MODELING OF SHIP DYNAMIC ON WAVES USE «KASTNER – PAULLING» CONCEPTION

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Abstract. The results of modeling of ship dynamics on wave for critical situations established as a result of experimental researches Kastner - Paulling are discussed. As against works of D.M.Ananiev, the mathematical model of broaching contains complex nonlinear spatial function of the righting moment, which continuously changes in time. The validation procedure of mathematical model is carried out with use of criteria of ship safety at movement on following waves in various extreme situations. The analysis within the framework of the classical Balchi circuit, extended at the expense of inclusion of a cycle connected to physical modeling of the researched phenomenon is carried out. The results of the comparative analysis with use of the given tests of self-propelled radiocontrolled models on natural waves are given.

INTRODUCTION

About 40 years ago extensive studies of ship stability on following sea in experimental model basin of many countries are carried out [Boroday I.K., Netsvetaev Yu.A. (1982)], [Kobylinski L., Kastner S. (2003)], [Nechaev Yu.I. (1989)], [Nechaev Yu.I. Zavyalova O.P. (2003)], [Pauling J., Kastner S., Schaffran S. (1973)]. Studies of physical view of self-propelled radiocontrolled models overturn carried out by S.Kastner and R.Paulling [Kastner S. (1962)], [Pauling J., Kastner S., Schaffran S. (1973)] produced special practical interest. These works becomes the impulse for in-depth study of ship overturn phenomenon and create of stability estimation methods for critical situation formulated in base of physical modeling data. Criteria basis for dangerous situation has been formulated and has been founded for national rationing arrangements.
Evolution of computing facilities today opened a new opportunities of mathematical modeling for complicated physical phenomenon of ship motion on following waves. Works devoted to parametric resonance research are intense interest [Belenky V.L., Sevastianov N.B. (2003)] and others. In present papers discussion of computational experiment concerned with modeling of ship dynamic on following sea for critical situation established in experimental research by S.Kastner – J.Paulling [Kastner S. (1962)], [Pauling J., Kastner S., Schaffran S. (1973)] are given. Procedure of mathematical model's validation is carried out with use of safety target of ship on following sea in various extreme situations [Nechaev Yu.I. (1989)], [Nechaev Yu.I. Zavyalova O.P. (2003)]. Analysis is realized within the framework of the classical Balchi circuit [Balci O. (1998)] extended at the expense of inclusion of a cycle connected to physical modeling of the researched phenomenon.

1. PROPOUNDING

Mathematical model of ship dynamic on following waves described by system of differential equations:

$$F_i(x_i'',x_i',xi,X_{i1},\ldots,X_{im},Y_{i1},\ldots,Y_{im}) = 0, \qquad (1)$$

$F_i(\cdot)$ – nonlinear function; x_i – linear and angle motion; $X_{i1},\ldots,X_{im}$ – parameters, characterizing of ship as dynamic system (inertia, dampfer and righting components);$Y_{i1},\ldots,Y_{in}$ – rouse force and moments.
The realization of system eqn.(1) in practical tasks of stability is usually reduced to integration of the differential equations systems, and sometimes and separate equations, suitable in each case for the description of a considered situation. This choice is determined by features of dynamics of ship interaction with external environment in various critical situations. At movement of a ship on following wave on the basis of results of tests of self-propelled radiocontrolled models on natural excitement allocate three critical situations [Nechaev Yu.I. (1989)]:

- total stability loss owing to sharp decrease of resistance of a ship external to heel loadings at reduction of the righting moment;

- resonance regimes of rolling (base and parametric resonance);
- loss of maneuverability and sudden capture of a ship (broaching).

Total stability loss. General form of mathematical model describing ship behavior on waves under the influence of wind force can be described by system of differential equations in the form [Boroday I.K., Netsvetaev Yu.A. (1982)], [Nechaev Yu.I. (1978, 1989)]:

$$(D/g + \mu_{\eta\eta})\eta'' + \lambda_{\eta\eta}{}^{*}\eta' + \lambda_{\eta\eta}{}^{**}\eta^2 = P(t); \qquad (2)$$

$$(J_x+\mu_{\theta\theta})\theta'' + M_R(\theta') + M(\theta,\varphi,t) = M_x(t), \qquad (3)$$

where $D/g + \mu_{\eta\eta}$ - ship mass with water added mass; $\mu_{\theta\theta}$ - moment of ship mass inertia with added moment of inertia relatively longitudinal central axis; $\lambda_{\eta\eta}{}^{*}$ and $\lambda_{\eta\eta}{}^{**}$- drift resistance coefficients for linear and quadratic laws; $M_R(\theta)$ - resistance forces moment; $M(\theta,\varphi,t)$ – righting moment; $M_x(t)$- heeling moment inclusive aerodynamic force moment $P(t)$ and hydrodynamic forces moments inertial and no inertial nature caused by drift.

Base and parametric resonance. The mathematical model describing resonant regime of a ship rolling on following waves, can be described by system eqn.(2),(3), or separate equation of ship rolling [Boroday I.K., Netsvetaev Yu.A. (1982)], [Nechaev Yu.I. (1978)]. In case of a parametrical resonance at a situation of a ship broad-side to waves the equation vertical of rolling is in used.

Broaching. In broaching regime instead of model (1) the following system of differential equations is used [Nechaev Yu.I. Zavyalova O.P. (2003)]:

$$\left.\begin{aligned}
&\left[\left(\frac{D}{g}\right)+\mu_{\xi\xi}\right]\left(\dot{v}\cos\beta^{*}-\beta^{*}v\sin\beta^{*}\right)+\left[\left(\frac{D}{g}\right)+\mu_{\eta\eta}\right]v\dot{\chi}\sin\beta^{*}=F_x=X(t)+P_e-R\\
&\left[\left(\frac{D}{g}\right)+\mu_{\eta\eta}\right]\left(\dot{v}\sin\beta^{*}-\beta^{*}v\cos\beta^{*}\right)+\left[\left(\frac{D}{g}\right)+\mu_{\xi\xi}\right]v\dot{\chi}\cos\beta^{*}=F_y=Y(t)+R_{yB}+R_{yR}\\
&\left(J_z+\mu_{\chi\chi}\right)\ddot{\chi}=M_z=M_z(t)+M_{zB}+M_{zP}\\
&\left(J_x+\mu_{\theta\theta}\right)\ddot{\theta}+M_R\left(\dot{\theta}\right)+M(\theta,\varphi,t)=M_G+M_{\hat{A}}
\end{aligned}\right\} \qquad (4)$$

where β^* – drift angle; χ– angle of yawing; $X(t)$, $Y(t)$, $M_X(t)$, $M_Z(t)$ – disturbing forces and moments; M_R and $M(\theta,\varphi,t)$ – damping and righting moment; R_{yB} and M_{zB} – force and moment caused by a rudder action; M_{xB} – moment of a rotational nature caused by a drift and rotation of a ship concerning a vertical axes; M_G – hydrodynamic heeling moment; M_A – wind heeling moment.

As objects of research two sea ships suffering failure on following waves are used:

Small trade vessel – seiner «PC-708», having the following characteristic: L=25m, B=6,2m, T=2,1m, H=3m, δ= 0,54, α= 0,79, β=0,84, h_0=0,42m.

The cargoship – «Poronaisk»: L=96m, B=6,75m, T=2,6m, H=3,4m, δ=0,5, α=0,78, β=0,74, h_0=0,33.

where L,B,T – ship principal dimensions; δ,α,β – coefficients displacement efficiency, waterline coefficient and midship coefficient.

3. CRITERIA BASIS

A. Stability criterion on following sea [Nechaev Yu.I. (1978)]:

$$K_w = k(\theta_w)M_{cw} / M_w, \ (h_0)_{min} \geq 0, \qquad (5)$$

where K_W – weather criterion; h_W – minimum value of metacentric height on following waves; $k(\theta_w)=F(\varphi)\theta_r$ – coefficient considering capsizing moment decrease at the expense of rolling influence on following waves (θ_w – rated rolling amplitude); θ_r - rolling amplitude calculated in accordance with Russia Register rules; M_{cw} and M_w – capsizing and heeling moments on following waves defined by formulas:

Capsizing moment:

$$M_{cw}=(M_{cw})min+\Delta M[(M_c)_{mid} - (M_c)_{min}]; \qquad (6)$$

$$\Delta M=0,27(\Delta Fr)_w + 8,13(\Delta Fr)_w^2 - 11,04(\Delta Fr)_w^3, \ \Delta Fr=0,40\text{-}Fr;$$

$$(M_{cw})_{mid}= 0,5[(M_c)_{min}+(M_c)_{max}],$$

where $(M_c)_{min}$ and $(M_c)_{max}$ – capsizing moments meanings defined by stability diagram at the ship position on the crest and bottom of wave.

Calculation of capsizing and heeling moments on the crest and bottom of wave carry out by means of special programs.
Heeling moment:

$$M_w = k(\varphi_W)\, M_v = 0{,}6 \cdot 10^3\, p_v A_v z, \tag{7}$$

where $k(\varphi_W)$ - factor taking into account angle course to a wind (the meaning of the moment at angle φ_W =30-40° on the data of purges in aerodynamic pipes makes about 0,5-0,7); M_v – heeling moment calculated in accordance with Russian Register rules; p_v, A_v, z – wind pressure, sail area and distance of gravity centre its sail area from operating waterline.
Metacentric height

$$(h_0)\min = h_0 + \Delta h_W \geq 0, \tag{8}$$

where h_0 – metacentric height of still water; Δh_W – the negative amendment which is taking into account decrease of size h_0 at the moment of passage of a wave top through of a ship middle:

$$\begin{gathered}
\Delta h_W = -\,BF_1(\lambda/L)(-0{,}575h_w/L - 0{,}0115X_1 - \\
0{,}02001X_2 + 0{,}0980X_3 + 0{,}0229X_4 + 0{,}1100X_5 + a_6X_6 + \\
+0{,}0023X_1^2 - 0{,}0046X_2^2 + 0{,}0712X_3^2 - 0{,}0207X_5^2 + 0{,}1195X_6^2 - 0{,}0265X_2X_3 - \\
-\,0{,}1805X_2X_4 - 0{,}0265X_1X_6 + 0{,}0078X_2^3 - 0{,}0528X_3^3 - 1{,}700X_5^3); \\
a_1 = 0{,}1610 \text{ at } Fr > 0{,}28,\ a_1 = -\,0{,}2080 \text{ at } Fr < 0{,}28,\ \ Fr = 0{,}514Vs/(gL)^{1/2}; \\
F_1(\lambda/L) = 1 + 0{,}87\lambda^* - 1{,}2\,(\lambda^*)^2 - 0{,}21(\lambda^*)^3,\ \lambda^* = (\lambda\,/L) - 1; \\
X_1 = (L/B) - 4{,}82,\ X_2 = (B/T) - 2{,}67,\ X_3 = (H/T) - 1{,}30; \\
X_4 = (\delta\,/\beta) - 0{,}692,\ X_5 = (\delta\,/\alpha) - 0{,}700,\ X_6 = Fr - 0{,}28.
\end{gathered} \tag{9}$$

B. Criteria for resonance regimes of rolling. Critical Froude number described by formula [Nechaev Yu.I. Zavyalova O.P. (2003)]:

$$(Fr)_{CR} = k_B(Fr)o,\ \ (Fr)_{CR} \notin \{(Q(Fr)^*),(Q(Fr)^{**})\}; \tag{10}$$

$$\begin{gathered}
(Fr)^* = \sqrt{\frac{\lambda}{L}}\left(\frac{1}{\sqrt{2\pi}} - \frac{k_\theta^*}{C\sqrt{g}}\sqrt{\frac{\lambda}{L}\frac{L}{B}\frac{h_\theta}{B}}\right), \\
(Fr)^{**} = \sqrt{\frac{\lambda}{L}}\left(\frac{k_\theta^{**}}{C\sqrt{g}}\sqrt{\frac{\lambda}{L}\frac{L}{B}\frac{h_\theta}{B}} - \frac{1}{\sqrt{2\pi}}\right).
\end{gathered} \tag{11}$$

where $k_B = f(L/B)$ – numerical coefficient; $(Fr)_o = V/(gL)^{1/2}$ – Froude number for ship speed in maximum load at still water V; k_θ^*, k_θ^{**} – factors describing resonance regime of ship rolling areas $Q(Fr)^*$ and $Q(Fr)^{**}$ on following waves: $k_\theta^* = (0{,}8 \div 1{,}2)$ for principal resonance; $k_\theta^{**} = (1{,}95 \div 2{,}05)$ for parametric resonance; $h_\theta = 0{,}5[(h_o)_{min} + (h_o)_{max}]$ – average value of metacentric height on longitudinal sea; $(h_o)_{min}$, $(h_o)_{max}$ – extreme value of metacentric height (on the crest and bottom of wave) for condition of capsizing moment $(M_c)_w$ calculating.

C. Criteria for broaching. Estimation of ship safety in broaching regime is reduced to testing of dynamic criteria together with critical Froude number [Nechaev Yu.I. Zavyalova O.P. (2003)]:

$$(M_c)_W \geq M_R,\ Fr \leq (Fr)_{CR}, \tag{12}$$

where $(M_c)_W$ – capsizing moment on following waves; M_R – heeling moments because of ship turn; $(Fr)_{CR}$ – critical Froude number described by eqn.(10).
Value of capsizing moment defined by expression [Nechaev Yu.I. (1978/1989)], [Nechaev Yu.I. Zavyalova O.P. (2003)]:

$$(M_c)_W = 0{,}5[M(\theta,\varphi,t)_{max} + M(\theta,\varphi,t)_{min}] = 0{,}5D[l(\theta,\varphi,t)_{max} + l(\theta,\varphi,t)_{min}], \tag{13}$$

where $M(\theta,\varphi,t)_{max}$, $M(\theta,\varphi,t)_{min}$ – extreme value of righting moment at ship position on crest and bottom of wave, wave length equal to ship length (λ=L), $l(\theta,\varphi,t)_{max}$, $l(\theta,\varphi,t)_{min}$ – corresponding value of stability arm, φ – course angle.
Calculated value of wave length equal to ship length (λ=L), steepness h_w/λ calculated by formula [Nechaev Yu.I. Zavyalova O.P. (2003)]:

$$h_w/\lambda\ = 0{,}05[1 + (160 - \lambda)/135]. \tag{14}$$

Heeling moments in broaching regime is defined by regression dependence:

$$M_R = F(L)Dl_R = D[1+0,023(30-L)+0,0005(30-L)^2] \times \times (0,15 - 0,05X_1 - 0,02X_2 + 0,03X_3 - 0,19X_4 + 0,06X_5 + + 0,007X_2^2 + 0.018X_3^2 + 0,12X_4^2 + 0,43X_5^2), \quad (15)$$

where D – ship displacement, expression in round brackets define heeling arm l_R; X_1– X_5 – non-dimensional characteristic, calculated by formulas:

$X_1 = L/B - 4,0$; $X_2 = B/T - 3,0$; $X_3 = H/T - 1,4$; $X_4 = \delta/\alpha - 0,7$; $X_5 = Fr - 0,32$;

L/B, B/T, H/T, δ/α – correlation of ship dimensions and vertical fullness coefficient.
Expression (13) is suitable for ship with length less then 60m, i.e. cover most essential diapason of small ships utmost liable to capsizing danger in broaching condition.

4. ANALYSIS OF RESULTS

The mathematical modeling was carried out according to the developed plan of experiment for two scenarios of storm development submitted on a fig.1. The first scenario (curve 1) assumed gradual increase and attenuation of storm, second scenario (curve 2) - slower increase passing in brightly expressed peak with the subsequent intensive attenuation. For a small fishing vessel the accounts were carried out for all critical situations. For the cargoship (L=96 m) the broaching situation was not investigated, as his length and Froude number are outside a dangerous range.
The results of modeling of a ship dynamics on waves are submitted in a fig.2 as curve dependence of considered of stability criterion from wave height of $(h_w)_{3\%}$ varied depending on the researched scenarios of storm development.
On the basis of the given fig.2 it is possible to note the following laws of stability change. The role of stability criteria during development and attenuation of storm is not identical and essentially depends on the characteristics of a ship and force of waves.
For a small ship (fig.2A) it is possible precisely enough to allocate areas dangerous from the point of view as of complete loss of stability (criterion K_2 (eqn.12)), and broaching occurrence (criterion K_1 (eqn.5)). And there is an area, in which both these critical situations (shade a strip) are equally dangerous. The criterion of metacentric height for a small ship in the accepted condition of loading is satisfied in all range of the considered heights of waves.

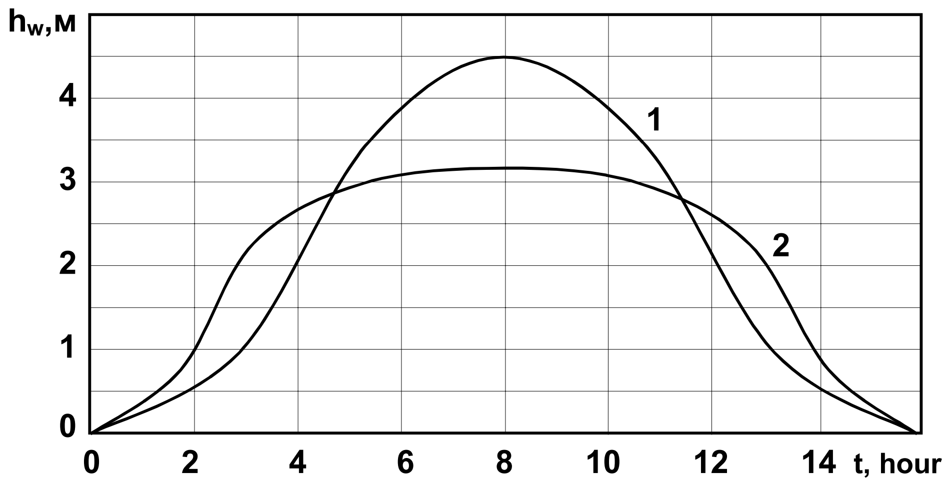

Fig. 1. Scenarios of storm: 1 - slower increase passing in brightly expressed peak;
2 - gradual increase and attenuation of storm.

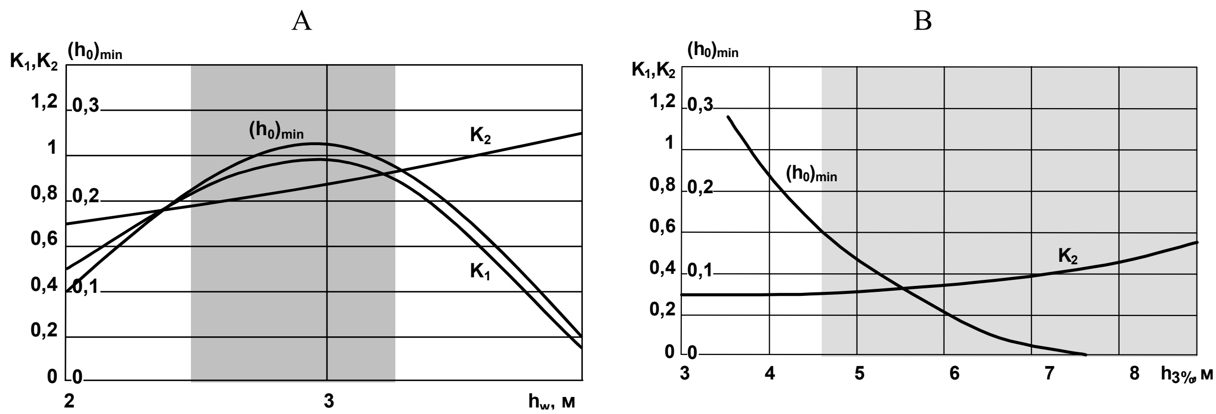

Fig.2. Change of the stability characteristics on waves for a small ship (A) and for the cargo-ship (B): the continuous curve characterizes the requirement to metacentric height ($(h_0)_{min}$), criterion of weather in a broaching regime (K_1), criterion of weathers connected to deterioration of stability on following waves (K_2); shade the strip determines adverse area.

For a cargo-ship having the S-figurative of stability diagram and small matacentric height there is a situation connected to sharp deterioration of stability (criterion K_2). However owing to low initial of stability there is a fast deterioration of metacentric height (criterion $(h_0)_{min}$), and at height of a wave $(h_w)_{3\%}\geq 7{,}5$м the size $(h_0)_{min}$ becomes equal to zero. In these conditions the vessel becomes crank, and it promotes occurrence of a significant roll and displacement of a cargo, that at the end and has resulted in heavy failure. The Froude number in view of decrease of speed on waves for the cargoship appears outside a critical range, and the broaching situation in this case does not represent danger.

Fig.3 characterizes probabilities of capture and capsizing in a broaching regime for the considered scenarios of storm development. As the process of development and attenuation of storm was simulated by symmetric functions, the curves are shown only up to the maximal point of storm development. From a fig.3 follows, that the character of storm development renders essential influence on probability of capture and capsizing. And, for the scenario 1 both curves have a characteristic maximum at certain of a wave height $h_{3\,\%}$, whereas the scenario 2 results in more monotonous change of curves in a wide range of heights of waves.

Storm scenario 1:
slower increase passing in brightly expressed peak;

Storm scenario 2:
gradual increase and attenuation of storm

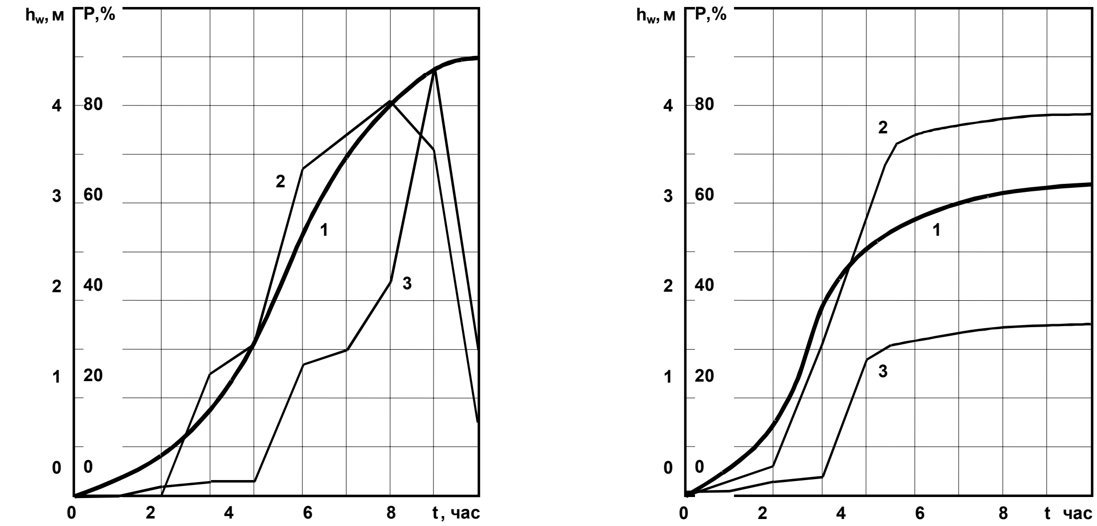

Fig.3. Broaching probability characteristic: 1 - scenarios of storm;
2 - probability of “capture”; 3 - probability of capsizing

The results of an estimation of rolling resonant regimes for the considered scenarios are submitted in the table 1,2.

Table 1: Froude number for small-ship. Input data: V=9,8 knots, Fr = 0,322

λ/L	Wave steepness 0,05		Wave steepness 0,11	
	Basic resonance n=0,8/1,2	Parametric resonance n=1,9/2,1	Basic resonance n=0,8/1,2	Parametric resonance n=1,9/2,1
1	0,136 / 0,223	0.018 / 0.062	0,148 / 0,231	0.001 / 0.041
1,25	0,117 / 0,227	0.075 / 0.130	0,237 / 0,132	0.051 / 0.104
1,5	0,094 / 0,225	0.137 / 0.203	0,112 / 0,237	0.108 / 0.171

Table 2: Froude number for cargo-ship. Input data: V=13,5 knots, Fr = 0,226

λ/L	Wave steepness 0,05		Wave steepness 0,11	
	Basic resonance n=0,8/1,2	Parametric resonance n=1,9/2,1	Basic resonance n=0,8/1,2	Parametric resonance n=1,9/2,1
1	0,212 / 0,274	-0,102 / -0,071	0,241 / 0,294	-0,149 / -0,123
1,25	0,212 / 0,29	-0,075 / -0,036	0,249 / 0,315	-0,134 / -0,101
1,5	0,208 / 0,301	-0,044 / 0,003	0,252 / 0,331	-0,114 / -0,074

Note: Mark the minus specifies, that the occurrence of a parametrical resonance arises only at movement of a ship on counter waves.

5. VALIDATION OF MATHEMATICAL MODELS

Verification task for mathematical model describing complicated ship evolution in broaching-to is multistage iterative process demonstrating of accuracy and correctness of conclusion about investigated dynamic system's behavior. One of the well-known schemes of model's validation is Balchi's diagram, first version of which there is presented in 1986 year. Diagram modernized by authors in compliance with development of specific application shown in fig.4.

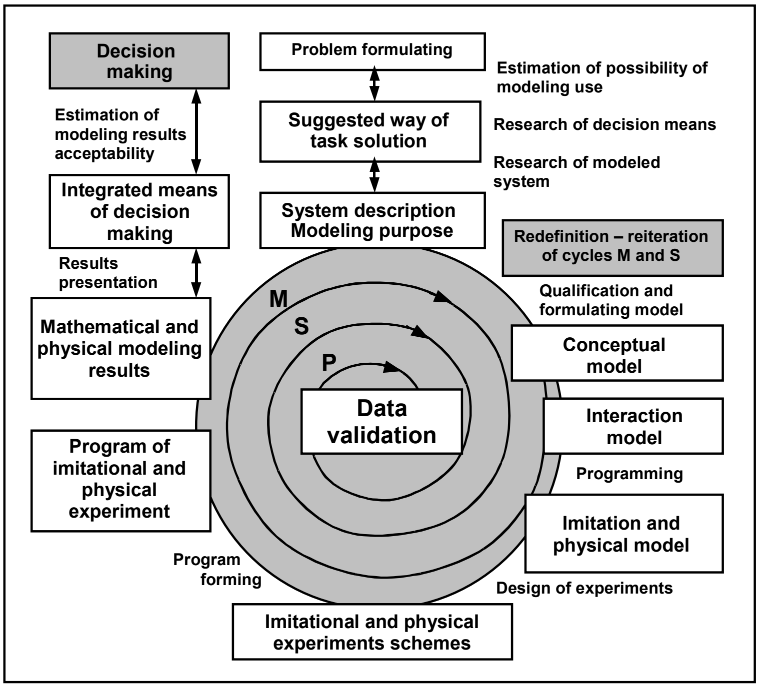

Fig.4. Methodological scheme.

At that basic idea of Balchi's diagram is conserved. However imitational model is regarded as component of practical application – modeling task of broaching danger estimation which has a three-cyclic structure. The first cycle is connected with development and designing of mathematical model (modeling – M). In process of this cycle estimation of generic structure and model's components is carried out. The second cycle is realization of corresponding mathematical (imitational) experiments (simulation – S). And the third cycle is physical experiments realization (physical – P). Indicated cycles are marked by symbols M, S and P.

We shall give short characteristic of term used in fig.4. Model *verification* concerned with confirmation that model can be transformed from one form to another with adequate accuracy, starting with formulating problem and ending with receiving of computer application. *Validation* is confirmation that model provide adequate accuracy in accordance with cycles M, S and P in considered data domain. Generally validation is control of accordance between imitational model behavior and researched dynamic system (i.e. results of physical modeling).

Conceptual model base on accepted assumption and define structure of researched dynamic system and links between its separate components. *Interaction model* represent system dynamic. Its verification is confirmation of ship and environment interaction's image correctness in researched situation. *Imitation model* is program image of conceptual model defined by one of computer language. Verification of imitation model is demonstration of possibility of program model realization as machine analog of conceptual model (on basis of its maximal similarity). Definition of the necessary similarity level is verification procedure's aim.

Experiment scheme is program realization of experiment scenario. Mathematical model validity depend on machine experiment scheme subject to initial condition defined corresponding limits. Therefore mathematical model and real system (data of physical modeling) can be adequate with respect to specified experiment scheme only. This condition is taking to account at comparison results of mathematical and physical modeling. Verification of experiment scheme is confirmation of compatibility experiment scheme with imitation model mainly. Furthermore accordance of experiment condition to concrete planned scenario have to be established.

Comparison results of mathematical and physical modeling. Comparison of calculation results with physical modeling data is important for estimation adequacy of models defining condition of dynamic heel and capsizing in broaching-to. Such comparison is realized for self-propelled radio-controlled model of seiner at natural waves. Comparison results are representing in table 3. As shown in the table for different situations (situation of ship capsizing and surf-to) fixed at tests, stability estimation by suggested criteria give satisfactory results. Corresponding graphics present at fig.5.

Table 3. Comparative data of capsizing estimation in broaching-to

Situations characteristic						**1**	**2**
Waves packet	$\overline{h}$,m	h_w/λ	φ, deg	V, m/s	l_k, m		
IW	2.5	0.11	180	5.0	0.07	o	o
IW	3.5	0.05	180	5.5	0.07	o	o
IW	3.5	0.11	180	5.0	0.07	•	•
IW	2.5	0.11	180	5.0	0.12	•	•
IW	3.5	0.11	170	5.0	0.12	•	•
CF	3.5	0.11	180	5.5	0.10	•	•
CF	3.5	0.11	170	6.0	0.10	•	•
IW	3.5	0.11	180	5.0	0.10	•	•
IW	3.5	0.05	180	5.5	0.12	•	•

Note: IW – irregular waves; CF – classical form; $\overline{h}$ - average mean of wave height in realization; h_w/λ - wave steepness; φ – course angle of wave; V – ship speed; l_k – maximum mean of hydrodynamic moment; 1 – experiment, 2 – calculation; • – capsize, o – «surf-to» without capsize.

Graphical representation of sub products and output data as well as animation of modeling process is important tool for validation of researched mathematical model describing dynamic of broaching-to. Bar chart, time graph for separate variables at modeling period, interdependency graph, circular graph and linear graph are used as the most effective tools of visual analysis.

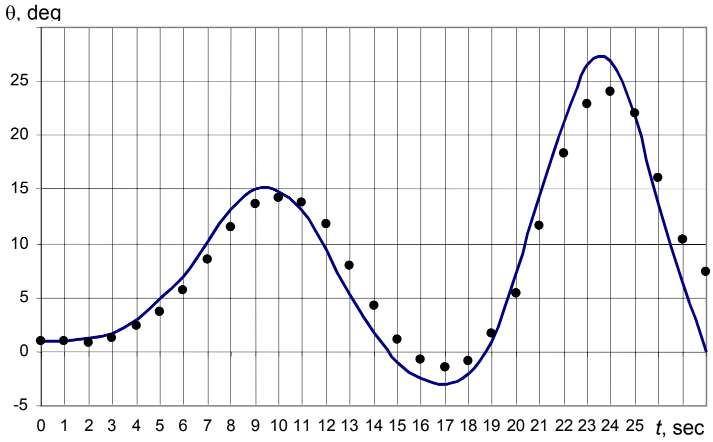

Fig.5. Heeling dynamic in broaching-to: point – physical experiment, solid curve – mathematical modeling results.

CONCLUSION

Realized research of ship dynamic on following sea at different scenarios of storm evolution allows establishing following facts and regularity:

- Danger of extreme situation on following sea depend on not only usually considered characteristics (sea loading, hull form parameters and ship loading, Froude number and course angle) but depend on scenarios of storm evolution.
- Control of mathematical model adequacy expedient to realize on base of rational combination of physical experiment data with self-propelled models in condition of experimental model basin and natural sea. Statistical procedures used on each stage of realization of adequacy estimation use methods of correlation, variance and regression analysis.
- Data received with help of describing procedures allow considering than used mathematical models and criteria of following sea danger sufficient describe particularity of considered extreme situations at different scenarios of storm evolution.

Correctness of used statistical procedures on a validation stage is confirmed by physical modeling results. Future trends of estimation task solution connect with development of researches of distributed technologies application in the domain of simulation modeling required development of new principles and approaches.

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