<u>Verification of CST Element Type</u>

This documentation provides an analytical calculation of the deformation and stress in a constant strain (CST) element. As structure, a thin and squared metal sheet is considered.

1. Formulas

1. Deformation:

The deformation of a metal sheet can be calculated using

$$\mathrm{u} = \frac{F \cdot L}{A \cdot E} \tag{1}$$

Where:

F: Applied force in Newton [N]

L: Edge length of the structure in meters [m]

A: Cross-sectional area in square meters [m²]

E: Modulus of elasticity in Pascal [Pa]

u: Change in length in meters [m]

2. Stress:

The stress sigma in the structure is given by:

$$\sigma = \frac{F}{A} \tag{2}$$

2. Example Calculation

The metal sheet is fixed at the bottom and subjected to a pure tension force in y-direction. Given the following parameters:

- Edge length of the squared metal sheet, L = 10 [m]
- Thickness of the metal sheet, h = 0.2 [m]
- Modulus of elasticity, E = 2.0E+05 [Pa]
- Applied force, F = 10000 [N]

1. Deformation:

Inserting into (1) leads to a deformation of

$$\mathrm{u} = \frac{F \cdot L}{A \cdot E} = \frac{10000 \cdot 10}{(10 \cdot 0.2) \cdot 2.0E05} = 0.25\ [m].$$

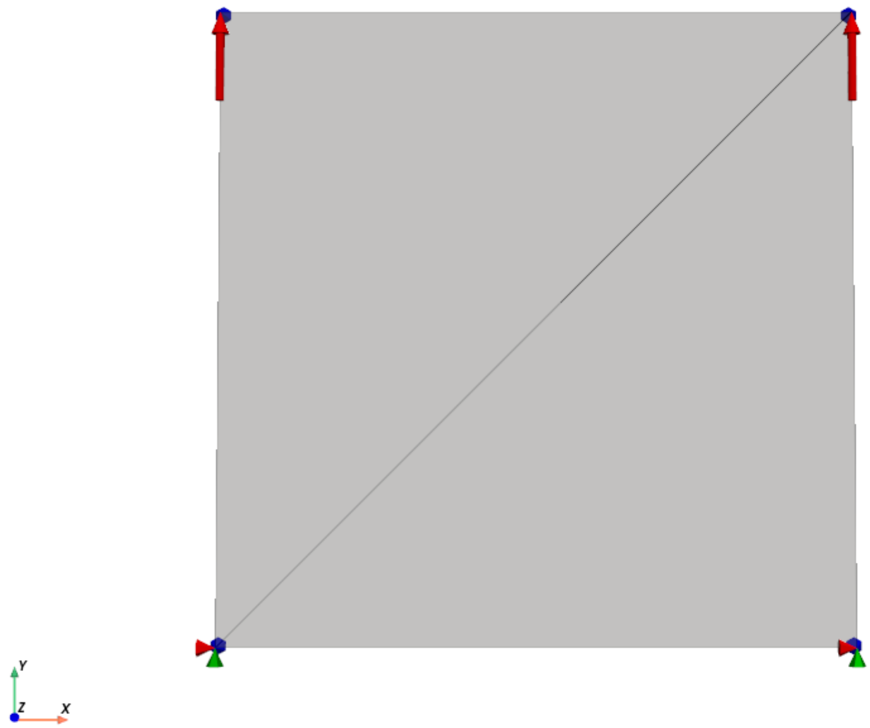
3. Stress:

Inserting into (2) leads to a stress of

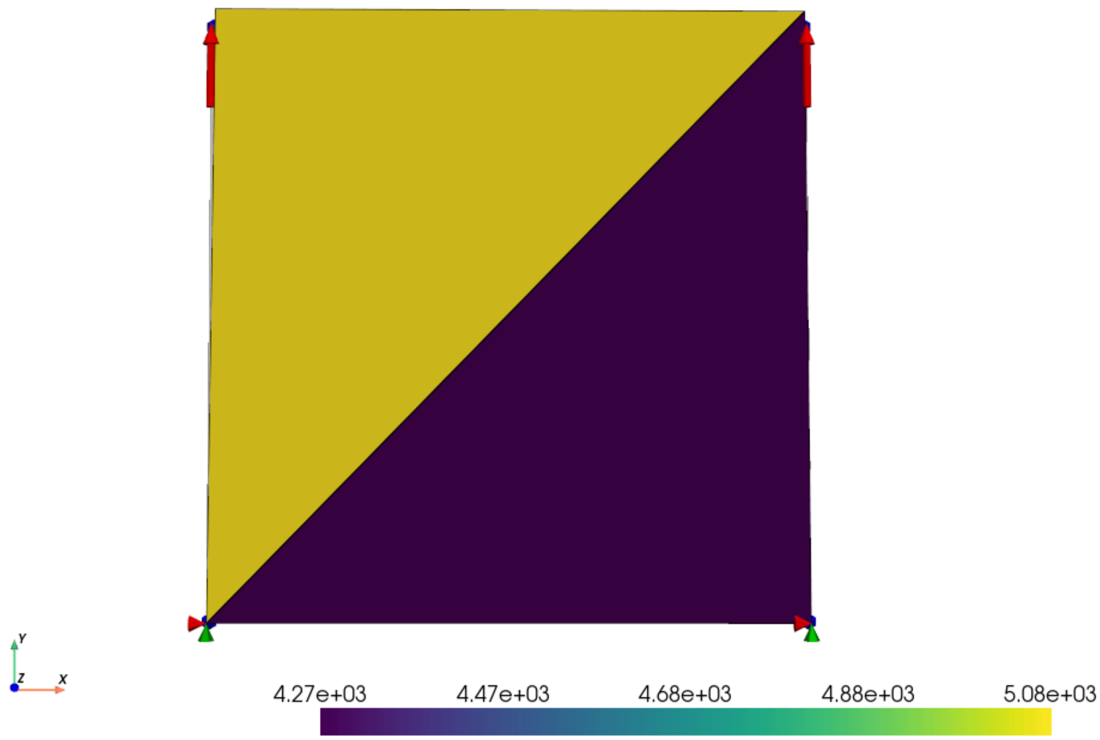
$$\sigma = \frac{F}{A} = \frac{10000}{10 \cdot 0.2} = 5.0E03\ \left[\frac{N}{m^2}\right].$$

3. FE-Model

The FE-Structure is provided in "Verify_Element_CST.py". The structure consists of four nodes and two linear CST elements. The bottom nodes are fixed and the top nodes are constrained in z-direction. The force is applied to the top nodes in y-direction. Plane stress mode is considered for the analysis.



Solving the structure using the direct stiffness algorithm leads to a mean stress in y-direction of 5.0E+03 [N/m²] as well as a displacement of 2.548E-01 [m]. The largest Von Mises stress is 5.083E+03 [N/m²]. The results agree well with the analytical solution. The resulting plot of the deformed structure using the Von Mises stress values for coloring is shown below. The force arrows are plotted on the undeformed structure, which is the reason for the small offset between them and the deformed structure.


Y
Z
X
4.27e+03
4.47e+03
4.68e+03
4.88e+03
5.08e+03