PRELIMINARY RESULTS OF EXPERIMENTAL VALIDATION OF PRACTICAL NON-ERGODICITY OF LARGE AMPLITUDE ROLLING MOTION

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SUMMARY

Practical non-ergodicity (correct math term is cyclic non-stationary quality) means that ergodic hypothesis cannot be used for any practical calculation of nonlinear irregular rolling. In other words, all the probabilistic characteristics have to be averaged over representative ensemble of realization and any result based on one realization is not correct. This was first stated on the base of numerical simulations. It was believed that fold bifurcation is responsible for the effect. It was not clear if contributions from other factors (like water on deck, impacts of breaking waves, influence of others degrees of freedom, etc.) might decrease this effect, so ergodic assumption might be still acceptable for practical purposes.

The model experiment was carried out in the towing tank of National Research Institute of Fisheries Engineering in Japan. There were two series of tests with two models. The first series of tests was done with free drifting model of Japanese purse seiner. Such test yields about 10 minutes realization. Since absence of ergodic qualities can be checked only on significant amount of time, the second series of tests was conducted with restrained model that was not able to drift. The second series produced 30 and 40 minutes realizations of model time that is close to quasi-stationary range of full-scale waves. It was meant that the first series can be used for validation of the second one, in other words, to estimate how significant these restraints are in statistical sense.

The results have shown significant difference in variance estimate on different wave realization that cannot be explained only by statistical errors that may constitute absence of ergodic qualities in practical sense. All the motion recorded were far enough from fold bifurcation region, and at the same time intensive deck flooding with episodic breaking wave hit were observed.

To make sure that GZ non-linearity is not the only reason for ergodicity the second model was tested. It was a rectangular pontoon with almost linear GZ curve.

The paper also rises some methodological issues concerning severe irregular rolling like estimation of "degree of non-ergodicity"

1. INTRODUCTION

Ship response in irregular seas is usually considered as a general stochastic process. This means that we use math abstraction to describe the real world phenomenon. As an abstraction, it is supposed that a stochastic process is infinite in time and has an infinite number of realizations. So, if we fix the time (or, in other words make a time section), we have a usual random number that has an infinite number of values and its own average, variance, distribution and other probabilistic characteristics. These characteristics might be different in different moments of time, however, there are group of stochastic processes, for which the probabilistic characteristic does not depend on time. Every moment brings exactly the same values for the probabilistic characteristics. These processes are considered to be stationary.

Strictly speaking both waves and ship response are not stationary, because waves are changing due to weather and ship response also depends on speed and course, which also are not constant. So, in order to simplify the problem, we consider a period of time when weather, speed and course could be considered as constants; this duration is usually called "period of quasi-stationary". It usually assumed that ship response time does not exceed the period of quasi-stationary.

Averaging the current value of one realization of stationary process, we got so-called time-average estimates for probabilistic characteristics. This way, each realization might have its own mean value, variance, distribution and other estimates. The true estimates for the whole stochastic process are averages of corresponding ones of the realizations.

This means that we can estimate probabilistic characteristics in two ways: using time section or by averaging estimates for each individual realization. Some processes however show identical (in statistical sense) estimates for individual realizations. This makes a problem much simpler – we need just one, long enough realization, to produce estimates. This kind of processes are called "ergodic", and the corresponding quality of such stochastic process – "ergodicity". Usually, sea waves are assumed to be ergodic.

There is a proof that a linear dynamical system, being excited by stationary ergodic process produces also stationary ergodic response. However, nothing like this is provided for nonlinear systems. It creates a question on ergodic qualities of nonlinear rolling and other ship motions that are essential for estimation of capsizing probability. (So far, we assume nonlinear rolling to be a stationary process at least, which is also questionable from point of view of pure math.)

2. ERGODICITY CHECK BY SIMULATION

Here we give a brief review of the previous results, mainly based by [1-2].

2.1 WAVES

Stochastic elevation of sea wave is usually expressed as

$$\zeta_w(t) = \sum_{i=0}^{N} a_i \sin(\omega_i t + \varphi_{0i}) \qquad (1)$$

Here a_i are amplitudes of components that are defined from spectrum, ω_i is a given set of frequencies and φ_{0i} is a set of random phase numbers distributed uniformly from 0 to 2π. The last figure is responsible for the generation of new realization. Every time the set of phases are calculated and substituted into one, the new realization of the wave process is created. All these realizations, however, would still produce the same spectrum, since amplitudes were not changed.

2.2 RESPONCES

2.2 (a) Linear System

The different realizations of the waves now have to be used for ship rolling simulation. One of the ways to check ergodicity is to calculate an estimate, for example, of the ship roll variance V for consecutive moments of time t_1, t_2, ... t_n: in a form of a function $V(t)$. The procedure has to be repeated for all available realizations, so we have a set of functions $\{V(t)\}_{j=0..k}$

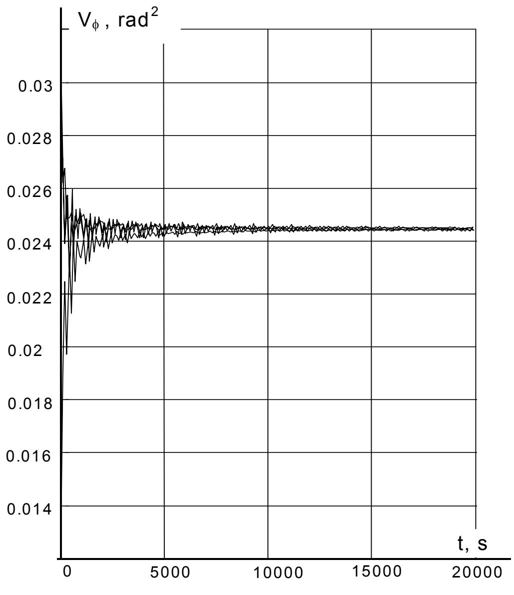

Figure 1. Ergodicity check of linear rolling

If the response is ergodic, all these curves must have one clear limit, as it shown on figure 1, where response of the linear system is shown.
The results on the figure 1 (taken from [1]) was obtained by simulation, so all numerical errors are included, nevertheless there is a clear limit reached in 20,000 seconds –16 hrs. 40 min., that is close to quasi-periodic period.

2.2 (b) Nonlinear System

Now let's check the same rolling-only system, but nonlinear restoring is introduced. The result changes dramatically, see figure 2 (taken from [1])

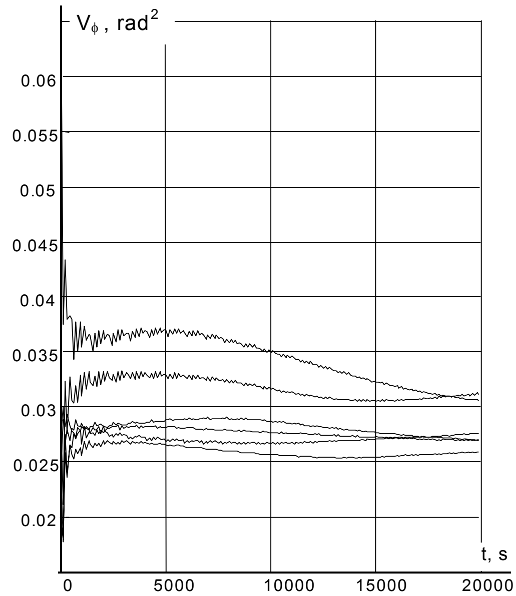

Figure 2 Ergodicity check of rolling with nonlinear restoring

As it is clear from figure 2, the limit is not reached for 16 plus hours, moreover, the shape of the curve might even question the stationary assumption...
A. Degtyarev and A. Boukhanovsky [1] checked this effect, using completely different model of waves and rolling, but finally came to the same conclusion.

3. MODEL TEST SET-UP

The purpose of the model test is to check if the effect of significant absence of ergodicity could be obtained in the conditions of model experiment.

3.1 NRIFE TOWING TANK

The model test was carried out in the towing tank of National Research Institute of Fisheries Engineering in Japan. The tanks dimensions are 137 x 6 x 3 m, equipped with rolling plate type wavemaker, wave absorption beach and self-propelled carriage. The schematic of the wave-maker is shown at figure 3.

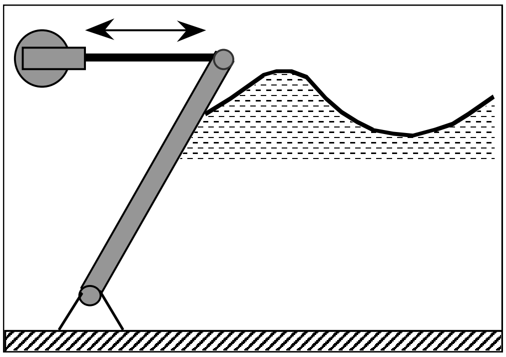

Figure 3: Schematic of the wavemaker

Control system of the wavemaker is capable of reproducing the given electric signal. The control signal elevations was calculated by formula (1) and then transformed to analog form using TEAC DR-F2M Digital Recorder.

3.2 MODELS

Two models were tested: Japanese purse seiner GT-80 (scale 12.6) and box shaped model. The box shaped model had depth larger than breadth, so GZ curve was almost linear. This allowed checking the hypothesis [1] that the absence of ergodicity was caused by rare jumps to higher amplitudes.
The model of GT-80 was tested for two different draughts, in order to check influence of nonlinear damping caused by deck entering water. Bulwark of the model was removed to minimize effect of the green water. Numerical characteristics of the models are given in tables 1 and 2 correspondingly.

Table 1. Characteristics of Purse Seiner GT-80

Characteristics		Full scale	Model
Length O. A., m		36.5	2.900
Length B. P., m		29.0	2.300
Breadth, m		6.80	0.540
Depth, m		2.60	0.206
Draught 1	Draught, m	2.19	0.174
	Displacement, ton	261	0.130
	KG, m	2.38	0.189
	KM, m	3.84	0.304
	GM, m	1.46	0.116
	CB	0.603	0.603
Draught 2	Draught, m	1.74	0.137
	Displacement, ton	180	0.090
	KG, m	2.52	0.200
	KM, m	3.86	0.306
	GM, m	1.33	0.106
	CB	0.528	0.528

The GZ curve for the box –shaped model is shown in figure 4. Both models were equipped with high-precision gyroscope for measuring roll angles. Measurements were recorded by TEAC digital recorder, decoded on a PC and stored in form of ASCII files.

Table 2 Characteristics of box shaped model

Characteristics	Value
Length B. P., m	1.500
Breadth, m	0.300
Depth, m	0.400
Draught	0.246
Displacement, ton	0.111
KG, m	0.087
KM, m	0.305
GM, m	0.0666

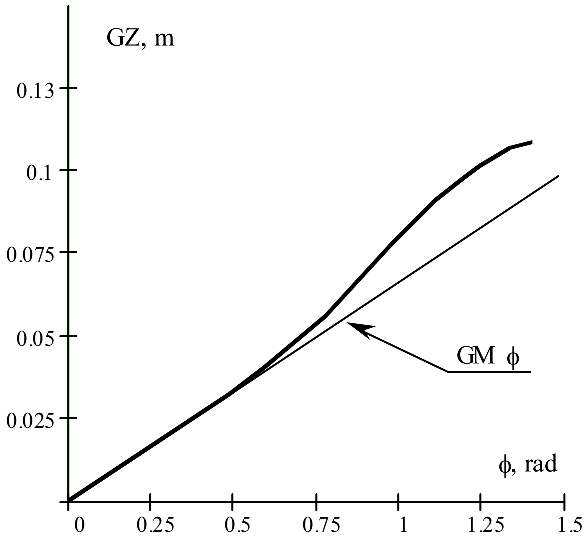

Figure 4: GZ curve of box shaped model

Wave heights were measured in close proximity of the model by string wavemeter. Output signal was amplified and then recorded by TEAC digital recorder, decoded on a PC and stored in a form of ASCII file. The model test layout is shown in figure 5. The model was free to sway, heave and roll, but was restricted in surging, pitching and yawing.

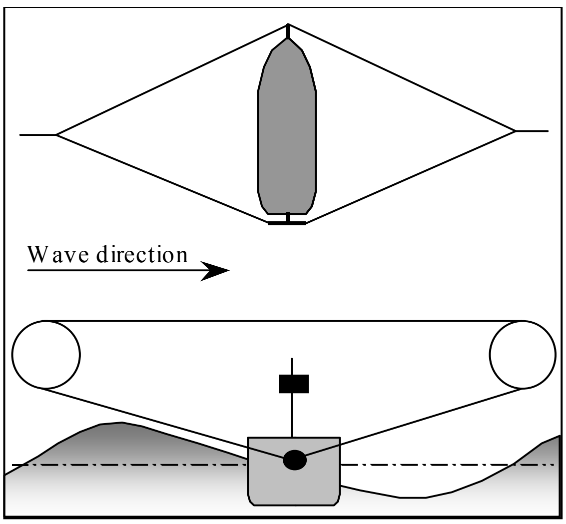

Figure 5: Model test layout

4. TEST PROGRAM

4.1 WAVE GENERATION

Wave spectrum density was calculated using the following formula:

$$s(\omega) = 9.43 \cdot \frac{V_w}{\omega_{wm}} \cdot \left(\frac{\omega_{max}}{\omega}\right)^6 \cdot \exp\left[-1.5 \cdot \left(\frac{\omega_{max}}{\omega}\right)^4\right] \quad (2)$$

Here, ω_{max} is cyclic frequency of spectrum's maximum and ω_{wm} is mean cyclic frequency. These two values are assumed to be related as:

$$\omega_{max} = 0.77\omega_{wm} \quad (3)$$

The entire model test was done for one value of the variance 22.375 cm^2 and frequency of spectrum's maximum 0.7 Hz.
The spectral density of control signal was obtained by multiplying expression (2) by transfer function of the wavemaker that was obtained during special calibration experiment. Then, the controlling signal was calculated by formula (1).
Different realizations were obtained using different set of random initial phases in formula (1). There were total 9 realizations, 30 minutes long each (1 hour 48 min of full-scale time) and 4 realizations, 40 minutes long each (2 hours 22 min full-scale time). The spectral density of one of them (both calculated by formula (2) and actually measured in the tank) is shown on figure 6.

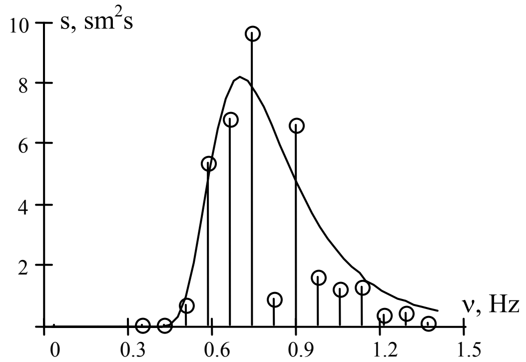

Figure 6. Input and measured wave spectral density

4.2 TEST PROCEDURE

As it was mentioned above there were two models: Japanese purse seiner GT-80 (two loading conditions) and box shaped model (one loading condition), which makes three series of test runs. Each series included three types of tests:

- Free rolling motion;
- Free drift test;
- Stop test.

The model was able to drift freely under action of incident waves during free drift test, however, the length of the tank limited the time of realization that might be recorded. This time (about 10 minutes – model time) is not enough to estimate probabilistic characteristics with accuracy, that would be enough for judgement on ergodicity. So we had to restrain drift of the model, which made it "stop test" and use free drift test results to check an influence of drift restraining. This analysis in not included in this paper.

5. PRELIMINARY ERGODICITY ANLYSIS

5.1 CUMULATIVE VARIANCE

We followed the procedure that was applied for simulation results in [1]. The "cumulative" variances time histories are shown on figure 7 for model GT-80, draught 1, figure 8 for model GT-80 draught 2 and on figure 9 for box shaped model. Four longer curves correspond to 40 minutes realization, nine shorter ones – to 30 minutes realizations.

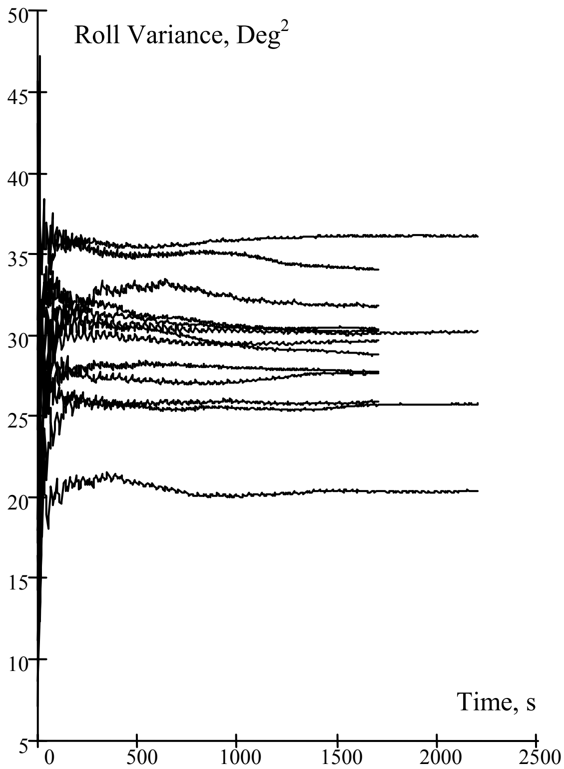

Figure 7. Cumulative variance for GT-80, draught 1

As it is quite clearly seen from figures 7, 8 and 9, all the responses cannot be considered ergodic, at least during testing time. Moreover, majority of the curves tends to almost horizontal asymptotes. This gives a background to a hypothesis, that the process is stationary and does not posses ergodic qualities during quasi-stationary period as well. However it cannot be considered as experimental evidence, yet. The waves in the tank might not be ergodic either, since the wavemaker might be nonlinear dynamic system as well and absence of ergodicity might be a simple reaction on non-ergodic excitation.

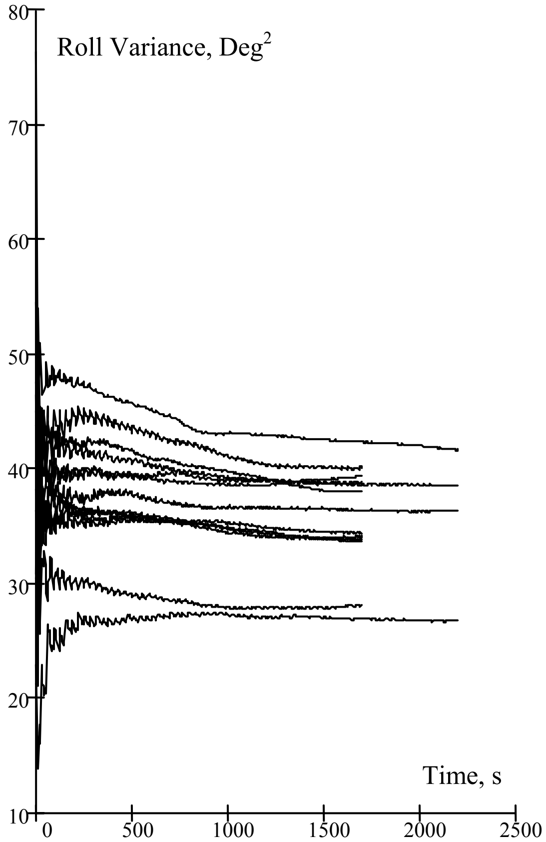

Figure 8. Cumulative variance for GT-80, draught 2

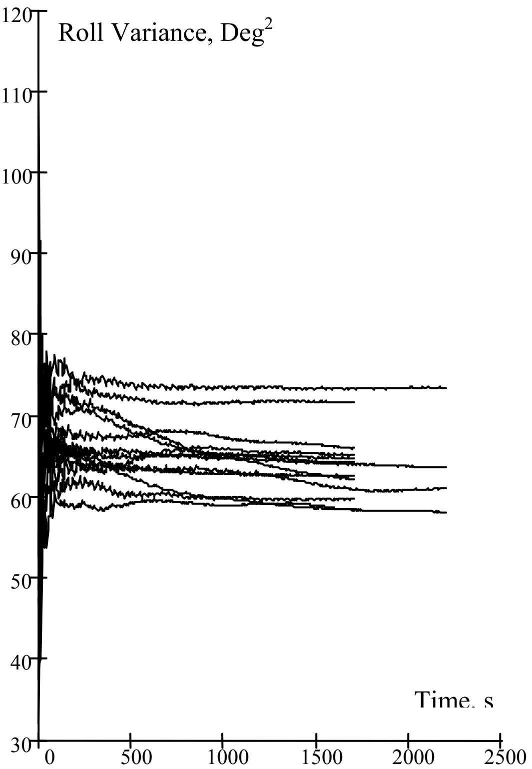

Figure 9. Cumulative variance box shaped model

5.3 ERGODIC QUALITIES OF WAVES

Basically, it is well known, that the waves in the model basin are not a perfect model of the sea waves: physics of wave generation is different. For example, spectrum might be dependent on where the measurements were done exactly in the basin. This error is considered acceptable for vast majority of the model test in irregular waves.

However, absence or presence of ergodicity is critical for this test, if the waves are ergodic, results at figures 7-9 would be enough to reject the hypothesis of ergodicity for tested cases and at least question such a hypothesis in general, when the nonlinear effect is important.

The figure 10 shows the cumulative variances of waves in the towing tank recorded along with the stop test. The wave transducer was located far enough before the model to exclude any influence of wave generated by the model.

As it clearly seen from figure 10, unfortunately, the waves also cannot be considered ergodic.

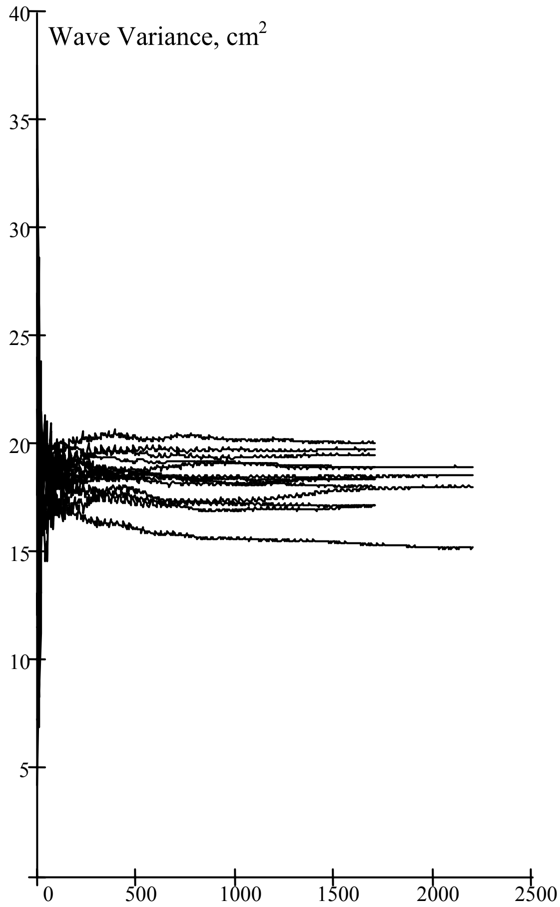

Figure 10. Cumulative variance of waves

So, to conclude anything on applicability of ergodicity hypothesis for nonlinear rolling we have to develop a criterion for ergodicity of the dynamical system exposed to non-ergodic excitation and somehow separate inherent and input non-ergodicity.

5.4 CRITERIA OF ERGODICITY

Derivation and testing of ergodicity criterion that possesses the above capability deserves separate paper. Here we give it brief consideration without any strict definitions and proofs from math point of view, rather being based on common sense. (Which does not always works but still good enough for preliminary analysis.) Following A. Degtyarev and A. Boukhanovsky [1], we take confidence interval as the main tool for the study.

Any estimate of variance or mean value obtained from finite set of statistics is a random value. Further we will be working with estimate of variance only, however the same method can be applied to any estimate of probabilistic characteristic. The confidence interval is a range that can be calculated for the estimate and contain true value of characteristic with certain given confidence probability β. Here we use β=0.9973.

For the purpose of preliminary analysis, we assume that both distributions of roll process and variance estimate are Gaussian. Then the confidence interval half-width can be calculated as [3]:

$$\Delta V = P_{inv}(\beta, m[V], V[V]) \qquad (4)$$

Here: P_{inv} inverse Gaussian cumulative probability

$m[V]$ – mean value of the variance estimate

$V[V]$ – variance of the variance estimate

Mean value of the variance estimate is the estimate itself if it is calculated with corrected formula:

$$m[V] = \tilde{V} = \frac{1}{N-1}\sum_{i=0}^{N}\left(\phi_i - m[\phi]\right)^2 \qquad (5)$$

Variance of variance estimate can be calculated via 4th moment of distribution, but since we assume the Gaussian distribution, this figure could be expressed as:

$$V[V] = \frac{2}{N-1}\cdot\tilde{V}^2 \qquad (6)$$

Finally, the variance estimate with confidence interval can be written as:

$$V = \tilde{V} \pm \Delta V\,, \qquad (7)$$

and:

$$P\left\{V \in \left[\tilde{V} - \Delta V, \tilde{V} + \Delta V\right]\right\} = \beta \qquad (8)$$

Here V is a true value of the variance and $\tilde{V}$ is its estimate. Again, formulae (4-8) give confidence interval in assumption that the only reason for the difference between estimate and true value is finite volume of statistics. Since we have clear difference between variances estimated by different realizations, we average the estimate over all realizations. Confidence interval also has to be calculated for the estimate averaged over the all available realizations. Also we used the same number of points for each realization to give them equal statistical weight. Results are summarized in the table 3 for rolling and table 4 for waves. Estimates of wave elevation variance were expected to be almost the same for the same realizations, but they are not. This represents influence of model waves, reflection and other factors related to wavemaker work. Repeatability of waves is also important topic, but it is out of scope of preliminary analysis, presented here and will be considered in future papers.

Table 3 Roll Variance Estimates

Variance estimate Deg^2	GT-80 draught 1	GT-80 draught 2	Box shaped model
Realization 1	29.61	38.75	65.07
Realization 2	30.36	37.86	64.14
Realization 3	28.72	33.99	61.99
Realization 4	30.19	39.26	58.17
Realization 5	25.86	27.97	59.59
Realization 6	31.74	40.00	62.35
Realization 7	27.57	33.6	64.64
Realization 8	27.68	34.24	71.5
Realization 9	33.95	33.72	65.85
Realization 10	25.64	36.26	63.91
Realization 11	30.04	38.54	73.25
Realization 12	36.06	42.23	58.29
Realization 13	20.35	26.86	60.71
Average	29.06	35.64	63.81
Variance of Estimate	$7.642\ 10^{-4}$	$1.15\ 10^{-3}$	$3.68\ 10^{-3}$
Confidence Interval	0.154	0.189	0.338

Table 4 Wave Elevation Variance Estimates

Variance estimate cm^2	GT-80 draught 1	GT-80 draught 2	Box shaped model
Realization 1	19.45	20.76	18.24
Realization 2	18.01	19.32	21.36
Realization 3	17.10	17.73	22.35
Realization 4	19.74	19.18	20.81
Realization 5	17.14	16.32	17.58
Realization 6	18.86	18.19	17.47
Realization 7	18.35	17.7	17.19
Realization 8	18.35	17.74	19.97
Realization 9	20.01	19.01	17.91
Realization 10	15.30	15.83	19.45
Realization 11	18.42	18.12	20.92
Realization 12	18.89	19.89	21.28
Realization 13	17.92	17.27	18.59
Average	18.27	18.24	19.47
Variance of Estimate	$3.02\ 10^{-4}$	$3.01\ 10^{-4}$	$3.43\ 10^{-4}$
Confidence Interval	0.0967	0.0965	0.103

It is quite evident from the both tables that were already visually clear from figures 7-10: vast majority of realization estimates does not belong to confidence interval. It is also illustrated on figure 11 and 12 for wave and roll processes correspondingly. Points that present realization variance estimates (with respective confidence interval) are spread far outside of the confidence interval of the estimate averaged over the whole ensemble. (Only results for GT-80 draught 2 are shown).

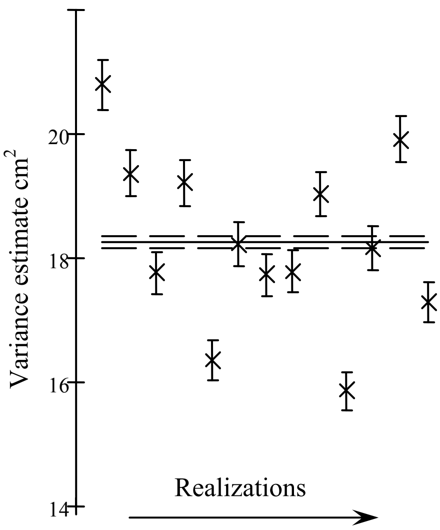

Figure 11 Wave realization estimates vs. wave ensemble estimate (dashed line shows confidence interval for ensemble estimate)

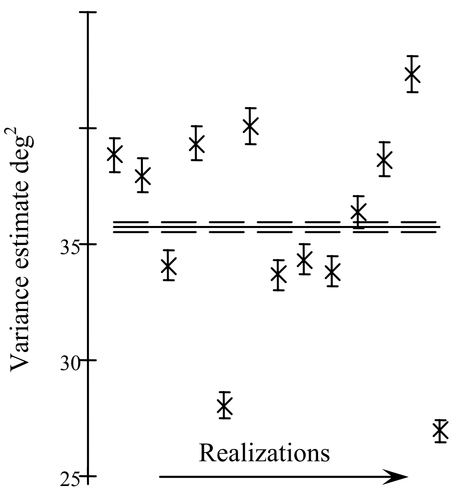

Figure 12 Roll realization estimates vs. roll ensemble estimate (dashed line shows confidence interval for ensemble estimate)

So, the deviation could not be explained just by finite number of statistics. Now let's consider non-ergodicity of the studied processes. For the purpose of preliminary analysis, we treat realization estimates as independent realization of a random number and use "direct" calculation of the variance of variance.

$$V_{NE}[V] = \frac{1}{K-1}\sum_{j=1}^{K}(\widetilde{V}_j - m_{NE}[V])^2 \quad (9)$$

Here m_{NE} is mean value estimate of realization variances calculated with the same assumptions:

$$m_{NE}[V] = \frac{1}{K}\sum_{j=1}^{K}\widetilde{V}_j \quad (10)$$

Again the summation are to be made over realizations, so K is number of recorded ones and equals 13. Now we can use formula (4) for calculation of confidence interval without ergodic assumption:

$$\Delta V_{NE} = P_{inv}(\beta, m_{NE}[V], V_{NE}[V]) \quad (11)$$

Now, criterion on non-ergodicity can be proposed in the following form:

$$E = \frac{\Delta V_{NE}}{\Delta V} \quad (12)$$

If the process is ergodic this criterion will tend to unity, since there no difference how to calculate estimates for ergodic process: by realization or by ensemble. Results of calculation are summarized in the table 5

Table 5 Calculation of ergodic criteria

Value	GT-80 draught 1	GT-80 draught 2	Box shaped model
Ensemble averaged wave variance (from table 4), cm^2	18.27	18.24	19.47
Non-ergodic wave variance of estimate, cm^4	1.58	1.914	3.07
Confidence interval for waves, cm^2	7.00	7.70	9.74
Ergodic criterion for waves	72.40	79.75	94.53
Ensemble averaged roll variance (from table 3), deg^2	29.06	35.64	63.81
Non-ergodic roll variance of estimate, deg^4	15.53	20.63	20.84
Confidence interval for roll, deg^2	21.93	25.27	25.40
Ergodic criterion for roll	142.55	133.97	75.21

Values of the proposed criteria indicate absence of ergodicity, what actually has been seen from figures 7-12. However, the proposed criterion estimates degree of non-ergodicity, in other words, how far an ergodic assumption would be from the reality.

Relative values of the criterion for wave and roll bear an important information. As we can see from the table 5, the criterion values for rolling of GT-80 is about twice as large in comparison of the same for waves. This might be interpreted as the dynamic system "adds" it own non-ergodicity (caused by non-linearity) to already non-ergodic input process.

Such interpretation was confirmed by numerical simulation. As we stated above, the scope of this paper does not provide an opportunity to give detailed description of the criterion testing. Four cases were simulated: ergodic input with linear system, non-ergodic input with linear system, ergodic input with nonlinear system and non-ergodic input with nonlinear system. It was found that linear system (or inherently ergodic system) does not increase the criterion. Contrary, if the system possesses significant non-linearities, the criterion increases sharply. More details will be available in future publications

Curious enough, that box shaped model actually demonstrates slight decreasing of the criterion. If this effect will be confirmed by more precise analysis, this might be interpreted as an indication that GZ curve is a major nonlinearly affecting on ergodicity of rolling in irregular seas.

6. CONCLUSIONS

The paper describes the model test carried out in towing tank of National Research Institute of Fisheries Engineering of Japan. The purpose of the test was to check applicability of ergodic assumption for severe rolling in beam irregular seas. As of today, preliminary analysis has shown up that:

- Irregular wave produced in the tank is not ergodic stochastic process.
- Roll response of the model tank is not ergodic stochastic process
- There is an indication that non-ergodicity of roll response is also contributed by the non-linearity of the dynamical system
- There is an indication that GZ curve may be the major non-linear factor affecting on ergodic qualities of roll response in beam seas.

7. ACKNOWLEDGEMENTS

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