Instituto Politécnico Nacional

Escuela Superior de Cómputo

Teoría de comunicaciones y señales.

Problemario 1° parcial.

Grupo: 3CV16

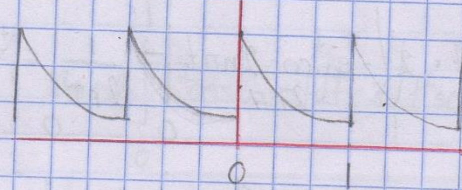
Alumno: Maximiliano Cazares Martínez

Profesora: Jacqueline Arzate Gordillo

Problemario 1er Departamental.

Problema 1. Expresar las señales e^{-t}, t^2, $2t$ como STF en el intervalo $(0,1)$.

$f(t) = e^{-t} \quad 0<t<1$



$T = 1 \quad W_0 = \frac{2\pi}{1} = 2\pi$

$$a_n = \frac{2}{1}\int_0^1 e^{-t}\cos 2n\pi t\,dt = 2\left\{ \frac{e^{-t}\,\text{sen}(2n\pi t)}{2n\pi}\Big|_0^1 + \frac{1}{2n\pi}\int_0^1 e^{-t}\,\text{sen}\,2n\pi t\,dt \right\}$$

$u = e^{-t} \quad du = -e^{-t}dt$ | $u = e^{-t} \quad du = -e^{-t}dt$

$dv = \cos 2n\pi t\,dt \quad v = \frac{\text{sen}\,2n\pi t}{2n\pi}$ | $dv = \text{sen}\,2n\pi t\,dt \quad v = -\frac{\cos 2n\pi t}{2n\pi}$

$$a_n = 2\left\{ \frac{e^{-1}\,\text{sen}(2n\pi)}{2n\pi} - \cancel{\text{sen}\,0}^{\,0} + \frac{1}{2n\pi}\left[\frac{-e^{-t}\cos 2n\pi t}{2n\pi}\Big|_0^1 - \frac{1}{2n\pi}\int_0^1 e^{-t}\cos 2n\pi t\,dt\right]\right\}$$

$$a_n = \frac{\cancel{\text{sen}(2n\pi)}^{\,0}}{en\pi} + \frac{1}{n\pi}\left[\frac{-e^{-1}\cos 2n\pi}{2n\pi} + \frac{e^{0}\cos 0}{2n\pi}\right] - \frac{1}{2n^2\pi^2}\int_0^1 e^{-t}\cos 2n\pi t\,dt$$

$$a_n = \frac{-e^{-1}}{2n^2\pi^2} + \frac{1}{2n^2\pi^2} - \frac{1}{2n^2\pi^2}\int_0^1 e^{-t}\cos 2n\pi t\,dt$$

$$2\int_0^1 e^{-t}\cos 2n\pi t\,dt = \frac{1-e^{-1}}{2n^2\pi^2} - \frac{1}{2n^2\pi^2}\int_0^1 e^{-t}\cos 2n\pi t\,dt$$

$$\left\{\int_0^1 e^{-t}\cos 2n\pi t\,dt\right\}\left\{2 + \frac{1}{2n^2\pi^2}\right\} = \frac{1-e^{-1}}{2n^2\pi^2} \Rightarrow \int_0^1 e^{-t}\cos 2n\pi t\,dt = \left(\frac{2n^2\pi^2}{4n^2\pi^2+1}\right)\left(\frac{1-e^{-1}}{2n^2\pi^2}\right)$$

$$a_n = \frac{1-e^{-1}}{4n^2\pi^2+1}$$

$$a_0=\int_0^1 e^{-t}dt=\left[-e^{-t}\right]_0^1=-e^{-1}+e^{0}=-e^{-1}+1=1-e^{-1}$$

$$b_n=\frac{2}{T}\int_0^1 e^{-t}\operatorname{sen}2n\pi t\,dt=2\left\{-\frac{e^{-t}\cos 2n\pi t}{2n\pi}\Bigg|_0^1-\frac{1}{2n\pi}\int_0^1 e^{-t}\cos 2n\pi t\,dt\right\}$$

$u=e^{-t}\quad du=-e^{-t}$ | $u=e^{-t}\quad du=-e^{-t}$

$dv=\operatorname{sen}2n\pi t\,dt\quad v=-\dfrac{\cos 2n\pi t}{2n\pi}$ | $dv=\cos 2n\pi t\,dt\quad v=\dfrac{\operatorname{sen}2n\pi t}{2n\pi}$

$$b_n=2\left\{\frac{-e^{-1}\cancelto{1}{\cos 2n\pi}}{2n\pi}+\frac{e^{0}\cancelto{1}{\cos\phi}}{2n\pi}-\frac{1}{2n\pi}\left[\frac{e^{-t}\operatorname{sen}2n\pi t}{2n\pi}\Bigg|_0^1+\frac{1}{2n\pi}\int_0^1 e^{-t}\operatorname{sen}2n\pi t\,dt\right]\right\}$$

$$b_n=\frac{-e^{-1}}{n\pi}+\frac{1}{2n\pi}-\frac{1}{n\pi}\left(\frac{e^{-1}\cancelto{0}{\operatorname{sen}2n\pi}}{\cancel{2n\pi}}-\cancelto{0}{\operatorname{sen}\phi}\right)-\frac{1}{2n^2\pi^2}\int_0^1 e^{-t}\operatorname{sen}2n\pi t\,dt$$

$$2\int_0^1 e^{-t}\operatorname{sen}2n\pi t\,dt=\frac{1-e^{-1}}{n\pi}-\frac{1}{2n^2\pi^2}\int_0^1 e^{-t}\operatorname{sen}2n\pi t\,dt$$

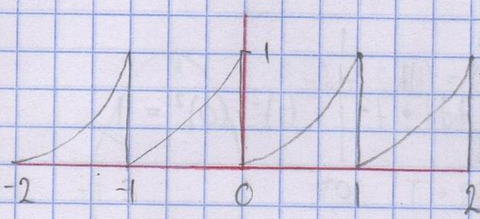
$$\left\{\int_0^1 e^{-t}\operatorname{sen}2n\pi t\,dt\right\}\left\{2+\frac{1}{2n^2\pi^2}\right\}=\frac{1-e^{-1}}{n\pi}\Rightarrow\int_0^1 e^{-t}\operatorname{sen}2n\pi t\,dt=\left(\frac{2n^2\pi^2}{4n^2\pi^2+1}\right)\left(\frac{1-e^{-1}}{n\pi}\right)$$

$$b_n=\frac{2n\pi\left(1-e^{-1}\right)}{4n^2\pi^2+1}$$

$$f=\left(1-\frac{1}{e}\right)+2\sum_{n=1}^{\infty}\left(\frac{1-e^{-1}}{4n^2\pi^2+1}\right)\cos(2n\pi t)+\left(\frac{2n\pi\left(1-e^{-1}\right)}{4n^2\pi^2+1}\right)\operatorname{sen}(2n\pi t)$$

$f(t) = t^2 \quad 0<t<1$ $\qquad u = t^2 \quad du = 2t\,dt$ $\qquad dv = \cos 2n\pi t\,dt \quad v = \dfrac{\operatorname{sen} 2n\pi t}{2n\pi}$ $\quad\Big|\quad u = t \quad du = dt$ $\qquad dv = \operatorname{sen} 2n\pi t\,dt \quad v = \dfrac{-\cos 2n\pi t}{2n\pi}$

$T = 1 \quad w_0 = 2\pi$



$$a_n = 2\int_0^1 t^2 \cos 2n\pi t\,dt = 2\left\{ \frac{t^2 \operatorname{sen} 2n\pi t}{2n\pi}\Bigg|_0^1 - \frac{1}{n\pi}\int_0^1 t \operatorname{sen} 2n\pi t\,dt \right\}$$

$$a_n = 2\left\{ \frac{(1)^2 \cancelto{0}{\operatorname{sen} 2n\pi}}{2n\pi} - \frac{(0)^2 \cancelto{0}{\operatorname{sen} \emptyset}}{2n\pi} - \frac{1}{n\pi}\left[\frac{-t\cos 2n\pi t}{2n\pi}\Bigg|_0^1 + \frac{1}{2n\pi}\int_0^1 \cos 2n\pi t\,dt \right] \right\}$$

$$a_n = -\frac{2}{n\pi}\left\{ \frac{-\cancelto{1}{\cos 2n\pi}}{2n\pi} + (0) + \frac{1}{2n\pi}\left[\cancel{\frac{\cancelto{0}{\operatorname{sen} 2n\pi}}{2n\pi}} - \cancel{\frac{\cancelto{0}{\operatorname{sen}(\emptyset)}}{2n\pi}} \right] \right\} = \frac{1}{n^2\pi^2}$$

$$a_0 = \int_0^1 t^2\,dt = \frac{t^3}{3}\Bigg|_0^1 = \frac{(1)^3}{3} - \frac{(\emptyset)^3}{3} = \frac{1}{3}$$

$$b_n = 2\int_0^1 t^2 \operatorname{sen} 2n\pi t\,dt = 2\left\{ \frac{-t^2\cos 2n\pi t}{2n\pi}\Bigg|_0^1 + \frac{1}{n\pi}\int_0^1 t\cos 2n\pi t\,dt \right\}$$

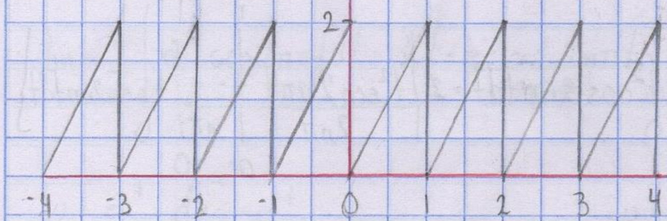
$$b_n = 2\left\{ \frac{-(1)^2 \cancelto{1}{\cos 2n\pi}}{2n\pi} + \cancel{\frac{(0)^2 \cancelto{0}{\cos(\emptyset)}}{2n\pi}} + \frac{1}{n\pi}\left[\frac{t \operatorname{sen} 2n\pi t}{2n\pi}\Bigg|_0^1 - \frac{1}{2n\pi}\int_0^1 \operatorname{sen} 2n\pi t\,dt \right] \right\}$$

$$b_n = -\frac{1}{n\pi} + \frac{2}{n\pi}\left\{ \cancel{\frac{\cancelto{0}{\operatorname{sen} 2n\pi}}{2n\pi}} - \cancel{\frac{(0)\cancelto{0}{\operatorname{sen}(\emptyset)}}{2n\pi}} - \frac{1}{2n\pi}\left[\cancel{\frac{-\cancelto{1}{\cos 2n\pi}}{2n}} + \cancel{\frac{\cancelto{1}{\cos(\emptyset)}}{2n}} \right] \right\} = -\frac{1}{n\pi}$$

$$f(t) = \frac{1}{3} + \sum_{n=1}^{\infty}\left(\frac{\cos(2n\pi t)}{n^2\pi^2} - \frac{\operatorname{sen}(2n\pi t)}{n\pi} \right)$$

$f(t) = 2t$

$T = 1 \quad W_0 = 2\pi$



$$a_0 = 2\int_0^1 t\,dt = t^2\Big|_0^1 = (1)^2 - (0)^2 = 1$$

$$a_n = 4\int_0^1 t\cos(2n\pi t)\,dt = 4\left\{\frac{t\,sen(2n\pi t)}{2n\pi}\Big|_0^1 - \frac{1}{2n\pi}\int_0^1 sen(2n\pi t)\,dt\right\}$$

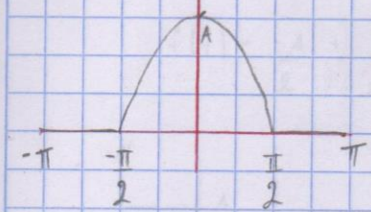
$$a_n = 4\left\{\cancel{\frac{sen(2n\pi)}{2n\pi}} - \cancel{\frac{(0)sen(0)}{2n\pi}} - \frac{1}{2n\pi}\left[-\frac{\cos(2n\pi)}{2n\pi} + \frac{\cos(0)}{2n\pi}\right]\right\} = 0$$

$$b_n = 4\int_0^1 t\,sen(2n\pi t)\,dt = 4\left\{-\frac{t\cos(2n\pi t)}{2n\pi}\Big|_0^1 + \frac{1}{2n\pi}\int_0^1 \cos(2n\pi t)\,dt\right\}$$

$$b_n = 4\left\{-\frac{\cos(2n\pi)}{\cancel{2n\pi}} + \cancel{\frac{(0)\cos(0)}{2n\pi}} + \frac{1}{2n\pi}\left[\cancel{\frac{sen\,2n\pi}{2n\pi}} - \cancel{\frac{sen(0)}{2n\pi}}\right]\right\} = \frac{-2}{n\pi}$$

$$f(t) = 1 + \sum_{n=1}^{\infty} \frac{-2\,sen(2n\pi t)}{n\pi}$$

Problema 2. Encontrar la serie trigonométrica de Fourier de cada una de las siguientes señales en el intervalo de $-\pi$ a π



$$f(t) = \begin{cases} 0 & -\pi < t < -\pi/2 \\ A\,sen(t+\pi/2) & -\pi/2 < t < \pi/2 \\ 0 & \pi/2 < t < \pi \end{cases}$$

$$T = 2\pi \qquad W_0 = \frac{2\pi}{2\pi} = 1$$

$$a_n = \frac{4}{2\pi}\int_0^{\pi/2} A\,sen(t+\pi/2)\cos(nt)\,dt = \frac{2A}{\pi}\int_0^{\pi/2}\frac{1}{2}\left[sen\left(t+\frac{\pi}{2}+nt\right)+sen\left(t+\frac{\pi}{2}-nt\right)\right]dt$$

$$a_n = \frac{A}{\pi}\int_0^{\pi/2} sen\left(t+\frac{\pi}{2}+nt\right)dt + \frac{A}{\pi}\int_0^{\pi/2} sen\left(t+\frac{\pi}{2}-nt\right)dt \qquad u = t+\frac{\pi}{2}\pm nt \quad du = (1\pm n)\,dt \quad dt = \frac{du}{1\pm n}$$

$$a_n = \frac{A}{(1+n)\pi}\int_0^{\pi/2} sen\,u\,du + \frac{A}{(1-n)\pi}\int_0^{\pi/2} sen\,u\,du = \frac{-A}{(1+n)\pi}\cos\left(t+\frac{\pi}{2}+nt\right)\Bigg|_0^{\pi/2} - \frac{A}{(1-n)\pi}\cos\left(t+\frac{\pi}{2}-nt\right)\Bigg|_0^{\pi/2}$$

$$a_n = \frac{-A}{(1+n)\pi}\left[\cos\left(\pi+\frac{n\pi}{2}\right) - \cancel{\cos\left(\frac{\pi}{2}\right)}^{0}\right] - \frac{A}{(1-n)\pi}\left[\cos\left(\pi-\frac{n\pi}{2}\right) - \cancel{\cos\left(\frac{\pi}{2}\right)}^{0}\right]$$

$$\cos(\pi \pm n\pi/2) = \underbrace{\cos(\pi)}_{-1}\cos(n\pi/2) \mp \underbrace{sen(\pi)}_{0}\,sen(n\pi/2) = -\cos(n\pi/2)$$

$$a_n = \frac{A\cos(n\pi/2)}{(1+n)\pi} + \frac{A\cos(n\pi/2)}{(1-n)\pi} = \frac{A}{\pi}\cos(n\pi/2)\left[\frac{1}{1+n}+\frac{1}{1-n}\right] = \frac{2A\cos(n\pi/2)}{(1-n^2)\pi} \quad \forall n \neq 1$$

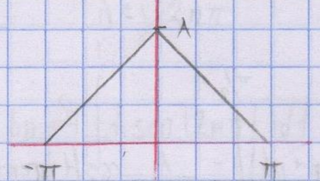
$$a_0 = \frac{2}{2\pi}\int_0^{\pi/2} A\,sen\left(t+\frac{\pi}{2}\right)dt = \frac{A}{\pi}\int_0^{\pi/2} sen\left(t+\frac{\pi}{2}\right) = -\frac{A}{\pi}\cos\left(t+\frac{\pi}{2}\right)\Bigg|_0^{\pi/2} = -\frac{A}{\pi}\left[\cancel{\cos\pi}^{-1} - \cancel{\cos\frac{\pi}{2}}^{0}\right] = \frac{A}{\pi}$$

$$a_1 = \frac{2A}{\pi}\int_0^{\pi/2} \operatorname{sen}\left(t+\frac{\pi}{2}\right)\cos(t)\,dt = \frac{A}{\pi}\int_0^{\pi/2}\left[\operatorname{sen}\left(t+\frac{\pi}{2}+t\right)+\operatorname{sen}\left(t+\frac{\pi}{2}-t\right)\right]dt \qquad u = 2t+\frac{\pi}{2} \quad du = 2dt \quad dt = \frac{du}{2}$$

$$a_1 = \frac{A}{2\pi}\int_0^{\pi/2} \operatorname{sen} u\,du + \frac{A}{\pi}\int_0^{\pi/2} dt = \frac{-A}{2\pi}\cos\left(2t+\frac{\pi}{2}\right)\Big|_0^{\pi/2} + \frac{A}{\pi}\,t\Big|_0^{\pi/2}$$

$$a_1 = \frac{-A}{2\pi}\left[\cancelto{0}{\cos\left(\frac{3\pi}{2}\right)} - \cancel{\cos\left(\frac{\pi}{2}\right)}\right] + \frac{A}{\pi}\left[\frac{\pi}{2} - 0\right] = \frac{A}{2}\cos(t)$$

$$f = \frac{A}{\pi} + \frac{A\cos(t)}{2} + \sum_{n=2}^{\infty} \frac{2A\cos(n\pi/2)}{\pi(1-n^2)}\cos(nt)$$



$$f(t) = \begin{cases} \dfrac{t+\pi}{\pi} & -\pi < t < 0 \\[2mm] \dfrac{\pi - t}{\pi} & 0 < t < \pi \end{cases} \qquad f(t) \text{ es par} \qquad T = 2\pi \quad \omega_0 = 1$$

$$a_n = \frac{4A}{2\pi}\int_0^{\pi}\left(\frac{\pi - t}{\pi}\right)\cos(nt)\,dt = -\frac{2A}{\pi^2}\int_0^{\pi} t\cos(nt)\,dt + \frac{2A}{\pi}\int_0^{\pi}\cos(nt)$$

$u = t \quad du = dt$

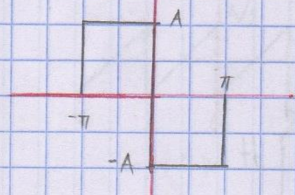
$dv = \cos(nt)\,dt \quad v = \frac{\operatorname{sen}(nt)}{n}$

$$a_n = -\frac{2A}{\pi^2}\left\{\frac{t\operatorname{sen}(nt)}{n}\Big|_0^{\pi} - \frac{1}{n}\int_0^{\pi}\operatorname{sen}(nt)\,dt\right\} + \frac{2A}{\pi n}\left[\cancelto{0}{\operatorname{sen}(n\pi)} - \cancelto{0}{\operatorname{sen}(\phi)}\right]$$

$$a_n = -\frac{2A}{\pi^2}\left\{\frac{\pi\cancelto{0}{\operatorname{sen}(n\pi)}}{n} - \phi - \frac{1}{n}\left[\frac{-\overset{(-1)^n}{\cos(n\pi)}}{n} + \frac{\overset{1}{\cos(\phi)}}{n}\right]\right\} = \frac{-2((-1)^n - 1)A}{n^2\pi^2}$$

$$a_0 = \frac{2}{2\pi}\int_0^{\pi} A\left(\frac{\pi - t}{\pi}\right)dt = \frac{A}{\pi^2}\int_0^{\pi}(\pi - t)\,dt = \frac{A}{\pi^2}\left(-\frac{t^2}{2}+\pi t\right)\Bigg|_0^{\pi} = \frac{A}{\pi^2}\left(-\frac{\pi^2}{2}+0+\pi^2-0\right) = \frac{A}{\pi^2}\left(\frac{\pi^2}{2}\right) = \frac{A}{2}$$

$$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{-2A\left((-1)^n - 1\right)}{n^2\pi^2}\cos(nt)$$



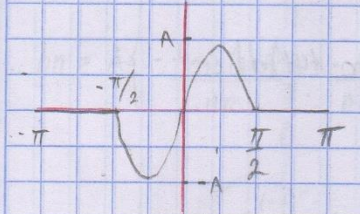
$$f(t) = \begin{cases} A & -\pi < t < 0 \\ -A & 0 < t < \pi \end{cases}$$

$a_0 = a_n = \emptyset$

$T = 2\pi \quad W_0 = 1$

$$b_n = \frac{4}{2\pi}\int_0^{\pi} -A\,\text{sen}(nt)\,dt = -\frac{2A}{\pi}\int_0^{\pi}\text{sen}(nt)\,dt = -\frac{2A}{\pi}\left[-\frac{\overset{(-1)^n}{\cancel{\cos(n\pi)}}}{n} + \frac{\overset{1}{\cancel{\cos(0)}}}{n}\right]$$

$$b_n = \frac{2A\left[(-1)^n - 1\right]}{n\pi} \qquad f(t) = \sum_{n=1}^{\infty}\frac{2A\left[(-1)^n - 1\right]}{n\pi}\,\text{sen}(nt)$$



$$f(t) = \begin{cases} 0 & -\pi < t < -\pi/2 \\ A\,\text{sen}(2t) & -\pi/2 < t < \pi/2 \\ 0 & \pi/2 < t < \pi \end{cases}$$

$f(t)$ impar $a_0 = a_n = 0$

$T = 2\pi \quad W_0 = 1$

$$b_n = \frac{4}{2\pi}\int_0^{\pi/2} A\,\text{sen}(2t)\,\text{sen}(nt)\,dt = \frac{2A}{\pi}\int_0^{\pi/2}\frac{1}{2}\left[\cos(2t-nt) - \cos(2t-nt)\right]dt$$

$u = 2t \pm nt \quad du = (2 \pm n)dt$

$dt = \frac{du}{2 \pm n}$

$$b_n = \frac{A}{\pi}\int_0^{\pi/2}\cos(2t-nt)\,dt - \frac{A}{\pi}\int_0^{\pi/2}\cos(2t+nt)\,dt$$

$$b_n = \frac{A}{\pi(2-n)}\,\text{sen}(2t-nt)\Bigg|_0^{\pi/2} - \frac{A}{\pi(2+n)}\,\text{sen}(2t+nt)\Bigg|_0^{\pi/2}$$

$$bn = \frac{A}{\pi(2-n)}\left[\operatorname{sen}\left(\frac{\pi - n\pi}{2}\right) - \cancel{\operatorname{sen}(\emptyset)}^{\,0}\right] - \frac{A}{\pi(2+n)}\left[\operatorname{sen}\left(\frac{\pi + n\pi}{2}\right) - \cancel{\operatorname{sen}(\emptyset)}^{\,0}\right]$$

$$\operatorname{sen}(\pi \pm n\pi/2) = \cancel{\operatorname{sen}(\pi)}^{\,0}\cos(n\pi/2) \pm \cancel{\cos(\pi)}^{\,-1}\operatorname{sen}(n\pi/2) = \mp \operatorname{sen}\left(\frac{n\pi}{2}\right)$$

$$bn = \frac{A\operatorname{sen} n\pi/2}{\pi(2-n)} + \frac{A\operatorname{sen} n\pi/2}{\pi(2+n)} = \frac{4A\operatorname{sen}(n\pi/2)}{\pi(4-n^2)} \quad \forall n \neq 2$$

$$b_1 = \frac{4}{2\pi}\int_0^{\pi/2} A\operatorname{sen}(2t)\operatorname{sen}(t)\,dt = \frac{2A}{\pi}\int_0^{\pi/2}\frac{1}{2}\left[\cos(\cancel{2t-t}^{\,t}) - \cos(\cancel{2t+t}^{\,3t})\right]dt$$

$$b_1 = \frac{A}{\pi}\int_0^{\pi/2}\cos t\,dt - \frac{A}{\pi}\int_0^{\pi/2}\cos(3t)\,dt = \frac{A}{\pi}\left[\cancel{\operatorname{sen}\left(\frac{\pi}{2}\right)}^{\,1} - \cancel{\operatorname{sen}(\emptyset)}^{\,0}\right] - \frac{A}{\pi}\left[\frac{\cancel{\operatorname{sen}\left(\frac{3\pi}{2}\right)}^{\,-1}}{3} - \frac{\cancel{\operatorname{sen}(\emptyset)}^{\,0}}{3}\right]$$

$$b_1 = \frac{A}{\pi} + \frac{A}{3\pi} = \left(\frac{3A+A}{3\pi}\right)\operatorname{sen}(nt) = \frac{4A\operatorname{sen} t}{3}$$

$$b_2 = \frac{2A}{\pi}\int_0^{\pi/2}\operatorname{sen}^2(2t) = \frac{2A}{\pi}\int_0^{\pi/2}\frac{1}{2}\left[1-\cos(4t)\right]dt = \frac{A}{\pi}\int_0^{\pi/2}dt - \frac{A}{\pi}\int_0^{\pi/2}\cos(4t)\,dt$$

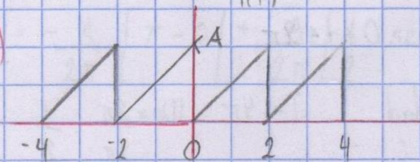
$$bn = \frac{A}{\pi}\left[\frac{\pi}{2} - 0 - \frac{\cancel{\cos(2\pi)}^{\,1}}{4} + \frac{\cancel{\cos(\emptyset)}^{\,1}}{4}\right] = \frac{A}{\pi}\left(\frac{\pi}{2}\right) = \frac{A}{2}$$

$$bn = \frac{A\operatorname{sen}(2t)}{2}$$

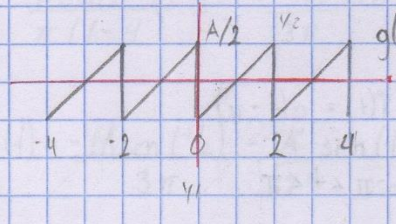
$$f(t) = \frac{4\operatorname{sen}(t)}{3} + \frac{\operatorname{sen}(2t)}{2} + \sum_{n=3}^{\infty}\frac{4\operatorname{sen}(n\pi/2)}{\pi(4-n^2)}\operatorname{sen}(nt)$$

Problema 3. Determinar la STF de cada una de las señales periódicas a continuación.

a)



$f(t) = \frac{At}{2} \quad 0<t<2 \quad f(t)$ no tiene paridad



$f(t) - \frac{A}{2} = g(t) \Rightarrow g(t) + \frac{A}{2} = f(t)$

$g(t)$ es impar $\quad T = 2 \quad w_0 = \frac{2\pi}{2} = \pi$

$$bn = \frac{4}{2}\int_0^1 \left(\frac{At}{2} - \frac{A}{2}\right) sen(n\pi t)\,dt = A\int_0^1 t\,sen(n\pi t)\,dt - A\int_0^1 sen(n\pi t)\,dt$$

$u = t \quad du = dt$

$dv = sen(n\pi t)\,dt \quad v = \frac{-\cos(n\pi t)}{n\pi}$

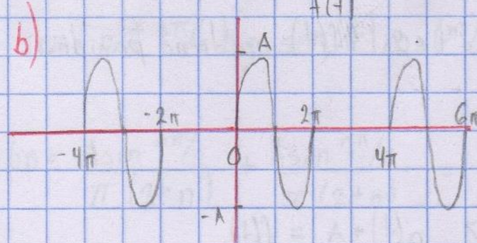
$$bn = A\left\{ -\frac{t\cos(n\pi t)}{n\pi}\Big|_0^1 + \frac{1}{n\pi}\int_0^1 \cos(n\pi t)\,dt \right\} - A\left[-\frac{1}{n\pi}\left(\cos(\overset{(-1)^n}{n\pi}) - \cos(\overset{1}{\emptyset})\right)\right]$$

$$bn = A\left\{ -\frac{\cos(\overset{(-1)^n}{n\pi})}{n\pi} + \emptyset + \frac{1}{n\pi}\left[\frac{1}{n\pi}\left(\cancel{sen(n\pi)} - \cancel{sen(\emptyset)}\right)\right]\right\} + \frac{A}{n\pi}\left[(-1)^n - 1\right]$$

$$bn = \frac{-A(-1)^n}{n\pi} + \frac{A}{n\pi}\left[(-1)^n - 1\right] = -\frac{A}{n\pi}$$

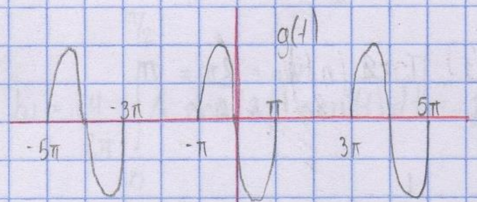
$$g(t) = \sum_{n=1}^{\infty}\left(\frac{-A}{n\pi}\right) sen(n\pi t)$$ pero $f(t) = g(t) + \frac{A}{2}$ entonces $$f(t) = \frac{A}{2} - \sum_{n=1}^{\infty} \frac{A\,sen(n\pi t)}{n\pi}$$

b)



$f(t) = A\,\text{sen}(t) \qquad 0 < t < 2\pi$

$f(t)$ no tiene paridad $\qquad T = 4\pi \quad W_0 = \frac{2\pi}{4\pi} = \frac{1}{2}$



$g(t) = f(t+\pi) \quad \Rightarrow \quad f(t) = g(t-\pi)$

$g(t) = -A\,\text{sen}(t) \qquad -\pi < t < \pi$

$g(t)$ es impar $\qquad u = t \pm \frac{nt}{2} \quad du = 1 \pm \frac{n}{2} = \frac{2 \pm n}{2}$

$$bn = \frac{4}{4\pi}\int_0^{\pi} -A\,\text{sen}(t)\,\text{sen}\left(\frac{nt}{2}\right)dt = -\frac{A}{\pi}\int_0^{\pi}\frac{1}{2}\left[\cos\left(t-\frac{nt}{2}\right) - \cos\left(t+\frac{nt}{2}\right)\right]dt$$

$$bn = \frac{-A}{2\pi}\int_0^{\pi}\cos\left(t-\frac{nt}{2}\right)dt + \frac{A}{2\pi}\int_0^{\pi}\cos\left(t+\frac{nt}{2}\right)dt = -\frac{A}{2\pi}\left\{\frac{2}{2-n}\,\text{sen}\left(t-\frac{nt}{2}\right)\Big|_0^{\pi}\right\} + \frac{A}{2\pi}\left\{\frac{2}{2+n}\,\text{sen}\left(t+\frac{nt}{2}\right)\Big|_0^{\pi}\right\}$$

$$bn = \frac{-A}{\pi(2-n)}\left[\text{sen}\left(\pi - \frac{n\pi}{2}\right) - \text{sen}(\emptyset)\right] + \frac{A}{\pi(2+n)}\left[\text{sen}\left(\pi + \frac{n\pi}{2}\right) - \text{sen}(\emptyset)\right]$$

$$\text{sen}(\pi \pm {}^{n\pi}/_{2}) = \underbrace{\text{sen}(\pi)}_{0}\cancel{\cos({}^{n\pi}/_{2})} \pm \underbrace{\cos(\pi)}_{-1}\,\text{sen}({}^{n\pi}/_{2}) = \pm\left(-\text{sen}({}^{n\pi}/_{2})\right)$$

$$bn = \frac{-A}{\pi(2-n)}\,\text{sen}({}^{n\pi}/_{2}) - \frac{A}{\pi(2+n)}\,\text{sen}({}^{n\pi}/_{2}) = \frac{-A\,\text{sen}({}^{n\pi}/_{2})}{\pi}\left[\frac{1}{2-n} + \frac{1}{2+n}\right]$$

$$bn = \frac{-4A\,\text{sen}({}^{n\pi}/_{2})}{\pi(4-n^2)} = \frac{4A\,\text{sen}({}^{n\pi}/_{2})}{\pi(n^2-4)} \qquad \forall n \neq 2$$

$$b_2 = \frac{4}{4\pi}\int_0^{\pi} -A\,sen^2(t)\,dt = -\frac{A}{\pi}\int_0^{\pi} sen^2 t\,dt = -\frac{A}{\pi}\int_0^{\pi}\left[\frac{1}{2}-\frac{1}{2}\cos(2t)\right]dt = -\frac{A}{2\pi}\int_0^{\pi} dt + \frac{A}{2\pi}\int_0^{\pi}\cos(2t)\,dt$$

$$b_2 = -\frac{A}{2\pi}\left[\pi - 0\right] + \frac{A}{2\pi}\left[\frac{1}{2}\left(\underbrace{sen(2\pi)}_{0} - \underbrace{sen(\emptyset)}_{0}\right)\right] = -\frac{A}{2}$$

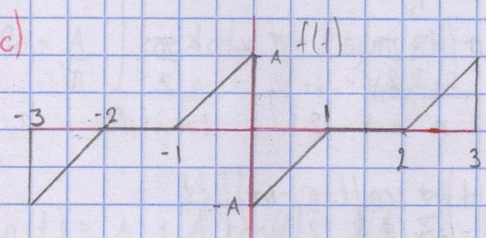
$$b_1 = \frac{4A\,sen(\pi/2)}{\pi(1-4)} = -\frac{4A}{3\pi}\,sen\left(\frac{t}{2}\right)$$

$$g(t) = \frac{-4A\,sen(t/2)}{3\pi} - \frac{A}{2}\,sen(t) + \frac{4A}{\pi}\sum_{n=3}^{\infty}\left(\frac{sen(n\pi/2)}{n^2-4}\right)sen\left(\frac{nt}{2}\right)$$

pero $f(t) = g(t-\pi)$

$$f(t) = \frac{-4A}{3\pi}\,sen\left(\frac{t-\pi}{2}\right) - \frac{A}{2}\,sen(t-\pi) + \frac{4A}{\pi}\sum_{n=3}^{\infty}\left(\frac{sen(n\pi/2)}{n^2-4}\right)sen\left(\frac{n}{2}(t-\pi)\right)$$

c)



$f(t)$ es impar $\quad T = 3 \quad w_0 = \frac{2\pi}{3}$

$$f(t) = \begin{cases} A(t+1) & -1<t<0 \\ A(t-1) & 0<t<1 \end{cases}$$

$$b_n = \frac{4}{3}\int_0^1 A(t-1)\,sen\left(\frac{2n\pi t}{3}\right)dt = \frac{4A}{3}\int_0^1 t\,sen\left(\frac{2n\pi t}{3}\right)dt - \frac{4A}{3}\int_0^1 sen\left(\frac{2n\pi t}{3}\right)dt$$

$u = t \quad du = dt$

$dv = sen\,\frac{2n\pi t}{3}\,dt \quad v = \frac{-3}{2n\pi}\cos\left(\frac{2n\pi t}{3}\right)$

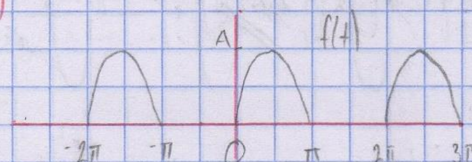
$$b_n = \frac{4A}{3}\left\{\frac{-3t}{2n\pi}\cos\left(\frac{2n\pi t}{3}\right)\Bigg|_0^1 + \frac{3}{2n\pi}\int_0^1\cos\left(\frac{2n\pi t}{3}\right)dt\right\} - \frac{4A}{3}\left[-\frac{3}{2n\pi}\cos\left(\frac{2n\pi t}{3}\right)\Bigg|_0^1\right]$$

$$b_n = -\frac{2A}{n\pi}\left[\cos\left(\frac{2n\pi}{3}\right) - \phi\right] + \frac{2A}{n\pi}\left\{\frac{3}{2n\pi}\left[\operatorname{sen}\left(\frac{2n\pi}{3}\right) - \operatorname{sen}(\phi)\right] + \frac{2A}{n\pi}\left[\cos\left(\frac{2n\pi}{3}\right) - \cos(\phi)\right]\right\}$$

$$b_n = \frac{3A}{n^2\pi^2}\operatorname{sen}\left(\frac{2n\pi}{3}\right) - \frac{2A}{n\pi}$$

$$f(t) = \sum_{n=1}^{\infty}\left(\frac{3A}{n^2\pi^2}\operatorname{sen}\left(\frac{2n\pi}{3}\right) - \frac{2A}{n\pi}\right)\operatorname{sen}\left(\frac{2n\pi t}{3}\right)$$

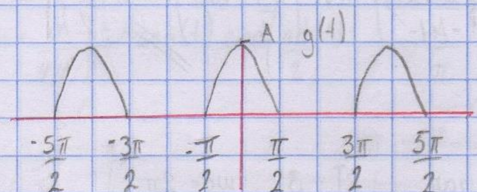
d)



$f(t)$ no tiene paridad

$f(t) = A\operatorname{sen} t \qquad 0 < t < \pi$

$T = 2\pi \qquad w_0 = 1$



$g\left(t + \frac{\pi}{2}\right) = f(t) \qquad g(t)$ es par

$g(t) = \operatorname{sen}\left(t + \frac{\pi}{2}\right) \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$

$$a_n = \frac{4}{2\pi}\int_0^{\pi/2} A\operatorname{sen}\left(t+\frac{\pi}{2}\right)\cos(nt)\,dt = \frac{2A}{\pi}\int_0^{\pi/2}\frac{1}{2}\left[\operatorname{sen}\left(t+\frac{\pi}{2}+nt\right) + \operatorname{sen}\left(t+\frac{\pi}{2}-nt\right)\right]dt$$

$$u = t + \frac{\pi}{2} \pm nt \qquad du = (1 \pm n)\,dt \qquad dt = \frac{du}{1 \pm n}$$

$$a_n = \frac{A}{\pi}\left\{-\frac{1}{1+n}\cos\left(t+\frac{\pi}{2}+nt\right)\Big|_0^{\pi/2}\right\} + \frac{A}{\pi}\left\{-\frac{1}{1-n}\cos\left(t+\frac{\pi}{2}-nt\right)dt\Big|_0^{\pi/2}\right\}$$

$$a_n = \frac{-A}{\pi(1+n)}\left[\cos\left(\pi + \frac{n\pi}{2}\right) - \cancelto{0}{\cos\left(\frac{\pi}{2}\right)}\right] - \frac{A}{\pi(1-n)}\left[\cos\left(\pi - \frac{n\pi}{2}\right) - \cancelto{0}{\cos\left(\frac{\pi}{2}\right)}\right]$$

$$\cos(\pi \pm n\pi/2) = \cancelto{-1}{\cos(\pi)}\cos(n\pi/2) \mp \cancelto{0}{\operatorname{sen}(\pi)}\operatorname{sen}(n\pi/2) = -\cos(n\pi/2)$$

$$a_n = \frac{A\cos(n\pi/2)}{\pi}\left[\frac{1}{1+n} + \frac{1}{1-n}\right] = \frac{2A\cos(n\pi/2)}{\pi(1+n^2)} \quad \forall n \neq 1$$

$$a_1 = \frac{4}{2\pi}\int_0^{\pi/2} A\,\mathrm{sen}\left(t+\frac{\pi}{2}\right)\cos(t)\,dt = \frac{2A}{\pi}\int_0^{\pi/2}\frac{1}{2}\left[\mathrm{sen}\left(2t+\frac{\pi}{2}\right) + \cancel{\mathrm{sen}}\left(\frac{\pi}{2}\right)^{\to 1}\right]dt$$

$$u = 2t + \frac{\pi}{2} \quad du = 2dt \quad dt = \frac{du}{2}$$

$$a_1 = \frac{A}{\pi}\left\{-\frac{1}{2}\cos\left(2t+\frac{\pi}{2}\right)\Big|_0^{\pi/2}\right\} + \frac{A}{\pi}\left[\frac{\pi}{2} - 0\right] = \frac{A}{\pi}\left\{-\frac{1}{2}\left[\cancel{\cos}\left(\frac{3\pi}{2}\right)^{\to 0} - \cancel{\cos}\left(\frac{\pi}{2}\right)^{0}\right]\right\} + \frac{A}{2} = \frac{A}{2}$$

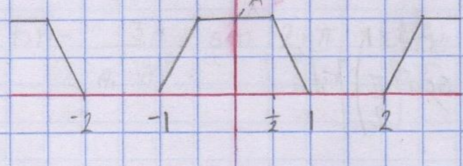
$$a_0 = \frac{2}{2\pi}\int_0^{\pi/2} A\,\mathrm{sen}\left(t+\frac{\pi}{2}\right)dt = \frac{A}{\pi}\int_0^{\pi/2}\mathrm{sen}\left(t+\frac{\pi}{2}\right)dt = \frac{A}{\pi}\int_0^{\pi/2}\left[\mathrm{sen}(t)\cancel{\cos}\left(\frac{\pi}{2}\right)^{\to 0} + \cos(t)\,\cancel{\mathrm{sen}}\left(\frac{\pi}{2}\right)^{\to 1}\right]dt$$

$$a_0 = \frac{A}{\pi}\int_0^{\pi/2}\cos t\,dt = \frac{A}{\pi}\left[\cancel{\mathrm{sen}}\left(\frac{\pi}{2}\right)^{1} - \cancel{\mathrm{sen}}(\emptyset)^{0}\right] = \frac{A}{\pi}$$

$$g(t) = \frac{A}{\pi} + \frac{A}{2}\cos(t) + \frac{2A}{\pi}\sum_{n=2}^{\infty}\frac{\cos(n\pi/2)}{1+n^2}\cos(nt)$$

pero $f(t) = g\left(t - \frac{\pi}{2}\right)$

$$f(t) = \frac{A}{\pi} + \frac{A}{2}\cos\left(t-\frac{\pi}{2}\right) + \frac{2A}{\pi}\sum_{n=2}^{\infty}\frac{\cos(n\pi/2)}{1+n^2}\cos\left[n\left(t-\frac{\pi}{2}\right)\right]$$



$f(t)$ es par $\quad T = 3 \quad W_0 = \dfrac{2\pi}{3}$

$$f(t) = \begin{cases} 0 & -3/2 < t < -1 \\ 2A(t+1) & -1 < t < -1/2 \\ A & -1/2 < t < 1/2 \\ 2A(1-t) & 1/2 < t < 1 \\ 0 & 1 < t < 3/2 \end{cases}$$

$$a_n = \frac{4}{3}\int_0^{1/2} A\cos\left(\frac{2n\pi t}{3}\right)dt + \frac{4}{3}\int_{1/2}^{1} 2A\cos\left(\frac{2n\pi t}{3}\right)dt - \frac{4}{3}\int_{1/2}^{1} 2At\cos\left(\frac{2n\pi t}{3}\right)dt$$

Integrando por separado cada integral

$$i = \frac{4A}{3}\int_0^{1/2}\cos\left(\frac{2n\pi t}{3}\right)dt = \frac{4A}{3}\left[\frac{3}{2n\pi}\,\text{sen}\left(\frac{2n\pi t}{3}\right)\right]_0^{1/2} = \frac{4A}{3}\left[\frac{3}{2n\pi}\left(\text{sen}\left(\frac{n\pi}{3}\right) - \text{sen}(0)\right)\right] = \frac{2A}{n\pi}\,\text{sen}\left(\frac{n\pi}{3}\right)$$

$$j = \frac{8A}{3}\int_{1/2}^{1}\cos\left(\frac{2n\pi t}{3}\right)dt = \frac{8A}{3}\left[\frac{3}{2n\pi}\,\text{sen}\left(\frac{2n\pi t}{3}\right)\right]_{1/2}^{1} = \frac{8A}{3}\left[\frac{3}{2n\pi}\left(\text{sen}\left(\frac{2n\pi}{3}\right) - \text{sen}\left(\frac{n\pi}{3}\right)\right)\right] = \frac{4A}{n\pi}\left[\text{sen}\left(\frac{2n\pi}{3}\right) - \text{sen}\left(\frac{n\pi}{3}\right)\right]$$

$$K = -\frac{8A}{3}\int_{1/2}^{1} t\cos\left(\frac{2n\pi t}{3}\right)dt = -\frac{8A}{3}\left[\frac{3t}{2n\pi}\,\text{sen}\left(\frac{2n\pi t}{3}\right)\Bigg|_{1/2}^{1} - \frac{3}{2n\pi}\int_{1/2}^{1}\text{sen}\left(\frac{2n\pi t}{3}\right)dt\right]$$

$u = t \quad du = dt \qquad dv = \cos\left(\dfrac{2n\pi t}{3}\right)dt \qquad v = \dfrac{3}{2n\pi}\,\text{sen}\left(\dfrac{2n\pi t}{3}\right)$

$$K = -\frac{8A}{3}\left\{\frac{3}{2n\pi}\left[\text{sen}\left(\frac{2n\pi}{3}\right) - \frac{1}{2}\text{sen}\left(\frac{n\pi}{3}\right)\right] - \frac{3}{2n\pi}\left[\frac{-3}{2n\pi}\left(\cos\left(\frac{2n\pi}{3}\right) - \cos\left(\frac{n\pi}{3}\right)\right)\right]\right\}$$

$$K = -\frac{4A}{n\pi}\left[\text{sen}\left(\frac{2n\pi}{3}\right) - \frac{1}{2}\text{sen}\left(\frac{n\pi}{3}\right)\right] - \frac{6A}{n^2\pi^2}\left[\cos\left(\frac{2n\pi}{3}\right) - \cos\left(\frac{n\pi}{3}\right)\right]$$

sumando i, j y K

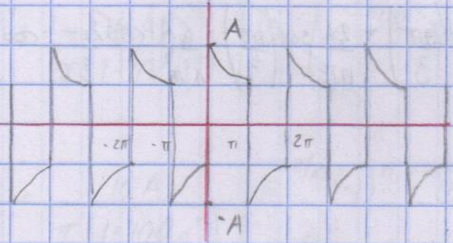
$$a_n = \frac{2A}{n\pi}\,\mathrm{sen}\left(\frac{n\pi}{3}\right) + \frac{4A}{n\pi}\,\mathrm{sen}\left(\frac{2n\pi}{3}\right) - \frac{4A}{n\pi}\,\mathrm{sen}\left(\frac{n\pi}{3}\right) - \frac{4A}{n\pi}\,\mathrm{sen}\left(\frac{2n\pi}{3}\right) + \frac{2A}{n\pi}\,\mathrm{sen}\left(\frac{n\pi}{3}\right) - \frac{6A}{n^2\pi^2}\left[\cos\frac{2n\pi}{3} - \cos\frac{n\pi}{3}\right]$$

$$a_n = -\frac{6A}{n^2\pi^2}\left[\cos\left(\frac{2n\pi}{3}\right) - \cos\left(\frac{n\pi}{3}\right)\right]$$

$$a_0 = \frac{2}{3}\int_0^{1/2} A\,dt + \frac{2}{3}\int_{1/2}^{1} 2A\,dt - \frac{2}{3}\int_{1/2}^{1} 2At\,dt = \frac{2At}{3}\Big|_0^{1/2} + \frac{4At}{3}\Big|_{1/2}^{1} - \frac{4A}{3}\left(\frac{t^2}{2}\right)\Big|_{1/2}^{1}$$

$$a_0 = \frac{2A}{3}\left[\frac{1}{2} - 0\right] + \frac{4A}{3}\left[1 - \frac{1}{2}\right] - \frac{4A}{3}\left[\frac{1}{2} - \frac{1}{8}\right] = \frac{A}{3} + \frac{A}{3} - \frac{A}{2} = A - \frac{A}{2} = \frac{A}{2}$$

$$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} -\frac{6A}{n^2\pi^2}\left[\cos\left(\frac{2n\pi}{3}\right) - \cos\left(\frac{n\pi}{3}\right)\right]\cos\left(\frac{2n\pi t}{3}\right)$$



$f(t)$ no tiene paridad

$$f(t) = \begin{cases} A e^{-t/10} & 0 < t < \pi \\ -A e^{-t/10} & \pi < t < 2\pi \end{cases}$$

$T = 2\pi \qquad w_0 = \dfrac{2\pi}{2\pi} = 1$

$$\boxed{\begin{array}{ll} u = \cos(nt) & du = -n\,\mathrm{sen}(nt)\,dt \\ dv = e^{-t/10}\,dt & v = -10\, e^{-t/10} \end{array}}$$

$$a_n = \frac{2}{2\pi}\int_0^{\pi} A e^{-t/10}\cos(nt)\,dt + \frac{2}{2\pi}\int_{\pi}^{2\pi} -A e^{-t/10}\cos(nt)\,dt$$

Resolviendo los integrales por separado

$$I = \frac{A}{\pi}\int_0^{\pi} e^{-t/10}\cos(nt)\,dt = \frac{A}{\pi}\left\{ -10\, e^{-t/10}\cos(nt)\Big|_0^{\pi} - 10n\int_0^{\pi} e^{-t/10}\,\mathrm{sen}(nt)\,dt \right\}$$

$$\boxed{\begin{array}{ll} u = \mathrm{sen}(nt) & du = n\cos(nt)\,dt \\ dv = e^{-t/10}\,dt & v = -10\, e^{-t/10} \end{array}}$$

$$I = \frac{A}{\pi}\left\{ -10\left[e^{-\pi/10}\overbrace{\cos(n\pi)}^{(-1)^n} - \overbrace{e^{0}\cos(0)}^{1} \right] - 10n\int_0^{\pi} e^{-t/10}\,\mathrm{sen}(nt)\,dt \right\}$$

$$I = \frac{A}{\pi}\left\{ -10\left[e^{-\pi/10}(-1)^n - 1\right] - 10n\left[-10\, e^{-t/10}\,\mathrm{sen}(nt)\Big|_0^{\pi} + 10n\int_0^{\pi} e^{-t/10}\cos(nt)\,dt \right] \right\}$$

$$I = \frac{A}{\pi}\left\{ -10\left[e^{-\pi/10}(-1)^n - 1\right] - 10n\left[-10\left(e^{-\pi/10}\underbrace{\mathrm{sen}(n\pi)}_{0} - e^{0}\underbrace{\mathrm{sen}(0)}_{0} \right) + 10n\int_0^{\pi} e^{-t/10}\cos(nt)\,dt \right] \right\}$$

$$I = \frac{A}{\pi}\left\{ -10\left[e^{-\pi/10}(-1)^n - 1\right] - 100n^2\int_0^{\pi} e^{-t/10}\cos(nt)\,dt \right\}$$

$$II = \frac{A}{\pi}\int_{0}^{2\pi} e^{-t/10}\cos(nt)\,dt = \frac{A\left(100n\,e^{-t/10}\,sen(nt) - 10e^{-t/10}\cos(nt)\right)}{\pi(1+100n^2)}\Bigg|_{\pi}^{2\pi}$$

$$II = \frac{10A}{\pi(1+100n^2)}\left[10n\left(e^{-2\pi/10}\,\underbrace{sen(2n\pi)}_{0} - e^{-\pi/10}\,\underbrace{sen(n\pi)}_{0}\right) - \left(e^{-2\pi/10}\underbrace{\cos(2n\pi)}_{1} - e^{-\pi/10}\underbrace{\cos(n\pi)}_{(-1)^n}\right)\right]$$

$$II = \frac{10A}{\pi(1+100n^2)}\left[e^{-\pi/10}(-1)^n - e^{-2\pi/10}\right]$$

Sumando I y II

$$I + II = \frac{10A}{\pi(1+100n^2)}\left[1 - e^{-\pi/10}(-1)^n - e^{-\pi/10}(-1)^n + e^{-2\pi/10}\right]$$

$$a_n = \frac{10A}{\pi(1+100n^2)}\left[1 - 2e^{-\pi/10}(-1)^n + e^{-2\pi/10}\right]$$

$$b_n = \frac{A}{\pi}\int_0^{\pi} e^{-t/10}\,sen(nt)\,dt - \frac{A}{\pi}\int_{\pi}^{2\pi} e^{-t/10}\,sen(nt)\,dt$$

$u = sen(nt)$ $\quad du = n\cos(nt)\,dt$

$dv = e^{-t/10}dt$ $\quad v = -10e^{-t/10}dt$

$u = \cos(nt)$ $\quad du = -n\,sen(nt)\,dt$

Resolviendo por separado las integrales

$$I = \frac{A}{\pi}\int_0^{\pi} e^{-t/10}\,sen(nt)\,dt = \frac{A}{\pi}\left\{-10e^{-t/10}\,sen(nt)\Big|_0^{\pi} + 10n\int_0^{\pi} e^{-t/10}\cos(nt)\,dt\right\}$$

$$I = \frac{A}{\pi}\left\{-10e^{-t/10}\,sen(nt)\Big|_0^{\pi} + 10n\left[-10e^{-t/10}\cos(nt)\Big|_0^{\pi} - 10n\int_0^{\pi} e^{-t/10}\,sen(nt)\,dt\right]\right\}$$

$$I = \frac{A}{\pi}\left\{\left[-10e^{-t/10}\,sen(nt) - 100n\,e^{-t/10}\cos(nt)\right]\Bigg|_0^{\pi} - 100n^2\int_0^{\pi} e^{-t/10}\,sen(nt)\,dt\right\}$$

$$\left(\int_0^{\pi} e^{-t/10}\,\mathrm{sen}(nt)\,dt\right)\left(1+100n^2\right) = \left(-10e^{-t/10}\,\mathrm{sen}(nt) - 100n\,e^{-t/10}\cos(nt)\right)\Bigg|_0^{\pi}$$

$$= \frac{-10}{(1+100n^2)}\left[\left(e^{-\pi/10}\cancel{\mathrm{sen}(n\pi)}^{\,0} - e^{0}\cancel{\mathrm{sen}(0)}^{\,0}\right) + 10n\left(e^{-\pi/10}\cancel{\cos(n\pi)}^{\,(-1)^n} - e^{0}\cancel{\cos(0)}^{\,1}\right)\right]$$

$$= \frac{-10}{1+100n^2}\left(10n\left(e^{-\pi/10}(-1)^n - 1\right)\right) = \frac{100n}{1+100n^2}\left(1 - e^{-\pi/10}(-1)^n\right)$$

$$\int_0^{\pi} e^{-t/10}\,\mathrm{sen}(nt)\,dt = \frac{100An}{(1+100n^2)\pi}\left(1 - e^{-\pi/10}(-1)^n\right)$$

$$\int_{\pi}^{2\pi} e^{-t/10}\,\mathrm{sen}(nt)\,dt = \frac{A}{\pi(1+100n^2)}\left(-10e^{-t/10}\,\mathrm{sen}(nt) - 100n\,e^{-t/10}\cos(nt)\right)\Bigg|_{\pi}^{2\pi}$$

$$= \frac{-10A}{\pi(1+100n^2)}\left[\left(e^{-2\pi/10}\cancel{\mathrm{sen}(2n\pi)}^{\,0} - e^{-\pi/10}\cancel{\mathrm{sen}(n\pi)}^{\,0}\right) + 10n\left(e^{-2\pi/10}\cancel{\cos(2n\pi)}^{\,1} - e^{-\pi/10}\cancel{\cos(n\pi)}^{\,(-1)^n}\right)\right]$$

$$= \frac{-100An}{\pi(1+100n^2)}\left(e^{-2\pi/10} - e^{-\pi/10}(-1)^n\right) = \frac{100An}{\pi(1+100n^2)}\left(e^{-\pi/10}(-1)^n - e^{-2\pi/10}\right)$$

$$bn = \mathrm{I} - \mathrm{II}$$

$$bn = \frac{100An}{\pi(1+100n^2)}\left(1 - e^{-\pi/10}(-1)^n - e^{-\pi/10}(-1)^n + e^{-2\pi/10}\right)$$

$$bn = \frac{100An}{\pi(1+100n^2)}\left(1 - 2e^{-\pi/10}(-1)^n + e^{-2\pi/10}\right)$$

$$a_0 = \frac{A}{2\pi}\int_0^{\pi} e^{-t/10}\,dt - \frac{A}{2\pi}\int_{\pi}^{2\pi} e^{-t/10}\,dt = \frac{A}{2\pi}\left\{-10\left(e^{-\pi/10}-e^{0}\right)+10\left(e^{-2\pi/10}-e^{-\pi/10}\right)\right\}$$

$$a_0 = \frac{5A}{\pi}\left(e^{-2\pi/10}-2e^{-\pi/10}+1\right)$$

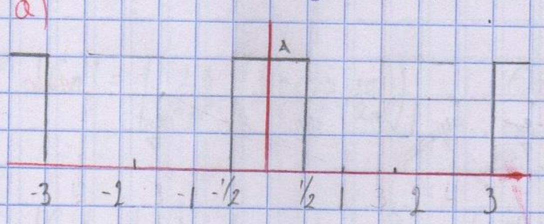
Finalmente

$$g(t) = \frac{5A}{\pi}\left(e^{-2\pi/10}-2e^{-\pi/10}+1\right) + \frac{10A}{\pi}\sum_{n=1}^{\infty}\left\{\frac{1}{1+100n^2}\left[\left(1-2e^{-\pi/10}(-1)^n+e^{-2\pi/10}\right)\left(\cos(nt)\right)\right.\right.$$

$$\left.\left. + 10n\left(1-2e^{-\pi/10}(-1)^n+e^{-2\pi/10}\right)\left(\operatorname{sen}(nt)\right)\right]\right\}$$

Problema 4. Calcular la serie exponencial de Fourier de cada una de las señales periódicas que se ilustran en la figura 3 y graficar los espectros de magnitud y fase.

a)



$$f(t) = \begin{cases} 0 & -7/4 < t < -1/2 \\ A & -1/2 < t < 1/2 \\ 0 & 1/2 < t < 7/4 \end{cases}$$

$$T = \frac{7}{2} \qquad w_0 = \frac{2\pi}{7/2} = \frac{4\pi}{7}$$

$$C_n = \frac{2A}{7}\int_{-1/2}^{1/2} e^{-i\frac{4n\pi t}{7}}\,dt = \frac{2A}{7}\left[\frac{-7}{i4n\pi}e^{-i\frac{4n\pi t}{7}}\Bigg|_{-1/2}^{1/2}\right] = \frac{-A}{i2n\pi}\left[e^{-i\frac{2n\pi}{7}} - e^{i\frac{2n\pi}{7}}\right]$$

$$C_n = \frac{4A}{n\pi}\left[\frac{e^{i\frac{2n\pi}{7}} - e^{-i\frac{2n\pi}{7}}}{i2}\right] = \frac{A}{n\pi}\operatorname{sen}\left(\frac{2n\pi}{7}\right)$$

$$f(t) = \frac{2A}{7} + \sum_{n=-\infty}^{\infty}\frac{A}{n\pi}\operatorname{sen}\left(\frac{2n\pi}{7}\right)e^{i\frac{4\pi t}{7}}$$

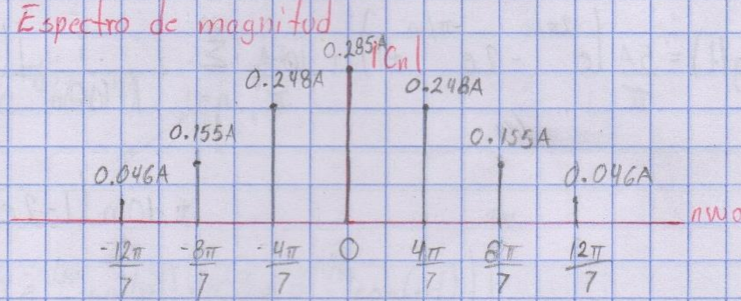
Espectros de magnitud y fase.

$$C_n = \frac{A}{n\pi}\,\text{sen}\left(\frac{2n\pi}{7}\right) \Rightarrow |C_n| = \left|\frac{A}{n\pi}\,\text{sen}\left(\frac{2n\pi}{7}\right)\right| \qquad |C_0| = \left|\frac{2A}{7}\right| = \frac{2A}{7}$$

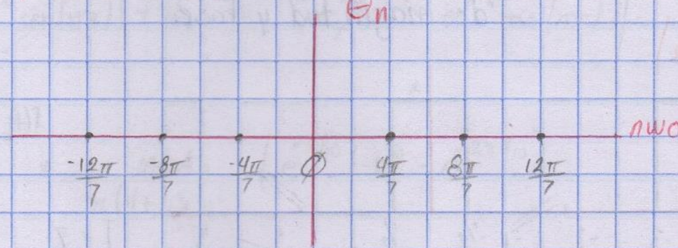
$$\Theta_n = \tan^{-1}\left[\frac{\phi}{\frac{A}{n\pi}\,\text{sen}\left(\frac{2n\pi}{7}\right)}\right] = \tan^{-1}(\phi) = \phi \qquad \Theta_\phi = \tan^{-1}\left[\frac{\phi}{\frac{2A}{7}}\right] = \phi$$

n	$n\omega_0$	$\|C_n\|$	Θ_n
-3	$-12\pi/7$	0.046 A	ϕ
-2	$-8\pi/7$	0.155 A	ϕ
-1	$-4\pi/7$	0.248 A	ϕ
0	0	0.285 A	ϕ
1	$4\pi/7$	0.248 A	ϕ
2	$8\pi/7$	0.155 A	ϕ
3	$12\pi/7$	0.046 A	ϕ

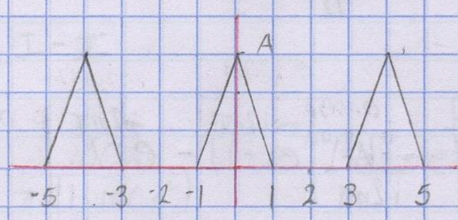
Espectro de magnitud



Espectro de fase



b)



$T = 4$

$\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$$f(t) = \begin{cases} 0 & -2 < t < -1 \\ A(t+1) & -1 < t < 0 \\ -A(t-1) & 0 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$$

$$C_n = \frac{A}{4}\int_{-1}^{0} (t+1)\, e^{-\frac{in\pi t}{2}}\, dt - \frac{A}{4}\int_{0}^{1} (t-1)\, e^{-\frac{in\pi t}{2}}\, dt$$

$u = t+1 \quad du = dt$

$dv = e^{-\frac{in\pi}{2}t}\, dt$

$u = t-1$

$du = dt$

$v = -\frac{2}{in\pi}\, e^{-\frac{in\pi}{2}t}$

$$C_n = \frac{2A}{4}\left\{ \frac{-2(t+1)}{in\pi} e^{\frac{-in\pi t}{2}} \Bigg|_{-1}^{0} + \frac{2}{in\pi}\int_{-1}^{0} e^{\frac{-in\pi t}{2}}\,dt + \frac{2(t-1)}{in\pi} e^{\frac{-in\pi t}{2}} \Bigg|_{0}^{1} - \frac{2}{in\pi}\int_{0}^{1} e^{\frac{-in\pi}{2}t}\,dt \right\}$$

$$C_n = \frac{A}{4}\left\{ -\frac{2}{in\pi} e^{0} + 0 + 0 + \frac{2}{in\pi} e^{0} + \frac{2}{in\pi}\left[-\frac{2}{in\pi} e^{\frac{-in\pi t}{2}} \Bigg|_{-1}^{0} + \frac{2}{in\pi} e^{\frac{-in\pi t}{2}} \Bigg|_{0}^{1} \right] \right\}$$

$$C_n = \frac{A}{4}\left\{ \frac{-4}{n^2\pi^2}\left[e^{0} - e^{\frac{in\pi}{2}} \right] - \frac{4}{n^2\pi^2}\left[e^{\frac{-in\pi}{2}} - e^{0} \right] \right\}$$

$$C_n = \frac{A}{n^2\pi^2}\left[1 - e^{\frac{in\pi}{2}} \right] - \frac{A}{n^2\pi^2}\left[e^{\frac{-in\pi}{2}} - 1 \right] = \frac{A}{n^2\pi^2}\left[1 - e^{\frac{in\pi}{2}} - e^{\frac{-in\pi}{2}} + 1 \right]$$

$$C_n = \frac{A}{n^2\pi^2}\left[2 - 2\cos\left(\frac{n\pi}{2}\right) \right] = \frac{2A}{n^2\pi^2}\left(1 - \cos\left(\frac{n\pi}{2}\right) \right) \quad \forall\, n \neq 0$$

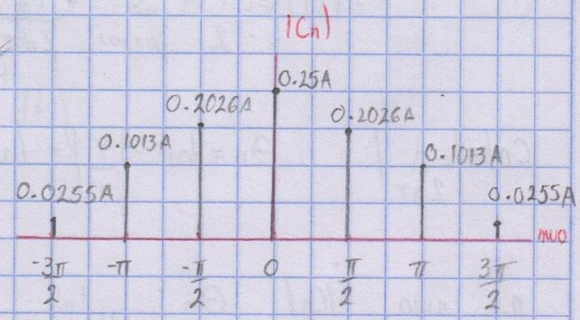
$$f(t) = \frac{A}{4} + \sum_{n=-\infty}^{\infty} \left(\frac{2A}{n^2\pi^2}\left(1 - \cos\left(\frac{n\pi}{2}\right) \right) \right) e^{\frac{in\pi t}{2}}$$

$$|C_n| = \left| \frac{2A}{n^2\pi^2}\left(1 - \cos\frac{n\pi}{2} \right) \right| \qquad \theta_n = \tan^{-1}\left| \frac{0}{\frac{2A}{n^2\pi^2}\left(1 - \cos\frac{n\pi}{2} \right)} \right| = 0$$

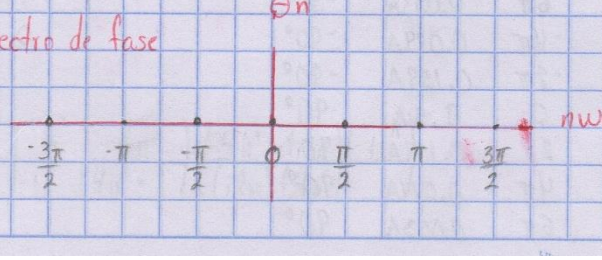
$$|C_0| = \left| \frac{A}{4} \right| = \frac{A}{4} \qquad \theta_0 = \tan^{-1}\left| \frac{0}{\frac{A}{4}} \right| = 0$$

n	$n\omega_0$	$\lvert C_n \rvert$	θ_n
-3	$-3\pi/2$	0.0225A	0
-2	$-\pi$	0.1013A	0
-1	$-\pi/2$	0.2026A	0
0	0	0.25A	0
1	$\pi/2$	0.2026A	0
2	π	0.1013A	0
3	$3\pi/2$	0.0225A	0

Espectro de magnitud

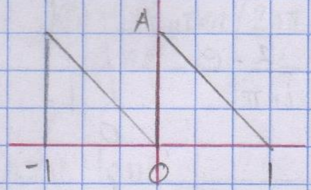


Espectro de fase



$$e^{-i2n\pi} = \cos(2n\pi) - i\,\text{sen}\,(2n\pi) = 1$$

c)



$T = 1$

$w_0 = 2\pi$

$f(t) = -A(t-1)$

$u = (t-1) \quad du = dt$

$dv = e^{-i2n\pi t}\,dt \quad v = -\dfrac{1}{i2n\pi} e^{-i2n\pi t}$

$$C_n = -A\int_0^1 (t-1)\, e^{-i2n\pi t}\, dt = -A\left\{ \frac{-(t-1)}{i2n\pi} e^{-i2n\pi t}\Big|_0^1 + \frac{1}{i2n\pi}\int_0^1 e^{-i2n\pi t}\, dt \right\}$$

$$C_n = -A\left\{ \emptyset - \frac{1}{i2n\pi} e^{0} + \frac{1}{i2n\pi}\left[-\frac{1}{i2n\pi} e^{-i2n\pi t}\Big|_0^1 \right]\right\}$$

$$C_n = -A\left\{ -\frac{1}{i2n\pi} + \frac{1}{i2n\pi}\left[-\frac{1}{i2n\pi}\left(e^{-i2n\pi} - e^{0}\right)\right]\right\} = -A\left\{ -\frac{1}{i2n\pi} + \frac{1}{4n^2\pi^2}\left(e^{-i2n\pi} - 1\right)\right\}$$

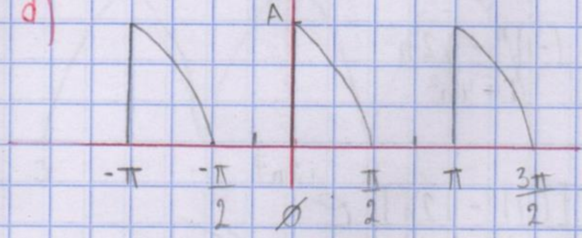
$$C_n = \frac{A}{i2n\pi} - \frac{A}{4n^2\pi^2}\left(\underbrace{e^{-i2n\pi}}_{1} - 1\right) = \frac{A}{i2n\pi} = -\frac{iA}{2n\pi} \quad \forall n \neq 0$$

$$f(t) = \frac{1}{2} + \sum_{n=-\infty}^{\infty} -\frac{iA}{2n\pi} e^{2n\pi t}$$

$$|C_n| = \left|\frac{A}{2n\pi}\right| \qquad \theta_n = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{\frac{A}{2n\pi}}{\emptyset}\right| = \begin{cases} \theta_n = 90^\circ & y > 0 \\ \theta_n = -90^\circ & y < 0 \end{cases}$$

n	nw_0	$\lvert C_n \rvert$	θ_n
-3	-6π	0.053A	-90°
-2	-4π	0.079A	-90°
-1	-2π	0.159A	-90°
0	0	0.5A	90°
1	2π	0.159A	90°
2	4π	0.079A	90°
3	6π	0.053A	90°

d)



$$f(t) = \begin{cases} 0 & -\pi/4 < t < 0 \\ A\cos(t) & 0 < t < \pi/2 \\ 0 & \pi/2 < t < 3\pi/4 \end{cases}$$

$T = \pi \qquad w_0 = \frac{2\pi}{\pi} = 2$

$$C_n = \frac{1}{\pi}\int_0^{\pi/2} A\cos(t)\, e^{-i2nt}\, dt = \frac{A}{\pi}\int_0^{\pi/2} \cos(t)\, e^{-i2nt}\, dt$$

$u = \cos(t) \quad du = -\operatorname{sen}(t)\,dt$

$dv = e^{-i2nt}\,dt \quad v = -\frac{e^{-i2nt}}{i2n}$

$$C_n = \frac{A}{\pi}\left\{ -\frac{\cos(t)\,e^{-i2nt}}{i2n}\Bigg|_0^{\pi/2} - \frac{1}{i2n}\int_0^{\pi/2} \operatorname{sen}(t)\, e^{-i2nt}\, dt \right\}$$

$u = \operatorname{sen}(t) \quad du = \cos(t)\,dt$

$$C_n = \frac{A}{\pi}\left\{ -\frac{1}{i2n}\left[\emptyset - \cos(\emptyset)\, e^{0}\right] - \frac{1}{i2n}\left[-\frac{\operatorname{sen}(t)\,e^{-i2nt}}{i2n}\Bigg|_0^{\pi/2} + \frac{1}{i2n}\int_0^{\pi/2} \cos(t)\, e^{-i2nt}\, dt \right] \right\}$$

$$C_n = \frac{A}{\pi}\left\{ \frac{1}{i2n} - \frac{1}{i2n}\left[\frac{-1}{i2n}\left[e^{-in\pi} - \emptyset\right] + \frac{1}{i2n}\int_0^{\pi/2} \cos(t)\, e^{-i2nt}\, dt \right] \right\}$$

$$C_n = \frac{A}{\pi}\left\{ \frac{1}{i2n} - \frac{1}{4n^2} e^{-in\pi} + \frac{1}{4n^2}\int_0^{\pi/2} \cos(t)\, e^{-i2nt}\, dt \right\}$$

$$\int_0^{\pi/2} \cos(t)\, e^{-i2nt}\, dt = \frac{1}{i2n} - \frac{1}{4n^2} \overset{(-1)^n}{e^{-in\pi}} + \frac{1}{4n^2}\int_0^{\pi/2} \cos(t)\, e^{-i2nt}\, dt$$

$$\int_0^{\pi/2} \cos(t)\, e^{-i2nt} = \left(\frac{4n^2}{4n^2-1}\right)\left(\frac{1}{i2n} - \frac{(-1)^n}{4n^2}\right) = \left(\frac{4n^2}{4n^2-1}\right)\left(\frac{4n^2 - (i2n)(-1)^n}{(i2n)(4n^2)}\right)$$

$$\int_0^{\pi/2} \cos(t)\, e^{-i2nt}\, dt = \frac{4n^2 - (i2n)(-1)^n}{(4n^2-1)(i2n)} = \frac{4n^2}{(4n^2-1)(i2n)} - \frac{(i2n)(-1)^n}{(4n^2-1)(i2n)}$$

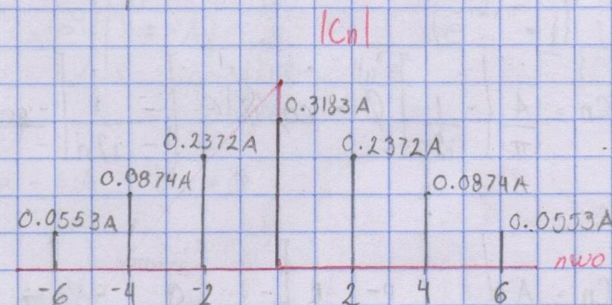
$$= i\,\frac{2n}{4n^2-1} - \frac{(-1)^n}{4n^2-1} = \frac{i2n - (-1)^n}{4n^2-1} = \frac{(-1)^n - i2n}{1-4n^2}$$

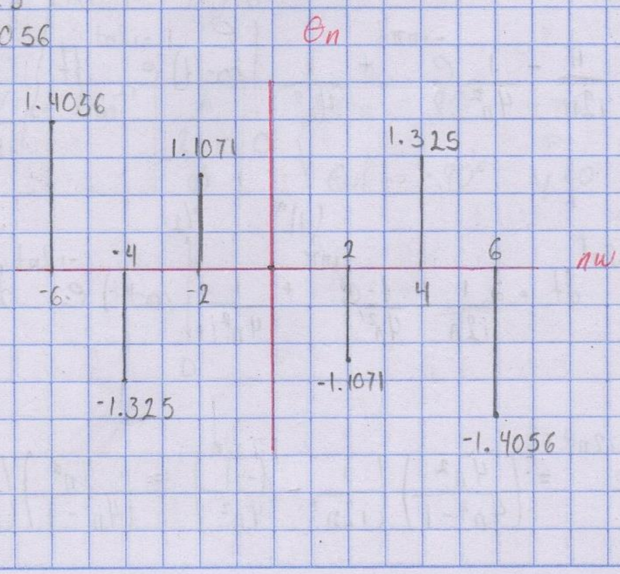
$$C_n = \frac{A\left[(-1)^n - i2n\right]}{\pi(1-4n^2)} \Rightarrow f(t) = \sum_{n=-\infty}^{\infty} \frac{A\left[(-1)^n - i2n\right]}{\pi(1-4n^2)}\, e^{i2nt}$$

$$|C_n| = \sqrt{\frac{A^2(-1)^{2n}}{\pi^2(1-4n^2)^2} + \frac{A^2(4n^2)}{\pi^2(1-4n^2)^2}} = \sqrt{\frac{A^2\left[(-1)^{2n} + 4n^2\right]}{\pi^2(1-4n^2)^2}} = A\sqrt{\frac{(-1)^{2n} + 4n^2}{\pi^2(1-4n^2)^2}}$$

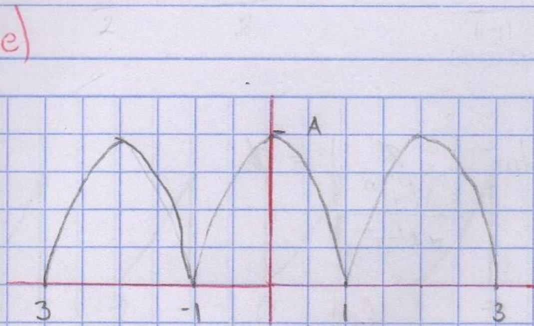
$$\theta_n = \tan^{-1}\left|\frac{\frac{2An}{\pi(1-4n^2)}}{\frac{A(-1)^n}{\pi(1-4n^2)}}\right| = \tan^{-1}\left|\frac{2An\cdot\pi(1-4n^2)}{A(-1)^n\cdot\pi(1-4n^2)}\right| = \tan^{-1}\left|\frac{2n}{(-1)^n}\right|$$

n	$n\omega_0$	$\lvert C_n\rvert$	θ_n
-3	-6	0.0553A	1.4056
-2	-4	0.0874A	-1.325
-1	-2	0.2372A	1.1071
0	0	0.3183A	0
1	2	0.2372A	-1.1071
2	4	0.0874A	1.325
3	6	0.0553A	-1.4056





e)



$$f(t) = A\cos w't \qquad -1 < t < 1$$

$$w' = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$T = 2 \qquad w_0 = \pi$$

$$C_n = \frac{1}{2}\int_{-1}^{1} A\cos\left(\frac{\pi t}{2}\right) e^{-in\pi t}\, dt = \frac{A}{2}\int_{-1}^{1}\left[\frac{e^{\frac{i\pi t}{2}} + e^{\frac{-i\pi t}{2}}}{2}\right] e^{-in\pi t}\, dt$$

$$C_n = \frac{A}{4}\int_{-1}^{1}\left[e^{i(1/2-n)\pi t} + e^{-i(1/2+n)\pi t}\right] dt = \frac{A}{4}\left\{\left.\frac{e^{i(1/2-n)\pi t}}{i(1/2-n)\pi}\right|_{-1}^{1} - \left.\frac{e^{-i(1/2+n)\pi t}}{i(1/2+n)\pi}\right|_{-1}^{1}\right\}$$

$$C_n = \frac{A}{4}\left\{\frac{1}{i(1/2-n)\pi}\left[e^{i(1/2-n)\pi} - e^{-i(1/2-n)\pi}\right] - \frac{1}{i(1/2+n)\pi}\left[e^{-i(1/2+n)\pi} - e^{i(1/2+n)\pi}\right]\right\}$$

$$C_n = \frac{A}{4}\left\{\frac{2}{i\pi(1-n)}\left[e^{i(1/2-n)\pi} - e^{-i(1/2-n)\pi}\right] + \frac{2}{i\pi(1+n)}\left[e^{i(1/2+n)\pi} - e^{-i(1/2+n)\pi}\right]\right\}$$

$$C_n = \frac{A}{4}\left\{\frac{2}{i\pi(1-n)}\left[e^{i\pi/2}\cdot e^{-in\pi} - e^{-i\pi/2}\, e^{in\pi}\right] + \frac{2}{i\pi(1+n)}\left[e^{i\pi/2}\cdot e^{in\pi} - e^{-i\pi/2}\cdot e^{-in\pi}\right]\right\}$$

$$e^{\pm i\pi/2} = \overset{0}{\cancel{\cos(\pi/2)}} \pm i\,\overset{1}{\cancel{\operatorname{sen}(\pi/2)}} = \pm i \qquad ; \qquad e^{in\pi} + e^{-in\pi} = 2\cos(n\pi)$$

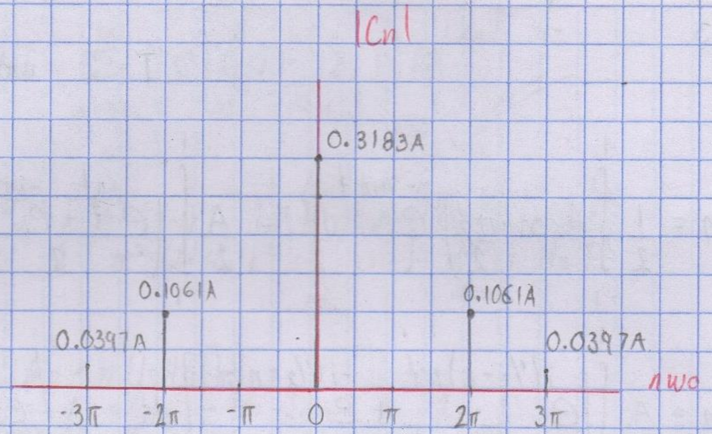
$$C_n = \frac{A}{4}\left\{\frac{2}{i\pi(1-n)}\left[ie^{-in\pi} + ie^{in\pi}\right] + \frac{2}{i\pi(1+n)}\left[ie^{in\pi} + ie^{-in\pi}\right]\right\}$$

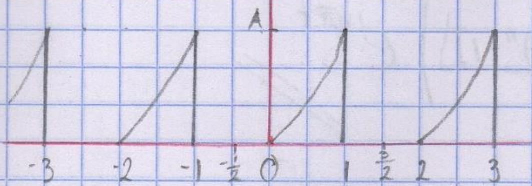
$$C_n = \frac{A}{4\pi}\left[\frac{4\cos(n\pi)}{1-n} + \frac{4\cos(n\pi)}{1+n}\right] = \frac{A\cos(n\pi)}{\pi(1-n^2)}$$

$$f(t) = \sum_{n=\infty}^{\infty} \frac{A(-1)^n}{\pi(1-n^2)} e^{in\pi t}$$

$$|C_n| = \left| \frac{A(-1)^n}{\pi(1-n^2)} \right| \qquad \theta_n = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left[\frac{\emptyset}{\frac{A(-1)^n}{\pi(1-n^2)}}\right] = \emptyset$$

n	nwo	$\lvert C_n \rvert$	θ_n
-3	-3π	0.0397A	$\emptyset$
-2	-2π	0.1061A	$\emptyset$
-1	$-\pi$		
0	0	0.3183A	$\emptyset$
1	π		
2	2π	0.1061A	$\emptyset$
3	3π	0.0397	$\emptyset$





$$f(t) = At^2 \qquad 0 < t < 1$$

$$u = t \qquad du = dt$$

$$T = 2 \qquad w_0 = \pi$$

$$u = t^2 \qquad du = 2t\,dt \qquad dv = e^{-in\pi t}\,dt \qquad v = -\frac{e^{-in\pi t}}{in\pi}$$

$$C_n = \frac{1}{2}\int_0^1 At^2 e^{-in\pi t}\,dt = \frac{A}{2}\left\{ -\frac{t^2 e^{-in\pi t}}{in\pi}\Bigg|_0^1 + \frac{2}{in\pi}\int_0^1 t e^{-in\pi t}\,dt \right\}$$

$$C_n = \frac{A}{2}\left\{ -\frac{1}{in\pi}\left[e^{-in\pi} - \emptyset\right] + \frac{2}{in\pi}\left[\frac{-te^{-in\pi t}}{in\pi}\Bigg|_0^1 + \frac{1}{in\pi}\int_0^1 e^{-in\pi t}\,dt\right]\right\}$$

$$C_n = \frac{A}{2}\left\{ \frac{-e^{-in\pi}}{in\pi} + \frac{2}{in\pi}\left[-\frac{1}{in\pi}\left[e^{-in\pi} - \emptyset\right] + \frac{1}{in\pi}\left[-\frac{1}{in\pi}\left[e^{-in\pi} - e^0\right]\right]\right]\right\}$$

$$C_n = \frac{A}{2}\left\{ \frac{-e^{-in\pi}}{in\pi} + \frac{2}{in\pi}\left[\frac{-e^{-in\pi}}{in\pi} + \frac{1}{n^2\pi^2}\left(e^{-in\pi} - 1\right)\right]\right\}$$

$$C_n = \frac{A}{2}\left\{ \frac{-e^{-in\pi}}{in\pi} + \frac{2e^{-in\pi}}{n^2\pi^2} + \frac{2}{in^3\pi^3}\left(e^{-in\pi} - 1\right)\right\}$$

$$e^{-in\pi} = \overset{\nearrow (-1)^n}{\cos(n\pi)} - i\,\overset{\nearrow 0}{\cancel{\operatorname{sen}}(n\pi)} = (-1)^n$$

$$C_n = \frac{A}{2}\left\{ \frac{i(-1)^n}{n\pi} + \frac{2(-1)^n}{n^2\pi^2} - \frac{i2}{n^3\pi^3}\left[(-1)^n - 1\right]\right\}$$

$$C_n = \frac{A}{2}\left\{ i\,\frac{n^2\pi^2(-1)^n - 2\left[(-1)^n - 1\right]}{n^3\pi^3} + \frac{2(-1)^n}{n^2\pi^2}\right\}$$

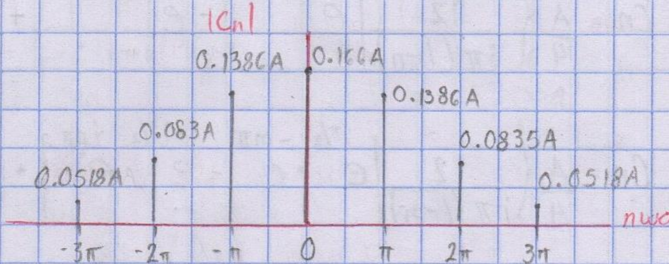
$$C_n = \frac{A(-1)^n}{n^2\pi^2} + i\,\frac{A\left\{n^2\pi^2(-1)^n - 2\left[(-1)^n - 1\right]\right\}}{2\pi^3 n^3}$$

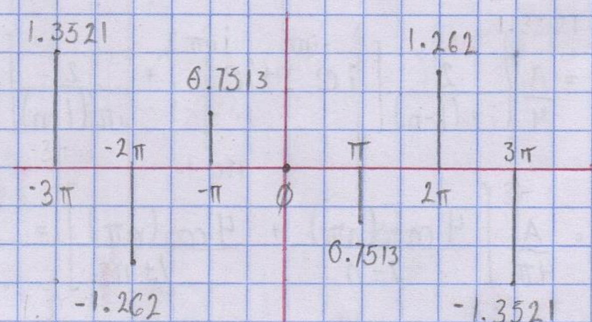
$$f(t) = \sum_{n=-\infty}^{\infty}\left(\frac{A(-1)^n}{n^2\pi^2} + \frac{iA\left\{n^2\pi^2(-1)^n - 2\left[(-1)^n - 1\right]\right\}}{2\pi^3 n^3}\right)e^{in\pi t}$$

$$|C_n| = \sqrt{\left(\frac{A(-1)^n}{n^2\pi^2}\right)^2 + \left(\frac{A\left\{n^2\pi^2(-1)^n - 2\left[(-1)^n - 1\right]\right\}}{2\pi^3\pi^3}\right)^2}$$

$$\theta_n = \tan^{-1}\left[\frac{\dfrac{A\left\{n^2\pi^2(-1)^n - 2\left[(-1)^n - 1\right]\right\}}{2\pi^3 n^3}}{\dfrac{A(-1)^n}{n^2\pi^2}}\right]$$

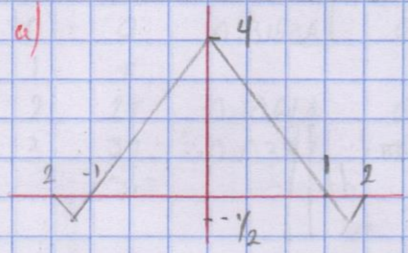
n	$n\omega_0$	$\lvert C_n \rvert$	θ_n
-3	-3π	0.0518A	1.3521
-2	-2π	0.0835A	-1.262
-1	$-\pi$	0.1386A	0.7513
0	0	0.1666A	Ø
1	π	0.1386A	-0.751
2	2π	0.0835A	1.262
3	3π	0.0518A	-1.3521





Problema 5. Obtener la transformada de Fourier de cada una de las señales de figura 4.

a)



$$f(t)=\begin{cases} -\frac{4}{3}t-\frac{8}{3} & -2<t<-\frac{3}{2} \quad \text{I} \\ 4t+4 & -\frac{3}{2}<t<0 \quad \text{II} \\ -4t+4 & 0<t<\frac{3}{2} \quad \text{III} \\ \frac{4}{3}t-\frac{8}{3} & \frac{3}{2}<t<2 \quad \text{IV} \end{cases}$$

$$\text{I}=\int_{-2}^{-3/2}\left(-\frac{4}{3}t-\frac{8}{3}\right)e^{-iwt}\,dt=\left[\frac{-\left(\frac{4}{3}t+\frac{8}{3}\right)e^{-iwt}}{iw}-\frac{\left(\frac{4}{3}\right)e^{-iwt}}{w^2}\right]_{-2}^{-3/2}$$

$$=\left[\frac{\left(\frac{4}{3}\left(-\frac{3}{2}\right)-\frac{8}{3}\right)e^{\frac{i3w}{2}}}{\frac{-\frac{12}{6}+\frac{16}{6}}{w}}-\frac{\left(\frac{4}{3}\right)e^{\frac{i3w}{2}}}{w^2}-\frac{\left(\frac{4}{3}(-2)+\frac{8}{3}\right)e^{i2w}}{iw}+\frac{\frac{4}{3}e^{-i2w}}{w^2}\right]$$

$$=\left[\frac{\left(-\frac{12}{6}+\frac{8}{3}\right)e^{\frac{i3w}{2}}}{iw}-\frac{4e^{\frac{i3w}{2}}}{3w^2}+\frac{4e^{i2w}}{3w^2}\right]=\frac{4e^{\frac{i3w}{2}}}{i6w}-\frac{4e^{\frac{i3w}{2}}}{3w^2}+\frac{4e^{i2w}}{3w^2}$$

$$\text{II}=\int_{-3/2}^{0}(4t+4)e^{-iwt}\,dt=4\left[\frac{-(t+1)e^{-iwt}}{iw}+\frac{e^{-iwt}}{w^2}\right]_{-3/2}^{0}=4\left[\frac{-e^{0}}{iw}+\frac{e^{0}}{w^2}-\frac{e^{\frac{i3w}{2}}}{i2w}-\frac{e^{i\frac{3}{2}w}}{w^2}\right]$$

$$=4\left[\frac{1}{iw}+\frac{1}{w^2}-\frac{e^{\frac{i3w}{2}}}{2iw}-\frac{e^{-\frac{i3w}{2}}}{w^2}\right]$$

$$II = \int_0^{3/2} (-4t+4)\, e^{-iwt}\, dt = 4\left[\frac{-(t+1)\,e^{-iwt}}{iw} - \frac{e^{-iwt}}{w^2}\right]\Bigg|_0^{3/2} = 4\left[\frac{e^{-\frac{i3w}{2}}}{i2w} - \frac{e^{-\frac{i3w}{2}}}{w^2} + \frac{1}{iw} + \frac{1}{w^2}\right]$$

$$III = \int_{3/2}^{2} \left(\frac{4}{3}t - \frac{8}{3}\right) e^{-iwt}\, dt = \left[\frac{-\left(\frac{4}{3}t - \frac{8}{3}\right) e^{iwt}}{iw} + \frac{4e^{iwt}}{3w^2}\right]\Bigg|_2^{3/2} = \frac{4e^{2iw}}{3w^2} + \frac{\left(\frac{4}{3}\left(\frac{3}{2}\right) - \frac{8}{3}\right) e^{\frac{i3w}{2}}}{iw} - \frac{4e^{\frac{i3w}{2}}}{3w^2}$$

$$= \frac{4e^{i2w}}{3w^2} - \frac{4e^{\frac{i3w}{2}}}{i6w} - \frac{4e^{\frac{i3w}{2}}}{3w^2}$$

I y III

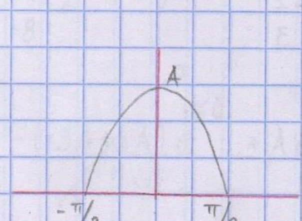
$$\frac{4e^{i2w}}{3w^2} - \frac{4e^{\frac{i3w}{2}}}{i6w} - \frac{4e^{\frac{i3w}{2}}}{w^2} + \frac{4e^{\frac{i3w}{2}}}{i6w} - \frac{4e^{\frac{i3w}{2}}}{3w^2} + \frac{4e^{i2w}}{3w^2} = \frac{8e^{i2w}}{3w^2} - \frac{8e^{\frac{i3w}{2}}}{3w^2}$$

II y III

$$\frac{-4}{iw} + \frac{4}{w^2} - \frac{4e^{\frac{i3w}{2}}}{i2w} - \frac{4e^{\frac{i3w}{2}}}{w^2} + \frac{4}{iw} + \frac{4}{w^2} + \frac{4e^{-\frac{i3w}{2}}}{i2w} - \frac{4e^{-\frac{i3w}{2}}}{w^2} = \frac{8}{w^2} - \frac{8e^{-\frac{i3w}{2}}}{w^2}$$

$$\therefore \mathcal{F}\{f(t)\} = \frac{8}{w^2} - \frac{16e^{-\frac{i3w}{2}}}{3w^2} + \frac{8e^{i2w}}{3w^2}$$

b)

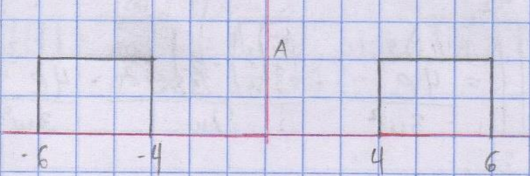


$$f(t) = A\cos t \qquad -\pi/2 < t < \pi/2$$

$$\int_{-\pi/2}^{\pi/2} A\cos t\, e^{-iwt}\, dt = A\left[\frac{e^{-iwt}\left[-iw\cos(t) + \operatorname{sen}(t)\right]}{1-w^2}\right]\Bigg|_{-\pi/2}^{\pi/2} = A\left[\frac{e^{-\frac{i\pi w}{2}}}{1-w^2} + \frac{e^{\frac{i\pi w}{2}}}{1-w^2}\right] = \frac{2A\cos\left(\frac{w\pi}{2}\right)}{1-w^2}$$

$$\therefore F\{f(t)\} = \frac{2A\cos\left(\frac{w\pi}{2}\right)}{1-w^2}$$

c)



$$f(t)=\begin{cases} A & -6<t<-4 \\ 0 & -4<t<4 \\ A & 4<t<6 \end{cases}$$

$$A\int_{-6}^{-4} e^{-i\omega t}\,dt + A\int_{4}^{6} e^{-i\omega t}\,dt = A\left[\frac{-e^{-i\omega t}}{i\omega}\Bigg|_{-6}^{-4} - \frac{e^{-i\omega t}}{i\omega}\Bigg|_{4}^{6}\right] = \frac{iA}{\omega}\left[e^{i4\omega} - e^{i6\omega} + e^{-i6\omega} - e^{-i4\omega}\right]$$

$$= \frac{iA}{\omega}\left(e^{i4\omega} - e^{-i4\omega}\right) - \frac{iA}{\omega}\left(e^{i6\omega} - e^{-i6\omega}\right) = -\frac{2A}{\omega}\operatorname{sen}(4\omega) + \frac{2A}{\omega}\operatorname{sen}(6\omega)$$

$$= \frac{2A}{\omega}\left[\operatorname{sen}(6\omega) - \operatorname{sen}(4\omega)\right]$$

d)

$$f(t)=\begin{cases} At^2 & 0<t<1 \\ 0 & 1<t<2 \\ At^2 & 2<t<3 \end{cases}$$

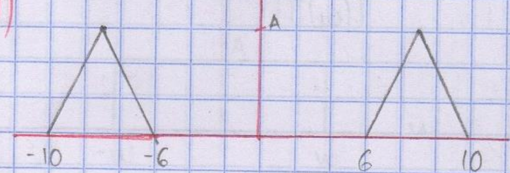
$$A\int t^2 e^{-i\omega t}\,dt = A\left[\frac{it^2 e^{-i\omega t}}{\omega} + \frac{te^{-i\omega t}}{\omega^2} - \frac{i2e^{-i\omega t}}{\omega^3}\right]\Bigg|_{2}^{3}$$

$$\mathrm{I} = \left\{\left[\frac{ie^{-i\omega}}{\omega} + \frac{e^{-i\omega}}{\omega^2} - \frac{i2e^{-i\omega}}{\omega^3}\right] - \left[-\frac{i2e^{0}}{\omega^3}\right]\right\} = \frac{ie^{-i\omega}}{\omega} + \frac{e^{-i\omega}}{\omega^2} - \frac{i2e^{-i\omega}}{\omega^3} + \frac{i2}{\omega^3}$$

$$\mathrm{II} = \left\{\left[\frac{i9e^{-i3\omega}}{\omega} + \frac{3e^{-i3\omega}}{\omega^2} - \frac{i2e^{-i3\omega}}{\omega^3}\right] - \left[\frac{i4e^{-i2\omega}}{\omega} + \frac{2e^{-i2\omega}}{\omega^2} - \frac{i2e^{-i2\omega}}{\omega^3}\right]\right\}$$

$$\mathcal{F}(f(t)) = A\left[\frac{ie^{-iw}}{w} + \frac{e^{-iw}}{w^2} - \frac{i2e^{-iw}}{w^3} + \frac{i2}{w^3} + \frac{i9e^{-i3w}}{w} + \frac{3e^{-i3w}}{w^2} - \frac{i2e^{-i3w}}{w^3} - \frac{i4e^{-i2w}}{w} - \frac{2e^{-i2w}}{w^2} + \frac{i2e^{-i2w}}{w^3}\right]$$

d)



$$f(t) = \begin{cases} A/2\,t + 5A & -10 < t < -8 & \text{I} \\ A/2\,t + 3A & -8 < t < -6 & \text{II} \\ A/2\,t - 3A & 6 < t < 8 & \text{III} \\ A/2\,t - 5A & 8 < t < 10 & \text{IV} \end{cases}$$

$$\text{I} = \int_{-10}^{-8} \left(\frac{A}{2}t + 5A\right) e^{-iwt}\, dt = \left[\frac{-\left(\frac{A}{2}t + 5A\right)e^{-iwt}}{iw} + \frac{Ae^{-iwt}}{2w^2}\right]\Bigg|_{-10}^{-8}$$

$$= \left\{\left[\frac{-(-4A+5A)e^{i8w}}{iw} + \frac{Ae^{i8w}}{2w^2}\right] - \left[\frac{-\cancel{(-5A+5A)}^{\,0}e^{i10w}}{iw} + \frac{Ae^{i10w}}{2w^2}\right]\right\}$$

$$= \frac{iAe^{i8w}}{w} + \frac{Ae^{i8w}}{2w^2} - \frac{Ae^{i10w}}{2w^2}$$

$$\text{II} = \int_{-8}^{-6} \left(\frac{A}{2}t + 3A\right) e^{-iwt}\, dt = \left[\frac{-\left(\frac{A}{2}t + 3A\right)e^{-iwt}}{iw} + \frac{Ae^{-iwt}}{2w^2}\right]\Bigg|_{-8}^{-6}$$

$$= \left\{\left[\frac{-\cancel{(-3A+3A)}^{\,0}e^{i6w}}{iw} + \frac{Ae^{i6w}}{2w^2}\right] - \left[\frac{-\cancel{(-4A+3A)}^{\,-A}e^{i8w}}{iw} + \frac{Ae^{i8w}}{2w^2}\right]\right\}$$

$$= \frac{Ae^{i6w}}{2w^2} + \frac{iAe^{i8w}}{w} - \frac{Ae^{i8w}}{2w^2}$$

$$\text{III} = \int_{6}^{8} \left(\frac{A}{2}t - 3A\right) e^{-iwt}\, dt = \left\{\left[\frac{-\cancel{(4A-3A)}^{\,A}e^{-i8w}}{iw} + \frac{Ae^{-i8w}}{2w^2}\right] - \left[\frac{-\cancel{(3A-3A)}^{\,0}e^{-i6w}}{iw} + \frac{Ae^{-i6w}}{2w^2}\right]\right\}$$

$$= \frac{iAe^{-i8w}}{w} + \frac{Ae^{-i8w}}{2w^2} - \frac{Ae^{i6w}}{2w^2}$$

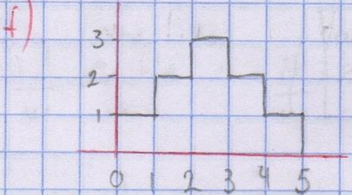
$$IV = \int_8^{10}\left(\frac{A}{2}t - 5A\right)e^{-iwt}\,dt = \left[\frac{-\overbrace{(5A-5A)}^{0}e^{-i10w}}{iw} + \frac{Ae^{-i10w}}{2w^2}\right] - \left[\frac{-\overbrace{(4A-5A)}^{-A}e^{-i8w}}{iw} + \frac{Ae^{-i8w}}{2w^2}\right]$$

$$I \text{ y } II = \frac{i2Ae^{i8w}}{w} - \frac{Ae^{i10w}}{2w^2} + \frac{Ae^{i6w}}{2w^2}$$

$$III \text{ y } IV = \frac{i2Ae^{-i8w}}{w} - \frac{Ae^{-i6w}}{2w^2} + \frac{Ae^{-i10w}}{2w^2}$$

$$\mathcal{F}\{f(t)\} = \frac{i4A}{w}\cos(8w) - \frac{iA}{w^2}\,\text{sen}(10w) + \frac{iA}{w^2}\,\text{sen}(6w)$$

f)



$$f(t) = \begin{cases} 1 & 0 < t < 1 \quad I \\ 2 & 1 < t < 2 \quad II \\ 3 & 2 < t < 3 \quad III \\ 2 & 3 < t < 4 \quad IV \\ 1 & 4 < t < 5 \quad V \end{cases}$$

$$\int_a^b Ae^{-iwt}\,dt = \left.-\frac{Ae^{-iwt}}{iw}\right|_a^b = \left.\frac{iAe^{-iwt}}{w}\right|_a^b$$

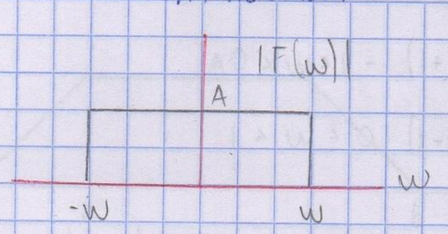
$$I = \frac{ie^{-iw}}{w} - \frac{i}{w}\,, \quad II = \frac{i2e^{-i2w}}{w} - \frac{i2e^{-iw}}{w}\,, \quad III = \frac{i3e^{-i3w}}{w} - \frac{i3e^{-i2w}}{w}$$

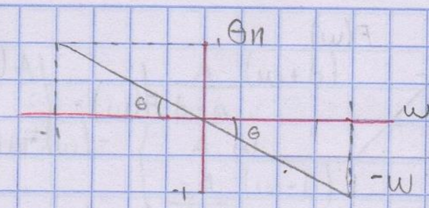
$$IV = \frac{i2e^{-i4w}}{w} - \frac{i2e^{-i3w}}{w} \qquad V = \frac{ie^{-i5w}}{w} - \frac{ie^{-i4w}}{w}$$

$$\mathcal{F}\{f(t)\} = -\frac{i}{w} - \frac{ie^{-iw}}{w} - \frac{ie^{-i2w}}{w} + \frac{ie^{-i3w}}{w} + \frac{ie^{-i4w}}{w} + \frac{ie^{-i5w}}{w}$$

Problema 6. Determinar cada una de las señales $f(t)$ cuya transformada se ilustra a continuación.

a)



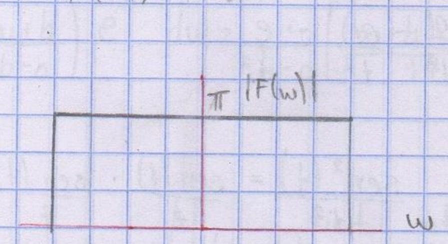


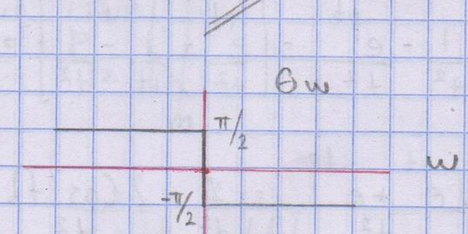
$$F(w) = \begin{cases} 0 \cdot e^{-iw} & w < -W \\ A e^{-iw} & -W < w < W \\ 0 \cdot e^{-iw} & w > W \end{cases} \qquad f(t) = \frac{A}{2\pi}\int_{-W}^{W} e^{-iw} \cdot e^{iwt}\, dw$$

$$f(t) = \frac{A}{2\pi}\int_{-W}^{W} e^{i(t-1)w}\, dw = \frac{A}{2\pi}\left[\frac{e^{i(t-1)w}}{i(t-1)}\right]_{-W}^{W} = \frac{A}{i2\pi(t-1)}\left[e^{i(t-1)w} - e^{-i(t-1)w}\right]$$

$$f(t) = \frac{A}{\pi(t-1)} \operatorname{sen}[i(t-1)w] = \frac{A}{\pi} w \operatorname{Sa}[i(t-1)w]$$

b)



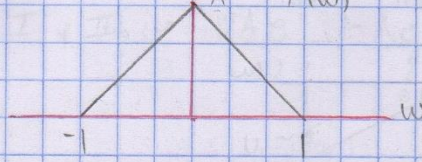


$$F(w) = \begin{cases} 0 \cdot e^{i\pi/2} & w < -W \\ \pi e^{i\pi/2} & -W < w < 0 \\ \pi e^{-i\pi/2} & 0 < w < W \\ 0\, e^{-i\pi/2} & w > W \end{cases} \qquad f(t) = \frac{\pi}{2\pi}\int_{-W}^{W} \left(e^{i\pi/2} + e^{-i\pi/2}\right) e^{iwt}\, dw$$

$$f(t) = \frac{1}{2} e^{i\pi/2}\int_{-W}^{0} e^{iwt}\, dw + \frac{1}{2} e^{-i\pi/2}\int_{0}^{W} e^{iwt}\, dw = \frac{e^{i\pi/2}}{i2t}\left[\overset{1}{\cancel{e^{0}}} - e^{-iwt}\right] + \frac{e^{-i\pi/2}}{i2t}\left[e^{iwt} - \overset{1}{\cancel{e^{0}}}\right]$$

$$f(t)\ \frac{e^{i\pi/2}}{i2t}\left[1 - e^{-iwt}\right] + \frac{e^{-i\pi/2}}{i2t}\left[e^{iwt} - 1\right] = \frac{1 - \operatorname{sen}(\pi/2 - wt)}{t} = \frac{1 - \cos(wt)}{t}$$

c)



$$F(w)=\begin{cases} A(w+1) & -1<w<0 \\ -A(w-1) & 0<w<1 \end{cases}$$

$$f(t) = \frac{A}{2\pi}\int_{-1}^{0}(w+1)e^{iwt}\,dw - \frac{A}{2\pi}\int_{0}^{1}(w-1)e^{iwt}\,dw$$

$u = w \pm 1 \quad du = dw$

$dv = e^{iwt}\,dw \quad v = \dfrac{e^{iwt}}{it}$

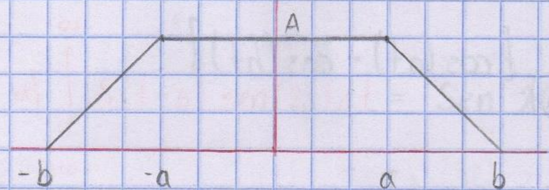
$$f(t) = \frac{A}{2\pi}\left\{\frac{(w\pm1)e^{iwt}}{it} + \frac{e^{iwt}}{t^2}\right\}\Bigg|_a^b = \left\{\left[\frac{(0+1)e^{0}}{it} + \frac{e^{0}}{t^2}\right] - \left[\frac{\cancel{(-1+1)}e^{-it}}{it} + \frac{e^{-it}}{t^2}\right] - \left[\frac{\cancel{(1-1)}e^{it}}{it} + \frac{e^{it}}{t^2}\right] - \left[\frac{(0-1)e^{0}}{it} + \frac{e^{0}}{t^2}\right]\right\}$$

$$f(t) = \left[\cancel{\frac{1}{it}} + \frac{1}{t^2} - \frac{e^{-it}}{t^2}\right] - \left[\frac{e^{it}}{t^2} + \cancel{\frac{1}{it}} - \frac{1}{t^2}\right] = \frac{2}{t^2} - \frac{e^{-it}}{t^2} - \frac{e^{it}}{t^2}$$

$$f(t) = \frac{2}{t^2} - \frac{(e^{it} + e^{-it})}{t^2} = \frac{2}{t^2} - \frac{2\,Cos(t)}{t^2} = \frac{sen^2(t)}{t^2} = \frac{sen(t)}{t}\cdot\frac{sen(t)}{t}$$

$$\therefore\ f(t) = \frac{A}{2\pi}\,Sa^2(t)$$

d)



$$F(w) = \begin{cases} \dfrac{A}{b-a}(w+b) & -b < w < -a \\ A & -a < w < a \\ \dfrac{-A}{b-a}(w-b) & a < w < b \end{cases}$$

$$f(t) = \frac{A}{2\pi}\int_{-b}^{-a}\left(\frac{w+b}{b-a}\right)e^{iwt}\,dw + \frac{A}{2\pi}\int_{-a}^{a} e^{iwt}\,dw - \frac{A}{2\pi}\int_{a}^{b}\left(\frac{w-b}{b-a}\right)e^{iwt}\,dw$$

I II III

$$\text{II} = \frac{A}{2\pi}\int_{-a}^{a} e^{iwt}\,dw = \frac{A}{2\pi(it)}\left[e^{iat} - e^{-iat}\right]$$

$u = w+b \quad du = dw$

$dv = e^{iwt}dw \quad v = \dfrac{e^{iwt}}{it}$

$$\int_{m}^{n}\left(\frac{w+b}{b-a}\right)e^{iwt}\,dw = \frac{1}{b-a}\left[\frac{(w+b)e^{iwt}}{it} + \frac{e^{iwt}}{t^2}\right]\Bigg|_{m}^{n}$$

$$\text{I y III} = \frac{A}{2\pi(b-a)}\left\{\left[\frac{(-a+b)e^{-iat}}{it} + \frac{e^{-iat}}{t^2} - \cancelto{0}{\frac{(-b+b)e^{-ibt}}{it}} - \frac{e^{-ibt}}{t^2}\right] - \left[\cancelto{0}{\frac{(b-b)e^{ibt}}{it}} + \frac{e^{ibt}}{t^2} - \frac{(a-b)e^{iat}}{it} - \frac{e^{iat}}{t^2}\right]\right\}$$

$$\text{I y III} = \frac{A}{2\pi(b-a)}\left[\frac{(b-a)e^{-iat}}{it} - \frac{(b-a)e^{iat}}{it} + \frac{e^{-iat}}{t^2} + \frac{e^{iat}}{t^2} - \frac{e^{-ibt}}{t^2} - \frac{e^{ibt}}{t^2}\right]$$

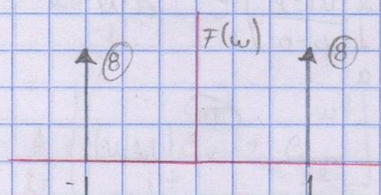
$$= \frac{A}{2\pi(b-a)}\left[\frac{-(b-a)}{it}\left(e^{iat} - e^{-iat}\right) + \frac{e^{iat} + e^{-iat}}{t^2} - \frac{e^{ibt} + e^{-ibt}}{t^2}\right]$$

$$I \text{ y } III = \frac{A}{2\pi(b-a)}\left[\frac{-2(b-a)}{t}\,\text{sen}(at) + \frac{2}{t^2}\cos(at) - \frac{2}{t^2}\cos(bt)\right]$$

$$f(t) = \frac{A}{\pi t}\,\text{sen}(at) - \frac{A}{\pi t}\,\text{sen}(at) + \frac{A}{\pi t^2(b-a)}\left[\cos(at) - \cos(bt)\right]$$

$$f(t) = \frac{A}{\pi t^2(b-a)}\left[\cos(at) - \cos(bt)\right]$$

e)



$$F(w) = \begin{cases} 8\,\delta(w+1) & -1 < w < 0 \\ 8\,\delta(w-1) & 0 < w < 1 \end{cases}$$

Teorema de muestreo $\displaystyle\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$

$$\frac{4}{\pi}\int_{-1}^{0} e^{iwt}\,\delta(w+1)\,dw + \frac{4}{\pi}\int_{0}^{1} e^{iwt}\,\delta(w-1)\,dw = \frac{4}{\pi}\left[e^{-it} + e^{it}\right]$$

$$f(t) = \frac{8}{\pi}\cos(t)$$

Problema 7. Por medio de la propiedad de muestreo de la función impulso, calcular las siguientes integrales.

a) $\int_{-\infty}^{\infty} \delta(t-5)\,sen(2t)\,dt = Sen\ 2(5) = Sen\ 10$

b) $\int_{-\infty}^{\infty} \delta(2-t)(t^5-3)\,dt = (2)^5 - 3 = 32 - 3 = 29$

c) $\int_{-\infty}^{\infty} e^{-x^2}\delta(x)\,dx = e^{-(0)^2} = e^0 = 1$

d) $\int_{-\infty}^{\infty} \delta(t-2)\cos[\pi(t-3)]\,dt = \cos[\pi(2-3)]$

$= \cos(-\pi) = -1$

e) $\int_{-\infty}^{\infty} \delta(t+2)\,e^{-2t}\,dt = e^{-2(-2)} = e^4$

f) $\int_{-\infty}^{\infty} e^{\cos t}\,\delta(t-\pi)\,dt = e^{\cos(\pi)} = e^{-1} = \frac{1}{e}$

g) $\int_{1}^{100} \log_{10}(t)\,\delta(t-10)\,dt = \log_{10}(10) = 1$

Problema 8. Considerando que $f(t)$ y $F(\omega)$ forman un par de transformadas, usando las propiedades de la transformada, encontrar la transformada de Fourier de las siguientes expresiones.

a) $f(2-t) \leftrightarrow ?$

$f(-t) \leftrightarrow F(-\omega)$

$f(2-t) \leftrightarrow F(-\omega)\,e^{i2\omega}$

b) $f[(t-3)-3] \leftrightarrow ?$

$f(t-6) \leftrightarrow ?$

$f(t-6) \leftrightarrow F(\omega)\,e^{-i6\omega}$

c) $\left[\frac{d}{dt} f(t)\right] \cdot sen(t) \leftrightarrow ?$

$f(t) \leftrightarrow F(\omega)$

$\frac{d\,f(t)}{dt} \leftrightarrow i\omega F(\omega)$

$\frac{d\,f(t)}{dt}\,sen(t) \leftrightarrow \frac{i\omega}{2}\left[F(\omega+1) - F(\omega-1)\right]$

d) $\frac{d}{dt}[f(-2t)] \leftrightarrow ?$

$f(t) \leftrightarrow F(w)$

$f(-2t) \leftrightarrow \frac{1}{|-2|} F\left(\frac{w}{-2}\right)$

$\frac{d}{dt}[f(-2t)] = \frac{iw}{2} F\left(-\frac{w}{2}\right)$

e) $t f(3t) \leftrightarrow ?$

$f(t) \leftrightarrow F(w)$

$f(3t) \leftrightarrow \frac{1}{3} F(w/3)$

$(it) f(3t) \leftrightarrow \frac{1}{3} \frac{d}{dt}[F(w/3)]$

$t f(3t) \leftrightarrow -\frac{i}{3} \frac{d}{dt}[F(w/3)]$

f) $(t-5) f(t) \leftrightarrow ?$

$f(t) \leftrightarrow F(w)$

$f(t+5) \leftrightarrow F(w) e^{i5w}$

$-it f(t+5) \leftrightarrow \frac{d}{dw}[F(w) e^{i5w}]$

$t f(t+5) \leftrightarrow i \frac{d}{dw}[F(w) e^{i5w}]$

$(t-5) f(t) \leftrightarrow i \frac{d}{dw}[F(w) e^{i5w}] e^{-i5w}$

g) $(t-3) f(-3t) \leftrightarrow ?$

$f(t) \leftrightarrow F(w)$

$f(t+3) \leftrightarrow F(w) e^{i3w}$

$f(-3t+3) \leftrightarrow \frac{1}{3} F(-w/3) e^{-iw}$

$t f(-3t+3) \leftrightarrow \frac{i}{3} \frac{d}{dw}[F(-w/3) e^{-iw}]$

$(t-3) f(-3t) \leftrightarrow \frac{i}{3} \frac{d}{dw}[F(-w/3) e^{-iw}] e^{-i3w}$

h) $t \cdot \frac{d}{dt} f(t) \leftrightarrow ?$

$f(t) \leftrightarrow F(w)$

$\frac{d}{dt} f(t) \leftrightarrow iw F(w)$

$-it \frac{d}{dt} f(t) \leftrightarrow i \frac{d}{dw}[w F(w)]$

$t \frac{d}{dt} f(t) \leftrightarrow -\frac{d}{dw}[w F(w)]$

i) $f(6-t) \leftrightarrow ?$

$f(-t) \leftrightarrow F(-w)$

$f(6-t) \leftrightarrow F(-w) e^{i6w}$

j) $(2-t) f(8-t) \leftrightarrow ?$

$f(-t) \leftrightarrow F(-w)$

$f(6-t) \leftrightarrow F(-w) e^{i6w}$

$-t f(6-t) \leftrightarrow -i \frac{d}{dw}[F(-w) e^{i6w}]$

$(2-t) f(8-t) \leftrightarrow -i \frac{d}{dw}[F(-w) e^{i6w}] e^{i2w}$

Problema 9. Completa en tiempo o frecuencia el par de transformada solicitado, usando las propiedades de la Transformada de Fourier.

a) $5\delta(t-1) \leftrightarrow ?$

$\delta(t) \leftrightarrow 1$

$5\delta(t) \leftrightarrow 5$

$5\delta(t-1) \leftrightarrow 5e^{-iw}$

b) $? \leftrightarrow 8\delta(w+1) + 8\delta(w-1)$

$\delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\delta(w)$

$\frac{1}{2\pi} \leftrightarrow \delta(w)$

$\frac{4}{\pi} \leftrightarrow 8(w)$

$\frac{4}{\pi}e^{\mp it} \leftrightarrow 8\delta(w\pm 1)$

$\frac{4}{\pi}\left[e^{-it} + e^{it}\right] \leftrightarrow 8[\delta(w+1) + \delta(w-1)]$

$\frac{8}{\pi}\cos(t) \leftrightarrow 8[\delta(w+1) + \delta(w-1)]$

c) $t \leftrightarrow ?$

$\delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\delta(w)$

$-it \leftrightarrow 2\pi \frac{d}{dw}\delta(w)$

$t \leftrightarrow i2\pi \frac{d}{dw}\delta(w)$

d) $t^2 \leftrightarrow ?$

$\delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\delta(w)$

$-t^2 \leftrightarrow 2\pi \frac{d^2}{dt^2}\delta(w)$

$t^2 \leftrightarrow -2\pi \frac{d^2}{dt^2}\delta(w)$

e) $2C_2(t)\cos(1000t) \leftrightarrow ?$

$A\,C_d(t) \leftrightarrow Ad\,Sa\left(\frac{wd}{2}\right)$

$A=2 \quad d=2$

$2C_2(t) \leftrightarrow 4Sa(w)$

$2C_2(t)\cos(1000t) \leftrightarrow 2[Sa(w+1000) + Sa(w-1000)]$

f) $? \leftrightarrow \cos(1000w)$

$\cos(1000t) \leftrightarrow \pi[\delta(w+1000) + \delta(w-1000)]$

$\frac{1}{2}[\delta(t+1000) + \delta(t-1000)] \leftrightarrow \cos(1000w)$

g) $? \leftrightarrow 5w$

$\delta(t) \leftrightarrow 1$

$\frac{d}{dt}\delta(t) \leftrightarrow iw$

$-i\frac{d}{dt}\delta(t) \leftrightarrow w$

$-i5\frac{d}{dt}\delta(t) \leftrightarrow 5w$

h) $? \leftrightarrow \delta(w)\,e^{-i5w}$

$\delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\,\delta(w)$

$\frac{1}{2\pi} \leftrightarrow \delta(w)$

$\frac{1}{2\pi} - 5 \leftrightarrow \delta(w)\,e^{-i5w}$

Problema 10. A partir de los siguientes pares de transformadas

$\delta(t) \leftrightarrow 1$ $\quad A\,C_d \leftrightarrow Ad\,Sa\left(\frac{wd}{2}\right)$ $\quad u(t) \leftrightarrow \pi\delta(w) + \frac{1}{iw}$

a) $? \leftrightarrow 3\,\mathrm{sgn}(4w-2)$

$\mathrm{sgn}(t) \leftrightarrow \frac{2}{iw}$

$3\,\mathrm{sgn}(t-2) \leftrightarrow \frac{6\,e^{-i2w}}{iw}$

$3\,\mathrm{sgn}(4t-2) \leftrightarrow \frac{3}{2}\cdot\frac{e^{-i2(w/4)}}{i(w/4)}$

$3\,\mathrm{sgn}(4t-2) \leftrightarrow \frac{6\,e^{-i\,w/2}}{iw}$

$\frac{6\,e^{-i\,t/2}}{it} \leftrightarrow 6\pi\,\mathrm{sgn}(4w-2)$

$\frac{3\,e^{-i\,t/2}}{i\pi t} \leftrightarrow 3\,\mathrm{sgn}(4w-2)$

b) $C_2\left(\frac{2t}{3}\right) \leftrightarrow ?$

$A\,C_d(t) \leftrightarrow Ad\,Sa\left(\frac{wd}{2}\right)$

$A = 1\ ,\ d = 2$

$C_2(t) \leftrightarrow 2\,Sa(w)$

$C_2\left(\frac{2t}{3}\right) \leftrightarrow \frac{2}{\left|\frac{2}{3}\right|}\,Sa\left(\frac{3w}{2}\right)$

$C_2\left(\frac{2t}{3}\right) \leftrightarrow 3\,Sa\left(\frac{3w}{2}\right)$

c) $2C_2(t)\cos(250)t \leftrightarrow ?$

$A\,C_d(t) \leftrightarrow A d\, Sa\left(\frac{\omega d}{2}\right)$

$2C_2(t) \leftrightarrow 4\, Sa(\omega)$

$2C_2(t)\cos(250t) \leftrightarrow 2\left[Sa(\omega+250) + Sa(\omega-250)\right]$

d) $u(10t-1)\,t \leftrightarrow ?$

$u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{i\omega}$

$u(t-1) \leftrightarrow \left[\pi\delta(\omega) + \frac{1}{i\omega}\right] e^{-i\omega}$

$u(10t-1) \leftrightarrow \frac{e^{-i\omega/10}}{10}\left[\pi\delta(\omega/10) + \frac{10}{i\omega}\right]$

$u(10t-1)\,t \leftrightarrow i\frac{d}{d\omega}\left\{\frac{e^{-i\omega/10}}{10}\left[\pi\delta(\omega/10) + \frac{10}{i\omega}\right]\right\}$

e) $e^{i7t}\,\delta(6t-1)\,t^3 e^{i5t} \leftrightarrow ?$

$\delta(t) \leftrightarrow 1$

$\delta(t-1) \leftrightarrow e^{-i\omega}$

$\delta(6t-1) \leftrightarrow \frac{e^{-i\omega/6}}{6}$

$i\,t^3\delta(6t-1) \leftrightarrow \frac{d^3}{d\omega^3}\left[\frac{e^{-i\omega/6}}{6}\right]$

$e^{i7t}\cdot e^{i5t}\,t^3\,\delta(6t-1) \leftrightarrow -i\frac{d^3}{d\omega^3}\left[\frac{e^{-i\frac{\omega-12}{6}}}{6}\right]$

f) $? \leftrightarrow \frac{4}{\pi} Sa(4w-2)$

$A\,C_d(t) \leftrightarrow Ad\left(\frac{wd}{2}\right)$

$A = 1/\pi \quad d = 4$

$\frac{1}{\pi} C_4(t) \leftrightarrow \frac{4}{\pi} Sa(2w)$

$\frac{1}{\pi} C_4(t)\, e^{i2t} \leftrightarrow \frac{4}{\pi} Sa(2w-2)$

$\frac{4}{\pi} Sa(2t-2) \leftrightarrow 2C_4(w)\, e^{-i2w}$

$\frac{4}{\pi} Sa(4t-2) \leftrightarrow C_4(w/2)\, e^{-iw}$

$C_4(t/2)\, e^{-it} \leftrightarrow 8\, Sa(4w-2)$

$\frac{1}{2\pi} C_4(t/2)\, e^{-it} \leftrightarrow \frac{4}{\pi} Sa(4w-2)$ //

h) $C_{\frac{4}{3}}(t+6) \leftrightarrow ?$

$A\,C_d(t) \leftrightarrow Ad\, Sa\left(\frac{wd}{2}\right)$

$A = 1 \quad d = 4/3$

$C_{4/3}(t) \leftrightarrow \frac{4}{3} Sa\left(\frac{2}{3}w\right)$

$C_{4/3}(t+6) \leftrightarrow \frac{4}{3} Sa(2/3\, w)\, e^{i6w}$ //

g) $? \leftrightarrow \left[\pi\delta\left(w+\frac{3}{4}\right) + \frac{1}{i\left(w+\frac{3}{4}\right)}\right](-w)\, e^{i1000w}$

$u(t) \leftrightarrow \pi\delta(w) + \frac{1}{iw}$

$u(t)\, e^{-\frac{i3w}{4}} \leftrightarrow \pi\delta\left(w+\frac{3}{4}\right) + \frac{1}{i\left(w+\frac{3}{4}\right)}$

$-\frac{d}{dt}\left[u(t)\, e^{-\frac{i3w}{4}}\right] \leftrightarrow -w\left[\pi\delta\left(w+\frac{3}{4}\right) + \frac{1}{i\left(w+\frac{3}{4}\right)}\right]$

$-\frac{d}{dt}\left[u(t+1000)\, e^{-i\,3/4(w+1000)}\right] \leftrightarrow -w\left[\pi\delta\left(w+\frac{3}{4}\right) + \frac{1}{i\left(w+\frac{3}{4}\right)}\right] e^{i1000w}$ //

i) $(3\delta(t-1)-3\delta(t+1))\cos(18t) \leftrightarrow ?$

$\delta(t) \leftrightarrow 1$

$3\delta(t\pm1) \leftrightarrow 3e^{\pm iw}$

$3\delta(t-1)-3\delta(t+1) \leftrightarrow 3e^{-iw}-3e^{iw}$

$(3\delta(t-1)-3\delta(t+1))\cos(18t) \leftrightarrow \frac{1}{2}\left\{\left[3e^{-i(w+18)}-3e^{i(w+18)}\right]+\left[3e^{-i(w-18)}-3e^{i(w-18)}\right]\right\}$ //

j) $? \leftrightarrow 2\cos(500w)$

$\cos(500t) \leftrightarrow \pi[\delta(w+500)+\delta(w-500)]$

$\pi[\delta(t+500)+\delta(t-500)] \leftrightarrow 2\pi\cos(500w)$

$\delta(t+500)+\delta(t-500) \leftrightarrow 2\cos(500w)$

k) $t+t^2+1 \leftrightarrow ?$
② ③ ①

① $\delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\,\delta(w)$

② $1 \leftrightarrow 2\pi\,\delta(w)$

$-it \leftrightarrow 2\pi \frac{d}{dw}\delta(w)$

$t \leftrightarrow i2\pi \frac{d}{dw}\delta(w)$

③ $1 \leftrightarrow 2\pi\,\delta(w)$

$-t^2 \leftrightarrow 2\pi \frac{d^2}{dw^2}\delta(w)$

$t^2 \leftrightarrow -2\pi \frac{d^2}{dw^2}\delta(w)$

$\therefore\ t+t^2+1 \leftrightarrow i2\pi\frac{d}{dw}\delta(w) - 2\pi\frac{d^2}{dw^2}\delta(w)+2\pi\delta(w)$ //

l) $\frac{i5}{t} \leftrightarrow ?$

$\mathrm{sgn}(t) \leftrightarrow \frac{2}{iw}$

$\frac{-i2}{t} \leftrightarrow -2\pi\,\mathrm{sgn}(w)$

$\frac{i5}{t} \leftrightarrow 5\pi\,\mathrm{sgn}(w)$ //

m) $? \;\leftrightarrow\; \dfrac{1}{w}$ n) $? \;\leftrightarrow\; \dfrac{1}{w} e^{-i4w}$

$$\operatorname{sgn}(t) \;\leftrightarrow\; \frac{2}{iw} \qquad \frac{i}{2}\operatorname{sgn}(t) \;\leftrightarrow\; \frac{1}{w}$$

$$\frac{i}{2}\operatorname{sgn}(t) \;\leftrightarrow\; \frac{1}{w} \qquad \frac{i}{2}\operatorname{sgn}(t-4) \;\leftrightarrow\; \frac{1}{w} e^{-i4w}$$

ñ) $5\, e^{-i\frac{7}{8}(t-3)} \;\leftrightarrow\; ?$

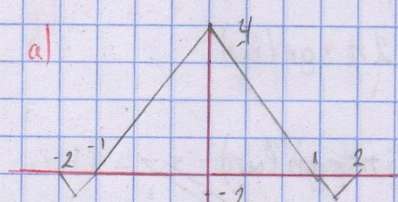
$$e^{-iw_0 t} \;\leftrightarrow\; 2\pi\,\delta(w + w_0)$$

$$e^{-i\frac{7}{8}t} \;\leftrightarrow\; 2\pi\,\delta\left(w + \frac{7}{8}\right)$$

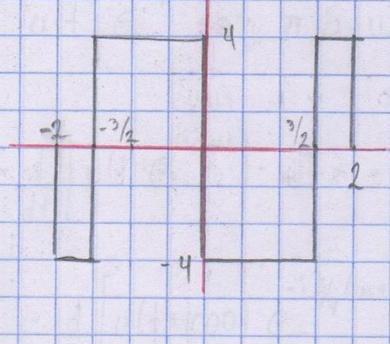
$$e^{-i\frac{7}{8}(t-3)} \;\leftrightarrow\; 2\pi\,\delta\left(w + \frac{7}{8}\right) e^{-i3w}$$

Problema 11. Aplicando las propiedades de la transformada de Fourier, determinar $F(w)$.

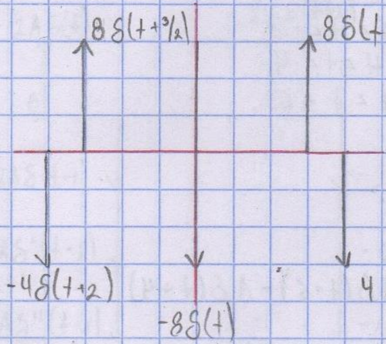
a)



$$f(t) = \begin{cases} -4t - 8 & -2 < t < -3/2 \\ 4t + 4 & -3/2 < t < 0 \\ -4t + 4 & 0 < t < 3/2 \\ 4t - 8 & 3/2 < t < 2 \end{cases}$$



$$\frac{df(t)}{dt} = \begin{cases} -4 & -2 < t < -3/2 \\ 4 & -3/2 < t < 0 \\ -4 & 0 < t < 3/2 \\ 4 & 3/2 < t < 2 \end{cases}$$



$$\frac{d^2 f(t)}{dt^2} = -4\delta(t+2) + 8\delta(t+3/2) - 8\delta(t) + 8\delta(t-3/2) - 4\delta(t-2)$$

$$\mathcal{F}\left\{\frac{d^2 f(t)}{dt^2}\right\} = -4e^{i2w} + 8e^{i3/2w} - 8 + 8e^{-i3/2w} - 4e^{-i2w}$$

$$\mathcal{F}\left\{\frac{d^2 f(t)}{dt^2}\right\} = -4\left(e^{i2w} + e^{-i2w}\right) + 8\left(e^{i3/2w} + e^{-i3/2w}\right) - 8$$

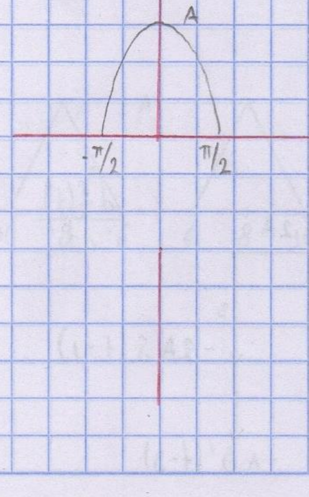
$$\mathcal{F}\left\{\frac{d^2 f(t)}{dt^2}\right\} = -8\cos(2w) + 16\cos\left(\frac{3w}{2}\right) - 8$$

si $f(t) \leftrightarrow F(w)$

$$\frac{d^2 f(t)}{dt^2} \leftrightarrow \underbrace{(iw)^2}_{-w^2} F(w)$$

$$F(w) = \frac{1}{w^2}\left[8\cos(2w) - 16\cos\left(\frac{3w}{2}\right) + 8\right]$$

b)

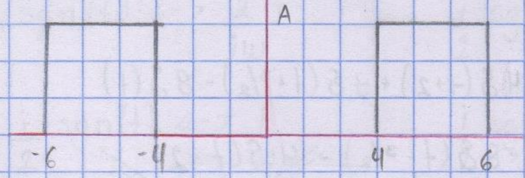


$f(t) = A\cos(t) \quad -\pi/2 < t < \pi/2$

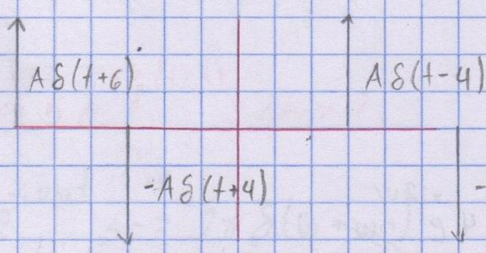
$$\cos(w_0 t) \leftrightarrow \pi\left[\delta(w+w_0) + \delta(w-w_0)\right]$$

$$A\cos(t) \leftrightarrow A\pi\left[\delta(w+1) + \delta(w-1)\right]$$

c)



$$f(t) = \begin{cases} A & -6 < t < -4 \\ 0 & -4 < t < 4 \\ A & 4 < t < 6 \end{cases}$$



$$\frac{df(t)}{dt} = A\delta(t+6) - A\delta(t+4) + A\delta(t-4) - A\delta(t-6)$$

$$\mathcal{F}\left\{\frac{df(t)}{dt}\right\} = Ae^{i6w} - Ae^{i4w} + Ae^{-i4w} - Ae^{-i6w}$$

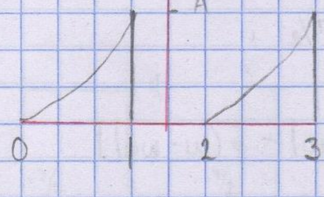
$$\mathcal{F}\left\{\frac{df(t)}{dt}\right\} = A\left(e^{i6w} - e^{-i6w}\right) - A\left(e^{i4w} - e^{-i4w}\right) = i2A\,\mathrm{sen}(6w) - i2A\,\mathrm{sen}(4w)$$

$$\frac{d}{dt} f(t) \leftrightarrow iw\,F(w)$$

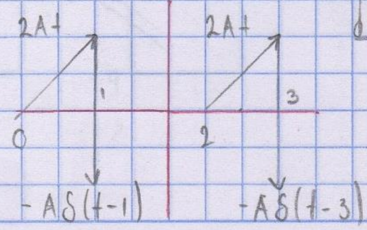
$$iw\,F(w) = i2A\left[\mathrm{sen}(6w) - \mathrm{sen}(4w)\right]$$

$$F(w) = \frac{2A}{w}\left[\mathrm{sen}(6w) - \mathrm{sen}(4w)\right]$$

d)

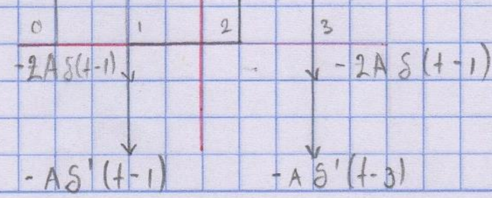


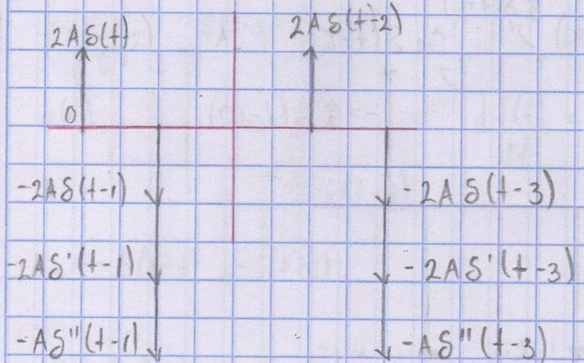
$$f(t) = \begin{cases} t^2 & 0 < t < 1 \\ 0 & 1 < t < 2 \\ t^2 & 2 < t < 3 \end{cases}$$



$$\frac{df(t)}{dt}$$

$$\frac{d^2f(t)}{dt}$$





$$\frac{d^3 f(t)}{dt^3}$$

$$\frac{d^3}{dt^3} f(t) = 2A\delta(t) - 2A\delta(t-1) - 2A\frac{d}{dt}\delta(t-1) - A\frac{d^2}{dt^2}(t-1) + 2A\delta(t-2) - 2A\delta(t-3)$$

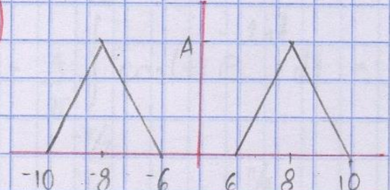
$$- 2A\frac{d}{dt}\delta(t-3) - A\frac{d^2}{dt^2}\delta(t-3)$$

$$\mathcal{F}\left\{\frac{d^3 f(t)}{dt}\right\} = 2A - 2Ae^{-iw} - i2Awe^{-iw} + w^2Ae^{-iw} + 2Ae^{-i2w} - 2Ae^{-i3w} + i2Awe^{-i3w} + w^2Ae^{-i3w}$$

$$\frac{d^3 f(t)}{dt} \longleftrightarrow -iw^3 F(w)$$

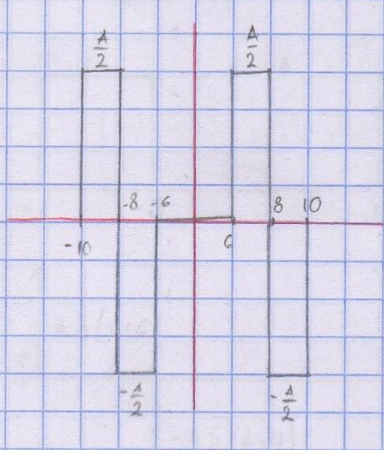
$$F(w) = \frac{i}{w^2}\left[2A - 2Ae^{-iw} - i2Awe^{-iw} + w^2Ae^{-iw} + 2Ae^{-i2w} - 2Ae^{-i3w} - i2Awe^{-i3w} + w^2Ae^{-i3w}\right]$$

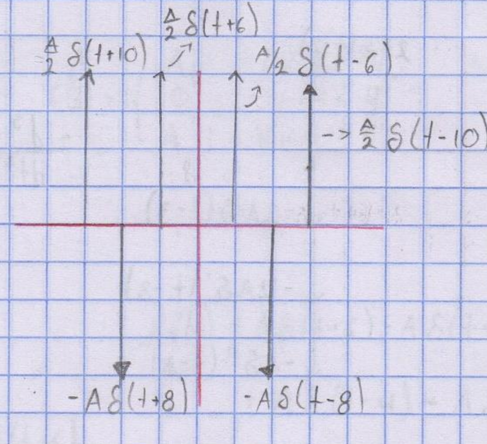
e)



$$f(t) = \begin{cases} A/2\, t + 5A & -10 < t < -8 \\ -A/2\, t - 3A & -8 < t < -6 \\ A/2\, t - 3A & 6 < t < 8 \\ -A/2\, t + 5A & 8 < t < 10 \\ 0 & -6 < t < 6 \end{cases}$$

$$\frac{df(t)}{dt} \qquad\qquad \frac{d^2}{dt^2} f(t)$$





$$\frac{d^2}{dt^2} f(t) = \frac{A}{2}\delta(t+10) - A\delta(t+8) + \frac{A}{2}\delta(t+6) + \frac{A}{2}\delta(t-6) - A\delta(t-8) + \frac{A}{2}\delta(t-10)$$

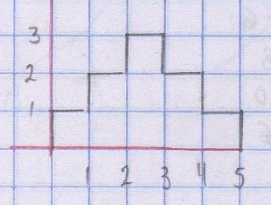
$$\mathcal{F}\left\{\frac{d^2}{dt^2} f(t)\right\} = \frac{A}{2}e^{i10w} - Ae^{i8w} + \frac{A}{2}e^{i6w} + \frac{A}{2}e^{-i6w} - Ae^{-i8w} + \frac{A}{2}e^{-i10w}$$

$$\frac{d^2}{dt^2} f(t) \leftrightarrow A\cos(10w) - 2A\cos(8w) + A\cos(6w)$$

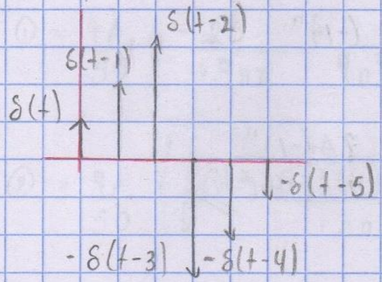
$$\frac{d^2}{dt^2} f(t) \leftrightarrow -w^2 F(w)$$

$$F(w) = -\frac{1}{w^2}\left[A\cos(10w) - 2A\cos(8w) + A\cos(6w)\right]$$

f)



$$f(t) = \begin{cases} 1 & 0 < t < 1 \\ 2 & 1 < t < 2 \\ 3 & 2 < t < 3 \\ 2 & 3 < t < 4 \\ 1 & 4 < t < 5 \end{cases}$$



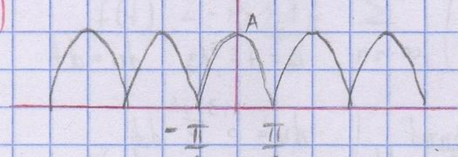
$$\frac{df(t)}{dt} = \delta(t)+\delta(t-1)+\delta(t-2)-\delta(t-3)-\delta(t-4)-\delta(t-5)$$

$$\mathcal{F}\left\{\frac{d}{dt}f(t)\right\} = 1 + e^{-iw} + e^{-i2w} - e^{-i3w} - e^{-i4w} - e^{-i5w}$$

$$\frac{df(t)}{dt} \;<\!-\!>\; iwF(w)$$

$$F(w) = \frac{1}{iw} + \frac{e^{-iw}}{iw} + \frac{e^{-i2w}}{iw} - \frac{e^{-3w}}{iw} - \frac{e^{-i4w}}{iw} - \frac{e^{-i5w}}{iw}$$

h)



$$f(t) = A\cos(t) \quad -\pi/2 < t < t$$

$$T = \pi \qquad w0 = \frac{2\pi}{\pi} = 2$$

$$\cos(t) = \frac{e^{it} - e^{-it}}{2}$$

$$f(t) \;<\!-\!>\; 2\pi \sum_{n=-\infty}^{\infty} C_n\,\delta(w-2n)$$

$$C_n = \frac{A}{\pi}\int_{-\pi/2}^{\pi/2} \cos(t)\, e^{-i2nt}\, dt = \frac{A}{2\pi}\int_{-\pi/2}^{\pi/2} \left(e^{it}+e^{-it}\right) e^{-i2nt}\, dt = \frac{A}{2\pi}\left\{\int_{-\pi/2}^{\pi/2} e^{i(1-2n)t}\, dt + \int_{-\pi/2}^{\pi/2} e^{-i(1+2n)t}\, dt\right\}$$

$$C_n = \frac{A}{2\pi}\left\{\left.\frac{e^{i(1-2n)t}}{i(1-2n)}\right|_{-\pi/2}^{\pi/2} - \left.\frac{e^{-i(1+2n)t}}{i(1+2n)}\right|_{-\pi/2}^{\pi/2}\right\} = \frac{A}{2\pi}\left\{\frac{1}{i(1-2n)}\left[e^{i(\pi/2-n\pi)} - e^{-i(\pi/2-n\pi)}\right] - \frac{1}{i(1+2n)}\left[e^{-i(\pi/2+n\pi)} - e^{+i(\pi/2+n\pi)}\right]\right\}$$

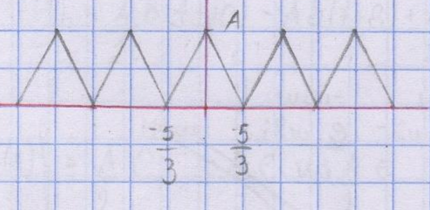
$$C_n = \frac{A}{2\pi}\left\{ \frac{2}{1-2n}\,sen\left(\pi/2 - n\pi\right) + \frac{2}{1+2n}\,sen\left(\pi/2 + n\pi\right)\right\}$$

$$sen\left(\pi/2 \pm n\pi\right) = \overset{1}{\cancel{sen(\pi/2)}}\,\overset{(-1)^n}{\cos(n\pi)} \pm \overset{0}{\cancel{\cos(\pi/2)}\,sen(n\pi)} = (-1)^n$$

$$C_n = \frac{A(-1)^n}{\pi}\left[\frac{1}{1-2n} + \frac{1}{1+2n}\right] = \frac{A(-1)^n}{\pi}\left[\frac{2}{1-4n^2}\right] = \frac{2A(-1)^n}{\pi(1-4n^2)}$$

$$f(t) \longleftrightarrow 4A \sum_{n=-\infty}^{\infty}\left(\frac{(-1)^n}{1-4n^2}\right)\delta(w-2n)$$

i)



$$f(t) = \begin{cases} \frac{3A}{5}\left(t+\frac{5}{3}\right) & -\frac{5}{3} < t < 0 \\ -\frac{3A}{5}\left(t-\frac{5}{3}\right) & 0 < t < \frac{5}{3} \end{cases}$$

$$T = \frac{10}{3} \qquad w_0 = \frac{2\pi}{T} = \frac{2\pi}{10/3} = \frac{3\pi}{5}$$

$$f(t) \longleftrightarrow 2\pi \sum_{n=-\infty}^{\infty} C_n\,\delta\left(w - \frac{3n\pi}{5}\right)$$

$u = t \pm 5/3 \qquad du = dt$

$dv = e^{-i\frac{3n\pi t}{5}}\,dt$

$v = -\frac{5e^{-i\frac{3n\pi t}{5}}}{i3n\pi}$

$$C_n = \frac{3}{10}\left(\frac{3A}{5}\right)\left\{\int_{-5/3}^{0}\left(t+\frac{5}{3}\right)e^{-i\frac{3n\pi t}{5}}\,dt - \int_{0}^{5/3}\left(t-\frac{5}{3}\right)e^{-i\frac{3n\pi t}{5}}\,dt\right\}$$

$$I = \frac{9A}{50}\left\{\left.\frac{-5(t\pm 5/3)\,e^{-i\frac{3n\pi t}{5}}}{i3n\pi}\right|_a^b - \frac{1}{i3n\pi}\left[\left.\frac{-5e^{-i\frac{3n\pi t}{5}}}{i3n\pi}\right]\right|_a^b\right\}$$

$$I = \frac{9A}{50}\left\{\left.\frac{-5(t\pm 5/3)\,e^{-i\frac{3n\pi t}{5}}}{i3n\pi}\right|_a^b - \left.\frac{5}{9n^2\pi^2}\,e^{-i\frac{3n\pi t}{5}}\right|_a^b\right\}$$

$$① = \frac{9A}{50}\left\{\left[\frac{-5(5/3)e^{0}}{i3n\pi} + \varnothing\right] - \left[\frac{5}{9n^2\pi^2}e^{0} - \frac{5}{9n^2\pi^2}e^{in\pi}\right]\right\}$$

$$① = \frac{9A}{50}\left\{-\frac{25}{i9n\pi} - \frac{5}{9n^2\pi^2} + \frac{5(-1)^n}{9n^2\pi^2}\right\}$$

$$② = \frac{9A}{50}\left\{\left[\varnothing + \frac{5(-5/3)e^{0}}{i3n\pi}\right] - \left[\frac{5e^{-in\pi}}{9n^2\pi^2} - \frac{5e^{0}}{9n^2\pi^2}\right]\right\}$$

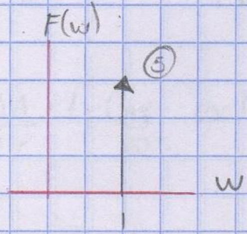
$$② = \frac{9A}{50}\left\{-\frac{25}{i9n\pi} - \frac{5(-1)^n}{9n^2\pi^2} + \frac{5}{9n^2\pi^2}\right\}$$

$$C_n = ① - ② = \frac{9A}{50}\left[-\frac{10}{9n^2\pi^2} + \frac{10(-1)^n}{9n^2\pi^2}\right] = \frac{9A}{50}\left[\frac{10(-1)^n - 10}{9n^2\pi^2}\right]$$

$$C_n = \frac{A\left[(-1)^n - 1\right]}{5n^2\pi^2}$$

$$f(t) \longleftrightarrow \frac{2A\pi}{5}\sum_{n=-\infty}^{\infty}\left(\frac{(-1)^n-1}{n^2\pi^2}\right)\delta\left(w - \frac{3n\pi}{5}\right)$$

j)



$$F(w) = 5\,\delta(w-1)$$

$$f(t) = \frac{5}{2\pi}\int_{-\infty}^{\infty}\delta(w-1)\,e^{iwt}\,dw = \frac{5}{2\pi}e^{-it}$$

$f(t) \longleftrightarrow F(w)$

$\delta(t) \longleftrightarrow 1$

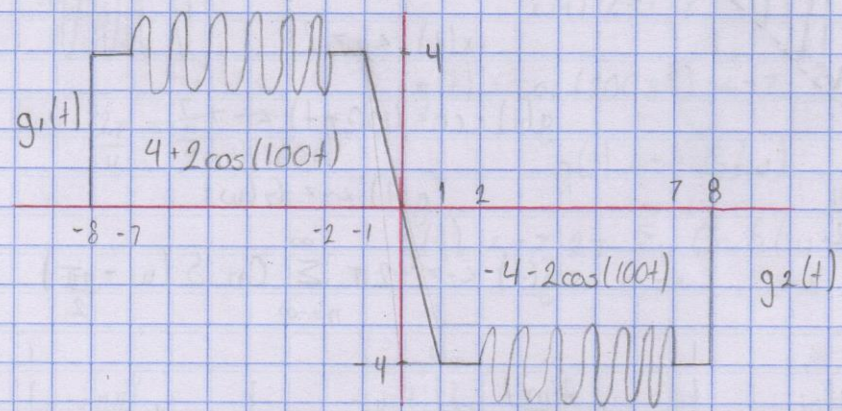
$5\,\delta(t) \longleftrightarrow 5$

$5\,\delta(t-1) \longleftrightarrow 5e^{-iw}$

$5e^{-it} \longleftrightarrow 10\pi\,\delta(w-1)$

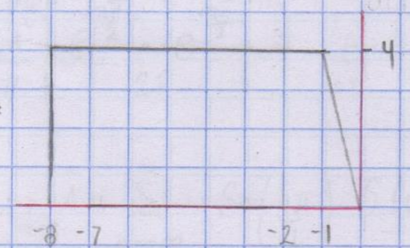
$$\frac{5}{2\pi}e^{-it} \longleftrightarrow 5\,\delta(w-1)$$

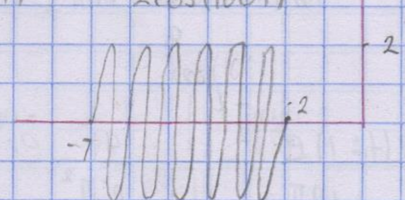
g)



Como $g(t)$ es impar, $g_1(t) = -g_1(-t)$ entonces $g(t) = g_1(t) - g_1(-t)$

Sea $g_1(t) = m(t) + n(t)$ donde

$m(t) =$ 

y $n(t) =$ $2\cos(100t)$ 

calculando $2C_5(t+9/2)\cos(100t)$

si $AC_d(t) \leftrightarrow Ad\,Sa\left(\frac{wd}{2}\right)$

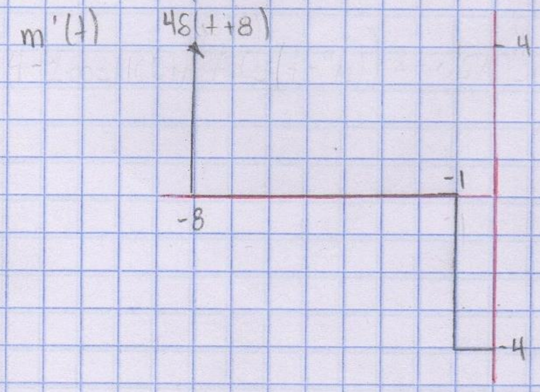
$A=2 \quad d=5$

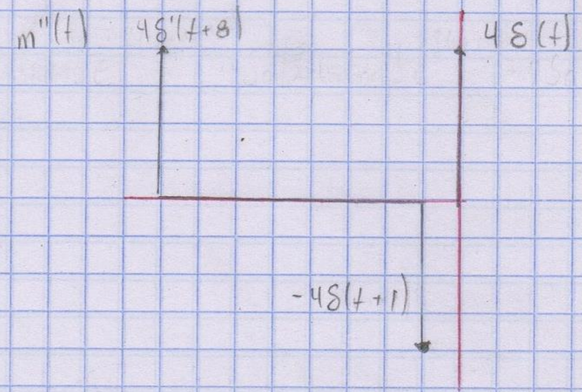
$2C_5(t) \leftrightarrow 10\,Sa\left(\frac{5}{2}w\right)$

$2C_5(t+9/2) \leftrightarrow 10\,Sa\left(\frac{5}{2}w\right)e^{i\frac{9}{2}w}$

$2C_5(t+9/2)\cos(100t) \leftrightarrow 5\left[Sa\left[\frac{5}{2}(w+100)\right]e^{i\frac{9}{2}(w+100)} + Sa\left[\frac{5}{2}(w-100)\right]e^{i\frac{9}{2}(w-100)}\right]$

calculando $m(t) \leftrightarrow M(w)$





$$m''(t) = 4\delta'(t+8) - 4\delta(t+1) + 4\delta(t)$$

$$\mathcal{F}\{m''(t)\} = i4w e^{i8w} - 4e^{iw} + 4 \qquad \frac{d^2}{dt^2} m(t) \leftrightarrow -w^2 M(w)$$

$$M(w) = \frac{4}{w^2}\left[e^{iw} - iwe^{i8w} - 1\right]$$

si $g_1(t) = m(t) + n(t)$ entonces $G_1(w) = M(w) + N(w)$

$$G_1(w) = \frac{4}{w^2}\left[e^{iw} - iwe^{i8w} - 1\right] + 5\left[Sa\left[\tfrac{5}{2}(w+100)\right] e^{i\frac{9}{2}(w+100)} + Sa\left[\tfrac{5}{2}(w-100)\right] e^{i\frac{9}{2}(w-100)}\right]$$

$g(t) \leftrightarrow G(w)$

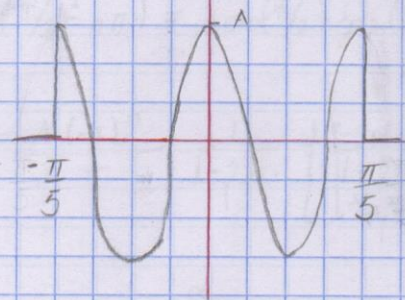
$$G(w) = G_1(w) - G_1(-w)$$

$$G_1(-w) = \frac{4}{w^2}\left[e^{-iw} + iwe^{-i8w} - 1\right] + 5\left[Sa\left[\tfrac{5}{2}(-w+100)\right] e^{i\frac{9}{2}(-w+100)} + Sa\left[\tfrac{5}{2}(-w-100)\right] e^{i\frac{9}{2}(-w-100)}\right].$$

$\therefore$

Problema 12. Aplicando el teorema de modulación encontrar la transformada de cada una de las señales moduladas.

a)



$f(t) = A\cos(20t) \quad -\pi/5 < t < \pi/5$

$h(t) = g(t)\, f(t)$

$g(t) = C_{\frac{2\pi}{5}}(t)$

$A\, C_{\frac{2\pi}{5}} \cdot \cos(20t) \; \leftrightarrow \; ?$

por propiedades de la transformada

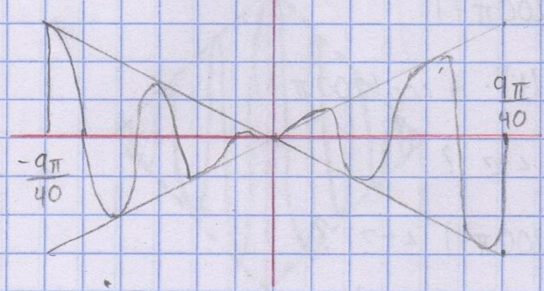
si $A\, C_d(t) \leftrightarrow A d \; Sa\left(\frac{wd}{2}\right)$

$A = A \quad d = \frac{2\pi}{5}$

$A\, C_{\frac{2\pi}{5}}(t) \leftrightarrow \frac{2A\pi}{5} Sa\left(\frac{w\pi}{5}\right)$

$A\, C_{\frac{2\pi}{5}}(t) \cdot \cos(20t) \leftrightarrow \frac{A\pi}{5}\left[Sa\left[\frac{\pi}{5}(w+20)\right] + Sa\left[\frac{\pi}{5}(w-20)\right]\right]$

b)



$$f(t) = -\frac{40A}{9\pi} t \cos(20t) \quad \left(-\frac{9\pi}{40} < t < \frac{9\pi}{40}\right)$$

$$f(t) \leftrightarrow ?$$

$$-\frac{40A}{9\pi} t \cdot \cos(20t) \cdot C_{\frac{9\pi}{20}}(t) \leftrightarrow ?$$

por propiedades de la transformada

$$A C_d(t) \leftrightarrow A d \, Sa\left(\frac{wd}{2}\right)$$

$$A = 1 \quad d = 9\pi/20$$

$$C_{\frac{9\pi}{20}}(t) \leftrightarrow \frac{9\pi}{20} Sa\left(\frac{9\pi w}{40}\right)$$

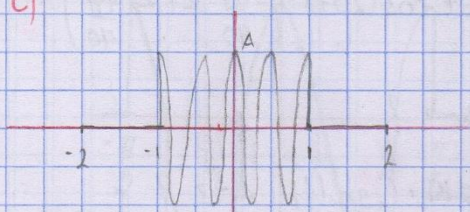
$$C_{\frac{9\pi}{20}}(t) \cdot \cos(20t) \leftrightarrow \frac{9\pi}{40}\left[Sa\left[\frac{9\pi}{40}(w+20)\right] + Sa\left[\frac{9\pi}{40}(w-20)\right]\right]$$

$$t \cdot C_{\frac{9\pi}{20}} \cdot \cos(20t) \leftrightarrow i \frac{9\pi}{40} \frac{d}{dw}\left\{Sa\left[\frac{9\pi}{40}(w+20)\right] + Sa\left[\frac{9\pi}{40}(w-20)\right]\right\}$$

$$-\frac{40A}{9\pi} \cdot t \cdot C_{\frac{9\pi}{20}} \cdot \cos(20t) \leftrightarrow -\frac{A}{i} \cdot \frac{d}{dw}\left\{Sa\left[\frac{9\pi}{40}(w+20)\right] + Sa\left[\frac{9\pi}{40}(w-20)\right]\right\}$$

Problema 12. Aplicando el teorema de modulación encontrar la transformada de cada una de las señales moduladas

c)



$$f(t) = A\cos(200\pi t)$$

$$x(t) = g(t)\cdot A\cos(200\pi t)$$

$$x(t) \leftrightarrow\ ?$$

$$g(t)\cdot\cos(200\pi t) \leftrightarrow\ ?$$

$$T = 4 \qquad \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$g(t) \leftrightarrow G(\omega)$$

$$g(t) \leftrightarrow 2\pi\sum_{n=-\infty}^{\infty} C_n\,\delta\left(\omega - \frac{n\pi}{2}\right)$$

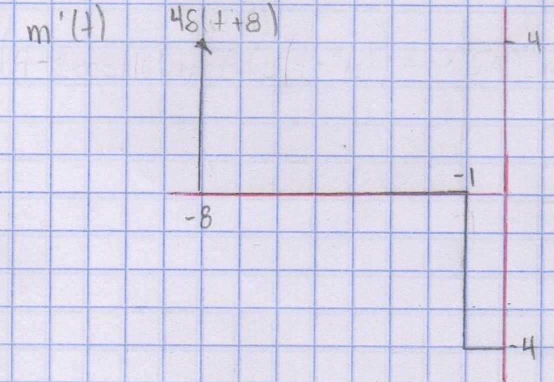
$$C_n = \frac{A}{4}\int_{-1}^{1} e^{-\frac{in\pi t}{2}}\,dt = \frac{A}{4}\left[\frac{-2}{in\pi}e^{-\frac{in\pi t}{2}}\right]_{-1}^{1} = \frac{A}{4}\left[\frac{-2}{in\pi}e^{-\frac{in\pi}{2}} + \frac{2}{in\pi}e^{\frac{in\pi}{2}}\right]$$

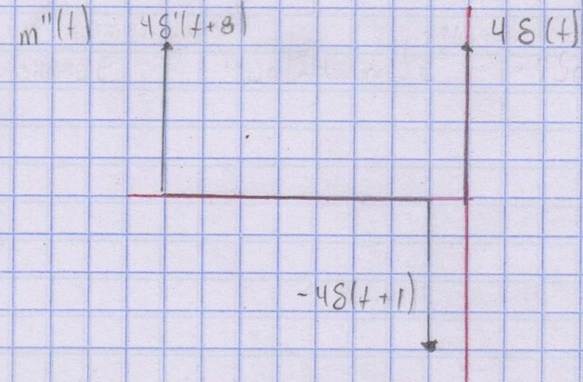
$$C_n = \frac{2A}{2n\pi}\left[\frac{e^{\frac{in\pi}{2}} - e^{-\frac{in\pi}{2}}}{2i}\right] = \frac{A}{n\pi}\cdot\operatorname{sen}\left(\frac{n\pi}{2}\right) = \frac{A}{2}Sa\left(\frac{n\pi}{2}\right)$$

$$g(t) \leftrightarrow A\pi\sum_{n=-\infty}^{\infty} Sa\left(\frac{n\pi}{2}\right)\delta\left(\omega - \frac{n\pi}{2}\right)$$

$$A\cdot g(t)\cdot\cos(200\pi t) \leftrightarrow \frac{A\pi}{2}\sum_{n=-\infty}^{\infty}\left\{Sa\left(\frac{n\pi}{2}\right)\left[\delta\left(\omega - \frac{n\pi}{2} + 200\pi\right) + \delta\left(\omega - \frac{n\pi}{2} - 200\pi\right)\right]\right\}$$

calculando $m(t) \longleftrightarrow M(w)$





$$m''(t) = 4\delta'(t+8) - 4\delta(t+1) + 4\delta(t)$$

$$\mathcal{F}\{m''(t)\} = i4w e^{i8w} - 4e^{iw} + 4 \qquad \frac{d^2}{dt^2} m(t) \longleftrightarrow -w^2 M(w)$$

$$M(w) = \frac{4}{w^2}\left[e^{iw} - iwe^{i8w} - 1 \right]$$

si $g_1(t) = m(t) + n(t)$ entonces $G_1(w) = M(w) + N(w)$

$$G_1(w) = \frac{4}{w^2}\left[e^{iw} - iwe^{i8w} - 1\right] + 5\left[Sa\left[\tfrac{5}{2}(w+100)\right] e^{i\frac{9}{2}(w+100)} + Sa\left[\tfrac{5}{2}(w-100)\right] e^{i\frac{9}{2}(w-100)}\right]$$

$g(t) \longleftrightarrow G(w)$

$$G(w) = G_1(w) - G_1(-w)$$

$$G_1(-w) = \frac{4}{w^2}\left[e^{-iw} + iwe^{-i8w} - 1\right] + 5\left[Sa\left[\tfrac{5}{2}(-w+100)\right] e^{i\frac{9}{2}(-w+100)} + Sa\left[\tfrac{5}{2}(-w-100)\right] e^{i\frac{9}{2}(-w-100)}\right].$$