Hybrid Models for Fast Time-Domain Simulation of Stability Failures in Irregular Waves With Volume-Based Calculations for Froude-Krylov and Hydrostatic Forces

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ABSTRACT

Development of the IMO second generation intact stability criteria requires applying a variety of mathematical models of ship motions in waves, from very simple cods for vulnerability criteria (mostly based on ordinary differential equations, or ODEs) to the state-of-the art advanced hydrodynamic codes for direct stability assessment. This paper describes an approach to mathematical modeling that stands between ODE and advanced codes. While retaining nonlinearity and inseparability of hydrostatic and Froude-Krylov forces, the "hybrid" model remains as simple as an ODE in all other aspects. The algorithm is based on 3-DOF (heave, roll, pitch) volume calculations. Calculation speed makes it attractive for validation of prediction methods for probability of stability failures.

KEYWORDS

Ship motions, irregular waves

INTRODUCTION

Linearity of the equations of ship motions was essential for the frequency domain approach that opened the way to computation of ship motions in irregular waves (St Denis & Pierson, 1953). The assumption of linearity (*i.e.*, small amplitude and wave slope) obviously limits application of the frequency domain approach. Assessment of dynamic stability in waves was one of those applications that required large-amplitude motions capabilities.

Nonlinearity in equations of ship motions comes from different sources for different degrees of freedom. Nonlinearity of roll motions, with respect to dynamic stability concerns, mainly comes from hydrostatic and incident wave (Froude-Krylov) forces. Most ships have two stable equilibria in roll: upright and capsized. Transition from roll motion around the upright position to the capsized position is the total stability failure and is the ultimate goal of dynamic stability assessment.

Thus, the simplest possible mathematical model for dynamic stability is an ordinary differential equation with several stable equilibria:

$$\ddot{\phi} + 2\delta\dot{\phi} + \omega_0^2 f(\phi) = f_E(t) \qquad (1)$$

where ϕ is roll angle, δ is the linearized damping coefficient, ω_0 is the natural roll frequency, $f(\phi)$ is the stiffness function (related to GZ curve in calm water), and $f_e(t)$ is the external forcing function. The latter actually is not limited by irregular wave excitation, but could come from, for example, an anchor-handling operation. The only modeling requirement for capsizing is the existence of two stable equilibria in roll, separated by the unstable equilibrium, which is the angle of vanishing stability; see Figure 1.

Equation (1) is the usual model for studying dynamic stability; by using different approximations for stiffness, a number of nonlinear phenomena have been studied, such as fold and flip bifurcation, sub and ultra-harmonic resonance, and so forth (*e.g.*, Nayfeh & Khdeir 1986, Cardo *et al.* 1981). Adding a periodic change of stiffness to the model allowed parametric resonance to be modeled (Paulling & Rosenberg 1959).

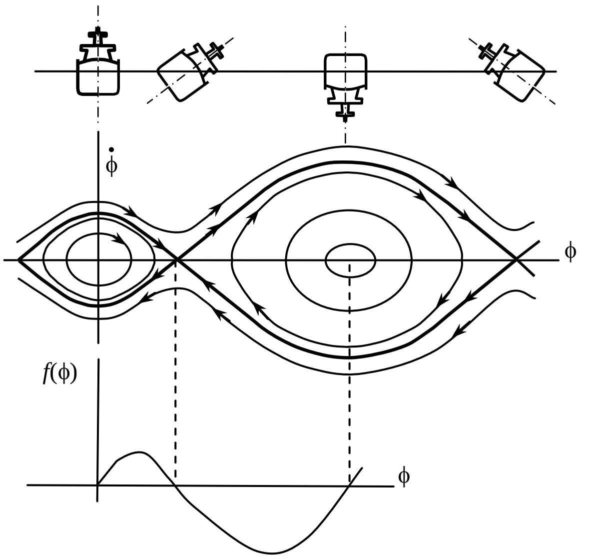

Fig. 1: The simplest model of capsize

The theoretical background for these and similar work was the nonlinear oscillators summarized by Andronov *et al.* (1966). The appearance of nonlinear dynamics (Guckenheimer & Holms 1983) gave ship dynamics a new motivation; for an example, see Spyrou (1996).

These works tremendously improved the understanding of nonlinear ship dynamics, but remained mostly qualitative. There is no strict way to derive Equation (1). Considering ship motions in waves, using potential flow assumption leads to the system of six integro-differential equations, and only a linear assumption (small waves slope, small-amplitude motions) will turn it into a system of ordinary differential equations. One would need some additional assumptions to reduce the problem to a single degree of freedom; then the nonlinearity is artificially reintroduced to arrive at Equation (1). The logic of these considerations was reproduced in Belenky & Sevastianov (2007).

Meanwhile, improvements in computational capabilities and numerical hydrodynamics have led to development of numerical codes capable of reproducing ship motions with fewer assumptions; an example is the Large-Amplitude Motion Program (LAMP) code, which is explained in Lin & Yue (1990) and Shin *et al.* (2003). A detailed review of the theoretical background of such codes from the hydromechanics point of view can be found in Beck & Reed (2001).

The cost of using fewer assumptions is not small. Interpreting results from numerical simulation is difficult, as the terms in the equations of motions are not expressed as explicit functions. In particular, stiffness (hydrostatic) and excitation (Froude-Krylov) cannot be separated. Nevertheless, some tools of nonlinear dynamics are also applicable to integro-differential equations, in particular the continuation method, explained in Spyrou *et al.* (2009) and Spyrou & Tigkas (2011). Nevertheless, extending the methods of nonlinear dynamics to hydrodynamic codes seems to be the natural next step.

THE CONCEPT OF A HYBRID MODEL

Stability variation in irregular waves is a difficult problem for ordinary differential equation (ODE) - based models of large-amplitude roll motions. When a wave is regular, its shape can be assumed sinusoidal; then the GZ curve for each position of the wave crest can be computed and used in nonlinear ODE as variable stiffness. Once it is approximated, the model can be treated numerically and analytically; for examples, see Sanchez & Nayfeh 1990, Bulian 2004, Spyrou 2005, Rodriguez & Neves 2011, and others).

The difficulties appear in irregular waves, when the shape of the wave is random. Indeed, a stability curve can also be characterized in irregular waves (Belenky & Weems 2007), but then its modeling within the ODE may be complex or require additional assumptions. Another way is to limit the consideration with

GM and use a spectral representation (Dunwoody 1989a, 1989b). Grim's effective wave (Grim 1961) is a popular technique; it involves substitution of the actual irregular wave with a sinusoidal wave with a length equal to the ship length and random height. Umeda *et al*. (1990) applies this approach to capsizing probability evaluation.

Heave and pitch may have significant influence on stability variation when a ship is not in following or stern-quartering waves (Paulling 2011). This circumstance was the immediate motivation for a "hybrid" model (Belenky *et al.* 2011). Heave and pitch were calculated in time domain, and the ship's attitude and position on the wave was used to evaluate stability in waves. These heave and pitch calculations were based on submerged volume only, while radiation and diffraction were included as coefficients. These simplifications are attractive from a regulatory standpoint, as the volume calculations can be easily verified.

The next step in this direction is to include roll into volume calculations as well. Such a model, while still being simple, should be capable of describing nonlinearity caused by hydrostatic and Froude-Krylov forces on a consistent basis. The remainder of the paper is focused on this particular aspect.

FORMULATION OF VOLUME-BASED CALCULATIONS

The non-linear wave forcing and restoring forces can generally be computed by integrating the incident wave and hydrostatic pressure over the instantaneous wetted hull surface (in the Earth-fixed frame):

$$\mathbf{F}_{FK+HS}(t) = -\rho \iint_{S_B(t)} \left(\frac{\partial \phi_0(x,y,z,t)}{\partial t} + gz \right) \hat{\mathbf{n}}\, ds \qquad (2)$$

where $\partial\phi_0(x,y,z,t)/\partial t$ is the pressure of the undisturbed incident wave field (Froude-Krylov pressure) and $S_B(t)$ is the instantaneous wetted portion of the hull surface up to the incident wave waterline $\eta(x,y,t)$. The key element of this expression is that it captures the geometric non-linearities due to large vertical motion relative to the wave surface, ranging from the effect of bow flare to full emergence or submergence of the bow and stern.

It should be noted that this expression can be used with linear or nonlinear incident wave models as long as the incident wave model expresses a pressure and velocity field in the body-nonlinear domain, that is up to $z=\eta(x,y,t)$. For the typical linear wave model – in which the wave is represented by a superposition of sinusoidal components – this can be accomplished by applying the Wheeler stretching technique, in which the exponential decay term in the expressions for pressure, velocity, and their derivatives is expressed as $e^{k(\eta-z)}$.

The force calculation in Equation (2) is fairly straightforward, and many seakeeping codes, including LAMP, have a scheme that cuts off the 3-D geometry model at the instantaneous waterline, evaluates the incident wave and hydrostatic pressure over a set of hull panels or points, and numerically integrates the pressure over the wetted hull surface. While straightforward, this calculation can be expensive because it may require many evaluations of the incident wave function, and each wave evaluation may require many terms. The latter will be particularly true for very long irregular wave simulations, as a large number of wave components are required to provide a statistically independent realization of the seaway (Belenky & Sevastianov 2007). However, time-domain calculations using such schemes can generally still run faster or even much faster than real time and are often limited by other parts of the calculation such as the solution of the wave-body disturbance forces.

However, for the analysis of the long-term characteristics of dynamic stability problems, it is desirable to have a capability that can capture the full body-linear restoring but which can run significantly faster than real time. To provide such a capability, a volume-based calculation scheme is considered, based on the submerged volume at each instant in time, and can be calculated with a minimal number of evaluations of the incident wave. It is, however, imperative that the scheme capture the effect of the longitudinal variation of the relative motion, as this is a principle driver in dynamic stability phenomena such as parametric roll and pure loss of stability in waves. To do so, Equation (2) is expressed as the sum of incremental forces calculated on asset of incremental sections distributed along the ship's length:

$$\mathbf{F}_{FK+HS}(t) = \sum \delta\mathbf{F}_{FK+HS}(x_i, t) \tag{3}$$

where $\delta\boldsymbol{F}_{FK+HS}(x_i,t)$ is force computed over an incremental submerged portion of the hull's surface $\delta S_B(x_i,t)$:

$$\delta\mathbf{F}_{FK+HS}(x_i,t) = -\rho\iint_{\delta S_B(x_i,t)} \left(\frac{\partial\phi_0(x,y,z,t)}{\partial t} + gz \right) \hat{\mathbf{n}}\, ds \tag{4}$$

Note that the incremental hull surface $\delta S_B(x_i,t)$ is considered to include both the wetted portion of the hull for that section as well as the "wetted" (below incident wave) portions of the planes separating this section from adjacent sections.

Within each section, a Taylor expansion (neglecting higher-order derivative) can be used to approximate the distribution of the incident wave pressure over an incremental hull section in terms of the value and derivatives of the pressure at a nominal point (x_0,y_0,z_0) on the section:

$$\begin{aligned}\frac{\partial\phi_0(x,y,z,t)}{\partial t} \cong \frac{\partial\phi_0(x_0,y_0,z_0,t)}{\partial t} &+ \\ \frac{\partial^2\phi_0(x_0,y_0,z_0,t)}{\partial t\partial x}(x-x_0) &+ \\ \frac{\partial^2\phi_0(x_0,y_0,z_0,t)}{\partial t\partial y}(y-y_0) &+ \\ \frac{\partial^2\phi_0(x_0,y_0,z_0,t)}{\partial t\partial z}(z-z_0) &\end{aligned} \tag{5}$$

The dynamic free surface can be used to relate the Froude-Krylov pressure at the free surface to the incident wave elevation:

$$\frac{\partial\phi_0(x,y,\eta,t)}{\partial t} = -g\eta(x,y,t) \tag{6}$$

If the evaluation point is chosen to be on the incident wave surface, $z_0=\eta$, then Equation (5) can be written as:

$$\begin{aligned}\frac{\partial\phi_0(x,y,z,t)}{\partial t} &\cong -g\eta(x_0,y_0,t) \\ &- g\frac{\partial\eta(x_0,y_0,t)}{\partial x}(x-x_0) \\ &- g\frac{\partial\eta(x_0,y_0,t)}{\partial y}(y-y_0) \\ &+ \frac{\partial^2\phi_0(x_0,y_0,\eta,t)}{\partial t\partial z}(z-\eta)\end{aligned} \tag{7}$$

Using an overbar to designate the mean or nominal value of the elevation, etc., for a section, the sectional force can be written:

$$\begin{aligned}\delta\mathbf{F}_{FK+HS}(x_i,t) \cong \rho\iint_{\delta S_B(t)} &(g\overline{\eta} - gz \\ &+ g\overline{\frac{\partial\eta}{\partial x}}(x-x_0) + g\overline{\frac{\partial\eta}{\partial y}}(y-y_0) \\ &+ \overline{\frac{\partial^2\phi_0}{\partial z\partial t}}(z-z_0))\hat{\mathbf{n}}\, ds\end{aligned} \tag{8}$$

Since the incremental surface $\delta S_B(x_i,t)$ includes the plane separating adjacent sections, and the pressure over the free surface above the section will be zero, Gauss's theorem can be applied to define the sectional force in terms of the integral of the gradient of the approximate pressure field of the incremental volume:

$$\delta\mathbf{F}(t) = \iint_{\delta S_B(t)} P\hat{\mathbf{n}}\, ds = -\iiint_{\delta V(t)} \nabla P dv \tag{9}$$

This results in a volume-based formula for the sectional incident wave and restoring force:

$$\begin{aligned}\delta\mathbf{F}_{FK+HS}(x_i,t) &\cong \rho g\delta V(x_i,t)\hat{\mathbf{k}} \\ &- \rho g\delta V(x_i,t)\overline{\frac{\partial\eta}{\partial x}}\hat{\mathbf{i}} \\ &- \rho g\delta V(x_i,t)\overline{\frac{\partial\eta}{\partial y}}\hat{\mathbf{j}}\end{aligned} \tag{10}$$

$$+\rho g \delta V(x_i,t)\overline{\frac{\partial^2 \phi_0}{\partial z \partial t}}\hat{\mathbf{k}} \tag{10}$$

$\delta V(x_i,t)$ is the instantaneous volume of the submerged portion of the i^{th} section of the hull up to the incident wave surface. The first term is the familiar buoyancy term, but with the volume integrated up to the incident wave surface. The second and third terms are longitudinal and side forces from the gradient of the incident wave pressure field, evaluated in terms of the incident wave slope. The final term can be considered to be a "correction" to the buoyancy force, using a linear approximation of the exponential decay of the incident wave pressure field.

Similarly, expressions for the moments can be derived by applying the relation:

$$-\iint_S (\hat{\mathbf{n}} \times P\mathbf{r})\, ds = \iiint_V \nabla \times P\mathbf{r} dv \tag{11}$$

This gives the following formula for the roll and pitch moments:

$$\begin{aligned}\delta Mx_{FK+HS}(x_i,t) \cong & -\rho g \delta V(x_i,t) y_{CV}(x_i,t) \\ & -\rho g \delta V(x_i,t)\overline{\frac{\partial \eta}{\partial x}} z_{CV}(x_i,t) \\ & +\rho g \delta V(x_i,t)\overline{\frac{\partial^2 \phi_0}{\partial z \partial t}} y_{CV}(x_i,t)\end{aligned} \tag{12}$$

$$\begin{aligned}\delta My_{FK+HS}(x_i,t) \cong & -\rho g \delta V(x_i,t) x_{CV}(x_i,t) \\ & +\rho g \delta V(x_i,t)\overline{\frac{\partial \eta}{\partial x}} z_{CV}(x_i,t) \\ & -\rho g \delta V(x_i,t)\overline{\frac{\partial^2 \phi_0}{\partial z \partial t}} x_{CV}(x_i,t)\end{aligned} \tag{13}$$

$x_{cv}(x_i,t)$, $y_{cv}(x_i,t)$ and $z_{cv}(x_i,t)$ are the coordinates of the center of the instantaneous submerged volume for the i^{th} section up to the incident wave waterline.

With these formulae, the body-nonlinear Froude-Krylov and hydrostatic restoring forces can be computed with a minimum number of evaluations of the incident wave. The only major assumption in the derivation of these formulae is the Taylor series expansion of the incident wave pressure in Equation (5). This expansion assumes that the wave slope is constant over the beam and incremental length of each section and can be considered a long-wavelength assumption in which the wave length is assumed to be long with respect to the beam and increment section length. This assumption should be quite reasonable for waves, or wave components in an irregular sea model, that are longer than two or three times the beam, but the linear approximation of the sinusoidal wave profile will become inaccurate for shorter waves.

However, the section-based implementation considers the variation of elevation and slope from section to section, so the wave is not assumed to be long relative to ship length and the variation of relative motion along the length of the ship can be considered.

The expansion also considers the vertical pressure gradient to be, at most, linear with depth, so the wave is also assumed to be long compared to the draft of the ship. The linear approximation of the exponential pressure decay will become quite inaccurate for shorter waves, so the implementation of $\partial^2\phi_0/\partial z\partial t$ must be very careful in the evaluation of short waves or wave components.

IMPLEMENTATION OF VOLUME-BASED CALCULATIONS

The implementation of the volume-based body-nonlinear Froude-Krylov and hydrostatics force calculation requires the time-calculation of the sectional submerged volume and volume center beneath the incident wave. For application to extreme motion problems, these sectional volume calculations should accommodate large amplitude heave and pitch including fully submerged and emerged sections, and large amplitude roll motions including a fully inverted ship. For consistency and speed, the volume calculations can also be implemented using a single evaluation of the wave elevation and slope for each section.

In the present calculation, the sectional volume calculations were implemented using a Bonjean curve-like approach in which a set of stations were cut through the hull and the volume and volume moments were pre-computed up to each station offset point. At each time of a simulation (or heel angle of a restoring curve calculation), the Froude-Krylov and hydrostatic restoring force are computed with the following steps:

1. Compute the incident wave elevation and slope at the centerline of each station.
2. Find the intersection of the incident wave surface and the section centerline considering the wave elevation and vertical motion of the station due to the ship's heave and pitch.
3. Find the port and starboard waterline points from the incident wave/center intersection, lateral wave slope, and ship roll angle.
4. Interpolate volume and volume moments from pre-computed Bonjean curves and correct for roll + wave slope.
5. Compute section forces and moments via Equations (10), etc.
6. Integrate force/moments along the length of the ship.

Figure 2, which shows a midship station of the ONR Topsides Series Tumblehome hull, illustrates how the centerline/incident wave intersection point and an effect heel angle (roll angle + wave slope) are used to find the waterline points and compute the submerged volume and center of a station.

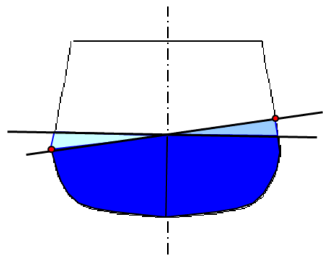

Fig. 2: Station offsets and waterline intersection points

Figure 3 shows the station offsets and the waterline intersection points of each station for a time instant from a simulation in stern oblique irregular waves.

In the initial implementation of the volume-based calculation, the $\partial^2\phi_0/\partial z\partial t$ term has *not* been included and the numerical studies have focused on wave lengths greater than twice the ship beam, with shorter waves being attenuated in the evaluation of the wave elevation and slope. Further work is required to explore the $\partial^2\phi_0/\partial z\partial t$ term and to develop a robust and accurate handling for shorter waves and irregular wave representation, including short wave components.

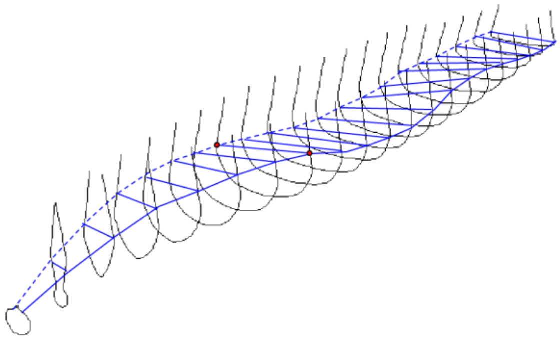

Fig. 3: Station/incident wave intersection points for ONR Tumblehome hull in stern oblique seas

VERIFICATION *VS.* 3-D SURFACE PRESSURE INTEGRATION

In order to verify the formulation and implementation of the sectional volume-based calculation, the roll restoring arm (GZ) curve was computed in both calm water and for the quasi-static wave-pass problem and compared to results from LAMP's 3-D surface pressure integration. Figure 4 compares the calm water restoring arm of the two calculations.

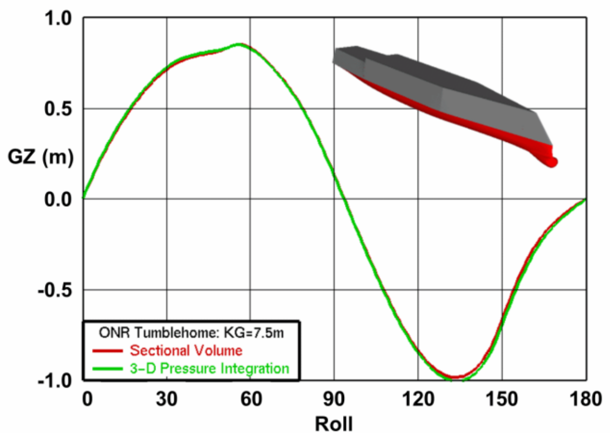

Fig. 4: Calm water roll restoring arm (GZ) curve for ONR Tumblehome hull

The volume-based Froude-Krylov and hydrostatic force calculation scheme has also been verified versus the conventional 3-D surface pressure integration approach for motion response predictions in regular and irregular waves. Figure 5 shows the roll response for a 3-DOF (heave, roll, pitch) simulation of a 100m x 20m x 6m rectangular barge in regular quartering waves with wave length equal to ship length. The roll responses are nearly identical, as were heave and pitch (not plotted).

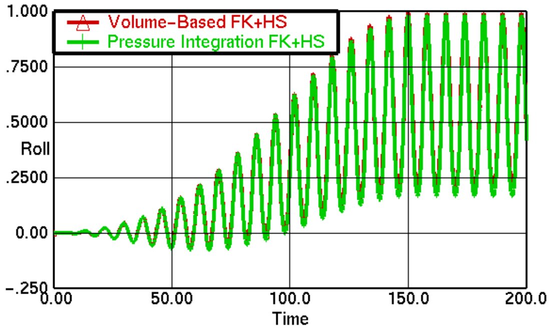

Fig. 5: Roll motion verification for rectangular barge in quartering regular seas, $\lambda=L_{OA}$ h=d/3

SIMULATION SPEED

In order to provide large volumes of realistic roll response data for characterizing extreme roll motion in irregular seas, a large series of time-domain 3-DOF (heave, roll, pitch) simulations have been made for the ONR Topsides Series Tumblehome hull in severe seas. In addition to the volume-based Froude-Krylov and hydrostatic forces, these simulations incorporate pre-calculated added mass coefficients and linear and non-linear damping models.

Figure 6 shows 20 records of the roll response for the ship at a low GM condition in large (Sea State 8) steep stern quartering waves. The seaway is modeled by 220 wave components to provide a statistically independent wave representation over each 30 minute realization.

The total calculation time for these 20 realizations was about 7 seconds on a single processor laptop computer.

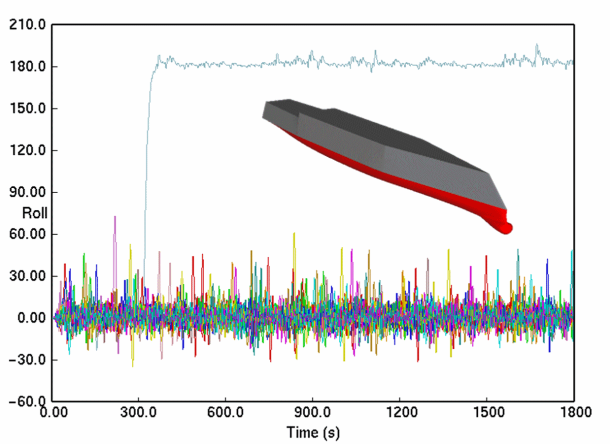

Fig. 6: Roll response of ONR Tumblehome hull in steep Sea State 8

SUMMARY

In order to provide simple, easily reproducible, robust, and very fast calculation of the body-nonlinear Froude-Krylov and hydrostatic forces for a ship undergoing large motions in a seaway, a sectional, volume-based approach has been developed. The approach requires a single evaluation of the incident wave per station but captures the effects of large relative motion and the longitudinal variation of the stability in waves, which are principle drivers in dynamic stability phenomena such as parametric roll and pure loss of stability in waves. The approach is nearly exact for waves which are long compared to the beam and draft, but care will need to be taken in its application to shorter waves or wave components.

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