Properties of elementary particles

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1 Introduction

The experiment "Properties of elementary particles" (E212), which has been running at our University since 1977, introduces students to basic techniques and concepts of experimental particle physics. Examples are cross section measurements, determination of a particle's spin and parity, kinematic reconstruction of particle reactions, etc. Photos from a bubble chamber showing the paths of particles within the chamber will be utilized to reconstruct particle reactions inside the chamber. In order to allow a full three-dimenisonal reconstruction of the particle's tracks, photos from different angles are taken for each particle reaction. Students will use large-area projection tables for the reconstruction. For an introdution to bubble chambers see the book by D. Perkins "Introduction to high Energy Physics".

In the second part of the experiment students will analyze data recorded in reactions of the type[1] $p\,p \rightarrow p p \pi^+ \pi^- \pi^0$. In such a reaction, the three pions might come from the decay of a short-lived particle – a so-called resonance. You will determine the properties of the ω resonance in the experiments.

1.1 Conserved Quantities and relativistic kinematics

All (known) particle reaction conserve energy and momentum, angular momentum, electric charge, baryonnumber and leptonnumber. These conservation laws are very useful for the kinematic reconstruction of reactions. Strong interactions also conserve strong isospin and the strangeness quantum number. For a discussion of these quantum number please consult any textbook on particle physics such as Griffith or Perkins.

According to Einstein's equation $E = mc^2$ energy can be converted to mass and vice versa. Particle physicists virtually watch nature producing and annihilating matter. The energies converted in these reactions range from 1 to 1000 GeV typically. Expressed in Joule this is not much, 1 TeV is equivalent to about 1.6 10^{-10} Joule, but it is an enourmous energy if concentrated on a single particle. The typical velocity of particles in our experiments comes close to the speed of light. In every reference frame massless particles move with the speed of light. But no particle can be faster than the speed of light. The energy of a particle is $E = \frac{mc^2}{\sqrt{1-v^2/c^2}}$, and its momentum is $\vec{p} = \frac{m\vec{v}}{\sqrt{1-v^2/c^2}}$. m indicates the rest mass of a particle here. We define $\beta = v/c$ and $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$. For a given energy you find for β and γ:

$$\beta = \frac{|\vec{p}|}{E} \text{ und } \gamma = \frac{E}{m}. \tag{1}$$

The transformation of coordinates from one reference frame moving uniformly (inertial frame) to another one is described by Lorentz transformations. If we combine space and time components of an event into a four-vector $\boldsymbol{x}$ with components

$$\boldsymbol{x} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

, then the Lorentz transformation can be displayed in matrix notation. A transformation in a reference frame moving with a relative velocity v in the direction of the x-axis has the form:

$$\Lambda = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

Under a transition from one inertial frame to another, a four-vector is transformed by a Lorentz transformation. The most frequently used four-vector in high energy physics is the energy-momentum four-vector or just four-momentum $\boldsymbol{p}$, with

$$\boldsymbol{p} = \begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

In collisions or in the decay of particles the momentum four-vector is conserved. That is to say, the sum of all four-vectors of the particles in the initial state is equal to the sum of all four-vectors of the particles in

[1] The p stands of course for proton.

the final state. For calculations in relativistic kinematics it is often more convenient to use methods, where the scalar products of four-vectors, which are Lorentz invariant, are calculated, rather than to apply Lorentz transformations explicitly[2]. For example, the scalar product of a momentum four-vector with itself yields the square of the particle's rest mass: $\boldsymbol{p}^2 = (E/c)^2 - \vec{p}^{\,2} = (mc)^2$. According to the natural units system the constant c kann be omitted, therefore: $E^2 - \vec{p}^{\,2} = m^2$.

1.2 Scattering

The most important concepts in scattering theory that we are going to use in this lab course are matrix element, cross section, luminosity and integrated luminosity. A good introduction of the matrix element or amplitude is given in the book by C. Berger, section 2.1. The matrix element gives information about the differential cross section. The most common unit of cross section is barn (1 barn = 10^{-28} m^2). The relation between the average reaction rate and the cross section is

$$\dot{N} = \sigma \cdot \mathcal{L} \tag{3}$$

, where $\dot{N}$ is the reaction rate (scatterings per second), σ the cross section and $\mathcal{L}$ the luminosity. The luminosity only depends on the settings of the scattering experiment (given in units of sec^{-1} cm^{-2}), while the cross section describes the physics of the reaction. The number of interactions is proportional to the integrated luminosity, i.e. to the time integral of the luminosity. Because of that the integrated luminosity often is given in inverse barn.

1.3 Tips for data analysis

Average π^0 multiplicity In order to calculate the average number of π^0 per event from the number of converted photons, one starts with the absorption length λ_{abs} of photons in liquid hydrogen, which is 1146 cm. The probability for that a conversion of a photon in the field of a nucleus into an electron-positron pair occurs at a distance d away from the creation point of the photon follows an exponential curve: $P(d) = \frac{1}{\lambda_{abs}} e^{-d/\lambda_{abs}}$. If the observable maximum path length of a photon is L_c, the probability that a photon created at point x_0 within the chamber converts within the chamber's sensitive volume is given by

$$P_{conv}(x_0) = \int_{x_0}^{L_c} P(L_c - x)\,\mathrm{d}x = 1 - e^{-(L_c - x_0)/\lambda_{abs}}$$

Since not all photons are created at the same position, we need to fold the above equation with the position of the hard reaction which we assume to be flat between zero and L_c.

$$P_{conv}^{av.} = \frac{1}{L_c} \int_0^{L_c} P_{conv}(x)\,\mathrm{d}x = 1 - \lambda_{abs}/L_c + (\lambda_{abs}/L_c) e^{-L_c/\lambda_{abs}}$$

To calculate the number of π^0 from the number of photons, please note that there are two photons per π^0 decay.

2 Bubble Chamber

A metal container is filled with a transparent liquid. The liquid's temperature is just below the boiling point. A fast expansion with the help of a piston leads to an overheating of the chamber liquid (metastable state). Ionizing particles and excited atoms but also asperities of the container surface cause the forming of bubble "seeds", which expand to bubbles in the overheated liquid. As soon as they have reached a visible size (0.01-0.1 mm), they are photographed by a flash light through the glass window. In order to reconstruct an event spatially, three to four cameras in different positions are used. Figure 1 shows the basic structure of a bubble chamber.

Figure (2) (German script of Gerhild Seul) depicts how a bubble chamber runs in time. The expansion of the liquid is started by a signal S from the accelerator, which ensures, that after 5 to 10 ms at the arrival time A of the beam particles the pressure has already dropped low enough to make the chamber liquid metastable and the bubble chamber is ready to detect particles. At a time B, when the bubbles have been expanding for about 10 ms, flash lights activate the cameras. After that, the recompression of the liquid is started in order to destroy the bubbles. All in all a complete cycle takes 100 ms of time. Due to the short time interval, in which

[2] The Lorentz transformation of a Lorentz invariant is a trivial operation, as its value does not change under this transformation.

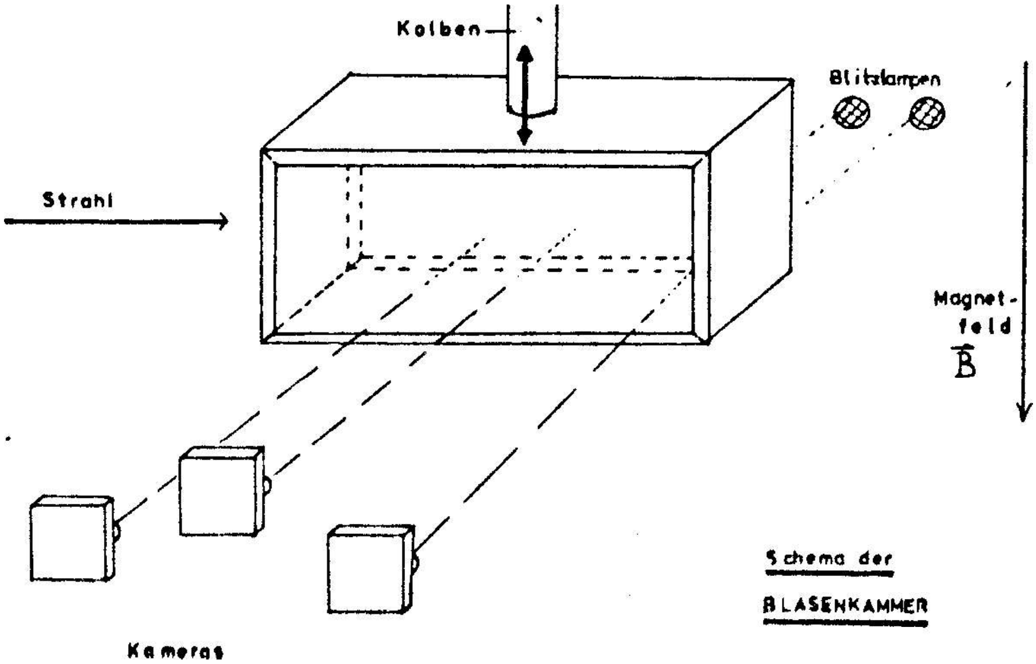

Figure 1: Basic structure of a bubble chamber

the chamber is sensitive, and the fast recompression the heat, which is supplied by ionization, is reduced by the re-liquification of the bubbles. In order to differentiate between the tracks of differently charged particles and to measure the particles' momenta from the curvature of the tracks a magnetic field can be imposed around the bubble chamber.

3 Evaluation of Bubble Chamber Photographs

The first goal for an evaluation of the bubble chamber photos is the exact description of each individual reaction together with the assignment of a mass to each track. This is done in three steps:
a) Geometrical Reconstruction:
In order to analyse a reaction in an accurate way, each event has to be spatially reconstructed. For this purpose several points are measured on each track. From their positions the exact track development and the momentum vector at the interaction point can be determined by a computer programme, which takes into consideration the magnification and the curvature of the track. In the case of relatively large measurement errors a repeated measurement is necessary.
b)Kinematical Analysis:
On the basis of energy-, momentum- and quantum number conservation laws another computer programme sets up different reaction hypotheses, that means it claims which track belongs to which particle.
c) Kinematical Decision:
In general, one solution out of many possible interpretations is unambiguously chosen on account of the bubble density and the ionization.
In order to avoid additional time and efford concerning technics, only events in which all tracks lie roughly in the same plane (inclination angle $\leq 18^o$) are chosen for the lab course. Thus the spatial reconstruction is superfluous here and it is sufficient to work with only one projection.
In this case the measurement accuracy for the momentum of the particles is about 10 percent. The picture scale on the projection table only needs to be determined once. In the following it will be explained how to determine the magnification of the bubble chamber pictures and how information about the sign of the charge and momentum vectors of the particles can be obtained from the tracks.

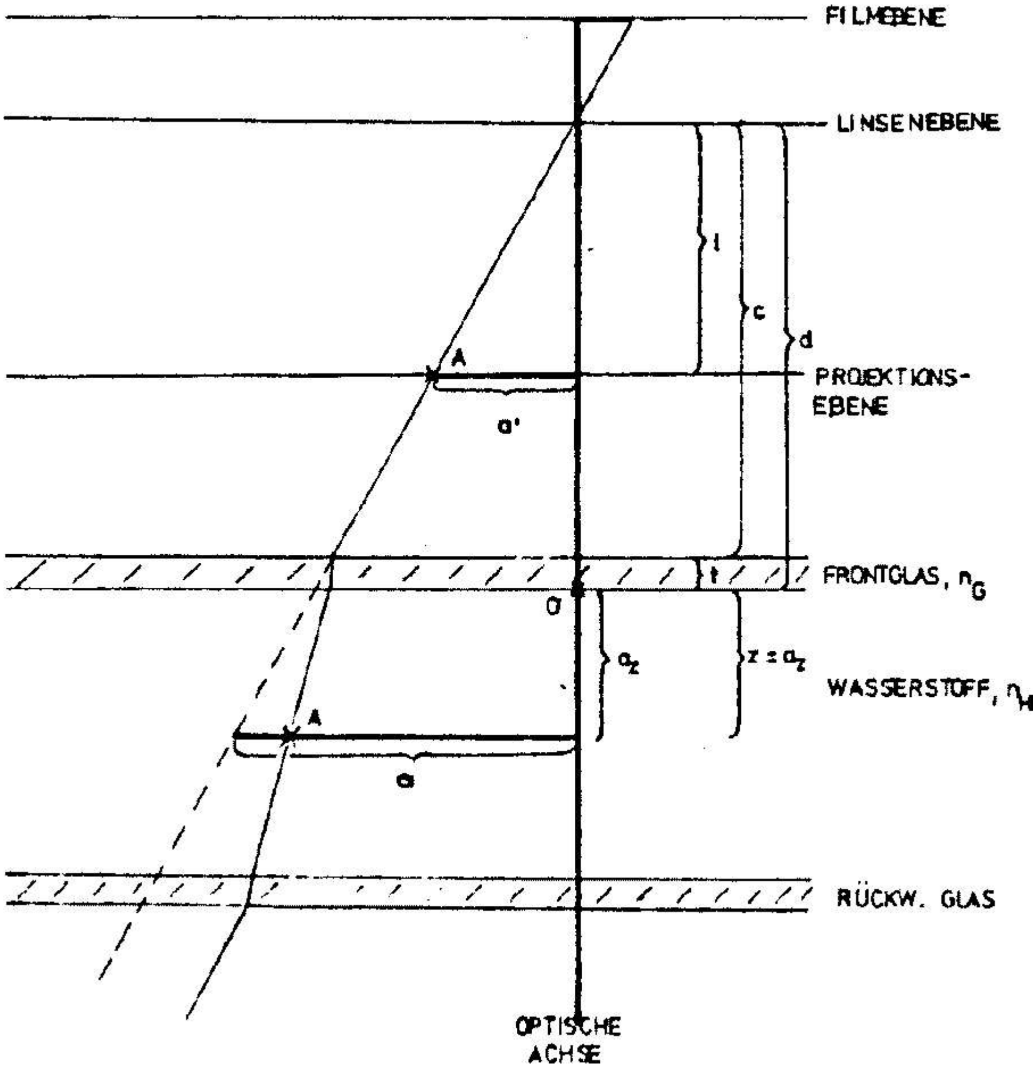

Figure 2: Cut view of a bubble chamber

3.1 Magnification

Figure 2 shows a scematic cut view of a bubble chamber. The incoming particle beam traverses the chamber perpendicular to the optical axis approximately running through the point A in a depth a_z. The scale, with which a point of the beam plane is projected onto the projection plane, depends on the depth a_z. As the beam traverses the chamber in a well-defined, approximately constant depth a_z, it is normally sufficient to determine the mapping scale just once, and to apply the same scale to all pictures.
In this section you will get familiar with a method, which determines the magnification in a first approximation (neglecting the refraction on the face-plate and in the hydrogen).
Using the ray law yields an expression for the magnification V_A at the point A:

$$V_A = \frac{a}{a'} = \frac{d + a_z n_H}{l} \tag{4}$$

$$\text{with} \quad d = c + t n_G \tag{5}$$

where n_G and n_H are the refraction indices of the glass plate and the hydrogen.
d and l only depend on the chamber geometry and the size of the projected picture. d and l can be calculated easily, if you measure the position of well-known reference signs[3] in the projection plane: The scheme is depicted

[3] These are crosses (+,x) drawn on the front and the back glass plate of the chamber. Their position is exactly known and they are needed to determine a fixed frame of reference.

in figure 3:

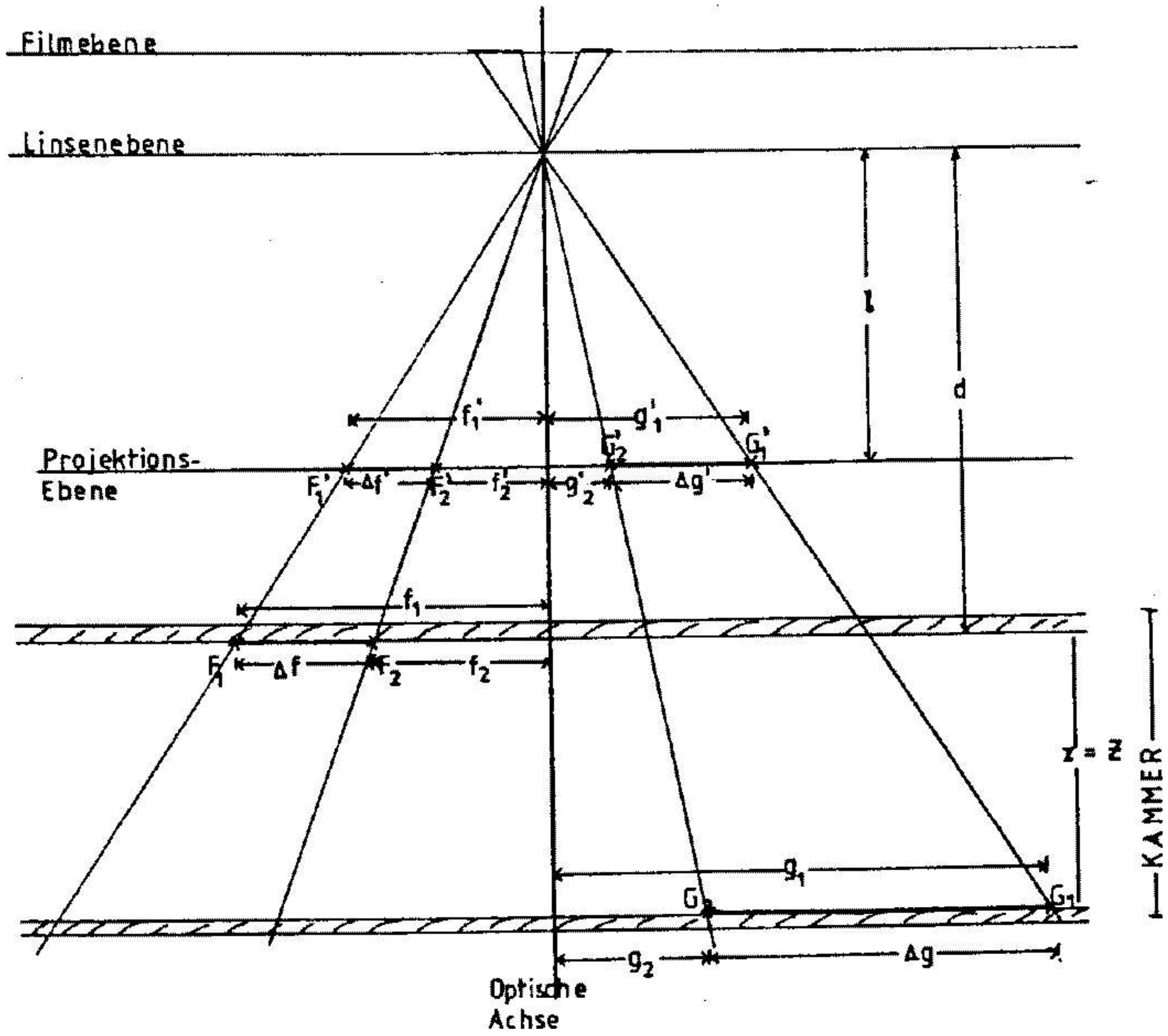

Figure 3: Cut view of the chamber

- F_1, F_2 Reference signs on the front glass (z = 0),
- f_1, f_2 their x-coordinates with respect to the optical axis.
- F_1', F_2' Pictures of the reference signs in the projection plane,
- f_1', f_2' their x-coordinates with respect to the optical axis.
- G_1, G_2 Reference signs on the back glass (z = Z),
- g_1, g_2 their x-coordinates with respect to the optical axis.
- G_1', G_2' Pictures of the reference signs in the projection plane,
- g_1', g_2' their x-coordinates with respect to the optical axis.

By using the ray law we get:

$$\frac{f_1}{f_1'} = \frac{d}{l} \quad \Rightarrow \quad f_1 = \frac{d}{l} f_1'$$
$$\frac{f_2}{f_2'} = \frac{d}{l} \quad \Rightarrow \quad f_2 = \frac{d}{l} f_2'$$

It follows:

$$f_1 - f_2 = \frac{d}{l}(f_1' - f_2')$$

With

$$\begin{aligned} f_1 - f_2 &= \Delta f \\ f_1' - f_2' &= \Delta f' \end{aligned}$$

we obtain the magnification V_F for the front reference signs:

$$\frac{\Delta f}{\Delta f'} = \frac{d}{l} = V_F \tag{6}$$

In a similar way we receive the magnification V_G for the reference signs on the back glass plate:

$$\frac{\Delta g}{\Delta g'} = \frac{d+z}{l} = V_G \tag{7}$$

If we eliminate d from equation 6 and 7, this yields:

$$l = \frac{Z}{V_G - V_F}$$

Therefore with equation 6 we get:

$$d = V_F \frac{Z}{V_G - V_F}$$

Finally, we receive according to 5 the magnification of a track at the point A:

$$V_A = \frac{V_F + (V_G - V_F) a_Z n_H}{Z} \tag{8}$$

$\Delta f'$ and $\Delta g'$ are directly measured on the projection, Δf and Δg are calculated from the coordinates of the reference signs.
Under the assumption that the event took place in the middle of the chamber, we may insert

$$\begin{aligned} a_Z \cdot n_H &= 0.5 \cdot Z \\ \text{into 8:} \quad V_A &= \frac{V_F \cdot Z + (V_G - V_F) 0.5 \cdot Z}{Z} = 0.5 \cdot V_F + 0.5 \cdot V_G \end{aligned}$$

$$V_A = \frac{V_F + V_G}{2} \tag{9}$$

3.2 Charge Sign

The fastest way to determine the charge sign is to observe the so called "δ-electrons". During the ionisation process electrons in the shell of the liquid atoms are knocked out along the track of the traversing particle. Sometimes, they receive so much energy, that they leave tracks by themselves. The orientation of the electron spirals tells the deflection direction of all negatively charged particles (Fig. (10), (11), German script of Gerhild Seul).

3.3 Momentum

Measuring the momenta of the involved particles is essential for an event analysis. In general, this is done by taking into account the curvature of the particle in the magnetic field and in the case of short tracks the energy range in the chamber liquid.

3.3.1 Momentum Determination from the Curvature Radius

A particle in a magnetic field is deflected under the influence of the Lorentz force:

$$\vec{K} = e \cdot \vec{v} \times \vec{B} \tag{10}$$

Where e=1. 6 10^{-19} C, v being the velocity of the particle and B denoting the magnetic field. As the particles enter the bubble chamber perpendicularly to the magnetic field, the track of a charged particle approximately follows a circle. Deviations from this circular trajectory appear:

- when the particle doesn't move in a plane perpendicular to the magnetic field,
- due to the energy loss through ionization,
- because of multiple scattering.

That means if the magnetic field B is known, the momentum p of a particle can be calculated from the curvature radius r by equalizing the centrifugal and the Lorentz force. ($\vec{v} \perp \vec{B}$)

$$\frac{mv^2}{r} = e \cdot v \cdot B \tag{11}$$

$$p = e \cdot B \cdot r \tag{12}$$

where $m = m_o\gamma$ is the relativistic mass of the particle.
In the units $B[kG]$, $p[MeV/c]$, $r[cm]$, $e[C]$ this results in:

$$p = 0.3 \cdot B \cdot r[kG \cdot cm] \tag{13}$$

We can determine the radius of curvature of a particle track with the help of radius stencils (These are foils, on which circular arcs are displayed together with the corresponding radii.) Another way to determine the radius is the "Sagitta-method", which is described in the appendix 1. In both methods, the accuracy on the measurement is inversely proportional to the radius.
From the radius measured in the projection plane r', the actual radius can be calculated according to equation 5:

$$r = V \cdot r' \tag{14}$$

This yields the following eqaution for the momentum p:

$$p = 0.3 \cdot B \cdot V \cdot r' \tag{15}$$

Now, it is quite simple to determine the momentum components in a coordinate system of the projection plane, which is given for example by reference marks or the beam direction:

$$\vec{p} = p \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \tag{16}$$

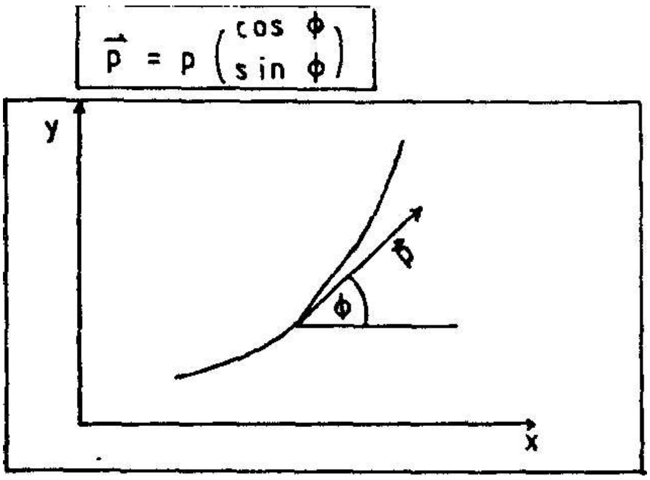

Figure 4: Momentum components

3.3.2 Determination of the Momentum from the Range of the Particle

The radius of curvature can not always be measured accurately enough, this is the case when

- a particle is too fast (p $\geq$ 100 GeV)
- the track is too short (the particle stops in the chamber).

In the first case the momentum is not measurable accurately. But in the second case the momentum can additionally be calculated from the energy range R of the particle.
A traversing particle constantly suffers energy losses in matter. The ionization per cm increases, the velocity decreases. R can be calculated from the energy loss relation:

$$R = -\int \frac{dE}{dE/dx} \tag{17}$$

The energy loss dE/dx for highly relativistic particles is approximately constant, it follows:

$$R = const \cdot E$$

For nonrelativistic particles, the energy loss is inversely proportional to the energy of the particle i. e. to the velocity squared, that means:

$$R = const \cdot E^2$$

The momentum corresponding to a certain range R can be determined by reading it from the energy range-momentum diagram (See Appendix A5, figure 14 and figure (4b),German script of Gerhild Seul).
Usually, this method is more accurate than measuring the momentum from the radius of curvature.

3.4 Velocity Measurement from the Bubble Density

If the momentum of a particle is known, its mass and energy can be determined from its velocity. The information about the velocity can be gained by determining the bubble density. It is known that:
The number of bubbles per track length $\sim \frac{1}{\beta^2}$
This relationship between momentum and bubble density is shown in fig. (5) (German script of Gerhild Seul).
A discrimination between two particles with the same momentum is only possible in the region between 0.5 GeV and 1.5 GeV. Below 0.5 GeV, velocities are very low, so that the bubble density increases too much. Above 1.5 Gev, particles become minimum ionizing particles at a β value of about $1/\beta^2 = 10/9$ and are indistinguishable as a consequence[4].

[4]The minimum is actually reached for $\beta\gamma \approx 3$.

By comparing the bubble density of a particle to the minimal density observed in all tracks $1/\beta^2$ can be determined. When also the momentum is known from the curvature we have a relation:

$$\frac{1}{\beta^2} = \frac{E^2}{p^2} = \frac{m^2 + p^2}{p^2} = \frac{m^2}{p^2} + 1$$

the mass can then be calculated as follows:

$$m = p\sqrt{\frac{1}{\beta^2} - 1} \tag{18}$$

4 Determination of the Neutrino Momentum in a $\pi\mu e$ -Decay

The $\pi\mu e$ decay has a characteristic signature in the bubble chamber: The slow muon has a very short range and one of its decay products is an electron or a positron with a low momentum. Since for the π^- it is much more likely to undergo a scattering process with charge exchange $\pi^- \to n\pi^0$, any $\pi\mu e$ decay that we observe is probably a π^+ decay, which takes place in the following way:

$$\begin{array}{ccl} \pi^+ & \rightarrow & \mu^+\nu_\mu \\ & & \downarrow \\ & & e^+\nu_e\overline{\nu_\mu} \end{array}$$

It is your task to derive the relation for the center-of-mass momentum of the neutrinos in a π-decay and to check your result on an example event. For this purpose an event was chosen, in which a pion decays at rest. As a consequence the CMS and the lab system are the same (compare fig. (22), German script of Gerhild Seul). The tracks of the pion and the muon are approximately located ($\lambda \approx 13^o$) in a plane parallel to the chamber windows.

4.1 Derivation of the Relation for the Neutrino Momentum in the π-CM-System

In its own Center-of-mass frame the π^+ has no momentum: $\vec{P}_\pi = \vec{0}$
Due to the momentum conservation law the decay products have opposite and equal momenta:

$$\vec{p}_\nu = -\vec{p}_\mu =: \vec{q}$$

The energy conservation law yields:

$$\begin{aligned} m_\pi &= \sqrt{m_\mu^2 + q^2} + q \\ (m_\pi - q)^2 &= m_\mu^2 + q^2 \\ m_\pi^2 + q^2 - 2m_\pi q &= m_\mu^2 + q^2 \\ q &= \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \end{aligned}$$

With the masses:

$$\begin{aligned} m_\pi &= 139.6\ MeV \\ m_\mu &= 105.7\ MeV \end{aligned}$$

we find for q :

$$q = 29.8\ MeV$$

4.2 Determination of the Momenta in a $\pi\mu e$ -Decay

The problem in determining the pion momentum is that the track has a spiral shape. In addition there is a small kink about 2.5 cm in front of the decaying point. With the help of a stencil the radius of curvature r' of the pion track at the kink position can be measured. Now the π momentum can be calculated using formula 13:

$$p = 0.3 \cdot B \cdot V \cdot r'$$

with

$$\begin{aligned} V &= 0.833 \pm 0.003 \\ B &= 17.4kG \\ r' &= (10.5 \pm 1)cm \\ \rightarrow p &= 45.7 \pm 4.4 MeV/c \\ \text{Error:} \quad \Delta p &= 0.3B\sqrt{(\Delta V r')^2 + (\Delta r' V)^2} \end{aligned}$$

The correlation between the range and the momentum reveals that a pion with this momentum flies about 2.5 cm. This value corresponds to the distance between the kink and the decay point: The pion decays at rest. Thus the lab and the CM system are the same. The momentum of the muon P_μ is expected to be about 30 MeV. Because of the short track length of the μ its momentum can not be figured out from its radius of curvature, but only from its range. That means the flight length l' of the muon needs to be measured:

$$l' = (0.7 \pm 0.1)cm$$

According to eq. 22 in the Appendix 2 the true length can be calculated:

$$\begin{aligned} l &= V \cdot l' \\ l &= (0.6 \pm 0.1)cm \\ \text{Error:} \quad \Delta l &= \sqrt{(\Delta V l')^2 + (V \Delta l')^2} \end{aligned}$$

The corresponding momentum can be read from the range-momentum diagram (fig. (4), German script of Gerhild Seul):

$$P_\mu \approx 26 MeV$$

The result agrees with the expected value within the large error ($\approx$ 15 percent) for the determination of l.

5 Identification of an Interaction pp $\rightarrow$ p $K^+\Lambda$

$\Lambda \rightarrow p\pi^-$ at 24 GeV/c Incident Momentum

This reaction was chosen here, because it is a example for an associated production, where the number of potentially appearing particles is restricted due to strangeness- and baryon-number conservation laws. The spatial reconstruction can be omitted as in this special event the V^0 particle and its decay products lie approximately in a plane parallel to the chamber windows. Furthermore an event with a very slow Λ was chosen (momentum < 500 MeV/c), which leads to the fact that the momenta can be measured to the accuracy of 6 percent via the curvature radii of the tracks 1 and 2 (see fig. 5). The event is shown in fig. (23) (German script of Gerhild Seul).
Your task is to investigate, if the V^0 particle is associated to the primary vertex and if it is a K^0 or a Λ.

5.1 Testing of Association

The association can be verified with the help of the momentum conservation law. The curvature orientation of the tracks 1 and 2 determines the charge sign.

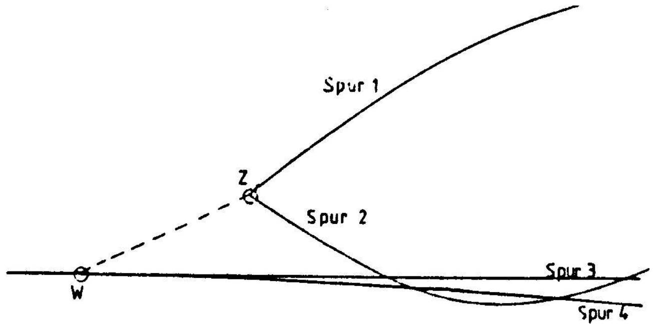

Figure 5: Testing of association

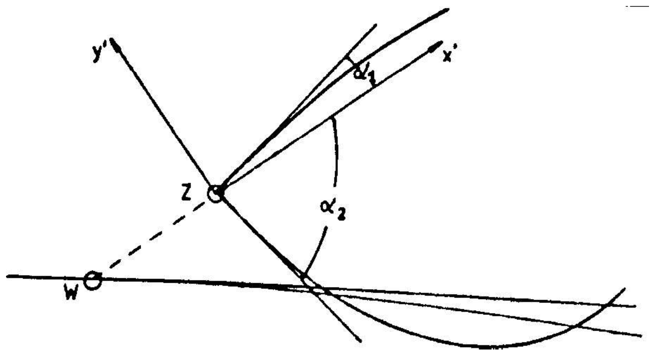

Figure 6: Interaction points

Track 1:positive

Track 2:negative

The curvature radius r' of each track is measured with the stencils. This determines the magnitude of the track momentum:

$$p = 0.3 \cdot B \cdot V \cdot r'$$

$$\text{with } V = 0.833 \pm 0.003$$
$$B = 17.4kG$$

$$r'_1 = (85 \pm 5)cm \longrightarrow p_1 = (370 \pm 22)MeV$$
$$r'_2 = (17.5 \pm 0.5)cm \longrightarrow p_2 = (76 \pm 3)MeV$$

$$\text{Error: } \Delta p = 0.3B\sqrt{(\Delta V r')^2 + (\Delta r' V)^2}$$

The direction of the line connecting the two interaction points W and Z is chosen to be the x'-axis for the V^o particle:

In order to determine the momentum vectors

$$\vec{p} = \begin{pmatrix} p_{x'} \\ p_{y'} \end{pmatrix} = \begin{pmatrix} p\cos\alpha \\ p\sin\alpha \end{pmatrix}$$

, the angles are measured, which are enclosed by the tangents on the tracks at the point Z and the x'-axis:

$$
\begin{aligned}
\alpha_1 &= 11^o \pm 0.2^o \\
\alpha_2 &= 78^o \pm 0.2^o
\end{aligned}
$$

$$
\begin{aligned}
\vec{p}_1 &= \begin{pmatrix} (363 \pm 22)MeV \\ (71 \pm 5)MeV \end{pmatrix} \\
\vec{p}_2 &= \begin{pmatrix} (16 \pm 1)MeV \\ (-74 \pm 3)MeV \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\text{Error:} \quad \cos(\alpha \pm 0.2) &= \cos\alpha \mp 0.2 \sin\alpha \\
\sin(\alpha \pm 0.2) &= \sin\alpha \pm 0.2 \cos\alpha
\end{aligned}
$$

$$
\begin{aligned}
\Delta p_{x'} &= \sqrt{\Delta p^2 \cos^2\alpha + (0.2 \sin\alpha)^2 p^2} \\
\Delta p_{y'} &= \sqrt{\Delta p^2 \sin^2\alpha + (0.2 \cos\alpha)^2 p^2}
\end{aligned}
$$

In order to check the association the sum of the momenta $\vec{p_0}'$ is formed:

$$
\vec{p}_1 + \vec{p}_2 = \begin{matrix} (379 \pm 22)MeV \\ (-3 \pm 6)MeV \end{matrix} = \vec{p'}_o
$$

$$
\begin{aligned}
\text{Error:} \quad \Delta(p'_{ox'}) &= \sqrt{(\Delta p_{1x'})^2 + (\Delta p_{2x'})^2} \\
\Delta(p'_{oy'}) &= \sqrt{(\Delta p_{1y'})^2 + (\Delta p_{2y'})^2}
\end{aligned}
$$

Since the y'-component is almost zero, the sum vector $\vec{p_0}'$ lies on the x' axis: Z is associated to W. The momentum magnitude p_0 of the V^0 particle is:

$$
\begin{aligned}
p_o &= |\vec{p}_o| = \sqrt{(p_{ox'})^2 + (p_{oy'})^2} \\
p_o &= (379 \pm 22)MeV \\
\text{Error:} \quad \Delta p_o &= \frac{1}{p_o}\sqrt{(\Delta p'_{ox'})^2 (p'_{ox'})^2 + (\Delta p'_{oy'})^2 (p'_{oy'})^2}
\end{aligned}
$$

5.2 Identification of the V^o Particle

For a kinematical analysis the masses of the occurring particles must be known. Therefore you should at first try to assign masses to the different tracks by taking into account quantum numbers and conservation laws. The number of these "mass hypotheses" is limited by the fact that we are dealing with the decay of a V^o particle rather than any reactions (because of charge conservation). Also, a process of pair production,

$$
\gamma p \rightarrow e^+ e^- p
$$

in which the recoil proton is not visible in the bubble chamber, can be excluded since the resulting angle is not 0^o but 89^o. In addition, the bubble density is very high, while for a electron positron pair it should be minimal.
Referring to the momentum analysis of the previous subsection it can be assumed that no neutral particles have been produced at the decay point Z.

In conclusion the following two-particle decays are possible:

$$\begin{array}{rcl}
{K^0}_s & \rightarrow & \pi^+\pi^- \\
{K^0}_s & \rightarrow & \mu^+\mu^- \\
{K^0}_L & \rightarrow & \begin{array}{c} e^+\mu^- \\ e^-\mu^+ \end{array} \\
\Lambda & \rightarrow & p\pi^- \\
\Sigma^0 & \rightarrow & \Lambda\gamma \\
 & & \downarrow \\
 & & p\pi^- \\
\Xi^0 & \rightarrow & p\pi^- \\
 & & \Lambda\pi^0 \\
 & & \downarrow \\
 & & p\pi^-
\end{array}$$

As it is already mentioned above, no electrons occur in this V^0, because they are minimum ionizing and the tracks in contrast reflect a high bubble density and end up in the chamber. Due to the range-momentum relation, see fig. (4b) (German script of Gerhild Seul) it can be deduced that the positive track can only be formed by a proton and the negative track can only result from a pion. Thus, only the following decay processes remain:

$$\begin{array}{rcl}
\Xi^0 & \rightarrow & p\pi^- \\
\Lambda & \rightarrow & p\pi^-
\end{array}$$

The mass m(x) of the V^0 particle is given by the following equation:

$$m(X)^2 = (E(p) + E(\pi^-))^2 - (\vec{p}_1 + \vec{p}_2)^2$$

With:

$$E = \sqrt{m^2 + p^2}$$

and with the masses:

$$\begin{array}{rcl}
m(p) & = & 938.3MeV \\
m(\pi^-) & = & 139.6MeV
\end{array}$$

we obtain:

$$\begin{array}{rcl}
m(X) & = & \sqrt{(\sqrt{m(p)^2 + {p_1}^2} + \sqrt{m(p)^2 + {p_2}^2})^2 - {p_o}^2} \\
m(X) & = & (1104 \pm 12)MeV
\end{array}$$

Error:

$$\Delta m(X) = \frac{1}{m(X)}\sqrt{\left[\left(\frac{p_1 \cdot \Delta p_1}{E(p)}\right)^2 + \left(\frac{p_2 \cdot \Delta p_2}{E(\pi)}\right)^2\right] \cdot (E(p) + E(\pi))^2 + (p_0 \cdot \Delta p_0)^2}$$

The value of the mass m(X) of the mysterious particle agrees within error bars with the mass of the Λ^0:

$$m(\Lambda^0) = 1115.6MeV$$

So finally, it has been proved within the restrictions, which are made in this case, that the V^0 particle is actually a Λ^0.

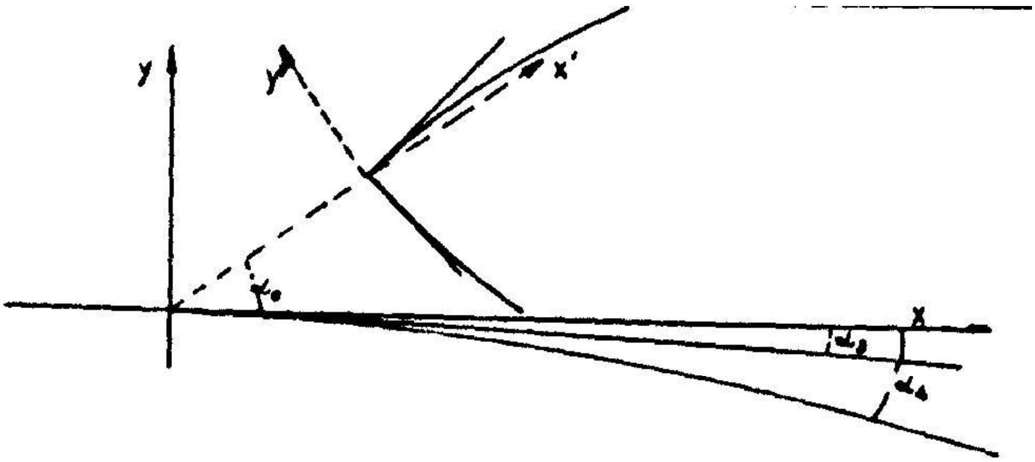

Figure 7: Flight direction

5.3 Testing of the Primary Reaction

Since strange particles always appear in pairs, another strange particle must have been created in the primary reaction, which can be determined in a similar way as the Λ :

a) geometrical reconstruction

Charge and momentum values are determined as done in the first subsection of this chapter:
Track 3:positive
Track 4:positive

$$\begin{aligned} p_o &= (379 \pm 22)MeV \\ r'_3 = (4000 \pm 500)cm \longrightarrow p_3 &= (17400 \pm 2200)MeV \\ r'_4 = (910 \pm 50)cm \longrightarrow p_4 &= (3950 \pm 220)MeV \end{aligned}$$

$$\text{Error:} \quad \Delta p = 0.3B\sqrt{(\Delta V r')^2 + (\Delta r' V)^2}$$

In order to determine the momentum vectors the flight direction of the incoming proton is chosen as the x-axis: (See fig. 7)

$$\begin{aligned} \alpha_o &= 42^o \pm 0.2^o \\ \alpha_3 &= 0.3^o \pm 0.1^o \\ \alpha_4 &= 2.7^o \pm 0.1^o \\ \vec{p_o} &= \begin{pmatrix} (282 \pm 16)MeV \\ (254 \pm 15)MeV \end{pmatrix} \\ \vec{p_3} &= \begin{pmatrix} (17400 \pm 2200)MeV \\ (-91 \pm 32)MeV \end{pmatrix} \\ \vec{p_4} &= \begin{pmatrix} (3950 \pm 220)MeV \\ (-186 \pm 12)MeV \end{pmatrix} \end{aligned}$$

The sum of these momentum vectors yields:

$$\vec{p_o} + \vec{p_3} + \vec{p_4} = \begin{pmatrix} (21650 \pm 2200)MeV \\ (-23 \pm 37)MeV \end{pmatrix} = \vec{p}$$

Error:

$$\begin{aligned}
\Delta p_x &= \sqrt{(\Delta p_{ox})^2 + (\Delta p_{3x})^2 + (\Delta p_{4x})^2} \\
\Delta p_y &= \sqrt{(\Delta p_{oy})^2 + (\Delta p_{3y})^2 + (\Delta p_{4y})^2}
\end{aligned}$$

This sum vector corresponds to the direction and magnitude of the momentum of the incoming proton (with an incident momentum of 23877 MeV) within the error bars. This allows the conclusion that the Λ was immediately created in the primary reaction and didn't result from any of the following decays:

$$\begin{array}{ccl}
\Sigma^0 & \longrightarrow & \Lambda\gamma \\
 & & \downarrow \\
 & & \\
\Xi^0 & \longrightarrow & \Lambda\pi^0 \\
 & & \downarrow \\
 & & p\pi^-
\end{array}$$

In addition, besides the Λ no other neutral particle can occur.

b) kinematical analysis

On the basis of the previous considerations it can be assumed that the event takes place in the following way:

$$\begin{array}{ccl}
pp & \longrightarrow X_3 X_4 & \Lambda \\
 & & \downarrow \\
 & & p\pi^-
\end{array}$$

Due to the conservation laws for charge, baryon number and strangeness, there are the following possibilities for a mass assignment to track 3 and 4:

	Track3	Track4	
pp $\longrightarrow$	p	K^+	Λ
pp $\longrightarrow$	K^+	p	Λ

Which of the tracks belongs to the proton and which one to the kaon can eventually be decided by using the energy conservation law ($m(K^+)$ =493.7 MeV):

$$
\begin{aligned}
E(\text{beam}) &= 23895 MeV \\
E(\text{target}) &= 938 MeV \\
\rightarrow\ E(\text{pp}) &= 24833 MeV \\
\\
E_3(p) &= \sqrt{m(p)^2 + {p_3}^2} = (17430 \pm 2200) MeV \\
E_4(K^+) &= \sqrt{m(K^+)^2 + {p_4}^2} = (3980 \pm 220) MeV \\
E(\Lambda) &= \sqrt{m(\Lambda)^2 + {p_o}^2} = (1178 \pm 8) MeV \\
\\
E(pK^+\Lambda) &= (22600 \pm 2200) MeV \\
\\
E_3(K^+) &= \sqrt{m(K^+)^2 + {p_3}^2} = (17400 \pm 2200) MeV \\
E_4(p) &= \sqrt{m(p)^2 + {p_4}^2} = (4060 \pm 220) MeV \\
E(\Lambda) &= (1178 \pm 8) MeV \\
\\
E(K^+p\Lambda) &= (22640 \pm 2200) MeV \\
\\
\text{Error:}\quad \Delta E &= \frac{p\Delta p}{E} \\
\\
\Delta(E(X_3X_4\Lambda)) &= \sqrt{(\Delta E(X_3))^2 + (\Delta E(X_4))^2 + (\Delta E(\Lambda))^2}
\end{aligned}
$$

As both energies agree with the total energy E(pp) within the error bars, an unambiguous mass assignment to the tracks is not possible. On the other hand, it is reasonable from a physical point of view to assign the proton to the track with the higher momentum. Therefore, the result of the analysis can be interpreted as follows (see fig. 8):

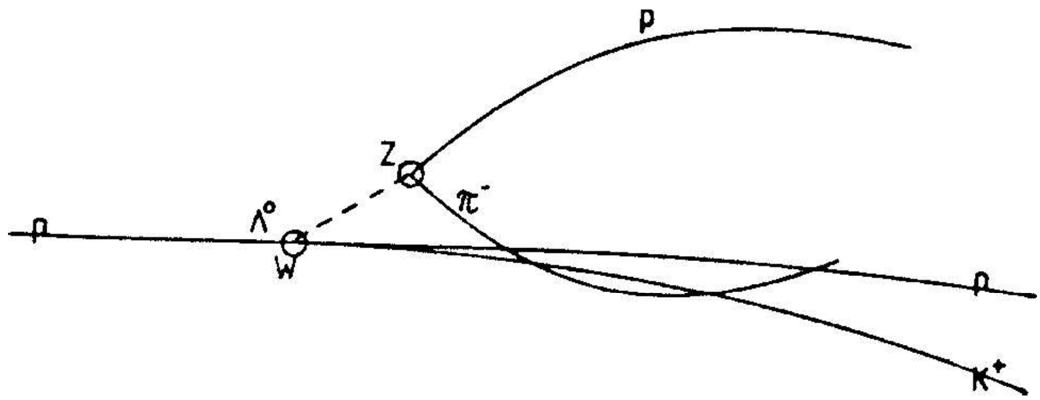

Figure 8: Track identification

$$
\begin{array}{lll}
pp & \longrightarrow pK^+ & \Lambda \\
 & & \downarrow \\
 & & p\pi^-
\end{array}
$$

A Appendix

A.1 Sagitta Method for the Measurement of the Track Curvature

With the help of the Sagitta method the curvature radius of a track can be determined without using radius stencils. It makes use of distances between fixed points, as it is depicted in figure 9:

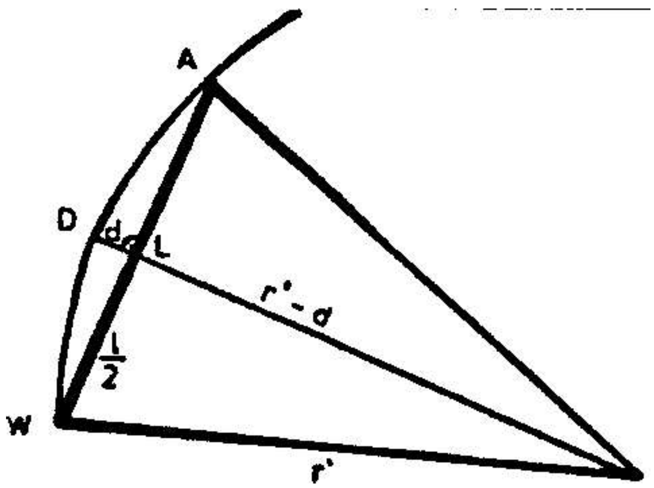

Figure 9: Sagitta method

Obviously, the following equation can be found:

$$(\frac{l}{2})^2 = r'^2 - (r' - d)^2 = 2r'd - d^2$$

This yields for the radius r':

$$r' = \frac{l^2 + 4d^2}{8d}$$

$$r' = \frac{l^2}{8d} + \frac{d}{2} \tag{19}$$

According to equation 14 the true radius r can then be calculated from the radius r' measured on the projection.

A.2 Event Analysis Considering the Inclination Angle

In order to analyse events with a large inclination angle ($> 18^o$) and for more accurate measurements it is necessary to perform a spatial reconstruction. For this purpose the respective track depth z must be known. It can be determined by the so called "stereo-shift" method. From the z-depth the distance between two points in the bubble chamber can be calculated, which is important for accurate measurements of the flight length of neutral particles, for example. You should determine the absolute values of the Σ and π momenta in the example decay process $K^-p \overset{\text{at rest}}{\longrightarrow} \pi^+\Sigma^-$.

"Stereo-shift" method

In order to determine the z coordinate of a point A on a track the "stereo-shift" method is used. Two projections (views) are needed here, which have to be superposed in such a way that both pictures of the reference sign F on the front glass (z= 0) overlap. Thus also all other points with the same z coordinate as F overlap. The larger the z component of other points, the further their pictures are shifted against each other: The shift s_A of point A needs to be measured as well as the shift s_G of a reference point G on the back glass in the projection plane. See also figure 10.

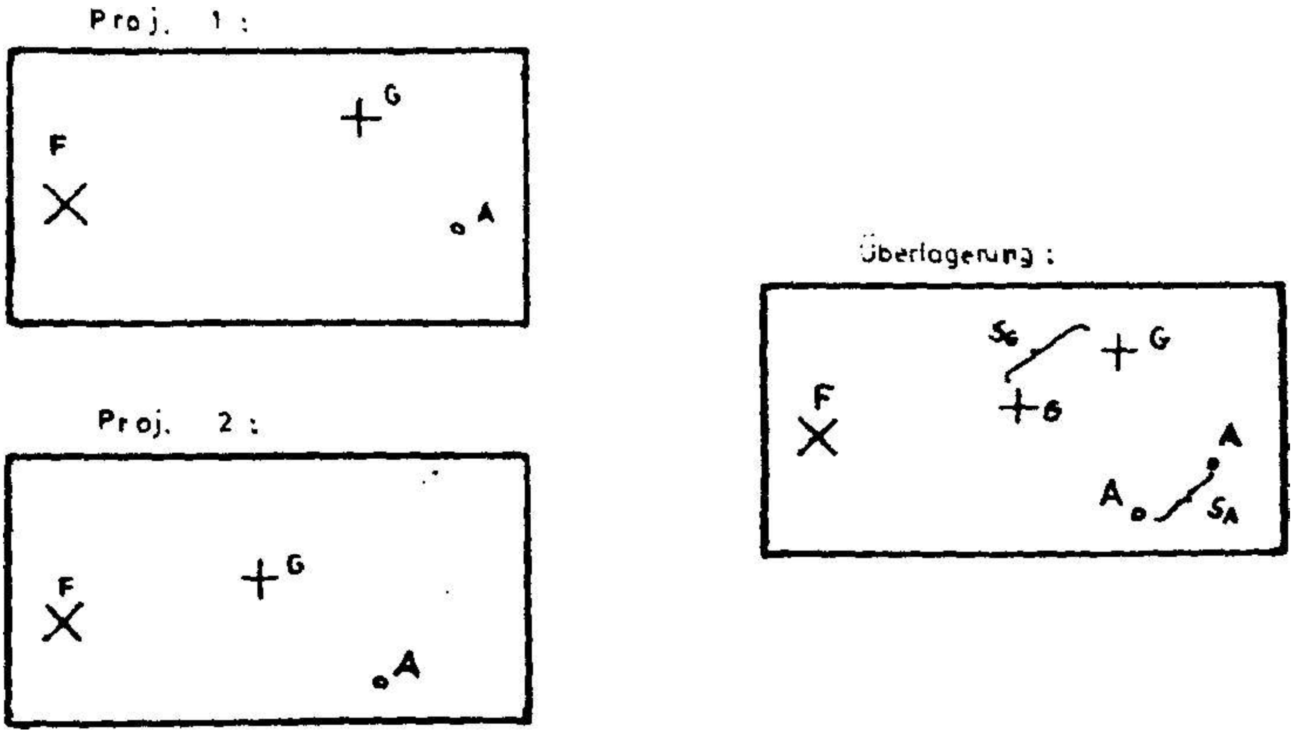

Figure 10: “Stereo shift” method

Here:

$$\frac{a_z}{s_A} = \frac{g_z}{s_G}$$

where g_z and a_z are the z coordinates of G and A.
It follows:

$$a_z = \frac{g_z s_A}{s_G} \tag{20}$$

Distance Between Two Points on a Track with an Inclination Angle

In order to determine the distance between two points A and B, their coordinates must be determined at first. A reference sign on the front glass is used as the origin of the coordinate system. (See figure 11) The z coordinates are obtained with the help of the “stereo-shift” method.

The coordinates of the point A are:

$$A = \begin{pmatrix} a_x + f_x \\ a_y + f_y \\ a_z \end{pmatrix}$$

Due to the magnification V_A in a depth a_Z we obtain according to equation 5:

$$A = \begin{pmatrix} V_A \alpha_x + f_x \\ V_A \alpha_y + f_y \\ a_z \end{pmatrix}$$

For B, the second point in a depth b_z, we obtain in an analogue way:

$$B = \begin{pmatrix} V_B \beta_x + f_x \\ V_B \beta_y + f_y \\ b_z \end{pmatrix}$$

Finally, the distance l between A and B is given by:

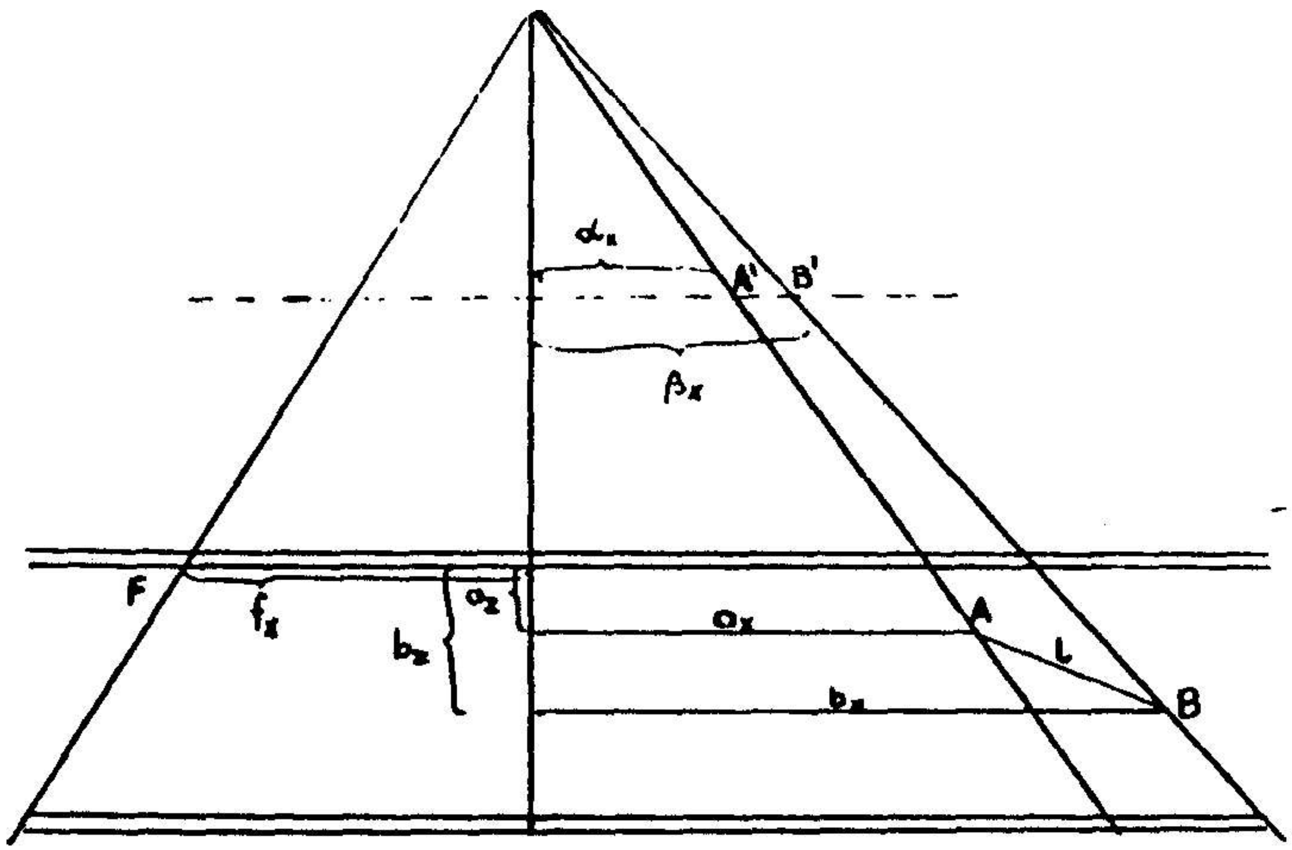

Figure 11: Distances of points A and B

$$l = \sqrt{(V_A\alpha_x - V_B\beta_x)^2 + (V_A\beta\alpha_y - V_B\beta_y)^2 + (a_z - b_z)^2} \tag{21}$$

If A and B lie at the same chamber depth ($a_z = b_z$, $V_A = V_B = V$), equation 22 is simplified:

$$l = Vl' \tag{22}$$

where $l' = \sqrt{(\alpha_x - \beta_x)^2 + (\alpha_y - \beta_y)^2}$ is the measured distance between A' and B' in the projection plane.

Momentum Vector of a Particle with an Inclined Trajectory

To determine the momentum vector $\vec{p}$ of a particle at the interaction point W the azimuthal and inclination angle of the track tangent at point W are needed, together with the absolute value p of the momentum. Looking at figure 12 you can find for $\vec{p}$:

$$\vec{p} = p_{xy}\begin{pmatrix}\cos\phi\\ \sin\phi\\ \tan\lambda\end{pmatrix} = p\begin{pmatrix}\cos\lambda\cos\phi\\ \cos\lambda\sin\phi\\ \sin\lambda\end{pmatrix} \tag{23}$$

The measurement of the angles is very complex and will not be explained here.
There are two optional methods depicted here how to determine the absolute value of the momentum p for a special case:

$$\begin{array}{ll} K^- p \stackrel{\text{at rest}}{\longrightarrow} \pi^+ & \Sigma^- \\ & \downarrow \\ & \pi^- n \end{array}$$

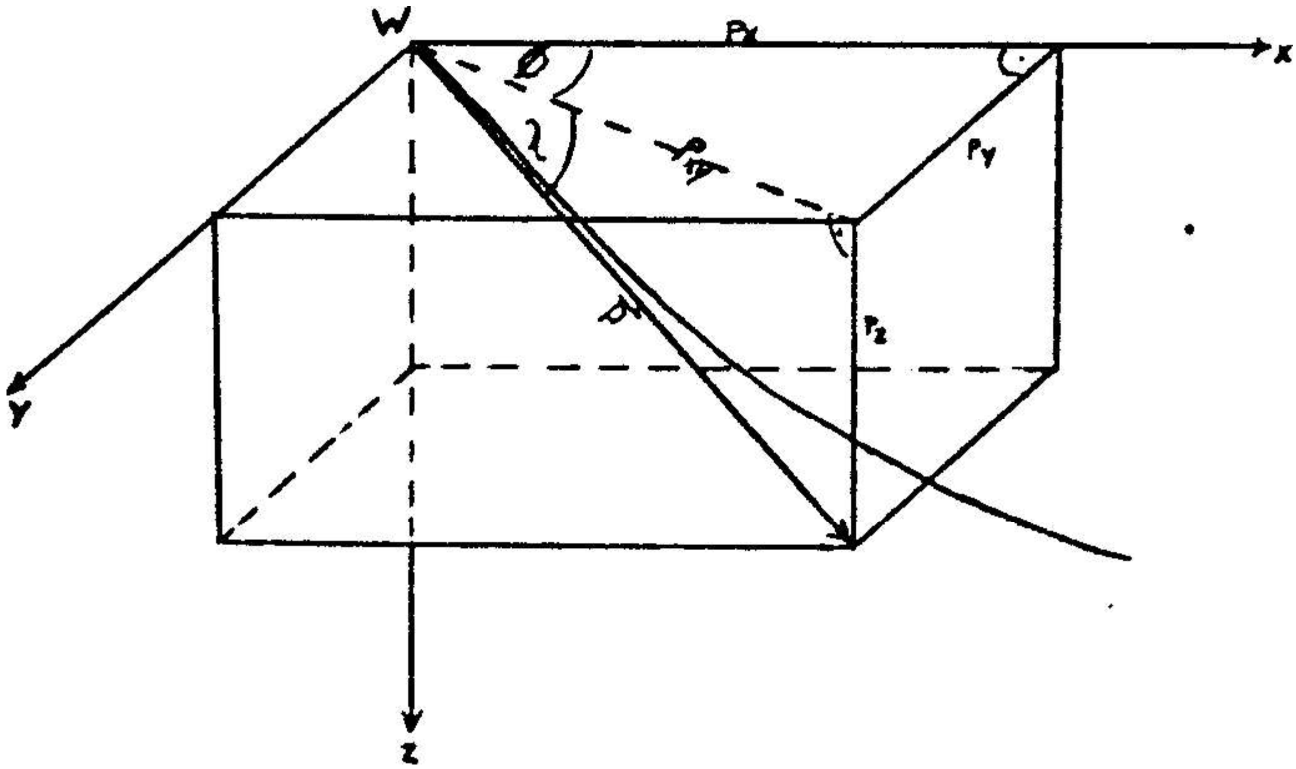

Figure 12: 3D

a)from the curvature radius r' of the π -track the radius projected onto the xy plane r_{xy} can be calculated for all events:

$$r_{xy} = Vr'$$

The momentum projected onto the xy plane p_{xy} can then be calculated:

$$p_{xy} = 0.3Br_{xy}$$

θ should now be the angle between the momentum vector and the magnetic field $\vec{B}$. See fig. 13.

Thus:

$$p_{xy} = p\sin\theta \tag{24}$$

As there are many events on the film, the average $\langle p_{xy}\rangle$ of all p_{xy} is used.

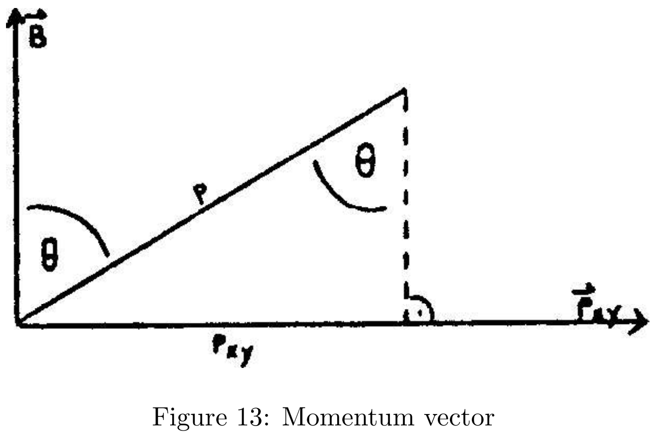

Figure 13: Momentum vector

$$\langle p_{xy}\rangle = \langle p\rangle\langle\sin\theta\rangle$$

Assuming that the momentum orientations are isotropically distributed we can obtain:

$$\langle\sin\theta\rangle = \frac{\int\sin\theta d\Omega}{\int d\Omega} = \frac{\pi^2}{4\pi} = \frac{\pi}{4}$$

For $\langle p\rangle$ this yields:

$$\langle p\rangle = \frac{\langle p_{xy}\rangle}{\langle\sin\theta\rangle}$$

$$\langle p\rangle = \frac{4\langle p_{xy}\rangle}{\pi}$$

b) another possibilty to determine the absolute momentum value is to plot the frequency H(r_{xy}) against r_{xy} in a histogram.

The larger the inclination angle of a track, the smaller the curvature radius appears in the projection plane. As a first approximation one can assume that the π -momenta of the events observed are uniformly distributed. That means the radius r_{xy}(max) belongs to events, in which the track of the π with the radius r lies in the xy plane of the bubble chamber. For these tracks we find then:

$$r_{xy}(max) = r$$

and the momentum value can be calculated according to equation 13:

$$p = 0.3Br$$

A.3 Mass Determination

Missing Mass

If the energies and momenta of the particles a_1, a_2, b_1, ... b_n in a reaction $a_1 + a_2 \longrightarrow b_1 + b_2 + ...b_n$ are known, the "missing" mass m_{b_n} of the particle b_n can be calculated from the energies and momenta of the rest of the particles:

$$m_{b_n} = \left[\left(E_{a_1} + E_{a_2} - \sum_{i=1}^{n-1} E_{b_i}\right)^2 - \left(\vec{p}_{a_1} + \vec{p}_{a_2} - \sum_{i=1}^{n-1} \vec{p}_{b_i}\right)^2\right]^{1/2} \tag{25}$$

Example:
In the event $K^-p \overset{\text{at rest}}{\longrightarrow} \Sigma^-\pi^+$, the mass of the Σ^- should be determined. Since K^- interacts with the proton "at rest", the following equations are true:

$$\begin{aligned} \vec{p_K} &= \vec{p_p} = 0 \\ \vec{p_K} &= \vec{p_p} = 0 \\ E_K &= m_K \\ E_\pi &= \sqrt{{m_\pi}^2 + {p_\pi}^2} \\ E_p &= m_p \end{aligned}$$

It follows for the mass of the Σ^-:

$$m_{\Sigma^-} = \sqrt{(m_K + m_p - \sqrt{{m_\pi}^2 + {p_\pi}^2})^2 - {p_\pi}^2}$$

A.4 Life Time

The number N of particles that crossed a distance of the length l in the bubble chamber and didn't decay until then is given by:

$$N = N_0 e^{-\frac{l}{\lambda}} \tag{26}$$

where N_0 is the number of incoming particles and λ is the decay length in the lab system. λ is the distance a particle can cross during its mean life time:

$$\lambda = vt$$

where $v = \beta c$ is the velocity in the lab system and t is the life time in the lab system. According to time dilation:

$$t = \gamma\tau$$

with τ being the mean life time in the center-of-mass system of the particle. Therefore:

$$\lambda = \beta\gamma\tau, \ \ [c = 1] \tag{27}$$

Since $\beta\gamma = \frac{p}{m}$ (p=momentum, m=rest mass), one obtains:

$$\lambda = \frac{p\tau}{m}$$

$$\tau = \frac{\lambda m}{p} \tag{28}$$

Charged particles suffer energy losses through ionization, thus they get slower. Assuming a uniform velocity for them is no longer justifiable. Therefore the result must be corrected by an additional term for charged particles:

$$+\frac{k\tau}{2\beta^3 m} \tag{29}$$

$k = 0.248 MeVcm^{-1}$ being the minimum ionization.

Example:

For the event $K^- p \overset{\text{at rest}}{\longrightarrow} \Sigma^- \pi^+$ the life time of the Σ should be obtained.

We use the equation 26. N_0 in this case denotes the number of all events on the film and N is the number of events, where the Σ crosses a distance of length l. Now the logarithmic histogram of the distances l in units of Δl against $\ln N$ should be plotted. The gradient is equal to $-\frac{1}{\lambda}$. Thus the decay length can be determined and finally the life time can be calculated using equation 28 and 29:

$$\tau_\Sigma = \frac{\lambda m_\Sigma}{p_\Sigma}(1 + \frac{k}{2\beta^3 m_\Sigma})$$

A.5 Energy Loss

A high-energetic charged particle traversing a medium steadily loses its energy through collisions with electrons of the target atoms. This leads to ionization and excitation of the atoms. The energy loss through ionization in a track section dx is given by:

$$-\frac{dE}{dx} = \frac{2\pi N_o Z z^2 e^4}{m_e v^2 A}[\ln(\frac{2m_e v^2 E'_{max}}{I^2(1-\beta^2)}) - 2\beta^2] \tag{30}$$

BETHE BLOCH EQUATION.

N_o = Avogadro's Number
Z = atomic number of medium
A = atomic weight of medium
ze = charge of incoming particle
v = velocity of the incoming particle
m_e = electron mass
E'_{max} = maximum energy transfer in a single collision
$E'_{max} \sim \gamma^2$
$I(Z)$ = effective ionisation potential of an atom with the atomic number Z, averaged over all electrons
$I \approx 10Z$ eV

(This formula refers to incoming particles without a spin, but effects of the spin on the mean enery loss can be neglected.) For a fixed z the energy loss is proportional to $1/v^2$ for a nonrelativistic particle:

$$\frac{dE}{dx} \sim \frac{1}{v^2}$$

With increasing velocity the interaction time per cm decreases (the probability of tranferring energy gets smaller) and thus also the energy loss decreases. This can be seen in the bubble chamber, as well, since the bubble density drops. See fig. (5) (German script of Gerhild Seul).
For relativistic particles we find:

$$\beta \approx 1 \quad , \quad \gamma \gg 1$$

The logaritmic part of the equation is no longer constant, i.e. the energy loss increases again: this is the so called relativistic rise. Because of the propotionality

$$E'_{max} \sim \gamma^2$$

the energy transfer value, which is permitted by kinematics, gets larger and high energetic electrons occur (δ). This results in additional ionisation. The relativistic rise reaches a constant value (for $\gamma > 10$ the plateau value is about 10 percent above the minimum).
There is also a relation between the range of a particle and its energy loss. The Bragg curve shows this dependence: (See fig. 14)

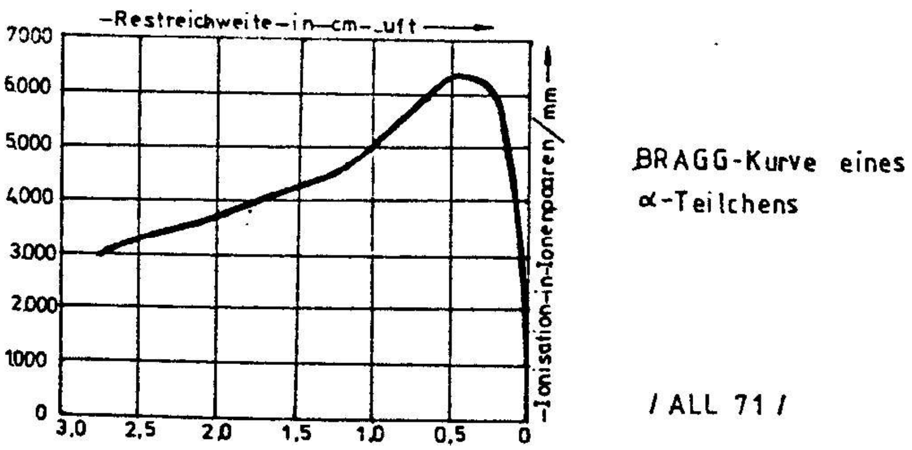

Figure 14: Bragg curve

By using the range the particle momentum can be determined. This dependence is shown in fig. (4b) (German script of Gerhild Seul), but usually tables are used for that purpose.

E N D