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Pure Mixed Convection inside Square Ventilated Cavity Containing Rotating Adiabatic Cylinder: Influence of Working Fluid

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Abstract

Mixed convection has been one of the major topics of research in the field of heat transfer for many years. Mixed convection from open or ventilated cavity deserves particular attention for their more common occurrence in practical field. In this paper, mixed convection in a square cavity, having inlet and outlet ports at bottom and top walls respectively and containing a rotating circular cylinder at center, is analyzed thoroughly under pure mixed convective regime for a plain fluid (water) and a nanofluid (Al_2O_3-water) with solid volume fraction (φ) of 0.1. For pure mixed convection (Richardson number, Ri =1), both aiding and opposing flow convection mechanisms are analyzed by varying speed (Ω = 0 – 100) and direction of rotation of cylinder keeping Reynolds (Re) and Grashof (Gr) numbers constant at 100 and 10^4. Moreover, effect of addition of nanofluid is observed by comparing average Nusselt number (Nu) at heated boundary and average fluid temperature (T_{av}), as well as, flow and thermal fields for both water and nanofluid. At Ri = 1, Gr is varied from 0.1 to 10^5 for both kinds of fluid to find out the impact of transition of mixed convection regimes. It has been observed that, for opposing flow mixed convection heat transfer rate is significantly low up to a certain value of Ω. However, significant increment of heat transfer rate is possible for high rotational speed of cylinder, higher Re and Gr. Almost invariably, addition of nanofluid augments heat transfer rate.

Keywords: Mixed convection, ventilated cavity, rotating cylinder, nanofluid, pure mixed convection.

1. Introduction

Pure mixed convection, combination of natural and forced convection, has many occurrences such as in nuclear reactor, solar receiver, heat exchanger, thermal storage, semiconductors, etc. [1], [2]. Other applications include rotating-tube heat exchangers, rotating shafts, drilling of oil-wells, nuclear reactor fuel rods, steel suspension bridge cables, etc. [3]

A nanofluid contains nanometer-sized particles, called nanoparticles, suspended in base fluids (water, ethylene glycol, etc). These nanoparticles are generally made of metals, metal oxides, carbides or carbon nanotubes. Roslan *et al.* [3] studied the effect of a rotating cylinder on heat transfer in a square enclosure, filled with nanofluid. They showed that the maximum heat transfer can be found at a high nanoparticle concentration. Teng *et al.* [4] made a study on the effect of Al_2O_3-water nanofluid particle size on thermal conductivity. The results showed a correlation between small nanoparticle size and higher temperature. Sebdani *et al.* [5] carried out a study on the effect of Al_2O_3-water nanofluid on mixed convection in a square cavity. They found that the rate of heat transfer decreased with the increase of nanoparticle volume fraction for a constant Reynolds (*Re*) number and high Rayleigh (*Ra*) numbers. Costa and Raimundo [6] studied on steady mixed convection in a differentially heated square enclosure with an active rotating circular cylinder. They found that the rotating cylinder affected the thermal performance and the thermo-physical properties of the cylinder were important on the overall heat transfer process.

Basak *et al*. [7] considered mixed convection in a square cavity with uniform and non-uniform heating of bottom wall. The local Nusselt (*Nu*) number plot showed that the heat transfer rate was very high at the edges of the bottom wall, but it decreased at the center of the wall for uniform heating. Rahman *et al*. [8] carried out a numerical study on opposing mixed convection in a ventilated enclosure. They found that the rate of heat transfer from the heated wall depended on the position of the inlet port. Mehrizi *et al*. [9] made a study on mixed convection in a ventilated cavity with hot obstacle. They showed that the heat transfer rate was the maximum with different locations of outlet port at different Richardson numbers.

In our present work, pure mixed convection inside a square ventilated cavity is studied. A rotating adiabatic cylinder is at the center of the cavity. The cavity has inlet and outlet ports with the flow of Al_2O_3–water

Nomenclature			
a	Length of inlet and outlet	*Greek symbols*	
C_p	Constant pressure specific heat	α	Thermal diffusivity
g	Gravitational acceleration	β	Coefficient of volume expansion
Gr	Grashof number	μ	Dynamic viscosity
k	Thermal conductivity	ν	Kinematic viscosity
L	Length of square cavity	ρ	Density
Nu	Nusselt number	ω	Angular rotational velocity
p	Pressure	Ω	Dimensionless Angular rotational velocity
P	Dimensionless pressure	θ	Dimensionless temperature
Pr	Prandtl number		
r	Radius of cylinder		
R	Dimensionless radius of cylinder	Subscripts	
Re	Reynolds number	c	Cold (lower value)
Ri	Richardson number	f	Fluid
T	Temperature	h	Hot (higher value)
u,v	Cartesian velocity components	nf	Nanofluid
U,V	Dimensionless velocity components	o	Initial value
x,y	Cartesian co-ordinates	0	At the center of the cylinder
X,Y	Dimensionless co-ordinates		

nanofluid. The cylinder can rotate with different speeds and directions. The Reynolds (*Re*) and Grashof (*Gr*) numbers are kept constant at 100 and 10^4 respectively, whereas the Richardson (*Ri*) number is kept 1. Qualitative analyses are carried out by comparing thermal and flow fields in terms of isotherm and streamlines while quantitative analyses are done by presenting Nusselt number (*Nu*) at the heated wall and average fluid temperature inside the cavity.

2. Problem Formulation

2.1. Physical Modeling

The present configuration consists of a two-dimensional square enclosure of the same width and height, L. A rotating adiabatic cylinder of radius R is placed at the center (X_0,Y_0) of the square cavity, which rotates with an angular velocity, ω. The physical model is shown in Fig. 1 along with the important geometric parameters. The origin of a Cartesian coordinate is at the lower left corner of the computational domain. Both the lower and the upper horizontal walls are assumed to be adiabatic along with an inlet at the left corner and an outlet at the right corner respectively. The vertical side left wall is assumed to have an isothermal temperature (T_h) higher than the temperature (T_c) of the opposite right wall. No slip condition is assumed at the walls. Under the influence of the vertical gravitation field, the vertical walls with different temperatures lead to a natural convection problem. Again, due to the non-slip boundary condition for velocity on its surface, the rotating adiabatic cylinder induces a forced flow. The overall situation results in a mixed convection problem. The free space of the cavity is filled with a water-based nanofluid containing Al_2O_3 nanoparticles. The flow is assumed to be laminar. Thermal radiation and viscous dissipation are neglected.

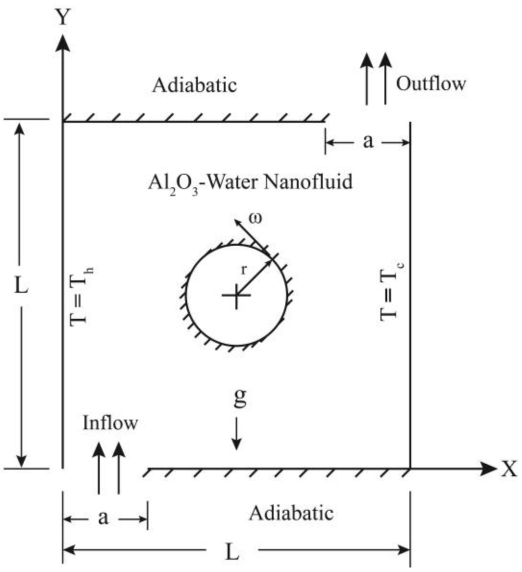


Fig. 1. Schematic diagram of the square ventilated cavity with problem specifications.

2.2. Mathematical Modeling

Mixed convection fluid flow inside the cavity follows the continuity equation that reads, in its dimensionless form

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

The momentum equations,

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{\mu_{nf}}{\rho_{nf}\nu_f}\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{\mu_{nf}}{\rho_{nf}\nu_f}\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}Ri\theta \tag{3}$$

The energy equation,

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\alpha_{nf}}{\alpha_f}\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

The dimensionless parameters appearing in the above equations are the space coordinates, the velocity components, the temperature and the driving pressure, defined, respectively, as

$$(X,Y)=(x,y)/L;(U,V)=(u,v)/u_o \tag{5}$$

$$\theta=\frac{T-T_c}{T_h-T_c};P=\frac{p}{\rho_{nf}u_o^2};\mathrm{Pr}=\frac{(\mu C_p)_f}{k_f};\mathrm{Re}=\frac{\rho_f u_o L}{\mu_f};Gr=\frac{g\rho_f^2\beta_f(T_h-T_c)L^3}{\mu_f^2};Ri=\frac{Gr}{\mathrm{Re}^2}. \tag{6}$$

For the rotating adiabatic cylinder, the dimensionless governing parameters are as

$$R=r/L;\Omega=\omega L^2/\alpha_f;U=-\Omega(Y-Y_0);V=-\Omega(X-X_0) \tag{7}$$

Over the surface of the cylinder, the absolute value of velocity can be evaluated as [3]

$$|V|=|\Omega|R. \tag{8}$$

Boundary conditions for analysis of pure mixed convection in a square ventilated cavity, containing rotating adiabatic cylinder are presented in non-dimensional form in Table 1.

Table 1. Boundary conditions for this problem in non-dimensional form

Boundary Wall	***Flow Field***	***Thermal Field***
Top Wall	U = V = 0	$\partial\Theta/\partial n = 0$
Bottom Wall	U = V = 0	$\partial\Theta/\partial n = 0$
Left Side Wall	U = V = 0	$\Theta = 1$

Right Side Wall	U = V = 0	Θ = 0
Rotating Cylinder	U= $-\Omega(Y-Y_0)$, V= $-\Omega(X-X_0)$	∂Θ/∂n = 0
Inlet	U = 0, V = 1	Θ = 0
Outlet	P = 0, ∂U/∂Y = 0, ∂V/∂Y = -∂U/∂X	Θ = Θ/A, Θ = -(Nuc* Θx)/A

The heat transfer rate from the high temperature vertical wall to fluid is represented by the overall Nusselt number (*Nu*), defined as

$$Nu = \frac{k_{nf}}{k_f} \int_0^L -\left(\frac{\partial \theta}{\partial X}\right)_{X=0} dY. \tag{9}$$

Average temperature of the flow field can be calculated as,

$$\theta_{av} = \frac{1}{A}\int_A \theta dA \tag{10}$$

Here, area of the cavity, $\mathrm{A} = L^2 - \pi r^2$ (11)

3. Numerical Scheme

3.1. Numerical Procedure

Galerkin weighted residual scheme of finite element method (FEM) has been used to get the solution of this specific problem. Six nodded triangular mesh has been employed to discretize the entire cavity. Fine mesh has been applied near the boundary walls. An iterative method has been used to find out the solution by the UMPFACK solver and relative tolerance of 10^{-5} has been set for the error estimation.

3.2. Grid Independency Test

A grid independency test has been performed to economize the solution process by taking least number of elements for satisfactory solution. The grid independency test result is presented in Fig.2. From the figure it is evident that for the lower grid element numbers, the Nusselt number varies significantly. Nusselt number is calculated at the left vertical heated wall. When the number of grid elements reaches to 5000, the Nusselt number becomes more or less constant. For higher elements the grid shows constant Nusselt number. So the grid having 5000 elements has been taken for the entire numerical simulation of the specified problem.

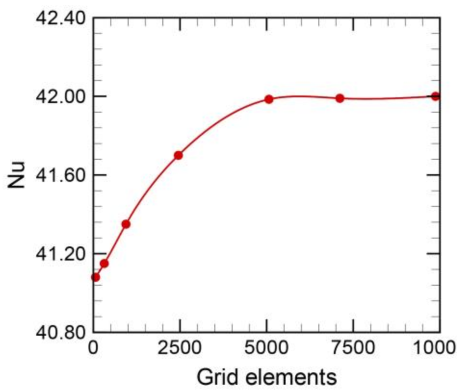
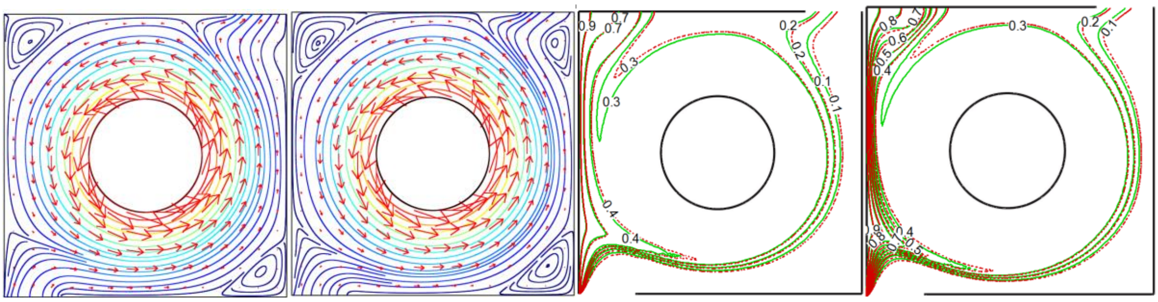


Fig. 2. Variation of Nusselt number with grid size for *Gr* = 10^4 and Ω = 50.

4. Results and Discussions

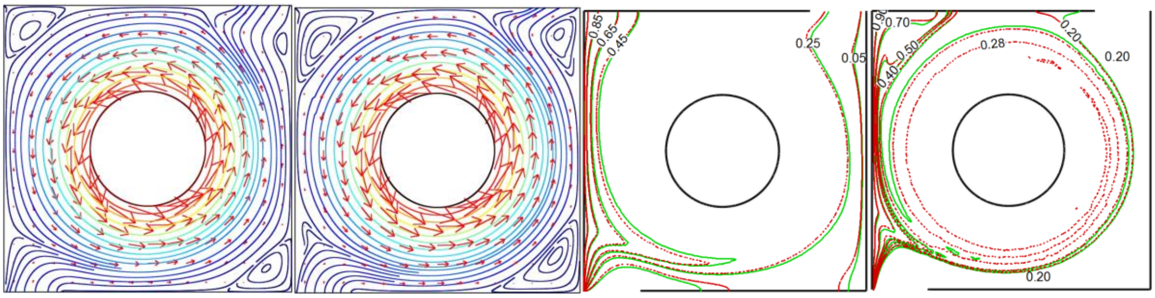
Results are generated for water as plain fluid and nanofluid (Al_2O_3 –Water) with 0.1 solid volume fraction (φ). Firstly isotherm and streamline contours for both fluids are presented for $0 < \Omega < 100$. Ω indicates the rotational speed of cylinder. Then isotherm and streamlines for both fluids are presented for different Grashof number, *Gr*. Then average temperature for the cavity and average Nusselt number at the heated left vertical wall between the geometries have been presented with varying *Gr* and Ω.

4.1. Effect of Rotational Speed (Ω) and Grashof Number (*Gr*)



(a) (b) (c) (d)

Fig. 3. Streamlines for (a) Ω = 50, (b) Ω =100. Isotherms for (c) Ω =50, (d) Ω =100. At *Ri* = 1, *Re* =100 and *Gr* = 10^4.

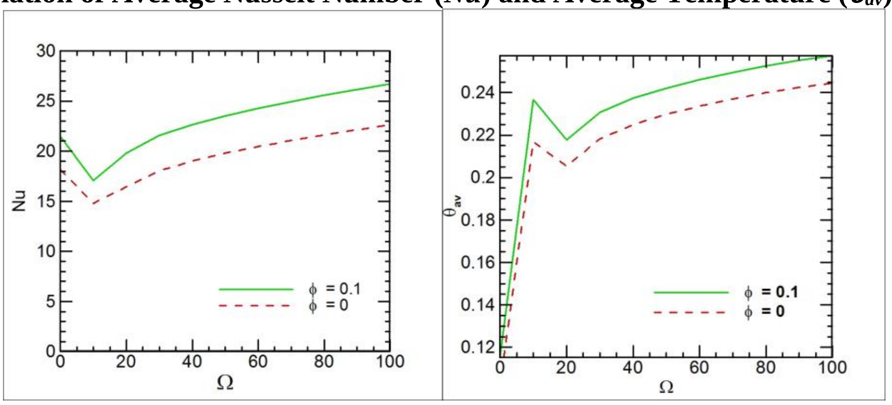


(a) (b) (c) (d)

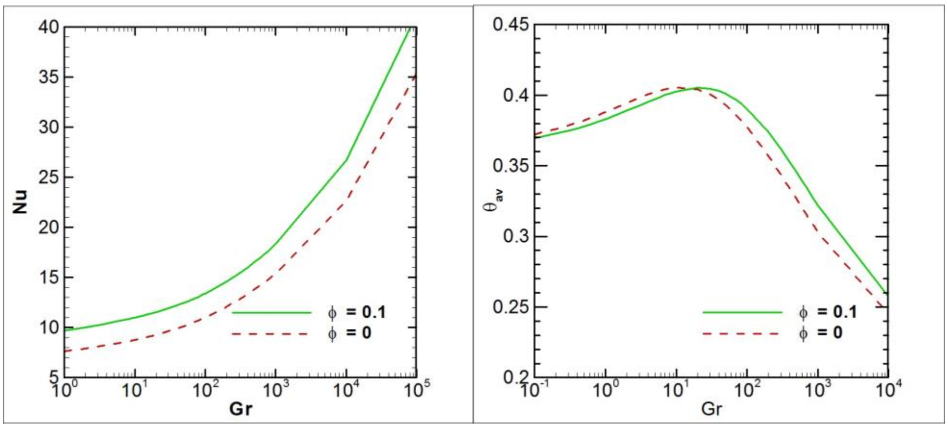
Fig. 4. Streamlines for (a) *Gr* = 10^3, (b) *Gr* =10^5. Isotherm for (c) *Gr* =10^3, (d) *Gr* =10^5 at *Ri* = 1, Ω=100.

Fig. 3 has been generated for different Ω, specifically for 50 and 100 in the range of 0< Ω <100. Also Grashof (*Gr*) and Reynolds (*Re*) numbers, *Re* are fixed to 10^4 and 100 respectively thus making Richardson number (*Ri*) equal to 1. Fluid flow takes place in the free space between the cylinder and the enclosure for pure mixed convection specifically the fluid flows towards the outlet port from the inlet port. The rotating motion of the cylinder creates a vortex which rotates in the counterclockwise direction. There is also a small vortex at the right corner of the cavity. At low rotational speed of cylinder this small vortex becomes weaker. Only primary vortex is created due to pure mixed convection. Streamline plots for Ω =50 and 100 are shown in Fig. 3(a) and 3(b) and isotherm plots are shown in Fig. 3(c) and 3(d). Fig. 4 has been generated for Grashof number 10^3 and 10^5. Streamline and isotherm plots in Fig. 4 indicate that, increasing the Grashof number from 10^3 to 10^5 the natural convection is increased compared to forced convection.

4.2. Variation of Average Nusselt Number (*Nu*) and Average Temperature ($\boldsymbol{\theta_{av}}$)



(a) (b)

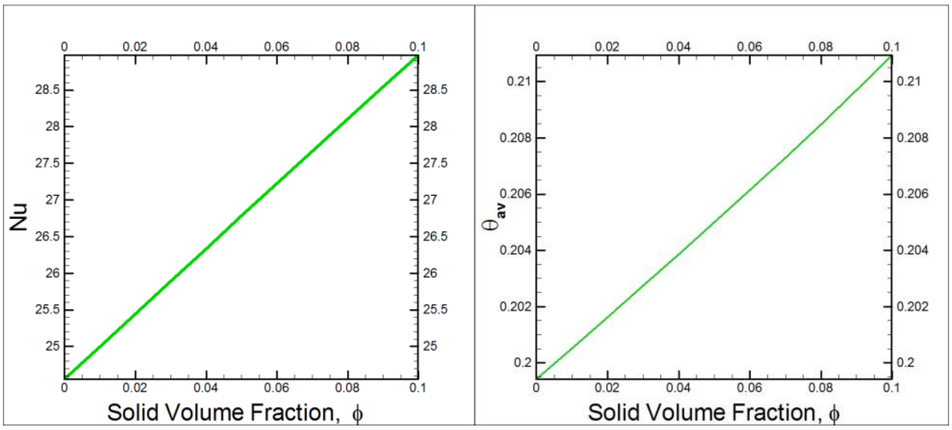


(c) (d)

Fig. 5. Variation of (a) *Nu* and (b) θ_{av} with Ω at Gr=10^3. Variation of (c) *Nu* and (d) θ_{av} with *Gr* at Ω =50. Dashed lines are for water and solid lines for Al_2O_3*-water*.

In the figure, variation of *Nu* and θ_{av} are analyzed. The solid volume fraction, φ in nanofluid is 0.1 and 0 in plain fluid. Ω was varied in the range of $0 < \Omega < 100$ for Fig. 5(a) and 5(b). *Gr* was varied in the range of $0.1 < Gr < 10^5$ for figure 5(c) and 5(d). With the increase of Ω, *Nu* and θ_{av} both increase. With the increase of *Gr*, *Nu* increases but θ_{av} decreases. Although, inclusion of solid nanoparticles yields the higher thermal conductivity that eventually leads to higher energy and thus increases the average temperature. So, in Fig. 5(a) and 5(c), it is seen that Nusselt number for nanofluid is greater than plain fluid. Numerically from Fig. 5(a), Nusselt number is 26.75 for nanofuid and 22.5 for plain fluid. In Fig. 5(b) and 5(d), it is seen that average temperature for nanofluid is greater than plain fluid.

4.3. Effect of Solid Volume Fraction (φ) on Average Nusselt Number (*Nu*) and Average Temperature ($\boldsymbol{\theta_{av}}$)



(a) (b)

Fig. 6. Variation of (a) *Nu* and (b) θ_{av} with φ at Ω = 50, *Gr* = 1000.

In Fig. 6, it is seen that addition of solid volume fraction increases the *Nu* and θ_{av} proportionally. The addition of solid nanoparticles increases the heat conductivity of the fluid and thus increases the energy of the fluid.

5. Conclusions

In this paper a thorough discussion has been made on the problem specification, its background literature and numerical solution procedures. From the result analysis the following conclusions can be drawn:

- Addition of rotating speed, Ω results in a recirculating cell in the cavity which circulates in the opposite to the natural convection flow.
- Higher value of Gr increases the natural convection rate rather than forced convection.
- Inclusion of nanoparticles increases the Nusselt number and average temperature of the fluid.

Since the geometry shows a lot of promise and has got attention lately, the authors hope that the present paper will help the researcher around the world to have a better insight on the problem.

6. Acknowledgement

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