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MHD Free Convection and Mass Transfer Flow over a Vertical Porous Plate in Presence of Ion-slip Current

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Abstract

In this dissertation have been devoted to the study of MHD free convection and mass transfer flow over a vertical porous plate with viscous dissipation and ion-slip currents in a rotating system. The finite difference method is used as main tool for the numerical approach. The governing equations are derived using the boundary layer and Boussinesqs' approximations. These equations with boundary conditions are converted using usual transformations. It is observed that the primary velocity increases for different values of ion-slip parameter, Eckert number while it decreases for different values of magnetic parameter, rotation parameter and Prandtl number. The secondary velocity increases for different values of magnetic parameter where as it decreases ion-slip parameter. Temperature increases for different values of magnetic parameter and Eckert number while it decreases for different values of Prandtl number.

Keywords: MHD, Ion-slip current, Rotation, Dissipation, Uniform magnetic field.

1. Introduction

Combined heat and mass transfer in fluid-saturated porous media finds applications in a variety of engineering processes such as heat exchanger devices, petroleum reservoirs, geothermal and geophysical engineering, moisture migration in a fibrous insulation and nuclear waste disposal. Viscous dissipation changes the temperature distributions by playing a role like an energy source, which leads to affected heat transfer rates. The merit of the effect of viscous dissipation depends on whether the plate is being cooled or heated. Prasanna Lakshmi et al. [1] studied MHD boundary layer flow of heat and mass transfer over a moving vertical plate in a porous medium with suction and viscous dissipation.

In an ionized gas where the density is low and/or the magnetic field is very strong, the effects of Hall and ion-slip currents play a significant role in the velocity distribution of the flow. The study of magnetohydrodynamic flows with Hall and ion-slip currents has important engineering applications in the problem of magnetohydrodynamic generators and of Hall accelerators as well as flight magnetohydrodynamics. Combined effects of Hall and ion-slip currents on free convective heat generating flow past a semi-infinite vertical flat plate have been investigated by Abo-Eldahab and Aziz [2]. The rotating flow of an electrical conducting fluid in the presence of magnetic field is encountered in geophysical and comical fluid dynamics. Study of the interaction of Coriolis force with electromagnetic force in porous media is important in some geophysical and astrophysical problems. Dileep and Priyanka [3] investigated the Hall effects on MHD slip flow and heat transfer through a porous medium over an accelerated plate in a rotating system. An investigation of the effect of Hall current and rotational parameter on dissipative fluid flow past a vertical semi-infinite plate was studied by Abuga et al. [4]. Emad M. Abo-Eldahab and, Mohamed A. El Aziz [5] studied viscous dissipation and Joule heating effects on MHD-free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ion-slip currents. An investigation of the effect of hall current and rotational parameter on dissipative fluid flow past a vertical semi-infinite plate investigated by Abuga et al. [6]. Satya Narayana [7] studied Hall current effects on free convection MHD flow past a porous plate. Ghosh et al. [8] investigated Hall effects on heat transfer and MHD flow in a rotating channel.

Objective of the present investigation is to study the effects of Hall and ion-slip currents, rotation on a hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible fluid past over a vertical semi-infinite plate embedded in a porous medium in the presence of viscous dissipation.

2. Governing Equations

The two dimensional unsteady flow of an electrically conducting incompressible viscous fluid past a semi-infinite vertical porous plate has been considered. The flow is assumed to be in the x-axis which is taken along the plate in the upward direction and y-axis is normal to it. Initially the fluids as well as the plate are at rest but for time $t > 0$ the whole system is allowed to rotate with a constant angular velocity Ω about the y-axis. It is assumed that, T_w, C_w are temperature and spices concentration at the wall and, T_∞, C_∞ are the temperature and the concentration of the spices outside the boundary layer respectively. The physical configuration of the problem is shown in Fig.1. A strong magnetic field is applied in the y-direction. The uniform magnetic field strength B_0 can be taken as $\mathbf{B} = (0,\ B_0, 0)$. The induced magnetic field is neglected, since the magnetic Reynolds number of a partially-ionized fluid is very small. The equation of conservation of electric charge $\overline{\nabla} \bullet \mathbf{J} = 0$ gives $J_y =$ constant because the direction of propagation is considered only along y-axis and $\mathbf{J}$ does not have any variation along the y-axis. The equations which govern the flow under the above consideration and Boussinesq's approximation are as follows:

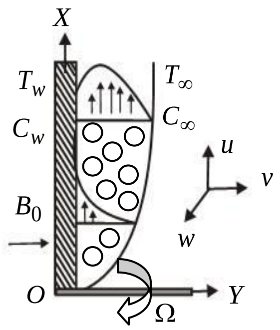

Fig.1 Physical configuration and coordinate system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g_0\beta(T - T_\infty) + g_0\beta^*(C - C_\infty) + 2\Omega w - \frac{\upsilon}{k}u - \frac{\sigma_e B_0^2}{\rho\left(\alpha_e^2 + \beta_e^2\right)}(\alpha_e u + \beta_e w) \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = \upsilon\frac{\partial^2 w}{\partial y^2} - 2\Omega u - \frac{\upsilon}{k}w + \frac{\sigma_e B_0^2}{\rho\left(\alpha_e^2 + \beta_e^2\right)}(\beta_e u - \alpha_e w) \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{c_p}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \tag{4}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} \tag{5}$$

where $\alpha_e = 1 + \beta_e\beta_i$, β_e (Hall parameter), β_i (ion-slip parameter), β (volumetric coefficient of thermal expansion), $\beta^\bullet$ (concentration expansion co-efficient), Ω (angular velocity), g_0 (acceleration due to gravity), υ (Kinematic viscosity), ρ (fluid density), k (permeability of the porous medium), c_p (Specific heat at constant pressure), κ (Thermal conductivity), D_m (Co-efficient of mass diffusivity), B_0 (uniform magnetic field), σ_e (Electrical conductivity), T (temperature in the boundary layer), T_∞ (temperature outside the boundary layer), C (concentration in the boundary layer), C_∞ (concentration outside the boundary layer), t (dimensional time).

The boundary conditions for the problems are;

$$\begin{aligned} &u = U_0, v = 0, w = 0, T = T_w,\ C = C_w \quad \text{at } y = 0 \\ &u = 0, v = 0, w = 0, T = T_\infty, C = C_\infty \quad \text{as } y \to \infty \end{aligned} \tag{6}$$

3. Mathematical Formulation

The problem is simplified by writing the equations in the non-dimensional form. Now introduce the following non-dimensional quantities

$$X = \frac{xU_0}{\upsilon},\ Y = \frac{yU_0}{\upsilon},\ U = \frac{u}{U_0},\ V = \frac{v}{U_0},\ W = \frac{w}{U_0},\ \tau = \frac{tU_0^2}{\upsilon},\ \overline{T} = \frac{T - T_\infty}{T_w - T_\infty},\ \overline{C} = \frac{C - C_\infty}{C_w - C_\infty} \tag{7}$$

Then introducing the dimensionless quantities (7) in equations (1)-(5) respectively, the following dimensionless equations are as follows;

$$\frac{\partial U}{\partial X}+\frac{\partial v}{\partial X}=0 \tag{8}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+G_r\overline{T}+G_m\overline{C}+2RW-\gamma U-\frac{M(\alpha_e U+\beta_e W)}{\alpha_e^2+\beta_e^2} \tag{9}$$

$$\frac{\partial W}{\partial \tau}+U\frac{\partial W}{\partial X}+V\frac{\partial W}{\partial Y}=\frac{\partial^2 W}{\partial Y^2}-2RU-\gamma W+\frac{M(\beta_e U-\alpha_e W)}{\alpha_e^2+\beta_e^2} \tag{10}$$

$$\frac{\partial \overline{T}}{\partial \tau}+U\frac{\partial \overline{T}}{\partial X}+V\frac{\partial \overline{T}}{\partial Y}=\frac{1}{P_r}\frac{\partial^2 \overline{T}}{\partial Y^2}+E_c\left[\left(\frac{\partial U}{\partial Y}\right)^2+\left(\frac{\partial W}{\partial Y}\right)^2\right] \tag{11}$$

$$\frac{\partial \overline{C}}{\partial \tau}+U\frac{\partial \overline{C}}{\partial X}+V\frac{\partial \overline{C}}{\partial Y}=\frac{1}{S_c}\frac{\partial^2 \overline{C}}{\partial Y^2} \tag{12}$$

The corresponding boundary conditions are as follows;

$$U=1,\ V=0,\ W=0,\ \overline{T}=1,\ \overline{C}=1\ \ at\ Y=0 \tag{13}$$

$$U=0,\ \ W=0,\ \overline{T}=0,\ \overline{C}=0\ \ as\ Y\to 0$$

where $G_r=\frac{g_0\beta(T_w-T_\infty)\upsilon}{U_0^3}$ (Grashof number), $G_m=\frac{g_0\beta^*(C_w-C_\infty)\upsilon}{U_0^3}$ (modified Grashof number),

$M=\frac{\sigma_e B_0^2\upsilon}{\rho U_0^2}$ (magnetic parameter), $P_r=\frac{\rho\upsilon c_p}{\kappa}$ (Prandtl number), $S_c=\frac{\upsilon}{D_m}$ (Schmidt number),

$R=\frac{\Omega\upsilon}{U_0^2}$ (rotation parameter), $\gamma=\frac{\upsilon^2}{kU_0^2}$ (permeability parameter), $E_c=\frac{U_0^2}{c_p(T_w-T_\infty)}$ (Eckert number).

4. Solution Technique

The governing second order non-linear coupled dimensionless partial differential equations have been solved numerically with the associated boundary conditions. The explicit finite difference method has been used to solve the coupled equations (8)-(12) with boundary conditions (13). To obtain the difference equations the region of the flow is divided into a grid or mesh of lines parallel to X and Y axes, where X-axis is taken along the plate and Y-axis is taken normal to the plate. Here the plate height $X_{\max}(80.0)$ is considered i.e. X varies from 0 to 80 and assumed $Y_{\max}(60.0)$ as corresponding $Y_{\max}\to\infty$ i.e. Y varies from 0 to 60. There are $m=300$ and $n=300$ grid spacing in the X and Y directions respectively and taken as follows $\Delta X=0.27(0\le X\le 80)$ and $\Delta Y=0.2(0\le Y\le 60)$ with the smaller time step $\Delta\tau=0.005$.

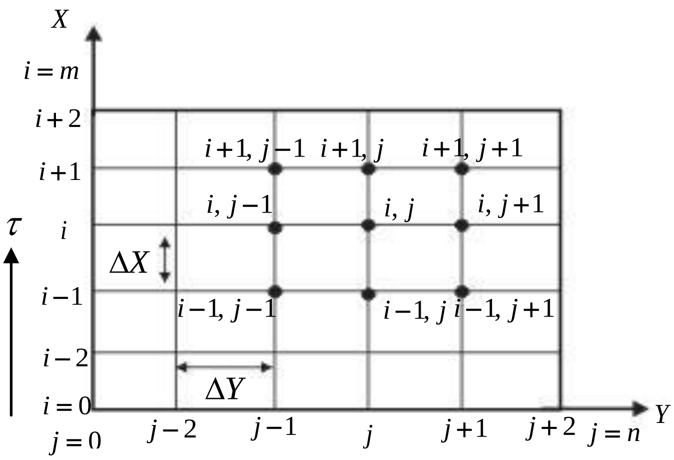

Fig.2 Finite difference grid space

5. Stability and Convergence Analysis

For the constant mesh sizes the stability criteria of the scheme has been established.

$$\frac{U\Delta\tau}{\Delta X}+\frac{|-V|\Delta\tau}{\Delta Y}+2\frac{\Delta\tau}{(\Delta Y)^2}\leq 1$$

$$\frac{U\Delta\tau}{\Delta X}+\frac{|-V|\Delta\tau}{\Delta Y}+\frac{2}{P_r}\frac{\Delta\tau}{(\Delta Y)^2}\leq 1$$

$$\frac{U\Delta\tau}{\Delta X}+\frac{|-V|\Delta\tau}{\Delta Y}+\frac{2}{S_c}\frac{\Delta\tau}{(\Delta Y)^2}\leq 1$$

$$\Delta X=0.27(0\leq X\leq 80)\,,\ \Delta Y=0.2(0\leq X\leq 60)\,,\ \Delta\tau=0.005$$

6. Results and Discussion

The numerical results has been carried out for dimensionless primary velocity (U), secondary velocity(W), temperature($\overline{T}$), species concentration ($\overline{C}$), local and average shear stresses in x-axis (τ_{LU}, τ_{AU}), local and average shear stresses in z-axis (τ_{LW}, τ_{AW}), local and average Nusselt numbers (N_{uL}, N_{uA}), local and average Sherwood numbers (S_{hL}, S_{hA}) for various values of the material parameters such as Hall parameter(β_e), ion-slip parameter(β_i), magnetic parameter(M), rotation parameter(R), Prandtl number(P_r), Schmidt number (S_c), permeability parameter(γ), Eckert number (E_c). The values for the parameters are chosen arbitrarily in most cases. Physically $P_r=0.71$ corresponds to air at 20^0C, $P_r=1.0$ corresponds to salt water at 20^0C and $P_r=1.63$ corresponds to glycerin at 50^0C. The importance of cooling problem in nuclear engineering in connection with the cooling of reactors, the values of G_r and G_m are taken positive. Throughout the calculations the values of G_r and G_m are taken very large ($G_r=5.0$ and $G_m=2.0$).

From Fig.3, it is seen that the primary velocity (U) decreases with the increase of magnetic parameter (M). An increase in the value of the magnetic parameter leads to increase in the magnitude of the Lorentz force which serves to retard the primary velocity. Fig.4 is illustrated that the secondary velocity (W) increases with increasing values of magnetic parameter. This is due to the fact that the resulting Lorentzian body force will not act as a drag force as in conventional MHD flows, but as an aiding body force. This will serve to accelerate the secondary fluid velocity. Fig.5 is illustrated that the temperature ($\overline{T}$) distributions increases with the increase of M. The effects of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive-type force called the Lorentz force. This force has the tendency to increase its temperature distributions.

Form Fig.6, it is seen that the primary velocity (U) increases with an increase of ion-slip parameter (β_i). This is due to fact that the effective conductivity decreases, which reduces the magnetic resistive force affecting on the primary flow. But secondary velocity (W) has decreasing effects with the increase of β_i which is shown in Fig.7.

It is seen from Fig.8, the primary velocity (U) decreases with the increase of rotational parameter (R).In fact rotation parameter defines the relative magnitude of the Coriolis force and the viscous force, thus rotation retards primary flow in the boundary layer.

The temperature increases with the increase of Eckert number (E_c) is shown in Fig.9. The effect of viscous dissipation on flow field is to increase the energy, yielding a greater fluid temperature and as a consequence greater buoyancy force. The increase in the buoyancy force due to an increase in the dissipation parameter enhances the temperature.

From Fig.10, it is seen that the primary velocity (U) decreases with an increases of Prandtl number (P_r). This is because in the free convection the plate velocity is higher than the adjacent fluid velocity and the momentum boundary layer thickness decreases. From Fig.11, it is illustrated that the temperature $(\overline{T})$ distribution decreases with increases of P_r . This is consistent with the well-known fact that the thermal boundary layer thickness decreases with increasing (P_r).

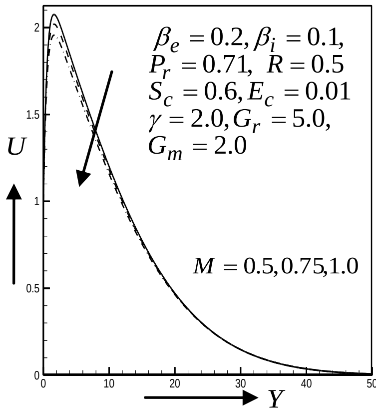

Fig.3 Primary velocity profiles for different values of M

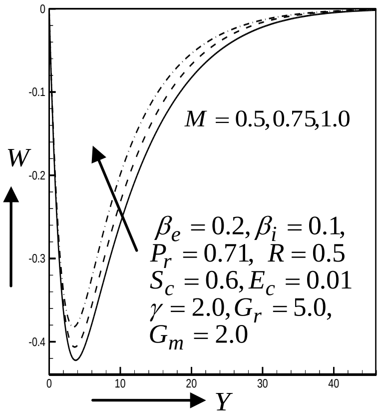

Fig.4 Secondary velocity profiles for different values of M

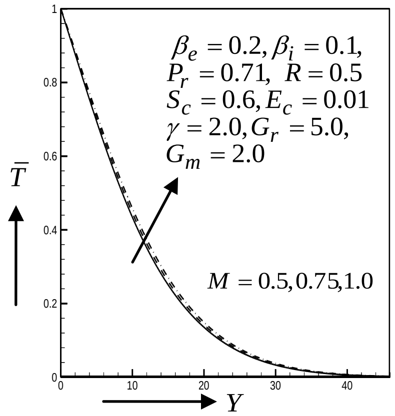

Fig.5 Temperature profiles for different values of M

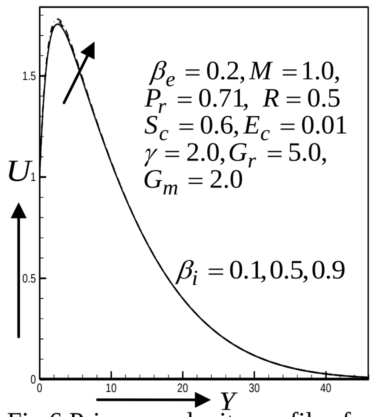

Fig.6 Primary velocity profiles for different values of β_i

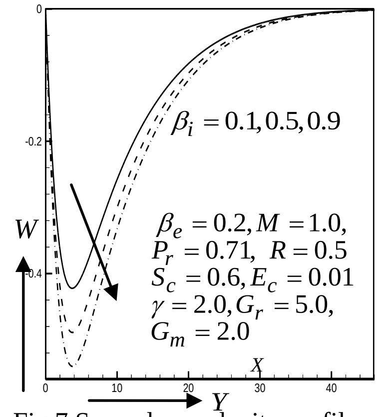

Fig.7 Secondary velocity profiles for different values of β_i

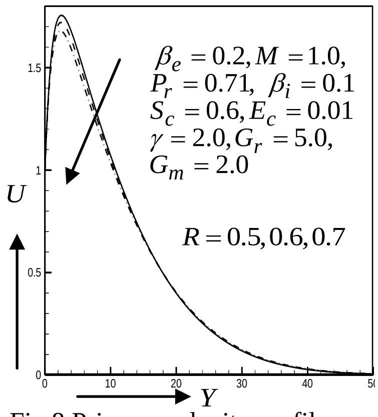

Fig.8 Primary velocity profiles for different values of R

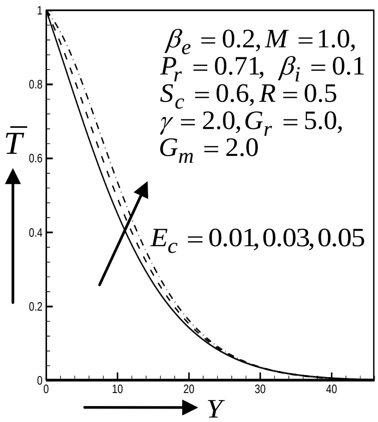

Fig.9 Temperature rofiles for different values of E_c

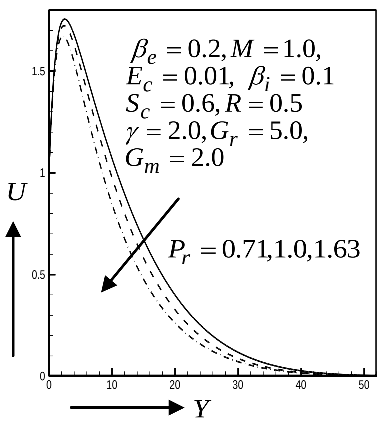

Fig.10 Primary velocity profiles for different values of P_r

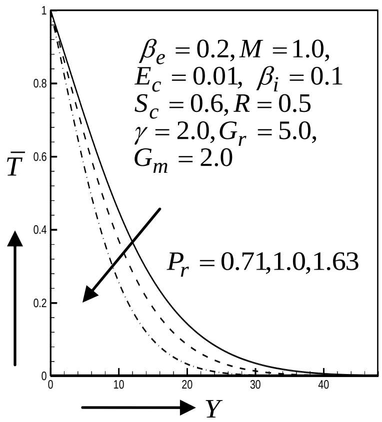

Fig.11 Temperature profiles for different values of P_r

Table 1. Local shear stress and local Nuselt number when
$P_r = 0.71,\ G_r = 5.0, G_m = 2.0, \beta_e = 0.2, \gamma = 2.0,\ S_c 0.6, R = 0.5, \beta_i = 0.1$

M	E_c	τ_x	N_u
0.5	0.01	0.488699	0.850075
0.75	0.01	0.402539	0.750120
1.0	0.01	0.321513	0.654271
1.0	0.01	0.675127	0.502832
1.0	0.03	0.774231	0.431258
1.0	0.05	0.802114	0.401276

7. Conclusion

The effects of governing physical parameters on the primary velocity and secondary velocity, temperature as well as shear stress in *x*-axis, Nusselt number computed and presented in graphical and tabular forms. It is observed that the primary velocity increases as the ion-slip current parameter increases. However the opposite behavior is predicted as the magnetic parameter, rotation parameter, Prandtl number is increased. The secondary velocity increases with the increase of magnetic parameter while it decreases with increasing values of ion-slip parameter. The temperature increases as the magnetic parameter, Eckert number increases but it is decreased for increasing values of Prandtl number. The shear stress in *x*-axis decreases for increasing values of magnetic parameter while Nusselt number decreases with increasing values of magnetic parameter and Eckert number.

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