

Ej1)

Ejecutar paso a paso, graficando las soluciones parciales, el algoritmo de Prim que computa el árbol generador mínimo sobre los grafos con nodos $\{1, 2, \dots, 8\}$ y costos dados por una función w :

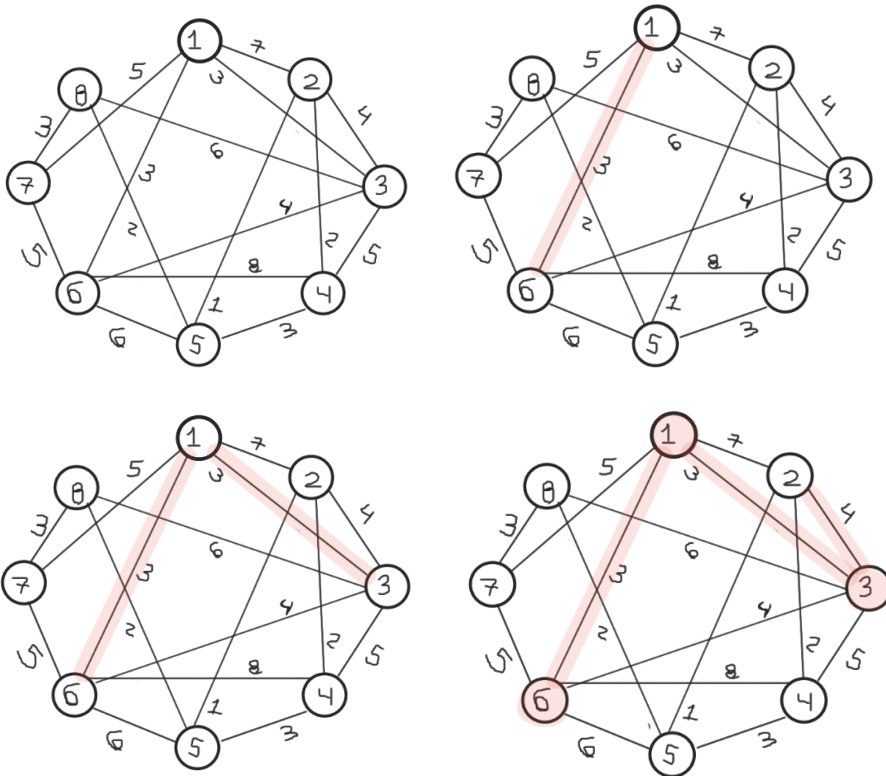
(a)

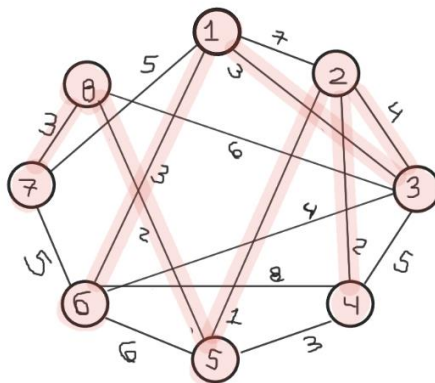
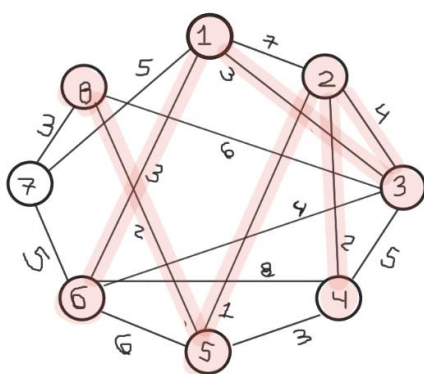
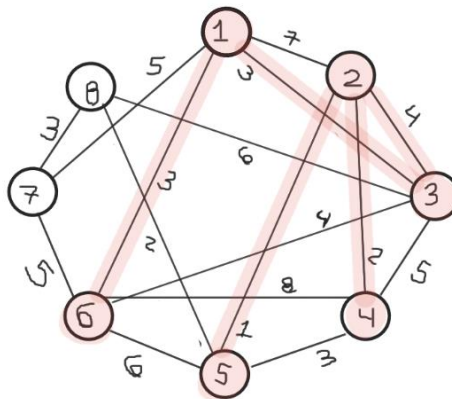
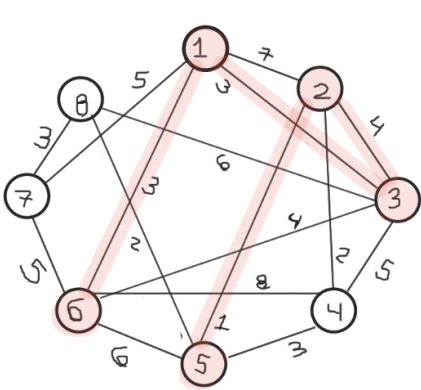
$$\begin{aligned} w((1, 2)) &= 7 & w((1, 6)) &= 3 & w((1, 7)) &= 5 & w((1, 3)) &= 3 & w((2, 3)) &= 4 & w((2, 4)) &= 2 \\ w((2, 5)) &= 1 & w((3, 4)) &= 5 & w((3, 6)) &= 4 & w((3, 8)) &= 6 & w((4, 6)) &= 8 & w((5, 4)) &= 3 \\ w((5, 6)) &= 6 & w((6, 7)) &= 5 & w((8, 5)) &= 2 & w((8, 7)) &= 3 \end{aligned}$$

(b)

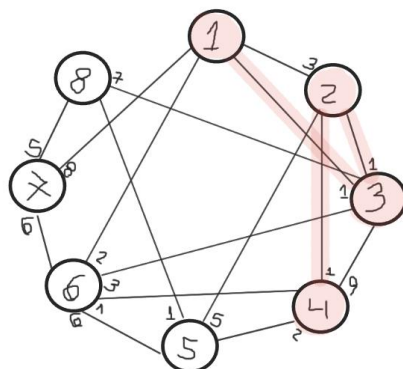
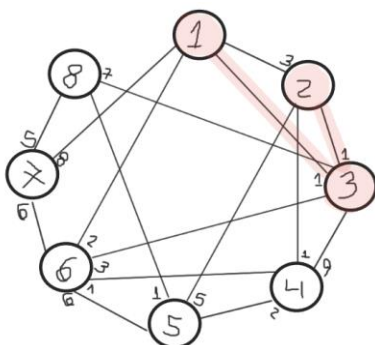
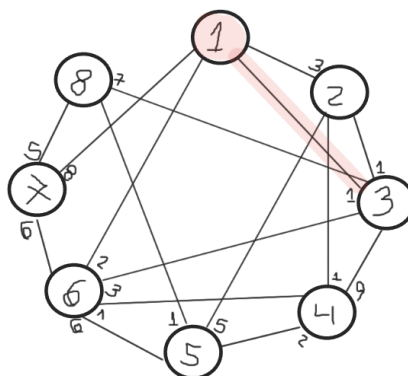
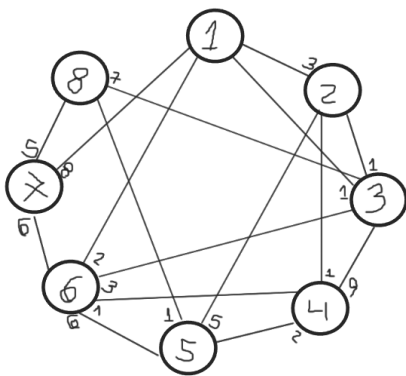
$$\begin{aligned} w((1, 2)) &= 3 & w((1, 6)) &= 2 & w((1, 7)) &= 8 & w((1, 3)) &= 1 & w((2, 3)) &= 1 & w((2, 4)) &= 1 \\ w((2, 5)) &= 5 & w((3, 4)) &= 9 & w((3, 6)) &= 3 & w((3, 8)) &= 7 & w((4, 6)) &= 1 & w((5, 4)) &= 2 \\ w((5, 6)) &= 6 & w((6, 7)) &= 6 & w((8, 5)) &= 1 & w((8, 7)) &= 5 \end{aligned}$$

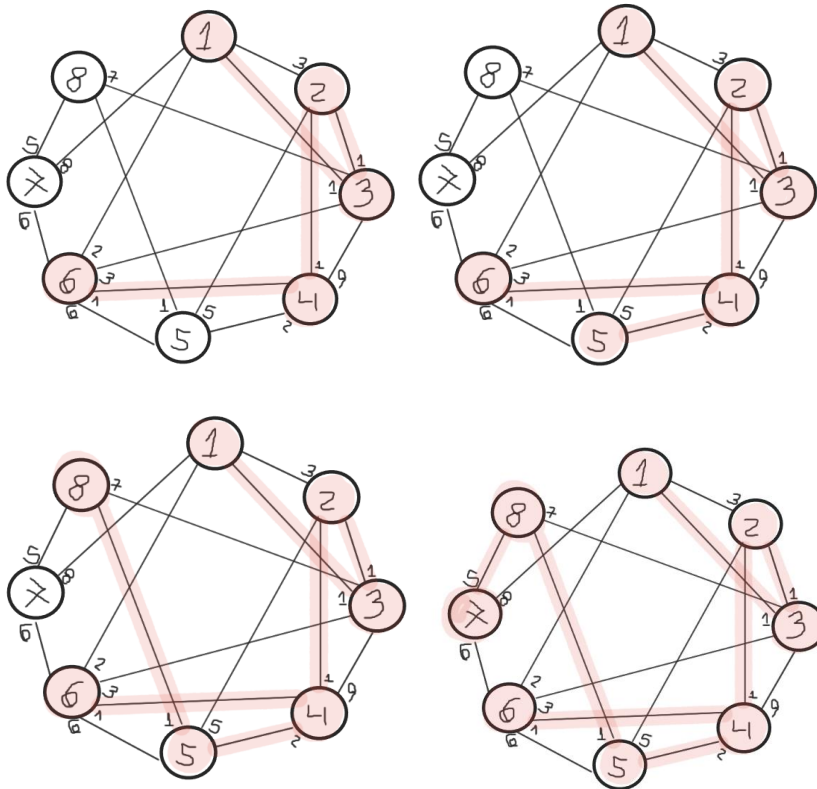
Solución a)





Solución b)

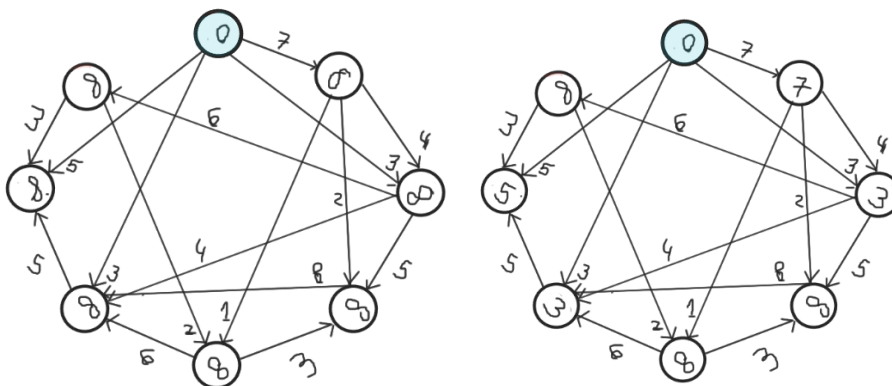


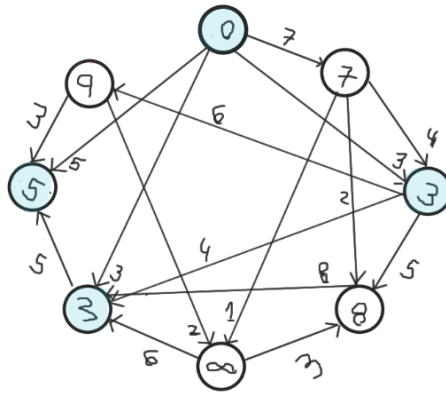
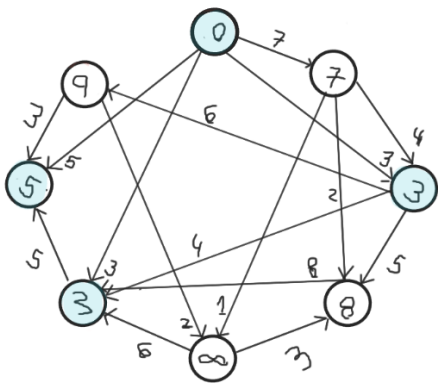
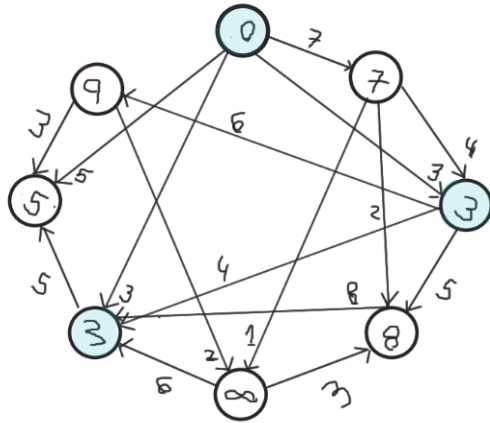
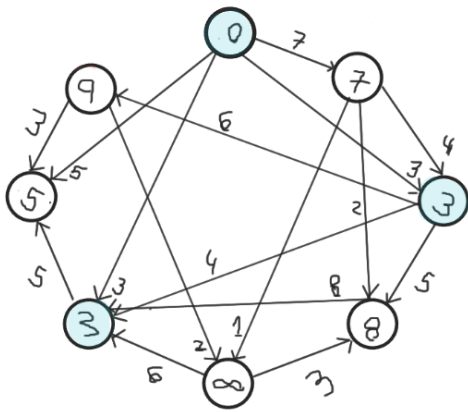
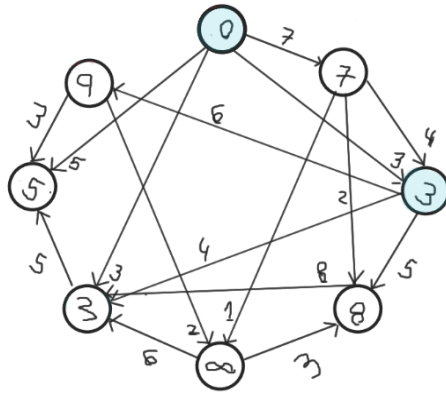
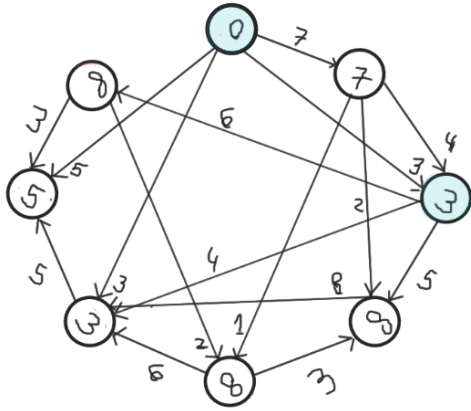


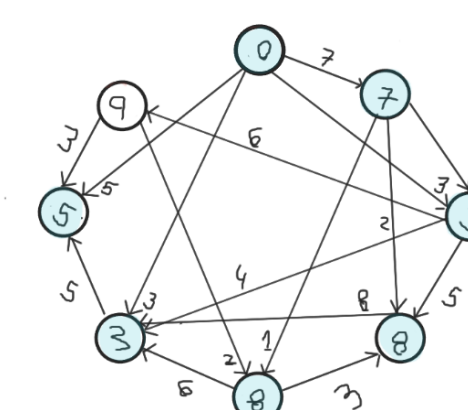
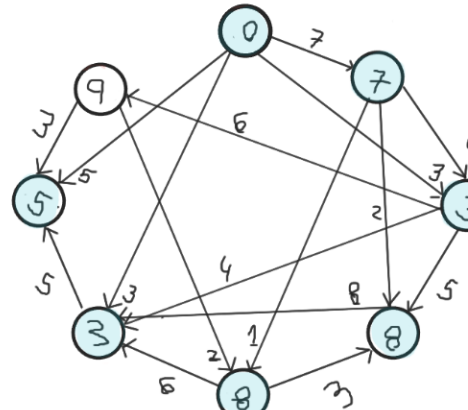
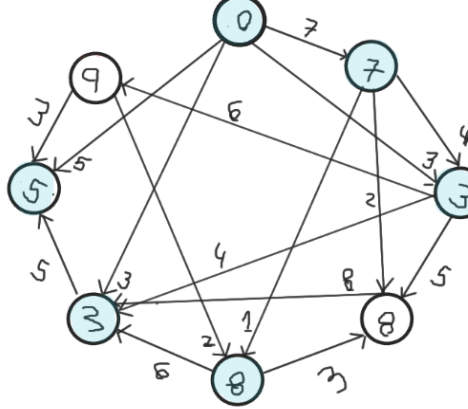
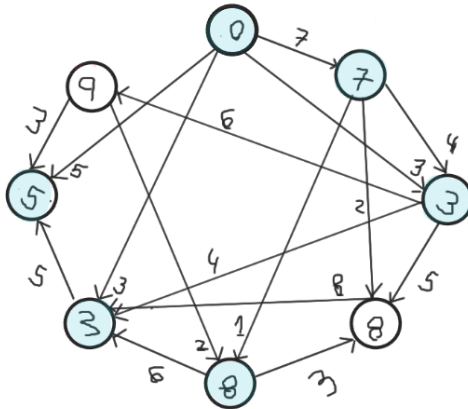
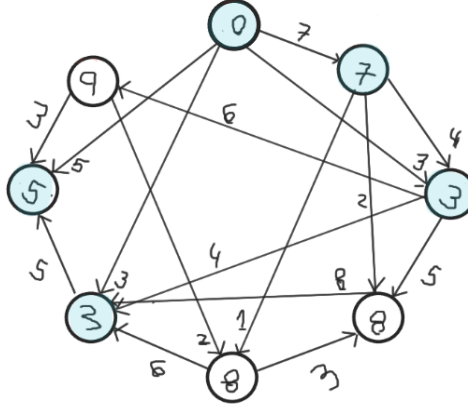
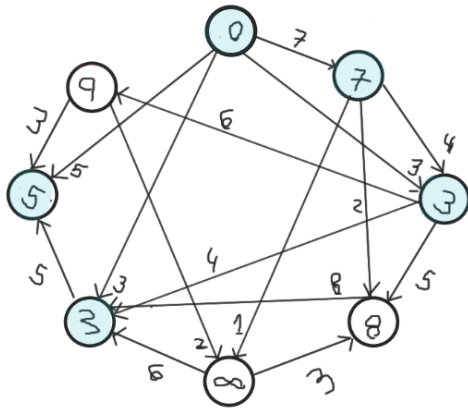
Ej2)

Ejecutar paso a paso el algoritmo de Dijkstra que computa el camino de costo mínimo entre un nodo dado y los restantes nodos de un grafo, sobre los dos grafos especificados en el ejercicio anterior. Considerar 1 como el nodo inicial. Explicitar en cada paso el conjunto de nodos para los cuales ya se ha computado el costo mínimo y el arreglo con tales costos

Solución a)







| | | | | | | | | |
|--|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--|---|---|---|---|---|---|---|---|

| | | | | | | | | |
|---|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | 0 | 7 | 3 | inf | inf | 3 | 5 | inf |
| 2 | inf | 0 | 4 | 2 | 1 | inf | inf | inf |
| 3 | inf | inf | 0 | 5 | inf | 4 | inf | 6 |
| 4 | inf | inf | inf | 0 | inf | 8 | inf | inf |
| 5 | inf | inf | inf | 3 | 0 | 6 | inf | inf |
| 6 | inf | inf | inf | inf | inf | 0 | 5 | inf |
| 7 | inf | inf | inf | inf | inf | inf | 0 | inf |
| 8 | inf | inf | inf | inf | 2 | inf | 3 | 0 |

$C = [2, 3, 4, 5, 6, 7, 8]$ arreglo de nodos

$D = [0, 7, 3, \text{inf}, \text{inf}, 3, 5, \text{inf}]$ Solución inicial

Primera interacción:

“elijo elemento c de C tal que $D[c]$ sea mínimo”

$c = 3$

“eliminó c de C ”

$C = [2, 4, 5, 6, 7, 8]$

“Actualizo D $D[j] := \min(D[j], D[c] + L[c, j])$ ”

$D = [0, 7, 3, \text{inf}, \text{inf}, 3, 5, \text{inf}]$

$D[2] = \min D[2] \text{ y } D[3] + L[3,2] = \min (7, 3 + \text{inf})$

$D[2] = 7$

$D = [0, 7, 3, \text{inf}, \text{inf}, 3, 5, \text{inf}]$

$D[4] = \min D[4] \text{ y } D[3] + L[3,4] = \min (\text{inf}, 3 + 5)$

$D[4] = 8$

$D = [0, 7, 3, 8, \text{inf}, 3, 5, \text{inf}]$

$D[5] = \min D[5] \text{ y } D[3] + L[3,5] = \min (\text{inf}, 3 + \text{inf})$

$D[5] = \text{inf}$

$D = [0, 7, 3, 8, \text{inf}, 3, 5, \text{inf}]$

$D[6] = \min D[6] \text{ y } D[3] + L[3,6] = \min (3, 3 + 4)$

$D[6] = 3$

$D = [0, 7, 3, 8, \text{inf}, 3, 5, \text{inf}]$

$D[7] = \min D[7] \text{ y } D[3] + L[3,7] = \min (5, 3 + \text{inf})$

$D[7] = 5$

$D = [0, 7, 3, 8, \text{inf}, 3, 5, \text{inf}]$

$D[8] = \min D[8] \text{ y } D[3] + L[3,8] = \min (\text{inf}, 3 + 6)$

$$D[8] = 9$$

D = [0, 7, 3, 8, inf, 3, 5, 9]

Segunda iteración:

“elijo elemento c de C tal que D[c] sea mínimo”

$$c = 6$$

“eliminó c de C”

C = [2, 4, 5, 7, 8]

“Actualizo D $D[j] := \min(D[j], D[c] + L[c, j])$ ”

D = [0, 7, 3, 8, inf, 3, 5, 9]

$$D[2] = \min D[2] \text{ y } D[6] + L[6, 2] = \min (7, 3 + \text{inf})$$

$$D[2] = 7$$

D = [0, 7, 3, 8, inf, 3, 5, 9]

$$D[4] = \min D[4] \text{ y } D[6] + L[6, 4] = \min (8, 3 + \text{inf})$$

$$D[4] = 8$$

D = [0, 7, 3, 8, inf, 3, 5, 9]

$$D[5] = \min D[5] \text{ y } D[6] + L[6, 5] = \min (\text{inf}, 3 + \text{inf})$$

$$D[5] = \text{inf}$$

D = [0, 7, 3, 8, inf, 3, 5, 9]

$$D[7] = \min D[7] \text{ y } D[6] + L[6, 7] = \min (5, 3 + 5)$$

$$D[7] = 5$$

D = [0, 7, 3, 8, inf, 3, 5, 9]

$$D[8] = \min D[8] \text{ y } D[6] + L[6, 8] = \min (9, 3 + \text{inf})$$

$$D[8] = 9$$

D = [0, 7, 3, 8, inf, 3, 5, 9]

Tercera iteración:

“elijo elemento c de C tal que D[c] sea mínimo”

$$c = 7$$

“eliminó c de C”

C = [2, 4, 5, 8]

“Actualizo D $D[j] := \min(D[j], D[c] + L[c, j])$ ”

D = [0, 7, 3, 8, inf, 3, 5, 9]

$$D[2] = \min D[2] \text{ y } D[7] + L[7, 2] = \min (7, 5 + \text{inf})$$

$$D[2] = 7$$

D = [0, 7, 3, 8, inf, 3, 5, 9]

$$D[4] = \min D[4] \text{ y } D[7] + L[7, 4] = \min (8, 5 + \text{inf})$$

$$D[4] = 8$$

$D = [0, 7, 3, 8, \text{inf}, 3, 5, 9]$

$D[5] = \min D[5] \text{ y } D[7] + L[7,5] = \min (\text{inf}, 5 + \text{inf})$

$D[5] = \text{inf}$

$D = [0, 7, 3, 8, \text{inf}, 3, 5, 9]$

$D[8] = \min D[8] \text{ y } D[7] + L[7,8] = \min (9, 5 + \text{inf})$

$D[8] = 9$

$D = [0, 7, 3, 8, \text{inf}, 3, 5, 9]$

Cuarta iteración:

“elijo elemento c de C tal que $D[c]$ sea mínimo”

$c = 2$

“eliminó c de C ”

$C = [4, 5, 8]$

“Actualizo D $D[j] := \min(D[j], D[c] + L[c,j])$ ”

$D = [0, 7, 3, 8, \text{inf}, 3, 5, 9]$

$D[4] = \min D[4] \text{ y } D[2] + L[2,4] = \min (8, 7 + 2)$

$D[4] = 8$

$D = [0, 7, 3, 8, \text{inf}, 3, 5, 9]$

$D[5] = \min D[5] \text{ y } D[2] + L[2,5] = \min (\text{inf}, 7 + 1)$

$D[5] = 8$

$D = [0, 7, 3, 8, 8, 3, 5, 9]$

$D[8] = \min D[8] \text{ y } D[2] + L[2,8] = \min (9, 7 + \text{inf})$

$D[8] = 9$

$D = [0, 7, 3, 8, 8, 3, 5, 9]$

Quinta iteración:

“elijo elemento c de C tal que $D[c]$ sea mínimo”

$c = 4$

“eliminó c de C ”

$C = [5, 8]$

“Actualizo D $D[j] := \min(D[j], D[c] + L[c,j])$ ”

$D = [0, 7, 3, 8, 8, 3, 5, 9]$

$D[5] = \min D[5] \text{ y } D[4] + L[4,5] = \min (8, 8 + \text{inf})$

$D[5] = 8$

$D = [0, 7, 3, 8, 8, 3, 5, 9]$

$D[8] = \min D[8] \text{ y } D[4] + L[4,8] = \min (9, 8 + \text{inf})$

$$D[8] = 9$$

D = [0, 7, 3, 8, 8, 3, 5, 9]

Sexta iteración:

“elijo elemento c de C tal que D[c] sea mínimo”

$$c = 5$$

“eliminó c de C”

C = [8]

“Actualizo D $D[j] := \min(D[j], D[c] + L[c, j])$ ”

D = [0, 7, 3, 8, 8, 3, 5, 9]

$$D[8] = \min D[8] \text{ y } D[5] + L[5, 8] = \min (9, 8 + \inf)$$

$$D[8] = 9$$

D = [0, 7, 3, 8, 8, 3, 5, 9]

Séptima iteración:

“elijo elemento c de C tal que D[c] sea mínimo”

$$c = 8$$

“eliminó c de C”

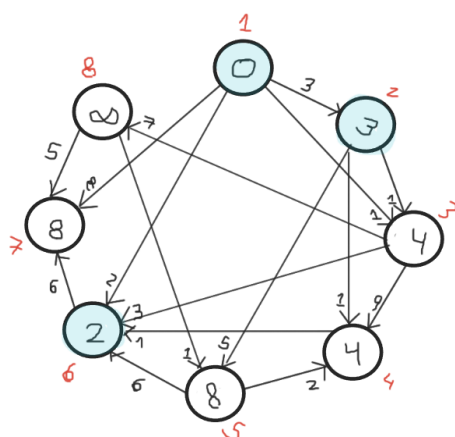
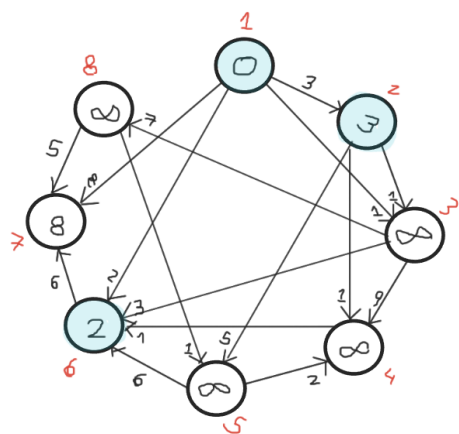
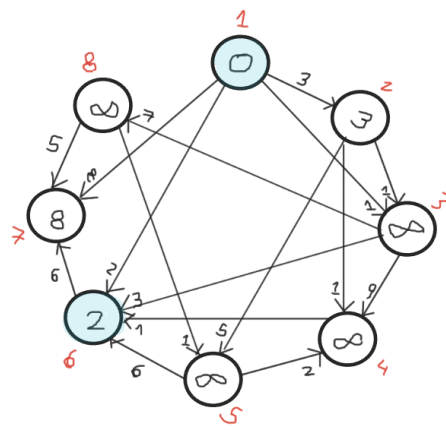
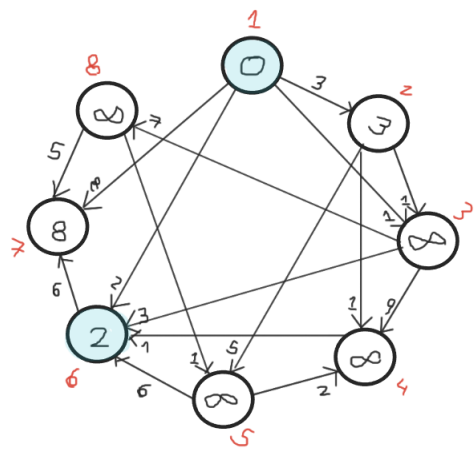
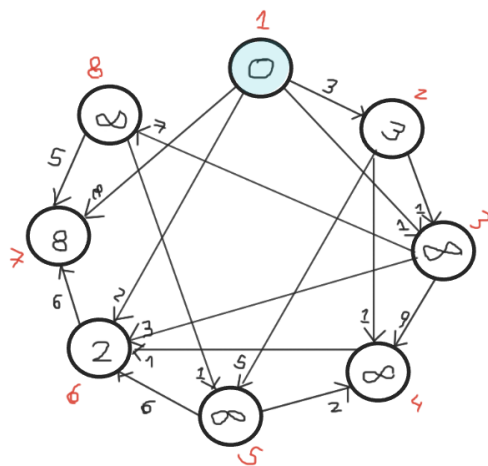
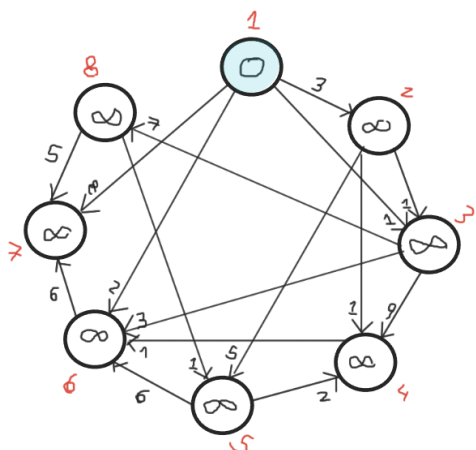
C = []

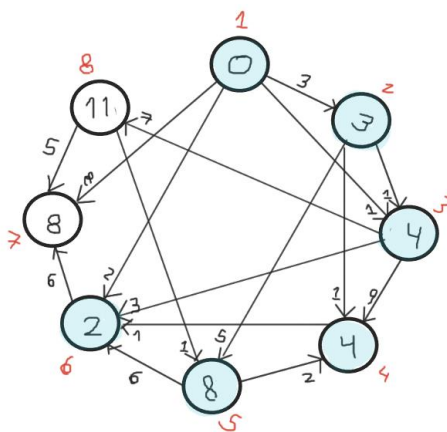
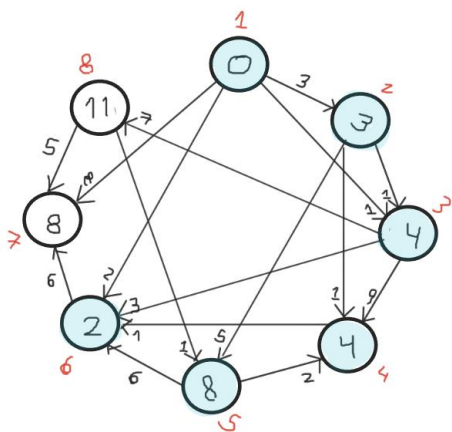
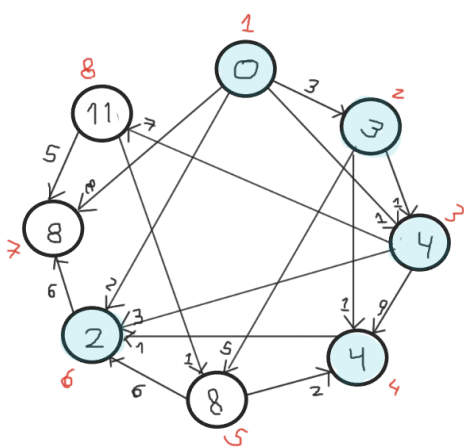
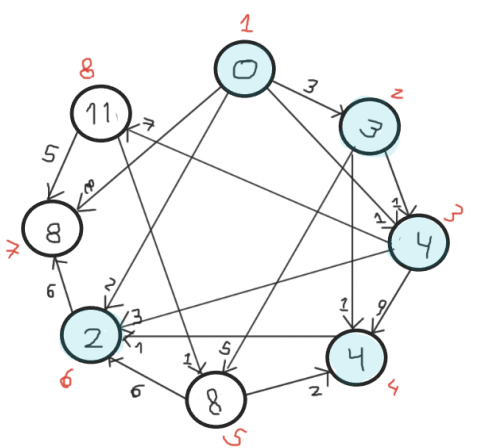
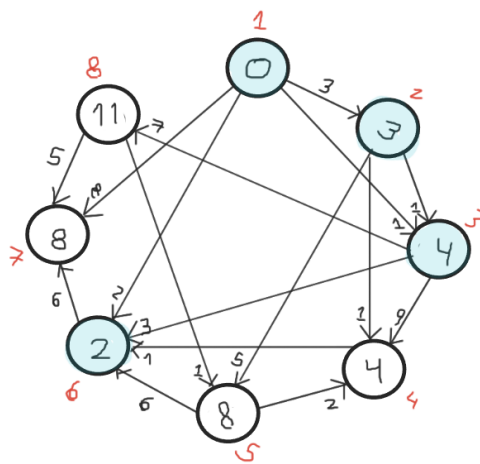
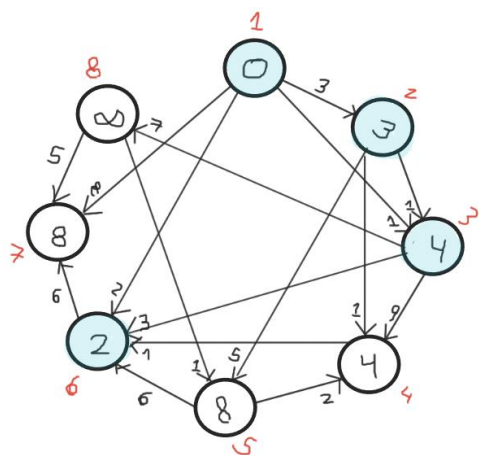
“Actualizo D $D[j] := \min(D[j], D[c] + L[c, j])$ ”

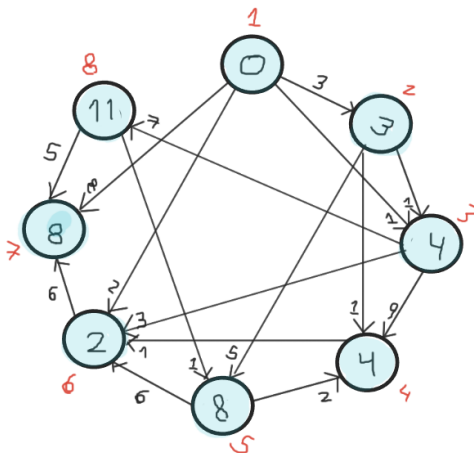
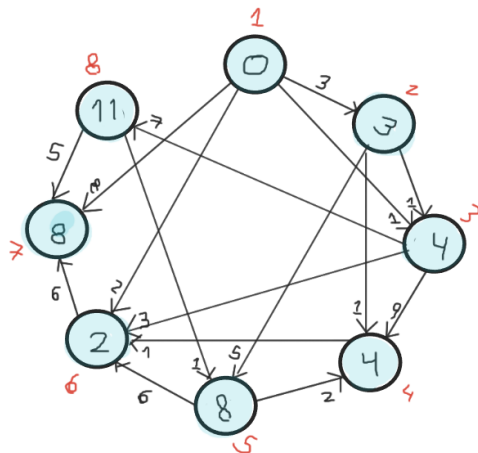
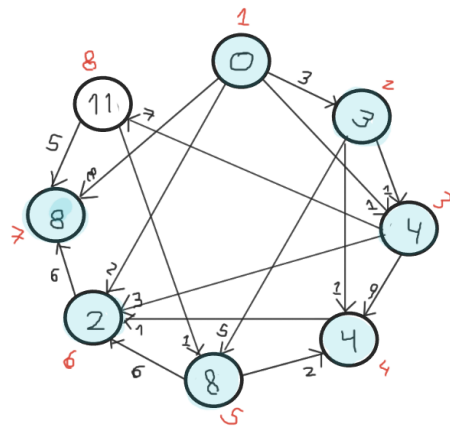
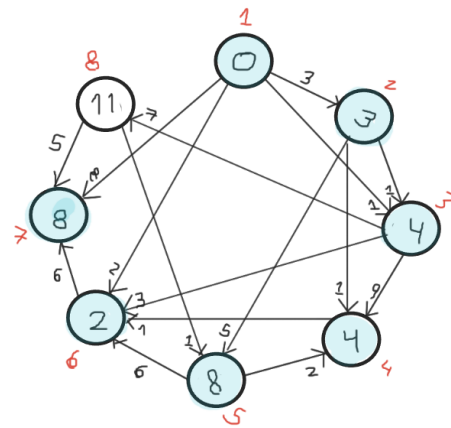
D = [0, 7, 3, 8, 8, 3, 5, 9]

D = [0, 7, 3, 8, 8, 3, 5, 9]

Solución b)







Inicio:

$C = [2, 3, 4, 5, 6, 7, 8]$

$D = [0, 3, \text{inf}, \text{inf}, \text{inf}, 2, 8, \text{inf}]$

Primera iteración:

$d[2] = \min(d[2], d[6] + \text{inf}) = d[2]$

$d[3] = \min(d[3], d[6] + \text{inf}) = d[3]$

$d[4] = \min(d[4], d[6] + \text{inf}) = d[4]$

$d[5] = \min(d[5], d[6] + \text{inf}) = d[5]$

$d[7] = \min(d[7], d[6] + 6) = (8, 8) = 8$

$d[8] = \min(d[8], d[6] + \text{inf}) = d[8]$

$C = [2, 3, 4, 5, 7, 8]$

$D = [0, 3, \text{inf}, \text{inf}, \text{inf}, 2, 8, \text{inf}]$

Segunda iteración:

$$d[3] = \min(d[3], d[2] + 1) = \min(\text{inf}, 3 + 1) = 4$$

$$d[4] = \min(d[4], d[2] + 1) = \min(\text{inf}, 3 + 1) = 4$$

$$d[5] = \min(d[5], d[2] + \text{inf}) = \min(\text{inf}, 3 + 5) = 8$$

$$d[7] = \min(d[7], d[2] + \text{inf}) = \min(8, 3 + \text{inf}) = 8$$

$$d[8] = \min(d[8], d[2] + \text{inf}) = d[8]$$

$$C = [3, 4, 5, 7, 8]$$

$$D = [0, 3, 4, 4, 8, 2, 8, \text{inf}]$$

Tercera iteración:

$$d[4] = \min(d[4], d[3] + 9) = \min(4, 4 + 9) = 4$$

$$d[5] = \min(d[5], d[3] + \text{inf}) = \min(8, 4 + \text{inf}) = 8$$

$$d[7] = \min(d[7], d[3] + \text{inf}) = \min(8, 4 + \text{inf}) = 8$$

$$d[8] = \min(d[8], d[3] + \text{inf}) = \min(\text{inf}, 4 + 7) = 11$$

$$C = [4, 5, 7, 8]$$

$$D = [0, 3, 4, 4, 8, 2, 8, 11]$$

Cuarta iteración:

$$d[5] = \min(d[5], d[4] + \text{inf}) = \min(8, 4 + \text{inf}) = 8$$

$$d[7] = \min(d[7], d[4] + \text{inf}) = \min(8, 4 + \text{inf}) = 8$$

$$d[8] = \min(d[8], d[4] + \text{inf}) = \min(11, 4 + \text{inf}) = 11$$

$$C = [5, 7, 8]$$

$$D = [0, 3, 4, 4, 8, 2, 8, 11]$$

Quinta iteración:

$$d[7] = \min(d[7], d[5] + \text{inf}) = \min(8, 8 + \text{inf}) = 8$$

$$d[8] = \min(d[8], d[5] + \text{inf}) = \min(11, 8 + \text{inf}) = 11$$

$$C = [7, 8]$$

$$D = [0, 3, 4, 4, 8, 2, 8, 11]$$

Sexta iteración:

$$d[8] = \min(d[8], d[7] + \text{inf}) = \min(11, 8 + \text{inf}) = 11$$

$$C = [8]$$

$$D = [0, 3, 4, 4, 8, 2, 8, 11]$$

Séptima iteración:

$$C = []$$

$$D = [0, 3, 4, 4, 8, 2, 8, 11]$$

3. Usted quiere irse de vacaciones y debe elegir una ciudad entre K posibles que le interesan. Como no dispone de mucho dinero, desea que el viaje de ida hacia la ciudad pueda realizarse con a lo sumo L litros de nafta.

- (a) Dé un algoritmo que, dado un grafo representado por una matriz $E : \text{array}[1..n, 1..n] \text{ of Nat}$, donde el elemento $E[i,j]$ indica el costo en litros de nafta necesario para ir desde la ciudad i hasta la ciudad j ; un conjunto C de vértices entre 1 y n , representando las ciudades que quieren visitarse; un vértice v , representando la ciudad de origen del viaje; y un natural L , indicando la cantidad de litros de nafta total que puede gastar; devuelva un conjunto D de aquellos vértices de C que puede visitar con los L litros.
- (b) Ejecute el algoritmo implementado en el inciso anterior para el grafo descrito en el siguiente gráfico, con vértices $1, 2, \dots, 11$, tomando $C = \{11, 5, 10, 7, 8\}$ como las ciudades de interés, disponiendo de $L = 40$ litros de nafta. ¿Cuáles son los posibles destinos de acuerdo a su presupuesto?

Ayuda: Utilice el algoritmo de Dijkstra.

Ej3)

Solución a)

- Tenemos un grafo con $1..n$ nodos, con costos en las aristas.
- Tenemos un nodo inicial v .
- Dado un subconjunto C de $\{1..n\}$, c_n (posiciones de C) queremos saber a cuales de esos vértices podemos ir con costo menor a L , partiendo de V
¿Cómo podemos usar Dijkstra?
Dijkstra nos permite obtener el costo del camino de costo mínimo desde v a cada uno de los vértices $1..n$

idea:

1. Llamo a Dijkstra y conseguí $C' = [c_1, c_2, \dots, c_n]$ los costos mínimos para ir desde v hacia $1, 2, \dots, n$.
2. Recorro el conjunto C : para cada elemento c en C , si $C'[c] < L$ lo agrego al conjunto solución D .
3. Me quedo con el elemento de C tq $C'[c_n] < L$

Código:

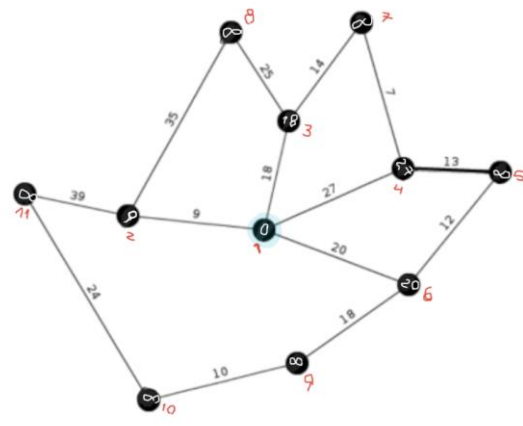
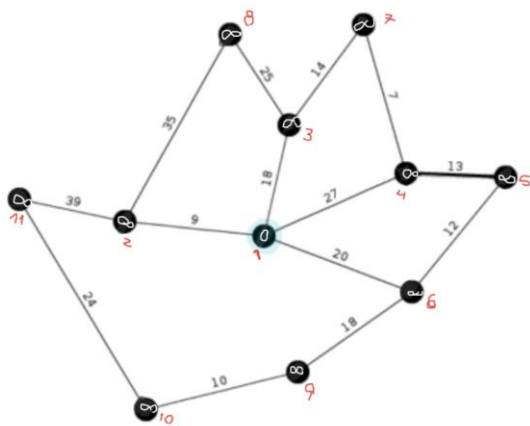
```

fun holidays(E: array[1..n,1..n] of nat, v: nat, C: Set of nat, L: nat)
ret D: Set of nat
    var costos: array[1..n] of nat
    var c_aux: Set of nat
    var c: nat
    c_aux := copy_set(C)
    D := empty_set()
    costos := Dijkstra(E,v)
    while(not is_empty_set(c_aux)) do
        c := get(c_aux)
        if(costos[c] ≤ L) then
            add(D,c)
        fi
        elim_set(c_aux,c)
    od
    Destroy(c_aux)
end fun

```

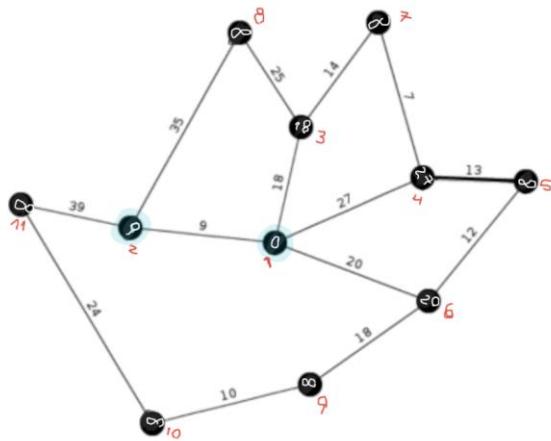
Solución b)

Corremos dikstra:



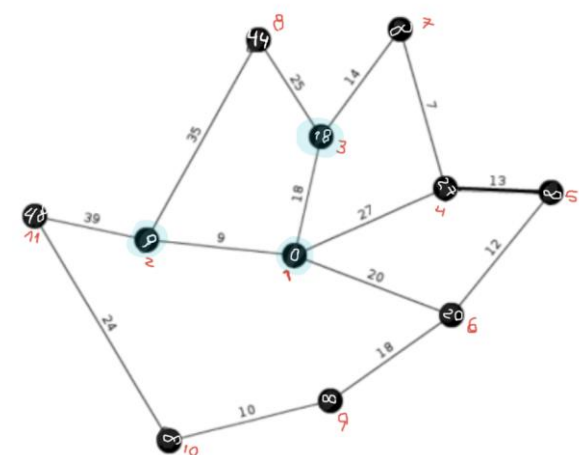
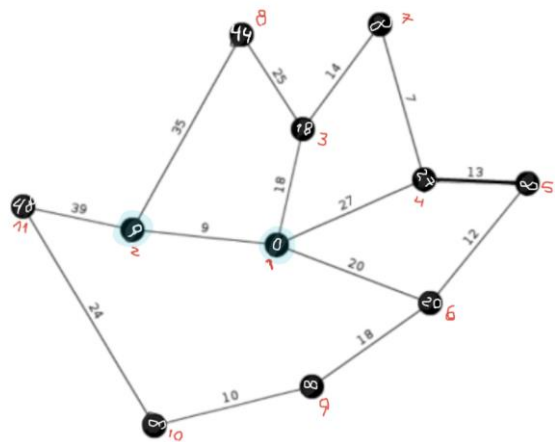
C = (2,3,4,5,6,7,8,9,10,11)

D = (0,9,18,27,inf,20,inf,inf,inf,inf,inf)



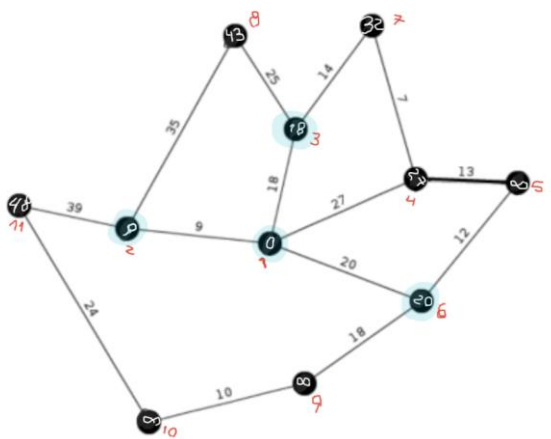
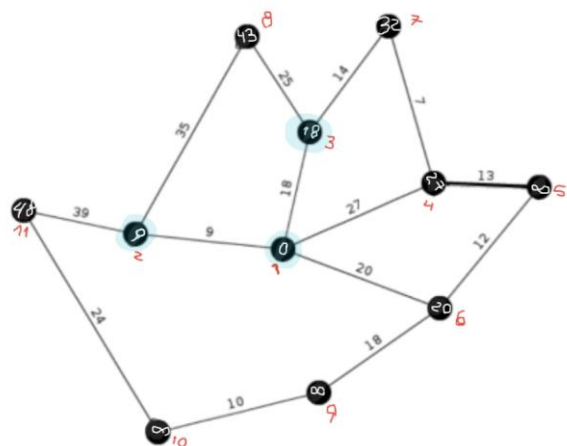
$C = (3,4,5,6,7,8,9,10,11)$

$D = (0,9,18,27,\text{inf},20,\text{inf},44,\text{inf},\text{inf},48)$



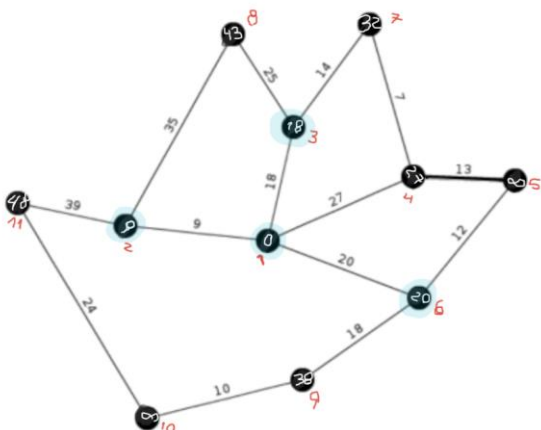
$C = (4,5,6,7,8,9,10,11)$

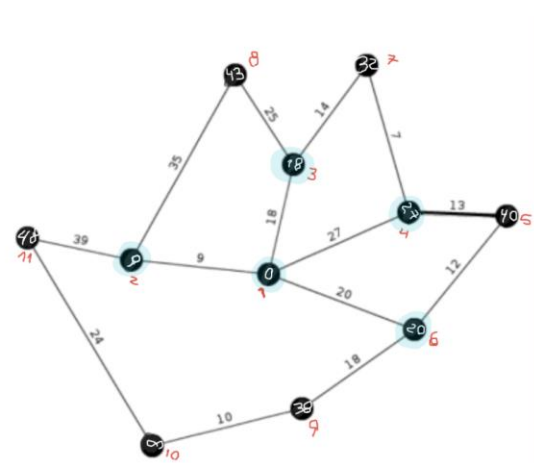
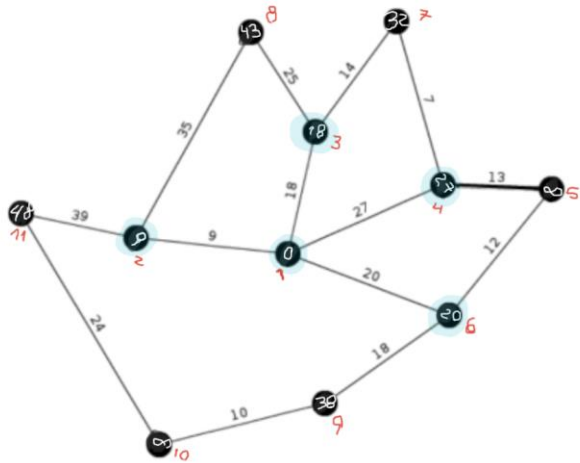
$D = (0,9,18,27,\text{inf},20,32,43,\text{inf},\text{inf},48)$



$C = (4,5,7,8,9,10,11)$

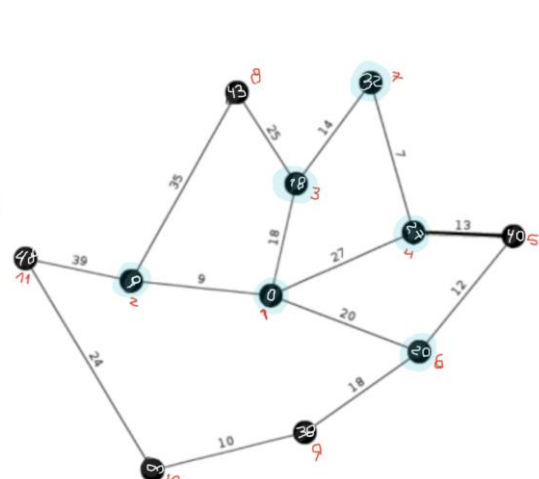
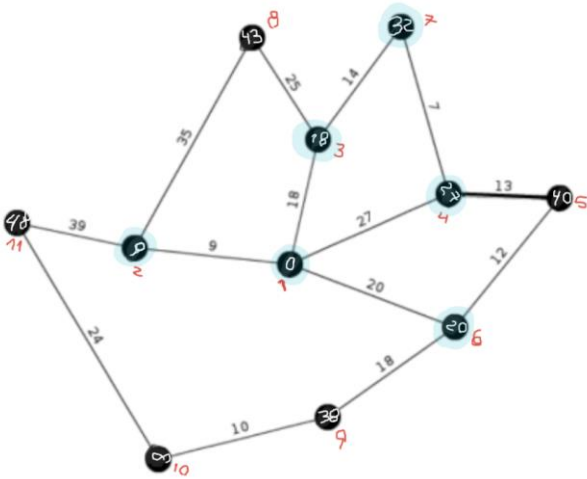
$D = (0,9,18,27,\text{inf},20,32,43,38,\text{inf},48)$





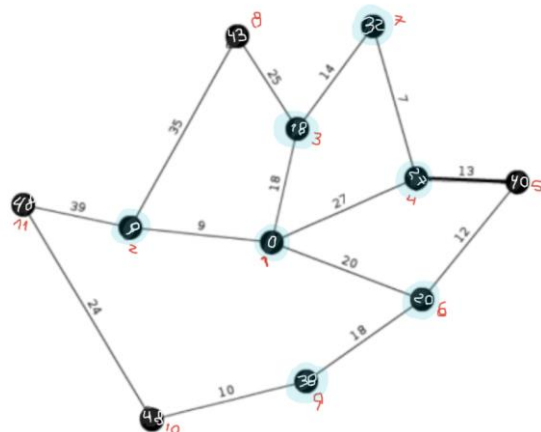
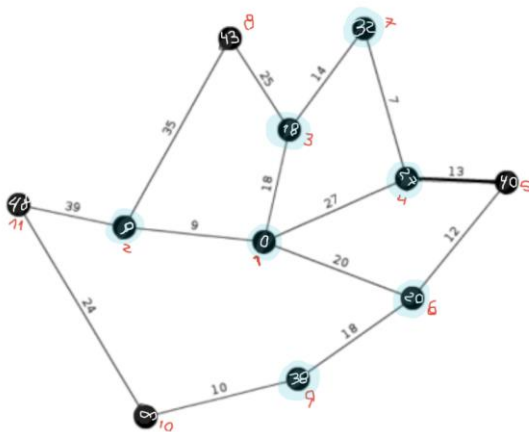
C = (5,7,8,9,10,11)

D = (0,9,18,27,40,20,32,43,38,inf,48)



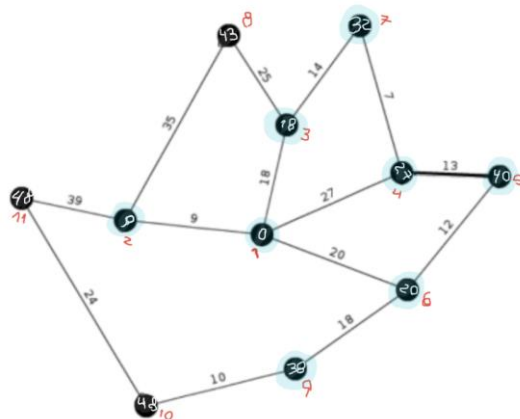
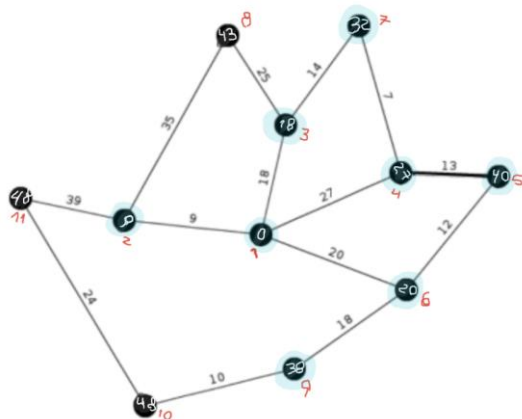
C = (5,8,9,10,11)

D = (0,9,18,27,40,20,32,43,38,inf,48)



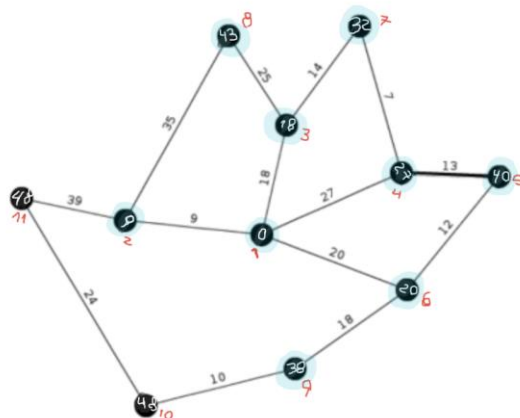
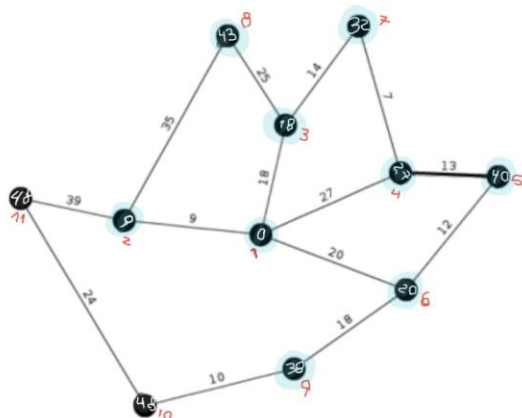
C = (5,8,10,11)

D = (0,9,18,27,40,20,32,43,38,48,48)



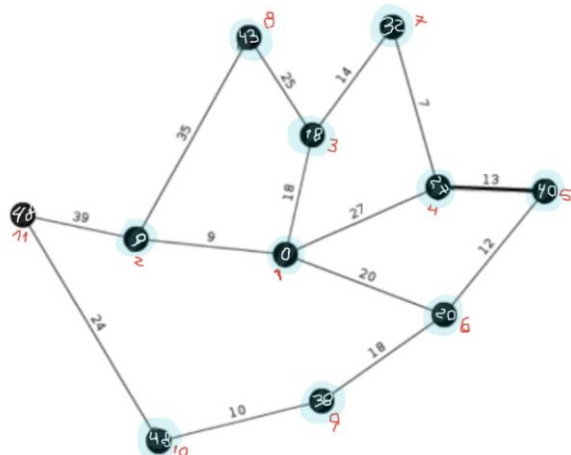
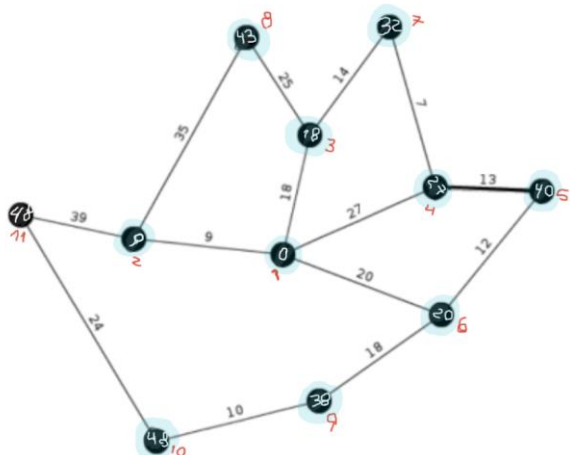
$C = (8, 10, 11)$

$D = (0, 9, 18, 27, 40, 20, 32, 43, 38, 48, 48)$



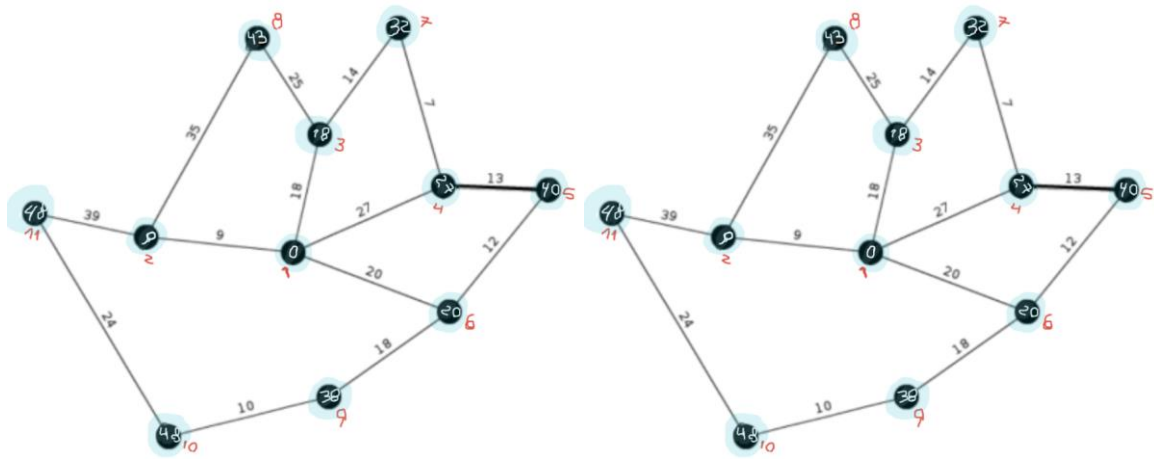
$C = (10, 11)$

$D = (0, 9, 18, 27, 40, 20, 32, 43, 38, 48, 48)$



$C = (11)$

$D = (0, 9, 18, 27, 40, 20, 32, 43, 38, 48, 48)$



$C = ()$

$D = (0, 9, 18, 27, 40, 20, 32, 43, 38, 48, 48)$

Ahora nos quedamos con las los elementos de $C = \{11, 5, 10, 7, 8\}$ tq

$D[\text{elemento de } c] \leq 40$

Solución: $S = (5, 7)$