Verification of Hexahedral Element Type

This document provides an approach of an analytical calculation of the deformation and stress in a hexahedral element. Both the linear hexahedral element with the standard algorithm as well as the linear hexahedral element using the selective reduced integration (SRI) algorithm are documented. As structure, a cube with equal side lengths is considered.

1. Formulas

1. Deformation:

The deformation of a cube-like structure can be calculated using

$$\mathrm{u} = \frac{F \cdot L}{A \cdot E} \tag{1}$$

Where:

F: Applied force in Newton [N]

L: Edge length of the structure in meters [m]

A: Cross-sectional area in square meters [m^2]

E: Modulus of elasticity in Pascal [Pa]

u: Change in length in meters [m]

2. Stress:

The axial stress sigma in the structure is given by:

$$\sigma = \frac{F}{A} \tag{2}$$

2. Example Calculation

The cube is fixed at the left side and subjected to a pure tension force in y-direction on the right side. Given the following parameters:

- Edge length of the cube, L = 10 [m]
- Modulus of elasticity, E = 2.0E+05 [Pa]
- Applied force, F = 40000 [N]

1. Deformation:

Inserting into (1) leads to a deformation of

$$\mathrm{u} = \frac{F \cdot L}{A \cdot E} = \frac{40000 \cdot 10}{(10 \cdot 10) \cdot 2.0E05} = 0.02\ [m].$$

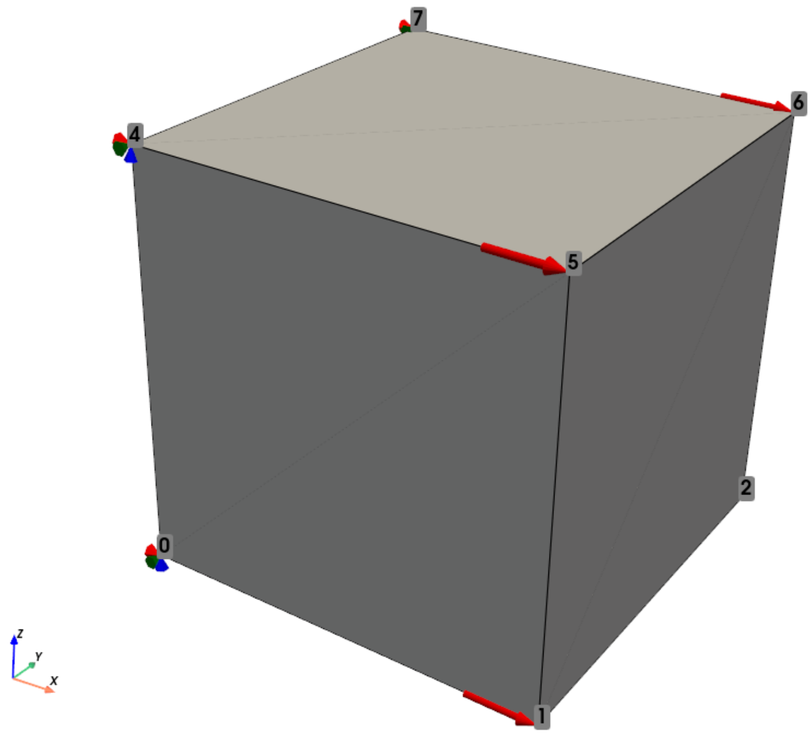
3. Stress:

Inserting into (2) leads to an axial stress of

$$\sigma = \frac{F}{A} = \frac{40000}{10 \cdot 10} = 400.0\ \left[\frac{N}{m^2}\right].$$

3. FE-Model

The FE-Structures are provided in "Verify_Element_Hexahedral.py" and "Verify_Element_Hexahedral_SRI.py" for both element types respectively. The structure consists of eight nodes and one linear hexahedral element. The left sided nodes are fixed and the force is applied to the right sided nodes in x-direction. The structure and its boundary conditions is shown below.



<u>Linear hexahedral element with standard algorithm</u>

Solving the structure using the direct stiffness algorithm leads to an axial stress in x-direction of 400.0 [N/m²] as well as a displacement of 1.712E-02 [m]. The approximation of the axial stress aligns well with the analytical solution, however, for the displacement, a noticeable difference can be seen. The element behaves too stiff which is probably due to the effect of traverse shear locking. Due to this problem, the linear hexahedral element using the SRI algorithm was implemented.

<u>Linear hexahedral element with SRI algorithm</u>

Solving the structure using the direct stiffness algorithm leads to an axial stress in x-direction of 400.0 [N/m²] as well as a displacement of 1.911E-02 [m]. Just as for the standard algorithm, the axial stress aligns well with the analytical solution. The displacement still differs from the analytical solution; however, the approximation is close to the actual value and better than using the standard algorithm.

The solution using the SRI algorithm is shown below, where the coloring is done using the Von Mises stress (const. through the element).