ALADS - Assignment - 4

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1 1

1.1 1.a

$$A = \begin{pmatrix} 5 & 5 & 0 & 4 \\ 1 & 1 & 5 & 0 \\ 3 & 2 & 0 & 4 \\ 5 & 3 & 0 & 5 \\ 0 & 0 & 4 & 0 \end{pmatrix}$$

$$A = U\Sigma V^T$$

$$U = \begin{pmatrix} -0.65 & -0.04 & -0.71 & 0.23 \\ -0.11 & 0.78 & -0.14 & -0.24 \\ -0.43 & -0.06 & 0.51 & 0.63 \\ -0.62 & -0.07 & 0.41 & -0.64 \\ -0.02 & 0.62 & 0.22 & 0.31 \end{pmatrix}$$

$$V^T = \begin{pmatrix} -0.63 & -0.49 & -0.05 & -0.6 \\ 0.01 & 0.04 & 0.99 & -0.12 \\ -0.05 & -0.75 & 0.11 & 0.66 \\ -0.78 & 0.45 & 0.04 & 0.44 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 12.32 & 0 & 0 & 0 \\ 0 & 6.42 & 0 & 0 \\ 0 & 0 & 1.92 & 0 \\ 0 & 0 & 0 & 0.53 \end{pmatrix}$$

$$V = \begin{pmatrix} -0.63 & 0.01 & -0.05 & -0.78 \\ -0.49 & 0.04 & -0.75 & 0.45 \\ -0.05 & 0.99 & 0.11 & 0.04 \\ -0.6 & -0.12 & 0.66 & 0.44 \end{pmatrix}$$

$$
\begin{aligned}
&A = U\Sigma V^T \\
&= \begin{pmatrix} -0.65 & -0.04 & -0.71 & 0.23 \\ -0.11 & 0.78 & -0.14 & -0.24 \\ -0.43 & -0.06 & 0.51 & 0.63 \\ -0.62 & -0.07 & 0.41 & -0.64 \\ -0.02 & 0.62 & 0.22 & 0.31 \end{pmatrix} \begin{pmatrix} 12.32 & 0 & 0 & 0 \\ 0 & 6.42 & 0 & 0 \\ 0 & 0 & 1.92 & 0 \\ 0 & 0 & 0 & 0.53 \end{pmatrix} \\
&\begin{pmatrix} -0.63 & -0.49 & -0.05 & -0.6 \\ 0.01 & 0.04 & 0.99 & -0.12 \\ -0.05 & -0.75 & 0.11 & 0.66 \\ -0.78 & 0.45 & 0.04 & 0.44 \end{pmatrix}
\end{aligned}
$$

1.2 1.b

$$
\begin{aligned}
A &= U\Sigma V^T \\
\implies AV &= U\Sigma
\end{aligned}
$$

$$
\begin{aligned}
Av_1 &= \sigma_1 u_1 \\
\implies A \cdot \begin{pmatrix} -0.63 \\ -0.49 \\ -0.05 \\ -0.6 \end{pmatrix} &= 12.32 \cdot \begin{pmatrix} -0.65 \\ -0.11 \\ -.043 \\ -0.62 \\ -0.02 \end{pmatrix} \\
\implies A \cdot (-v_1) &= \sigma_1 \cdot (-u_1) \\
&= \begin{pmatrix} 8.00 \\ 1.37 \\ 5.27 \\ 7.62 \\ 0.20 \end{pmatrix}
\end{aligned}
$$

Based on our calculation, we know that Ali, Johan and Elsa are interested in Drama 1 type of movies.

$$
\begin{aligned}
Av_2 &= \sigma_2 u_2 \\
\implies A \cdot (-v_2) &= \sigma_2 \cdot (-u_2) \\
&= \begin{pmatrix} 0.27 \\ -5.00 \\ 0.39 \\ 0.46 \\ -3.97 \end{pmatrix}
\end{aligned}
$$

Based on our calculation, we know that Beatrix and Chandra dislike Drama 2 type of movies very much.

1.3 1.c

- The column space $\mathbf{C}(A)$ is spanned by the columns of U, and it represents how different movies contribute to each latent viewer stereotype. The first few columns are the most important because they correspond to relatively large singular values.
- The row space $\mathbf{C}(A^T)$ is spanned by the columns of V^T, and shows how each viewer's preferences align with the significant patterns identified by V^T.

2 2

2.1 2.a

As I gradually ran the algorithm, the diagonal elements of the matrix B now gradually approached the eigenvalues of the initial matrix B.

```
QR_Algorithm B eigenvalues:
 10.964315909149002 3.3633527179506095 1.5686034468629337 0.1037279260374569
numpy.linalg.eigh(B) B eigenvalues:
 10.964321891452668 3.3634213783919993 1.568528804117882 0.10372792603744915
```

Figure 1: Output Question 2.a

2.2 2.b

Schur decomposition Matrix B is gradually transformed into an upper triangular matrix by continuous computation.

```
Approximate diagonal matrix of eigenvalues:
 [[ 6.6508118548 -1.9807429352  0.7740059854  0.9356350624]
 [ 0.            3.144829962  -0.4969163949 -1.7734732435]
 [ 0.            0.            0.8774125062  0.458285872 ]
 [ 0.            0.            0.            0.326945677 ]]
Matrix of eigenvectors:
 [[ 0.0790927763 -0.4123575069  0.7659625062  0.486833707 ]
 [ 0.3678456218 -0.628741687  -0.5937344422  0.3418375371]
 [ 0.3125372429 -0.4636484983  0.2252429095 -0.7978823056]
 [ 0.8722123612  0.4686952609  0.1002321451  0.097590299 ]]
```

Figure 2: Output Question 2.b

2.3 2.c

By calculating the function, the eigenvalues of the matrix A can be obtained, but I am puzzled why there is a difference in the sign of the eigenvalues, i.e. the difference between positive and negative values.

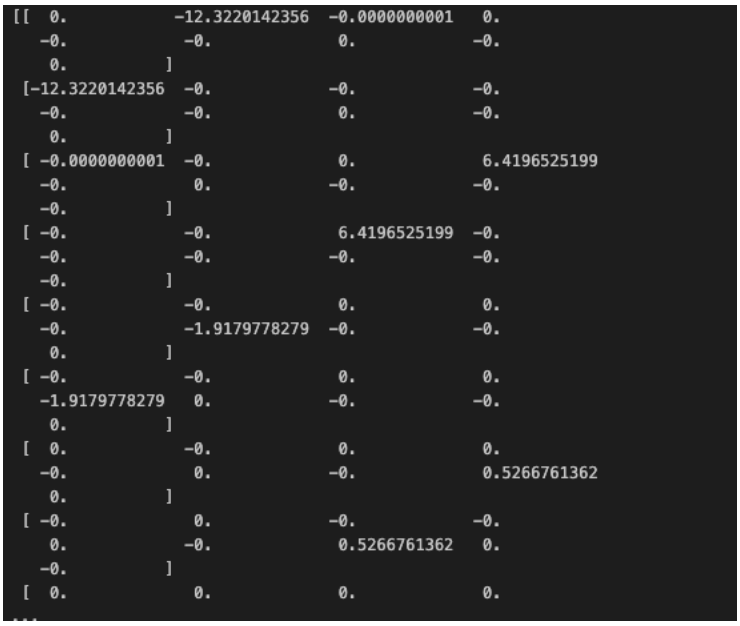

Figure 3: Output Question 2.c

3 Appendix

Code