Instituto Politécnico Nacional

Escuela Superior de Cómputo

Teoría de comunicaciones y señales.

Problemario 2

Grupo: 3CV16

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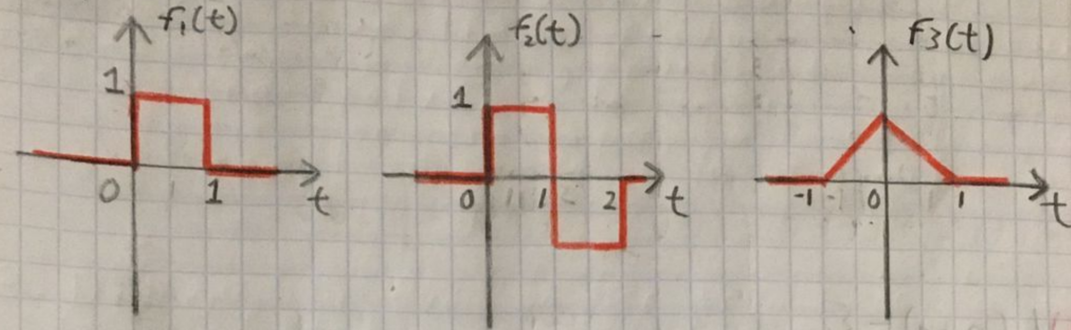
Ejercicios de Bermejo López Axel Nahir

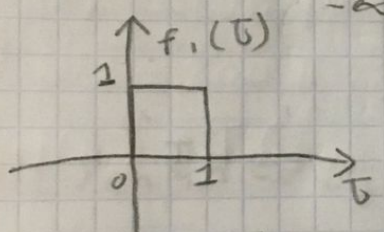
Sección 1

Problema 2. Evalúe las funciones de convolución para las señales mostradas en la figura 2.

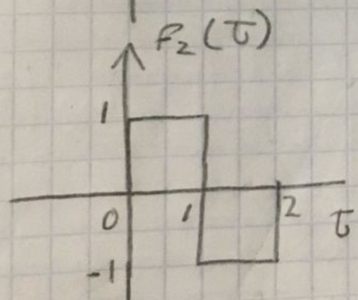


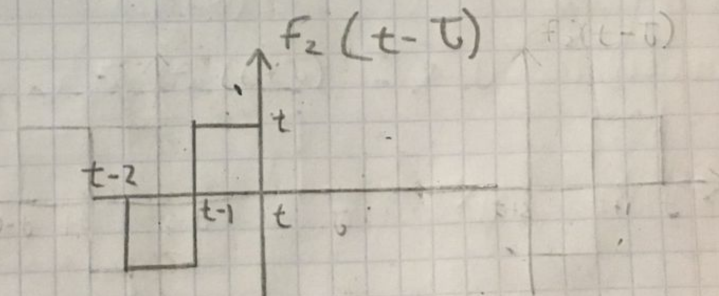
a) $f_1(t) * f_2(t)$.

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) \cdot f_2(t-\tau)\, d\tau$$

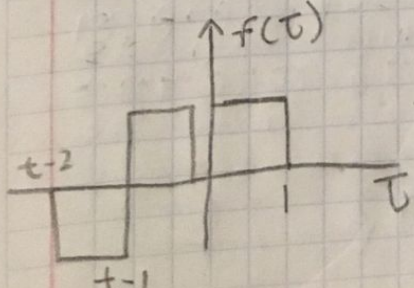


$$f_1(\tau) = \begin{cases} 1, & 0 < \tau < 1 \\ \emptyset, & \text{otro caso} \end{cases}$$



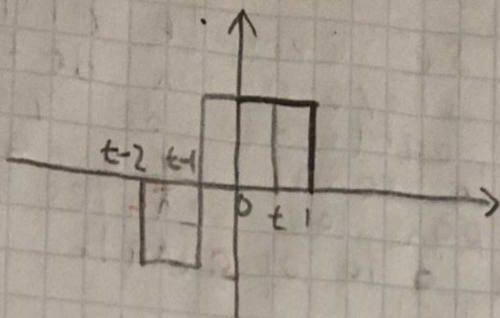


Caso $\emptyset$:



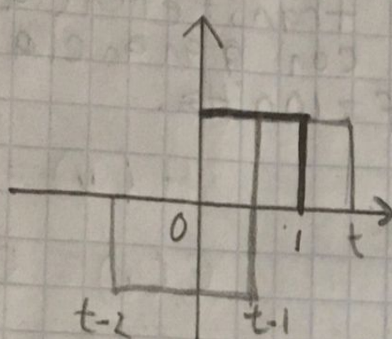
$$\int_{-\infty}^{\infty} f_1(\tau) \cdot f_2(t-\tau)\, d\tau = \emptyset$$

Caso 1:



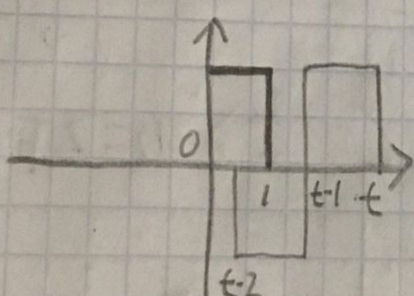
$$\int_0^t (-1)\cdot(1)\,d\tau \quad \begin{matrix} t>0 \\ t\le 1 \end{matrix} \Rightarrow \underline{0<t\le 1}$$

Caso 2:



$$\int_0^{t-1} (1)(-1)\,d\tau + \int_{t-1}^{1} (1)(1)\,d\tau \quad \begin{matrix} t>1 \\ t-1\le 1 \end{matrix} \Rightarrow \underline{1<t\le 2}$$

Caso 3:



$$\int_{t-2}^{1} (1)(-1)\,d\tau \quad \begin{matrix} t-1>1 \\ t-2\le 1 \end{matrix} \Rightarrow \underline{2<t\le 3}$$

Por lo tanto:

$$\underbrace{f_1(t) * f_2(t) = \underbrace{\int_0^1 (-1)(1)\,d\tau}_{0<t\le 1} + \underbrace{\int_0^{t-1} 1(-1)\,d\tau + \int_{t-1}^{1} (1)(1)\,d\tau}_{1<t\le 2} + \underbrace{\int_{t-2}^{1} (1)(-1)\,d\tau}_{2<t\le 3}}$$

Sección 2

3. ¿A qué se refiere el Efecto Alias?

R= Surge cuando no se cumple el teorema de muestra y provoca un deformamiento en la señal, partiendo de que se quiera volver a la señal original, es decir, se traslapan las señales y nos da una señal distinta a la original.

8. Dibuje la función de transferencia $H(\omega)$ de un filtro pasabajas con ganancia 5 y frecuencia de corte $f = 100$ Hz.

Tenemos que:

$H(\omega) = 5$

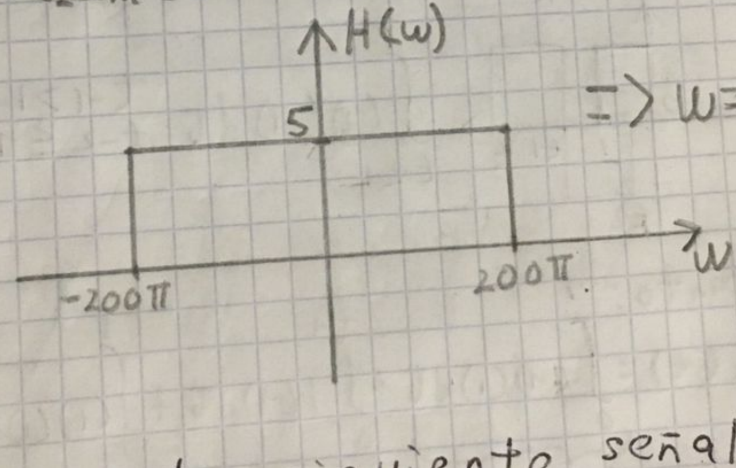
$C(\omega) = 5_{400}\, C(\omega)$

$f = 100$ Hz

$f_m = 100$ Hz

$\omega_m = 200\pi \ \frac{rad}{s}$

$H(\omega) = T\, C_{2\omega_m}(\omega) = 5\, C_{400\pi}$



$\Rightarrow \omega = 2\pi(100\ \text{Hz})$

13. Grafique la siguiente señal $y(n)$ que es una suma de sinusoides, indique su periodo de la suma es una periodo. ¿Cuál es el periodo de periodo N_1 y N_2? de dos sinusoides

$$y(n) = 10\,\text{sen}\left(\frac{3}{2}\pi n\right) + 5\cos\left(\frac{4}{9}\pi n\right)$$

Tenemos que:

$N_1 \to 10 \operatorname{sen}\left(\frac{3}{2}\pi n\right)$

$N_2 \to 5 \cos\left(\frac{4}{9}\pi n\right)$

Ahora tenemos que:

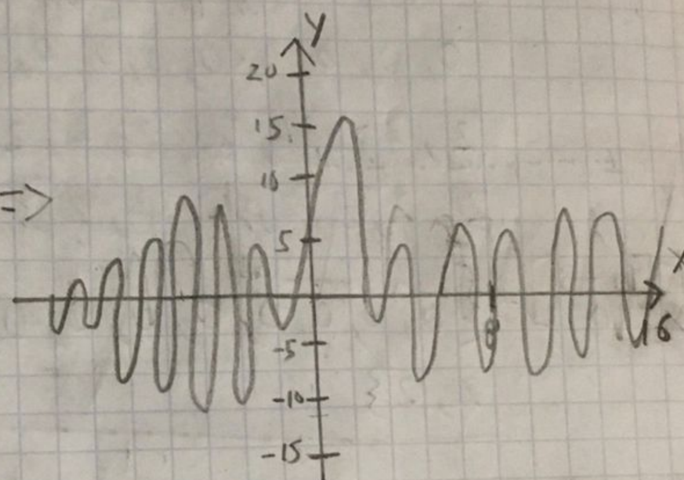
$N_1 = 4$

$N_2 = 18$

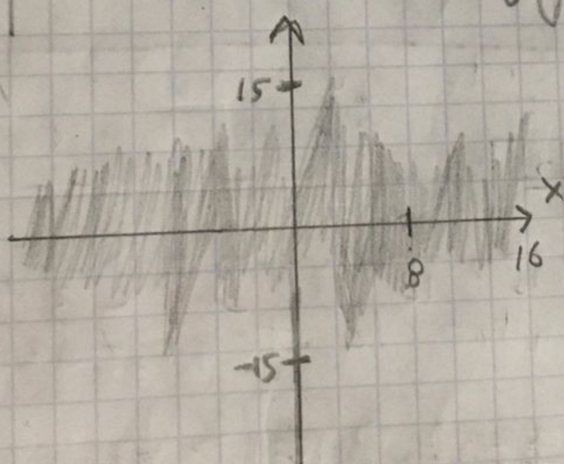
Por tanto:

4	18	2
2	9	2
1	9	3
1	3	3
1	1	

> 4, > 9 $> 36 \Rightarrow$



$\Rightarrow$



Sección 3

e) $\frac{1}{2} y[n+3]$:

$y[n+3] = \{2, 4, 8, 16, 32, 0, 0, \bar{0}\}$

$\frac{1}{2} y[n+3] = \{1, 2, 4, 8, 16, 0, 0, \bar{0}\}$

3.1.10

j) $k\left[\frac{4n-3}{10}\right]$:

$k[n] = \{\bar{0}, 0, 2, 2, 2, \dots\}$

$k[n-3] = \{\bar{0}, 0, 0, 0, 0, 2, 2, 2\}$

$k\left[\frac{n-3}{10}\right] = \{\bar{0}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$
$2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,$
$2, 2, 2, 2, 2, 2, 2\}$

$k\left[\frac{4n-3}{10}\right] = \{\bar{0}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2,$
$2, 2\}$

3.1.15

ñ) $g\left[\frac{3n-1}{2}\right] * y\left[\frac{2n}{2}+2\right]$

$g(3n) = \{\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \bar{\frac{1}{12}}, \frac{1}{15}, \frac{1}{18}, \dots\} = \{\frac{1}{3}, \bar{\frac{1}{12}}\}$

$g(3n-1) = \{\bar{\frac{1}{3}}, \frac{1}{12}\}$

$g\left(\frac{3n-1}{2}\right) = \{\bar{\frac{1}{3}}, \frac{1}{3}, \frac{1}{12}, \frac{1}{12}\}$

$y(2n) = \{2, 4, 8, 16, \overline{32}, \} = \{2, 8, \overline{32}\}$

$y\left(\frac{2n}{2}\right) = \{2, 2, 8, 8, \overline{32}, 32\}$

$y\left(\frac{2n}{n}+2\right) = \{2, 2, 8, 8, 32, 32, \bar{0}\}$

$g\left(\frac{3n-1}{2}\right) * y\left(\frac{2n}{2}+2\right) = \{\frac{1}{3}, \frac{8}{9}, \frac{86}{45}, \frac{59}{15}, \frac{118}{45}, \frac{76}{15}, \frac{136}{45}, \bar{\frac{16}{9}}, 0, 0\}$

Sección 4

Problema 2: Halle la transformada de Fourier de:

$$y(n) = \begin{cases} a^{n} & n \geq 0 \\ a^{-n} & n < 0 \end{cases}$$

$$y(n) = \sum_{n=-\infty}^{\infty} \Rightarrow y(n) = \sum_{n=-\infty}^{-1} a^{-n} e^{-i\Omega n} + \sum_{n=0}^{\infty} a^{n} e^{-i\Omega n}$$

$$y(n) = \sum_{n=1}^{\infty} a^{n} e^{i\Omega n} + \sum_{n=0}^{\infty} (ae^{-i\Omega})^{n} = \sum_{n=1}^{\infty} (ae^{i\Omega})^{n} + \sum_{n=0}^{\infty} (ae^{-i\Omega})^{n}$$

Por series geométricas tenemos que:

$$y(n) = \frac{ae^{i\Omega}}{1-ae^{i\Omega}} + \frac{1}{1-ae^{-i\Omega}} \quad \text{para} \quad |a| < 1$$

$$y(n) = \frac{ae^{i\Omega}(1-ae^{i\Omega}) + 1 - ae^{i\Omega n}}{(1-ae^{i\Omega})(1-ae^{-i\Omega})}$$

$$y(n) = \frac{ae^{i\Omega} - a\cancel{e^{0}}^{1} + 1 - ae^{i\Omega}}{1 - ae^{i\Omega} - ae^{i\Omega} + a^{2}e^{0}} = \frac{1-a^{2}}{1+a^{2}-2a\cos\Omega}$$

4.1.7

Problema 7: Halle la FFT para la siguiente secuencia:

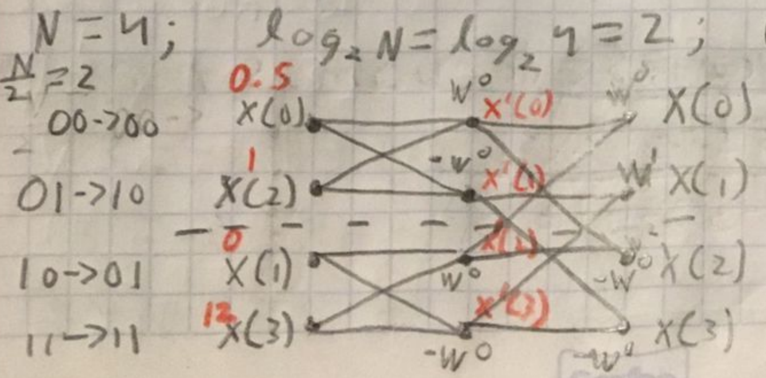
$$x[n] = [0.5,\ 1,\ 0,\ 12]$$

$X(0)$, $X(1)$, $X(2)$, $X(3)$

Tenemos que:

$N = 4$; $\log_2 N = \log_2 4 = 2$; $w^{2} = \left(e^{-i\frac{2\pi}{4}}\right)^{2} = \left(e^{-i\frac{\pi}{2}}\right)^{2} = -1$

$\frac{N}{2} = 2$



1ra etapa:

$X'(0) = 0.5 + W^0(1) = 1.5$ $\quad$ $X'(1) = 0.5 - W^0(1) = -0.5$

$X'(2) = 0 + W^0(12) = 12$ $\quad$ $X'(3) = 0 - W^0(12) = -12$

2da etapa:

$X(0) = X'(0) + W^0 X'(2) = 1.5 + 12 = 13.5$

$X(1) = X'(1) + W^1 X'(3) = -0.5 + (-i)12 = -0.5 + j12$

$X(2) = X'(0) - W^0 X'(2) = 1.5 - 12 = -10.5$

$X(3) = X'(1) - W^1 X'(3) = -0.5 - (-i)(-12) = -0.5 - 12j$

FFT hay muestras complejas conjugadas.

Demostrando:

$X(0)$

$X(1)$

$\cancel{X(2)}$

$X(3)$

=> Es cierto que $X(1)$ y $X(3)$ se reflejan es decir, son conjugados

4.1.12

Problema 12: Encuentre la transformada Z inversa:

$$X(z) = \frac{z(z+1)(z-5)}{z^3 - 5z^2 + 7z - 3} \qquad |z| > \frac{1}{3}$$

$$X(z) = \frac{z^3 - 4z^2 - 5z}{z^3 - 5z^2 + 7z - 3}$$

Dividiendo:

$$\begin{array}{r|l} z^3-5z^2+7z-3 & \begin{array}{l} 1 \\ \hline z^3-4z^2-5z \\ \underline{-z^3+5z^2-7z+3} \\ 0+z^2-12z+3 \end{array} \end{array}$$

$$X(z) = 1 + \underbrace{\frac{z^2-12z+3}{z^3-5z^2+7z-3}}$$

$$\frac{z^2-12z+3}{(z-1)^2(z-3)}$$

$$\frac{z^2-12z+3}{(z-1)^2(z-3)} = \frac{A}{(z-3)} + \frac{B}{(z-1)} + \frac{C}{(z-1)^2}$$

$$z^2-12z+3 = A(z-1)^2 + B(z-1)(z-3) + C(z-3)$$

- $z = 1$

$\Rightarrow 1-12+3 = C(-2)$

$-8 = -2C$

$4 = C$

- $z = 3$

$\Rightarrow 9-36+3 = A(4)$

$-24 = 4A$

$-6 = A$

- $z = 0, \quad A = -6, \quad C = 4$

$3 = -6 + B(3) + 4(-3)$

$3 = -6 + 3B - 12$

$3 + 18 = 3B \rightarrow B = \frac{21}{3} \rightarrow B = 7$

$\boxed{A = -6}$

$\boxed{B = 7}$

$\boxed{C = 4}$

$$\frac{z^2-12z+3}{(z-1)^2(z-3)} = \frac{-6}{(z-3)} + \frac{7}{(z-1)} + \frac{4}{(z-1)^2}$$

$$X(z) = 1 + \frac{z(z^2-12z+3)}{z^3-5z^2+7z-3}$$

$$X(z) = 1 - \frac{6z}{(z-3)} + \frac{7z}{(z-1)} + \frac{4z}{(z-1)^2}$$

$$Z^{-1}\{X(z)\} = Z^{-1}\{1\} + 6Z^{-1}\left\{\frac{z}{z-3}\right\} + 7Z^{-1}\left\{\frac{z}{z-1}\right\}$$

$$+ 4z^{-1}\left\{\frac{z}{(z-1)^2}\right\}$$

$$x(n) = \underline{\delta(n) + 6(3)^n + 7 + 4n(1)^n} \quad \text{con } n \geq 0$$

Ejercicios de Cazares Martínez Maximiliano

Sección 1. Convolución Continua.

Problema 1. Calcular las siguientes integrales de convolución.

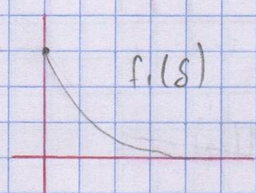
c) $e^{-t}u(t) * e^{-3t}u(t)$

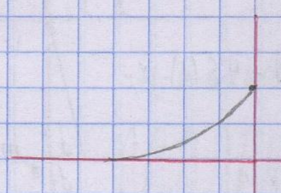
si $f_1(t) = e^{-t}u(t)$

$f_2(t) = e^{-3t}u(t)$

$$\Rightarrow \int_{-\infty}^{\infty} f_1(\delta)\cdot f_2(t-\delta)\,d\delta$$

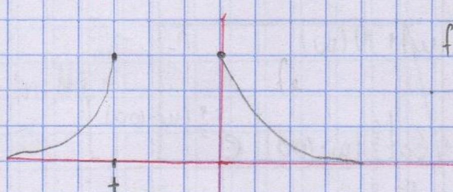
$f_1(\delta) = e^{-\delta}u(\delta)$ $\qquad$ $f_2(-\delta) = e^{3\delta}u(-\delta)$





$$f_2(t-\delta) = e^{3(t-\delta)}u(-\delta)$$

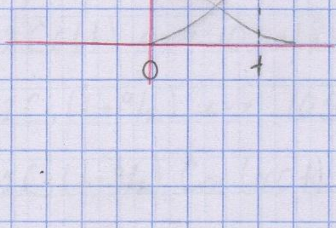
Caso $\emptyset$:



$f_2(t-\delta) < 0$

$$\int_{-\infty}^{\infty} f_1(t)\cdot f_2(t-\delta)\,d\delta = \emptyset$$

caso 1:



$$\int_0^t f_1(t)\cdot f_2(t-\delta)\,d\delta = \int_0^t e^{3(t-\delta)}\cdot e^{-\delta}\,d\delta = \int_0^t e^{3t-4\delta}\,d\delta$$

$$= \frac{1}{3t-4t}e^{3t-4t} - \frac{1}{3t}e^{3t} = -\frac{e^{-t}}{t} - \frac{e^{3t}}{3t} \quad \forall\, t \geq 0$$

Sección 2. Teorema de Muestreo y Tiempo discreto.

2) Enuncie el teorema de muestreo.

Toda señal limitada en banda que no contiene frecuenciales mayores a Hz y rad/s esta completamente representada por el conjunto de sus muestras formadas a intervalos uniformes no mayores de ½ fm (s).

Es decir $T \leq \frac{1}{2fm}$ ó $f \geq 2fm$

7) Aquella frecuencia que de acuerdo con el teorema de muestreo, debe ser excedida por la frecuencia de muestreo se llama razón de Nyquist. Determine la razón de Nyqist correspondiente a cada una de las siguientes señales.

a) $x(t) = 10\,sen(wt) + 5\,sen(2wt)$

$10\,sen(wt) \leftrightarrow ?$

$10\,sen(wt) \leftrightarrow i10\pi\,[\delta(\omega+w) - \delta(\omega-w)]$

$5\,sen(2wt) \leftrightarrow ?$

$5\,sen(2wt) \leftrightarrow i5\pi\,[\delta(w+2w) - \delta(\omega-2w)]$

La componente frecuencia $W_m = 2w$

así $fm = \frac{2w}{2\pi} = \frac{w}{\pi}$ $\quad f \geq 2fm \Rightarrow f \geq 2\left(\frac{w}{\pi}\right)$

$f_N = \frac{2w}{\pi}$ Hz

b) $x(t) = 1 + \cos(2000\pi t) + \text{sen}(4000\pi t)$

$1 \leftrightarrow ?$

$\delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\,\delta(\omega)$

$\cos(2000\pi t) \leftrightarrow ?$

$\cos(2000\pi t) \leftrightarrow \pi[\delta(\omega+2000\pi) + \delta(\omega-2000\pi)]$

$\text{sen}(4000\pi t) \leftrightarrow ?$

$\text{sen}(4000\pi t) \leftrightarrow i\pi[\delta(\omega+4000\pi) - \delta(\omega-4000\pi)]$

$x(t) = 2\pi\delta(\omega) + \pi[\delta(\omega+2000\pi) + \delta(\omega-2000\pi) + i\delta(\omega+4000\pi) - i\delta(\omega-4000\pi)]$

La componente frecuencia más grande es $\omega_m = 4000\pi$

$f_m = \dfrac{\omega_m}{2\pi} = \dfrac{4000\pi}{2\pi} = 2000\ Hz$

$f \geq 2f_m \Rightarrow f \geq 4000\ Hz \therefore f_N = 4000\ Hz$

12) Analice las siguientes secuencias (esto es, su frecuencia digital), e indique si son o no periódicas. En caso de ser periódicas, halle su periodo.

a) $x(n) = \cos\left(\frac{2\pi n}{3}\right) + e^{\pi n}$

$X_a(n) = \cos\left(\frac{2\pi n}{3}\right)$ es periodica

$X_b(n) = e^{\pi n}$, no es periodica

$\therefore$ $x(n)$ no es periodica

b) $y(n) = 2 + Re\left(e^{\frac{i\pi n}{3}}\right) + \cos\left(\frac{3\pi n}{2}\right)$

$y_1(n) = 2 = \{\ldots 2, 2, 2, \bar{2}, 2, 2, 2, \ldots\}$

$y_2(n) = Re\left(e^{\frac{i\pi n}{3}}\right) = \cos\left(\frac{\pi n}{3}\right) = \left\{\ldots, 1, \frac{1}{2}, \frac{1}{2}, -1, \frac{1}{2}, \frac{1}{2}, \bar{1}, \frac{1}{2}, \frac{1}{2}, -1, \frac{1}{2}, \frac{1}{2}, \ldots\right\}$

$N = 6$

$Y_3(n) = \cos\left(\frac{3\pi n}{2}\right) = \{\ldots, -1, 0, 1, 0, -1, 0, 1, 0, -1, 0, \ldots\}$

$n = 1$

$\therefore N = 12$

c) $10 \operatorname{sen}\left(\frac{3\pi n}{2}\right)$

$10 \operatorname{sen}\left(\frac{3\pi n}{2}\right) = 10\cos\left(\frac{3\pi n}{2} + \frac{3\pi}{2}\right)$

$2\pi F_N = \frac{3}{2}\pi N$

$F = \frac{3}{4} \Rightarrow N = 4$

b) $5\cos\left(\frac{4\pi n}{9}\right)$

$5\cos\left(\frac{4\pi n}{9}\right) \Rightarrow A\cos(2\pi f_N + d)$

$2\pi F_n = \frac{4}{9}\pi n$

$F = \frac{4}{18} \Rightarrow N = 18$

e) $2\cos\left(\frac{4n}{9}\right)$

$2\cos\left(\frac{4n}{9}\right) \Rightarrow A\cos(2\pi F n + \phi)$

$2\pi F = \frac{4n}{9}$

$F = \frac{4}{18\pi}$ $\therefore$ No es periodica

Sección 3. Operaciones básicas entre secuencias.

Problema 1. Considere las siguientes secuencias y realice con ellas las siguientes operaciones.

$x(n) = \{\bar{0}, 1, 2, 3, 4, 5, 6, 7, 8\}$ $\quad y(n) = \{2, 4, 8, 16, \bar{32}\}$

$z(n) = \sum_{k=-3}^{3} \delta(n-k)$ $\quad g(n) = \left\{\frac{1}{3}, \frac{1}{6}, \bar{\frac{1}{12}}, \frac{1}{15}, \frac{1}{18}, \dots\right\}$

$K(n) = 2u(n-2)$

d) $y(n-6) = \{\bar{0}, 0, 2, 4, 8, 16, 32\}$

i) $y\left(\frac{n-3}{3}\right) = \left\{2, \frac{8}{3}, \frac{10}{3}, 4, \frac{16}{3}, \frac{20}{3}, 8, \frac{32}{3}, \frac{40}{3}, \bar{16}, \frac{64}{3}, \frac{80}{3}, 32\right\}$

n) $x\left(\frac{n}{2}\right) * y\left(\frac{n}{3}\right) =$

$x\left(\frac{n}{2}\right) = \{\bar{0}, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8\}$

$y\left(\frac{n}{3}\right) = \{2, 2, 2, 4, 4, 4, 8, 8, 8, 16, 16, 16, \bar{32}, 32, 32\}$

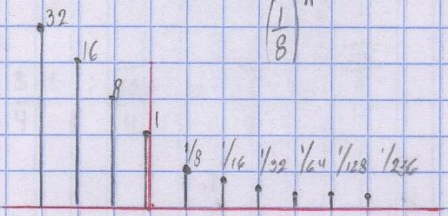
$x\left(\frac{n}{2}\right) * y\left(\frac{n}{3}\right) = \{0, 0, 2, 4, 8, 14, 22, 32, 48, 66, 90, \bar{124}, 164, 214, 286, 308,$
$472, 559, 640, 709, 788, 832, 896, 936, 960, 960, 970,$
$896, 832, 766, 256\}$

Sección 4. Transformada de Fourier y transformada Z.

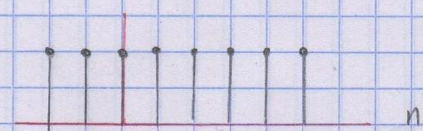
1) Halle la transformada de Fourier de $y(n) = \left(\frac{1}{8}\right)^n u(n+2) + 2^n u(-n)$

sea

$\left(\frac{1}{8}\right)^n$



$u(n+2)$

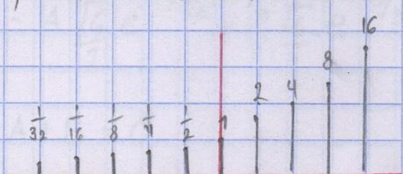


$$y_1(n) = \begin{cases} 0 & n < -2 \\ (1/8)^n & n \geq -2 \end{cases}$$

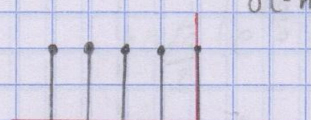
$$Y_1(\Omega) = \sum_{n=-2}^{\infty} \left(\frac{1}{8}\right)^n e^{-i\Omega n} = \sum_{n=-2}^{\infty} \left(\frac{1}{8} e^{-i\Omega}\right)^n \qquad n+2<0$$

$$\left|\frac{1}{8} e^{-i\Omega}\right| = \frac{1}{8} < 1 \Rightarrow Y_1(\Omega) = \frac{\left(\frac{1}{8} e^{-i\Omega}\right)^{-2}}{1 - \frac{1}{8} e^{-i\Omega}} = \frac{8^2 e^{-i2\Omega}}{\frac{8 - e^{-i\Omega}}{8}} = \frac{8^3 e^{2i\Omega}}{8 - e^{-i\Omega}}$$

$$y_2(n) = 2^n u(-n) = \begin{cases} 0 & n > 0 \\ 2^n & n \leq 0 \end{cases}$$



$u(-n)$



$$Y_2(\Omega) = \sum_{n=-\infty}^{0} 2^n e^{-i\Omega n} = \sum_{n=-\infty}^{0} \left(2 e^{-i\Omega}\right) = \sum_{n=0}^{n=\infty} \left(2 e^{-i\Omega}\right)^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{2} e^{i\Omega}\right)^n$$

$$\left|\frac{1}{2} e^{i\Omega}\right| = \frac{1}{2} < 1 \Rightarrow Y_2(\Omega) = \frac{1}{1 - \frac{1}{2} e^{i\Omega}} = \frac{2}{2 - e^{i\Omega}}$$

$$\therefore\ Y(n) = Y_1(n) + Y_2(n) = \frac{8^3 e^{2i\Omega}}{8 - e^{-i\Omega}} + \frac{2}{2 - e^{i\Omega}} = \frac{(8^3 e^{2i\Omega})(2 - e^{i\Omega}) + 2(8 - e^{-i\Omega})}{(8 - e^{-i\Omega})(2 - e^{i\Omega})}$$

$$= \frac{(2 \cdot 8^3) e^{2i\Omega} - 8 e^{3i\Omega} + 16 - 2 e^{-i\Omega}}{17 - 8 e^{i\Omega} - 2 e^{-i\Omega}}$$

11) Encuentre la transformada Z inversa.

$$Y(z) = \frac{z}{2z^2 - 3z + 3} \qquad |z| > \frac{3}{2}$$

$$X(z) = \frac{z}{2z^2 - 3z + 3} = \frac{z}{\left(z - \left[\frac{3}{4} + \frac{\sqrt{15}}{4} i\right]\right)\left(z - \left[\frac{3}{4} - \frac{\sqrt{15}}{4} i\right]\right)}$$

$$\frac{X(z)}{z} = \frac{1}{\left[z - \left(\frac{3}{4} + i\frac{\sqrt{15}}{4}\right)\right]\left[z - \left(\frac{3}{4} - i\frac{\sqrt{15}}{4}\right)\right]} = \frac{A}{z - \left(\frac{3}{4} + i\frac{\sqrt{15}}{4}\right)} + \frac{B}{z - \left(\frac{3}{4} - i\frac{\sqrt{15}}{4}\right)}$$

$$\frac{3}{4} \pm i\frac{\sqrt{15}}{4} \Rightarrow r = \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{\sqrt{15}}{4}\right)^2} = \frac{\sqrt{6}}{2} \; ; \quad \theta = \tan^{-1}\left(\frac{\sqrt{15}/4}{3/4}\right) = \frac{\sqrt{15}}{3}$$

$$\frac{3}{4} \pm i\frac{\sqrt{15}}{4} = \frac{\sqrt{6}}{2} e^{\pm i\frac{\sqrt{15}}{3}}$$

$$\frac{X(z)}{z} = \frac{1}{\left(z - \frac{\sqrt{6}}{2} e^{i\sqrt{15}/3}\right)\left(z - \frac{\sqrt{6}}{2} e^{-i\sqrt{15}/3}\right)} = \frac{A}{z - \frac{\sqrt{6}}{2} e^{i\sqrt{15}/3}} + \frac{B}{z - \frac{\sqrt{6}}{2} e^{i\sqrt{15}/3}}$$

$$1 = A\left(z - \frac{\sqrt{6}}{2} e^{-i\sqrt{15}/3}\right) + B\left(z - \frac{\sqrt{6}}{2} e^{i\sqrt{15}/3}\right)$$

$$1 = (A+B)z - A\frac{\sqrt{6}}{2} e^{-i\sqrt{15}/3} - B\frac{\sqrt{6}}{2} e^{i\sqrt{15}/3}$$

$$-A\frac{\sqrt{6}}{2} e^{-i\sqrt{15}/3} - B\frac{\sqrt{6}}{2} e^{i\sqrt{15}/3} = 1$$

$$A + B = 0$$

$$-A\frac{\sqrt{6}}{2} e^{-i\frac{\sqrt{15}}{3}} - B\frac{\sqrt{6}}{2} e^{i\frac{\sqrt{15}}{3}} = 1$$

$$\cancel{A\frac{\sqrt{6}}{2} e^{-i\frac{\sqrt{15}}{2}}} + B\frac{\sqrt{6}}{2} e^{-i\frac{\sqrt{15}}{3}} = 0$$

$$-B\frac{\sqrt{6}}{2}\left(e^{i\frac{\sqrt{15}}{3}} - e^{-i\frac{\sqrt{15}}{3}}\right) = 1$$

$$-B\frac{\sqrt{6}}{2}\left[i2\,\mathrm{sen}\left(\frac{\sqrt{15}}{3}\right)\right] = 1$$

$$-i\sqrt{6}\,B\,\mathrm{sen}\left(\frac{\sqrt{15}}{3}\right)=1$$

$$B=\frac{i}{\sqrt{6}\,\mathrm{sen}\left(\frac{\sqrt{15}}{3}\right)}\quad ; \quad A=-\frac{i}{\sqrt{6}\,\mathrm{sen}\left(\frac{\sqrt{15}}{3}\right)}$$

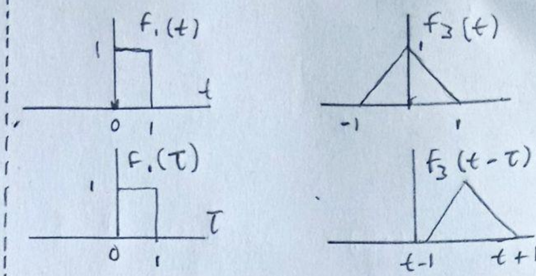
$$X(z)=\frac{-i}{\sqrt{6}\,\mathrm{sen}\left(\frac{\sqrt{15}}{3}\right)}\left[\frac{z}{z-\left(\frac{3}{4}+i\frac{\sqrt{15}}{4}\right)}\right]+\frac{i}{\sqrt{6}\,\mathrm{sen}\left(\frac{\sqrt{15}}{3}\right)}\left[\frac{z}{z-\left(\frac{3}{4}-i\frac{\sqrt{15}}{4}\right)}\right]$$

$$x(n)=\frac{-i}{\sqrt{6}\,\mathrm{sen}\left(\frac{\sqrt{15}}{3}\right)}\left(\frac{3}{4}+i\frac{\sqrt{15}}{4}\right)^{n}+\frac{i}{\sqrt{6}\,\mathrm{sen}\left(\frac{\sqrt{15}}{3}\right)}\left(\frac{3}{4}-i\frac{\sqrt{15}}{4}\right)^{n}$$

Ejercicios de Martínez Medrano Gerardo

S1 P2 b) 1.2.2

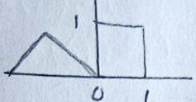
b) $f_1(t) * f_3(t) = \int_{-\infty}^{\infty} f_1(\tau)\, f_3(t-\tau)\, d\tau$



$f_3(\tau) \begin{cases} (\tau+1) & -1<\tau<0 \\ -(\tau-1) & 0<\tau<1 \\ \emptyset & \text{otros casos} \end{cases}$

$f_3(-\tau+t) \begin{cases} (-\tau+t+1) & -1<-\tau+t<0 ; \ell_1 \\ -(-\tau+t-1) & 0<-\tau+t<1 ; \ell_2 \end{cases}$

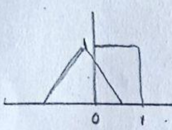
caso $\emptyset$



$\underbrace{\int_{-\infty}^{\infty} f_1(\tau)\, f_3(t-\tau)\, d\tau = \emptyset}_{t<-1}$

$t+1<0 \rightarrow t<-1$

caso I

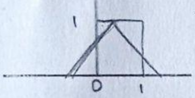


$\int_{-\infty}^{\infty} f_1(\tau) f_3(t-\tau)\, d\tau = \underbrace{\int_0^{t+1} (1)\, \ell_2\, d\tau}_{-1<t<\emptyset}$

$t<0 \ , \ t+1>0 \rightarrow t>-1$

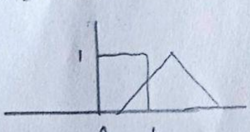
caso II

$t-1<0 \rightarrow t<1$



$\int_{-\infty}^{\infty} f_1(\tau) f_3(t-\tau) = \underbrace{\int_0^{t} (1)\, \ell_1 + \int_t^{1} (1)\, \ell_2\, d\tau}_{0<t<1}$

caso III



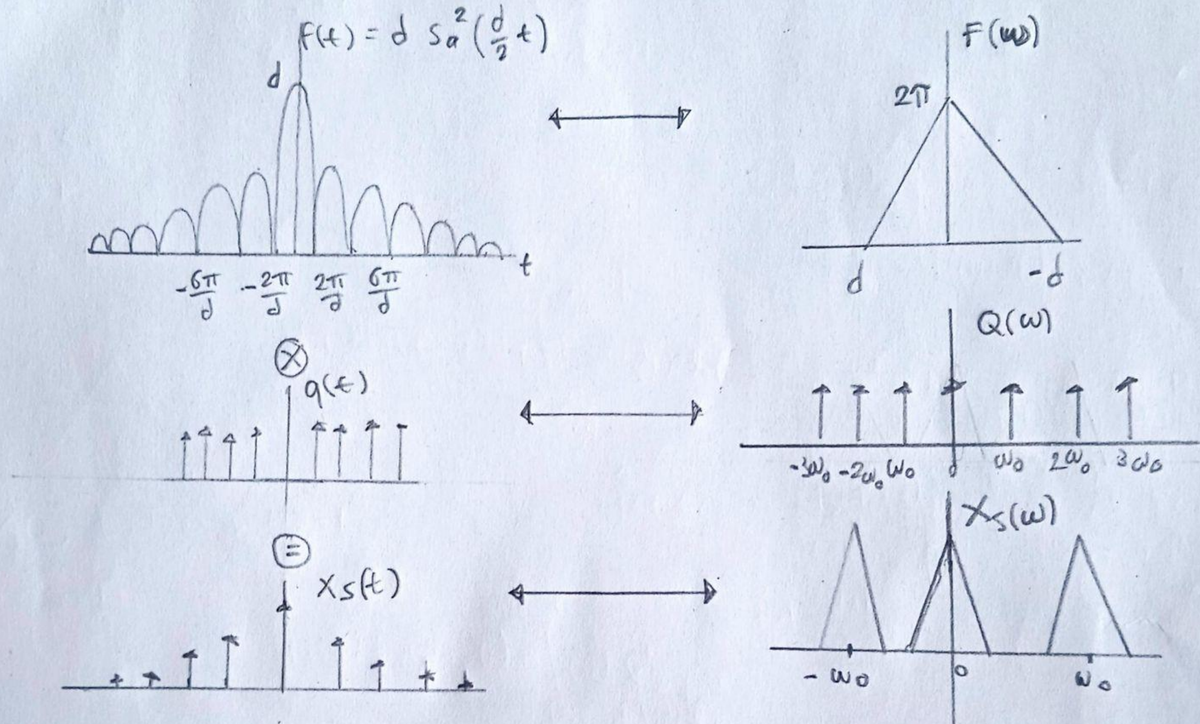
$= \underbrace{\int_{t-1}^{1} (1)\, \ell_1\, d\tau}_{1<t<2}$

$f_1(t) * f_3(t) = \underbrace{\int_0^{t+1} \ell_2}_{-1<t<0} + \underbrace{\int_0^{t} \ell_1 + \int_t^{1} \ell_2}_{0<t<1} + \underbrace{\int_{t-1}^{1} \ell_1}_{1<t<2}$

$f_1(t) * f_3(t) = -\underbrace{\int_0^{t+1} (-\tau+t-1)\, d\tau}_{-1<t<0} + \underbrace{\int_0^{t} (-\tau+t+1)\, d\tau}_{0<t<1} - \underbrace{\int_t^{1} (-\tau+t-1)\, d\tau}_{0<t<1} + \underbrace{\int_{t-1}^{1} (-\tau+t+1)\, d\tau}_{1<t<2}$

S2 P4) 2.1.4

4. Considere la siguiente funcion en el tiempo $f(t)$ y su transformada $F(\omega)$. Desarrolle grafica y matematicamente el muestreo ideal de $f(t)$



$$X_s(t) = \sum_{n=-\infty}^{\infty} f(nT)\,\delta(t-nT)$$

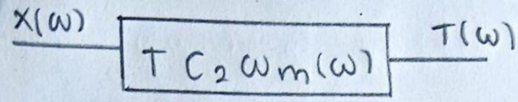
$$Q(\omega) = 2\pi \sum_{n=-\infty}^{\infty} C_n\,\delta(\omega - n\omega_0)$$

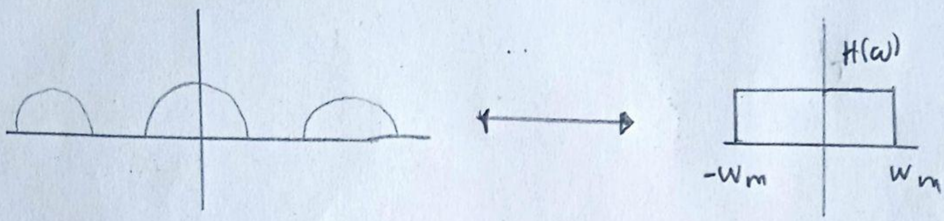
$$= \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

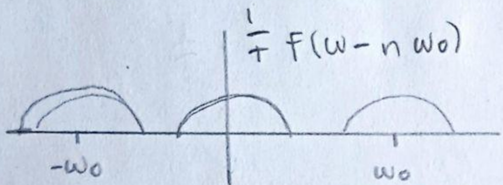
$$X_s(t) \longleftrightarrow \frac{1}{T} \sum_{n=-\infty}^{\infty} F(\omega - n\omega_0)$$

S2 P9) 2.1.9

9. Dibuje la funcion caracteristica $h(t)$ del filtro anterior







$\omega_0 < 2\omega m$

S3 P a) 3.1.1

a) $g[-n] = \left\{\ldots, \frac{1}{18}, \frac{1}{15}, \bar{\frac{1}{12}}, \frac{1}{9}, \frac{1}{6}, \frac{1}{3}\right\}$

S3 P K) 3.1.11

K) $K\left[-\frac{n}{4}+10\right]$

$\quad$ 1 2 3 4

$K[n] = 2u(n-2) = [\bar{0}, 0, 2, 2, \ldots]$

$K\left[\frac{n}{4}\right] = [\bar{0}, 0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 2, 2, 2, 2, \ldots]$

$K\left[\frac{n}{4}+10\right] = [2, 2, \bar{2}, 2, 2, 2, 2, 2, \ldots]$

$K\left[-\frac{n}{4}+10\right] = [\ldots, 2, 2, 2, 2, 2, \bar{2}, 2; 2]$

S3 P F) 3.1.6

F) $x[n-2] = \{\bar{0}, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$

S3 P2 3.2.1

Encuentre la grafica de la secuencia de convolucion de dos secuencias definidas como: $x(n)=\{1, \bar{2}, 3, 4, 3, 2, 1\}$ y $y(n)=\{-2, -1, \bar{0}, 1, 2\}$ Algoritmo Soma por colomnas

$$
\begin{array}{rrrrrrrrrrr}
 & & & & 1 & \bar{2} & 3 & 4 & 3 & 2 & 1 \\
 & & & & & & -2 & -1 & \bar{0} & 1 & 2 \\
\hline
 & & & & -2 & -4 & -6 & -8 & -6 & -4 & -2 \\
 & & & -1 & -2 & -3 & -4 & -3 & -2 & -1 & \\
 & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
 & 1 & 2 & 3 & 4 & 3 & 2 & 1 & & & \\
2 & 4 & 6 & 8 & 6 & 4 & 2 & & & & \\
\hline
\end{array}
$$

$-2, -5, -8, -\bar{10}, -6, 0, 6, 10, 8, 5, 2$

$x(n) * y(n) = \{-2, -5, -8, -\bar{10}, -6, 0, 6, 10, 8, 5, 2\}$

S4 P3) 4.1.3

3. Encuentre la transformada inversa de Fourier de $X(\Omega)$

$$X(\Omega) = \begin{cases} 2i & 0<\Omega<\pi \\ -2i & -\pi<\Omega<0 \end{cases}$$

$$x(n) = \int_{-\pi}^{\pi} X(\Omega)\, e^{i\Omega n}\, d\Omega = \int_{-\pi}^{0} -2i\, e^{i\Omega n}\, d\Omega + \int_{0}^{\pi} 2i\, e^{i\Omega n}\, d\Omega$$

$$x(n) = 2i \int_{0}^{\pi} e^{i\Omega n}\, d\Omega - 2i \int_{-\pi}^{0} e^{i\Omega n}\, d\Omega$$

$$x(n) = 2\cancel{i}\left[\frac{1}{\cancel{i}n} e^{i\Omega n}\right]\Big|_{0}^{\pi} - 2\left[\frac{1}{n} e^{i\Omega n}\right]\Big|_{-\pi}^{0}$$

$$x(n) = \frac{2}{n}\left[e^{i\pi n} - \cancel{e}^{1}\right] - \frac{2}{n}\left[\cancel{e}^{1} - e^{-i\pi n}\right]$$

$$x(n) = \frac{2}{n}\left(e^{i\pi n} + e^{-i\pi n} - 2\right) \cdot \frac{2}{2}$$

$$x(n) = \frac{4}{n}\left(\cos(\pi n) - 1\right)$$

S4, P8) 4.1.8

8. Encuentre la transformada z y grafique la region de convergencia de $x(n)$

$$x(n) = \left(\frac{1}{4}\right)^n u(n) + 2\left(\frac{1}{2}\right)^n u(-n-1)$$

Transformada z

$$X(z) = Z\{x(n)\} = \sum_{n=\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = \underbrace{\sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n}}_{(I)} + \underbrace{2\sum_{n=-\infty}^{-1} \left(\frac{1}{2}\right)^n z^{-n}}_{(II)}$$

(I)

$$\sum_{n=0}^{\infty} \left(\frac{1}{4} z^{-1}\right)^n \Rightarrow \sum_{n=0}^{\infty} \left(\frac{1}{4z}\right)^n$$

$$X(z) = \frac{1}{1-\frac{1}{4z}}, \text{si } \left|\frac{1}{4z}\right| < 1 \Rightarrow X(z) = \frac{4z}{4z-1} \quad \text{si } \frac{|1|}{|4||z|} < 1$$

$$\Rightarrow \frac{4z}{4z-1} \quad \text{si } \underbrace{\frac{1}{4} < |z|}_{\text{R.O.C.}} \qquad z = \frac{1}{4} \text{ es un polo}$$

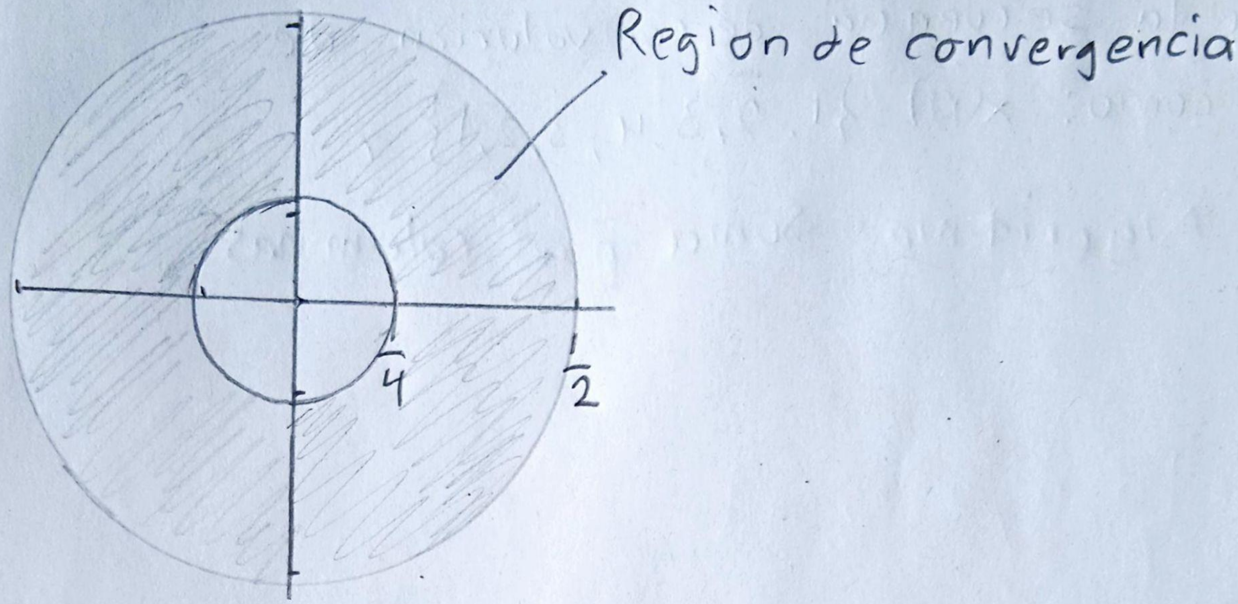
(II)

$$2\sum_{n=-\infty}^{-1} \left(\frac{1}{2z}\right)^n \Rightarrow 2\sum_{n=1}^{\infty} \left(\frac{1}{2z}\right)^{-n} \Rightarrow 2\sum_{n=1}^{\infty} (2z)^n$$

$$\sum_{n=m}^{\infty} \alpha^k = \frac{\alpha^m}{1-\alpha} \quad |\alpha| < 1$$

$$\Rightarrow 2\,\frac{2z}{1-2z} \quad \text{si } |2z| < 1$$

$$\Rightarrow \frac{4z}{1-2z} \quad \text{si } |z| < \frac{1}{2} \qquad \text{Polo } z = \frac{1}{2}$$



Region de convergencia
1/4
1/2

Ejercicios de Ramos Nieves Adrián

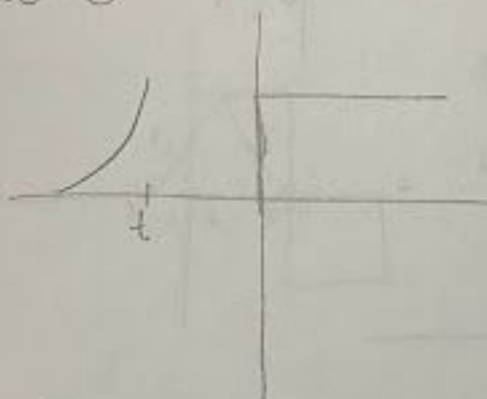
Problemario

Problema 1- Calcular las siguientes integrales de convolución

a) $u(t) * e^{-t} u(t)$

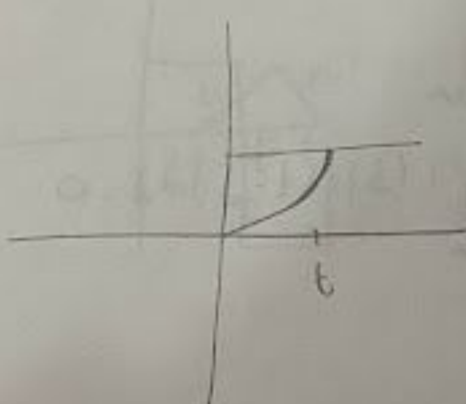
$$Z(t) = \int_{-\infty}^{\infty} f(S) f_2(t-S)\, dS$$

Caso 0



$$\int_{-\infty}^{\infty} f(S) f(t-S)\, dS = 0$$

Caso 1



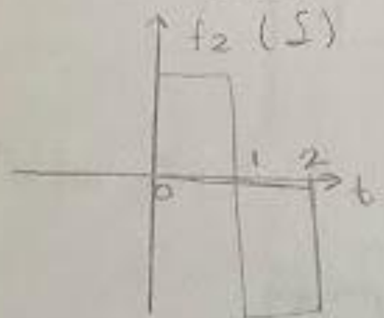
$$\int_0^t e^{t \cdot S}\, dS = \frac{1}{2t} e^{2t} - \frac{1}{t} e^{t}$$

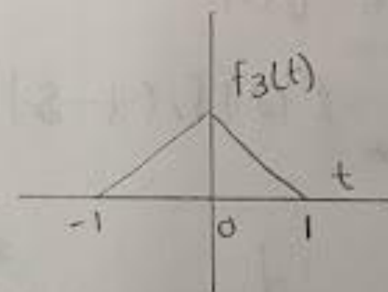
$$= \frac{e^t}{2t} [e^t - t]$$

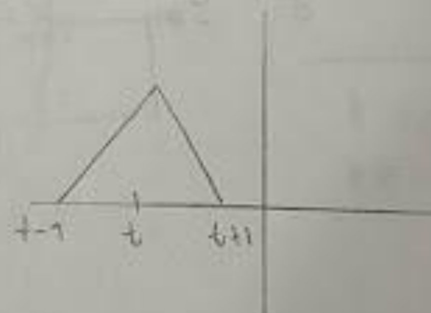
$$t \geq 0$$

Problema 2 Evalúe las funciones de convolución para las señales mostradas en la figura 2

c) $f_2(t) * f_3(t)$

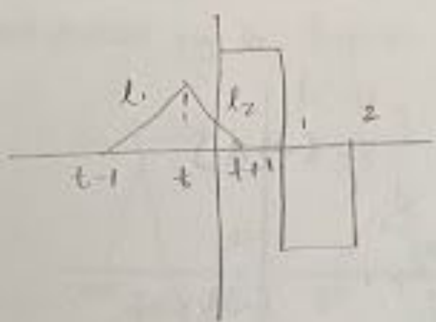






Caso 0

$$\int_{-\infty}^{\infty} f(\delta) f(t-\delta)\, d\delta = 0$$

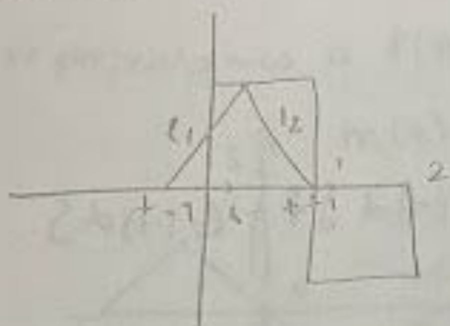


$$\int_0^{t+1} l_2(1)\,d\delta$$

$t+1>0$
$t \leq 0$

$t < 1 \leq 0$

Caso 2



$$\int_0^{t} l_1(1)\,d\delta + \int_t^{1} l_2(1)\,d\delta$$

$t>0$
$t+1 \leq 1$

$0 < t \leq 0$

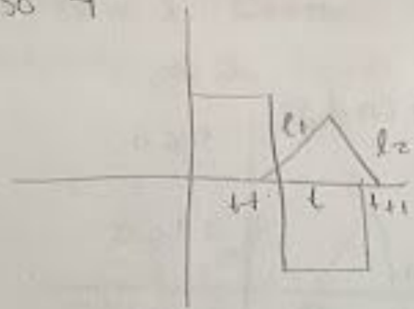
Caso 3

$$\int_{t-1}^{1} l_1\,d\delta + \int_1^{t+1} l_2(-1)\,d\delta$$

$t+1 > 1$
$t \leq 1$

$0 < t \leq 1$

Caso 4

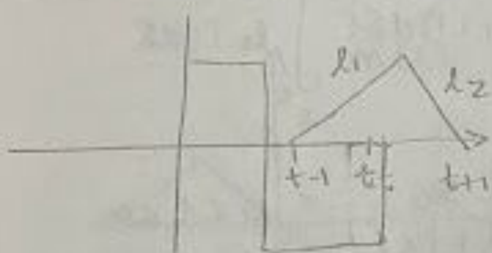


$$\int_{t-1}^{1} l_1\,dS + \int_{1}^{t} l_2(-1)\,dS$$

$t > 1$ $\quad 1 < t \leq 2$

$t-1 \leq 1$

Caso 5

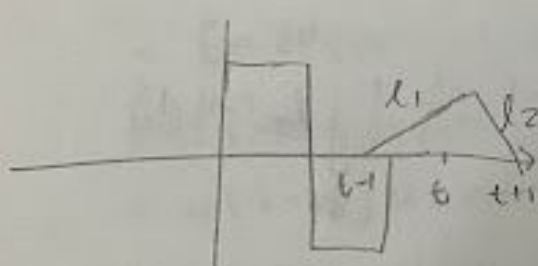


$$\int_{1}^{t} l_1(-1)\,dS + \int_{t}^{2} l_2(-1)\,dS$$

$t-1 > 1$ $\quad 2 < t \leq 2$

$t \leq 2$

Caso 6



$$\int_{t-1}^{2} l_1(-1)\,dS$$ $t > 2$, $t-1 \leq 2$

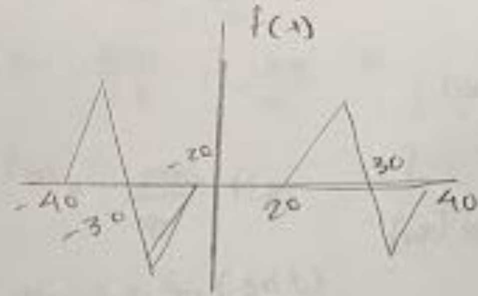
$2 < t \leq 3$

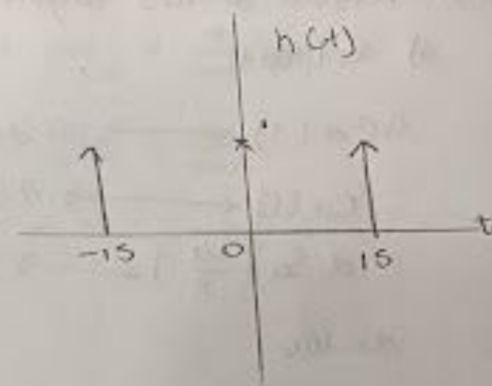
Por lo tanto

$$z(t) * f_3(t) = \int^{t+1} l_2\,dS + \int_{t-1}^{1} l_1\,dS + \int^{t+1} l_2(-1)\,dS + \int_{t-1}^{1} l_1\,dS + \int_{1}^{2} l_2(1)\,dS + \int_{t+1}^{2} l_1(-1)\,dS$$

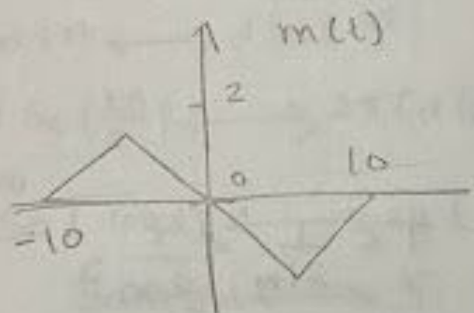
$-1 < t < 0$ $\quad 0 < t \leq 1$ $\quad 1 < t \leq 2$ $\quad 2 < t \leq 3$

Problema 3. Obtener y dibujar $f(t) * h(t)$ para las funciones mostradas en la figura 3

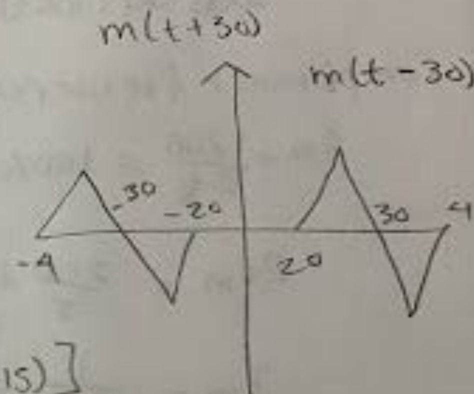




Si representamos a $f(t)$ como



$\Rightarrow$



$$f(t) * m(t) = f(t) * [\delta(t+15) + \delta(t-15)]$$

$$= [m(t+30) + m(t-30)] * [\delta(t+15) + \delta(t-15)$$

$$= m(t+30) * \delta(t+15) + m(t+30) * \delta(t-15) + m(t-30) * \delta(t+15)$$

$$+ m(t-30)\delta(t-15)$$

$$= m(t+30+15) + m(t+30-15) + m(t-30+15) + m(t-30-15)$$

$$= \underline{m(t+45) + m(t+15) + m(t-15) + m(t-45)}$$

5: Determine la rapidez minima de muestreo y el intervalo de Nyquist de las siguientes señales

a) $Sa(100t)$

$$A C_d(t) \longleftrightarrow A d\, Sa\left(\frac{wd}{2}\right)$$

$$C_d(t) \longleftrightarrow d\, Sa\left(\frac{wd}{2}\right)$$

$$d\, Sa\left(\frac{td}{2}\right) \longleftrightarrow 2\pi C_d(w)$$

$d = 100$

$$200\, Sa(100t) \longleftrightarrow 2\pi C_{200}(w)$$

Componente frecuencial

$$f_m = \frac{200}{2\pi} = 100/\pi \text{ Hz}$$

$$2f_m = \frac{200}{\pi} \text{ Hz} \qquad T \leq \frac{1}{2f_m} = \frac{\pi}{200} \text{ s}$$

$$T_N = \frac{1}{2f_m} = \frac{\pi}{200} \text{ s}$$

b) $Sa^2(100t)$

$$A\left(\frac{t}{d}\right) \longleftrightarrow d\, Sa^2\left(\frac{wd}{2}\right)$$

$$d\, Sa^2\left(\frac{td}{2}\right) \longleftrightarrow 2\pi \Lambda\left(\frac{w}{d}\right)$$

$d = 200$

$$200\, Sa^2(100t) \longleftrightarrow 2\pi \Lambda\left(\frac{w}{200}\right)$$

$$Sa^2(100t) \longleftrightarrow \frac{\pi}{100} \Lambda\left(\frac{w}{200}\right)$$

Componente frecuencial

$\omega = 200 \ \frac{rad}{s}$

$f_m = \frac{200}{2\pi} = \frac{100}{\pi} \ Hz$

$T \leq \frac{1}{2f_m} = \frac{\pi}{200} \ seg$

$T_N = \frac{1}{2f_m} = \frac{\pi}{200} \ s$

$2f_m = \frac{200}{\pi} \ Hz$

c) $Sa(100t) + Sa(50t)$

$Sa(100t) \longleftrightarrow ?$

$C_d(t) \longleftrightarrow d\,Sa\left(\frac{\omega d}{2}\right)$

$d\,Sa\left(\frac{td}{2}\right) \longleftrightarrow 2\pi\,C_d(\omega)$

$d = 200$

$200\,Sa(100t) \longleftrightarrow 2\pi\,C_{200}(\omega)$

$Sa(100t) \longleftrightarrow \frac{\pi}{100}\,C_{200}(\omega)$

$Sa(50t) \longleftrightarrow ?$

$C_d(t) \longleftrightarrow d\,Sa\left(\frac{\omega d}{2}\right)$

$d\,Sa\left(\frac{td}{2}\right) \longleftrightarrow 2\pi\,C_d(\omega)$

$d = 100$

$100\,Sa(50t) \longleftrightarrow \frac{2\pi}{}\,C_{100}(\omega)$

$Sa(50t) \longleftrightarrow \frac{\pi}{50}\,C_{100}(\omega)$

$Sa(100t) + Sa(50t) \longleftrightarrow \frac{\pi}{100}\,C_{200}(\omega) + \frac{\pi}{50}\,C_{100}(\omega)$

Componente frecuencial

$\omega_m = 200 \text{ rad/s}$

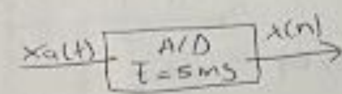
$f_m = \frac{200}{2\pi} = \frac{100}{\pi} \text{ Hz}$

$2f_m = \frac{200}{\pi} \text{ Hz}$

$T \leq \frac{1}{2f_m} = \frac{\pi}{200} \text{ S}$

$T_N = \frac{1}{2f_m} = \frac{\pi}{200} \text{ seg}$

10. Considere el sistema mostrado en la figura siguiente. La señal de entrada al sistema $x_a(t) = 3\cos 100\pi t + 2\,\text{sen}\,250\pi t$. Determine la versión discreta de $x_a(t)$. ¿Es posible recuperar la señal original a partir de $x(n)$ usando un filtro pasa bajas adecuado?



$f_m = \frac{100\pi}{2\pi} = 50$ $\qquad T_n = \frac{1}{2f_m} = \frac{1}{200}\,s$

$2f_m = 100\,Hz$ $\qquad f_n = 2f_m = 100\,Hz$

∴ Si es posible recuperar la señal usando un filtro paja bajas

Sección 3- Operaciones Básicas entre secuencias.

Problema 1 Considere las siguientes secuencias y realice con ellas las operaciones indicadas.

Si

$x[n] = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ $\qquad y[n] = \{2, 4, 8, 16, 32\}$

$z[n] = \sum_{k=-3}^{3} \delta(n-k)$ $\qquad g[n] = \{\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15}, \frac{1}{18}, \dots\}$

$k[n] = 2u(n-2)$

b) $z[n] + y[n]$

$z[n] + y[n] = \{1, 1, 1, 1, 2, 3, 4, 4, 5, 6, 7, 8\}$

g) $x[3n-3]$

$x[n-3] = \{\bar{0}, 0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$

$x[3n-3] = \{\bar{0}, 0, 3, 6\}$

1) $x\left[\frac{3n-3}{3}\right] - g\left[-\frac{8n-7}{3}\right]$

$x[3n] = \{\bar{0}, 1, 2, 3, 4, 5, 6, 7, 8\} = \{\bar{0}, 3, 6\}$

$x[3n-3] = \{\bar{0}, 0, 0, 0, 3, 6\}$

$x\left[\frac{3n-3}{3}\right] = \{\bar{0}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 3, 3, 3, 6, 6, 6, 6]$

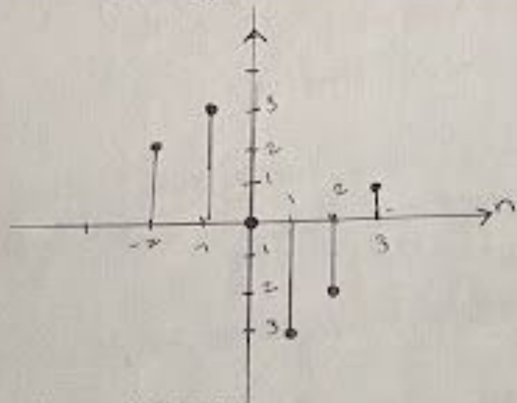
$g[8n] = \{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6},$
$\frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}$
$\frac{1}{12}, \frac{1}{12}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}\}$

$g[-8n-7] = \{\frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}$
$\frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9},$
$\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}$
$\frac{1}{3}, \frac{1}{3}\}$

$g\left[\frac{-8n-7}{3}\right] = \{\frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}$

$x\left[\frac{3n-3}{3}\right] - g\left[-\frac{8n-7}{3}\right] = \{\ldots, -\frac{1}{18}, -\frac{1}{9}, \frac{2}{3}, 2, 3, 4, 5, 6, 7, 8\}$

Problema 3. A partir de la secuencia mostrada en la figura



Encuentre $x\left(\frac{3n+3}{-4}\right)$ y $g(n) = x(n) * [x[\frac{n}{2}+1] - (u[n+1] - 4[n-3]]$

$$X(n) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$$

$$X(n) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n} = x(-2)e^{j2\Omega} + x(-1)e^{-j\Omega} + x(0)e^{0} + x(1)e^{-j\Omega} + x(2)e^{-2j\Omega}$$

$$= 2(e^{j2\Omega} + e^{-2j\Omega}) + e^{j\Omega} + e^{-j\Omega}$$

$$= 4\cos 2\Omega + 2\cos\Omega$$

Sección 4. Transformada de fourier y transformada Z

Problema 4. Calcule la transformada discreta de fourier (DFT) de $x(n)$

$$x(n) = [0, \tfrac{1}{2}, 1, 2, 3, -4]$$

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}Kn}$$

$N=6$ $\quad x(k) = \sum_{n=0}^{6} x(n) e^{i \frac{2\pi}{N} kn}$

con $k=0$

$$x(0) = \sum_{n=0} x(n) e^{0}$$

$$x(0) = x(0) + x(1) + x(2) + x(3) + x(4) + x(5)$$

$$x(0) = 0 + \tfrac{1}{2} + 1 + 2 - 3 - 4 = -7/2$$

$k=1$

$$x(1) = \sum_{n=0}^{5} x(n) e^{-i \frac{2\pi}{5} n}$$

$$x(1) = x(0) e^{0} + x(1) e^{-i\frac{2\pi}{5}} + x(2) e^{-i\frac{4\pi}{5}} + x(3) \cancel{e^{-i\frac{2\pi}{6}}}^{0} + x(4) \cancel{e^{-i\frac{8\pi}{6}}}^{1}$$

$$+ x(5) \cancel{e^{-i3\pi}}^{1}$$

$$X(1) = 0 - \tfrac{1}{2}i - 1 - i - 3i - 4 = -5 - \tfrac{9}{2} i$$

$k=2$

$$x(2) = \sum_{n=0}^{5} x(n) e^{-i \frac{4\pi}{5} n}$$

$$x(2) = x(0) e^{0} + x(1) e^{-i\frac{4\pi}{5}} + x(2) e^{-i\frac{8\pi}{5}} + x(3) e^{-i\frac{12\pi}{5}} + x(4) e^{-i\frac{16}{5}\pi}$$

$$+ x(5) e^{-i3\pi} = -\frac{3}{2} + \frac{3}{2} i$$

$$x(3) = -1 - 4i$$

$$x(4) = x(0) e^{0} + x(1) e^{-i\frac{8}{5}\pi} + x(2) e^{-i\frac{16}{5}\pi} + x(3) e^{-i\frac{24\pi}{5}} + x(4) e^{-i\frac{32\pi}{5}}$$

$$+ x(5) e^{-18\pi}$$

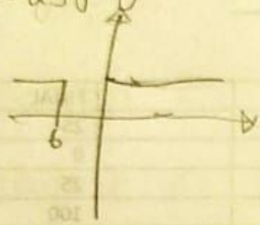
$$x(4) = \frac{1}{2} - 1 - i - 2 - 2i + 3i - 4 = \frac{15}{2}$$

Ejercicios de Rivero Enriquez Daniel Alejandro

Tema 1

b) $u(t) * u(t)$

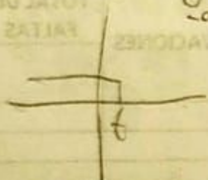
Caso 0



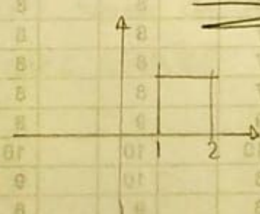
$u(t) * u(t) = \int_{-\infty}^{\infty} u(\tau * u(t-\tau) = 0 ; \forall t<0$

Caso 1

$u(t) * u(t) = \int_{-\infty}^{\infty} u(\tau * u(t-\tau) = \int_0^t 1 \, d\tau = \tau \Big|_0^t = t-0 = t \quad \forall t>0$



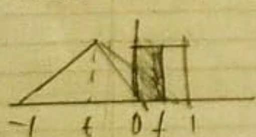
$u(t) * u(t) = \begin{cases} 0 & t<0 \\ t & t>0 \end{cases}$



Tema 2

b) $f_1(t) * f_3(t)$

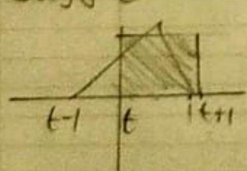
Caso 1



$f_1 * f_3 = \int_0^{t+1} I^2(\tau-1)\, d\tau = t+1>0 \quad t\le 0$

$-1<t\le 0$

Caso 2

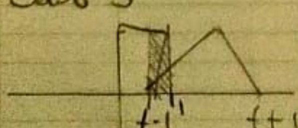


$= \int_0^1 I^2(\tau+1)\, d\tau - \int_t^1 I^2(\tau-1)\, d\tau$

$t+1>1 \quad t\le 1$

$0<t\le 1$

Caso 3



$= \int_{t-1}^1 I^2(\tau+1)\, d\tau$

$t>1 \quad t-1\le 1$

$1<t\le 2$

$$f_1(t) * F_3(t) = -\int_0^{t+1} I^2(\tau-1)\,d\tau + \int_0^{t} I^2(\tau+1)\,d\tau \quad 0<t\le 1$$

$-1<t\le 0$

$0<t\le 1$

$$-\int_t^{1} I^2(\tau-1)\,d\tau + \int_{t-1}^{0} I^2(\tau+1)\,d\tau \quad 1<t<2$$

Seccion 2

(2) enuncie el teorema del Muestreo

"Toda señal limitada en banda que no contiene componentes de frecuencia mayores al limite esta representada por el conjunto de sus muestras tomadas a intervalos T no mayores a $\frac{1}{2f_m}$ seg o sea $T \le \frac{1}{2f_m}$ dond t es el intervalo del muestreo

(5) determina la rapidez minima de muestreo y el intervalo Nysquist de las siguientes señales

a) $Sa(100t)$

$\omega_{max} = 100$ rad/seg

$F_{max} = \frac{100}{2\pi} = 16\ Hz$ $\quad T_N = \frac{1}{32\ Hz} = 31{,}25\ ms$

$F_N = 2(16\ Hz) = 32\ Hz$

b) $Sa^2(100t)$

$F_N = 32\ Hz \quad T_N = 31{,}25\ ms$

c) $Sa(100t) + Sa(50t)$

$F_N = 32\ Hz \quad T_N = 31.25\ ms$

8 dibuje la funcion de transferencia $H(w)$ de un filtro pasabajas con una ganancia de 5 y una frecuencia de corte de 100 Hz

$X_s(w)$

$X_s(w) \longrightarrow \boxed{\text{Filtro pasabajas } H(w) = T C_{2wm}(w)} \longrightarrow X_s(w) \cdot Hw$

Funcion de transferencia

$T = 5$

$w_m = ?$

$f_m = 100 \text{ Hz}$

$w_m = 2\pi f_m \ (\text{rad}/\text{seg})$

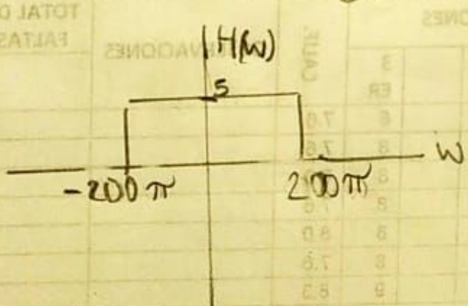
$= 2\pi (100 \text{ Hz})$

$w_m = 200\pi \ \text{rad}/\text{seg}$

$H(w) = T \ C_{2wm}(w)$

$= 5 \ C_{2(200\pi)}(w)$

$= 5 \ C_{400\pi}(w)$



Seccion 3

a) $g(-n) = \left\{ \ldots, \frac{1}{18}, \frac{1}{15}, \frac{1}{12}, \frac{1}{9}, \frac{1}{6}, \frac{1}{3} \right\}$

d) $x[n-6] = [\bar{0}, 0, 2, 4, 8, 16, 32]$

g) $x[3n-3] = ?$

$x[n-3] = [\bar{0}, 0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8]$

$x[3n-3] = [\bar{0}, 0, 3, 6]$

j) $K\left[\frac{4n-3}{10}\right]$

$K(n) = [\bar{0}, 0, 2]$

$K(n-3) = [\bar{0}, 0, 0, 0, 0, 2]$

$K(4n-3) = [\bar{0}, 0, 0, 0, 0, 8]$

$K\left(\frac{4n-3}{10}\right) = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8]$

n) $x\left[\frac{n}{2}\right] * y\left[\frac{n}{3}\right]$

$x\left[\frac{n}{2}\right] = \{\bar{0}, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8\}$

$y\left[\frac{n}{3}\right] = \bar{2}, 2, 2, 4, 4, 4, 8, 8, 8, 16, 16, 16, \overline{32}, 32, 32\}$

$x\left[\frac{n}{2}\right] * y\left[\frac{n}{3}\right] = [0, 0, 0, 2, 4, 8, 14, 22, 32, 48, 66, 90, 124, 164,$

$214, 286, 368, 472, 554, 640, 704, 788, 832, 896$

$936, 960, 979, 896, 832, 736, 512, 256]$

Seccion 4

(2) Halle la transformada de Fourier de

$$y(n) \begin{cases} a^{n} & n > 0 \\ a^{-n} & n < 0 \end{cases}$$

$$Y(\Omega) = \sum_{n=-\infty}^{\infty} y(n) e^{-j\Omega n}$$

$$= \sum_{n=-1}^{\infty} a^{-n} e^{-j\Omega n} + \sum_{n=0}^{\infty} a^{n} e^{-j\Omega n}$$

$$= \sum_{n=1}^{\infty} (a e^{j\Omega})^{n} + \sum_{n=0}^{\infty} (a e^{-j\Omega})^{n}$$

$$Y(n) = \frac{a e^{j\Omega}}{1 - a e^{j\Omega}} + \frac{1}{1 - a e^{-j\Omega}} \text{ para } |a| < 1$$

$$Y(n) = \frac{a e^{jn}(1 - a e^{-jn}) + 1 - a e^{jn\Omega}}{(1 - a e^{jn})(1 - a e^{-jn})}$$

$$= \frac{a e^{j\Omega} - a e^{0} + 1 - a e^{j\Omega}}{1 - a e^{j\Omega} - a e^{j} + a^{2} e^{0}}$$

$$= \frac{1 - a^{2}}{1 + a^{2} - 2a \cos \Omega}$$

⑤ Usando la forma matricial de la DFT calcule

$g(n) = [0.5, 0, 2, 5]$

$$W_n = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & i & 1 & -i \end{bmatrix} \begin{bmatrix} 0.5 \\ 0 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 0.5+2+5 \\ 0.5-2+5i \\ 0.5+2-5 \\ 0.5-2-5i \end{bmatrix} = \begin{bmatrix} 7.5 \\ -1.5+5i \\ -2.5 \\ -1.5-5i \end{bmatrix}$$