The Modeling of the Excitation of Large Amplitude Rolling in Beam Waves

Alberto Francescutto & Giorgio Contento [1]

ABSTRACT

In this paper the effect of the excitation modeling on the fitting capability of the nonlinear roll motion equation to experimental data is studied. Several frequency dependent and constant effective wave slope coefficients are derived for five different scale models corresponding to different ship typologies by a Parameter Identification Technique. The frequency domain behavior of the obtained coefficients is discussed and compared with linear diffraction (strip theory) results and I.M.O. suggested values. It appears that a mathematical modeling with constant damping parameters and frequency dependent excitation could give very good results. As regards the excitation parameters, a common trend for slender bodies is evidenced.

Key words: *Nonlinear Dynamics, Roll Motion, Simulation, Parameter Identification.*

MATHEMATICAL MODELLING OF SHIP ROLLING

In this paper some results regarding a simplified mathematical modeling of ship rolling, possibly using as much as possible constant coefficients, are reported in synthesis. More details can be found in [1-3].

The incident wave is assumed long enough to be described by the local slope $\alpha(t)$ where

a) Absolute angle description

$$(I_G + \delta I)\ddot{\varphi} + N\dot{\varphi} + D_1|\dot{\varphi}|\dot{\varphi} + D_2\dot{\varphi}^3 + \ldots + \Delta\overline{GZ}(\varphi) = E(t)$$

The excitation E(t) following Blagoveshchensky [4]:

$$E(t) = \chi_\varphi{}^{I}\Delta\overline{GM}\alpha + \chi_\varphi{}^{II} I_\nabla\ddot{\alpha} + \chi_\varphi{}^{III} N\dot{\alpha} + \chi_\varphi{}^{IV}\delta I\ddot{\alpha}$$

I_∇ is the inertia of the 'liquid hull';

$\chi_\varphi{}^{j}$ ($0 \le \chi_\varphi{}^{j} \le 1$) terms included to account for the hull shape/coefficients and for the ratio between wave parameters and hull breadth and draught.

In normalized form as follows:

$$\ddot{\varphi} + 2\mu\dot{\varphi} + \delta_1|\dot{\varphi}|\dot{\varphi} + \delta_2\dot{\varphi}^3 + \ldots + \omega_0{}^2\varphi + a_3\varphi^3 + a_5\varphi^5 + \ldots = e(t)$$

with

$$e(t) = \pi s_w\left[\left(\alpha_1\omega_0{}^2 - \alpha_2\omega^2\right)\cos(\omega t) - 2\mu\alpha_3\omega\sin(\omega t)\right]$$

where α_2 accounts for both terms $\chi_\varphi{}^{II} I_\nabla$ and $\chi_\varphi{}^{IV}\delta I$

The last contribution in e(t), proportional to the linear damping, is of order O(1/ω) with respect to the former and can be neglected:

$$e(t) = \pi s_w\omega_0{}^2\left[\alpha_1 - \alpha_2\left(\frac{\omega}{\omega_0}\right)^2\right]\cos(\omega t)$$

The roll excitation reminds the Morison's approach to wave loads. Here the incident flow characteristics are represented by the local wave slope $\alpha(t)$ and by its derivatives $\dot{\alpha}(t), \ddot{\alpha}(t)$ instead of the traditional velocity and acceleration in a representative point. Each term in

[1] DINMA - University of Trieste, via A. Valerio, 10 - 34127 Trieste, Italy
e-mail: francesc@univ.trieste.it

e(t) has been derived under the *Froude-Krilov hypotheses* and *long wave approximation*. The presence of the hull on the incident flow is accounted for only in some so called 'effective wave coefficients' $\chi_\varphi{}^j$. In this way an important feature of wave load is lost when the wave length becomes comparable with the transversal dimension of the body, i.e. in the diffraction regime. A tentative formulation is here proposed as follows:

$$e(t) = \pi s_w \omega_0{}^2 e^{-\left[\omega/\alpha_1\right]^{\alpha_2{}^2}} \cos(\omega t)$$

Often, no explicit dependence of the amplitude of the excitation on the wave frequency appears. In this case e(t) reads as follows:

$$e(t) = \pi s_w \omega_0{}^2 \alpha_1 \cos(\omega t)$$

Summarizing:

$$\ddot{\varphi} + 2\mu\dot{\varphi} + \delta_1|\dot{\varphi}|\dot{\varphi} + \delta_2\dot{\varphi}^3 + \ldots + \omega_0{}^2\varphi + \sum_{i=1}^{M} a_{2i+1}\varphi^{2i+1} =$$
$$\pi s_w \omega_0{}^2 \alpha_0{}^* \cos(\omega t)$$

$$\alpha_0{}^*(\omega) = \begin{cases} \alpha_1 - \alpha_2\left(\dfrac{\omega}{\omega_0}\right)^2 \\ \alpha_1 \\ e^{-\left[\omega/\alpha_1\right]^{\alpha_2{}^2}} \end{cases}$$

b) Relative angle description

Assuming the same hydrodynamic model for the damping and the same notations as in the previous section, the equation of roll motion in the relative angle approach can be written following Wright-Marshfield [5]:

$$I_G\ddot{\varphi} = -\delta I(\ddot{\varphi} - \ddot{\alpha}) - N(\dot{\varphi} - \dot{\alpha}) - M_R(\varphi - \alpha)$$

where $M_R(\varphi - \alpha)$ is the restoring moment. Introducing the relative angle $\vartheta = \varphi - \alpha$ and dividing by $I_G + \delta I$, we obtain

$$\ddot{\vartheta} + 2\mu'\dot{\vartheta} + \delta'_1|\dot{\vartheta}|\dot{\vartheta} + \delta'_2\dot{\vartheta}^3 + \ldots + \omega_0{}^2\vartheta + \sum_{i=1}^{M} a_{2i+1}\vartheta^{2i+1} =$$
$$-\frac{I_G}{I_G + \delta I}\pi s_w \omega^2 \cos(\omega t)$$

The two approaches are equivalent since they correspond only to a different grouping of the terms. Some differences are due to the fact that:

- in the absolute angle approach only the first term in the development of $\overline{GZ}(\varphi - \alpha) - \overline{GZ}(\varphi)$ is retained corresponding to the fact that the nonlinear dependency on the wave slope is neglected;
- because of the relative angle approach, the inertia loads are accounted for only by the term δI

Moreover, in the assumed relative angle approach only one "unknown" coefficient

$$\alpha_0{}^* = \frac{I_G}{I_G + \delta I}$$

is left to account for the "reduction" of the wave slope effectiveness. In this sense the relative angle model is equivalent to the "constant" wave slope reduction.

DISCUSSION AND CONCLUSIONS

The application of an efficient Parameter Identification Technique, developed on the basis of an idea from Haddara and Bennet [6], to a large series of experiments conducted at the University of Trieste allowed to obtain the following results:

a) Overall capability of the proposed excitation models

Absolute angle: As far as the form of the excitation is concerned, it can be seen from the graphs and from the residual χ^2 in Table 2 that frequency dependent effective wave slope coefficients work better than the constant. This becomes particularly evident outside the peak zone where the difference between estimated and measured values can exceed 100% even if at these frequencies the absolute roll amplitudes are quite small (few degrees). In Fig. 1 and Fig. 2 the results obtained from a highly non-linear restoring, giving rise to bifurcation, and from a mildly non-linear one respectively are reported.

Relative angle: quite similar results with some shortcoming in the mathematical modeling. The $\alpha_0{}^*$ are indeed quite small, so that the introduction of a double factor could be more adapt.

b) Dependency on the righting arm modeling

In some cases the Parameter Identification Technique exhibited an "hyper-sensitivity" to the GZ description mostly in the presence of a strongly nonlinear behavior.

c) Identification of a common trend

An evident frequency dependence of α_0^* is observed (Fig. 3,5). A common trend is evidenced that could be used as a default, at least for slender bodies.

d) Identified damping

The identified values of the damping coefficients, while different for the different ship typologies, show great stability with respect to the excitation modeling.

If the relative angle approach is adopted, then the values of the identified damping coefficients differ from those in the absolute angle.

Several mathematical models were tried. Basically the linear-plus-quadratic and linear-plus-cubic showed almost the same fitting capability, so that using one or the other is a matter of preference.

e) Use of perturbative solution in the PIT

This method was attempted and in some cases gave quite good results, in comparison with the numerical exact solution, at the cost of much less computing time. On the other hand, the results were not so reliable in the cases where high non-linearity was present in the righting arm.

f) Comparison with diffraction theory and I.M.O. suggestions

The frequency dependence of α_0^* is still observed when this parameter is obtained through a linear diffraction-strip theory based code (Fig. 6). However the forecasts of $\alpha_0^*(\omega)$ from the strip theory are usually quite higher than the results from the Parameter Identification method here used with the notable exception of the fishing vessel.

Finally, for sake of completeness, the I.M.O. suggested value of α_0^*, typically indicated with "r", for the roll amplitude computation in the Weather Criterion is compared with experimental results (Fig. 4,7). Here again, with the exception of the fishing vessel, the "r" values are sensibly higher than the observed ones. The deviation of course is on the right side, leading to an overestimation of the rolling amplitude when applying the Weather Criterion. High values of "r" are probably due to the fact that usually this coefficient is obtained by measuring the heeling moment due to waves at fixed model.

REFERENCES

1. Contento, G., Francescutto, A., Piciullo, M., 1996, On the effectiveness of constant coefficients roll motion equations. *Ocean Engineering*, Vol. 23, pp. 597-618.

2. Francescutto, A., Contento, G., 1997, An Investigation on the Applicability of Simplified Mathematical Models to the Roll-Sloshing Problem. *Proc. 7th International Conference on Offshore and Polar Engineering - ISOPE'97*, Honolulu, The Int. Society of Offshore and Polar Engineering, Vol. 3, pp. 507-514.

3. Francescutto, A., Contento, G., Biot, M., Schiffrer. L., Caprino, G., 1998, The Effect of Excitation Modeling in the Parameter Estimation of Nonlinear Rolling. *Proc. 8th International Conference on Offshore and Polar Engineering - ISOPE'98*, Montreal, The Int. Society of Offshore and Polar Engineering, Vol. 3, pp. 490-498.

4. Blagoveshchensky, S.N., 1962, *Theory of Ship Motions*. Dover Publications, Inc., New York, Vol. 2.

5. Wright, J. H. G., Marshfield, B. W., 1980, Ship Roll Response and Capsize Behavior in Beam Seas. *Trans. RINA*, Vol. 122, pp. 129-148.

6. Haddara, M.R., Bennett, P., 1989, A Study of the Angle Dependence of Roll Damping Moment. *Ocean Engineering*, Vol. 16, pp. 411-427.

AKNOWLEDGMENTS

This Research has been developed with the financial support of the Italian National Research Council under Contract 97.03187.CT07.

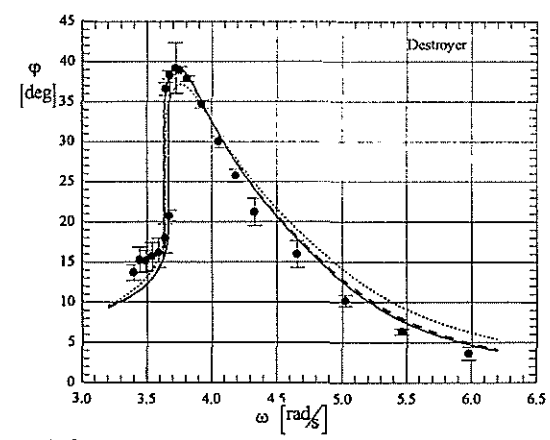

Figure 1. Steady roll amplitudes of a scale model of a destroyer in a regular beam sea. [● experimental data; ——— (eqns 9, 8.1); ·········· (eqns 9, 8.2); – – – – – (eqns 9, 8.3)].

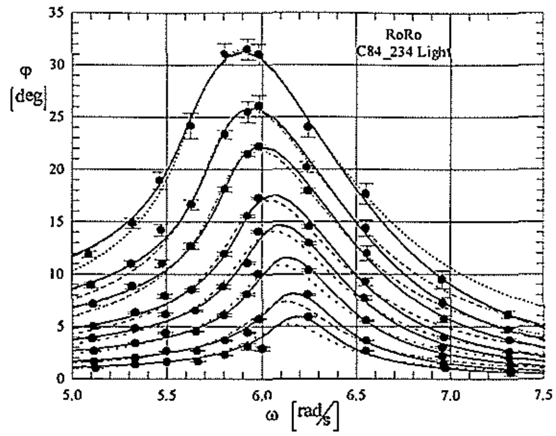

Figure 2. Steady roll amplitudes of a scale model of a RoRo (C84_234 Light) in regular beam sea. [● experimental data; —— (eqns 9, 8.1); ········ (eqns 9, 8.2); – – – – (eqns 9, 8.3)

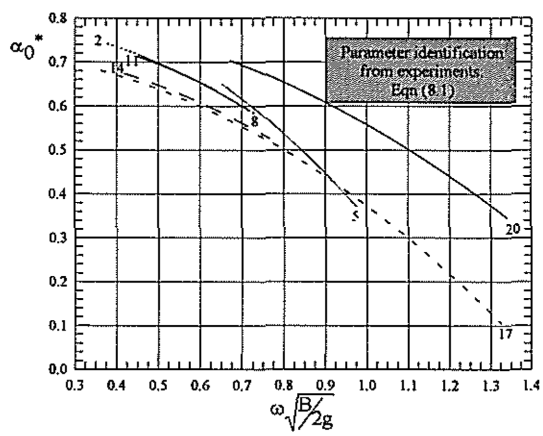

Figure 3. Effective wave slope coefficient α_0^* derived by PIT from eqn (8.1) versus the non-dimensional wave frequency for the different ship models (see Table 2 for curve labels).

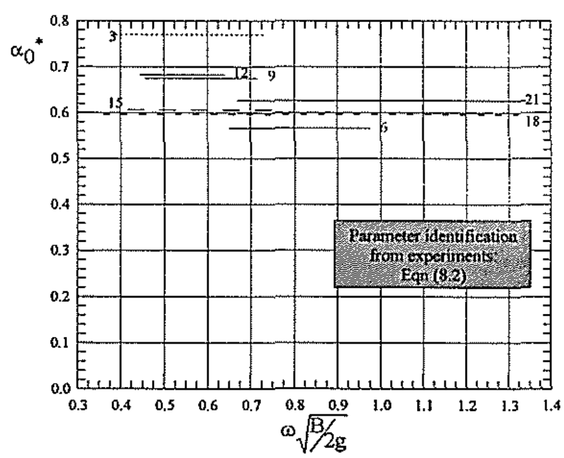

Figure 4. Effective wave slope coefficient α_0^* derived by PIT from eqn (8.2) versus the non-dimensional wave frequency for the different ship models (see Table 2 for curve labels).

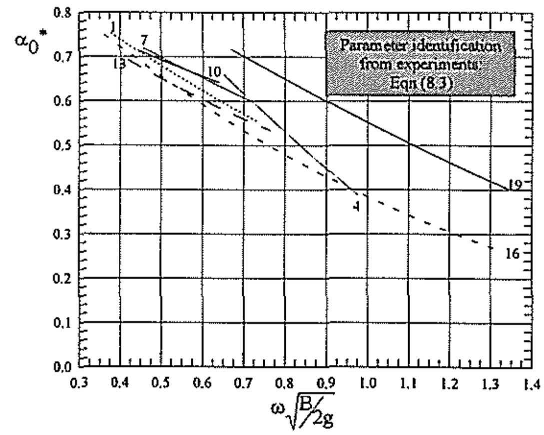

Figure 5. Effective wave slope coefficient α_0^* derived by PIT from eqn (8.31) versus the non-dimensional wave frequency for the different ship models (see Table 2 for curve labels).

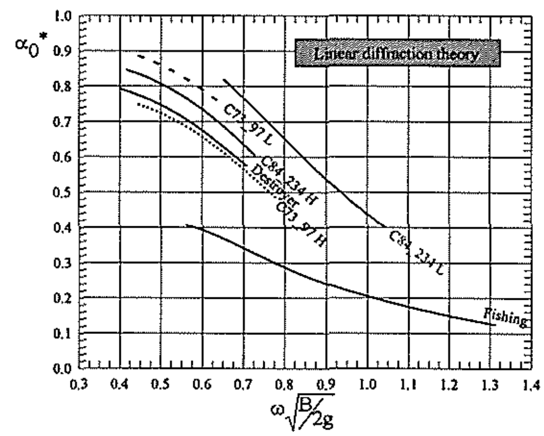

Figure 6. Effective wave slope coefficient α_0^* from a linear diffraction code (strip theory) versus the non-dimensional wave frequency for the different ship models.

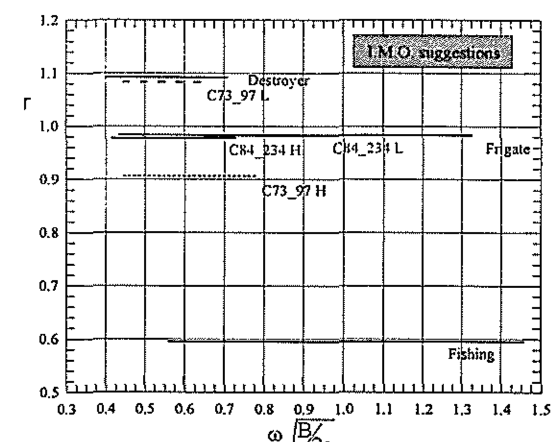

Figure 7. Parameter "r" as given by I.M.O. for the Weather Criterium versus the non-dimensional wave frequency for the different ship models.

Table 1.

	Destroyer C83_227	Frigate C84_236	Fishing C86_253	RORO (A) C73_97		RORO (B) C84-234	
Lpp $[m]$	2.532 ± 0.001	2.400 ± 0.001	2.000 ± 0.001	2.464 ± 0.001		1.752 ± 0.001	
B $[m]$	0.273 ± 0.0005	0.285 ± 0.0005	0.552 ± 0.0005	0.380 ± 0.0005		0.333 ± 0.0005	
Scale	1:50	1:50	1:12.5	1:50		1:30	
LOADING CONDITION				**LIGHT**	**HEAVY**	**LIGHT**	**HEAVY**
Displacement Δ $[N]$	262.4 ± 0.2	253.0 ± 0.2	1243.5 ± 0.2	464.3 ± 0.2	605.2 ± 0.2	251.3 ± 0.2	316.06 ± 0.2
T $[m]$	0.0800 ± 0.0005	0.0810 ± 0.0005	0.2450 ± 0.0005	0.0970 ± 0.0005	0.1175 ± 0.0005	0.0800 ± 0.0005	0.0950 ± 0.0005
Trim $[m]$	0.0000 ± 0.0005	0.0000 ± 0.0005	0.0000 ± 0.0005	0.0000 ± 0.0005	0.0000 ± 0.0005	0.0000 ± 0.0005	0.0000 ± 0.0005
$\overline{KG}$ $[m]$	0.1083 ± 0.0005	0.1155 ± 0.0005	0.1900 ± 0.0005	0.1541 ± 0.0005	0.1521 ± 0.0005	0.1130 ± 0.0005	0.1350 ± 0.0005
$\overline{GM}$ $[m]$	0.0217 ± 0.0005	0.0289 ± 0.0005	0.1020 ± 0.0005	0.0320 ± 0.0005	0.0384 ± 0.0005	0.0660 ± 0.0005	0.0330 ± 0.0005
Natural frequency ω_0 $[rad/s]$	4.6045 ± 0.005	5.020 ± 0.005	5.240 ± 0.005	3.696 ± 0.005	4.217 ± 0.005	6.296 ± 0.005	4.379 ± 0.005
Wave frequency. range ω $[rad/s]$	$3.30 \leq \quad \leq 6.00$	$3.79 \leq \quad \leq 10.80$	$4.22 \leq \quad \leq 7.82$	$3.27 \leq \quad \leq 4.42$	$3.49 \leq \quad \leq 5.24$	$5.10 \leq \quad \leq 7.31$	$3.62 \leq \quad \leq 5.29$
Nondim. frequency range $\omega\sqrt{B/2g}$	$0.39 \leq \quad \leq 0.71$	$0.46 \leq \quad \leq 1.30$	$0.71 \leq \quad \leq 1.31$	$0.46 \leq \quad \leq 0.62$	$0.49 \leq \quad \leq 0.73$	$0.66 \leq \quad \leq 0.95$	$0.47 \leq \quad \leq 0.69$
Wave steepness Sw	1/30	1/20	1/125, 1/50	1/90, 1/50, 1/30	1/30, 1/20	1/300, 1/200, 1/125, 1/90, 1/70, 1/50, 1/40, 1/30	1/300, 1/200, 1/125, 1/90, 1/70, 1/50, 1/40, 1/30

Table 2.

Ship model	Load cond.	Resid.	Damping model			Excitation model				Restoring					
		χ^2	μ	δ_1 quad.	δ_2 cubic		α_1	α_2	r (IMO)	GZ	ωo	a3	a5	a7	a9
Destroyer		0.03239	0.3050	0	0	expo	10.011	1.051	**1.092**	FRT	**4.6045**	**-56.498**	**182.04**	**-305.52**	**213.508**
		0.03532	0.3131	0	0	quad	0.800	0.120	**1.092**	FRT	**4.6045**	**-56.498**	**182.04**	**-305.52**	**213.508**
		0.07123	0.3464	0	0	const.	0.7701	0	**1.092**	FRT	**4.6045**	**-56.498**	**182.04**	**-305.52**	**213.508**
RoRo C84_234	Light	0.0070	0.1216	0.1630	0	expo	7.688	1.426	**0.9826**	PIT	6.216	-26.104	48.770	0	0
		0.0070	0.1251	0.1649	0	quad	0.8759	0.3478	**0.9826**	PIT	6.218	-27.00	52.167	0	0
		0.0122	0.1728	0.1249	0	const.	0.5662	0	**0.9826**	PIT	6.1599	-23.87	46.61	0	0
	Heavy	0.01104	0.0746	0	0.0613	expo	10.9736	0.9870	**0.9775**	FRT	**4.379**	**3.799**	**-15.248**	**0**	**0**
		0.01096	0.0752	0	0.0614	quad	0.8001	0.1331	**0.9775**	FRT	**4.379**	**3.799**	**-15.248**	**0**	**0**
		0.01274	0.0775	0	0.0616	const.	0.6750	0	**0.9775**	FRT	**4.379**	**3.799**	**-15.248**	**0**	**0**
RoRo C73_97	Light	0.0112	0.0812	0	0.2039	expo	13.422	0.8707	**1.083**	linear	**3.696**	**0**	**0**	**0.**	**0.**
		0.0111	0.0820	0	0.2043	quad	0.7878	0.0986	**1.083**	linear	**3.696**	**0**	**0**	**0.**	**0.**
		0.0121	0.0798	0	0.2031	const.	0.6818	0	**1.083**	linear	**3.696**	**0**	**0**	**0.**	**0.**
	Heavy	0.0032	0.0552	0	0.1740	expo	9.2662	0.9429	**0.9066**	PIT	**4.217**	-16.97	69.76	-48.01	0
		0.0035	0.0576	0	0.1744	quad	0.7365	0.1230	**0.9066**	PIT	**4.217**	-18.28	83.13	-81.27	0
		0.0063	0.0274	0	0.1896	const.	0.6059	0	**0.9066**	PIT	**4.217**	-28.45	188.15	-346.35	0
Frigate		0.00851	0.3918	0	0	expo	8.603	1.086	**0.9834**	PIT	**5.02**	-19.19	-67.80	309.46	-309.50
		0.01027	0.3857	0	0	quad	0.7272	0.1298	**0.9834**	PIT	**5.02**	-25.52	-13.54	156.69	-172.59
		0.01625	0.3620	0	0	const.	0.5961	0	**0.9834**	PIT	**5.02**	-40.02	113.13	-209.07	168.91
Fishing		0.00451	0.1632	0.2514	0	expo	8.5376	1.202	**0.5953**	linear	**5.24**	**0.**	**0.**	**0.**	**0.**
		0.00425	0.1668	0.2517	0	quad	0.8240	0.2058	**0.5953**	linear	**5.24**	**0.**	**0.**	**0.**	**0.**
		0.00807	0.1683	0.2553	0	const.	0.6260	0	**0.5953**	linear	**5.24**	**0.**	**0.**	**0.**	**0.**