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Mode-I Failure Investigation of Eccentric Elliptical Hole Containing Crack with Varying Eccentricity and Crack Length.

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Abstract

Thin plate with an eccentric hole containing crack is a more common machine element in contrast to center hole crack element. After certain cycle during the plate's life time, the hole becomes elliptical in shape due to continuous loading applied to it. In this investigation, stress intensity factor (SIF) due to mode–I failure (K_I) of such plate is calculated with finite element analysis (FEA) as the plate experiences loading perpendicular to the crack orientation. Node displacement and Stress Extrapolation methods are employed to calculate the SIF values with linear and quadratic elements. The eccentricity of the hole on a thin plate is varied along the crack length. Change in SIF is recorded with varying hole eccentricity and crack length. Result suggests, the SIF values increases with the increment of crack length and hole eccentricity. The results are compared with the analytical solution for cracked thin plates. The investigation concludes analytical solution for cracked thin plates is a good estimation of SIF values from FEA results as the results agree with acceptable accuracy.

Keywords: Mode-I failure, Stress Intensity Factor, Node Displacement, Stress extrapolation.

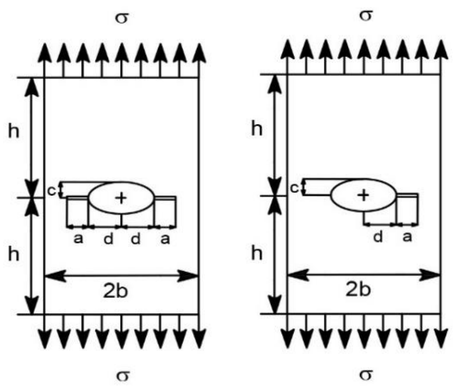
1. Introduction

Machine members often fail due to static and variable loading when a flaw or crack forms and propagates throughout the material. Cracks are formed preferentially at the specimen edges, grooves, shoulders and periphery of holes where stress is highly localized. In order to assess the integrity of structures containing cracks and its sustainability, an analysis tool, linear elastic fracture mechanics (LEFM) is used. In LEFM, stress intensity factor (SIF) is an important parameter which depends on the size, shape and geometry of the crack and the applied loading condition. The singular stress contribution in the vicinity of the crack tip is characterized by SIF and a good enumeration about the generation of fatigue crack is also found by SIF at the crack tip by Paris law[1].

Among three distinct modes of crack propagation, the SIF for mode I, the opening crack propagation mode, denoted by K_I is also reported in related handbooks[2], [3]. These theoretic values of SIF are found from eminently complicated mathematical and graphical analysis, which are applicable for ideal type of loading and geometry. But, in structural engineering the fatigue cracks usually are of complicated geometries and different types of fluctuating loads are applied. So, finite element analysis (FEA) methods are used to evaluate SIF of cracks of complex configurations in an easier numerical technique. In some previous studies, new methods to analyze stresses in the region of cracks are proposed on the basis of finite element solutions[4]–[6]. Besides, FEA methods are also used for different criteria like crack emanating at 45°orientation from a hole in pressurized cylinder[7], for cracked bridge roller bearings[8] and for evaluation of mixed mode SIFs in functionally graded materials etc. Afterwards, some new and more efficient methods are generated for SIF calculation, among these, Node Displacement method(NDM)[9],[10] and Stress Extrapolation method(SEM)[11], [12] are applied to find out the SIF for linear and quadratic elements in this paper. Different physical quantities like stresses, nodal locations and displacements are needed to find SIF by using these methods. In this paper, an investigation is presented for SIF of eccentric elliptical hole containing single and dual cracks in its periphery on structural steel plates. The SIFs are evaluated for MODE-I with FEA as the applied loading on the plate produces stresses normal to the crack direction. The crack length and eccentricity of the elliptical hole on the thin plate are varied here. The major and minor diameters of the hole are always kept fixed throughout the analysis. Finally, to determine the accuracy of the results found from these two methods, a comparison is made with theoretical results.

2. Problem Specification and Analytical Formulation

Here, cracks are emanating from two opposite sides of an eccentric elliptical hole's periphery on a thin plate. The crack nearer to middle vertical axis is denoted as inner crack and another one nearer to the edge is denoted as outer crack. But, for centric elliptical hole, cracks are equidistant from the center axis and edge of the plate. Here, the plate in uniform tension, containing crack emanating from an elliptical hole, the SIF at the crack tip can be determined by $K_1 = A\sigma\sqrt{\pi a}$ according to the handbook[3], in which 'A' is a parameter called stress intensity modification factor, related with the geometry of the hole and crack. In this analytical solution, eccentricity is considered zero. Here, the full width of the thin plate is 2b = 200mm and length, 2h = 600mm. The major and minor radii of the elliptical hole are denoted by d and c respectively. For dual cracks, d = 20 mm, c = 10 mm, c/d= 1/2 and for single crack, d = 15 mm, c = 5 mm, c/d= 1/3 are considered because the values of 'A' are available only for these c/d ratios in this handbook[3]. The length of crack is, a= 6 mm and the material of the plate is considered structural steel (ASTM –A36) with Young's Modulus, E = 2×10^5 MPa and Poisson's ratio, ν = 0.30. The magnitude of applied uniform tensile stress along the Y- axis is σ = 200 MPa, as shown in Fig.1. Theoretical SIF solutions at the crack tip of the single and dual outer cracks shown in Fig. 1 are listed in Table 1.



(a) Dual cracks (b) Single Crack

Fig. 1. Geometries of cracks emanating from elliptical hole.

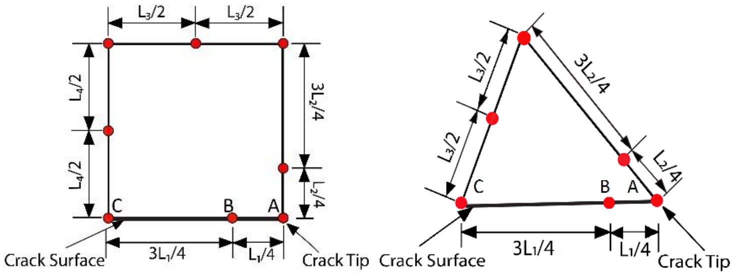


Fig. 2. Rectangular and Triangular quarter-point (singular) elements.

Table 1: Analytical solutions of SIF for dual cracks and single crack

	Dual Cracks	Single Crack
A	2.143	1.7153
K_1 ($MPa\sqrt{mm}$)	1860.813	1489.43

3. Theoretical Background

3.1 Node Displacement Method:

The fracture of a plate usually occurs due to the development of certain displacement in discontinued surfaces within the material under the action of stress. The Stress intensity factors of a particular discontinuity can be computed directly from nodal displacements. According to Barsoum [13], to have a more precise displacement and stress results by finite element analysis, singular elements can be used at the vicinity of crack tip. These triangular and rectangular quarter point elements (Fig. 2.) also help to obtain a good representation of the stress field around the crack-tip. The $1/\sqrt{r}$ linear-elastic singularity for stress is obtained by shifting the mid-side nodes a quarter to the crack tip of all the elements. So, using node displacement method, the SIF, K_1 can be evaluated by Eq. (1)

$$K_1 = \frac{E}{12}\sqrt{\frac{2\pi}{L}}(8V_B - V_C) \tag{1}$$

Here, E is the Young's Modulus, V_B and V_C are the displacements normal to crack propagation at node B and C respectively and L is the length of the element as shown in Fig. 2 along with the crack.

3.2 Stress Extrapolation Method:

The Stress Extrapolation method uses the results of the general elastic solutions of the crack-tip stress. For Mode 1, the stress vector expressions of stress field in the vicinity of the crack tip, can be expressed by the equations (2)[12], where σ_x and σ_y are normal stresses at node A, parallel and perpendicular to the direction of crack propagation respectively. Here, node A is a node closer to the crack tip which can be located by r and θ as shown in Fig. 3.

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} cos\frac{\theta}{2}(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}) \tag{2}$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} cos\frac{\theta}{2}(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}) \tag{3}$$

The normal stress σ_y along the crack face can be used to get the value of SIF by using the above equation as,

$$K_I = \sigma_{y,\theta=0}\sqrt{2\pi r} \tag{4}$$

By plotting the values of K_I with the values of r, the SIF K_I at r = 0, at the crack tip can be determined by extrapolation method using the different values of SIFs at different nodes along the crack propagation.

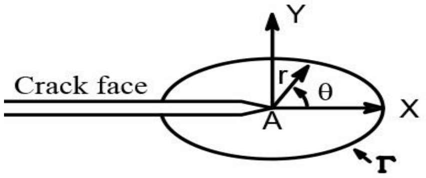


Fig. 3. Crack Tip coordinates

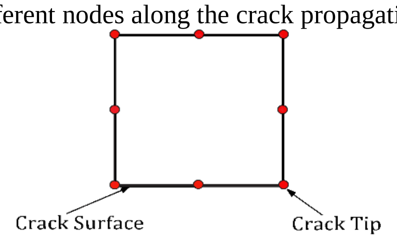


Fig. 4. A 8-node quadratic element

4. Finite Element Modeling

In all cases, where an expression for the stress-intensity factor cannot be obtained from existing solutions, FEA can be used to determine SIF. Trends of K_I can be described by using finite size effect on stress concentration. Here, finite element analysis of our cracked thin plate of finite area was conducted with finite element tool ANSYS[14]. We had considered Plane-183 as our solid structural element [15], which adjust nicely with quadratic displacement behavior. Here, for node displacement method special triangular quarter-point elements (Fig. 2.) were used at the crack tip because of the singularity of crack tip and 8-node quadratic elements (Fig. 4.) in other areas for both Node Displacement and Stress Extrapolation method. Element size 1mm was taken and the model was processed with 178219 nodes and 59120 elements.

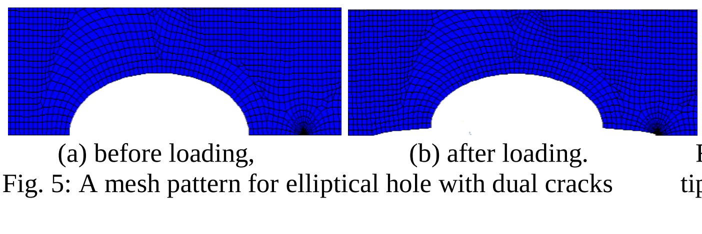

(a) before loading, (b) after loading.
Fig. 5: A mesh pattern for elliptical hole with dual cracks

Fig.6: Stress distribution (in MPa) around crack tips, colors follow the stress concentration values

Here, the cracks are subjected to uniform tensile stress which works normal to the crack propagation. As the taken geometry was symmetric, boundary condition of symmetry was applied here. Mapped meshing is used for both methods, where stress of higher amplitude was established at the crack tip. A meshed pattern for dual cracks with un-deformed shape of before loading is shown in Fig. 5(a). After loading, cracked plate was deformed as shown in Fig. 5(b). Stress distribution with values around the crack tips are shown in Fig. 6. Here stress is concentrated and has been maximum around the crack tip region.

5. Results and Discussion

The FEA method enables us to determine the stress intensity factor for different criteria like crack at one inner side, one outer side and both outer and inner sides at a time. Here, for changing eccentricity, the position of the elliptical hole varies. The major and minor radii are fixed, and for dual cracks, they are 20mm and 10mm and for single crack, they are 15mm and 5mm respectively. Trends of SIF found from Node Displacement and Stress Extrapolation method for dual cracks are shown in Fig. 7a as a function of normalized eccentricity (eccentricity is normalized by half plate width). The figure shows that the SIF is larger for the outer crack tip of dual cracks of SEM method compared to inner crack tip and NDM method. For the normalized eccentricity value of 0.00 (center hole), SIF for both crack tip points are around 1974 MPa√mm for NDM method. But, using SEM method, 2007MPa√mm and 1605 MPa√mm are found for outer and inner crack tip respectively at zero eccentricity. The value remains nearly unchanged for eccentricity value of 0.18. After that, change in SIF values are significant and start to increase with increasing eccentricity reaching at 2729 MPa√mm and 2623 MPa√mm for the outer crack tip, and 1830 MPa√mm and 2249 MPa√mm for inner crack tip at eccentricity 0.6502 by SEM and NDM respectively. From Fig. 7b, it's observed that single crack tip follows the same pattern for lower values of SIF. For zero eccentricity, SIF is 1568 MPa√mm for both crack tips and these remain nearly constant upto 0.3 eccentricity for NDM method. After that, change is significant and SIF increases in parabolic way up to 1649 MPa√mm and 1784 MPa√mm for inner and outer crack tip respectively.

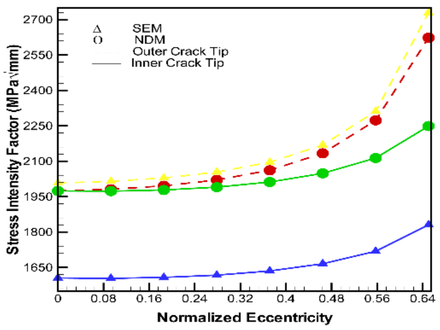


Fig. 7a. SIF values for dual cracks at different eccentricity with fixed crack length of 6 mm

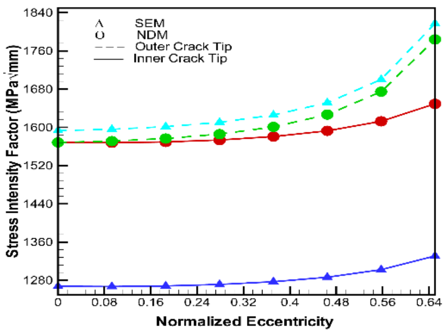


Fig. 7b. SIF values for single crack at different eccentricity with fixed crack length of 6 mm

Here, it's also observed for the values executed by Stress Extrapolation method, there is a significant difference in between the outer and inner crack tips. The SIF values are nearly constant up to eccentricity value of 0.18 and after that SIF starts to increase and maximum value is 1817 MPa√mm and 1330 MPa√mm for outer and inner crack tip of single crack respectively. Here, at zero eccentricity, we get the lowest value 1267 MPa√mm for inner crack tip and for outer tip the value is 1593 MPa√mm by Stress Extrapolation method.

To observe the change of SIF with respect to crack length at a fixed eccentricity of 46.44mm, we considered 7 different normalized crack lengths (by dividing crack length by 10).From Node Displacement method with 0.3 normalized value of crack length, the value of SIF is 1942 MPa√mm for outer crack tip and 1878 MPa√mm for inner crack tip in dual cracks. Then, for 1.2 crack length, SIF get increased upto 2506 MPa√mm and 2334 MPa√mm for outer and inner crack tips of dual cracks respectively. For Stress Extrapolation method with 0.3 value of normalized crack length the value of SIF is minimum and they are 1982 MPa√mm for outer and 1527 MPa√mm for inner crack tip. Then, values of SIFs increased linearly upto 2555 MPa√mm for outer crack tip and 1890 MPa√mm for inner crack tip at 1.2 crack length. These results are summarized in Fig.8a.

From the Fig. 8b, it’s perceived that, the SIF values for single cracks also increased gradually with increasing crack length both for Node Displacement and Stress Extrapolation method.

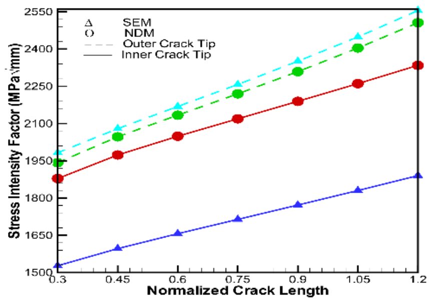


Fig. 8a: SIF values for dual cracks at different crack lengths with fixed eccentricity

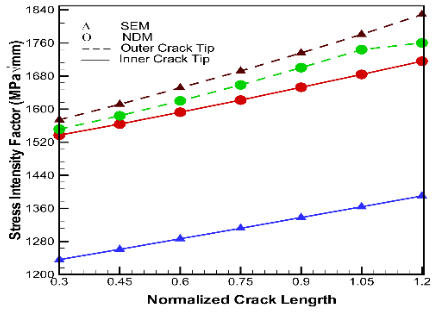


Fig. 8b: SIF values for single crack at different crack lengths with fixed eccentricity

Here, we get minimum values for 0.3 crack length, and they are 1551 MPa√mm and 1537 MPa√mm for outer and inner crack tip respectively by Node Displacement method. Afterward, for increasing crack length SIF increases linearly and the final results are 1760 MPa√mm and 1715 MPa√mm for outer and inner crack tip respectively for normalized crack length of 1.2. Applying Stress Extrapolation method, for 0.3 crack length the values from simulation are 1574 MPa√mm for outer crack tip and 1235 MPa√mm for inner crack tip. And, the final values done by FEA method, are 1828 MPa√mm for outer crack tip and 1390 MPa√mm for inner crack tip at 1.2 normalized crack length for single crack.

The compendium of the comparison of two different finite element methods with analytical solution is represented in Table 2. It is remarkable that Node Displacement method for single cracked system gives more accurate values than both for dual cracked system and Stress Extrapolation method.

Table 2: Comparison of results by Stress Extrapolation and Node Displacement method with analytical solution.

Solution Method	Dual Cracks	Difference with analytical	Single Crack (Outer tip)	Difference with analytical
Analytical	1860.813	-	1489.43	-
Node Displacement	1974	6.13%	1568	5.28%
Stress Extrapolation	2007	7.9%	1593	6.95%

Reducing element size may increase the efficiency but it will take more time and effort in simulation. In this study, stress is applied to the plate of finite area containing elliptical hole. Generally, crack growth occurs when the force release rate from applied tensile loading is greater than the rate of energy for crack growth. Vertical lines of forces created for tensile stress, are get diverted as stress cannot pass through any kind of discontinuity like cracks. Thus, stresses get increased in the immediate vicinity of the cracks. Similarly, when two cracks are present for an eccentric elliptical hole, they produce more disturbance for the lines of forces in a short region and there stress intensity factor increases highly along with stress. The accuracy of the Stress Extrapolation method highly dependent on the values of these stresses of the nodes and distances of nodes from crack tip.

6. Conclusion

Node Displacement and Stress Extrapolation methods are occupied to evaluate the stress intensity factor in this work. SIFs for 4 different crack tips on elliptical hole is represented here which is evaluated for 1mm 8-node quadratic elements because it gives closer values to analytical solutions than 4-node linear elements. Effects of varying eccentricity and crack length are studied here. Following conclusions are drawn through this analysis:
(1)Obtained results by Node Displacement and Stress Extrapolation methods agree with analytical solution within acceptable accuracy. Though, change of types of elements around crack tips and more calculation work

may improve the results. (2)By changing eccentricity, for all 4 crack tips, SIF remains fixed for 0 to 0.18 normalized eccentricity and then increases in a parabolic way. (3)For variable crack lengths, trend of SIF as a function of normalized crack length increases linearly for single and dual cracks. Finally, to summarize, it can be said that elliptical hole with high eccentricity and large crack length has a bigger value of SIF and is at higher risk of failure.

7. Acknowledgement

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8. Nomenclature

Symbol	Meaning	Unit
a	Crack length	(mm)
b	Plate half width	(mm)
h	Plate half length	(mm)
d	Hole's major radius	(mm)
c	Hole's minor radius	(mm)
σ	Normal stress	(MPa)
E	Young's modulus	(MPa)
K_1	Stress intensity factor for mode I crack	$(\mathrm{MPa}\sqrt{\mathrm{mm}})$
V	Nodal displacement perpendicular to crack growth direction	(mm)
μ	Poisson's ratio	Dimensionless
σ_x	Normal stress parallel to crack growth direction	(MPa)
σ_y	Normal stress perpendicular to crack growth direction	(MPa)

9. References

[1] P. Paris and F. Erdogan, "A Critical Analysis of Crack Propagation Laws," *J. Basic Eng.*, vol. 85, no. 4, pp. 528–533, Dec. 1963.

[2] B. Michel, "Murakami, Y. (ed.), Stress Intensity Factors Handbook. In 2 Volumes. Oxford etc., Pergamon press 1987. XLIX, XXXIX, 1456 pp., £ 200.–/$ 300.–. ISBN 0-08-034809-2," *ZAMM - J. Appl. Math. Mech. Z. Für Angew. Math. Mech.*, vol. 68, no. 6, pp. 259–260, Jan. 1988.

[3] H. Tada, P. C. Paris, and G. R. Irwin, *The Stress Analysis of Cracks Handbook, Third Edition*. Three Park Avenue New York, NY 10016-5990: ASME, 2000.

[4] Y. Yamamoto and N. Tokuda, "Determination of stress intensity factors in cracked plates by the finite element method," *Int. J. Numer. Methods Eng.*, vol. 6, no. 3, pp. 427–439, Jan. 1973.

[5] "Determination of stress intensity factor for mode I fatigue crack based on finite element analysis," *ResearchGate*. [Online].

[6] P. Weißgraeber, J. Felger, D. Geipel, and W. Becker, "Cracks at elliptical holes: Stress intensity factor and Finite Fracture Mechanics solution," *Eur. J. Mech. - ASolids*, vol. 55, pp. 192–198, Jan. 2016.

[7] A. A, "Evaluation of S.I.F for Crack Emanating at 45 Orientation from a Hole in Pressurised Cylinder Using FEA."

[8] P. Noury and K. Eriksson, "Determination of stress intensity factors for cracked bridge roller bearings using finite element analyses," *Eng. Fract. Mech.*, vol. Complete, no. 169, pp. 67–73, 2017.

[9] L. S. Chen and J. H. Kuang, "A modified linear extrapolation formula for determination of stress intensity factors," *Int. J. Fract.*, vol. 54, no. 1, pp. R3–R8, Mar. 1992.

[10] G. V. Guinea, J. Planas, and M. Elices, "KI evaluation by the displacement extrapolation technique," *Eng. Fract. Mech.*, vol. 66, no. 3, pp. 243–255, Jun. 2000.

[11] K. B. Broberg, *Cracks and Fracture*. Academic Press, 1999.

[12] S. K. Chan, I. S. Tuba, and W. K. Wilson, "On the finite element method in linear fracture mechanics," *Eng. Fract. Mech.*, vol. 2, no. 1, pp. 1–17, Jul. 1970.

[13] R. S. Barsoum, “On the use of isoparametric finite elements in linear fracture mechanics,” *Int. J. Numer. Methods Eng.*, vol. 10, no. 1, pp. 25–37, Jan. 1976.
[14] “ANSYS®, Release 15.0 - © SAS IP, Inc.2015.”
[15] “ANSYS Mechanical APDL Theory Reference. Release 15.0. ANSYS, Inc.2015.”