

Factory planning

Constants

P	set of products
M	set of types of machines
T	set of periods (months)
p_j	profit of j -th product ($j \in P$)
A_{ij}	time that product j must spend on machine i ($i \in M, j \in P$)
$MAXS_{tj}$	maximum number of product $j \in P$ that can be sold on period $t \in T$
MC_{it}	number of machines of type $i \in M$ available in period $t \in T$

Variables

x_{tj}	units of j -th product made in period $t \in T$
s_{tj}	units of the j -th product sold in period $t \in T$
i_{tj}	stock of product $j \in P$ at the end of period $t \in T$

$$\max z = \sum_{j \in P} \sum_{t \in T} (p_j s_{tj} - 0.5 i_{tj}) \quad (1)$$

Flow conservation

$$i_{tj} = i_{t-1,j} + x_{tj} - s_{tj} \quad t \in T, j \in P \quad (2)$$

Inventory limits

$$i_{tj} \leq 100 \quad t \in T, j \in P \quad (3)$$

$$i_{0j} = 0 \quad j \in P \quad (4)$$

$$i_{|T|j} = 50 \quad j \in P \quad (5)$$

Maximum sales

$$s_{tj} \leq MAXS_{tj} \quad t \in T, j \in P \quad (6)$$

Production time

$$\sum_{j \in P} a_{ij} x_{tj} \leq 16 * 24 MC_{it} \quad i \in M, t \in T \quad (7)$$

$$x_{tj}, i_{tj}, s_{tj} \geq 0 \text{ integer} \quad t \in T, j \in P \quad (8)$$

Analysis

1. What is the price we can pay to have one more machine ?

An additional machine some kind is useful to increase the profit if in the current scenario all its production time is used.

We start by relaxing the integrality constraints and look at the slack variables of constraints (7), which refers to the machine production time restrictions, and we analyze the respective attributes.

PT[January,Grinding]	slack =	0.000	RHS =	1152	DUAL =	8.571
PT[February,Grinding]	slack =	431.000	RHS =	1536	DUAL =	0.000
PT[March,Grinding]	slack =	1456.000	RHS =	1536	DUAL =	0.000
PT[April,Grinding]	slack =	1116.000	RHS =	1536	DUAL =	0.000
PT[May,Grinding]	slack =	602.000	RHS =	1152	DUAL =	0.000
PT[June,Grinding]	slack =	806.000	RHS =	1536	DUAL =	0.000
PT[January,Vertical drilling]	slack =	330.286	RHS =	768	DUAL =	0.000
PT[February,Vertical drilling]	slack =	398.000	RHS =	768	DUAL =	0.000
PT[March,Vertical drilling]	slack =	528.000	RHS =	768	DUAL =	0.000
PT[April,Vertical drilling]	slack =	154.000	RHS =	384	DUAL =	0.000
PT[May,Vertical drilling]	slack =	104.000	RHS =	384	DUAL =	0.000
PT[June,Vertical drilling]	slack =	383.000	RHS =	768	DUAL =	0.000
PT[January,Horizontal drilling]	slack =	746.000	RHS =	1152	DUAL =	0.000
PT[February,Horizontal drilling]	slack =	0.000	RHS =	384	DUAL =	0.625
PT[March,Horizontal drilling]	slack =	1152.000	RHS =	1152	DUAL =	0.000
PT[April,Horizontal drilling]	slack =	732.000	RHS =	1152	DUAL =	0.000
PT[May,Horizontal drilling]	slack =	612.000	RHS =	1152	DUAL =	0.000
PT[June,Horizontal drilling]	slack =	658.000	RHS =	768	DUAL =	0.000
PT[January,Boring]	slack =	231.343	RHS =	384	DUAL =	0.000
PT[February,Boring]	slack =	261.000	RHS =	384	DUAL =	0.000
PT[March,Boring]	slack =	0.000	RHS =	0	DUAL =	200.000
PT[April,Boring]	slack =	302.000	RHS =	384	DUAL =	0.000
PT[May,Boring]	slack =	256.000	RHS =	384	DUAL =	0.000
PT[June,Boring]	slack =	241.500	RHS =	384	DUAL =	0.000
PT[January,Planing]	slack =	356.289	RHS =	384	DUAL =	0.000
PT[February,Planing]	slack =	364.325	RHS =	384	DUAL =	0.000
PT[March,Planing]	slack =	384.000	RHS =	384	DUAL =	0.000
PT[April,Planing]	slack =	347.000	RHS =	384	DUAL =	0.000
PT[May,Planing]	slack =	361.000	RHS =	384	DUAL =	0.000
PT[June,Planing]	slack =	0.000	RHS =	0	DUAL =	800.000

- The 'Vertical drilling' are the only machine that have spare capacity on all periods (min 104 in May). Therefore adding one of these machines *do not* allow to increase the production, and consequently the gain.

- All the other machines have one period with zero residual time. Adding one of these machines could allow to increase the production and the gain.
- Recalling the theory of duality in linear programming, we know that the dual value associate to a constraint is the increment in the objective function due to the increment of the r.h.s. of one unit. It follows that we gain 8.571, 0.625, 200 and 800 pounds, respectively, by adding one Grinding, Horizontal drilling, Boring or Planing, in the corresponding month.
- The maximum gain is obtained by adding a 'Planing': 800
- If we add one 'Planing', we add it to all periods (6 constraints), so the gain is not limited to 800. Running again the program we will see a gain by 4330

Returning to the model with integer variables we have.

PT[January,Grinding]	slack =	0.100	RHS =	1152
PT[February,Grinding]	slack =	431.000	RHS =	1536
PT[March,Grinding]	slack =	1456.000	RHS =	1536
PT[April,Grinding]	slack =	1116.000	RHS =	1536
PT[May,Grinding]	slack =	602.000	RHS =	1152
PT[June,Grinding]	slack =	806.000	RHS =	1536
PT[January,Vertical drilling]	slack =	330.600	RHS =	768
PT[February,Vertical drilling]	slack =	398.000	RHS =	768
PT[March,Vertical drilling]	slack =	528.000	RHS =	768
PT[April,Vertical drilling]	slack =	154.000	RHS =	384
PT[May,Vertical drilling]	slack =	104.000	RHS =	384
PT[June,Vertical drilling]	slack =	383.000	RHS =	768
PT[January,Horizontal drilling]	slack =	744.400	RHS =	1152
PT[February,Horizontal drilling]	slack =	0.400	RHS =	384
PT[March,Horizontal drilling]	slack =	1152.000	RHS =	1152
PT[April,Horizontal drilling]	slack =	732.000	RHS =	1152
PT[May,Horizontal drilling]	slack =	612.000	RHS =	1152
PT[June,Horizontal drilling]	slack =	658.000	RHS =	768
PT[January,Boring]	slack =	231.230	RHS =	384
PT[February,Boring]	slack =	261.000	RHS =	384
PT[March,Boring]	slack =	0.000	RHS =	0
PT[April,Boring]	slack =	302.000	RHS =	384
PT[May,Boring]	slack =	256.000	RHS =	384
PT[June,Boring]	slack =	241.500	RHS =	384

PT[January,Planing]	slack =	356.200	RHS =	384
PT[February,Planing]	slack =	364.330	RHS =	384
PT[March,Planing]	slack =	384.000	RHS =	384
PT[April,Planing]	slack =	347.000	RHS =	384
PT[May,Planing]	slack =	361.000	RHS =	384
PT[June,Planing]	slack =	0.000	RHS =	0

- There are not machines/periods with a zero slack for Grinding and Horizontal drilling, as in the continuous case. However we notice that the remaining production time for Grindings (0.1 hours) is not enough to produce a single unit of any product. So increasing the number of Grindings will allow to increase the production
- The same does not applies for Horizontal drilling, where the residual time (0.4) is enough to produce two units of P1. However being restricted to consider integer solutions does not allow us to state that the increase of Horizontal drilling is useless.