Gaussian and Non Gaussian Response of Ship Rolling in Random Beam Waves

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ABSTRACT

Gaussian and Non Gaussian response of nonlinear ship rolling with softening spring in random beam waves is studied by moment equations. Stochastic rolling excitation is represented through a cascading filter driven by pure Gaussian white noise. The Gaussian and Non Gaussian Cumulant neglect method is applied to close the infinite hierarchy of moment equations. In this paper, an automatic neglect tool is developed to handle the complex and untraceable higher order cumulant neglect method and capture the Non Gaussian effect of the nonlinear rolling phenomena. With the automatic neglect tool, we can automatically generate moment equations for the state space format stochastic dynamical system with any number of dimensions at any closure level. Both transient and stationary responses of statistical moments are obtained through the solution of the closed moment equations.

KEYWORDS

Nonlinear Rolling; Random Waves; Filtered white noise; Moment Closure; Non Gaussian Cumulant Neglect Method

INTRODUCTION

Due to the randomness of realistic waves in the ocean, rolling motions are always considered as a stochastic process. For the marine and offshore industry, the nonlinear effect of loads and responses lead to lots of complicated phenomena, bifurcation, chaos, non-Gaussian statistics, etc. Basically, there are two different types of nonlinearity, i.e. loading nonlinearity, such as the 100 or 1000 year return period storm conditions; and system nonlinearity, the most typical case is the nonlinear ship rolling equation which can be decoupled from other modes.

The linearity of small amplitude response indicates that they are Gaussian or normal. All higher order statistics can be derived from second order moments. However, large amplitude rolling motion with nonlinear damping and stiffness needs more advanced method to analyze the higher order response statistical moments to capture the non-Gaussian effect.

The Markov assumption to describe the rolling process is the most popular procedure for random nonlinear dynamical analysis. Markov methods include the stochastic averaging method (Roberts 1982; Roberts and Vasta 2000), moment closure methods (Francescutto and Naito 2004) and also the direct solution of the Fokker-Planck-Kolmogorov equation(Naess and Moe 2000). Moment closure methods have been studied in many fields. In this method, the differential equations governing the response process are first determined. Itô differential rule is then applied to the governing equations to form the moment equations. If the system has nonlinear terms, the moment equations up to Nth order will include N+1, N+2 order or higher order moments, which is called the infinite hierarchy. Higher order moments have to be closed by some closure method, like the moment neglect, the Cumulant neglect, the Hermite moment closure(Ness, McHenry et al. 1989), etc. The cumulant neglect method which discards cumulants higher than a particular order N is adopted in this paper to close moment equations. If N equals 2, then the method is defined as Gaussian closure, if N greater than 2, it is non Gaussian closure. By setting the higher order cumulant to zero, the higher order moments can be expressed by the lower order moments to form the closed format equations. Response moments can be further used to generate the probability density function by Fourier transforming its characteristic function (Wojtkiewicz 2000), or maximum entropy (Sobczyk and Trcebicki 1999), etc.

For the ship rolling problems, most previous papers about moment equations only consider

Gaussian cumulant closure due to the difficulty in tracking the higher order closure see e.g. (Francescutto 1990; Francescutto and Naito 2004), especially for the higher order linear filter. Cumulant neglect becomes tedious and untraceable when increasing the neglect order. This motivates us to develop an automatic tool to address this difficulty(Wojtkiewicz, Spencer et al. 1996)to handle the higher dimensional state space stochastic model and higher order closure level. Ship rolling response with strongly nonlinear terms results in non-Gaussian effects. Higher order cumulant neglect will help to analysis higher order moment effect, like skewness and kurtosis, or even higher statistical moments. In this paper, we extend the neglect order to fourth order by developing an automatic neglect tool. Higher order moments response will benefit our understanding of the non Gaussian effect of nonlinear ship rolling and further understanding of capsizing.

MODELING OF ROLLING MOTION WITH FILTER APPLICATION

Modeling of Ship Rolling

Single degree freedom of rolling motion is always described as a second ordinary differential equation:

$$\begin{aligned}&(I_{44}+A_{44}(\omega))\ddot{\phi}+B_{44}(\omega)\dot{\phi}+B_{44q}(\omega)\dot{\phi}\left|\dot{\phi}\right|\\&+\Delta(C_1\phi+C_3\phi^3+..)=f(t)\end{aligned} \tag{1}$$

Where ϕ represents the rolling angle and $\dot{\phi}$ is the roll velocity. I_{44} and $A_{44}(\omega)$ represent the roll inertia and added inertia of vessel. $B_{44}(\omega)$ and $B_{44q}(\omega)$ are the linear and quadratic damping coefficient from hydrodynamic and viscous effect. Δ is the displacement of the ship, C_1 and C_2 are the linear and nonlinear restoring force coefficients. f(t)represents the random wave excitation. All hydrodynamics coefficient can be calculated using the strip theory hydrodynamics program, e.g. SHIPMO.BM (Beck and Troesch 1990). The nonlinear damping term, including the absolute value sign, will be changed to polynomial format by least square method, which is required by the moment equation, see e.g. (Dazell 1978)

State Space Formation of Rolling Motion

Rolling motion in random sea can be modeled in a state space form. Any excitation spectrum could be reproduced using some designed filter from Gaussian White noise. Fourth order linear differential equation could be designed as a very accurate filter. This filter is also viewed as cascade of two linear filters (Spanos 1983; Francescutto and Naito 2004).

$$\begin{aligned}&\ddddot{f}(t)+\lambda_3\dddot{f}(t)+\lambda_2\ddot{f}(t)+\lambda_1\dot{f}(t)+\lambda_0 f(t)\\&=\gamma_3\ddot{W}(t)\end{aligned} \tag{2}$$

The linear filter above could be applied to equation (1) to generate random wave moment with any spectrum.

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-2\mu x_2-\delta x_2^{\ 3}-\omega_0^2 x_1-\alpha_3 x_1^{\ 3}+\varepsilon x_3(t)\\ \dot{x}_3=x_4-\lambda_3 x_3\\ \dot{x}_4=x_5-\lambda_2 x_3+\gamma_3 W(t)\\ \dot{x}_5=x_6-\lambda_1 x_3\\ \dot{x}_6=-\lambda_0 x_3\end{cases} \tag{3}$$

Where $x_3=f$ and W(t) represents Gaussian white noise. With Markov assumption, equation (3) forms the Itô's differential equations

$$dX=F(X,t)dt+G(X,t)dB \tag{4}$$

$\frac{dB}{dt}=W(t)$ and $W(t)$ is white noise excitation. $B(t)$ is defined as Wiener process or Brownian motion. And $F(X,t)$ is defined as the drift coefficient, and $G(X,t)$ is the diffusion coefficient of dynamical system. And $G(X,t)$ is $n\times 1$ matrix, n is the dimension of equation (3); $G(X,t)$ is $n\times M$ matrix, here $n=6$, $M=1$.

CUMULANT NEGLECT CLOSURE METHOD

For N dimensional Itô differential equation, $N(N+1)(N+2)\ldots(N+M-1)/M!$ moment equations can be generated for M-th order moment. 6 1st order, 21 2nd order, 56 3rd order and 126 4th moment equations can be generated from equation (3). Gaussian cumulant neglect method means the algorithm will analyze orders of moment up to two, all higher order moments will be estimated by second and lower order moments. Itô's differential rule could be used to generate moment equations governing the statistical response of stochastic dynamical system. Let's set φ be a scalar valued real function and $\varphi = x_1^{k_1} x_2^{k_2} \cdots x_n^{k_n}$, where $X = [x_1, x_2, \cdots x_n]^T$ is the state space vector in equation (3). $E(\varphi)$ represents expectation of the combination φ. Following the Itô's differential rule (Itô 1951), moments equation for φ is formed in equation (5).

$$\begin{aligned} \frac{\partial E(\varphi)}{\partial t} &= E\left(\sum_{i=1}^{N} F^{(i)} \frac{\partial \varphi}{\partial x_i}\right) \\ &+ \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} E\left((GQG^T)_{ij} \frac{\partial^2 \varphi}{\partial x_i \partial x_j}\right) \end{aligned} \tag{5}$$

Q is the intensity of white noise. It is clear that the moment's equations will not be closed due to the nonlinear term of the dynamical system. Cumulant neglect methods will be adopted. Cumulant higher than some particular order N will be neglected and all higher order moments can be expressed by lower order moment. Moment equations will form closed equations after application of closure of the Cumulant. Cumulant neglect, or Cumulant discard method was first applied to turbulence theory(Beran 1965). More details can be found in the books of Lin (Lin and Cai 2004), Lutes (Lutes and Sarkani 2004) and Ibrahim (Ibrahim 2007).

Cumulant Neglect Method with Automatic Neglect Tool

In the cumulant closure technique, the response cumulants higher than some closure level are assumed smaller in comparison to those cumulants below closure order and then can be neglected. To establish the relation between cumulants and ordinary moments, we consider the random vector $X^T = [x_1, x_1 \ldots\ldots x_n]$ with characteristic function(Soong and Grigoriu 1993) as follows.

$$\phi(u_1, \cdots, u_n) = E\{\exp[j \sum_{i=1}^{n} u_i x_i]\} \tag{6}$$

Where $j = \sqrt{-1}$, n is the number of state space in the diffusion process, e.g. in equation (3), n=6. It can be expanded by Taylor expansion in terms of ordinary moments $E\{x_i x_j \cdots\}$ or cumulants $\kappa\{x_i, x_j \cdots\}$. The expansion has two forms as showed in equation (7) and (8). Here 'C' in equation (7) and (8) is the closure level and is a number. ϕ_1 and ϕ_2 are same function with different expressions. Moments and cumulants can thus be found through characteristic functions through equation (9) and (10).

$$\begin{aligned} \phi_1(u_1, \cdots, u_n) = \exp\Big\{ & j \sum_{i=1}^{n} u_i \kappa(x_i) + \frac{j^2}{2!} \sum_{i=1}^{n} \sum_{j=1}^{n} u_i u_j \\ & \kappa(x_i, x_j) + \cdots + \frac{j^C}{C!} \sum_{i=1}^{n} \cdots \\ & \sum_{C=1}^{n} u_i \cdots u_C \kappa(x_i, \cdots, x_C) + \cdots \Big\} \end{aligned} \tag{7}$$

$$\begin{aligned} \phi_2(u_1, \cdots, u_n) = \exp\Big(& 1 + j \sum_{i=1}^{n} u_i E(x_i) + \frac{j^2}{2!} \sum_{i=1}^{n} \sum_{j=1}^{n} u_i u_j \\ & E(x_i x_j) + \cdots + \frac{j^C}{C!} \sum_{i=1}^{n} \cdots \sum_{C=1}^{n} u_i u_j \cdots \\ & u_C E(x_i x_j \cdots x_C) + \cdots \Big\} \end{aligned} \tag{8}$$

$$E(x_1^{k_1} \cdots x_n^{k_n}) = \frac{1}{j^k} \frac{\partial^k \phi_1}{\partial u_1^{k_1} \cdots \partial u_n^{k_n}} \Bigg|_{u_1 = \cdots u_n = 0} \tag{9}$$

$$\kappa(x_1^{k_1}, \cdots, x_n^{k_n}) = \frac{1}{j^k} \frac{\partial^k \ln \phi_2}{\partial u_1^{k_1} \cdots \partial u_n^{k_n}} \Bigg|_{u_1 = \cdots u_n = 0} \tag{10}$$

where $k = k_1 + k_2 + \cdots k_n$, cumulants of order higher than closure order can be set zero and moments higher than closure order will be written in terms of moments with order less or equal to the closure level. Non Gaussian cumulant neglect method up to 4th order will involve 209 equations, which is extremely large, complex and impossible to find solution by hand. The automatic neglect procedure of algorithm is described in Fig. 1. Steps 1 to 7 in Fig. 1 generate all closed moments equations by MAPLE (MAPLE 2009) and Step 8 takes the efficiency and convenience of MATLAB ordinary differential equation solvers. In Fig. 2 and Fig 3, root mean square of rolling displacement and velocity by Gaussian and Non Gaussian closure are compared. Gaussian neglect (2nd order cumulant neglect) seems underestimate the response moments. Kurtosis could be verified larger than 3,which shows non Gaussian effect of rolling motion response and soft spring effect (Winterstein 1988).

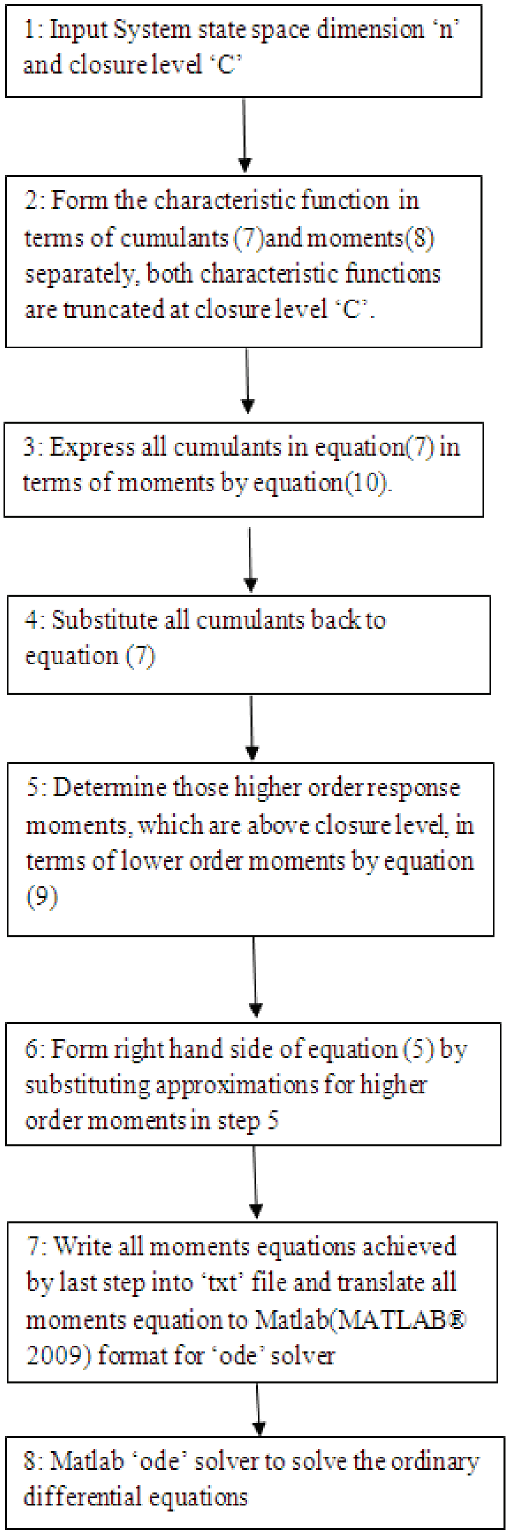

Fig. 1 Automatic Cumulant Neglect Tool

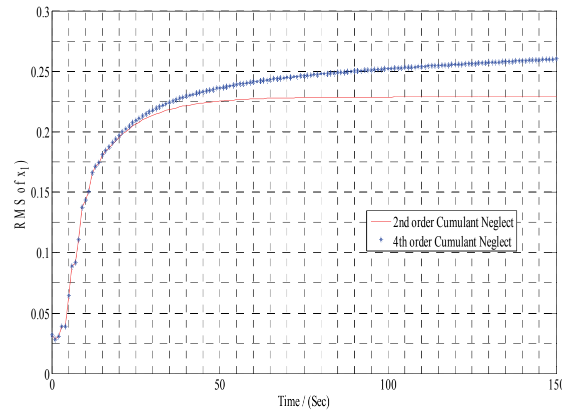

Fig. 2 Evolution of the Root Mean Square of the Rolling Displacement

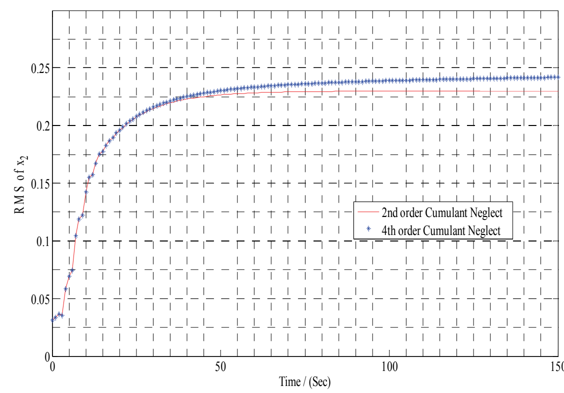

Fig. 3 Evolution of the Root Mean Square of the Rolling Velocity

CONLUSIONS

We successfully solved the higher order moment equation by the cumulants neglect method with an automatic neglect tool. This algorithm will greatly help us to analyze higher dimensional stochastic dynamical system at higher closure levels. Although not investigated here, system with parametric stochastic excitation (multiplicative noise) can be analyzed with the same algorithm as well as external stochastic excitation (additive noise) without any loss of generality.

Moments information will be used to generate joint probability density function (JPDF) of the rolling response approximately. The capsizing probability of ship in realistic seas could be predicted by computing probability outside of the safe basin, which is defined by the heteroclinic separatrix.

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