Comparison of global technique and direct evaluation of capsizing probability on French frigates

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ABSTRACT

The dynamic stability of warships is analyzed by evaluating the propension of the ships to overcome a prescribed rolling angle. One technique is the evaluation of the probability to reach that angle. If the dynamical behavior can reduced to a time differential equation, Melnikov theory and the analysis of the integrity of attraction basin is also useful. In the present study, it is shown that all three techniques allows to class French warships in terms of their capsizability.

KEYWORDS

Capsize, capsizing probability, dynamical stability, global analysis.

INTRODUCTION

Capsizing probability evaluation is a tedious task involving numerous parameters and a rather long computational effort. Moreover the result of the computation is, but for comparison purposes, biased by an arbitrary choice of some of those parameters. Among those parameters the capsizing angle above which the ship is considered to capsize has not received a clear definition. Chosen constant in a previous work (Beaupuy et al., 2012) and equal to 45° as measured on one of the ships involved in the comparison, this parameter decreased drastically the operability of the other ships and thus biased the computed values of operability. In the present work we propose a presentation of two methods the goal of which is the classification of ships by risk level. The first section presents the method of operability previously introduced (Beaupuy et al., 2012) and, in the second section, a global technique. Then we focus on the estimation of the capsizing angle based on the global technique and then used to define more precisely the operability of each ship.

OPERABILITY DEFINITION

Operability, p_{op}, can be defined as the probability for the ship to be operated safely from her launching to the end of her operative life. If p_{cap} is the probability of capsizing determined for the same period, we can write:

$$p_{op} = 1 - p_{cap}$$

To define the operability the following hypothesis are followed:

- The ship is supposed to be operated in the Atlantic Ocean. She has an equal probability to be everywhere in the ocean.

- The sea state is mono directional.
- Her headings are spread from 0 to 360 with an equal probability.
- Her speed is spread between 5 and her maximum speed with an equal probability.
- Each simulation is realized for duration of 3600 s.
- The limit roll angle for capsize is arbitrarily fixed at 45°.
- Yaw DDL has been blocked to avoid broaching.

For each run the successive maximum of roll angle are kept and post treated using a Weibull method proposed by (Derbanne, Leguen, Dupau, & Hamel, 2008) in order to define the capsize probability for particular conditions. In this method probabilities are applied to the roll angles and fitted by a Weibull law of probability. Extrapolation to the defined value of roll angle (45° in our case) allow the evaluation of capsize probability (green line). This process is illustrated on figure 1.

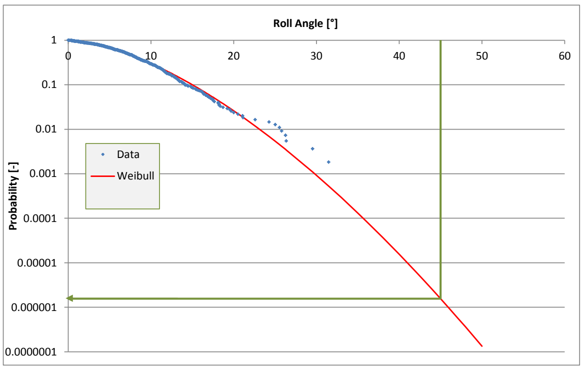

Fig 1: Rough data and Weibull law of probability fitted on the data.

For each run the ship speed is evaluated in waves and kept constant in mean by a slight variation of the rotational velocity of the shafts. This long and tedious procedure avoid bias effects that may impact the evaluation and the representation of capsizing risk as reported on polar plots of Figure 2.

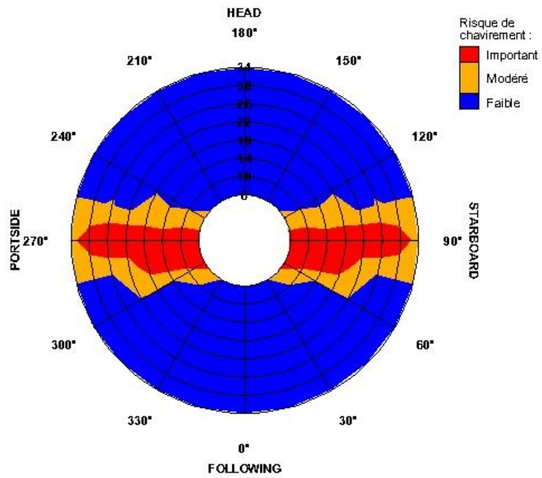

Fig 2: Polar plot of capsize probability (Hs = 4.5 m, Tp = 8 s, T = 3600 s).

For the evaluation of long term probability, an atlas of sea state describing the relation Hs-Tp for North Atlantic has been used (US Coast Guards Table) and a Pierson-Moskovitz sea spectrum is assumed. The long term probability if then evaluated by:

$$P_{cap}\left(X,T\right)=\left(\sum_{i}\alpha_{i}\left(1-p_{i}\left(X\right)\right)^{\frac{T_{SS}}{T_{Z_i}}}\right)^{\frac{T}{T_{SS}}}$$

Where α_i is the probability of occurrence of conditions i (issued from the atlas), T_{Zi} is period of zero up-crossing of the roll movement on the i^{st} condition, T_{SS} is the characteristic duration of one sea state, 4 hours in our case, and T is the total duration of the probability estimation, 30 years in the present study. From these computations a general operability has been deduced both function of the operability on a specific sea state and of the probability of appearance of the sea state in the chosen atlas of waves. This value, presented in figure 3 for North Atlantic, is thus a function of the chosen atlas.

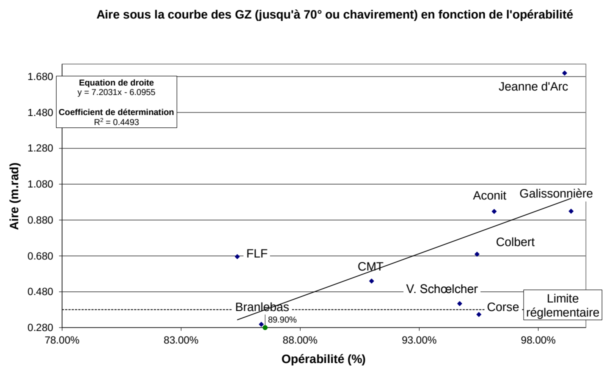

Fig 3: Correlation between GZ area curva and operability.

GLOBAL TECHNIQUE

The dynamic stability of several warships is analyzed and compared according to their respective rolling dynamics due to beam seas. Mathematically the dynamical system reduces to a second order differential equation. The excitation is due to a regular wave. Nonlinearities are introduced in the restoring moment and a quadratic damping moment. Standard hydrostatic softwares provide the restoring moment. As a result this moment (Virgin, 1987), represented with a discrete function, is fitted with a high order polynomials. It appears that an eleventh order polynomials is high enough to cover the wide range of ships. The other coefficients of the differential equation follow from standard seakeeping codes: "in air" and added inertia, linear and quadratic damping coefficients and excitation moment. In the present analysis, all coefficients of the differential equation are constant.

Among the methods that allow a global analysis of nonlinear dynamical system, Melnikov's method (Guckenheimer & Holmes, 1983) is proposed to quantify the loss of stability by measuring the distance between stable and unstable orbits. This distance is known as Melnikov's function and its zeroes define the Melnikov's criterion. The latter provides an easy way to link all the coefficients which define the dynamical system, provided those coefficients are constant in time.

The space of these coefficients (or parameters) can be hence separated in "safe" or "unsafe" areas and that separation follows from an analytical formula. Indeed the computation of that formula does not require a great amount of computational resources. For engineering purpose, as shown in Spyrou (Spyrou, 2011), such formula is of crucial interest since it allows to easily class ships with respect to pre-defined criteria. However it is well known that Melnikov's criterion might be too conservative.

For example, given the period of excitation, the predicted critical wave height above which the ship may capsize, is too small. Then it is an issue to correlate Melnikov's criterion with other approaches. One of those approaches is rather simple but might require tremendous computational resources if the implemented algorithms are not optimized. Here the technique which analyses the erosion of attraction basin (Thompson, 1990) is combined with an interpolated Cell-to-Cell Mapping(Tongue & Gu, 1988). The illustrative results of such a method are the typical "Dover cliffs" plots, showing the area of the eroded basin plotted in terms of two arbitrary parameters among: the wave amplitude, the wave frequency, the damping coefficients, the polynomials coefficients of the restoring moment,...

It is then observed that, in many situations, Melnikov's criterion follows the edge of the "Dover Cliff". However that depends on several conditions such that:

- The type of selected items listed above,
- The thresholds which define whether the computed orbits in the phase space escapes or not from the attraction basin.

The typical differential equation which simulates the dynamics of the rolling motion reads:

$$I\ddot{\phi}+B_1\dot{\phi}+B_2\dot{\phi}\left|\dot{\phi}\right|+C\phi\left(1-\frac{\phi^2}{{\phi_v}^2}\right)Q\left(\frac{\phi^2}{{\phi_v}^2}\right)=Af\sin(\Omega\theta)$$

(1)

where single over dot and double over dots denote first and second derivative with respect to time θ respectively. Polynomial Q(y) is non dimensional. It never vanishes whatever the value of ϕ and it reads

$$Q(y)=1+\sum_{i=1}^{N} a_i y^{2i}$$

so that the restoring force is represented with a polynomials of order 2N + 3. In the present study N = 4 and the highest polynomials is of order eleven. The solutions of time differential equation are separated in two categories: either bounded or unbounded. Indeed the present choice of polynomials lead to asymptotic infinite value of roll angle after the angle vanishing stability is reached. That is not physical but that essentially means that capsizing is a dramatic and not reversible event. That is why the word unbounded means that the rolling angle is greater than a threshold.

In practice it suffices to set this threshold at (or close to) the angle of vanishing stability.

Equation (1) is made non dimensional by introducing the following parameters

$$x=\frac{\phi}{\phi_v},\ \theta=\tau\sqrt{\frac{I}{C}},\ a=\frac{f}{\phi_v C},\ b_1=\frac{B_1}{\sqrt{CI}},\ b_2=\frac{\phi_v B_2}{I},\ \omega=\Omega\sqrt{\frac{I}{C}}$$

Hence yielding

$$\ddot{x}+b_1\dot{x}+b_2\dot{x}\left|\dot{x}\right|+x\left(1-x^2\right)Q(x)=Aa\sin(\omega\tau)\quad(2)$$

where from now on the over dots denote the derivatives with respect to τ.

It appears that equation (2) covers all the possible ships. Hence that makes it possible to compare them. Figure (4) shows the polynomials $p(x)=x\left(1-x^2\right)Q(x)$ for 8 of the studied warships.

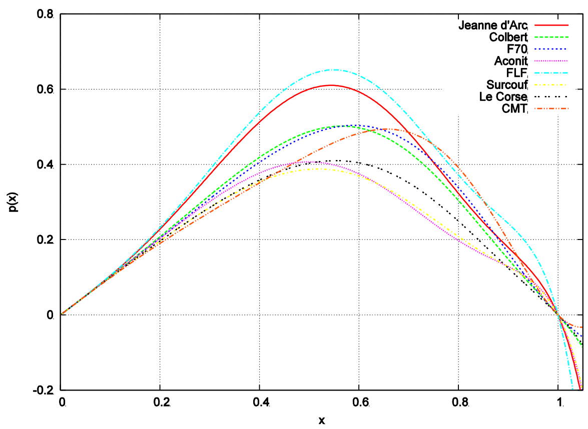

Fig 4: Variation of the restoring moment p(x) with the non dimensional rolling angle x.

It should be noted that such variations could not be fitted with simple third or quintic order polynomials. The Hamiltonian of the unperturbed dynamical system follows from the sum of the kinetic energy $E_c=\frac{1}{2}\dot{x}^2$ and potential energy $E_p(x)=\int_0^x \xi\left(1-\xi^2\right)Q(\xi)\,d\xi$.

The heteroclinic orbits which links the two symmetric equilibrium points ($x=\pm1, \dot{x}=0$) follow from the integration of the conservation law

$$E_c+E_p=E_p(1)=\frac{1}{4a_4}\left(1+2\sum_{n=1}^{4}\frac{a_n}{(n+1)(n+2)}\right)$$

(3)

Equation (3) is space integrated yielding the time variation of x and $\dot{x}$, hence the boundaries of the unperturbed attraction basin, denoted S_0. In practise the Melnikov function depends on the following two integrals

$$D_k=\int_{-\infty}^{+\infty}\left|\dot{x}(t)\right|^{k+1}dt,\quad k=1,2\quad(4)$$

$$F(\omega)=\int_{-\infty}^{+\infty}\dot{x}(t)\cos\omega t\,dt$$

We end up with Melnikov criterion which reads

$$A=\frac{D_1}{F}\frac{uB_1}{f}+\frac{D_2}{F}\frac{u^2B_2}{f},\ (5)$$

with $F=F\left(\Omega\sqrt{\frac{I}{Ca_4}}\right)$ and $u=\phi_v\sqrt{\frac{Ca_4}{I}}$

where we isolate the critical amplitude A in terms of the dimensional parameters. This simple equation is of crucial interest for designers since it separates the space of parameters into "safe" and "unsafe" areas. It can be concluded that the smaller the function

F, the safer the corresponding ship. In that direction, it is worth reminding that $F(\omega)$ may vanish for an infinite set of frequencies ω as soon as $a_1 \neq 0$ (Scolan, 1997). However in the present context that character is of little interest. In the three dimensional space of parameters B_1, B_2, A, the higher the coefficients of (B_1, B_2) the higher the critical wave height and the safer the corresponding ship. Hence, we can compare or classify ships depending on the criterion (5). Figure 5 shows the classification of 9 warships by plotting their associated coefficients (D_1, D_2). It is worth noting that D_1 and D_2 can be easily calculated analytically. The former is exactly half the area of the undisturbed attraction basin

$$D_1 = \int_{-\infty}^{+\infty} |\dot{x}(t)|^2 dt = \int_{-1}^{1} \dot{x}\, dx = \sqrt{8}\int_0^1 \sqrt{E_p(1) - E_p(x)}\, dx$$

The latter is the area below the potential energy curve,

$$D_2 = \int_{-\infty}^{+\infty} |\dot{x}(t)|^3 dt = \int_{-1}^{1} \dot{x}^2 dx = 4E_p(1) - 4\int_0^1 E_p(x) dx$$

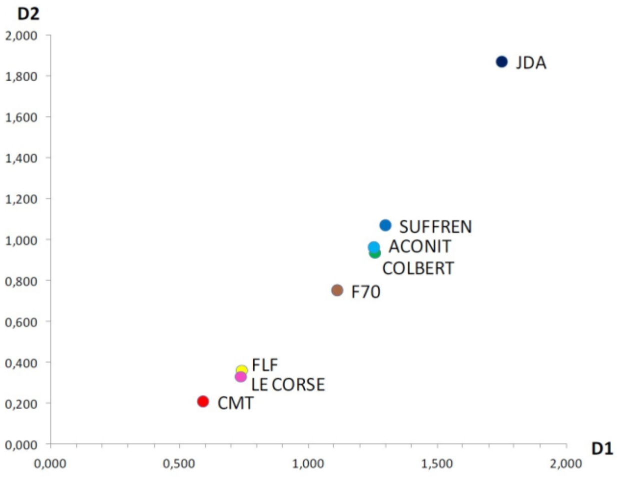

Fig 5: Coefficients (D_1, D_2) (see equation 4) for 9 warships of the French Navy.

As it is done in (Bikdash, Balachandran, & Nayfeh, 1994), a parametrical analysis of D_i in terms of a_i would bring new insights into dynamical stability criteria.

We perform a first parametric study in terms of the threshold of the Cell-to-Cell mapping. It is decided that the rolling motion is supposed to be critical above a threshold between 70% up 120% of the vanishing stability angle ϕ_V. Figure 6 shows how fast the erosion occurs depending on that modified threshold. It is observed that above 100% of ϕ_V, the erosion varies similarly with A. Below, the erosion occurs more and more abruptly. That can be correlated with the direct assessment of capsizing probability.

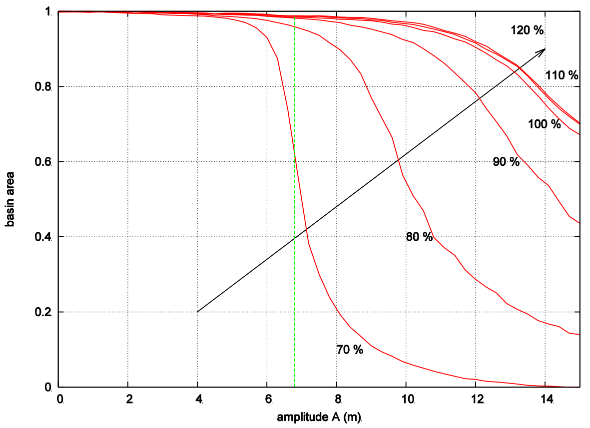

Fig 6: Variation of the area of the attraction basin with the amplitude of the excitation wave for warship F70. The arrow corresponds to an increasing threshold from 70% up 120% of the angle of vanishing stability.

The next parametric study confirms that Melnikov's criterion may be too conservative. Figures (7 a) shows the variation of the attraction basin area S(A,ω) with the wave amplitude A and the nondimensioned circular frequency ω for three vessels: jda (Jeanne d'Arc), F70 and CMT. Indeed the superimposed curve which corresponds to the equation (5) linking A and ω follows the edge of the "Dover cliff" at least for low frequencies.

The rate at which erosion occurs is a feature which has been barely explored so far. In figure (7 b), shows the variations of the quantity $\|\vec{\nabla} S\|$; that is the modulus of the gradient of the function S(A, ω). A criterion which can range the ship follows from the location of the maximum of $\|\vec{\nabla} S\|$ in the parameter space (A, ω), *i.e.* the sea state. The previous classification of the ships regarding the location of the greatest rate of erosion relatively to the highest wave amplitude A, is hence confirmed.

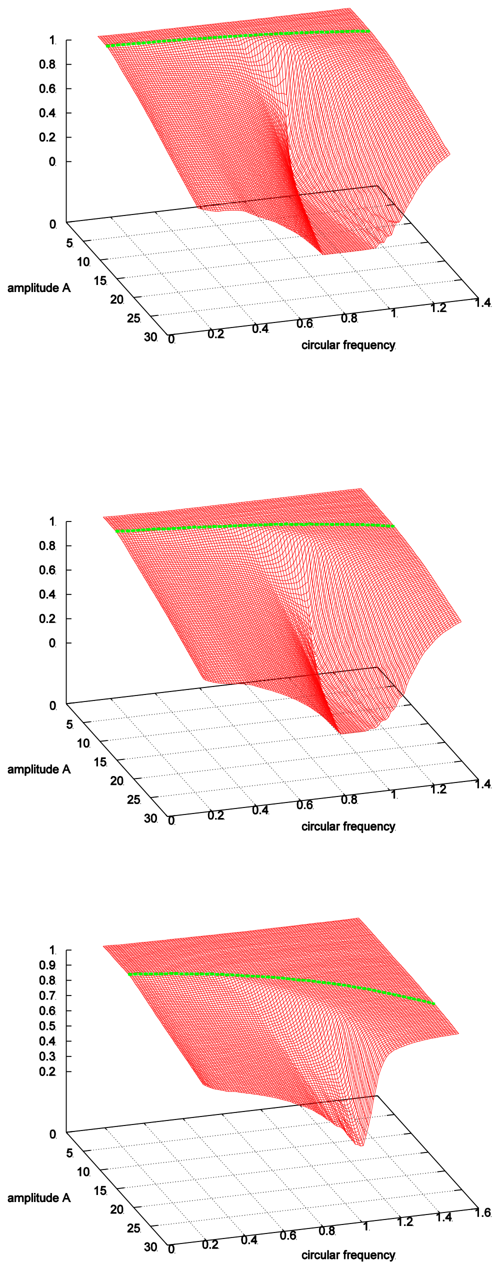

Fig 7 a: variation of the area of the attraction basin S(A, ω) made nondimensioned with S_0. Melnikov's criterion is superimposed with a green line. From top to bottom: CMT, F70, Jeanne d'Arc.

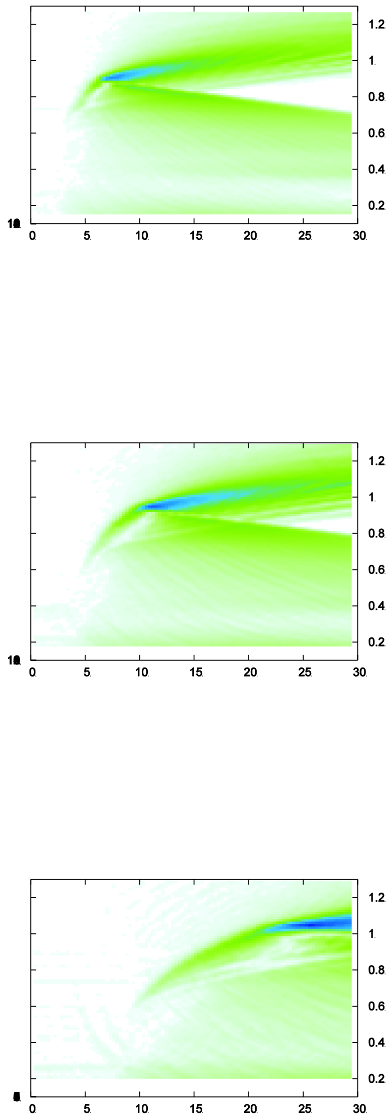

Fig 7 b: isocontour of the gradient $\left\|\vec{\nabla}S\right\|$ in the parameter space (A, ω), horizontal and vertical axes espectively. From top to bottom: CMT, F70, Jeanne d'Arc.

CONCLUSIONS

Different conclusions can be drawn from this research:

- The two methods based on completely different physical hypothesis lead to the same classification of the ships. The analytical method prevails by its rapidity and the second one allows an evaluation of the capsize risk;

- As far as the analytical method takes into account the ship characteristics and a periodic excitation we can infer that the two phenomena that control the probability of capsizing for frigates are the amplitude of excitation and the nonlinear behavior of the oscillator. The random character of the swell acts secondary;
- The rate of erosion of the attraction basin gives an interesting insight in the importance of the choice of the angle of capsizing. From this observation a perspective work is the determination of the angle of capsizing that can be different for each ship in the database. This association of a angle of capsizing different for each ship is consistent with the well-known variation of this parameter from one ship to another.

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