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Effect of magnetic field on natural convection in a rectangular enclosure with thin fin attached and having *Cu*-water nanofluid inside

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Abstract

In this study, a rectangular enclosure with thin fin attached and filled with Cu-water nanofluid under external magnetic field is numerically analyzed. A highly conductive thin fin is attached at the bottom wall. Numerical simulations have been carried out for wide variety of Rayleigh numbers ($Ra = 10^3 \sim 10^6$), Hartmann number ($Ha = 0$-100) and solid volume fraction ($\varphi = 0 \sim 0.15$). Galerkin weighted residual method of finite element analysis is used in this investigation to find the solution. Numerical accuracy is ensured by the grid independency test. Flow and thermal behavior are discussed on the basis of streamlines, isothermal lines respectively. Nusselt number (Nu) plot on cold wall and average temperature of the working fluid are shown to quantify the overall heat transfer rate. Results show that heat transfer rate increases with the increment of Rayleigh number, solid volume fraction and decreases with the increment of Hartmann number. More than 40% better heat transfer is recorded for lower strength of magnetic field. There is no significant change in overall heat transfer at $Ha > 40$, even with increasing Ra

Keywords: Magnetic field, Nusselt number, Rayleigh number, Nanofluid, Fin

1. Introduction

Heat transfer in natural convection mode is very crucial in this modern era of science and engineering. In order to meet the growing demand from various industries, engineers and scientists have to come up with numerical analysis of heat transfer and investigate the effect of different pertinent parameters [1-3]. Heat transfer analysis under external magnetic field has grown more attentions in this consequence. In recent times many researchers have investigated and successfully evaluated the effect of external magnetic field on natural convection. Amir Houshang Mahmoudi *et al*. [4] studied the MHD natural convection in a trapezoidal enclosure using *Cu*–water nanofluid considering two dimensional trapezoidal enclosure with partially activated wall, they found out that at Rayleigh number, $Ra = 10^4$ and 10^5 the Nusselt number increases with the Hartmann number due to the presence of nanoparticles but at higher Rayleigh number, Nusselt number reduction was also observed. Hakan F. Oztop *et al*. [5] has showed that for MHD buoyancy induced laminar in a non-isothermally heated square enclosure for different parameters as amplitude of sinusoidal function, increasing heat transfer rate is observed and Hartmann number can also be a control parameter for heat transfer rate and flow field. Recent numerical investigation also shows the importance of external magnetic field effect on heat transfer with different shapes of cavity and boundary conditions [6-8]. In this regard, Ghasemi *et al.* [9] showed that for a square enclosure filled with a water-Al_2O_3 which is influenced by magnetic field exerts stronger flow circulations and intensified isotherms near the vertical walls at higher Raleigh Number (*Ra*), lower Hartmann number (*Ha*) at a fixed solid volume fraction.

From industrial point of view energy saving and enhancement of heat transfer is a must. This calls for the replacement of conventional heat transfer fluids with nanofluid. Nanofluids are fluids containing nanometer-sized particles of metals, carbides, oxides, nitrides, or nanotubes. Related studies [10-12] show the enhanced thermophysical properties such as higher thermal conductivity and heat transfer coefficient compared to the base fluid.

Fins are thin components or appendage attached to a larger body or structure for increasing convection. Aminossadati *et al.* [13] showed in their study of MHD natural convection in a square cavity with a thin fin that the position of the fin has more effect on the hot wall then the cold wall at a higher values of Rayleigh number where the heat transfer is mainly due to convection. Also related investigations [14-16] shows that not only the positon but also the size and shape of the fin has severe impact in heat transfer mechanism.

The purpose of this investigation is to analyze natural convection under external magnetic field in a rectangular cavity. Effect of different pertinent parameters such as Rayleigh number (*Ra*), Hartmann number (*Ha*), solid volume fraction (φ) have been shown in terms of streamline, isotherm contours and related plots. Mid-plane velocity profile is shown to justify the flow behavior under external magnetic field. Nusselt number (Nu) at the cold wall and average temperature of the nanofluid is illustrated to investigate the heat transfer.

2. Problem specification

Fig. 1 shows a schematic diagram of the rectangle enclosure filled with *Cu* – water nanofluid. The two vertical left and right walls have been kept adiabatic and the bottom wall is kept at high temperature ($T = T_h$) and the top wall is kept at low temperature ($T = T_c$) where $T_c < T_h$. Here, Boussinesq approximation is assumed to hold true for buoyancy effect and walls are assumed no slip. The gravitational effect is shown along the negative *Y*-axis. *Cu* - Water nanofluid is considered as the working fluid. Radiation and viscous dissipation heat transfer effects are assumed to be negligible for the simplification of the investigation.

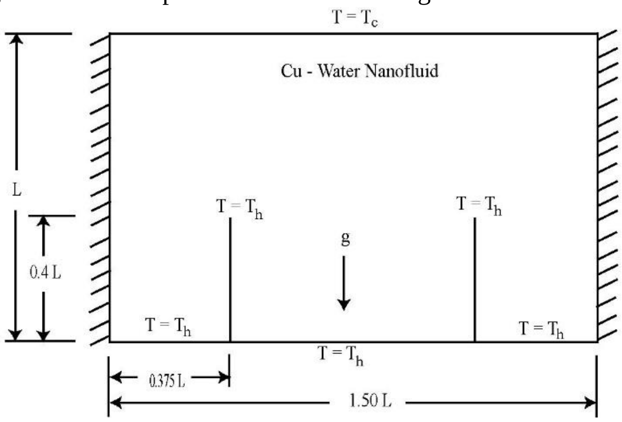

Fig. 1. Schematic diagram of a rectangular enclosure filled with *Cu*-Water nanofluid with thin fin attached

3. Mathematical formulation

The steady state equations that govern the conservation of energy, momentum and continuity for the natural convection of fluid in the presence of a magnetic field can be written in the following non dimensional forms.

$$\frac{\partial(U\delta)}{\partial X}+\frac{\partial(U\delta)}{\partial Y}=\frac{\partial}{\partial X}\left(\Gamma_\delta\frac{\partial\delta}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\Gamma_\delta\frac{\partial\delta}{\partial Y}\right)+S_\delta. \tag{1}$$

Table 1 describes the summarized form [17].

Table 1. A summary of the terms of the non-dimensional governing equations (1).

Equation	δ	Γ_δ	S_δ
Continuity	*1*	0	0
U-momentum	U	$\mu_{nf} / \rho_{nf}\alpha_f$	$-\partial P/\partial X$
V-momentum	V	$\mu_{nf} / \rho_{nf}\alpha f$	$-\partial P/\partial Y + (\rho\beta)_{nf} RaPr\Theta / (\rho_{nf}\beta f) - Ha^2PrV$
Energy	Θ	α_{nf} / α_f	0

Scales are obtained to find the non-dimensional form of the governing equations which can be defined as-

$$X=\frac{x}{L},Y=\frac{y}{L},U=\frac{u}{U_o},V=\frac{v}{U_o},P=\frac{p}{\rho_p {U_o}^2},\Theta=\frac{T-T_c}{T_h-T_c}. \tag{2}$$

The non-dimensional governing parameters Rayleigh number (Ra) is an indication of the effects of buoyancy forces, Prandtl number (Pr) is the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity and Hartmann number (Ha) corresponds to the effects of magnetic forces, can be expressed as

$$Ra=\frac{g\beta_f(T_h-T_c)L^3}{\nu_f^2}\mathrm{Pr},\mathrm{Pr}=\frac{\nu_f}{\alpha_f},Ha=B_0L\sqrt{\frac{\sigma_{nf}}{\rho_{nf}\nu_f}}. \tag{3}$$

The thermo-physical characteristic parameters are expressed below

Effective density of nanofluid, $\rho_{nf}=\left(1-\varphi\right)\rho_f+\varphi\rho_p$. (4)

Specific heat coefficient, $(\rho C_p)_{nf}=\left(1-\varphi\right)(\rho C_p)_f+\varphi(\rho C_p)_p$. (5)

Thermal expansion coefficient, $(\beta\rho)_{nf}=\left(1-\varphi\right)(\beta\rho)_f+\varphi(\rho\beta)_p$. (6)

Dynamic viscosity [18], $\mu_{nf}=\mu_f\left(1-\varphi\right)^{-0.25}$. (7)

Thermal conductivity [19], $$k_{nf} = k_f\left[\frac{(k_p+2k_f)-2\varphi(k_f-k_p)}{(k_p+2k_f)+\varphi(k_f-k_p)}\right]. \quad (8)$$

Here, subscript p, nf and f represent nanoparticle, nanofluid and base fluid respectively. Average verage Nusselt number can be expressed as,

$$\overline{Nu} = \frac{hL}{k_f} = -\frac{k_{nf}}{k_f}\int_0^1 \frac{\partial\Theta}{\partial Y}.dX \quad (9)$$

Boundary conditions illustrated in Fig. 1 can be written in dimensionless forms are presented in Table 2.

Table 2. Boundary condition in non-dimensional form

Boundary wall	Flow field	Thermal field
Bottom and fin wall	U = 0,V = 0	$\Theta = 1$
Top wall	U = 0,V = 0	$\Theta = 0$
Side wall	U = 0,V = 0	$\frac{\partial\Theta}{\partial X} = 0$

Here, U, V, Θ represent non dimensional x-direction velocity, y-direction velocity and temperature respectively. Again σ, υ, β and B_o represent electrical conductivity, kinematic viscosity, thermal expansion co-efficient and magnetic flux density respectively.

4. Numerical procedure

Grid Independency test

To examine the accuracy of the numerical results grid independency test is carried out. Mid-plane velocity profile of the vertical mid-plane is inspected at Ra = 10^4, Ha = 30, φ = 0.15 (see Fig 2). As shown in the following Fig. 2 mesh elements for the test are 1334, 2548, 6329, 10377. From the figure it is observed that mid-plane x-velocity profiles for 6329 and 10377 mesh elements are almost overlapping each other. As a result we take 6329 mesh elements as the optimum grid elements for present study.

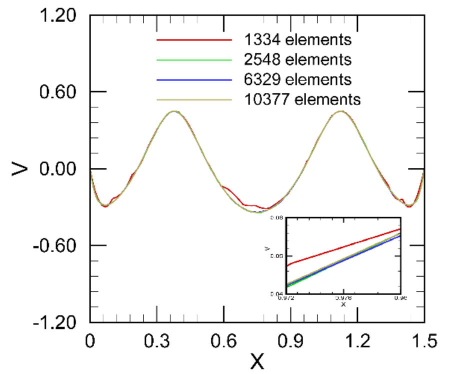

Fig. 2. Variation of mid-plane Y-velocity (V) with X at Ha = 20, Ra = 10^5 and φ = 0.15

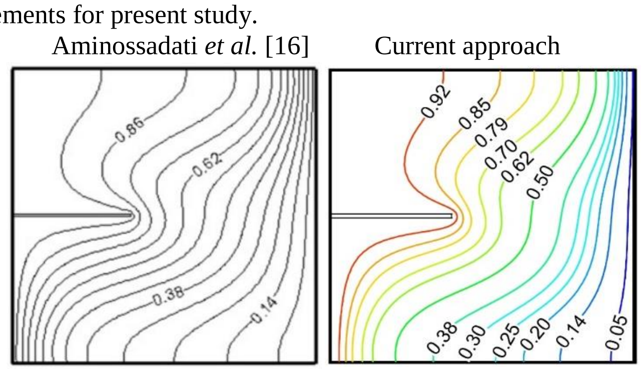

Fig. 3. Comparison of isotherm contours between Aminossadati *et al.* [16] and present code at Ra =10^5, Ha = 50, L_p = 0.4, Y_p = 0.5

Code Validation

The non-dimensional governing equations along with the boundary conditions are solved numerically. The previous study by Aminossadati *et al*. [16] is compared with the present code (see Fig. 3). For comparing the results isothermal lines are plotted for Ra = 10^5 and Ha = 50 when the length of the fin L_P = 0.4 and position of the fin Y_P = 0.5. According to figure, we can say that the validation study holds a very good agreement.

5. Result and discussion

Effect of Hartmann number (*Ha*) on streamlines and isotherm contours

From the Fig. 4, it is evident that that increment of Hartmann number (Ha) tends to decrease the strength of flow field no matter what the Rayleigh number (Ra) is. This is due to the fact that, the fluid particles do have the unique magnetic susceptibility property which results in Lorentz force. So domination of this force opposes the reason of its generation. Thus the net force applied on the working fluid is affected which eventually weakens the flow field in terms of values of stream function. Thus weaker vortices can be found at lower Ra = 10^4 and higher at Hartmann number (Ha = 50). But at higher Ra, the vortices previously formed in the cavity tend to absorb more energy, get larger in size thus get accumulate within the cavity. Stronger vortices with higher value of stream function created

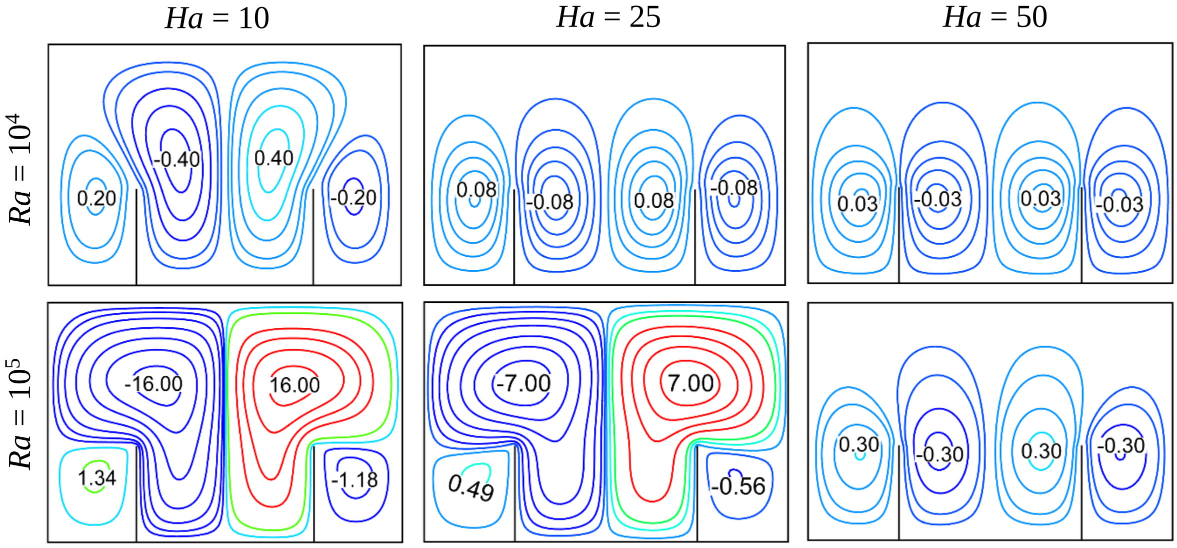

Fig. 4. Effect of *Ha* on streamlines in upper row for *Ra* $=10^4$ and in lower row for *Ra* $=10^5$ at $\varphi = 0.15$

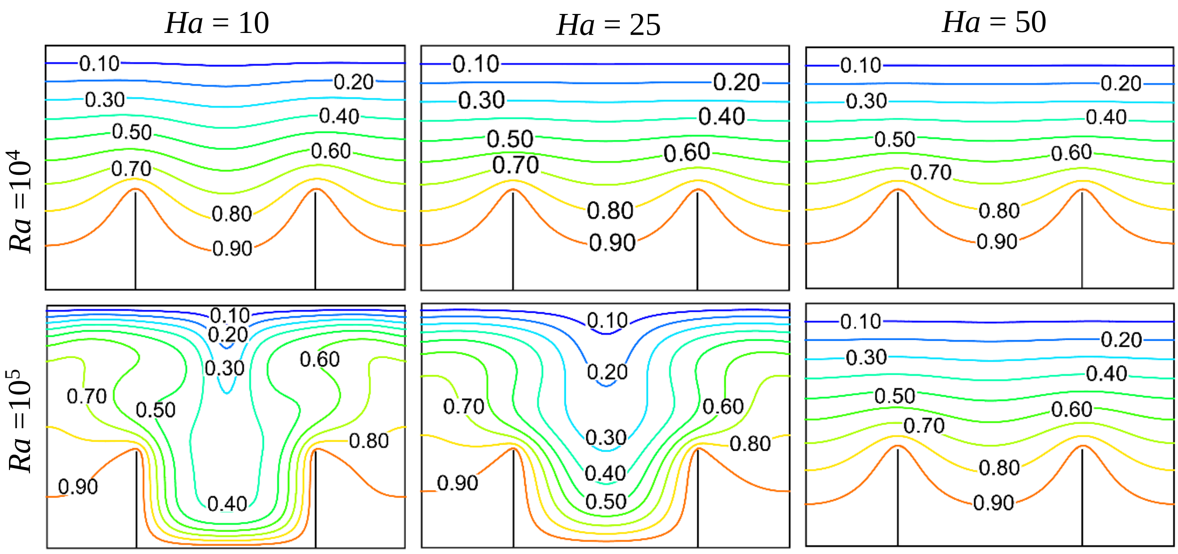

Fig. 5. Effect of *Ha* on isotherm contours in upper row for *Ra* $= 10^4$ and in lower row for *Ra* $=10^5$ at $\varphi = 0.15$

at *Ra* $= 10^5$ certainly have impact on the overall heat transfer rate which will be backed up by Nusselt number plots.

Fig. 5 represents the thermal field of the working fluid inside the cavity. At lower *Ra* $= 10^4$, isotherm patterns do not attain much variation with Hartmann number (*Ha*). This is because, in this case buoyancy force is too low to be suppressed by external magnetic field. But at higher *R*a $= 10^5$, when buoyancy flow is higher, more distorted lines are found which results in convective heat transfer phenomenon. With the increment of *Ha*, parallel lines suggest conduction mode of heat transfer. One interesting note is revealed that, being exposed to higher magnetic field (*Ha* = 50), isotherm contours become similar in pattern for *R*a $= 10^4$ and 10^5. Thus the thermal field suggests that higher magnetic field has always a negative impact on the heat transfer mechanism.

Effect Hartmann number on mid-plane velocity profile

Variation of Y-component of the velocity field of the working fluid is illustrated in Fig. 6. As *Ha* is increased, the velocity profile tends to be diminished and shows almost no variation for higher Hartmann number (*Ha* = 50). This is because when the *Ha* is increased, an influence of Lorentz force is occurred on the convective flow. In this case, lower Rayleigh number (*Ra* $= 10^4$) is chosen because the influence of magnetic field is more prominent on weaker convective flow.

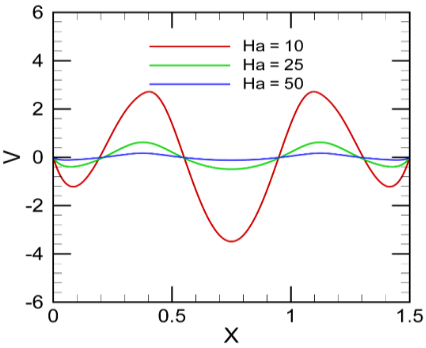

Fig. 6. Variation of Y-velocity (V) at mid-section of the enclosure with X-length for $Ra = 10^4$, $\varphi = 0.15$

Effect of Hartmann number on average Nusselt number and average temperature of the working fluid

Fig. 7 depicts the effect of Hartmann number on average Nusselt number (*Nu*). Nusselt number is quantified at the cold wall. This is because, the isothermal lines are more intensified at that wall. It is seen that *Nu* shows no variation at lower Rayleigh numbers ($Ra = 10^3$, 10^4) due to less convection phenomena. For higher Rayleigh number ($Ra = 10^5$) and $Ha = 0$, temperature gradient in the cold wall is higher and therefore Nusselt number (*Nu*) is maximum. It is observed that *Nu* tends to fall rapidly upto $Ha = 40$. An interesting observation is made that for $Ha > 40$, *Ra* shows no significant effect on overall heat transfer which is due to the dominance of conductive regime.

Table 3 shows the effect of *Ra* and *Ha* on the average temperature of the fluid. For inspecting the effect, *Ha* =10, *Ha* = 25 and *Ha* = 50 have been observed where *Ra* is varied from 10^3 to 10^5. It is evident that for *Ha* = 10, *Ha* = 25 and *Ha* = 50 the value of the average temperature does not change that much with the increase of *Ra* from 10^3 to 10^5 but slightly decrease at higher value of Hartmann number.

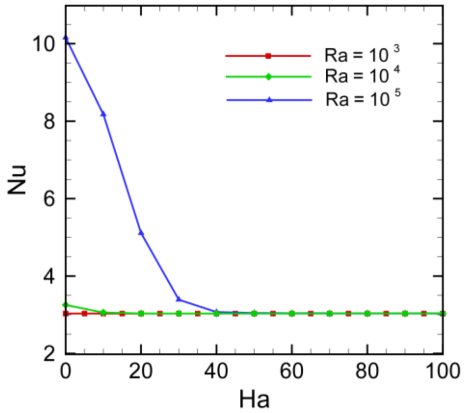

Fig. 7. Variation of average Nusselt number with *Ha* at $\varphi = 0.15$

Table 3. Variation of average temperature of the fluid for different *Ha* and *Ra* at $\varphi = 0.15$

Ra	*Ha*		
	10	25	50
1×10^3	0.907	0.908	0.908
5×10^3	0.906	0.907	0.908
1×10^4	0.904	0.907	0.907
5×10^4	0.902	0.889	0.906
1×10^5	0.902	0.877	0.905

Effect of solid volume fraction of nanofluid on average Nusselt number

Fig. 8 illustrates the effect of solid volume fraction (φ) on the average Nusselt number (*Nu*). It illustrates that value of *Nu* increases with the increase of φ and the rate of increase of *Nu* increases with the increase of φ. The addition of solid nanoparticle (*Cu*) to the base fluid (water) results in an increase of μ_{nf}/ρ_{nf} in the diffusion term which describes better buoyancy circulation. Such buoyancy current eventually increases the heat transfer rate. Thus maximum average Nusselt number at the cold wall is attained at solid volume fraction ($\varphi = 0.15$)

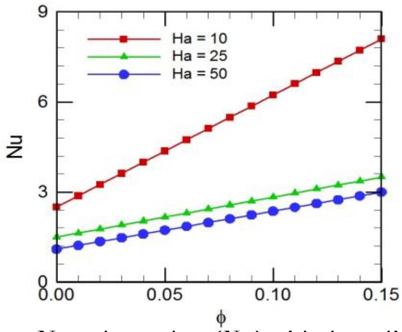

Fig. 8. Variation of average Nusselt number *(Nu)* with the solid volume fraction (φ) at $Ra = 10^5$

6. Conclusion

From the results discussed before, the following conclusion for this investigation can be drawn-

- Study shows the strength of flow decreases with the increment of magnetic field.
- The isothermal lines get less distorted as the magnetic field gets stronger. It signifies the increase of conduction mode of heat transfer.
- Strength of magnetic field has severe effect on amplitude of velocity profile of the working fluid.
- More than 40% better heat transfer is recorded for lower strength of magnetic field.
- Solid volume fraction of nanoparticle has a positive impact on heat transfer rate in terms of average Nusselt number.
- There is no significant variation in overall heat transfer at Ha > 40, even with increasing Ra.

7. Acknowledgement

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