

IMPORTANCE OF MEMORY EFFECT FOR CAPSIZING PREDICTION

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SUMMARY

A two-stage approach [7] to determination of non-linear motions of ship in waves is used in evaluating dynamic stability of ship A-1 of the ITTC benchmark study. Two different models of radiation forces are used. Both are based on the linearity assumption. In the first model radiation forces include the flow memory effect represented by the retardation function. In the second model constant added mass and damping concept is used to represent radiation forces.

1. INTRODUCTION

Linear models of ship dynamics in waves are well established. In most cases they result in a sufficiently accurate prediction of loads and ship motions. Perhaps the biggest benefit of using the linear models is that prediction of exceeding certain level of load or response can be easily derived.

The biggest shortcoming of the linearity assumption is that it precludes prediction of certain classes of ship responses. The linear models cannot predict the loss of ship stability in waves, parametric roll resonance of roll and asymmetry of sagging and hogging. Evaluation of these kind responses requires a proper non-linear modeling of ship dynamics and hydrodynamics. Moreover, the analysis has to be conducted in time domain.

In the two-stage approach [7] to determination of non-linear motions of ship in waves, the fully non-linear model represents the restoring forces and the Froude-Krylov part of wave forces while radiation and diffraction forces are regarded to be sufficiently well represented by the linear approximation. Ship dynamic behavior is represented by a rigid body dynamics having six degrees of freedom. There are no restrictions set on the motion's magnitude. There are two options for evaluating radiation forces. The first one is based on the approach of Cummins [2], which allows to evaluate the radiation forces in time domain without any assumption concerning motion frequency. This approach represents properly the memory effect on the radiation forces. In the second, simplified model, radiation forces are directly related to the added masses and damping coefficients.

Ship behavior in regular waves was evaluated by both approaches of representing the radiation forces. This was done for a container ship model of the Osaka University. The results are compared to the model test results [5].

2 AN OUTLINE OF THE TWO-STAGE APPROACH TO DETERMINATION OF NON-LINEAR MOTIONS OF SHIP IN WAVES

In this chapter only an-outline of the two-stage approach is presented. More detailed description of the approach is presented in [7].

2.1 CO-ORDINATE SYSTEMS USED IN EVALUATING SHIP MOTION

Ship is regarded as a rigid body possessing in general six degrees of freedom. In the following we focus our attention on the general theoretical model of rigid body motion.

Four co-ordinate systems are used for describing general ship motion. These are presented in Figure 1.

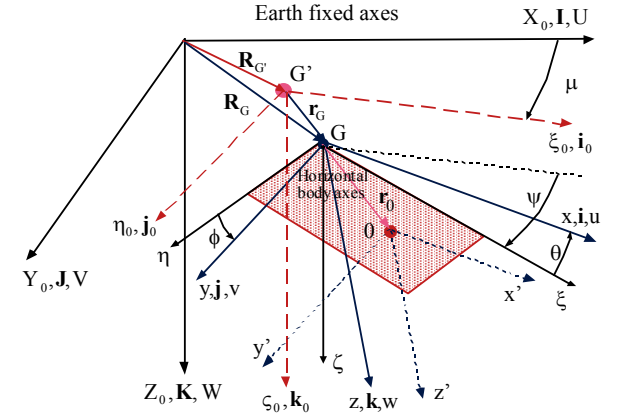


Fig. 1 Co-ordinate systems used in ship dynamics [7].

Inertial co-ordinate system fixed to Earth is denoted by $X_0Y_0Z_0$. X_0 -axis points in the wave propagation direction. The X_0Y_0 plane coincides with the still water level. Ship is on course μ with respect to waves. Course or encounter angle is a time-averaged or initial orientation of ship with respect to the direction of wave propagation. This time-averaged position defines the co-ordinate system $\xi_0\eta_0\zeta_0$. G' is the origin of this co-ordinate system and it is the time-averaged position of the ship's center of gravity. Axis ξ_0 points in the

direction of ship velocity vector V_s . The average position of ship is given by the position vector $R_G = X_G \mathbf{i} + Y_G \mathbf{j}$.

The origin of two other Cartesian co-ordinate systems is located at the instantaneous position of ship's origin (point G in Fig.1). Co-ordinate system xyz is fixed to the ship so that the x-axis points towards ship bow. This co-ordinate system is called the body-fixed co-ordinate system. The so-called horizontal body axes co-ordinate system [4] denoted as $\xi\eta\zeta$ moves with ship so that the ξ - η plane stays horizontal that is it is parallel to the plane X_0 - Y_0 and ζ -axis stays at ship centreplane. Both the body fixed and horizontal axes co-ordinate systems move with ship with a velocity

Fourth co-ordinate system, denoted by $x'y'z'$, is also body-fixed but with the origin located in other point denoted by 0. In the linear seakeeping theory usually origin 0 lies on the vertical plane that comprises the center of gravity and being the intersection of this plane with the centerplane plane and still waterplane.

Instantaneous position of ship's center of gravity is given by the following displacement components: surge (ξ_0 or x_1), sway (η_0 or x_2) and heave (ζ_0 or x_3). These are the motion components of the center of gravity in the moving with ship velocity V_s inertial co-ordinate system $\xi_0\eta_0\zeta_0$. Translational motion is defined as the motion of ship's origin 0 in the inertial co-ordinate system

$$\mathbf{r}_G = \xi_0 \mathbf{i}_0 + \eta_0 \mathbf{j}_0 + \zeta_0 \mathbf{k}_0. \quad (1)$$

The velocity of the origin of ship is given as

$$\mathbf{U} = \dot{\mathbf{r}}_G = \dot{\xi}_0 \mathbf{i}_0 + \dot{\eta}_0 \mathbf{j}_0 + \dot{\zeta}_0 \mathbf{k}_0 = u \mathbf{i} + v \mathbf{j} + w \mathbf{k}.$$

Angular position of the ship is given by the so-called ship Euler angles denoted in Fig. 1 as ψ, θ and ϕ . These angles bring vehicle from the reference (initial) orientation to the actual orientation of the body-fixed co-ordinate system. The orientation of the body-fixed co-ordinate system varies in time. It is given by the Euler angles. The following matrix relation [1, 3] gives the projection of the velocity expressed in body-fixed co-ordinate system on the Earth-fixed co-ordinates

$$\begin{Bmatrix} \dot{\xi}_0 \\ \dot{\eta}_0 \\ \dot{\zeta}_0 \end{Bmatrix} = \begin{bmatrix} \cos \cos & \cos \sin \sin & \cos \sin \cos \\ -\sin \cos & +\sin \sin & \\ \sin \cos & \sin \sin \sin & \sin \sin \cos \\ +\cos \cos & -\cos \sin & \\ -\sin & \cos \sin & \cos \cos \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \quad (3)$$

Angular velocity $\boldsymbol{\Omega}$ of ship can be expressed in terms of the time derivatives of roll, pitch and yaw as follows

$$\boldsymbol{\Omega} = P \mathbf{i} + Q \mathbf{j} + R \mathbf{k}. \quad (4)$$

The dependence of the derivatives of the Euler angles and angular velocity components expressed in the moving frame is as follows [1]

$$\begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 1 & \sin \tan & \cos \tan \\ 0 & \cos & -\sin \\ 0 & \sin / \cos \theta & \cos / \cos \theta \end{bmatrix} \begin{Bmatrix} P \\ Q \\ R \end{Bmatrix}. \quad (5)$$

2.2 GENERAL EQUATIONS OF MOTION

Equations of motion are given by the set of six non-linear ordinary differential equations [3]

$$\begin{aligned} X_g - mg \sin \theta &= m(\dot{u} + Qw - Rv) \\ Y_g + mg \cos \theta \sin \phi &= m(\dot{v} + Ru - Pw) \\ Z_g + mg \cos \theta \cos \phi &= m(\dot{w} + Pv - Qu) \\ K_g &= I_x \dot{P} - I_{xy} \dot{Q} - I_{xz} \dot{R} + (I_z R - I_{zx} P - I_{zy} Q)Q \\ &\quad - (I_y Q - I_{yz} R - I_{yx} P)R \\ M_g &= -I_{yx} \dot{P} + I_y \dot{Q} - I_{yz} \dot{R} + (I_x P - I_{xy} Q - I_{xz} R)R \\ &\quad - (I_z R - I_{zx} P - I_{zy} Q)P \\ N_g &= -I_{zx} \dot{P} - I_{zy} \dot{Q} + I_z \dot{R} + (I_y Q - I_{yz} R - I_{yx} P)P \\ &\quad - (I_x P - I_{xy} Q - I_{xz} R)Q. \end{aligned} \quad (6)$$

In equations 6, X_g , Y_g , Z_g , K_g , M_g and N_g depict the components of global reaction force and moment vectors acting on the ship. These are given in the-body fixed co-ordinate system xyz. m and I_{ij} mean ship's mass and the components of the mass moment of inertia. (2)

2.2 LINEAR APPROXIMATION OF THE EQUATIONS OF MOTION

The method starts with a linear approximation of motion estimate in irregular or regular waves. The linear approximation takes care of the diffraction forces and added parameters dependence upon the frequency of motion. Linear approximation of the responses in terms of the velocities

$$\mathbf{U}_L = u_L \mathbf{i} + v_L \mathbf{j} + w_L \mathbf{k} \quad (7)$$

$$\mathbf{L} = P_L \mathbf{i} + Q_L \mathbf{j} + R_L \mathbf{k} = \dot{\phi}_L \mathbf{i} + \dot{\theta}_L \mathbf{j} + \dot{\psi}_L \mathbf{k} \quad (8)$$

is obtained by the standard method such as for instance covered by reference [6]. Note that linear approximation does not distinguish between the inertial and body-fixed co-ordinate system. Motions are given in the co-ordinate system with the origin in the ship's center of gravity. The linearised equations of ship motion can be presented as follows

$$\begin{aligned}
m\dot{u}_L &= X_L = X_{\text{rad}} + X_{\text{diff}} + X_{\text{F.K.,L}} \\
m\dot{v}_L &= Y_L = Y_{\text{rad}} + Y_{\text{diff}} + Y_{\text{F.K.,L}} \\
m\dot{w}_L &= Z_L = Z_{\text{restoring,L}} + Z_{\text{rad}} + Z_{\text{diff}} + Z_{\text{F.K.,L}} \\
I_x \dot{P}_L - I_{xz} \dot{R}_L &= K_L = K_{\text{restoring,L}} + K_{\text{rad}} + K_{\text{diff}} + K_{\text{F.K.,L}} \\
I_y \dot{Q}_L &= M_L = M_{\text{restoring,L}} + M_{\text{rad}} + M_{\text{diff}} + M_{\text{F.K.,L}} \\
I_z \dot{R}_L - I_{zx} \dot{P}_L &= N_L = N_{\text{rad}} + N_{\text{diff}} + N_{\text{F.K.,L}}.
\end{aligned} \tag{9}$$

The indices rad, diff, F.K and restoring stand for radiation, diffraction, the so-called Froude-Krylov and restoring forces and moments. Index L depicts linear approximation to the forces and moments. In the linear approximation wave excitation is assumed to comprise the diffraction and Froude-Krylov forces and moments. The latter are evaluated from the pressures in and undisturbed oncoming wave. In the integration ship hull is assumed to have a constant velocity V_s pointing in the x-direction and integration is conducted up to the still water level.

The terms depicted by the indexes restoring,L are the z-directional force and moments acting on a ship in still water due to infinitely small and slow forced heaving displacement and angular inclination along x- and y-axes. The initial stability model is used to represent them.

2.3 THE NON-LINEAR PART OF THE RESPONSE

At the second stage, non-linear part of ship motions is evaluated in the time domain. This motion takes into account non-linearities of ship hydrostatics and non-linearities of wave loads at large amplitudes of motion. The only motion component that is not decomposed into the linear and non-linear part, is surge. Total surge motion is evaluated using the 1st of equations (6). The effect of added wave resistance, propulsor action and rudder forces are included in this equation. Total ship motion, or other type of response, being a sum of linear approximation and a non-linear part is thus obtained. In other words total responses in terms of velocities are written in the following form

$$\begin{aligned}
U &= ui + (v_L + v)j + (w_L + w)k \\
\Omega &= (P_L + P)i + (Q_L + Q)j + (R_L + R)k,
\end{aligned} \tag{10}$$

where variables without subscripts depict non-linear part of the response.

Subtracting the equations (9) of the linear approximation model from equations (6) yields the equations for the non-linear part of response

$$\begin{aligned}
m[\dot{u} + (Q_L + Q)(v_L + v) \\
- (R_L + R)(v_L + v) + g \sin(\theta_L + \theta)] &= X \\
m[\dot{v} + (R_L + R)u - (P_L + P)(w_L + w) \\
- g \cos(\theta_L + \theta) \sin(\phi_L + \phi)] &= Y \\
m[\dot{w} + (P_L + P)(v_L + v) - (Q_L + Q)u \\
- g \cos(\theta_L + \theta) \cos(\phi_L + \phi)] &= Z
\end{aligned} \tag{11}$$

$$\begin{aligned}
I_x \dot{P} + [I_z(R_L + R) - I_{zx}(P_L + P) - I_{zy}(Q_L + Q)](Q_L + Q) - I_{xy} \dot{Q} \\
- I_{xz} \dot{R} - [I_y(Q_L + Q) - I_{yz}(R_L + R) - I_{yx}(P_L + P)](R_L + R) = K \\
I_y \dot{Q} - I_{yx} \dot{P} + [I_x(P_L + P) - I_{xy}(Q_L + Q) - I_{xz}(R_L + R)](R_L + R) \\
- I_{yz} \dot{R} - [I_z(R_L + R) - I_{zx}(P_L + P) - I_{zy}(Q_L + Q)](P_L + P) = M \\
I_z \dot{R} - I_{zx} \dot{P} + [I_y(Q_L + Q) - I_{yz}(R_L + R) - I_{yx}(P_L + P)](P_L + P) \\
- I_{zy} \dot{Q} - [I_x(P_L + P) - I_{xy}(Q_L + Q) - I_{xz}(R_L + R)](Q_L + Q) = N.
\end{aligned}$$

Equations (11) govern non-linear part of the rigid body motion in six degrees of freedom. In order to solve them we need to specify the non-linear part of the external (fluid) forces X, Y, Z and moments K, M, N acting on a body. These are presented in bigger detail in reference [7]. Moreover we use equations (3) and (5) to express body velocities in the inertial co-ordinate system. Numerical integration of these equations together with the division of responses given by equations (10) yields the instantaneous position of ship in the inertial co-ordinate system $X_0 Y_0 Z_0$. Additional, thirteenth ordinary differential equation of a first order representing the action of auto-pilot is used to control the rudder angle. Integration is conducted using the 4th order Runge-Kutta scheme with an integration step being $\Delta t = 100$ ms. Computation is conducted for a full-scale ship. Linear approximation of responses and forces is related to ship's actual position in waves. It takes into account instantaneous heading angle. The zero initial conditions are used for all equations with an exception of surge velocity, which is set initially to a prescribed ship velocity in calm water. In order to dampen the spurious transients, wave amplitude is gradually increased from zero to the prescribed final value $A_{w,final}$ using the expression

$$\begin{aligned}
A_w(t) &= A_{w,final} \left[1 - \left(\cos \frac{\pi t}{2T_f} \right)^2 \right] \text{ for } t < T_f, \\
A_w(t) &= A_{w,final} \text{ for } t \geq T_f,
\end{aligned} \tag{12}$$

where t is time and with $T_f = 50$ seconds in full scale being used.

3 RADIATION FORCES

Radiation forces are approximated by a quasilinear model making use of the added mass and damping concept. These forces can be expressed in the general form as [8]

$$F_i = - \sum_{j=1}^6 (a_{ij} \dot{X}_j + b_{ij} U_j) \tag{13}$$

for $i = 1, 2, \dots, 6$ depicting degrees of freedom, or as follows

In equations 14 a_{ij} and b_{ij} depict added masses and damping coefficients referred to the origin located in the center of gravity (G in Fig. 1). These are frequency dependent values. In the present method these

$$\begin{aligned}
X_{\text{rad}} &= -a_{11}\dot{u} - b_{11}(u - V_S) - a_{15}\dot{Q} - b_{15}Q \\
Y_{\text{rad}} &= -a_{22}\dot{v} - b_{22}v - a_{24}\dot{P} - b_{24}P - a_{26}\dot{R} - b_{26}R \\
Z_{\text{rad}} &= -a_{33}\dot{w} - b_{33}w - a_{35}\dot{Q} - b_{35}Q \\
K_{\text{rad}} &= -a_{44}\dot{P} - b_{44}P - a_{46}\dot{R} - b_{46}R - a_{42}\dot{v} - b_{42}v \\
M_{\text{rad}} &= -a_{55}\dot{Q} - b_{55}Q - a_{53}\dot{w} - b_{53}w - a_{51}\dot{u} - b_{51}(u - V_S) \\
N_{\text{rad}} &= -a_{66}\dot{R} - b_{66}R - a_{64}\dot{P} - b_{64}P - a_{62}\dot{v} - b_{62}v.
\end{aligned} \tag{14}$$

coefficients are evaluated by a standard linear seakeeping theory based computer program [6]. Note that radiation forces are oriented in the body-fixed co-ordinate system.

3.1 MEMORY EFFECT INCLUDED USING THE RETARDATION FUNCTION CONCEPT

The radiation forces model represented by the equations 14 is good for a frequency domain linear analysis. Time domain approach requires the so-called convolution integral representation of the radiation forces [2]. In this time approach radiation forces vector \mathbf{X}_{rad} is represented by an expression:

$$\mathbf{X}_{\text{rad}}(t) = -\mathbf{a}_{\infty}\ddot{\mathbf{x}}(t) - \int_{-\infty}^t \mathbf{k}(t-\tau)\dot{\mathbf{x}}(\tau)d\tau, \tag{15}$$

where \mathbf{a}_{∞} is the matrix comprising of the added masses coefficients for an infinite frequency and \mathbf{x} is the response vector. Matrix function \mathbf{k} is the so-called retardation function which takes into account the memory effect of the radiation forces. This function can be evaluated as follows

$$\mathbf{k}(t) = \frac{2}{\pi} \int_0^{\infty} \mathbf{b}(\omega) \cos(\omega t) d\omega \tag{16}$$

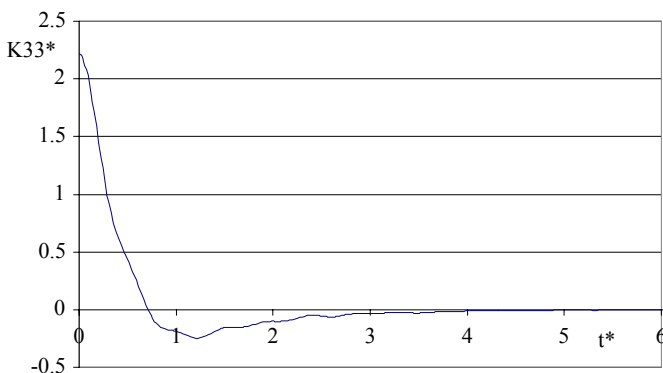
where \mathbf{b} is the frequency dependent added damping matrix. The $\mathbf{k}(t)$ functions have to be evaluated before the simulation. The Fast Fourier Transform algorithm is used when evaluating discrete values of the retardation functions as follows

$$K_{k,ij}(k\Delta t) = \frac{N\Delta\omega}{\pi} \mathbf{FFT}(g_{ij}(x)), \tag{17}$$

where the original added damping discrete functions are substituted by a 'double-sided function' $g(x)$ as follows:

$$\begin{aligned}
g_{ij}(x) &= b_{ij}(x) \text{ for } x = \Delta\omega, \Delta\omega N/2 \\
g_{ij}(N\Delta\omega - x) &= b_{ij}(x) \text{ for } x = 0, \Delta\omega(N/2 + 1).
\end{aligned} \tag{18}$$

Note that as a result the retardation function 16 is obtained at $N/2$ discrete time instants with a time step Δt . FFT analysis is conducted with $N = 2048$. As a result the



retardation functions are represented by 1024 discrete values covering the period of 102.4 seconds. An example of the retardation function for heave is given below.

Fig. 2 Non-dimensional heave memory (retardation) function $K_{33}^* = K_{33} / (m\sqrt{g/L})$ as a function of non-dimensional time $t^* = t / \sqrt{g/L}$, where L is waterline length of ship.

3.2 SIMPLIFIED MODEL WITH NO MEMORY EFFECT

The simplified model, which does not take flow memory into account, is based on the assumption that added masses and damping coefficients are constant. For this model two options are used. In case 2a added masses and damping values are evaluated for the prescribed frequency of encounter with an exception of roll coefficients (including cross-coupling of roll with other motion components), which are evaluated for the natural frequency of roll. In case 2b all coefficients are evaluated for the frequency of encounter.

4 RESULTS OF SIMULATION

Model test experiments of the containership conducted at the Osaka University [5] were simulated using three options for the radiation forces modelling. Each simulation run was of a time length 720 seconds full-scale for no-capsizing vessel. If ship capsizes, integration is terminated and time record is shorter. Wave condition is same in all cases. Amplitude of regular wave is $A_W = 4.5$ [m] and length $\lambda = 225$ [m], that is $\lambda = 1.5 * L$. The varied quantities are ship speed and heading. Summary of the simulation is presented in Table below.

Table Summary of the results.

F_n	0.2	0.2	0.3	0.4
Heading [deg]	0	45	30	30
Experiment	capsize	no-capsize	no-capsize	capsize
Case 1	no-capsize	no-capsize	no-capsize	capsize
Case 2a&b	no-capsize	no-capsize	capsize	capsize

Selected time histories of the simulated responses are presented in the following. Simulations do not predict ship capsizing in following regular waves and ship speed $F_n = 0.2$. Although after several wave encounters ship starts to roll heavily (see Fig. 3), this rolling motion is restricted to approximately 12.5 [deg].

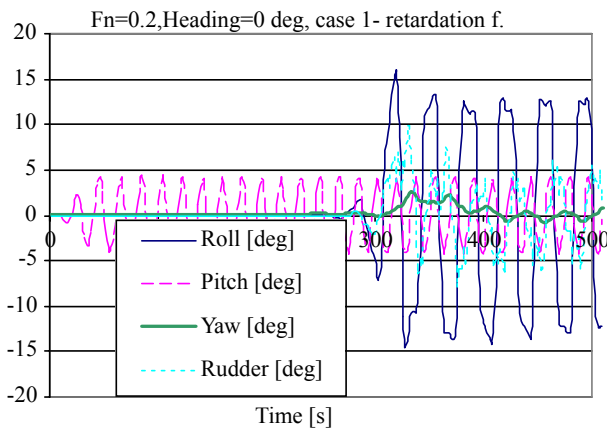


Fig. 3. Angular motions of ship in regular following in the radiation forces.

The case of heading being 30 [deg] and $F_n = 0.3$ is the one where considering memory effect in radiation forces has a positive effect on ship behaviour prediction. As it seen from Figs. 4 and 5, ship survives in this condition both in simulation, in which retardation function is used,

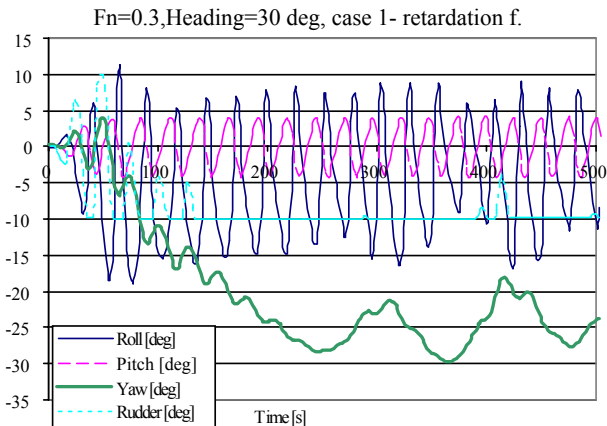


Fig. 4. Angular motions of ship in regular quartering waves (heading = 30 [deg]). Ship speed is $F_n = 0.3$. Memory effect is included in the radiation forces.

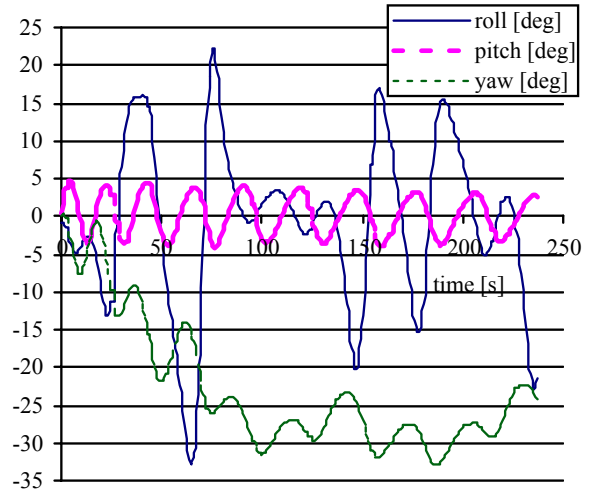


Fig. 5. Angular motions of ship in regular quartering waves (heading = 30 [deg]). Ship speed is $F_n = 0.3$. Model test result scaled to full-scale and yaw defined as a deviation from the initial course. [5].

Both constant added masses and damping models wrongly predict ship capsizing in this condition (see Fig. 6).

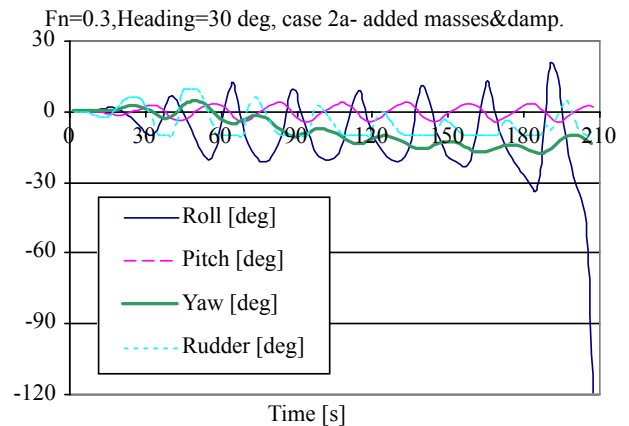


Fig. 6. Angular motions of ship in regular quartering waves. Ship speed is $F_n = 0.3$ and heading 30 [deg]. Radiation forces are represented by constant added masses and damping coefficients.

The case of highest speed ($F_n = 0.4$) and heading 30 [deg] is shown in Figs. 7, 8 and 9. In model test experiments (Fig. 7) ship capsizes. Same is predicted by simulations (Fig. 8 and 9). In computations it takes longer time for the model to capsize. The reason for this may be in the initial conditions of simulations.

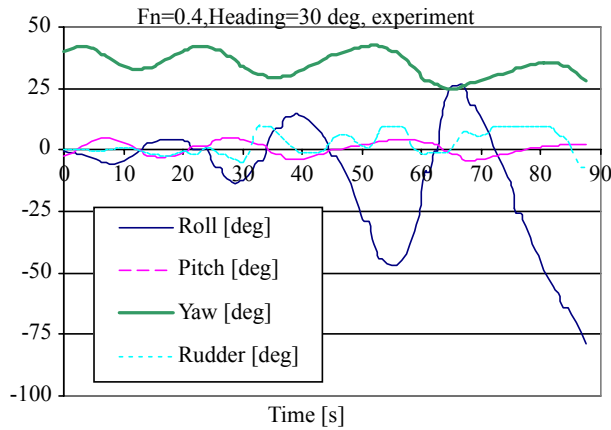


Fig. 7. Model running at $F_n = 0.4$ capsizes in regular quartering regular waves (heading 30 [deg]). Model test result scaled to full-scale. [5].

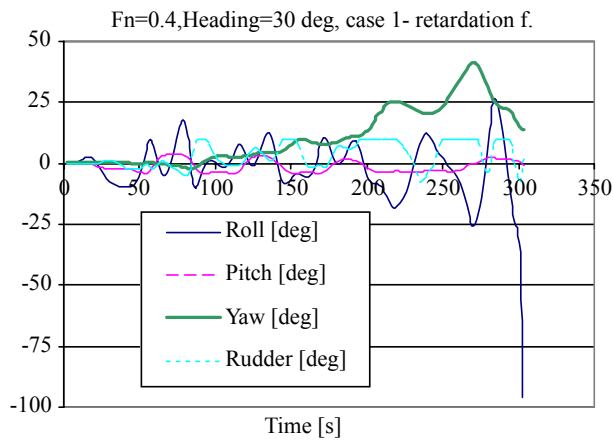


Fig. 8. Containership running at $F_n = 0.4$ capsizes in regular quartering regular waves (heading 30 [deg]). Simulations include the memory effect.

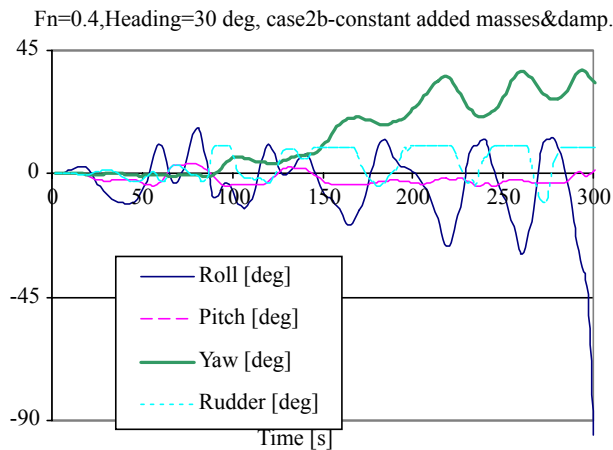


Fig. 9. Containership running at $F_n = 0.4$ capsizes in regular quartering regular waves (heading 30 [deg]). Simulations conducted using constant added masses and damping approach.

5 CONCLUSIONS

Considering the memory effect when modelling radiation forces yields better results of dynamic behaviour of ship in the context of two-stage approach of prediction non-linear ship motions.

This conclusion is not a surprising one. In time domain analysis, added mass and damping model is in principle applicable for harmonic monochromatic motions only. Dynamic stability of ship is characterized by non-linearities and transient type behavior. Although the retardation function approach implies the linearity assumption, it takes properly into account flow memory effect important in case of transient type behavior.

In the presented method, maneuvering hull forces are represented by the retardation functions and convolution integrals involving them. This potential flow model does not necessarily include all relevant flow features governing yaw and sway motion components. Moreover, this model is based on the linearity assumption. This may be the reason for a poor prediction of ship capsizing in following waves.

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