Type Error Debugging in Hazel

Computer Science Tripos, Part II



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Declaration of Originality

Declaration Here.

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Original Aims of the Project

Aims Here. Concise summary of proposal description.

Work Completed

Work completed by deadline.

¹ Calculated by texcount. Including: tables and footnotes. Excluding: the front matter, bibliography, and appendices

²Calculation method here

Special Difficulties

Any Special Difficulties encountered

Acknowledgements

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Chapter 1

Introduction

Brief overview here

1.1 Motivation

Include motivating examples. Give a brief overview of how type errors would be debugged with the tools produced by this project.

1.2 Previous Work

1.3 Contributions

 $Brief\ overview\ of\ the\ project\ contributions.$

1.4 Dissertation Outline

Explain structure – what is covered in each chapter briefly.

Chapter 2

Preparation

In this chapter I present the core semantics and a larger overview of the Hazel language.

2.1 The Hazel Language

What is Hazel and who is developing it here.

2.1.1 Overview & Vision

Detail the vision of the Hazel project and main features of Hazel.

Finish with the state of the subset of Hazel for which the project was implemented.

2.1.2 Core Hazel: Formal Semantics

For reference, the established semantics and type system for Hazel is presented. Derived from Omar et al. [1]. The paper itself goes into deeper depth into the intuition of the rules and the formal properties satisfied by the calculus. ¹

Syntax

The syntax, in Fig. 2.1, consists of types τ , external expressions e, and internal expressions d. Here, ? is the dynamic type, (e) is a non-empty hole containing e, and $\langle \tau_1 \Rightarrow \tau_2 \rangle$, $\langle \tau_1 \Rightarrow ? \Rightarrow \tau_2 \rangle$ are casts and cast errors from τ_1 to τ_2 respectively. The external language is a locally inferred [2] surface syntax for the language, and is statically elaborated to (explicitly typed) internal expressions, in a similar way to Harper and Stone's [3] approach to defining Standard ML as elaboration to an explicitly typed internal language, XML [4].

```
\tau ::= b \mid \tau \to \tau \mid ?

e ::= c \mid x \mid \lambda x : \tau . e \mid \lambda x . e \mid e(e) \mid \textcircled{\parallel}^{u} \mid \textcircled{(e)}^{u} \mid e : \tau

d ::= c \mid x \mid \lambda x : \tau d \mid d(d) \mid \textcircled{\parallel}^{u}_{\sigma} \mid (d)^{u}_{\sigma} \mid d\langle \tau \Rightarrow \tau \rangle \mid d\langle \tau \Rightarrow ? \Rightarrow \tau \rangle
```

Figure 2.1: Syntax: types τ , external expressions e, internal expressions d. With x ranging over variables, u over hole names, σ over $x \to d$ internal language substitutions/environments, b over base types and c over constants.

External Language: Type System

The static semantics in Fig. 2.2 of the *external language* is a bidirectionally typed system in the style of Pierce and Turner [2], and Dunfield and Krishnaswami [5]. There are two typing

¹Come up with and detail some good intuitions of what each type of value & expression is and what the cast calculus and elaboration does.

judgement modes: $\Gamma \vdash e \Rightarrow \tau$ which synthesises a type τ , algorithmically thought of as an output, and $\Gamma \vdash e \Leftarrow \tau$ which analyses against a type τ as an input.

$$\begin{array}{|c|c|c|}\hline \Gamma \vdash e \Rightarrow \tau & e \text{ synthesises type } \tau \text{ under context } \Gamma \\ \hline \text{SConst} & \hline \Gamma \vdash c \Rightarrow b & \text{SVar} & \frac{x : \tau \in \Gamma}{\Gamma \vdash x \Rightarrow \tau} & \text{SFun} & \frac{\Gamma, x : \tau_1 \vdash e \Rightarrow \tau_2}{\Gamma \vdash \lambda x : \tau_1 . e \Rightarrow \tau_1 \to \tau_2} \\ \hline & \Gamma \vdash e_1 \Rightarrow \tau_1 & \tau_1 \blacktriangleright_{\rightarrow} \tau_2 \to \tau \\ \hline \text{SApp} & \hline \Gamma \vdash e_1 \Leftrightarrow \tau_2 & \text{SEHole} & \hline \Gamma \vdash (0)^u \Rightarrow ? \\ \hline \text{SNEHole} & \hline \Gamma \vdash e \Leftrightarrow \tau & \text{SAsc} & \hline \Gamma \vdash e \Leftrightarrow \tau \\ \hline \Gamma \vdash e \Leftrightarrow \tau & e \text{ analyses against type } \tau \text{ under context } \Gamma \\ \hline \hline \Gamma \vdash e \Leftrightarrow \tau & \text{AFun} & \hline \Gamma, x : \tau_1 \vdash e \Leftrightarrow \tau_2 \\ \hline \Gamma \vdash \lambda x . e \Leftrightarrow \tau & \text{ASubsume} & \hline \tau \sim \tau' \\ \hline \Gamma \vdash e \Leftrightarrow \tau' & \hline \Gamma \vdash e \Leftrightarrow \tau' \\ \hline \end{array}$$

Figure 2.2: Bidirectional typing judgements for *external expressions*

These rules use a type consistency relation, \sim in Fig. 2.3, with types being consistent if they are equivalent up to the locations of the dynamic type. The type consistency relation is standard in gradual type systems [6], [7], and is similar to a subtyping relation but is *not* transitive.

$$\begin{array}{c|c} \hline \tau_1 \sim \tau_2 & \tau_1 \text{ is consistent with } \tau_2 \\ \\ \hline \text{TCDyn1} \hline \begin{array}{c} \tau_1 \sim \tau_1 & \tau_2 \sim \tau_2' \\ \hline \begin{array}{c} \tau_1 \sim \tau_1' & \tau_2 \sim \tau_2' \\ \hline \end{array} \end{array} }$$

Figure 2.3: Type consistency

Finally, a (function) type matching relation, $\blacktriangleright_{\rightarrow}$ in Fig. 2.4, matches the argument and return types from a function type, which for the dynamic type is ? $\blacktriangleright_{\rightarrow}$? \rightarrow ?.

$$\tau \blacktriangleright_{\to} \tau_1 \to \tau_2$$
 τ has arrow type $\tau_1 \to \tau_2$

MAFun $\tau_1 \to \tau_2 \blacktriangleright_{\to} \tau_1 \to \tau_2$

Figure 2.4: Type Matching

Elaboration

Elaboration to the *internal language* is possible for well-typed external expressions and consists of cast insertion, maintaining a hole context, and inserting initial identity hole environments. Each of these are used in the internal language type assignment Δ ; $\Gamma \vdash e : \tau$ and the dynamic semantics. Fig. 2.5 defines the elaboration judgements and Fig. 2.6 defines the internal language type assignment categorical judgement [8].

$$\Gamma \vdash e \Rightarrow \tau \leadsto d \dashv \Delta$$
 e syntheses type τ and elaborates to d

$$\operatorname{ESConst} \frac{x : \tau \in \Gamma}{\Gamma \vdash c \Rightarrow b \leadsto c \dashv \emptyset} \quad \operatorname{ESVar} \frac{x : \tau \in \Gamma}{\Gamma \vdash x \Rightarrow \tau \leadsto x \dashv \emptyset}$$

$$\operatorname{ESFun} \frac{\Gamma, x : \tau_1 \vdash e \Rightarrow \tau_2 \leadsto d \dashv \Delta}{\Gamma \vdash \lambda x : \tau_1 . e \Rightarrow \tau_1 \to \tau_2 \leadsto \lambda x : \tau_1 . d \dashv \Delta}$$

$$\Gamma \vdash e_1 \Rightarrow \tau_1 \qquad \tau_1 \blacktriangleright \to \tau_2 \to \tau$$

$$\Gamma \vdash e_1 \Leftrightarrow \tau_2 \to \tau \leadsto d_1 : \tau_1' \dashv \Delta_1 \qquad \Gamma \vdash e_1 \Leftrightarrow \tau_2 \leadsto d_2 : \tau_2' \dashv \Delta_2$$

$$\Gamma \vdash e_1(e_2) \Rightarrow \tau \leadsto (d_1 \langle \tau_1' \Rightarrow \tau_2 \to \tau \rangle) (d_2 \langle \tau_2' \Rightarrow \tau_2 \rangle) \dashv \Delta_1 \cup \Delta_2$$

$$\operatorname{ESEHole} \frac{\Gamma \vdash e \Rightarrow \tau \leadsto d \dashv \Delta}{\Gamma \vdash (e)^u \Rightarrow ? \leadsto (d)^u_{\operatorname{id}(\Gamma)} \dashv u :: ()[\Gamma]}$$

$$\operatorname{ESNEHole} \frac{\Gamma \vdash e \Leftrightarrow \tau \leadsto d \dashv \Delta}{\Gamma \vdash (e)^u \Rightarrow ? \leadsto (d)^u_{\operatorname{id}(\Gamma)} \dashv \Delta, u :: ()[\Gamma]}$$

$$\operatorname{ESAsc} \frac{\Gamma \vdash e \Leftrightarrow \tau \leadsto d : \tau' \dashv \Delta}{\Gamma \vdash e : \tau \Rightarrow \tau \leadsto d \langle \tau' \Rightarrow \tau \rangle \dashv \Delta}$$

 $\Gamma \vdash e \Leftarrow \tau_1 \leadsto d : \tau_2 \dashv \Delta$ e analyses against type τ and elaborates to d of consistent type τ_2

$$\begin{split} \tau & \blacktriangleright \rightarrow \tau_1 \rightarrow \tau_2 \\ \text{EAFun} & \frac{\Gamma, x : \tau_1 \vdash e \Leftarrow \tau_2 \leadsto d : \tau_2 \dashv \Delta}{\Gamma \vdash \lambda x. e \Leftarrow \tau \leadsto \lambda x : \tau_1. d : \tau_1 \rightarrow \tau_2' \dashv \Delta} \\ & \frac{e \neq \bigoplus^u \qquad e \neq \bigoplus^u \leq e' \bigoplus^u}{\Gamma \vdash e \Rightarrow \tau' \leadsto d \dashv \Delta \qquad \tau \sim \tau'} \\ \text{EASubsume} & \frac{\Gamma \vdash e \Rightarrow \tau' \leadsto d \dashv \Delta \qquad \tau \sim \tau'}{\Gamma \vdash e \Leftarrow \tau \leadsto d : \tau' \dashv \Delta} \\ & \text{EAEHole} & \frac{\Gamma \vdash \bigoplus^u \Leftarrow \tau \leadsto \bigoplus^u_{\mathrm{id}(\Gamma)} : \tau \dashv u :: \tau[\Gamma]}{\Gamma \vdash \bigoplus^u \Leftarrow \tau \leadsto \bigoplus^u_{\mathrm{id}(\Gamma)} : \tau \dashv u :: \tau[\Gamma]} \end{split}$$

Figure 2.5: Elaboration judgements

$$id(x_1:\tau_1,\ldots,x_n:\tau_n):=[x_1/x_1,\ldots,x_n/x_n]$$
 $\Delta;\Gamma\vdash\sigma:\Gamma'$ iff $\mathrm{dom}(\sigma)=\mathrm{dom}(\Gamma')$ and for every $x:\tau\in\Gamma'$ then: $\Delta;\Gamma\vdash\sigma(x):\tau$

Figure 2.7: Identity substitution and substitution typing

Figure 2.6: Type assignment judgement for internal expressions

Where ground types are base types or one-level unrollings of the dynamic type (each being the *least specific* type for each compound type).

$$\tau$$
 ground τ is a ground type
$$\frac{\text{GBase} - b \text{ ground}}{b \text{ ground}} = \frac{\text{GDynFun} - \frac{2}{3} - \frac{2}{3} \text{ ground}}{2}$$

Figure 2.8: Ground types

This elaboration is proven to produce unique internal expressions and hole contexts, and to preserve well-typedness.

Internal Language: Dynamic Semantics

In order to support the ability to evaluate expressions around holes and cast errors, Hazel defines multiple syntax-directed classes of final forms in Fig. 2.9. *Final forms* are irreducible expressions.

- Values Constants or functions.
- Boxed values Values or boxed values in one of the two cast forms. These must be unboxed (downcast) before reducing.
- Indeterminate forms Irreducible terms containing holes or are casts errors. Substitution of holes may make these reducible.
- Final All final forms.

Figure 2.9: Final forms

The small-step contextual dynamics [9] is defined on the internal expressions. The general idea is to consider two classes of casts: injections – casts from a ground type to the dynamic types, and projections – casts from the dynamic type to a ground type. These two classes of casts can be eliminated upon meeting if the ground types are equal or to a cast error if not. Function casts are dealt with by separating into two casts on the argument and return value. Finally, compound types can be cast to their least specific ground type specified by the

ground matching relation in Fig. 2.10.

$$\tau \blacktriangleright_{\text{ground}} \tau'$$
 τ matches ground type τ'
$$\frac{\tau_1 \to \tau_2 \neq ? \to ?}{\tau_1 \to \tau_2 \blacktriangleright_{\text{ground}}? \to ?}$$

Figure 2.10: Ground type matching

$$\boxed{d \longrightarrow d'} \quad d \text{ takes and instruction transition to } d'$$

$$\text{ITFun} \frac{}{(\lambda x : \tau.d_1)(d_2) \longrightarrow [d_2/x]d_1} \quad \text{ITCastId} \frac{}{d\langle \tau \Rightarrow \tau \rangle \longrightarrow d}$$

$$\text{ITAppCast} \frac{\tau_1 \to \tau_2 \neq \tau_1' \to \tau_2'}{d_1\langle \tau_1 \to \tau_2 \Rightarrow \tau_1' \to \tau_2' \rangle (d) \longrightarrow (d_1(d_2\langle \tau_1' \Rightarrow \tau_1 \rangle)) \langle \tau_2 \Rightarrow \tau_2' \rangle}$$

$$\text{ITCast} \frac{\tau \text{ ground}}{d\langle \tau \Rightarrow ? \Rightarrow \tau \rangle \longrightarrow d} \quad \text{ITCastError} \frac{\tau_1 \text{ ground}}{d\langle \tau_1 \Rightarrow ? \Rightarrow \tau_2 \rangle \longrightarrow d\langle \tau_1 \Rightarrow ? \Rightarrow ? \rangle} \frac{\tau_1 \neq \tau_2}{d\langle \tau_1 \Rightarrow ? \Rightarrow \tau_2 \rangle \longrightarrow d\langle \tau_1 \Rightarrow ? \Rightarrow ? \rangle}$$

$$\text{ITGround} \frac{\tau \blacktriangleright_{\text{ground}} \tau'}{d\langle \tau \Rightarrow ? \rangle \longrightarrow d\langle \tau \Rightarrow \tau' \Rightarrow ? \rangle} \quad \text{ITExpand} \frac{\tau \blacktriangleright_{\text{ground}} \tau'}{d\langle ? \Rightarrow \tau \rangle \longrightarrow d\langle ? \Rightarrow \tau' \Rightarrow \tau \rangle}$$

Figure 2.11: Instruction transitions

Variable substitution, used in ITFun, is capture avoiding and also substitutions are recorded in each hole's environment σ by (over)writing $[d/x]\sigma$.²

 $^{^2}$ Create a figure for this, maybe in appendix.

Context syntax:

$$E ::= \circ \mid E(d) \mid d(E) \mid (E)_{\sigma}^{u} \mid E \langle \tau \Rightarrow \tau \rangle \mid E \langle \tau \Rightarrow ? \Rightarrow \tau \rangle$$

$$d = E[d] \quad d \text{ is the context } E \text{ filled with } d' \text{ in place of } \circ$$

$$ECOuter \frac{d}{d = \circ[d]} \quad ECApp1 \frac{d_1 = E[d'_1]}{d_1(d_2) = E(d_2)[d_1]} \quad ECApp2 \frac{d_2 = E[d_2]}{d_1(d_2) = d_1(E)[d'_2]}$$

$$ECNEHole \frac{d = E[d']}{(d)_{\sigma}^{u} = (E)_{\sigma}^{u}[d']} \quad ECCast \frac{d = E[d']}{d \langle \tau_1 \Rightarrow \tau_2 \rangle = E \langle \tau_1 \Rightarrow \tau_2 \rangle [d']}$$

$$ECCastError \frac{d = E[d']}{d \langle \tau_1 \Rightarrow ? \Rightarrow \tau_2 \rangle = E \langle \tau_1 \Rightarrow ? \Rightarrow \tau_2 \rangle [d']}$$

$$d \mapsto d' \quad d \text{ steps to } d'$$

$$Step \frac{d_1 = E[d_2]}{d_1 \mapsto d'_1} \quad d_2 \longrightarrow d'_2 \quad d'_1 = E[d'_2]}{d_1 \mapsto d'_1}$$

Figure 2.12: Contextual dynamics of the internal language

The instruction transitions, Fig. 2.11, and evaluation context, Fig. 2.12, specify a non-deterministic evaluation order, which is a more suitable choice for supporting dynamic hole instantiation, as will be used by the search procedure.³

Casts are associative, so bracketing is omitted.

Hole Substitutions

Hole substitutions will be extensively used by the search procedure, so their semantics is given here. These correspond to contextual substitutions for meta-variables in closures in contextual modal type theory [10]. Intuitively, these are delayed substitutions with the holes being closures with environment σ and meta-variable/hole name u. A hole \bigoplus_{σ}^{u} being substituted

³Define Substitution in appendices?

with d (written $\llbracket d/u \rrbracket () \rrbracket_{\sigma}^{u}$) is d with it's environment assignment σ applied, after first substituting the hole $(\llbracket d/u \rrbracket \sigma)$ in σ .

Hole/contextual substitutions are mostly similar variable substitutions, except the substituted values may refer to bound variables without restriction. See Fig. 2.13 for full definition.

 $\llbracket d/u \rrbracket d' = d''$ d" is d' with each hole u substituted with d in the respective hole's environment σ .

 $\llbracket d/u \rrbracket \sigma = \sigma'$ of is σ with each hole u in σ substituted with d in the respective hole's environment.

$$[\![d/u]\!] \cdot = \cdot$$

$$[\![d/u]\!] \sigma, d'/x = [\![d/u]\!] \sigma, ([\![d/u]\!] d')/x$$

Figure 2.13: Hole substitution

Think in detail about CMTT and hole refinement. It is very relevant to the search procedure. In particular think about box and letbox in CMTT.

2.1.3 Hazel Codebase

2.2 The Project

2.2.1 Cast Slicing

Cast slicing is, to my knowledge, a new concept and is a method in which selected casts can be linked back to source code by creating a slice of all code contributing to the cast.

Slicing methods have been researched extensively since Wadler first proposed static program slicing [11], and later others proposed dynamic program slicing [12] and error slicing [13]. Cast slicing combines ideas from both dynamic slicing, tracking of casts during evaluation, and error slicing, tracking casts during static elaboration. But is relatively less expressive in terms of slicing criterion, only considering casts, and no other expressions.

2.2.2 Search Procedure

Search procedure for witnesses of type errors has been considered by Seidel et al. [14] for OCaml. Their approach creates a non-deterministic semantics for ill-typed OCaml with a hole in place of a function argument being dynamically type inferred and instantiated in a most general manner until evaluation gets stuck. The hole instantiation at this point is the error witness.

Adapting this search procedure to Hazel, which natively supports evaluation of ill-typed expressions and hole substitution allows for stronger semantic foundations. In particular hole substitution and refinement, borrowing from contextual modal type theory⁴.

Further, the use of dynamic types, casts, and the more general notion of holes will allow better handling of non-parametric

⁴See Chapter 5 of [1]

function types – stated as a problem in Seidel et al's. paper. Hence, very different semantics will need to be devised, but the general principle of dynamic inference and hole instantiation will remain the same.

2.3 Starting Point

Concepts

The foundations of most concepts in understanding Hazel from Part IB Semantics of Programming (and Part II Types later). The concept of gradual typing briefly appeared in Part IB Concepts of Programming Languages, but was not formalised. Dynamic typing, gradual typing, holes, and contextual modal type theory were not covered in Part IB, so were partially researched leading up to the project, then researched further in greater depth during the early stages. Similarly, Part IB Artificial Integlligence provided some context to search procedures, however nothing was directly relevant. Primarily, the OCaml search procedure for ill-typed witnesses Seidel et al. [14] and the Hazel core language [1] were researched over the preceding summer.

Tools and Source Code

My experience in OCaml was mostly from the Part IA Foundations of Computer Science course and small-scale personal projects. The Hazel source code had not been inspected in detail until after starting the project.

- 2.4 Requirement Analysis
- 2.5 Software Engineering Methodology
- 2.6 Legality

Chapter 3

TEMPORARY: Implementation Plan

3.1 Cast Slicing

Consider options of: annotating typing derivations, annotating casts (and splitting in evaluation), or reverse unevaluating casts [15].

See constraint free error slicing [16].

See the gradualizer to understand how casts work [17].

See Blame tracking to see how common type location transfer is handled [18].

Casting:

Casting goes from the terms actual type to coerce it into a new type. Casts are inserted by elaboration at appropriate points (where the type system uses consistency or pattern matching¹). Casts between compound types may be decomposed by the cast calculus. See gradualizer dynamic semantics

 $^{^{1}}$ See the gradualizer for intuition and direction of consistency casts

[19] for intuition and [18] for intuition on polarity affecting blame.

Consider deeply type constructor polarities [20, pg.473]. Covariant types give positive blame (blame value) and contravariant give negative (blame context).

GENERAL STRUCTURE:

More work needed on the treatment of type analysis. Understanding type flows, input/output mode, and type polarity will likely help. Also when considering types derived from joins.

- Slice code contributing to a type. In particular, compound types should each have their own sub-slice. e.g. with $\tau = \tau_1 \to \tau_2$ then τ would have a slice consisting of τ_1 's slice and τ_2 ' slice.
- Annotate slices onto the types themselves.
- Casts are then the combination of the two slices.
- Cast calculus (splitting of casts etc.) can be done easily.

Might be better to change this from slicing to marking, allowing marking of parts of terms (which are not terms themselves), i.e. lambda abstraction annotations

Why are casts elaborated

A cast is inserted at a point of either, for a subexpression $e:\tau$:

- Pattern Matching $\tau \triangleright \tau'$ Wrap e in a cast $\langle \tau \Rightarrow \tau'' \rangle$ where τ'' is the cast destination of τ'
- Flows $\tau \leadsto \tau'$ Wrap e in a cast $\langle \tau \Rightarrow \tau' \rangle$

A cast destination for τ is the result of applying flows recursively on all positive type positions and reversed flows on negative type positions in τ .

Flows relate to invocations of consistency and joins, with direction dictated by type polarity and modality:

- Producers flow to their final type
- Final types flow to the consumers
- Input variables are replaced with the final type.

Where the final type is an annotated *type*, output consumers, or the join of all producers. Note also that final types have output mode.

Slicing of Producers, Consumers, Final Types

Producer types correspond to synthesised types. Consumer types correspond to analysed types. Final types are one of the above or additionally, joins of types.

A slice should be all the code that contributes to the cast. Formally 2 :

• If any subterm in the slice were removed (replaced with hole or dynamic annotation etc.), then the cast would not be required or be different. The slice should ideally be minimal.

The source of any cast on a sub-term d is τ such that d has type τ , that Δ ; $\Gamma \vdash d : \tau$. The destination is as dependent on pattern matching and flows as above.

A cast being different/unneeded for a slice would imply that either the source or destination have different types. By the gradual guarantee, the new source must be more general but consistent. Also the destination would be more generalCheck..

Obtaining slices for producers:

²This is the key thing to define in a useful way. Think in more detail

Obtaining slices for consumers: Obtaining slices for joins:

Why are casts evaluated

REDO THIS!

Elaboration

First I will list the ways in which casts can be inserted by the elaboration judgements (2.5):

- 1. Ascription An annotation trivially inserts a cast by ESAsc.
- 2. Function Application A term $e_1(e_2)$ is elaborated by type synthesis for e_1 giving τ_1 , decomposed to $\tau_1 \blacktriangleright_{\rightarrow} \tau_2 \rightarrow \tau$. Both e_1 and e_2 are then analysed to d_1, d_2 of consistent types τ_1 and τ'_2 . Hence, the corresponding casts on the function: $\langle \tau'_1 \Rightarrow \tau_2 \rightarrow \tau \rangle$ and argument $\langle \tau'_2 \Rightarrow \tau_2 \rangle$, are inserted.

Blaming the annotation for ascriptions is perfectly reasonable. However, simply blaming the function and argument for their inserted casts respectively is too general. A better solution would be to somehow describe *why* a consistent, but different, type is produced under elaboration.

Notably, consistent but different types are produced during elaboration by use of the subsumption (EASubsume) rule. In a sense, type synthesis needs to produce a slice itself.

Consider EASubsume, then, as τ is an input it doesn't need a slice, it is obtained via some outer context. But τ' is synthesised via $\Gamma \vdash e \Rightarrow \tau' \leadsto d \dashv \Delta$, or equivalently via external typing directly $\Gamma \vdash e \Rightarrow \tau'$ (by elaboration generality Theorem 3.4 in paper).

As some rules (e.g. ESApp) use synthesis directly, the slice generation must be added to *external typing*, but may be reused for elaboration.

Type synthesis slices in External Typing

Listing a reasonable idea for each rule. I will draft a with a slice output, which may allow proofs about slicing.

I will represent slices simply as an expression with holes for terms which don't contribute to the type. More sophisticated methods including: dependencies between sub-slices or overlapping slices, tagging with type and/or type context, may be needed?

- SConstS $\frac{1}{\Gamma \vdash c \Rightarrow b \parallel c} c$ is clearly the only expression to blame here.
- SVarS $\frac{x : \tau \in \Gamma}{\Gamma \vdash x \Rightarrow \tau \parallel x}$ similar

• SLamS
$$\frac{\Gamma, x : \tau_1 \vdash e \Rightarrow \tau_2 \parallel s}{\Gamma \vdash \lambda x : \tau_1.e \Rightarrow \tau_1 \rightarrow \tau_2 \parallel \lambda x : \tau_1.s}$$

$$\Gamma \vdash e_1 \Rightarrow \tau_1 \parallel s_1 \quad \tau_1 \blacktriangleright_{\rightarrow} \tau_2 \rightarrow \tau$$

• SAppS $\frac{\Gamma \vdash e_1 \Rightarrow \tau_1 \parallel s_1 \quad \tau_1 \blacktriangleright_{\rightarrow} \tau_2 \to \tau}{\Gamma \vdash e_2 \Leftarrow \tau_2} - \text{for fresh } u.$ Discarding the argument potentially makes sense, in that the function contains synthesises the type information.

• SEHoleS
$$\Gamma \vdash ()^u \Rightarrow ? \parallel ()^u$$

• SNEHoleS
$$\frac{\Gamma \vdash e \Rightarrow \tau}{\Gamma \vdash (e)^u \Rightarrow ? \parallel (e)^u}$$

• SAsc
$$\frac{\Gamma \vdash e \Leftarrow \tau}{\Gamma \vdash e \Rightarrow \tau \parallel \bigcirc^u : \tau}$$
 – For fresh u .

• ALamS
$$\frac{\tau \blacktriangleright_{\rightarrow} \tau_1 \rightarrow \tau_2 \quad \Gamma, x : \tau \vdash e \Leftarrow \tau_2 \parallel s}{\Gamma \vdash \lambda x.e \Leftarrow \tau \parallel \lambda x.s} - \text{DEBAT-ABLE}$$

• ASubsumeS $\frac{\Gamma \vdash e \Rightarrow \tau \parallel s \quad \tau \sim \tau'}{\Gamma \vdash e \Leftarrow \tau' \parallel s}$ -Somehow a notion of slicing why subsumption is used is needed.

In a sense this can be thought of as replacing annotated expressions with holes, and recording uses of subsumption.

Properties for proof:

Clearly the sliced term should synthesise the same type.

Any change of a subterm to an inconsistent type should result in type checking failing or producing an inconsistent type. This requires slices to be passed through analysis mode. Look into Haack to see what they define, probably not this.

It should also be in some sense minimal – maybe that replacing any sub-expression with a hole results in a less specific type synthesised.

It should ideally also be the smallest term by size – but this is probably strictly dependent on the slicing method, other ways seem possible, including using constraints; the best idea is probably to conform with the bidirectional type checking method.

Cast Insertion

There will be a lot of redundancy so a good data-structure will be required for minimising duplicate sub-terms in AST forests. Further, use of holes (which has overhead) could be replaced with a simpler non-substitutable hole construct.

Dynamics

Splitting of casts can't really work with the slices defined above. But dependencies between casts can be traced and then unevaluated e.g. as in **FuctionalProgExplain**.

Maybe one idea is to have casts inherit their slices, then also produce an accompanying cast dependency graph tracking back to the original cast where the slice originates. Ideally, simplification of slices upon cast splitting would be conducted, but this may be difficult. Consider how this would generalise to sum, product types, operations etc..

3.2 Search Procedure

Track dynamic type refinement via provenances, so the same dynamic type is always refined in the same manner [21].

Look into Idris etc. etc.

Chapter 4 Implementation

Implementation Here.

Chapter 5 Evaluation

Evaluation Here.

Chapter 6 Conclusions

Conclusions Here.

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Appendix A

Formal Semantics and Proofs

Example Appendix Here

Appendix B Another Appendix

Another Example Appendix Here

Look at books for index structure reference. List key concepts, e.g. Core Hazel, Zipper, CMTT, Gradual Types.

Index

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Core Hazel, 3
Core Hazel syntax, 3
External language type system, 3
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Project Proposal

Description

This project will add some features to the Hazel language [22]. Hazel is a functional research language that makes use of gradual types to support unusual features such as: holes (code placeholders) to give type meaning to incomplete programs. Importantly for this project, all Hazel programs, even ill-typed or incomplete programs, are evaluable. This allows dynamic reasoning about ill-typed programs via evaluation traces with the potential to improve the user's understanding of why ill-typed programs go wrong. See example below:

But evaluation is still possible; see below a (compressed) trace to a stuck value exhibiting a cast error:

```
sum(2) \mapsto^* 2 + sum(1) \mapsto^* 2 + (1 + true^{\langle Bool \Rightarrow Int \rangle})
```

This project aims to exploit further this potential by providing some extra features to both: aid with finding values/inputs

that demonstrate why type-errors were found (type-error witnesses) and linking the evaluation traces back to source code. But is not expected to directly measure the usefulness of such evaluation traces themselves in debugging, nor is the design space for a Hazel debugger inspecting and interacting with traces to be explored.

Searching for type-error witnesses automatically is the main feature provided by this project, inspired by Seidel et al. [14]. The intended use of this is to automatically generate values (for example, function arguments) that cause ill-typed programs to 'go wrong' (lead to a cast error). More specifically, the search procedure can be thought of as evaluating a special hole which refines its type dynamically and non-deterministically instantiates itself to values of this type to find a value whose evaluation leads to a general cast error – 'general' meaning excluding trivial cast errors such as generating a value that doesn't actually have the refined expected type.

Such a search procedure is undecidable and subject to path explosion, hence the success criteria (detailed below) does not expect witnesses to be provided in general, even if they do exist. Sophisticated heuristics and methods to limit path explosion to support large code samples is not a core goal.

Formal semantics of this procedure and associated proofs is an extension goal, consisting of preservation proofs and witness generality proofs (formalising the notion of generality mentioned previously).

Secondly, cast slicing will track source code that contributed to any cast throughout the cast elaboration and evaluation phases. In particular, this allows a cast involved in a cast error relating to a type-error witness to point back to offending code. This is expected in some sense to be similar to blame tracking [18], error and dynamic program slicing [12], [13], although these are not directly relevant for this project.

Work required for the creation of an evaluation corpus of

ill-typed hazel programs, requiring manual work or creation of automated translation and/or fuzzing tools, is timetabled.

Starting Point

Only background research and exploration has been conducted. This consists of reading the Hazel research papers [22] and various other related research topics including: gradual types, bidirectional types, symbolic evaluation, OCaml error localisation and visualisation techniques.

More research, into the Hazel codebase in particular, and concrete planning is required and is timetabled accordingly.

Success Criteria

Core goals are the minimum expected goals that must be completed to consider this project a success. This corresponds to a working tool for a large portion of Hazel.

Extension goals will be timetabled in, but are relatively more difficult and not required for the project to be considered a success.

First, I give some definitions of terms:

- Core Calculus The formal semantics core of Hazel as referred to by the Hazel research papers [1].
- Basic Hazel A Hazel subset consisting of the core calculus, product and sum types, type aliases, bindings, (parametric) lists, bools, int, floats, strings, and their corresponding standard operations.
- Full Hazel Hazel, including Basic Hazel plus pattern matching, explicit impredicative system-F style polymorphism and explicitly recursive types.

- Core Corpus A corpus of ill-typed Hazel programs that are similar in complexity and size to student programs being taught a functional language, e.g. (incorrect) solutions to the ticks in FoCS. This will include examples in Basic or Full Hazel as required.
- Extended Corpus A corpus of ill-typed Hazel programs that are larger in size, more akin to real-world code.
- Evaluation Criteria Conditions for the search procedure to meet upon evaluation:
 - 1. Must have reasonable coverage success in finding an *existing* witness which is correct and general.
 - 2. Must find witnesses in an amount of time suitable for interactive debugging in-line with build-times for a debug build of existing languages.

Core Goals

- Success criteria for Cast Slicing Cast slicing must be correct (slices must include all code involved in the cast) and work for all casts, including casts involved in cast errors. Informal reasoning in evidence of satisfying these conditions is all that will be required.
- Success criteria for the Search Procedure The procedure must work for **Basic Hazel**, meeting the **Evaluation Criteria** over the **Core Corpus**. Analysis of some classes of programs for which witnesses could not be generated is also expected.

Extension Goals

- Search Procedure Extensions Support for **Full Hazel** under the same criteria as above.
- Search Procedure Performance Extensions Meeting of the **Evaluation Criteria** over an **Extended Corpus**
- Formal Semantics The specification of a formal evaluation semantics for the search procedure over the Core Calculus. Additionally, a preservation and witness generality proof should be provided.

Work Plan

21st Oct (Proposal Deadline) - 3rd Nov

Background research & research into the Hazel semantics, cast elaboration, type system, and codebase. Produce implementation plan for cast slicing and the search procedure for the **Core Calculus**. This includes an interaction design plan, expected to be very minimal.

Milestone 1: Plan Confirmed with Supervisors

4th Nov – 17th Nov

Complete implementation of Cast Slicing for the **Core Calculus**. Write detailed reasoning for correctness, including plan for **Basic Hazel**. Add unit testing.

Milestone 2: Cast slicing is complete for the **Core Calculus**.

18th Nov – 1st Dec (End of Full Michaelmas Term)

Complete implementation of the search procedure for the **Core** Calculus.

Milestone 3: Search Procedure is complete for the Core Calculus.

2nd Dec – 20th Dec

Extension of both cast slicing and the search procedure to **Basic Hazel**.

Milestone 4: Cast slicing & search procedure are complete for $\boldsymbol{Basic\ Hazel}$

21st Dec – 24th Jan (Full Lent Term starting 16th Jan)

Basic UI interaction for the project. Drafts of Implementation chapter. Slack time. Expecting holiday, exam revision, and module exam revision. Should time be available, the **Formal Semantics** extension will be attempted.

 ${\it Milestone~5:~Implementation~chapter~draft~complete}.$

25th Jan – 7th Feb (Progress Report Deadline)

Writing of Progress Report. Planning of evaluation, primarily including decisions and design of tools to be used to collect/create the **Core Corpus** and planning the specific statistical tests to conduct on the corpus. Collected corpus and translation method will be one of:

- 1. Manual translation of a small ill-typed OCaml program corpus into ill-typed Hazel.
- 2. Manual insertion of type-errors into a well-typed Hazel corpus.
- 3. Collection of a well-typed Hazel corpus.

 Tools: A Hazel type fuzzer to make the corpus ill-typed.
- 4. Collection of a well-typed OCaml corpus.

 Tools: OCaml-; Hazel translator/annotator which works with well-typed OCaml. A Hazel type fuzzer.
- 5. Collection of an ill-typed OCaml corpus.

 Tools: OCaml -¿ Hazel translator which works with ill-typed OCaml. This would NOT be expected to be an implicitly typed Hazel front-end which maintains desireable properties like parametricity.

Milestone 6: Evaluation plan and corpus creation method confirmed with supervisors.

Milestone 7: Underlying corpus (critical resource) collected.

8th Feb - 28th Feb

Implementation of the required tools for evaluation as planned. Some existing code or tools may be re-used, such as the OCaml type-checker.

Milestone 8: Core Corpus has been collected.

1st Mar - 15th Mar (End of Full Lent Term)

Conducting of evaluation tests and write-up of evaluation draft including results.

Milestone 9: Evaluation results documented.

Milestone 10: Evaluation draft complete.

16th Mar -30th Mar

Drafts of remaining dissertation chapters. If possible, collection and evaluation of **Extended Corpus** using the same tools as the **Core Corpus**.

Milestone 11: Full dissertation draft complete and sent to supervisors for feedback.

31st Mar - 13th Apr

Act upon dissertation feedback. Exam revision.

Milestone 12: Second dissertation draft complete and send to supervisors for feedback.

$14 ext{th Apr} - 23 ext{rd Apr}$ (Start of Full Easter Term)

Act upon feedback. Final dissertation complete. Exam revision.

Milestone 13: Dissertation submitted.

24th Apr – 16th May (Final Deadline)

Exam revision.

Milestone 14: Source code submitted.

Resource Declaration

• Underlying Corpus of either: Well-typed OCaml programs, Ill-typed OCaml programs, Hazel programs. For

use in evaluation. The required tools or manual translation to convert these into the ill-typed Hazel Core Corpus are detailed and allocated time in the timetable.

- Hazel source code. Openly available with MIT licence on GitHub [23].
- My personal laptop will be used for development, using GitHub for version control and backup of both code and dissertation. I accept full responsibility for this machine and I have made contingency plans to protect myself against hardware and/or software failure. A backup pc is available in case of such failure.