

Double Diffusive Natural Convection in a Porous Wavy Triangular Enclosure Filled with Nanofluid in Presence of Magnetic Field

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Abstract

The current numerical study is conducted to analyze double diffusive natural convection flow in a triangular enclosure with a sinusoidal wavy bottom surface filled in a porous medium saturated by nanofluid in presence of magnetic field. The wavy bottom surface of the cavity is heated constantly and maintained at higher concentration while left and right inclined walls are at cold temperature and maintained at lower concentration. The transport governing equations are solved by finite element formulation based on the Galerkin method of weighted residuals. The implications of thermal Rayleigh number (Ra_T), Hartmann number (Ha), Lewis number (Le) and number of undulations (λ) on the flow structure, heat and mass transfer characteristics are investigated in detail while, Prandtl number (Pr), buoyancy ratio (N) and solid volume fraction (ϕ) is considered fixed. The results are presented in terms of streamlines, isotherms, isoconcentrations, Nusselt number (Nu) and Sherwood number (Sh). Results of this investigation illustrates that the rate of heat and mass transfer are affected by the considering parameters.

Keywords: Double diffusion, Natural convection, Sinusoidal wavy triangular enclosure, Magnetic field, Nanofluid.

1. Introduction

Natural convective heat and mass transfer in cavities becomes most important issues from industrial and energy perspectives. In the past few decades, experimentally, analytically and numerically extensive researches have been performed on this topic by many researchers [1]-[4]. Combined convective heat and mass transfer is referred to buoyancy driven flows by both temperature and concentration gradients. Combined heat and mass transfer occurs in a wide range of applications in both nature and industry. In nature, such flows are occurred in oceans, lakes, shallow coastal water, and the atmosphere. In industry, these type of flows are the chemical process, crystal growth, solidification, food processing and migration of impurities in non-isothermal material processing applications. Ostrach [5] and Viskanta et al. [6] have reported complete reviews on the subject.

Double-diffusive natural convection in nanofluids is an important fluid dynamics topic that describes the convection driven by two different density gradients with different rates of diffusion [4]. Thermal buoyancy-induced flow and heat transfer inside a porous medium has been investigated enormously in the literature due to its relevance in many natural and industrial applications. The magnetic field effect on the convective heat transfer and the free convection flow of the fluid are very important in engineering. Effect of magnetic field and heat generation on free convection in a porous media filled equilateral triangular cavity has been studied by Chowdhury et al. [7]. Adjlout et al. [8] have studied laminar natural convection in an inclined cavity with a wavy wall. Molla et al. [9] have investigated natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/observation. Das and Mahmud [10] have conducted numerical investigation of natural convection inside a wavy enclosure. Das et al. [11] have analyzed effect of surface waviness and aspect ratio on heat transfer inside a wavy enclosure. Amiri et al. [12] have investigated effect of sinusoidal wavy bottom surface on mixed convection heat transfer in a lid-driven cavity.

Heat transfer in a triangular model can be a simple model for many engineering applications. Although many studies have reported natural convection in triangular cavities [7], studies in combined heat and mass transfer in wavy triangular enclosures filled with nanofluid are very limited. The aim of the present study is to investigate the flow pattern, heat and mass transfer in a porous wavy triangular enclosure filled with nanofluid exposed to both temperature and concentration gradients. Double-diffusive conditions is maintained by taking the bottom wall as heated wall and the source for solute concentration and the left and right inclined wall is cold and lower concentration.

Nomenclature		Greek Letters	
B_0	magnetic field [T]	α	thermal diffusivity [ms^{-2}]
c	concentration [$mol\ m^{-3}$]	β_T	volumetric coefficient of thermal expansion [K^{-1}]
C	dimensionless concentration	β_s	volumetric coefficient of solutal expansion [$m^3 kg^{-1}$]
C_p	specific heat [$Jkg^{-1} K^{-1}$]	λ	undulation number
D	mass diffusivity [m^2/s]	μ	dynamic viscosity [$kgm^{-1} s^{-1}$]
g	gravity acceleration [m/s^2]	θ	dimensionless temperature
k	thermal conductivity [$Wm^{-1} K^{-1}$]	ν	kinematic viscosity [$m^2 s^{-1}$]
L	height of the triangle [m]	ρ	density [kgm^{-3}]
N	buoyancy ratio, $\beta_s \nabla c / \beta_T \nabla T$	ϕ	nanoparticle volume fraction
p	fluid pressure [Pa]	Subscript	
P	dimensionless fluid pressure	c	cold
u, v	x, y component of velocity [ms^{-1}]	h	hot
U, V	x, y component of dimensionless velocity	f	fluid
T	temperature [K]	p	nanoparticle
		nf	nanofluid

2. Governing Equations

Fig. 1 shows the schematic diagram of wavy triangular enclosure subjected to the non-dimensional boundary conditions. The bottom wall of the cavity is heated uniformly at temperature T_h , the left and right inclined wall is considered to be cold at temperature T_c . The concentration c_h , is higher at wavy bottom wall and lower c_c , at left and right inclined wall. The working fluid consider in the system is water based Al_2O_3 nanofluid. The properties of water and Al_2O_3 are presented in Table 1. The physical properties of the fluid are assumed to be constant except the density in the buoyancy force term. The non-dimensional governing equations for nanofluids can be written as:

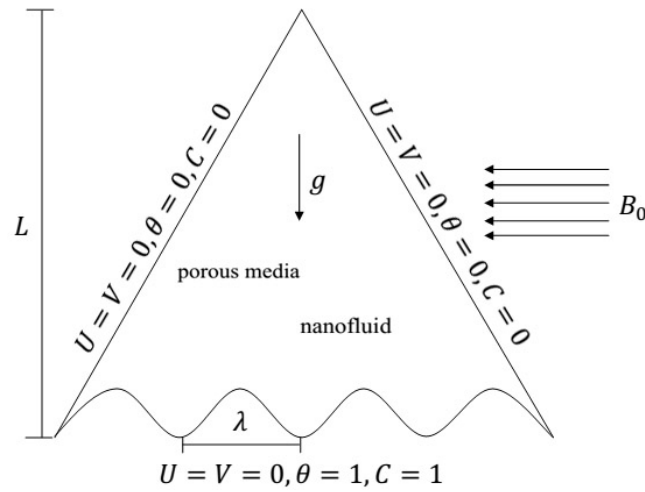


Fig. 1: Physical model of the problem with corresponding non-dimensional boundary conditions.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$\left(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = -\frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \frac{1}{Da} U \quad (2)$$

$$\left(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = -\frac{\partial P}{\partial Y} + \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \frac{1}{Da} V + Ra_T Pr (\theta + NC) - Ha^2 V \quad (3)$$

$$\left(U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} \right) = \frac{\alpha_{nf}}{\alpha_f} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (4)$$

$$\left(U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} \right) = \frac{1}{Le} \left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right) \quad (5)$$

where the thermal Rayleigh number $Ra_T = \frac{g \beta_f \rho_f (T_h - T_c) L^3}{\alpha_f \mu_f}$, the solutal Rayleigh number $Ra_S = \frac{g \beta_s \rho_f (C_h - C_c) L^3}{\alpha_f \mu_f}$,

Prandtl number $Pr = \frac{\nu_f}{\alpha_f}$, Darcy number $Da = \frac{K}{L^2}$, Hartmann number $Ha = B_0 L \sqrt{\frac{\sigma_f}{\rho_{nf} \alpha_f}}$, Buoyancy ratio $N = \frac{Ra_S}{Ra_T}$ and Lewis number $Le = \frac{\alpha_f}{D_f}$.

The non-dimensional boundary conditions are:

On the left and right wall: $U = V = 0, \theta = 0, C = 0$

On the bottom sinusoidal wall: $U = V = 0, \theta = 1, C = 1$.

The following dimensionless variables are used to non-dimensionalize the governing equations:

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha_f}, V = \frac{vL}{\alpha_f}, P = \frac{pL^2}{\rho_{nf} \alpha_f^2}, \theta = \frac{T-T_c}{T_h-T_c}, C = \frac{C-C_c}{C_h-C_c} \quad (6)$$

Table 1: Thermophysical properties of water and nanoparticles

Physical Properties	$C_p(J/kgK)$	$\rho(kg/m^3)$	$k(W/mK)$	$\beta_T(1/K)$
Water	4179	997.1	0.613	21×10^{-5}
Al_2O_3	765	3970	40	0.8×10^{-5}

The effective thermal conductivity of the nanofluid is approximated by the Hamilton-Crosser model [13] expressed as

$$k_{nf} = k_f \frac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f - \phi(k_f - k_p)}$$

The viscosity of nanofluid $\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$, volumetric coefficient of thermal expansion $(\rho\beta_T)_{nf} = (1-\phi)(\rho\beta_T)_f + \phi(\rho\beta_T)_p$, density $\rho_{nf} = (1-\phi)\rho_f + \phi\rho_p$, thermal diffusivity $\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}$, and heat capacitance $(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_p$.

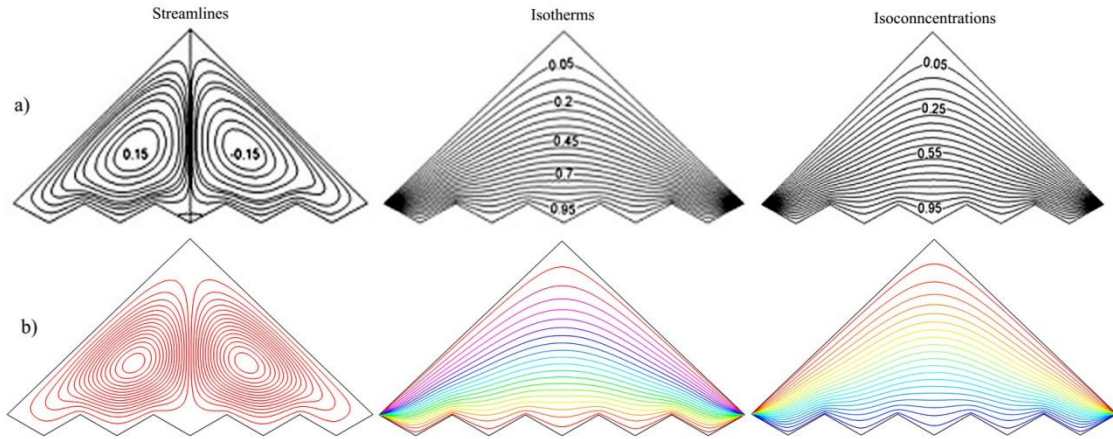


Fig. 2: A comparison for streamlines (left column), isotherms (middle column) and isoconcentration (right column) between Rahman et al. [14] (top row) and present study (bottom row) for $Ra_T = 10^4, Pr = 7.0, \phi = 0, Le = 2$ and $N = 0$.

The local and average Nusselt number along the heated bottom wall can be calculated as

$$Nu_b = -\frac{k_{nf}}{k_f} \left(\frac{\partial \theta}{\partial n} \right), Nu = -\left(\frac{k_{nf}}{k_f} \right) \frac{1}{L} \int_0^L \left(\frac{\partial \theta}{\partial n} \right) dL \quad (7)$$

The local and average Sherwood number along the heated bottom wall can be calculated as

$$Nu_b = -\left(\frac{\partial C}{\partial n} \right), Nu = -\frac{1}{L} \int_0^L \left(\frac{\partial C}{\partial n} \right) dL. \quad (8)$$

3. Numerical Method

The numerical procedure used in this study is based on the Galerkin weighted residual method of finite element method. In this method, the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Then the nonlinear governing partial differential equations are transferred into a system of integral equations by applying Galerkin weighted residual method. The integration involved in each term of these equations is performed by using Gauss's quadrature method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using Newton's method. Finally, these linear equations are solved by using Triangular Factorization method.

4. Validation

A test has been performed to validate the present study by comparing with earlier study Rahman et al. [14]. Fig. 2 represents the results in terms of streamlines, isotherms and isoconcentrations obtained by the present code and the results presented by [14]. The comparison shows in Fig. 2 and it can be clearly seen that there is good agreement between the results.

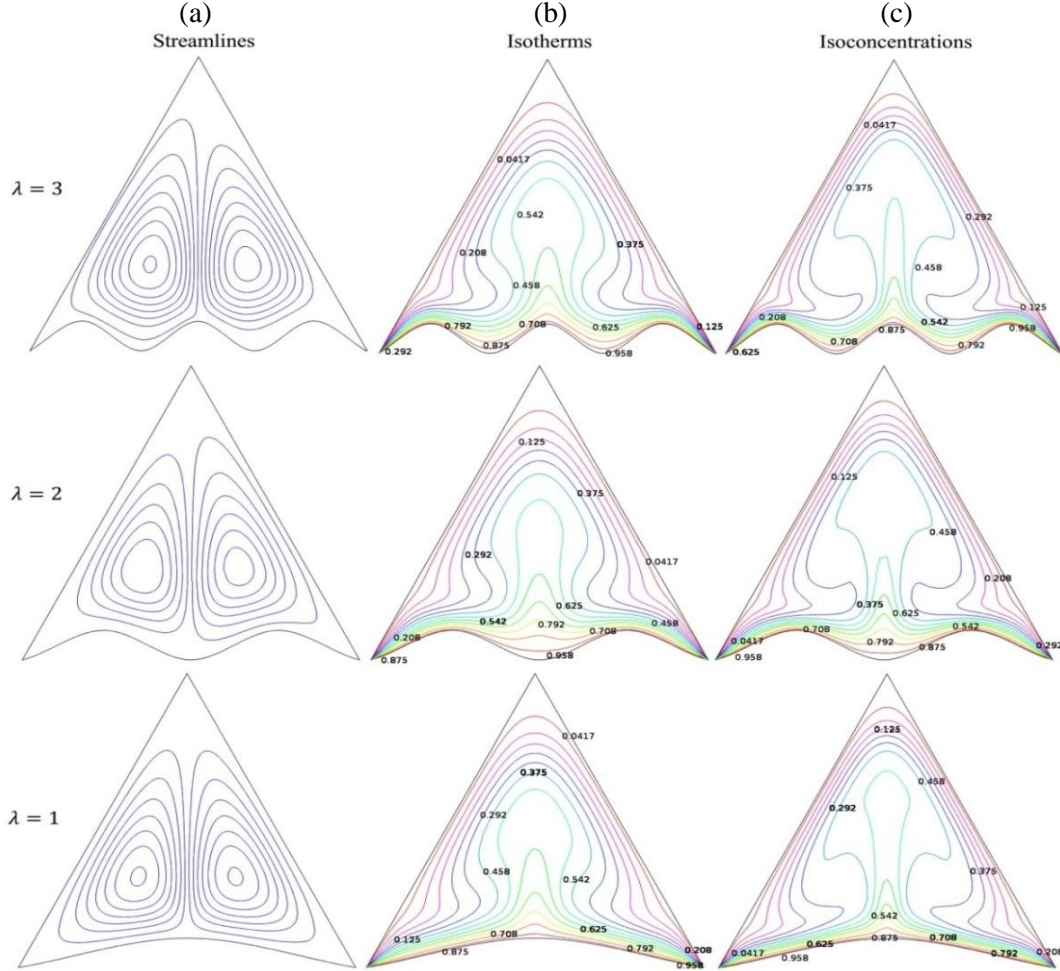


Fig. 3: a) Streamlines, b) Isotherms and c) Isoconcentrations for different values of undulation number λ with $Ra_T = 10^5$, $Pr = 7.0$, $\phi = 10\%$, $Le = 2$, $Ha = 25$, $N = 1$, $Da = 10^{-3}$.

5. Result and Discussion

The natural convection inside a porous wavy triangular cavity filled with nanofluid including magnetic effect is influenced by the controlling parameters $10^4 \leq Ra_T \leq 10^6$, $0 \leq \lambda \leq 3$, $0 \leq Ha \leq 50$ and $1 \leq Le \leq 10$. The results are represented in terms of streamlines, isotherms, isoconcentrations, average Nusselt number (Nu) and average Sherwood number (Sh) for controlling parameters while $Pr = 7.0$, $\phi = 10\%$, $N = 1$, $Da = 10^{-3}$ are kept fixed.

Fig. 3 (a)-(c) represents the flow strength, heat transport and species transport profile for different values of undulation number λ while $Ra_T = 10^5$, $Pr = 7.0$, $\phi = 10\%$, $Le = 2$, $Ha = 25$, $N = 1$, $Da = 10^{-3}$. At Fig. 3 (a), it can be shown that two symmetrically distributed cells are created within the cavity for $\lambda = 1$. The cell near the left inclined wall rotates clockwise direction while the other cell rotates in anticlockwise direction. The cell of rightside increases and dominates the cell of leftside for $\lambda = 2$. The opposite results can be shown for further increasing of undulation number ($\lambda = 3$) where the right cells dominates the left one. In Fig. 3 (b), a symmetrical plumwise temperature distribution is started due to moving fluid to the upper direction of the cavity as buoyancy driven flow moves the fluid from the hotter to colder. This flow becomes stronger with increasing the undulation number. Hat of the plume becomes thinner for concave wave ($\lambda = 2$) and wider for convex wave ($\lambda = 1, \lambda = 3$) at the centre of the bottom wall. A subplume are started from the wavy bottom wall and moves to the upper direction of the cavity for all values of λ . The similar distribution can be shown for concentration that can be noticed from the Fig. 3. A wavy solutal boundary layer forms for all undulation

numbers. The symmetrical temperature and concentration distribution occurs as the left and right inclined wall are maintained at the similar boundary condition. The thermal and solutal boundary layer near the bottom

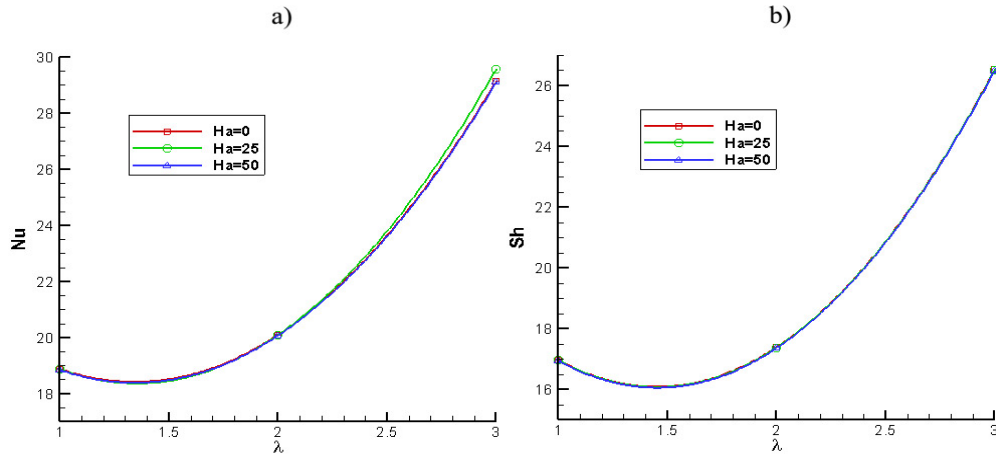


Fig. 4: The variation of a) average Nusselt number (Nu) and b) average Sherwood number (Sh) at the heated bottom wavy wall for different undulation number and Ha with $Pr = 7.0, \phi = 10\%, Ra_T = 10^5, Le = 2, N = 1$ and $Da = 10^{-3}$.

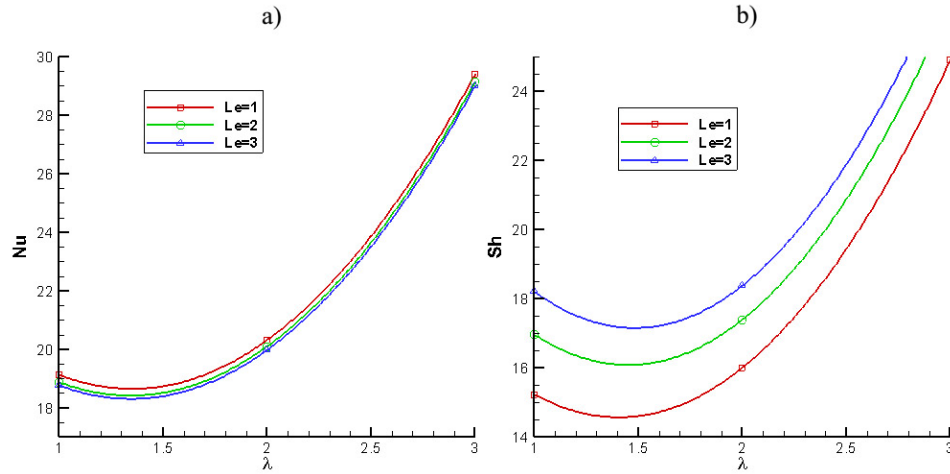


Fig. 5: The variation of a) average Nusselt number (Nu) and b) average Sherwood number (Sh) at the heated bottom wavy wall for different undulation number and Le with $Pr = 7.0, \phi = 10\%, Ra_T = 10^5, Ha = 25, N = 1$ and $Da = 10^{-3}$.

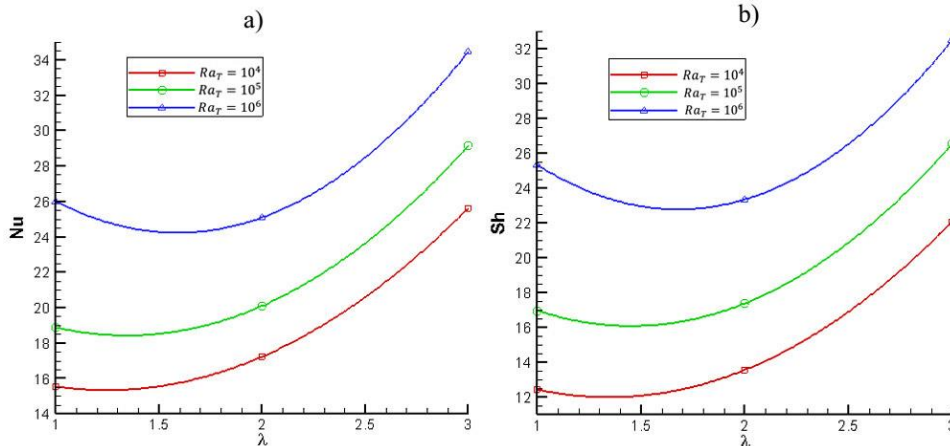


Fig. 6: The variation of a) average Nusselt number (Nu) and b) average Sherwood number (Sh) at the heated bottom wavy wall for different undulation number and Ra_T with $Pr = 7.0, \phi = 10\%, Le = 2, Ha = 25, N = 1$ and $Da = 10^{-3}$.

surface becomes slightly compressed when the number of waves increases.

Fig. 4-6 describes the investigation of the effect of Hartmann number, Lewis number and thermal Rayleigh number on the heat and mass transfer rate on the wavy bottom wall which is heated uniformly for different values of undulation number while $Pr = 7.0, \phi = 10\%, N = 1, Da = 10^{-3}$. In the Fig. 4-6, the graph demonstrates that the Nusselt number and Sherwood number on the heated bottom wavy wall increase

remarkably with the increase of λ for all Hartmann number, Lewis number and thermal Rayleigh number. The effect of Hartmann number is negligible for Nu and does not affect on the average Sherwood number that can be shown in Fig. 4 (a)-(b). Fig. 5 demonstrates that The average nusselt number is slightly decreased while the average Sherwood number increase considerably with the increasing of Lewis number. The average Sherwood number increases as the thermal resistance is higher than the solutal resistance and therefore the mass transfer rate is higher than the heat transfer rate for increasing the value of Le . The average Nusselt number and the average Sherwood number increase considerably with the increasing of thermal Rayleigh number that represents by the graphs in Fig. 6. This is because the flow strength increases with the increasing of thermal Rayleigh number.

6. Conclusion

In this study, a numerical simulation has been conducted to investigate the effect of Hartmann number, thermal Rayleigh number, undulation number and Lewis number on flow pattern, temperature and concentration profile in a wavy triangular cavity filled porous media saturated nanofluid. The outcomes of the existing analysis are as follows:

- The flow pattern, temperature and solutal profile changes with raising the number of waves in the bottom wall.
- The average Nusselt number and Sherwood number increases for increasing the undulation number and thermal Rayleigh number.
- The magnetic field effect is negligible on the average Nusselt number and Sherwood number.
- The heat transfer rate slightly decreases while the mass transfer rate increase significantly for increasing Lewis number.

7. Acknowledgements

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