Towards a theory of surf-riding in two-frequency and multi-frequency waves

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ABSTRACT

Steps are taken towards extending the theory of surf-riding for multi-chromatic waves. New bifurcation phenomena are identified and classified that are intrinsic to the presence of extra frequencies in the excitation. Alternative types of surf-riding are discovered. Chaotic transients seem to be quite a common feature of ship surge motion in extreme following seas.

Keywords: ship motions, surf-riding, Lagrangian coherent structures, basin erosion, chaos

1. INTRODUCTION

The theory explaining the nonlinear surging and surf-riding of ships in steep following waves has been built upon the assumption of monochromatic waves (Spyrou 1996). Many tacitly take for granted that these phenomena endure, in almost identical form, in irregular seas too. Nevertheless, the multi-chromatic sea renders the phase space flow of the underlying dynamical system time-dependent, a fact bearing many new possibilities of dynamic behaviour. For example, a ship can appear transferring randomly, in finite time intervals, between ordinary surging and surf-riding-like behaviour. Then, the concept of surf-riding equilibrium that had been the basis for explaining involuntary high speed runs in following waves is gone [Spyrou et al. 2014, Belenky et al. 2016; Themelis et al. 2016].

It is greatly desirable all yet undocumented motions types that can be realized in irregular seas to be systematically identified, evaluated and classified. However, conventional computational techniques that had been, up to now, successfully applied for studying the effect of monochromatic seas are not sufficient and a novel set of state-of-art computational tools will be required.

Driven by these observations, the first results from an ongoing exploration into the unsteady phase space of ship surging under bi-chromatic and multi-chromatic excitation will be presented; on the one hand demonstrating the approach; and on the other, identifying and analyzing new extreme types of ship behaviour, in relation to the frequency content and intensity of wave excitation.

2. DESCRIPTION OF APPROACH

Unidirectional wave fields are considered, created by the superposition of two or more wave components, propagating in the direction of ship motion. A standard mathematical model that can reproduce asymmetric surging and surf-riding has been employed, incorporating multi-frequency excitation (Spyrou et al. 2014). The examined ship was a tumblehome topside vessel, from the ONR series, with length L=154 m, beam B=18.8 m and mean draft T=5.5m.

Our analysis is focused on the identification of system's hyperbolic *Lagrangian coherent structures* (LCS) in phase space. The analysis is not constrained by the number of frequencies in the excitation, nor by the nature of it ("regular" or "irregular"). The LCS are phase space objects of a separatrix nature that can be considered as analogous to the stable and unstable manifolds of hyperbolic fixed points of autonomous dynamical systems. Hence, they indicate basins of attraction and, in general, they expose the skeleton of the flow. The LCS concept came about from the interbreeding of nonlinear dynamics and fluid mechanics (Haller & Yuan 2000; Shadden 2011). In a physical flow, LCS appear as cores of trajectory patterns, identified as being, locally, the strongest attracting/repelling material surfaces advected with the flow. A few approaches have been proposed for their identification, which vary in their robustness, potential for handling multi-dimensional phase space, in terms of computational cost, etc. Here we have implemented a scheme based on the calculation of the largest finite-time Lyapounov exponent (FTLE) field (Shadden et al. 2005; Kontolefas et al. 2016). Alternative approaches (not reported here) are also under evaluation.

For the bi-chromatic sea in particular, supplementary calculations were performed; specifically, a massive campaign of time-domain simulations. The goal was to capture the mean and the amplitude of the surge velocity oscillation, at steady state, in order to evaluate how these relate with characteristic reference velocities, such as the nominal speed and the celerities of the participating wave components.

3. PRINCIPAL FEATURES OF THE UNSTEADY PHASE-SPACE FLOW

Bi-chromatic waves

The ship is excited by two harmonic waves, defined as follows: the first (identified from now on as the "primary"), has fixed length λ_1 equal to the ship length L and its steepness is set at $s_1 = 0.035$. The other ("secondary"), can be regarded as a perturbation effect; nonetheless, its height will be allowed sometimes to become large. It will have a comparable frequency value, while its steepness will be varied according to the scenario.

The arrangement of system's LCS right upon the inception of global surf-riding is revealed through the two time shots of Fig. 1. Some differences from the monochromatic case are noticed: firstly, crossings of LCS (i.e. essentially of manifolds) appear, accompanied by the usual, in these cases, stretching and folding process. Secondly, as evidenced from Fig. 2, surf-riding is oscillatory (the power spectrum of the motion is also shown). In fact, this is a universal feature of surf-riding in bichromatic waves. It will be revealed later that the celerity of the primary wave dictates the mean value of ship velocity during surf-riding. The perturbing wave on the other hand, is responsible for velocity's oscillation around the celerity of the primary wave.

The crossing of LCS brings along the fractalisation of basin boundaries and subsequently, basin erosion. In the series of graphs of Fig. 3, the steepness of the secondary wave is raised from a very low value, in order to observe the successive transformations of phase space, as the effect of the secondary wave is intensified. The steepness of the primary wave is set lower than previously; in such a way that, in the absence of the secondary wave, coexistence of surging and surf-riding would exist (this fact is basically confirmed by the first graph of Fig. 3).

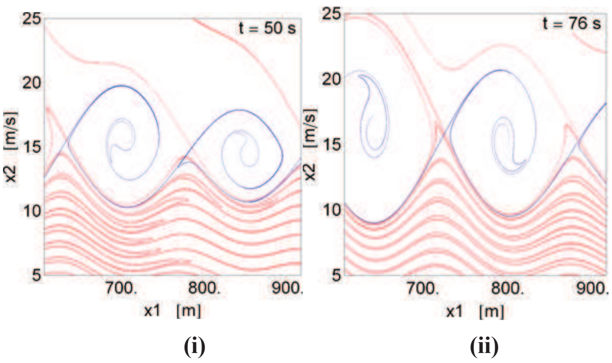


Figure 1: Phase-space portraits at different time instants, for bi-chromatic wave excitation. The attracting and repelling LCS (blue and red curves respectively) are shown. Parameters were set to the following values: $(\lambda_1, s_1, \omega_2/\omega_1, s_2/s_1, u_{nom}) = (L, 0.035, 0.9, 0.3, 12)$.

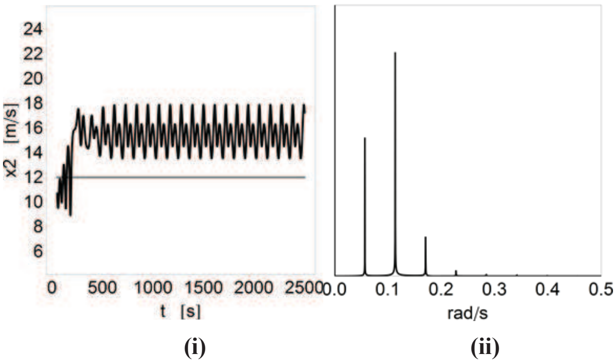


Figure 2: Character of surf-riding in bichromatic waves. (i) Time history of surge velocity (black curve) contrasted to the nominal speed (grey line). (ii) The discrete Fourier transform of the time history of surge velocity.

The fact that basin boundaries become fractal is verified by zooming successively onto a small area enclosing a basin boundary segment, revealing the well-known self-similarity pattern (see Fig. 4). The erosion of surf-riding's basins bears an important consequence: surging becomes motion destination from areas deep into surf-riding's domain, in a rather unpredictable manner. Two time-domain simulation examples, shown in Fig. 5, verify this behaviour. The particularly long, seemingly chaotic, transient of case 2 should be noticed.

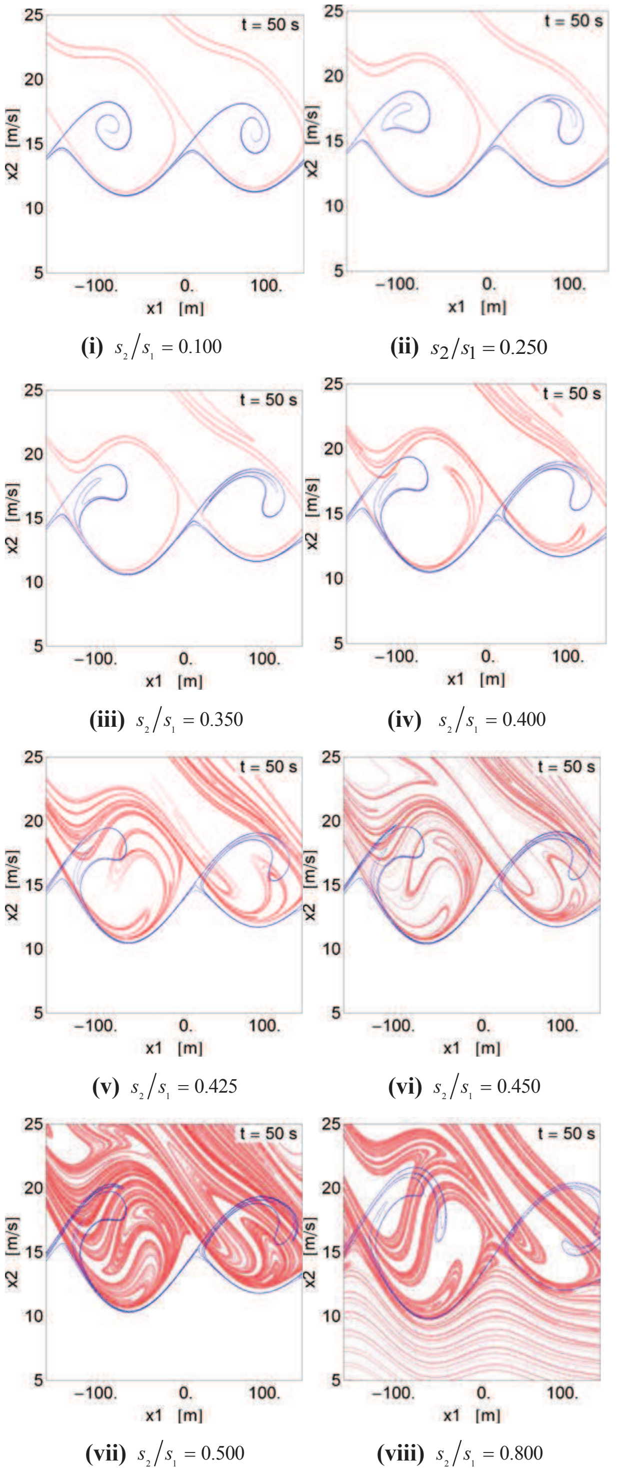


Figure 3: Transformation of the phase space as the steepness of the secondary wave is increased, due to tanglings of the attracting and the repelling LCS (blue and red curves respectively). The time shot is always at 50 s. Parameters have been assigned the following values: $(\lambda_1, s_1, \omega_2/\omega_1, u_{nom}) = (L, 0.02, 0.85, 12.5)$.

A strong hint about the arrangement of surf-riding and surging domains is offered from the graphs of Fig. 6, representing the field produced by the integration of phase-space-particles squared velocity along trajectories. The process of fractal destruction of the surf-riding domain is confirmed. Although the ship was very close to global surf-riding when the secondary wave excitation was established, this extra forcing did not lead to global surf-riding but to the fractal erosion of the surf-riding domain.

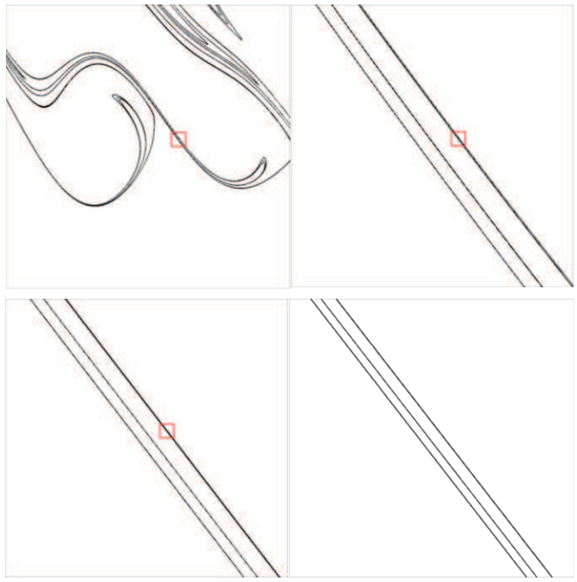


Figure 4: Self-similarity is revealed by successive enlargements of small rectangles placed on a surf-riding basin boundary [it corresponds to Fig. 3(iv)].

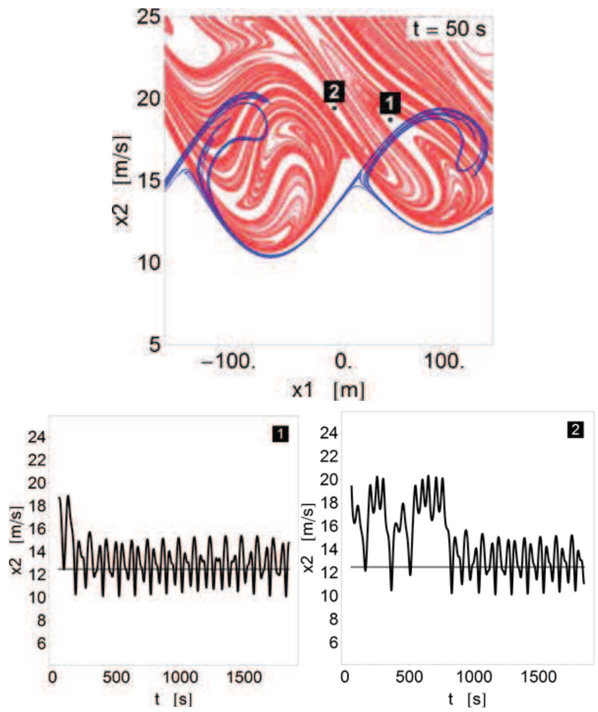


Figure 5: The erosion of surf-riding basins creates possibility of initiating surging from deep within the surf-riding area $(\lambda_1, s_1, \omega_2/\omega_1, s_2/s_1, u_{nom}) = (L, 0.02, 0.85, 0.500, 12.5)$.

Behaviour for "irregular" wave excitation

The time-changing LCS for wave excitation deriving from a JONSWAP spectrum, are shown in Fig. 7. We considered a frequency band with width $0.5\omega_p$, centred on spectrum's peak ω_p=0.598 rad/s.

The significant wave height was H_S=5.5 m. The spectrum was discretized through 48 components. Ship's nominal speed was 12 m/s. Substantial time variation of phase space flow can be noticed and, at first reading, the flow shows less coherence. In Fig. 7 is illustrated, in addition, the evolution of two groups of initial conditions (the green and the red) separated by a repelling LCS segment. Their initial placement is shown in the first of these graphs. The green points are found directed towards lower velocities (they should be identified as engaged in surging) compared to the red points that seem like being trapped at a higher velocity region. As a result, eventually, the green points lag behind the red points.

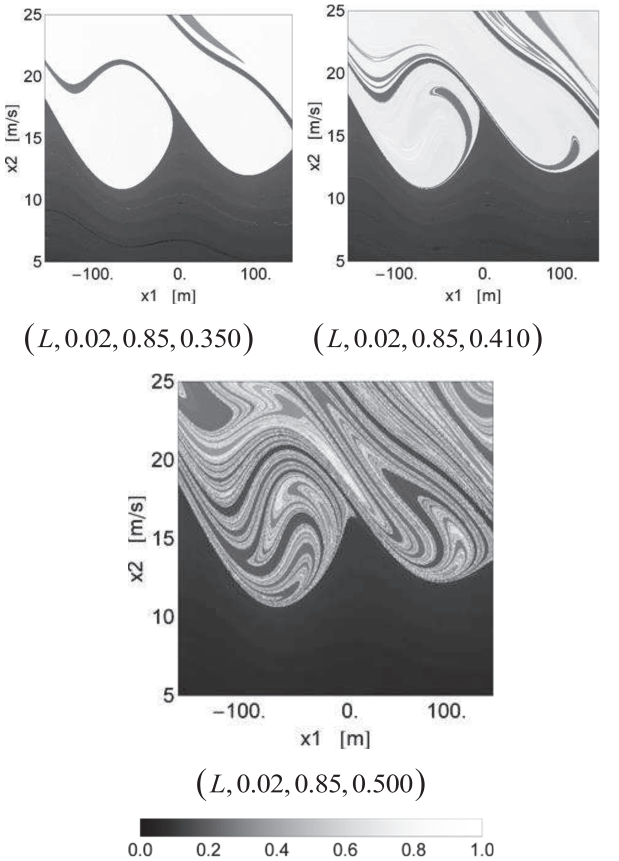


Figure 6: First row: Areas of surging (dark) and surf-riding (pale). Second row: Surging has dominated the entire phase space (pale regions indicate high-velocity transients not ending on surf-riding). The values of the parameters λ_1, s_1, ω_2/ω_1 and s_2/s_1 are indicated below the corresponding graph. Nominal speed is 12.5 m/s.

In the final investigation targeting the phase space, an irregular perturbation (calculated from a spectrum) was superimposed to a harmonic excitation, in such a way that, the wave energy content (based on the amplitudes of the participating discrete harmonics including the primary one) was maintained constant. The excitation was computed by applying a filter that had one of its parameters working as a control knob, gradually raising the amplitudes of the perturbation harmonics while lowering primary's (see Fig. 8). The spectrum (of JONSWAP type) had T_P=9.93 s and H_S=7 m. The number of participating harmonics was n=74 and ship's speed was set at 12 m/s.

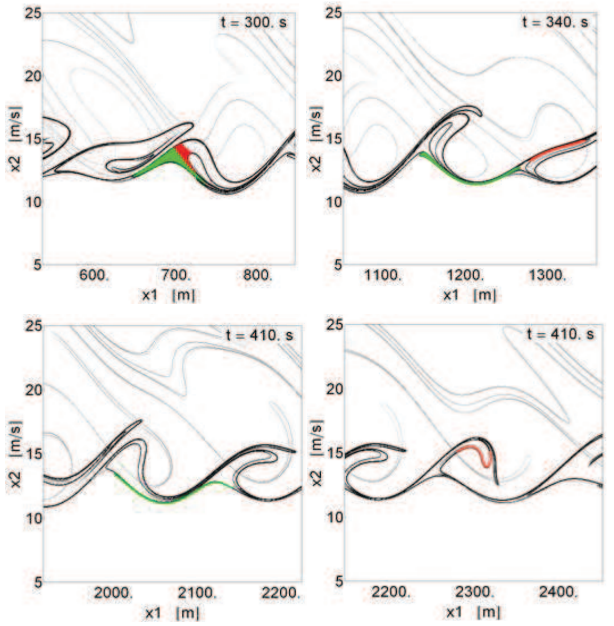


Figure 7: Portrait of phase space flow for JONSWAP spectrum. Two selected sets of initial conditions (appearing as green and red areas) evolve into different velocity ranges.

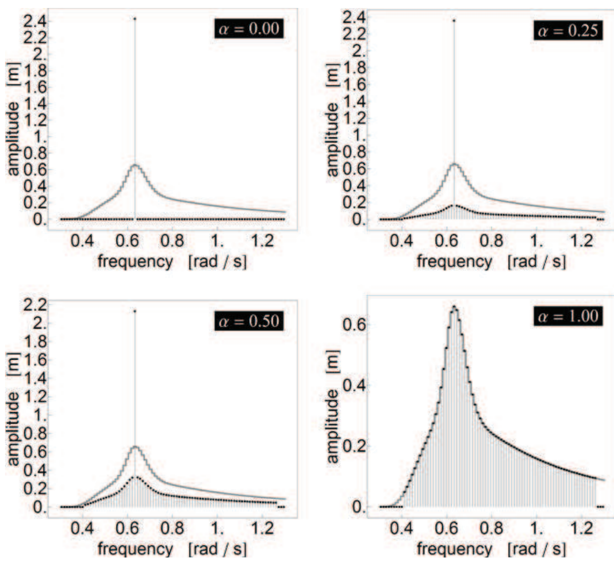
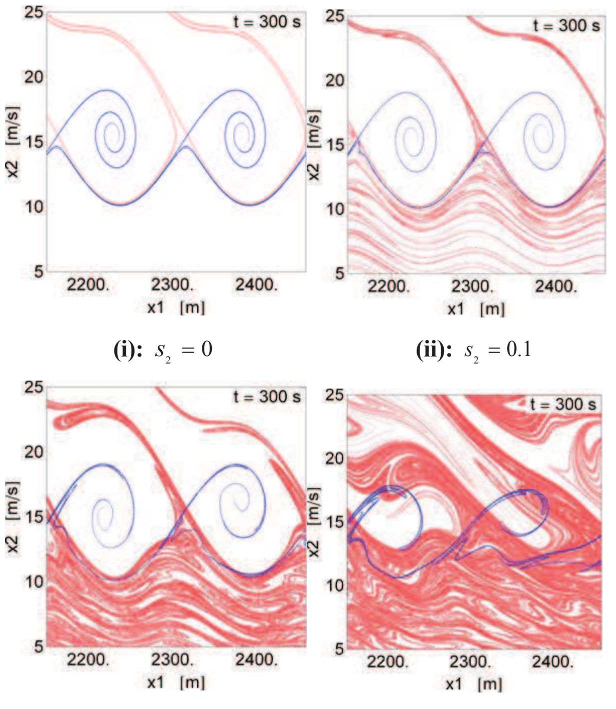


Figure 8: Wave amplitudes (black dots) obtained from a JONSWAP spectrum on the basis of energy equivalence, compared to the amplitude of the primary harmonic, as the control parameter *a* is gradually increased from 0 to 1. The intact spectrum (defining the energy level) is shown in grey.

In Fig. 9 are illustrated successive transformations of the phase space, which are provoked by the gradual turning of the excitation from mainly regular to mainly irregular. We have started, again, from a condition very close to the beginning of global surf-riding. Whilst, this time, global surf-riding did truly happen, it was followed by an erosion process of the surf-riding basins, provoked by LCS tanglings corrsponding to neighboring surf-riding basins.



(i): $s_2 = 0$ (ii): $s_2 = 0.1$

(iii): $s_2 = 0.5$ (iv): $s_2 = 1$

Figure 9: Transformations of phase space arrangement as one moves from a regular to an irregular excitation.

This is a new event where a surf-riding basin intrudes into another basin of the same kind. This makes uncertain the destination where the ship will settle, although surf-riding remains as the certain outcome.

It is evident therefore that, several new phenomena of behaviour become possible when one looks beyond the monochromatic sea; implying that, the probabilistic evaluation of a ship's tendency for surf-riding in irregular seas becomes an even more daring task.

4. CHARACTERIZATION OF HIGH SPEED RUNS

The final aspect considered was the characterization of the encountered types of surf-riding. Consider once more the idea of having a steep primary wave, perturbed by a secondary harmonic that is kept initially at a very low height. Naturally, one would expect to see a perturbed version of surf-riding, ruled by the celerity of the first wave. When the two wave components start having comparable magnitudes however, the outcome becomes difficult to predict. Three examples, corresponding to frequency ratios 0.8, 0.9 and 1.05, are shown, respectively, in Figs 10, 11 and 12. For frequency ratio 0.8, and as the steepness ratio is raised, the mean surge velocity falls initially perfectly on the celerity of the primary wave. Later however there is a jump to the celerity of the secondary wave, returning shortly to intermediate values (in-between the two celerities). Further increase of the steepness leads to domination of the celerity of the secondary wave. A look into the fluctuating surge velocity reveals period doublings and chaos. Some surf-riding oscillations are extremely large, driving the ship, in repeating short spells, to very high speed values. Similar patterns are noticed for the other two frequency ratios. It should be also noticed that the reference system is moving with the wave celerity c_1 of the prime wave ($\lambda = L$), thus the horizontal axis of the figures of the the mean surge velocity corresponds to c_1.

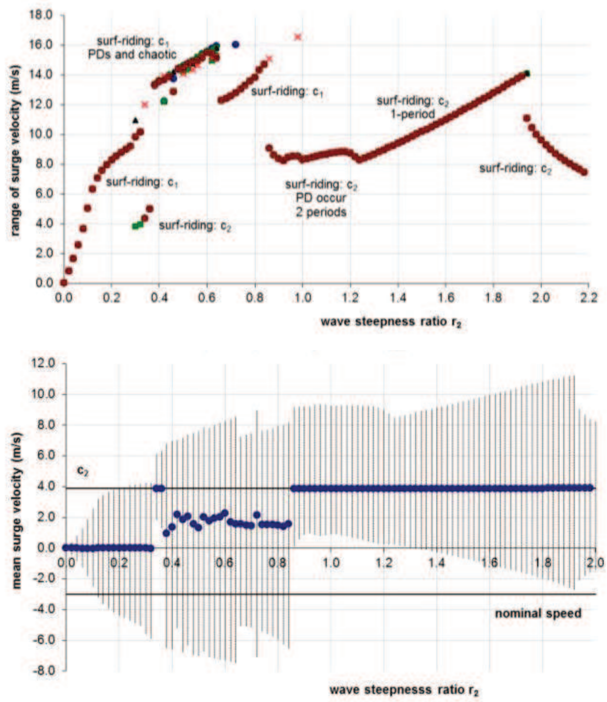


Figure 10: Range of surge velocity (upper) and mean value of surge velocity (down), for frequency ratio (of secondary to primary wave) 0.8, steepness of primary wave 1/30, nominal speed 12.5 m/s and initial surge velocity 10.5 m/s.

5. CONCLUSIONS

Several new phenomena of ship surge dynamics were observed when two or more frequencies were included in the excitation. In bi-chromatic waves, different types of oscillatory surf-riding exist, governed either by the first or by the second wave component. However, no coexistence of these two types was noticed as stable motions. Moreover, chaotic motions were identified in the intermediate range, sometimes extending to very high surge velocity values. They are preceded by homoclinic/heteroclinic tanglings of LCS found, creating fractalization of the surf-rising basin boundaries. Such phenomena were noticed in bichromatic as well as in multichromatic waves and seem to be quite common. In general, the exhibited dynamic behavior is very rich.

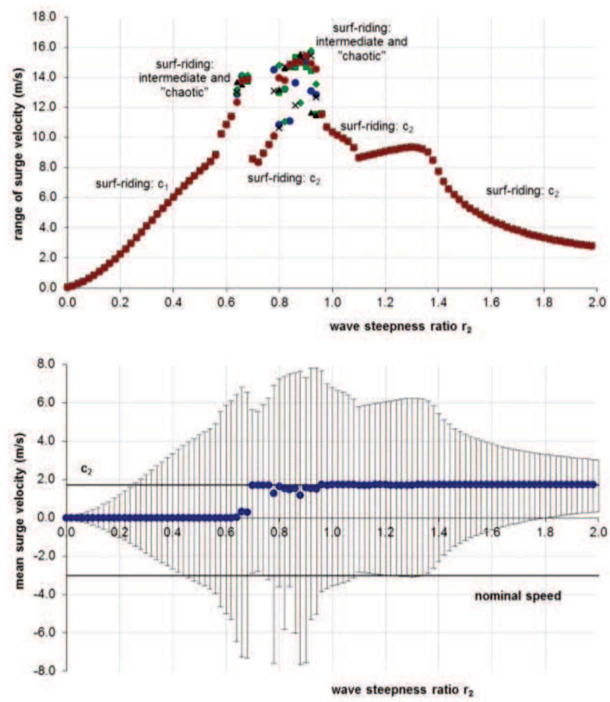


Figure 11: As Fig. 10, with frequency ratio 0.9.

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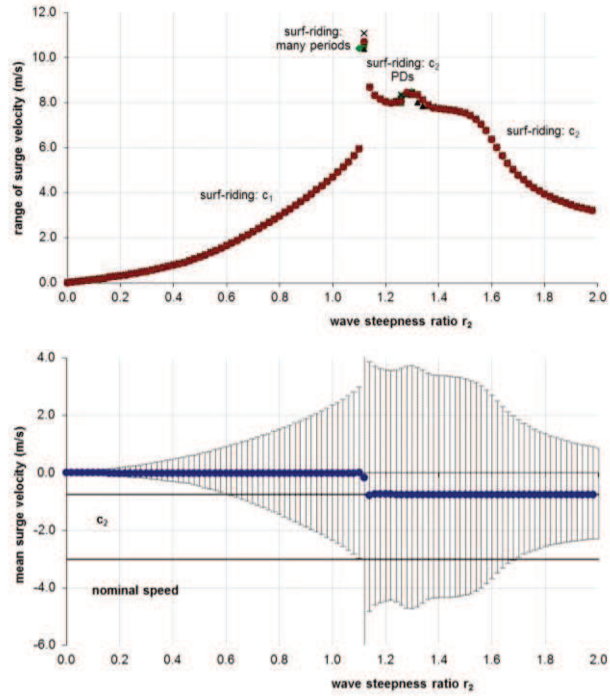

Figure 12: As Fig. 10, with frequency ratio 1.05.

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