Assignment 3 Feedback

Applied Linear Algebra
for Data Science

Q1 and Q2

- Q1: The idea was to show how you get from model (polynomial) to the matrix A and to compare the condition number, don't need to solve it.

  The condition number is "destroyed" when forming the normal equations: $cond_2(A) = 69.28$, $cond_2(A^T A) = 4799.7$

  Scaling and centering makes it more well-conditioned: $cond_2(A) = 1.00$, $cond_2(A^T A) = 1.00$

  Std in numpy:

```
dev = np.std(A[:,1], axis=0, ddof=1)
dev = np.std(A[:,1], axis=0)
```

  $\frac{1}{n-1}\sum_{i=1}^{n}(x_i - x)^2$

  $\frac{1}{n}\sum_{i=1}^{n}(x_i - x)^2$

- Q2:
  For example show $\|Q\|_2 = 1$ and $\|Q^{-1}\|_2 = 1$ using $\frac{\|Qx\|_2}{\|x\|_2}$

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Q3

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Q3 a, b

- Keeping the zeros or not?
- Draw the polynomial as a continuous curve (not scatter plot)
  Define an "denser" x-axis from min x-value to max x-value, and evaluate the polynomial in the x-axis values, for example

```
xaxis = np.arange(np.amin(x),np.amax(x),step=0.1)
yy_NE = np.polyval(np.flip(xNE), xaxis)
yy_QR = np.polyval(np.flip(xQR), xaxis)
plt.plot(xaxis,yy_NE,'-')
```

  Note: polyval presuppose the order

$$a_2x^2 + a_1x + a_0$$

  so might have to flip the order (depending on how matrix is constructed)

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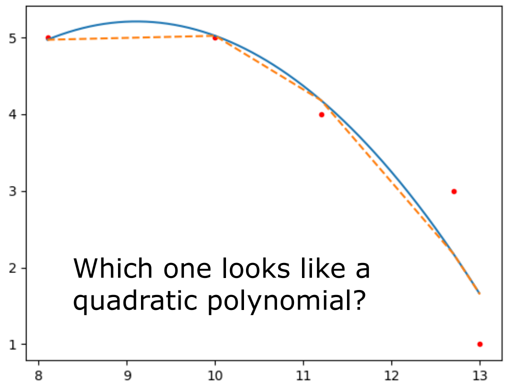
Q3 a, b

Example) Problem in Q1, quadratic polynomial plotted in two different ways
Evaluated in “dense” x-axis (blue) and data-points (orange), respectively



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Q3 a, b

- Use appropriate (efficient) solvers
  - Do not use inverse
  - Normal equations, use Cholesky

```
X_NE = scipy.linalg.solve(ATA, ATy, sym_pos=True)
```

  - QR-decomposition, use back substitution

```
X_QR = scipy.linalg.solve(R,Q.T@y,lower=False)
```


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Exercise 3c, condition number

- What does the condition number mean? Worst case scenario?
  Use formula rel error$\leq cond_2(A) \cdot$ rel error in rhs

  Here: $cond_2(A) \approx 1.6 \cdot 10^6$ and rel error in rhs $\varepsilon_M \approx 10^{-16}$

- Compare with the ”real” error, how much does normal equations destroy the solution compared with the “exact” solution?

  $\|x_{lstsq} - x_{NE}\|_2 / \|x_{lstsq}\|_2 \approx 10^{-13}$ so not as bad as it could be

  $\|x_{QR} - x_{NE}\|_2 / \|x_{QR}\|_2 \approx 1.9 \cdot 10^{-15}$, better result

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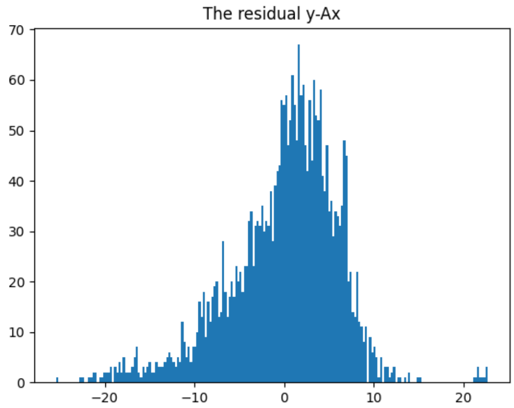
Q3



The residual should be roughly normal distributed

Q5

- Left nullspace, $\boldsymbol{N}(A^T)$ is so big in relation to $\boldsymbol{C}(A)$ => very likely that right-hand-side have components in $\boldsymbol{N}(A^T)$ => no exact solution

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Q6

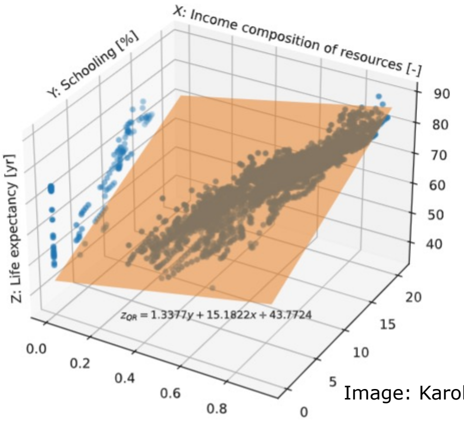

Image: Karol Sierszen

Multiple linear regression will give a 2D plane that fit the two datasets (not in the graph here).
Problem in this case is the different scales, so it does not not really work properly here.

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