Determination of Quantum Numbers of the ω -meson

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February 6, 2009

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1 About The Experimental Determination of The Quantum Numbers of The ω Meson

A thousand events of the type $pp \longrightarrow pp\pi^+\pi^-\pi^0$ with an invariant mass $m_{3\pi}$ smaller than 1.2 GeV were examined. The η ($m_\eta = 0.5488GeV$) and ω resonance ($m_\omega = 0.7826GeV$) lie in thi smass range. The upper limit of the invariant mass was set to 1.2 GeV,in order to save place in the institute computer. Four-momenta of incoming and outgoing particles in the Center-of-mass system (CMS) of the five outgoing particles were recorded which can then be obtained with the help of a menu programme and be analysed. The programme yields four-vectors for each individual events. It also provides four-vectors of the same events, which are calculated in the CMS of the three pions. The programme creates mass-histograms[1] for a certain number of events in order to examine the ω mass and -halftime,as well as radial histograms to determine spin and parity of ω meson. Since the programme yields lists of masses and radia, students could understand every analysis-step. The examinations and their limits are described in the following sections.

1.1 The $m_{3\pi}$ -effective Mass Distribution

The invariant mass of $m_{3\pi}$ of a three pion system is defined via the 4-momenta $q_i = (E_i, \vec{p_i})$ of the three pions (see also chapter 3).

$$m_{3\pi}^2 := (\sum_{i=1}^{3} E_i)^2 - (\sum_{i=1}^{3} \vec{p_i})^2 \tag{1}$$

The figure 1 shows a histogram of the invariant masses of 1000 events using a width of the individual bins of 16 MeV each. One can easily see the resonance signal of the ω meson at an invariant mass of 780 MeV. The signal from η meson at 550 MeV can be seen,as well, but is so weak in this measurement that there is no reasonable way to obtain data about it.

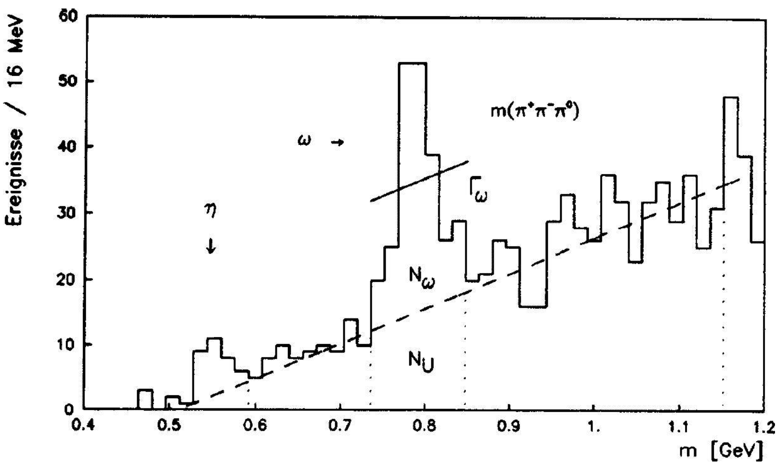

Figure 1: Masshistogram for 1000 events at 1.2 GeV

The first step when evaluating data as seen in the histogram is to determine the optimum bin width. Physically this width and the lifetime of the mesons obtained from the resonance signal are independent. It is clear though, that above a certain width, there will be a correlation between the two quantities. On the other hand too small bins will contain only very few events and thus contain high statistical errors.

In order to determine the FWHM[2] it is necessary to know the background level. The background distribution U(m) reflects the distribution of the invariant masses of three pion systems not resulting from the decay of an ω meson. U(m) can be approximated by linear function in the considered mass range :

$$U(m) = A \cdot m + B \tag{2}$$

[1]A histogram is a tool to summarize and display data. The range of values of one quantity, here it is the mass, is subdivided into several bins. For each bin number of entries in the data sample is calculated and the number of entries per bin versus the mass is graphically displayed.

[2]Full width of the signal peak at its half maximum amplitude

This linear fit can be done numerically or just by a ruler using those mass intervals as an orientation ,in which there are no resonance peaks - in our case that is 590 - 720 MeV and 840 - 1150 MeV.

The ideal value for the bin width is between 10 and 20 MeV. The following considerations are based on a value of 16 MeV.

1.1.1 Determination of the ω mass

In the ω resonance region the histogramm can be approximated as follows :

$$H(m) = U(m) + \alpha R(m)U(m) = U(m)(1 + \alpha R(m)) \tag{3}$$

U(m) describes the linear background, α is a normalization factor which is determined as the ratio of resonant events N_ω and background events N_U in the resonance region :

$$\alpha = \frac{N_\omega}{N_U} \tag{4}$$

From equation (3) the resonance distribution R(m) is given by

$$R(m) = \frac{1}{\alpha}(\frac{H(m)}{U(m)} - 1) \tag{5}$$

R(m) is shown in picture2. Mass and FWHM are read off from the histogram by for ,example, fitting with triangular distribution :

$$m_\omega = (780 \pm 8)MeV \Gamma_\omega = (35 \pm 8)MeV \tag{6}$$

The errors are given by halfwidth of a bin . Furthermore picture 2 shows a Breit-Wigner distribution BW(m) :

$$BW(m) = \frac{\Gamma_\omega}{2\pi} \frac{1}{(m - m_\omega)^2 + (\frac{\Gamma_\omega}{2})^2} \tag{7}$$

This distribution describes the signal shape for a particle decaying with mean lifetime τ of

$$\tau\Gamma \cong \hbar \tag{8}$$

1.1.2 Estimate for the ω lifetime and the apparative resolution

The measurement of momenta on bubble chamber photographs has finite resolution. This causes broadening of the resonance signals. Thus the measured FWHM of the ω signal does not directly give you the lifetime .

Assuming the relative error in the momentum measurement to be about 1 per cent for the charged pions and about 3 to 5 per cent for the neutral pion one can estimate the apparative FWHM Γ^{app} to be about 20 to 25 MeV. Provided the apparative resolution is also distributed according to a Breit-Wigner, the following equation yields for the natural FWHM of the ω meson :

$$\Gamma_\omega = \Gamma_\omega^{measured} - \Gamma^{app} \simeq 10 - 15MeV \tag{9}$$

This is a value which is in accordance with the PDG value from 1986 of $\Gamma_\omega^{lit} = 9,8MeV$. The mean lifetime is then given by :

$$\tau_\omega \cong \frac{\hbar}{\Gamma_\omega} \cong 3 \cdot 10^{-23}s \tag{10}$$

1.2 Dalitz-Stevenson distribution for the 3-pion-system

Spin and parity of resonances with 3-body-decays can be told from the Dalitz-Stevenson-distribution. Every event corresponds to a point in a two dimensional plot where the x and y coordinates are determined from the event kinematics of the three pions. In the case where all pions have the same energy one obtains the origin of the plot. This plot as well as everything contained in the next subsection is explained more thoroughly in the following chapter.

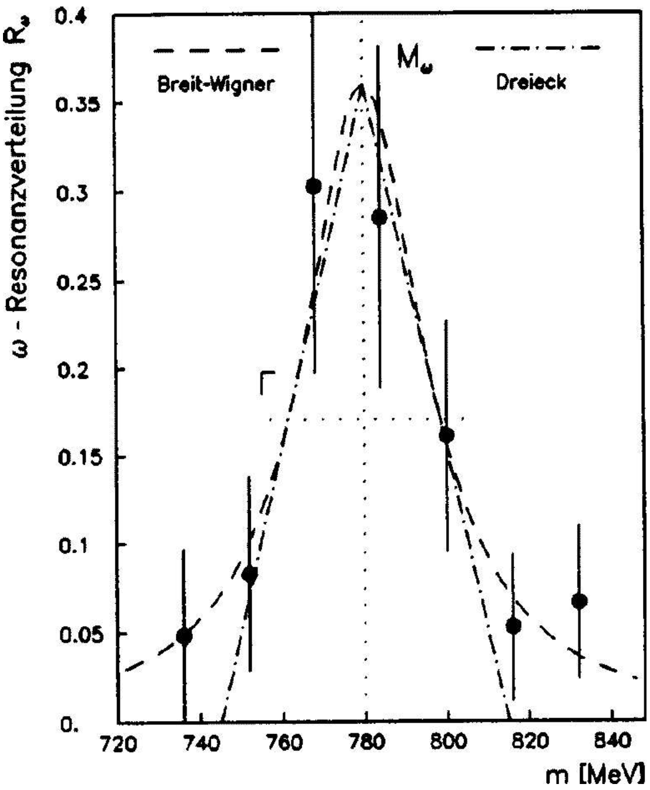

Figure 2: Resonance distribution compared to Breit-Wigner-Distribution

1.2.1 Radial density distribution, Spin and Parity of the ω meson

To simplify the analysis it is not the two dimensional density distribution in the (r, ϕ) plane that shall be regarded. In the following we will rather look at the one dimensional radial density distribution which is obtained by projection and depends only on r. In order to do this projection you first create a histogram for the distance of the individual events from the origin. Every bin, which has a given finite width,in the histogram reflects an area in the two dimensional (r, ϕ) plane. To get the desired radial density distribution from the histogram first,you must normalize number of events in each bin to the corresponding area. This means you have to divide the number of events in a bin by the area of the ring around the origin that is reflected by the bin. As long as the area of the rings are completely within the boundary curve (see the next chapter)of the Dalitz plot ,you get the radial density distribution. Some radia on the rings will contain areas which have no physical meaning as they are outside the boundary curve. For these bins outside, normalization to the ring area systematically makes the radial density distribution to small.

As the final step the density distribution is normalized to the overall number of events. The following figure shows the radial density distribution for a total of 245 events from a mass interval between 720 and 840 MeV. The statistical error per bin is about 20 per cent (Fig. 3)

For a 1^- meson you expect a centered distribution ($\sim 1 - 1.4r^2$) as shown in picture 4. Since there are approximately 50 per cent background events in the considered mass interval, the following fit is the best for the experimental data :

$$(1 - r^2) \sim 0,5 \cdot const + 0,5 \cdot (1 - 1.4r^2) \tag{11}$$

Using a χ^2 test this $(1-r^2)$ hypothesis could be verified to a confidence level of 95 per cent. This also establishes the $J^P = 1^-$ hypothesis. Testing in the neighboring interval without resonance, yielded a confidence level below 20 per cent.

1.3 Limits of the existing experiment

The limits of the existing experiment resulted from the low statistics of only 1000 events.

1. The size of the data set prescribes a certain minimum value for the bin width. The relative statistical

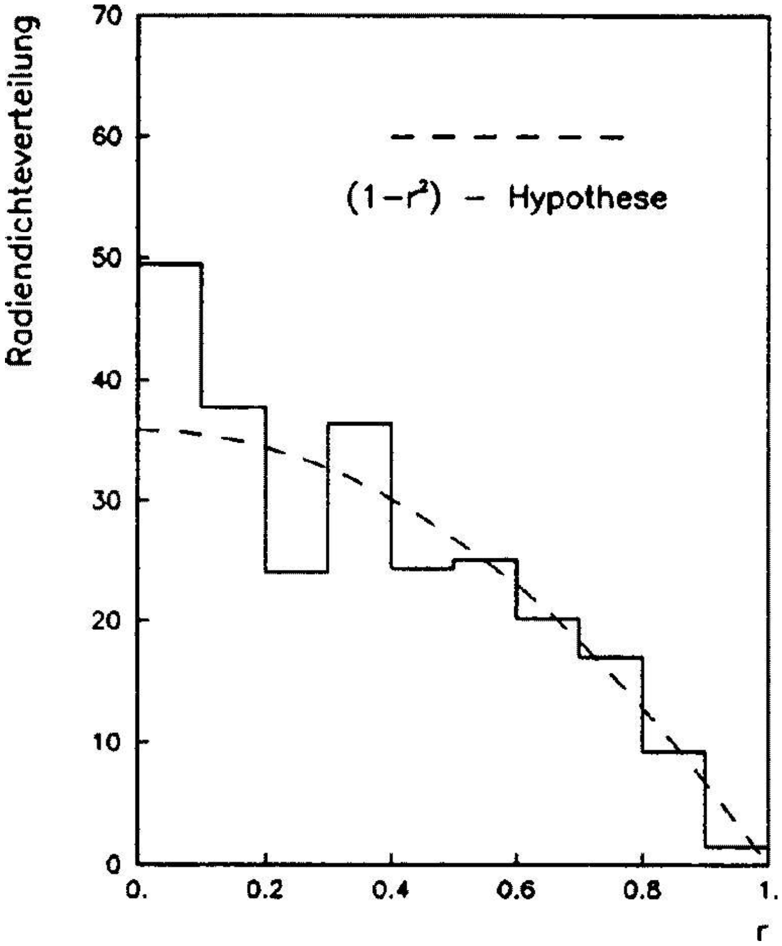

Figure 3: Radial distribution

errors for each bin are proportional to $1/\sqrt{n_{evt}}$, where n_{evt} is the number of events in this bin. Obviously they are diminished when using more data.

2. The apparative resolution in a histogram of invariant masses can me measured in a better way when using the η resonance. The η decays electro magnetically and thus has a much lower decay width (corresponding to a much longer lifetime) than the ω. Neglecting this small decay width the η resonance can be used to measure the apparative resolution. This value can consequently be used when regarding the ω resonance.

3. Due to lack of data a detailed analysis of the Dalitz distribution is impossible. Using six times as many data points it is possible to estimate the background of the radial distribution. Thus the resulting corrected distribution can be tested against the hypothesis of the meson having $J^P = 1^-$. Additionally also the angular Dalitz distribution could be tested.

2 Basics on the determination of the quantum numbers of the ω meson

Mass and lifetime of the ω meson are determined from the mass distribution. This is a histogram of the invariant masses of all events. When interpreting the distribution, the following distinctions are made concerning the origin of the three pions:

1. For the mass distribution of the pions that are created in a reaction $pp \to pp\pi^+\pi^-\pi^0$ one expects a phase space distribution, i.e. distribution according to energy and momentum conservation. For energies in the laboratory frame of above 8 GeV one finds a behaviour which follows the so called longitudinal phase space

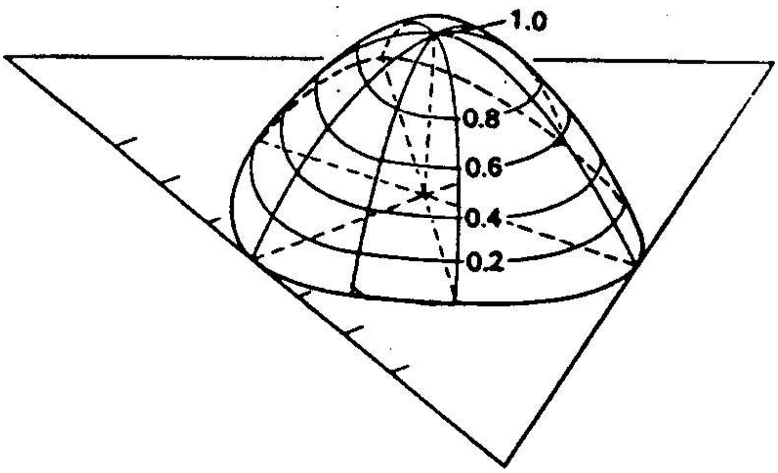

Figure 4: Theoretical Dalitz-Stevenson-distribution of a vectormeson

model[3], since the transverse momentum of the secondary particles is restricted : $< p_T > \approx 300 MeV/c$.

2. The pions originating from a resonance, form peaks corresponding mainly to Breit-Wigner distributions.

3. Sometimes also events occur where one or two pions come from resonance dacays other than the one mentioned above (reflection).

All events stemming from events other than a ω decay are defined to be the background distribution. The remaining resonance signal then allows the measurement of ω mass and lifetime.

In the following sections the theoretical background of the measurement of J^P for resonances decaying into three pions shall be clarified. This is done by using an example of the ω decay. As charged and uncharged pions have approximately the same mass and the pionic resonance thus decays into three equally heavy particles a modified Dalitz distribution is introduced : The Dalitz-Stevenson distribution.

2.1 The effective mass distribution

2.1.1 Statistical model, longitudinal phase space

In the following a reaction with k particles in the initial state and n particles in the final state is investigated $(k \leq n)$. The invariant mass is defined as follows :

$$(M_k^n)^2 := (\sum_{i=1}^{k} E_i)^2 - (\sum_{i=1}^{k} \vec{p_i})^2 \tag{12}$$

The four momenta of the k initial particles are denoted as $q_i = (E_i, \vec{p_i})$ The value of the effective mass is in the interval

$$m_1 + ... + m_k \leq M_k^n \leq E_{CM} - (m_{k+1} + ... + m_n) \tag{13}$$

The distribution of the effective mass is a density distribution

$$H = \frac{dN_k}{dM_k^n} \tag{14}$$

In case the k initial particle are created directly, dN_k denotes the number of states in the effective mass interval $[M_k^n, M_k^n + dM_k^n]$. The quantity N_k is called phase space. It is calculated as follows:

The state of a particle is represented by a vector $(x_1, x_2, x_3, p_1, p_2, p_3)$ in a six-dimensional space. According to Heisenbergs uncertainty relation :

$$\Delta x_i \Delta p_i \geq 2\pi\hbar\delta_{ij} \tag{15}$$

[3]In the context of the quark model, there is a natural explanation of this behaviour.

Thus a unit cell of the size

$$\prod_{j=1}^{3} \Delta x_j \Delta p_j = (2\pi\hbar)^3 \tag{16}$$

is defined. The geometric volume is restricted to $V = \prod_{j=1}^{3} \Delta x_j$:

$$\prod_{j=1}^{3} \Delta p_j = \frac{(2\pi\hbar)^3}{V} \tag{17}$$

Using the four vectors $q_i = (E_i, \vec{p_i})$ of the k particles and the 4 momentum of the centre of mass $Q = (E, \vec{P}) = \sum_{i=1}^{n} q_i$ the Lorentz invariant phase space $N_k(Q)$ is determined as :

$$N_k(Q) = (\frac{V}{(2\pi\hbar)^3})^k (\prod_{i=1}^{k} S_i) \int \prod_{i=1}^{k} \frac{d^3\vec{p_i}}{2E_i} \delta^4(\sum q_i - Q) \tag{18}$$

Here S_i denotes the number of spin states of the i-th particle. In appendix A the phase space of 2 and 3 particle systems (not including volume and spin factors) are calculated explicitly.

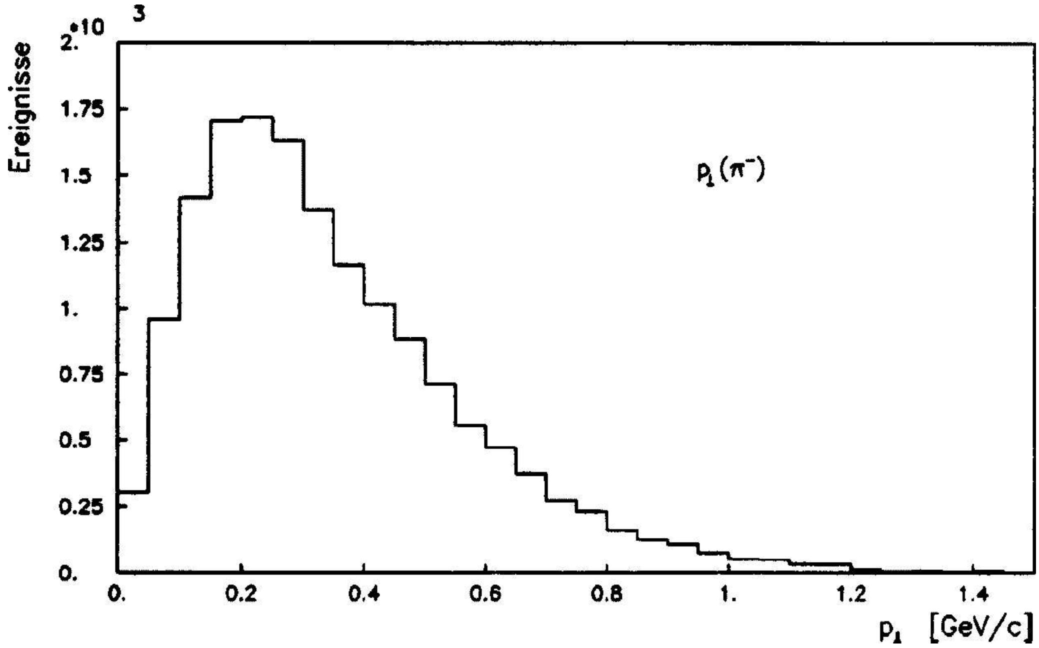

Figure 5: Transversal momentum distribution of a pion from 15813 events

Experimentally one found that for laboratory energies above 8 GeV there is a transverse momentum of the initial particles of only 300 to 400 MeV/c. The particles thus do not any more distribute isotropically in the centre of mass frame. The empirically determined mean value of the transverse momentum , above laboratory energy of 8 GeV, is almost independent of energy. Due to this restriction of the transverse momentum the density of states $H = \frac{dN_k^{long}}{dM_k^n}$ for high laboratory energies is described by the so called "longitudinal phase space" N_k^{long} :

$$H(E_{lab>8GeV}) = \frac{dN_k^{long}}{dM_k^n} \tag{19}$$

A simple model for the longitudinal phase space of k massless initial particles is :

$$N_k^{long}(Q) = \int \prod_{i=1}^{n} e^{-ap_{i\perp}} \frac{d^3\vec{p_i}}{2E_i} \delta^4(\sum q_i - Q) \tag{20}$$

$p_{i\perp}$ denotes the transverse momentum of the i-th particle. For a better understanding of this approach picture 5 shows the $p_{i\perp}$ distribution of the π^- from 15813 events of the form $pp \to pp\pi^+\pi^-\pi^0$. Up to about 200 MeV the distribution is rising linearly with the transverse momentum. The maximum is at about 250 MeV/c and from 400 MeV/c on the distribution behaves as $e^{-ap_\perp^2}$. The factor $F = e^{-ap_\perp^2}$ in the above formula describes this behaviour.

2.1.2 Resonance signals, Breit-Wigner distribution

A resonance is an excited state of a quark system. This system has a defined mass and can be characterized by a set of quantum numbers. According to the partial wave formalism it is the property of resonances that the phase shift δ_l of the l-th partial wave , which is an outgoing spherical wave, is given by $2\delta_l = k\pi$. As the system is not in it's ground state and decays with a lifetime τ the energy is a complex number :

$$E_{res} = E_0 + i\frac{\Gamma}{2} \tag{21}$$

Here, $\tau\Gamma \simeq \hbar$. Performing a Fourier transformation of the probability distribution $\mid \Psi \mid^2$ to energy space the normalized energy spectrum is obtained:

$$BW(E) = \frac{\Gamma}{2\pi}\frac{1}{(E-E_0)^2 + (\frac{\Gamma}{2})^2} \tag{22}$$

This curve is called a Breit-Wigner resonance-curve. The quantity Γ corresponds to the FWHM.

A resonance can by identified in an effective mass spectrum as a peak above the background signal (see figure 5). Figure 6 is meant to clarify the composition of the general mass distribution H(m) for our problem $pp \to pp\pi^+\pi^-\pi^0$.

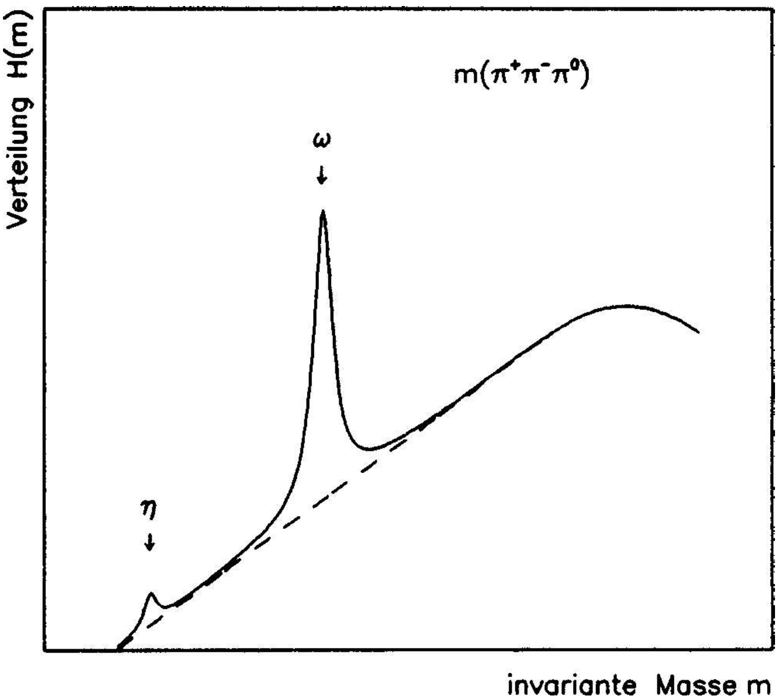

Figure 6: invariant mass

1. Let $H^{nr}(m)$ be the normalized distribution of those events in which no pions originate from a resonance. $H^{nr}(m)$ is then determined only by the longitudinal phase space.

$$H^{nr}(m) \sim \frac{dN^k_{long}}{dm} \tag{23}$$

2. If the three pions come from a ω or η resonance with masses m_i and FWHM Γ_i (i= ω, η) (see picture8) you get the resonance distribution $H_i^{res}(m)$ according to Fermis golden rule:

$$H_i^{res}(m) \sim \mid \frac{\Gamma}{(m-m_i)^2 + (\frac{\Gamma_i}{2})^2} \mid^2 \frac{dN^k_{long}}{dm} \tag{24}$$

3. If at least one, but not all pions come from a resonance one talks about reflection, which is caused by resonances of other particle systems. It might be, for example, that two pions come from resonances like the ρ or the f or from baryonic resonances as N* or Δ (see picture 9 showing an example of a Δ^{++} resonance). An explicit identification of the actual reflection is very difficult. Resonances of other particle systems (apart from $\pi^+\pi^-\pi^0$) are discussed in appendix D.

To be precise the mass distribution H(m) had to be derived from the wave function. In order to do this, not only the amplitudes but also the phases must to be known. As they are actually unknown, all that remains is the incoherent addition of the parts:

$$H(m) = c^1 H^{nr}(m) + \sum_i c_i^2 H_i^{res}(m) + \sum_j c_j^3 H_j^{ref}(m) \tag{25}$$

Here c^1, c^2, c^3 denote the intensity coefficients. When investigating a resonance, it is desirable to be able to read off the resonance distribution R(m) from the total distribution H(m). This is done by declaring all events not stemming from the resonance as background:

$$H(m) = U(m) + \alpha R(m) \frac{dN_k^{long}}{dm} \tag{26}$$

U(m) denotes the background distribution as defined above, α is the resonance intensity. In the following different trial solutions for $\frac{dN_k^{long}}{dm}$ shall be investigated:

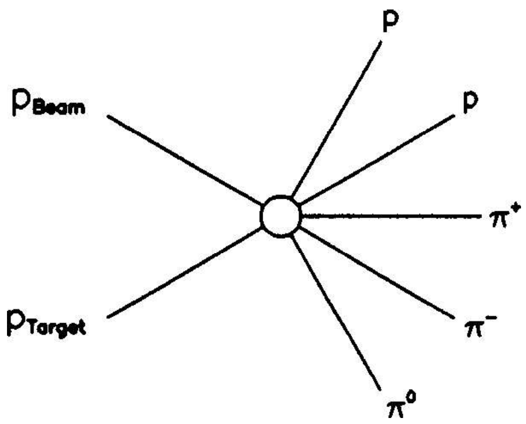

Figure 7: direct pion production

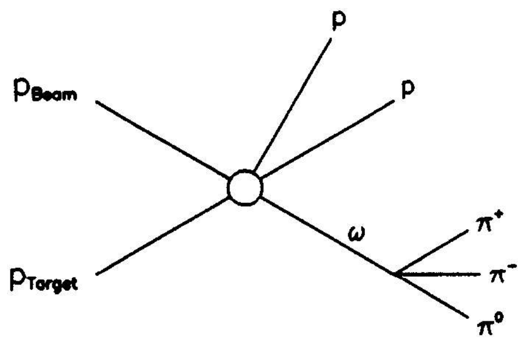

Figure 8: pion production through omega decay

When investigating small mass intervals, the simplest assumption to be made within one interval is :

$$\frac{dN_k^{long}}{dm} = const := N_U \tag{27}$$

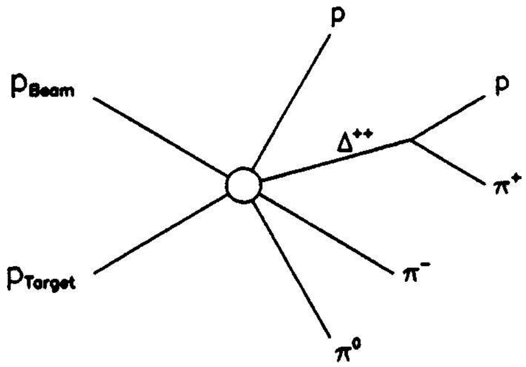

Figure 9: pion production through delta decay

This corresponds to the the first order Taylor expansion . N_U is the number of background events in the considered interval. For the effective mass spectrum of a resonance you find the following :

$$H(m) = U(m) + \alpha N_U R(m) \tag{28}$$

which is equivalent to

$$R(m) = \frac{1}{\alpha N_U}(H(m) - U(m)) \tag{29}$$

As an improved approximation it can be assumed that $\frac{dN_k^{long}}{dm}$ is proportional to the background distribution:

$$\frac{dN_k^{long}}{dm} \sim U(m) \tag{30}$$

Using this one obtains:

$$H(m) = U(m) \cdot (1 + \alpha R(m)) \tag{31}$$

yielding

$$R(m) = \frac{1}{\alpha}(\frac{H(m)}{U(m)} - 1^{)} \tag{32}$$

2.1.3 Background underneath resonance signals

There is no complete description of the background distribution, since not all wave functions of the resonances and reflections in the reaction channel are known. Therefore one has to start with an trial solution for the background distribution underneath the resonance signal. In case of the ω meson it shows that the background can be approximated by a linear distribution in a mass interval between 0.6 and 1.1 GeV (see pictures 3.2 and 4.2).

2.2 The Dalitz-Stevenson-distribution

2.2.1 Introduction

Let the three particles in the decay have equal masses $m_1 = m_2 = m_3 = m$ and the following kinetic energies in the CM system:

$$T_1(= T_{\pi^+}), T_2(= T_{\pi^-}), T_3(= T_{\pi^0})$$

The geometric foundation of the Dalitz-Stevenson plot is given in the following theorem: The sum of the distances of any given point in an equilateral triangle from the sides ,is a constant. This constant is the height of the triangle. Let $Q = T_1 + T_2 + T_3 > 0$ be the kinetic energy of the three particles. Every event is thus represented by a point in the triangle, where the distances of the point to the sides are given by $\frac{T_1}{Q}, \frac{T_2}{Q}, \frac{T_3}{Q}$ (see picture 10). Due to conservation of momentum not all points in the triangle represent physically possible events:
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1. In the non relativistic case the border line is the circle with radius $\frac{1}{3}$ being inscribed into the triangle.

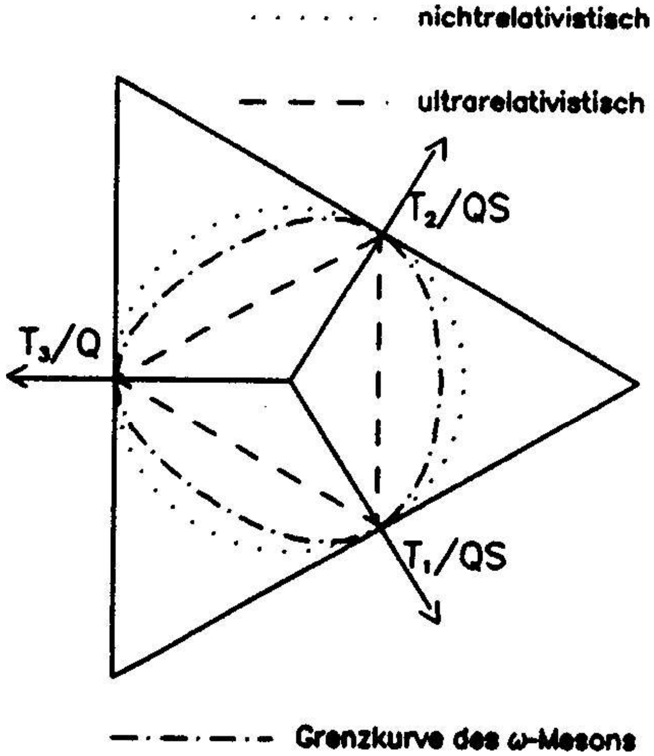

Figure 10: Dalitz-Triangle

2. In the extremely relativistic case the border line is the inscribed triangle having sides half as long as the Dalitz triangle.

3. In the general case one has a smooth curve between the two cases listed above.

If Q is given, T_3 is determined by T_2 and T_1 via the relation $\frac{T_3}{Q} = 1 - \frac{T_1+T_2}{Q}$. This triangular coordinate system can be described in spherical coordinates (r, ϕ). The origin in these coordinates is the centre of mass of the triangle and corresponds to $T_1 = T_2 = T_3$:

$$(r, \phi) \Leftrightarrow \frac{1}{Q}(T_1, T_2, T_3) = \frac{1}{3}(1 + r\cos(\phi - \frac{2\pi}{3}), 1 + r\cos(\phi + \frac{2\pi}{3}), 1 + r\cos(\phi)) \tag{33}$$

where

$$r = ((\frac{3}{Q} - 1)^2 + 3(\frac{1}{Q}(T_1 - T_2))^2)^{\frac{1}{2}} \tag{34}$$

$$\phi = \arccos(\frac{\frac{3T_1}{Q} - 1}{r} + \frac{2\pi}{3} \tag{35}$$

In /HAG63/ it is shown that the border line fulfills the following equation:

$$1 - (1 + \frac{2\sigma}{(2-\sigma)^2}r^2 - \frac{2\sigma}{(2-\sigma)^2}r^3\cos 3\phi = 0 \tag{36}$$

Here, $\sigma = \frac{Q}{M_{res}}$ and M_{res} denotes the mass of the resonance. In the non relativistic case $\sigma = 0$ in the relativistic case $\sigma = 1$. For the ω resonance, where $M_\omega = 783$ MeV one finds $\sigma = 0,47$.

From the distribution of points within the Dalitz Stevenson plot one sees the geometrical meaning of this plot:

A constant density of points in the Dalitz plot corresponds to an equal distribution of events in the Lorentz invariant phase space. Or, to put it in a different way:

A pure phase space distribution of the effective mass requires an equal distribution of points within the Dalitz triangle.

This means that in case one does not have an equal distribution, there will be one or several resonances or reflections in the considered mass interval. In the following chapter it is investigate how the distribution will look like for several values of spin and parity of the resonances.

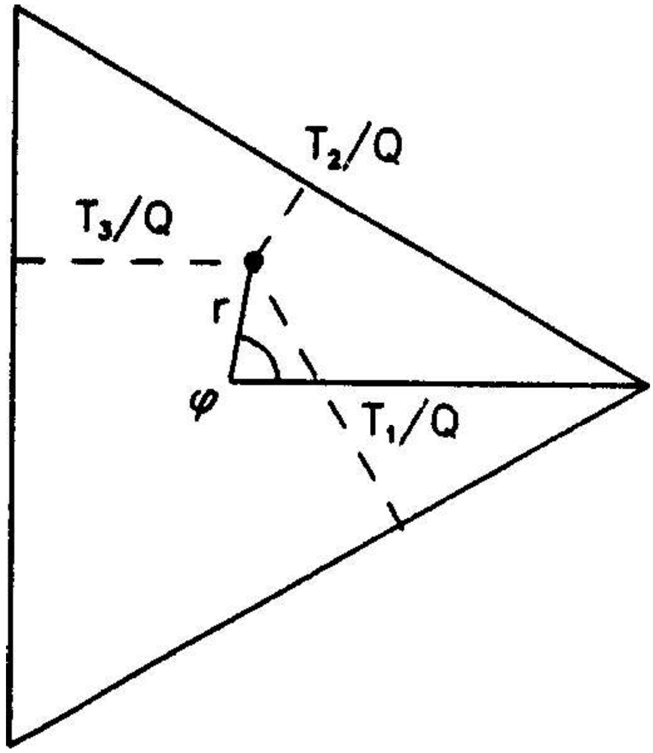

Figure 11: each event can be described in spherical coordinates

2.2.2 Dalitz-Stevenson-distribution for different J^P values of the 3π system

In general the wave function Ψ of a particle is composed from the spatial- the spin- and the isospin-wavefunction:

$$\Psi = \Psi_R \chi_J \phi_I \tag{37}$$

For the wavefunction of the ω meson the following considerations are valid :

1. Pions are bosons with spin 0. For this reason the ω is also a boson and has a symmetric overall wavefunction Ψ and a symmetric spin wavefunction χ_J.

2. Experimentally no charged states of the ω meson were found. Therefore the isospin I is zero and the isospin wavefunction is a scalar : $\Phi_\omega = \Phi_1(\Phi_2 \times \Phi_3)$. It is antisymmetric and vanishes for the (not observed) decay $\omega \rightarrow 3\pi^0$ (Fig. 12).

3. The spatial wavefunction is thus antisymmetric. The density distribution becomes symmetric with respect to the three symmetry axis of the triangle. Therefore it is possible to project all events onto one sector and thus achieve higher point density. Furthermore due to the antisymmetry,the density will be zero at those points where the pion has the maximum kinetic energy of 353 MeV.

The non constant transition matrix is a function of the four momenta q_i of the three pions:

$$M_{ae} = M_{ae}(q_1, q_2, q_3) \tag{38}$$

When deriving M_{qe} one has to pay attention to the following properties of the transition matrix:

1. Due to the antisymmetry of the spatial wavefunction Ψ_R the transition matrix $M_{ae}(q_1, q_2, q_3)$ has to be anti symmetric under exchange of two indices.

2. As the ω decays under the strong interaction parity has to be conserved. Pions have parity -1, so that the parity of P_ω of the ω meson can be seen from the following equation:

$$M_{ae}((E, \vec{p})_j) = (-1)^3 P_\omega M_{ae}((E, \vec{p})_j) \tag{39}$$

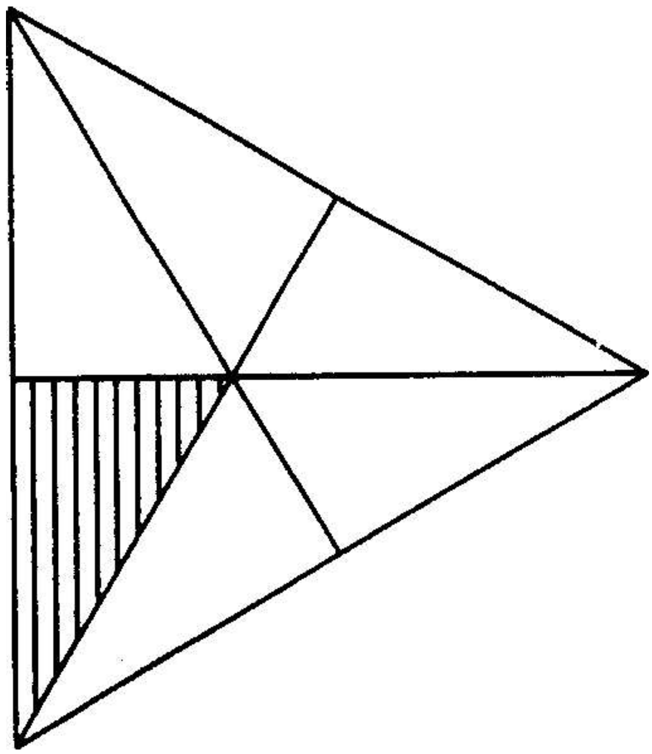

Figure 12: due to mirrorsymmetry of the spatial wave function,the events in the Dalitz triangle can be folded in one section

$(-1)^3$ is the inner parity of the three pion system. If M_{ae} is spatially symmetric, then $P_\omega =$ -1. An antisymmetric transition element yields positive parity.

From the possible spin parity combinations one gets the following distributions:

1. $J^P = 0^+$. As the spin is 0, the transition matrix is a scalar. Mesons with P=+ are also called "scalar" mesons. The simplest ansatz for a momentum independent transition matrix is :

$$M_{ae} = (\vec{p_1} \times \vec{p_2})\vec{p_3} \tag{40}$$

The ω meson cannot be scalar, as in the CMS $\vec{p_1}, \vec{p_2}, \vec{p_3}$ form a plane , furthermore parity is conserved in strong decays. This means that scalar particles cannot have three decay products.

2. $J^P = 0^-$. Particles with these quantum numbers are called pseudoscalars. The transition matrix is spatially symmetric and in the simplest case independent of momentum:

$$M_{ae}((E, \vec{p})_j) = (E_1 - E_2)(E_2 - E_3)(E_3 - E_1) \tag{41}$$

M_{ae} is spatially symmetric when interchanging two indices j. Thus all areas in the Dalitz triangle are depopulated where $E_i = E_j$ for $i \neq j$ (especially in the centre). In spherical coordinates (r, ϕ) one gets the Dalitz-Stevenson distribution

$$\mid M_{ae}(r, \phi) \mid^2 \sim r^6 \sin^2(e\phi) \tag{42}$$

3. $J^P = 1^+$. The transition matrix is a vector when J=1. Therefore particles with the given $J^P = 1^+$ are called pseudo vectors. For the transition matrix one makes the following ansatz:

$$(M_{ae})_{(ae=-1,0,1)} = (\vec{p_1} - \vec{p_2})E_3 + (\vec{p_2} - \vec{p_3})E_1 + (\vec{p_3} - \vec{p_1})E_2 \tag{43}$$

Using $\sum E_i = M_\omega$ and $\sum \vec{p_i} = 0$ one gets:

$$(M_{ae})_{ae=-1,0,1} = \vec{p_1}(M_\omega - 3E_2) - \vec{p_2}(M_\omega - 3E_1) \tag{44}$$

(M_{ae}) vanishes everywhere, where $q_1 = q_2$. Algebraic manipulation delivers:

$$\mid M_{ae}(r, \phi) \mid^2 \sim r^2(1 - \frac{\sigma}{2} - r\cos(3\phi)) \tag{45}$$

4. $J^P = 1^-$. The transition matrix of these vector particles has to have the following matrix element :

$$(M_{ae}) = (\vec{p_1} - \vec{p_2}) \times \vec{p_3} + (\vec{p_2} - \vec{p_3}) \times \vec{p_1} + (\vec{p_3} - \vec{p_1}) \times \vec{p_2} \tag{46}$$

   Using $\sum \vec{p_1} = 0$:

$$(M_{ae}) = 6\vec{p_2} \times \vec{p_1} \tag{47}$$

   This means that the transition matrix vanishes for two mesons having parallel momenta. This is especially important at the border of the Dalitz diagram. For the distribution one obtains:

$$\mid M_{ae}(r, \phi) \mid^2 = 1 - r^2(1 + \frac{2\sigma}{(2-\sigma)^2} - \frac{2\sigma}{(2-\sigma)^2} r^3 \cos(3\phi) \tag{48}$$

In general mesonic resonances have the following properties:

1. If the parity is $(-1)^J$ the distribution vanishes along the border line, as these are the points where particles are emitted with parallel momenta.
2. The density is zero in the centre of the triangle if the parity is positive and the spin J ¡ 3.
3. For even spin quantum numbers the density in the centre vanishes.

Figure 13 beneath shows the density distribution for $J^P = 0^-, 1^+, 1^-$.

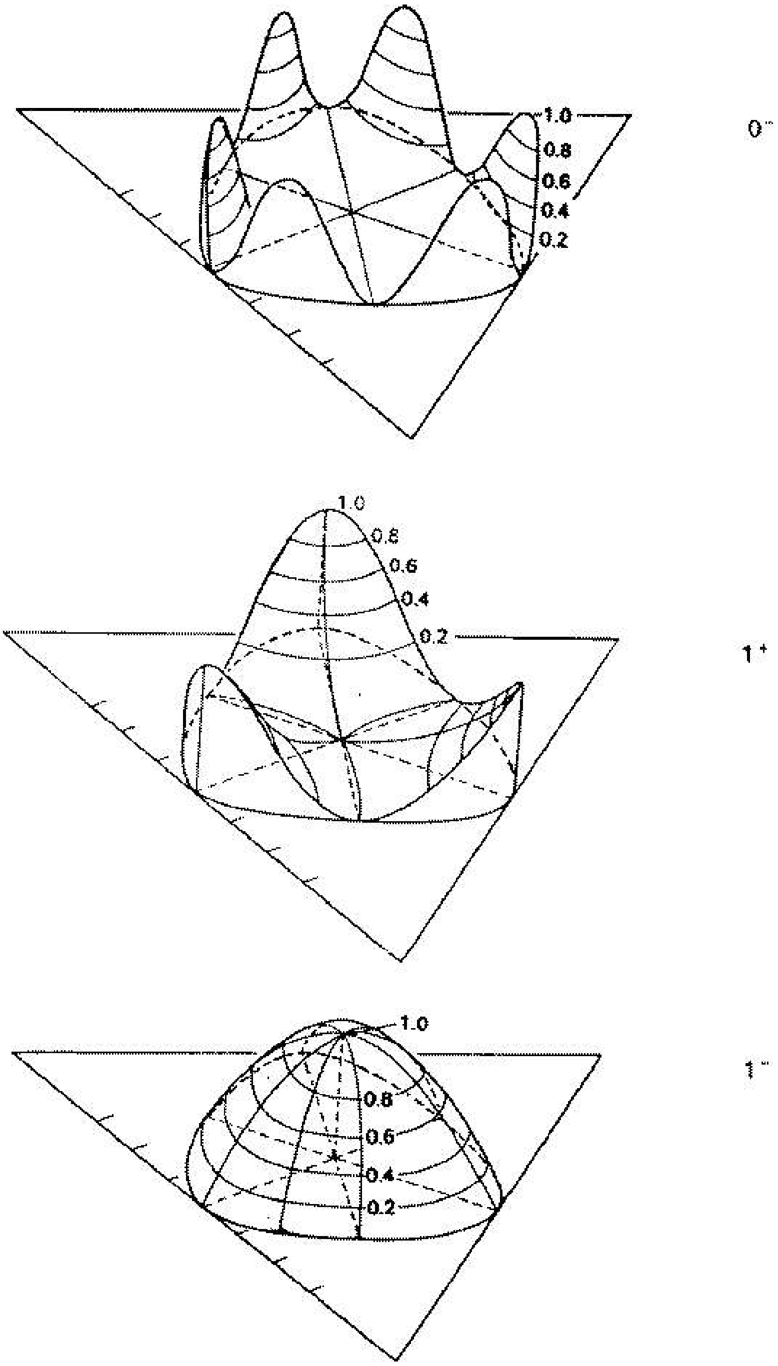

Figure 13: theoretical density distribution for 0-,1+,1-