison of streamline at various Hartmann number and power law index for Pr = 10 and Ra = 10^5

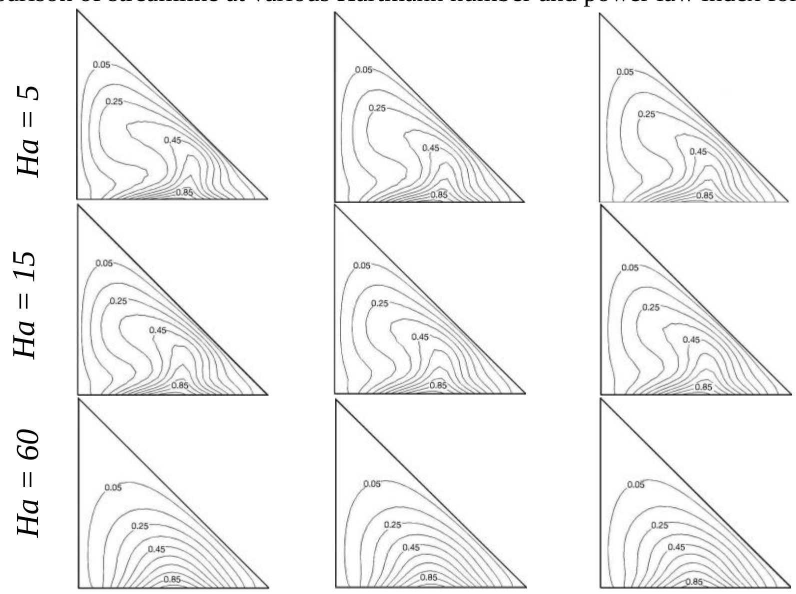

Fig. 5. Comparison of isotherm at various Hartman number and power law index for Pr = 10 and Ra = 10^5

It is evident from the isotherm (Fig. 5.) that the convective pattern is decreased due to the increase of Hartmann number. The more distortion of isotherm ensures convective dominance thus better heat transfer at lower Hartmann number. This is because the higher magnetic field distorts the vortices by suppressing buoyancy force with the increase of power law index, the isotherm gets less distorted. So heat transfer becomes weaker. The stream function also shows a decreasing trend of value at *Ha* = 5, 15.But for *Ha* = 60, it doesn't maintain the pattern. The stream function decreases with the increase of power law index. But for *Ha* = 60, it rises. So it ensures a different behavior at *Ha* = 60. Because higher magnetic field can affects the heat transfer.

Effect of Hartmann number on average Nusselt number at different parameter

In Fig. 6(a) and 6 (b) average Nusselt number is plotted against Hartmann number at different Rayleigh number and power law index respectively for *Pr* = 10. From fig. 6(a), it can be said that Hartmann number has less effect on average Nusselt number at $Ra = 10^3$ & 10^4 as the buoyancy force was not significant for those cases. A significant change of rate of heat transfer is observed for $Ra = 10^5$ where maximum value of *Nu* = 4.8037 is obtained in the absence of magnetic field. This is because of the higher temperature gradient at the heated wall. But as the effect of magnetic field starts to persist, the heat transfer rate diminishes gradually and becomes nearly constant at *Ha* > 75.

In fig. 6(b), it is observed that in the absence of magnetic field highest heat transfer is obtained for pseudo-plastic fluid (*n* = 0.5) with *Nu* = 4.8037 and lowest heat transfer is obtained for dilatants fluid (*n* = 1.5) with *Nu* = 3.6331. With the intensification of magnetic field, heat transfer rate shows a decreasing nature. An important change is found at nearly *Ha* = 35 where all the curves intersect. The effect Hartmann number is almost nullified and the heat transfer rate for pseudo-plastic fluid becomes lesser than that of the Newtonian (*n* = 1.0) and dilatants fluid. So it is also noticed that the lowest possible heat transfer rate is obtained for pseudo-plastic fluid at *Ha* = 100 with *Nu* = 1.9454.

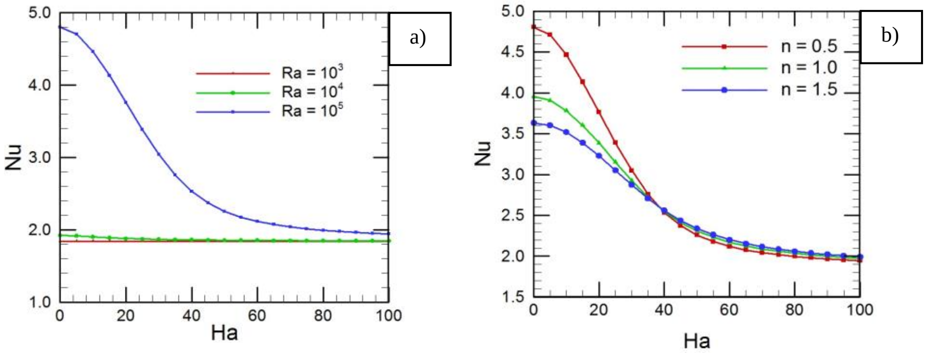

Fig. 6. Variation of the average Nusselt number due to the change of Hartmann number at different (a) Rayleigh number for *n* = 0.5 (b) Power law index for $Ra = 10^5$ at *Pr* = 10

Effect of Rayleigh number on average Nusselt number at different power law index

Fig 7 demonstrates the effect of Rayleigh number on average Nusselt number at different power law index for *Ha* = 30. It is followed that there is less effect on heat transfer rate due to the increment of Rayleigh number

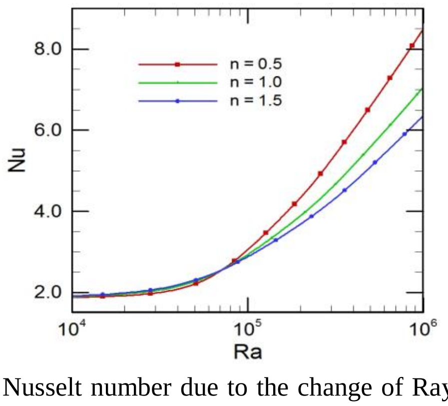

Fig. 7. Variation of average Nusselt number due to the change of Rayleigh number at different power law index for *Ha* = 30

at first. An intersection is found at near $Ra = 10^5$ for all fluids and a huge increase of heat transfer begin with the increment of Rayleigh number. The rate of increase of heat transfer for the pseudo plastic fluid is evidently the highest among all but the opposite is true for $Ra < 8 \times 10^4$.

6. Conclusion

Different pertinent parameters on this heat transfer investigation lead us to the following conclusion:

- Magnetic field is effective on overall heat transfer for higher Rayleigh number $Ra = 10^5$.

- Pseudo-plastic fluid (n = 0.5) should be used at lower magnetic field (Ha < 40) with higher buoyancy effect ($Ra > 8 \times 10^4$).
- Dilatant fluid (n = 1.5) would be feasible for higher intensity of magnetic induction (Ha > 40) at lower Rayleigh number ($Ra < 8 \times 10^4$).
- To obtain better heat transfer (more than 32%) non-Newtonian pseudo-plastic (n = 0.5) fluid can be used in absence of magnetic field.
- At an optimum magnetic induction (Ha = 40), identical heat transfer rate is found for both Newtonian fluid (n = 1.0) and non-Newtonian fluid (n = 0.5, 1.5).

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