# Computer Intensive Statistics
# Individual HWA1: Random Number and Monte Carlo

October 30, 2024

1. (1p) We have derived the Metropolis algorithm during the lecture. In this task, derive the Metropolis-Hastings algorithm.

2. (1p) Program your own $\hat{R}$ to evaluate your MCMC algorithm above. Program also your own confidence interval for your MCMC integral.

3. (2p) Find the asymptotic distribution of your normalized importance sampling estimator.

4. (1p) Suppose that $X$ and $Y$ are independent $N(0,1)$. Approximate $\mathrm{P}(X/Y < 1)$ using Rao-Blackwellization by conditioning on $Y$. Compare the variance with the naive Monte Carolo.

5. (1p) Suppose that the price of a stock at time $t$ satisfies the stochastic differentiable equation

$$\frac{dS(t)}{S(t)} = rdt + \sigma dW(t)$$

such that

$$S(t) = S(0)\exp\left[\left(r - \frac{1}{2}\sigma^2\right)t + \sigma W(t)\right].$$

A barrier option value is

$$\mathrm{E}\left[e^{-rT}S(T)\prod_{m=1}^{M}1_{(L<S_{t_m}<U)}(S_{t_m})\right],$$

for the monitoring time $\{t_m = m/M\}$, $m = 1, 2, ....$ We can simulate the process $S(t)$ by

$$S(t_{m+1}) = S(t_m)\exp\left\{\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}Z_{m+1}\right\},$$

where $\{Z_m\}$ are iid $N(0,1)$. We let $S(0) = 1$,$r = 0.05$, $\sigma = 0.2$, $L = 0.8$, $U = 1.2$, and $\Delta t = 0.01$ for simplicity. Approximate such barrier option value by Monte Carlo. Include also your algorithm. Do you encounter any issues if we let $T$ to be very large.