Maximiliano Cazares Martínez   Grupo: 3CV16

P1

$$f(t) = \begin{cases} 0 & -7 < t < -6 \\ 2 & -6 < t < -4 \\ -\tfrac{1}{2}t & -4 < t < 0 \\ \tfrac{1}{2}t & 0 < t < 4 \\ 2 & 4 < t < 6 \\ 0 & 6 < t < 7 \end{cases}$$

$T = 14$

$W_0 = \frac{2\pi}{T} = \frac{2\pi}{14} = \frac{\pi}{7}$

$f(t)$ es par

$b_n = \emptyset$

$$a_n = \frac{4}{14}\int_0^4 \left(\frac{t}{2}\right)\cos\left(\frac{n\pi t}{7}\right)dt + \frac{4}{14}\int_4^6 (2)\cos\left(\frac{n\pi t}{7}\right)dt$$

$u = t \quad du = dt$

$dv = \cos\left(\frac{n\pi t}{7}\right)dt \quad v = \frac{7}{n\pi}\operatorname{sen}\left(\frac{n\pi t}{7}\right)$

$$a_n = \frac{1}{7}\int_0^4 t\cos\left(\frac{n\pi t}{7}\right)dt + \frac{4}{7}\int_4^6 \cos\left(\frac{n\pi t}{7}\right)dt$$

$$a_n = \frac{1}{7}\left[\frac{7t}{n\pi}\operatorname{sen}\left(\frac{n\pi t}{7}\right)\Big|_0^4 - \frac{7}{n\pi}\int_0^4 \operatorname{sen}\left(\frac{n\pi t}{7}\right)dt\right] + \frac{4}{7}\left[\frac{7}{n\pi}\operatorname{sen}\left(\frac{n\pi t}{7}\right)\Big|_4^6\right]$$

$$a_n = \left[\frac{t}{n\pi}\operatorname{sen}\left(\frac{n\pi t}{7}\right)\Big|_0^4 + \frac{7}{n^2\pi^2}\cos\left(\frac{n\pi t}{7}\right)\Big|_0^4\right] + 4\left[\frac{1}{n\pi}\operatorname{sen}\left(\frac{n\pi t}{7}\right)\Big|_4^6\right]$$

$$a_n = \left\{\left[\frac{4}{n\pi}\operatorname{sen}\left(\frac{4n\pi}{7}\right) - \emptyset\right] + \frac{7}{n^2\pi^2}\left[\cos\left(\frac{4n\pi}{7}\right) - 1\right]\right\} + 4\left[\frac{1}{n\pi}\left[\operatorname{sen}\left(\frac{6n\pi}{7}\right) - \operatorname{sen}\left(\frac{4n\pi}{7}\right)\right]\right]$$

$$a_n = \frac{7}{n^2\pi^2}\left[\cos\left(\frac{4n\pi}{7}\right) - 1\right] + \frac{4}{n\pi}\operatorname{sen}\left(\frac{6n\pi}{7}\right)$$

$$a_0 = \frac{2}{14}\int_0^4 \frac{t\,dt}{2} + \frac{2}{14}\int_4^6 2\,dt = \frac{1}{14}\left(\frac{t^2}{2}\right)\Big|_0^4 + \frac{2}{7}(t)\Big|_4^6 = \frac{1}{28}\left[16-0\right] + \frac{2}{7}\left[6-4\right]$$

$$a_0 = \frac{16}{28} + \frac{4}{7} = \frac{4}{7} + \frac{4}{7} = \frac{8}{7}$$

$$f(t) = \frac{8}{7} + \sum_{n=1}^{\infty}\left\{\frac{4}{n\pi}\operatorname{sen}\left(\frac{6n\pi}{7}\right) + \frac{7}{n^2\pi^2}\left[\cos\left(\frac{4n\pi}{7}\right)-1\right]\right\}\cos\left(\frac{n\pi t}{7}\right)$$

Problema 2.

Sabiendo que $C_n = \frac{1}{2}(a_n - ib_n)$ entonces,

$$C_n = \frac{1}{2}\left[\frac{4}{n\pi}\operatorname{sen}\left(\frac{6n\pi}{7}\right) + \frac{7}{n^2\pi^2}\left[\cos\left(\frac{4n\pi}{7}\right)-1\right] + i(\phi)\right]$$

$$C_n = \frac{2}{n\pi}\operatorname{sen}\left(\frac{6n\pi}{7}\right) + \frac{7}{2n^2\pi^2}\left[\cos\left(\frac{4n\pi}{7}\right)-1\right]$$

$$\therefore \quad f(t) = \sum_{n=-\infty}^{\infty}\left\{\frac{2}{n\pi}\operatorname{sen}\left(\frac{6n\pi}{7}\right) + \frac{7}{2n^2\pi^2}\left[\cos\left(\frac{4n\pi}{7}\right)-1\right]\right\} e^{\frac{in\pi t}{7}}$$

Problema 3

$f(t) = A\operatorname{sen}(t) \quad 0 < t < \pi$

por tabla de transformadas, tenemos que:

$f(t) \leftrightarrow F(w)$

$\operatorname{sen}(w_0 t) \leftrightarrow i\pi\left[\delta(w+w_0) - \delta(w-w_0)\right]$

$\operatorname{sen}(t) \leftrightarrow i\pi\left[\delta(w+1) - \delta(w-1)\right]$

$A\operatorname{sen}(t) \leftrightarrow iA\pi\left[\delta(w+1) - \delta(w-1)\right]$

Problema 4.

$$\frac{t}{1-it} + Sa(2t-1) \leftrightarrow ?$$
① ②

① si $e^{-at}u(t) \leftrightarrow \frac{1}{a+iw}$

$$\frac{1}{a+it} \leftrightarrow e^{aw}u(-w)(2\pi)$$

$$\frac{1}{1+it} \leftrightarrow e^{aw}u(-w)(2\pi)$$

$$\frac{1}{1-it} \leftrightarrow e^{-aw}u(w)(2\pi)$$

$$\frac{t}{1-it} \leftrightarrow i2\pi\frac{d}{dw}\left[e^{-aw}u(w)\right]$$

② si $AC_d(t) \leftrightarrow Ad\,Sa\left(\frac{wd}{2}\right)$

$A=1$ y $d=2$

$$C_2(t) \leftrightarrow 2\,Sa(w)$$

$$2\,Sa(t) \leftrightarrow 2\pi\,C_2(w)$$

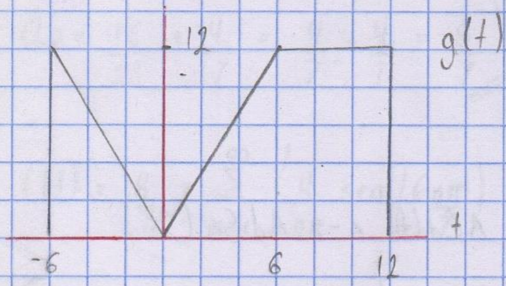
$$Sa(t) \leftrightarrow \pi\,C_2(w)$$

$$Sa(t-1) \leftrightarrow \pi\,C_2(w)e^{-iw}$$

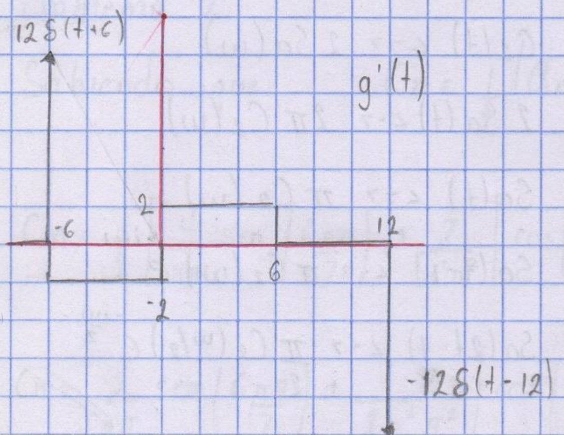
$$Sa(2t-1) \leftrightarrow \frac{\pi}{2}C_2(w/2)e^{-\frac{iw}{2}}$$

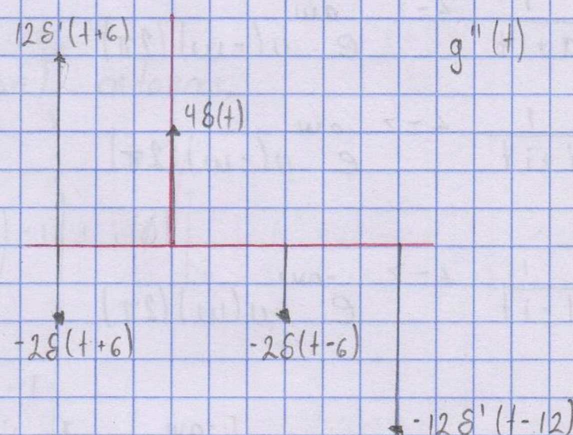
$$\frac{t}{1-it} + Sa(2t-1) \leftrightarrow i2\pi\frac{d}{dw}\left[e^{-aw}u(w)\right] + \frac{\pi}{2}C_2(w/2)e^{-\frac{iw}{2}}$$

problema 5.



$$g(t) = \begin{cases} -2t & -6 < t < 0 \\ 2t & 0 < t < 6 \\ 12 & 6 < t < 12 \end{cases}$$





$$\frac{d^2}{dt^2} g(t) = 12\delta'(t+6) - 2\delta(t+6) + 4\delta(t) - 2\delta(t-6) - 12\,\delta'(t-12)$$

$$A\delta'(t-t_0) \leftrightarrow iAw\,e^{-it_0w} \qquad A\delta(t-t_0) \leftrightarrow A\,e^{-it_0w}$$

$$\mathcal{F}\left\{\frac{d^2}{dt^2} g(t)\right\} = i12w\,e^{i6w} - 2e^{i6w} + 4 - 2e^{-i6w} - i12w\,e^{-i12w}$$

$$\text{si } \mathcal{F}\left\{\frac{d^2}{dt^2} g(t)\right\} = -w^2 F(w)$$

$$F(w) = -\frac{2}{w^2}\left[-i6w\,e^{i6w} + e^{i6w} - 2 + e^{-i6w} + i6w\,e^{-i12w}\right]$$

Problema 6

$$t e^{-t} u(t) \leftrightarrow \frac{1}{(1+iw)^2}$$

Expresamos $\frac{1}{(1+iw)^2}$ en formato magnitud y fase:

$$\frac{1}{(1+iw)^2} = \frac{1 \cdot e^{i\phi}}{\left[\sqrt{1+w^2} \cdot e^{i\tan^{-1} w}\right]^2}$$

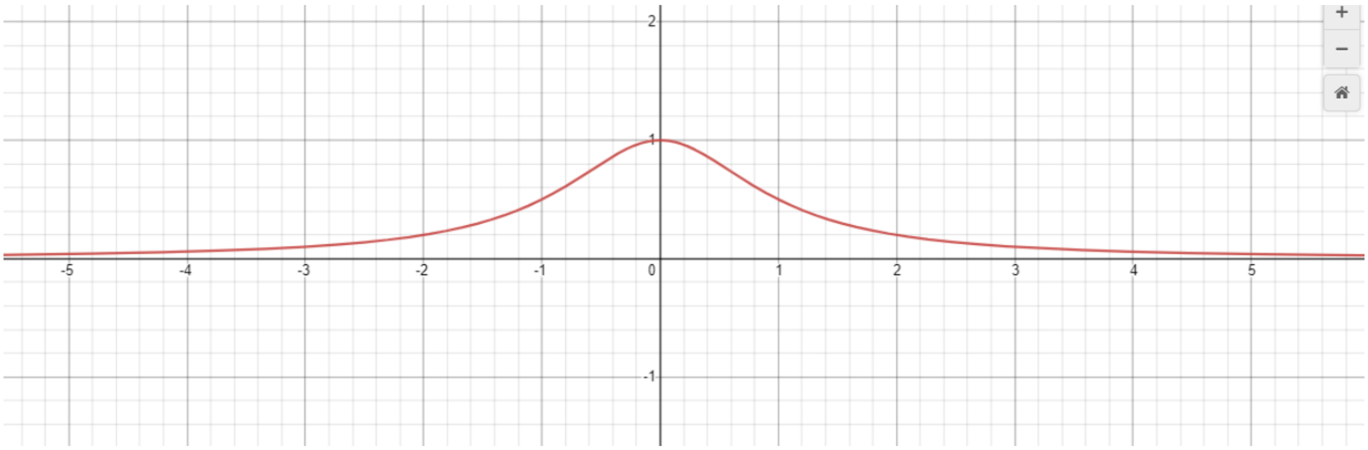
$$\frac{1}{(1+iw)^2} = \frac{e^{i\theta}}{(1+w^2) \cdot e^{i2\tan^{-1} w}}$$

$$F(w) = \frac{1}{1+w^2} e^{-i2\tan^{-1} w}$$

Finalmente:

$$|F(w)| = \frac{1}{1+iw^2} \quad y \quad \theta(w) = -2\tan^{-1}(w)$$

Espectro de magnitud



Espectro de fase