The study of ship capsize on random beam waves

Xianglu Huang
Shanghai Jiao Tong University

Abstract

The problem of ship capsize on a random beam waves was studied both theoretically and numerically. The theoretical method used in the study is the theory of Markov process. Based on such approach a concept of time before capsize was purposed. It seems in the framework of this method still remain some fundamental problems which has to be clarified. Another approach is to find the probability of capsize from the PDF of ship rolling amplitude and the condition of ship capsize. By using the technique of energy level and stochastic average, the FPK equations govern such PDF were established. From such equations and the method of Melnikov function., the probability of capsize for a given random wave train can be estimated from the rolling amplitude probability distribution function. Some numerical results were presented.

Key words Ship capsize Random beam waves Melnikov function

Introduction

The problem of a ship capsized on a random wave train involves two problems. First is to find the conditions of the happening of ship capsize, the other is at what time the conditions are fulfilled or in other word the capsize happen. As the basic framework of study, it is considered that the time domain method of the probability calculation is appropriate, which in our case is the method of Markov process theory. For a dynamic system behave nonlinearly, we can constitute the corresponding Ito stochastic equations according to the original motion equations of the system. Then find out the PDF of ship motions in time domain by solving the corresponding FPK equations. This approach has been applied to the problem of ship oscillation in random beam waves, and there are no obstacles to apply such method to any of the ship oscillation modes and motion conditions. But, the complexity, and the amount of calculation effort may prevent it from realize. We have applied such approach and the concept of first passage in searching the probability of time before capsize of a ship in a given random beam waves, and got satisfactory results[1]. The key point of the method of time domain approach is to determine the boundary of ship capsize. In other words, to get a criterion at which the capsize will happen. Such criterion is a combination of all the factors involved. Actually, such determination must involve the mechanism of capsize. As a first step, we assume that the criterion of capsize is simply the stability vanishing angle in the calculation of the time before capsize. But it is not so simple if the dynamic of ship rolling are taken into consideration. In order to investigate such problem more precisely, let us recall two points. One is that we are now only have factors as rolling angle, velocity and wave height and time derivatives of the elevation of wave. All of the values in hand are temporary values. The second is it seems the energy relation should be taken into consideration as the stability problem are involved. The stability boundary can be determined by the computation prior to the calculation of solving the corresponding FPK-equation. The principle of such calculation is that all of the arguments at the beginning time instant of a time step are considered as the initial values. Then solve the equations to get the values at the end of time step . The initial values will become the boundary of capsize if finally the rolling angle reaches the stability vanishing angle. By a serial computation of variety of ship rolling angle, velocity, wave height and time derivative of wave elevation a boundary of capsize which composed of those motion factors were obtained. Then such information will be applied in the calculation of solving FPK-equation and determine the time before capsize. Obviously, the amount of such computation is tremendous

Another way of determine the probability of capsize is the frequency domain method. In order to explain the basic idea of such approach, we at first consider the process of an oscillation of a ship on waves. From the relation of energy, it is believed that the oscillation will finally reach a stationary state in which the energy of the motion itself and the work done by all the external forces and are balanced. If the balance has not achieved, then the oscillation amplitude will increase or decrease according to the sigh of excess energy. This is very simple to employ such relation in regular wave, but is more complicated if the random waves are involved. One of the approaches to solve the problem is frequency domain method. It is easy to employ if the problem is in linear sense. That is the common way we treated the oscillation of a ship on a random waves In nonlinear cases the problem is not very easy to solve. Roberts has proposed a method which study the process of ship oscillation on a random beam waves in a new motion plane. That is the so called energy level and phase angle. According to the motion equation constituted in such sense, and by using the method of stochastic averaging the corresponding drifting and diffusion coefficients of a corresponding Ito equations can be determined. The physical meaning of such approach is that the wave is random but more regular so that the period of wave oscillation is changing very slowly. It seems the wave has the form of amplitude modification. So, in an interval of wave trace the wave can be considered as regular, the motion of the ship can be determined as the solution of corresponding Duffing equation. The PDF of such oscillation are governed by the corresponding FPK equations, provided the corresponding drift and diffusion coefficients were given. But in principle, the solution of such FPK equation the PDF of the large amplitude rolling should be a function of time. Only on some assumptions we can assume that it is stationary. From this point, it is possible for us to derive the PDF of capsize by combine the PDF of large amplitude rolling and Melnikov criterion. We will discuss this point later.

The main problem remain in the calculation of time before capsize

Time before capsize was defined in a previous paper as a random variable, which characterize the possibility of capsize of a ship on random beam waves. From the theoretical point of view, the method to determine the time before capsize is very simple. But there remain a point difficult to be judged by the time domain consideration. That is the determination of the threshold at which the ship will capsize. We have in our previous calculation simply determined as the stability vanishing angle.

In order to estimate the time before capsize. It is essential to determine the boundary of capsize. In other words, at what condition the ship will capsize. Because of the probability space now is extended by the four variables, rolling angle, rolling velocity wave elevation and wave velocity. The condition or boundary of capsize should be composed of those variables. The motion equation of a ship on a random beam wave train with given stochastic property can be expressed as follow:

$$\begin{cases} (I_{xx}+J_{xx})\ddot{\theta}(t)+\lambda_{44}^{(1)}\dot{\theta}(t)+\lambda_{44}^{(2)}\dot{\theta}(t)\left|\dot{\theta}(t)\right|+D(C_1\theta(t)+C_3\theta^3(t))=M_\theta\zeta(t) \\ \ddot{\zeta}(t)+\alpha\dot{\zeta}(t)+\beta\zeta(t)=\gamma W(t) \end{cases} \tag{1}$$

in which: θ (t) is the rolling angle, $\varsigma(t)$ is the wave elevation.

The condition for the happening of ship capsize will link with the rolling angle over the angle of the vanishing of stability. According to the theory of non-linear oscillation, the angle of stability vanishing is a sandal point. In Hamilton case this point determines the boundary between the stability and instability flow. The existence of wave excitation and damping will change the stability of this boundary and make the stability boundary move, which can be identified by the value of Melnikov function. Also due to the dynamic effect the existence of the velocity and wave excitation also will lead ship to capsize in such small time interval. So we integral the motion equation in time domain to determine what combination of the initial values will result in capsize within such time step length.

$$\begin{cases} \dot{x}_1=x_2 \\ \dot{x}_2=-\dfrac{\lambda_{44}^{(1)}}{I_{xx}+J_{44}}x_2-\dfrac{\lambda_{44}^{(2)}}{I_{xx}+J_{44}}x_2\left|x_2\right|-\dfrac{DC_1}{I_{xx}+J_{44}}x_1-\dfrac{DC_3}{I_{xx}+J_{44}}{x_1}^3+M_\theta x_3 \\ \dot{x}_3=x_4 \end{cases} \tag{2}$$

The integration is on the motion equation (2),in which the equation including white noise are ignored. This is because the inclusion of white noise in the motion equation is to take the randomness of external excitation into consideration. While the determination of the capsize boundary is a deterministic problem. So we don’t need to include the white noise in the expression. The boundary of capsize obtained in this way are shown in fig 1 ~6. Each figure corresponds to a wave elevation. It is very interesting to point out that for some area the angle of roll large over the stability vanishing angle can be reduced to the safe zone ,because of the direction of the oscillation velocity.

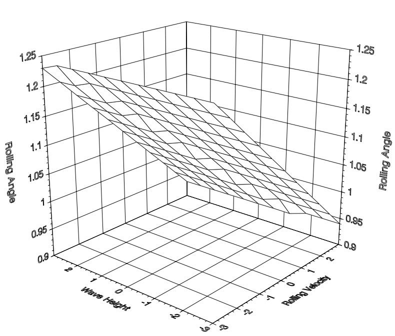

Fig 1

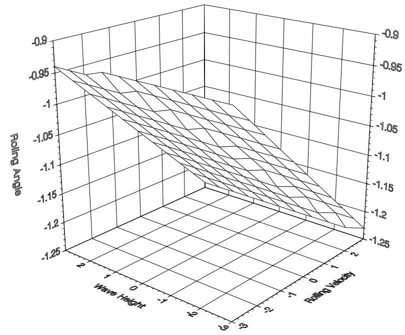

Fig 2

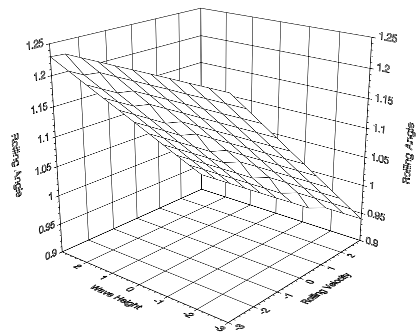

Fig 3

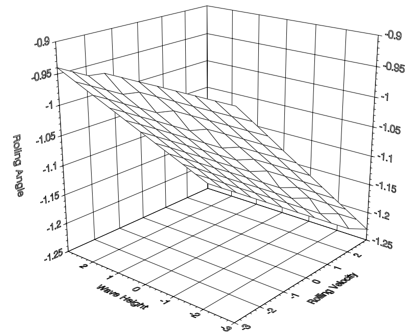

Fig 4

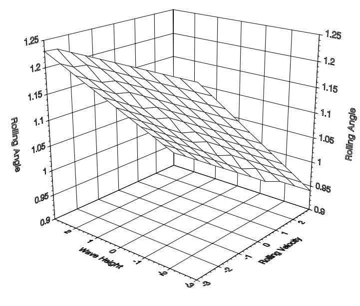

Fig 5

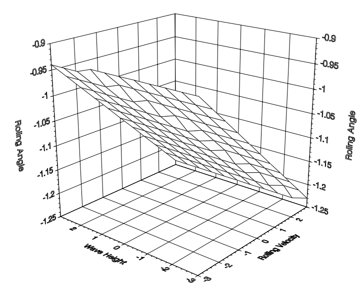

Fig 6

This is only from the intuitive thinking. Because the capsize of a ship involve the dynamics of ship oscillation, the determination of such threshold should consider the dynamic behavior. So, it seems helpful to study the rolling of a ship on random waves in phase plane. We have in the following survey the random oscillation of ship rolling on random beam waves by the method of energy level, and find some interesting results. In the meantime, evidence support our previous consideration to take the stability vanishing angle as the capsize threshold.

Some remarks on the solution of FPK equation of large amplitude rolling:

The FPK equation of the large amplitude rolling in beam waves derived by Roberts has the form as follows

$$\frac{\partial p}{\partial t}=\frac{\partial}{\partial V}\{[\beta A_1(V)-\pi\alpha(V)]p\}+\pi\frac{\partial^2}{\partial V^2}\{V\mu(V)p\} \tag{2}$$

in which: $V----$ The energy level of ship rolling, β -------A parameter quantify the damping function, $\alpha(V)$, $\mu(V)$ is function related to the solution of correspond Hamilltonian of the equation. and the external wave excitations. The theory and its derivation can be found in [2].

In principle, the solution of this FPK equation will depend on time. In other words, the PDF of the rolling amplitude is not stationary. Only in some wave train of low amplitude, the solution can be assumed as stationary. The reason of it is due to the problem of stability. The PDF of the rolling amplitude may obtain by solving the corresponding FPK equation directly. But, before solving such FPK equations, we have to set the initial condition of the probability. It can be determined as the solution of FPK equation assuming that the left term $\partial p/\partial t$ as zero. This is just the solution of stationary PDF. Start from such initial PDF the evolution of the PDF in succeeded time instant can be found by solving the FPK equation together with the condition of capsize.

It is well known that from the FKP equations, the probability conservation law can be expressed as

$$\int_{a_1}^{a_2} p_{t+\iota}(a)da-\int_{a_1}^{a_2} p_t(a)da=G(a_1)\iota-G(a_2)\iota \tag{3}$$

in which $G(V)=\{[\beta A_1(V)-\pi\alpha(V)]p(V,t)\}+\pi\frac{\partial}{\partial V}\{V\mu(V)p(V,t)\}$

If we extend the interval a_1, a_2 to the infinite, then the right hand of equation (3) will become zero provided the oscillation is stationary. But in our case since the capsize will happen when the oscillation amplitude or energy level over some threshold. As we can see in such cases the probability flow will goes out or in other words the probability of such oscillation should be subtracted from the total PDF.

The numerical solution of such problem should be possible. But, we will take a simplest way to solve it using a straight forward consideration. If we consider an amplitude or corresponding energy level in the initial PDF, then it will vary subsequently under the action of wave excitation and damping. The amplitude will increase if there is energy excess. The amount of increase of amplitude is determined by the amount of energy. By using the criterion of Melnikov.. The ship will capsize if the amplitude finally reaches the stability vanishing angle. By using such arguments we are able to determine the PDF of capsize from the initial PDF.

The method of energy level:

As we have mentioned in previous section , Roberts has developed a motion equations system to substitute the ordinary motion equations in which instead of the derivatives of the motion the derivatives of the energy level (the total energy of an oscillation) and phase angles are included. The motion equations has the form as:

$$\dot{V}=-\beta^2 f(V,\theta)-(2V)^{\frac{1}{2}}\sin\theta Y(t) \tag{4}$$

$$\dot{\theta}=-\frac{\beta f(V,\theta)}{(2V)^{\frac{1}{2}}}\cos\theta-\frac{\cos\theta}{(2V)^{\frac{1}{2}}}Y(t)+\frac{g(V,\theta)}{(2V)^{\frac{1}{2}}\cos\theta} \tag{5}$$

in which: $V=\frac{\dot{x}^2}{2}+U(x)$ --- energy level $\dot{x}$ ---velocity $U(x)=\int_0^x G(\xi)d\xi$ ---- Potential energy of oscillation

θ --- phase angle of oscillation β --- the parameter of smallness represent the damping

$f(V,\theta)$ --- function of damping term $g(V,\theta)$ --- function of restoring term and $Y(t)$ is the wave excitation.

By assuming that the damping term is small and the wave excitation is relatively large, Roberts introduces a smallness parameter ε ,which is $\beta = \varepsilon^2$ and assume that the wave excitation is one order higher than damping term, that means

$$y(t) = \varepsilon^{-1} Y(t)$$

Roberts got the final equations as

$$\dot{V} = -\varepsilon^2 a_1(V,\theta) - \varepsilon b_1(V,\theta,y) \tag{6}$$

$$\dot{\theta} = -\varepsilon^2 a_2(V,\theta) - \varepsilon b_2(V,\theta,y) + C(V,\theta) \tag{7}$$

in which:

$$a_1 = -f(V,\theta)(2V)^{\frac{1}{2}} \sin\theta \qquad b_1 = (2V)^{\frac{1}{2}} \sin\theta y(t)$$

$$a_2 = -\frac{f(V,\theta)\cos\theta}{(2V)^{\frac{1}{2}}} \qquad b_2 = \frac{\cos\theta y(t)}{(2V)^{\frac{1}{2}}} \qquad C = \frac{g(V,\theta)}{(2V)^{\frac{1}{2}}\cos\theta}$$

After explored the method of asymptotic averaging developed by Bogoliubov, Roberts finally got the motion equations, which satisfy the condition of stochastic averaging. So, the corresponding Fokker-Planck equations govern the PDF of both the energy level and phase angle were obtained. It should be noticed that this FPK equation is different from the ordinary equation system as the variables in the equation are not motion displacement and velocity, but the energy level and phase angle. It represent the quantity related to the oscillation orbit in phase plan. It implies that the oscillations are some large amplitude oscillation with integrated orbit in phase plane. Refer to the discussion in previous section, we at first solve the FPK equation with the initial condition set as $\partial p / \partial t = 0$. Unlike Roberts, we solve such stationary PDF without any restriction. It give us the initial PDF of rolling, which is only used in subsequent discussion. Then after solving the correspond Fokker-Planck equations, the final expression of stationary PDF of roll amplitude was obtained as

$$p_s(V) = \frac{K_1}{V\mu(V)} \exp\left\{ -\int_0^V \frac{[\beta A_1(\xi) - \pi\alpha(\xi)]}{\pi\xi\mu(\xi)} d\xi \right\} \tag{8}$$

in which: $A_1(\xi)$ ---- The term corresponds to a_1 after averaging.

$\alpha(\xi)$ ---- The term related to the wave excitation which are calculated as

$$\alpha(V) = \frac{\pi}{2\varepsilon^2} \sum_{n=1}^{\infty} (s_n^2 + c_n^2) S_Y(n\omega(V))$$

in which :

s_n ----The coefficients of the Fourier components of $\sin\theta(t)$

c_n ----The coefficients of the Fourier components of $\cos\theta(t)$

$S_Y(n\omega(V)$ --- The spectra of wave excitation

$$\mu(V) = \frac{\pi}{2\varepsilon^2} \sum_{n=1}^{\infty} c_n^2 S_Y(n\omega(V))$$

It is interesting to see that the expression obtained so far is very suitable for the analysis of capsizing because it use the same phase plan as the calculation of Melnikov function. For example, if you look at the term in the numerator term in the integral of the RHS of formula (8), it is similar to Melnikov function calculation. It is known that the oscillation orbit passing through the saddle point which called hetro-clinic orbit, is the boundary of stability of ship oscillation. The stability vanishing point which is the saddle point on the hetro-clinic orbit will become unstable if Melnikov function on it is less than zero. So, if the energy level reaches the stability vanishing angle and at the same time this term become positive. the ship will capsize . It is possible for us to use such consideration to determine the probability of ship capsize. From such consideration, we can predict ship capsize by calculate correspond Melnikov function of the energy level.

It has been proved that a ship will capsize if it reaches the hetro-clinic orbit and the Melnikov function positive. But, from the graphics of eroded safe basin it shows the capsize will happen inside the hetero-clinic orbit if the excitation over some level. In other words the ship can be capsized in an oscillation the amplitude is lower than stability vanishing angle. In order to survey the condition of capsize when the oscillation level is less than hetro-clinic orbit, It is appropriate to discuss further more about such situations. It is know that for a system of Duffing equation with soft spring, the solution of its Hamiliton system in phase plan are a series of close orbit around the zero point inside the hetro-clinic orbit.

The Melnikov function of the sub-orbit of the rolling oscillation can be calculated by following equations[3]

$$M^{m/n}(t_0,\delta,f) = \int_0^{mT}[-\delta y_k(t) + f\cos\omega(t+t_0)]y_k(t)dt$$
$$= -\delta J_1(m,n) + fJ_2(m,n)\cos\omega t_0 \tag{9}$$

in which

$$J_1(m,n) = \frac{8n}{3(1+k^2)^{3/2}}[(k^2-1)K(k) + (1+k^2)E(k)]; \tag{10}$$

$$J_2(m,n) = \begin{cases} 0 & n \neq 1 \quad m \ is \quad even \\ 2\sqrt{2}\pi\omega \csc h\dfrac{\pi m K'(k)}{2K(k)} & n = 1 \quad m \ is \quad odd \end{cases} \tag{11}$$

m,n are order of sub-harmonics, k is the modules of correspond Jacob Elliptic function which depends on the amplitude of oscillation $K(k), E(k)$ are complete Elliptic integrals.

Fig8– Fig10 show the PDF of the rolling amplitude on random beam waves of a ship (the parameters of which were shown in table1) and the corresponding Melnikov functions.

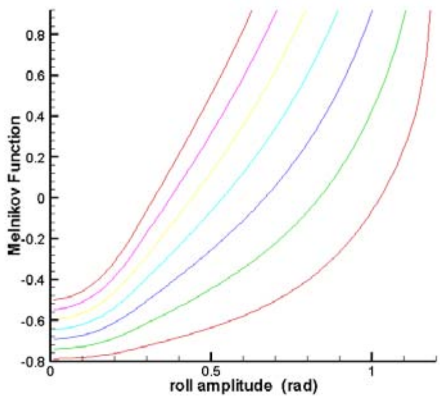

Fig 7 The Melnikov Function of different wave height

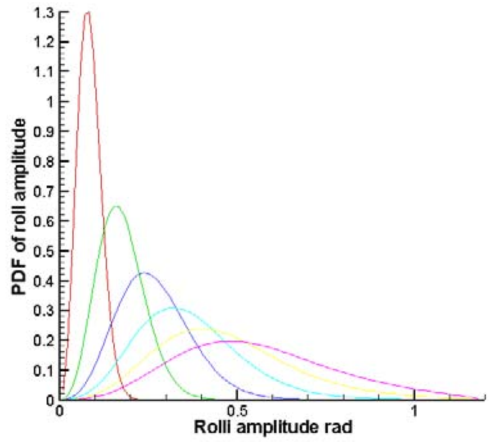

Fig 8. Calculated PDF of a ship on random beam waves with different wave height From 0.2 m to 1.2m 3 second period

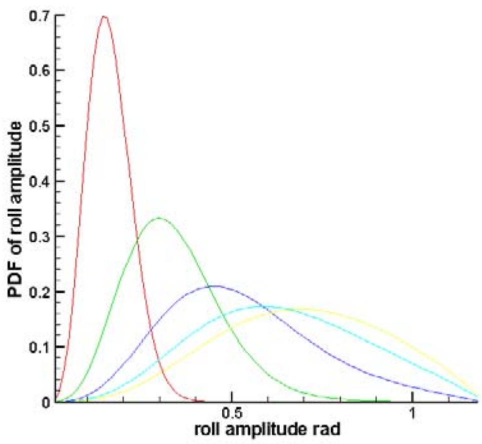

Fig 9 PDF of rolling amplitude of ship on random waves with 5sec period and wave height1m 2m 3m4m 5m from left to right

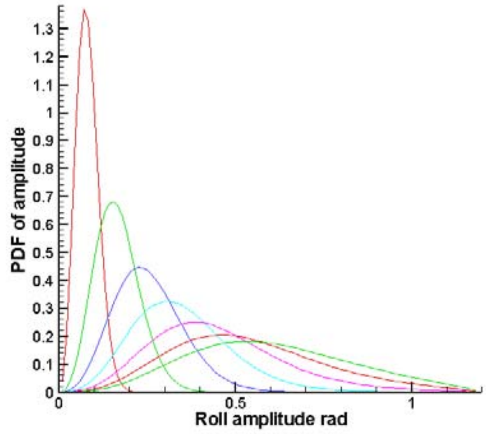

Fig 10 PDF of rolling amplitude on random waves with 7sec period and wave height 3m 4m 5m 6m 7m from left to right

Table 1: Parameters of the ship

A	D	N	C	C3
990ton	4800KN	40	1m	-0.7

A--Inertial coefficient, D--Displacement of ship, N--Damping coefficient,
C--Linear restoring coefficient, C3—restoring coefficient of third order

The relationship between capsize and Melnikov function:

It is well known that the capsize of a ship or in other words the stability of a large amplitude oscillation is closely linked with the Melnikov function which is defined as an integral of external forces on the hetro-clinic orbit of the oscillation, in our case it is on the orbit through the stability vanishing angle of the ship. By investigating the value of the Melnikov function on hetro-clinic orbit, the happening of capsize can easily be identified. But, such criterion only confirm that the ship on such wave is not stable, it does not give us any knowledge about the PDF of capsize. In order to have some idea about capsize, we have surveyed the Melnikov function on the orbits which has an amplitude lower than stability vanishing angle. It shows that in a range of amplitude lower than the value of stability vanishing angle, the Melnikov function calculated on such sub-orbits will have positive value. It means the sub-orbit of large amplitude oscillation exists. But it is not easy to determine immediately if such orbit is stable or not. In the following we try to estimate the stability of such orbit by considering the energy relationship in the oscillation. Coordinate with the PDF of large amplitude rolling we have obtained so far, makes us possible to estimate the probability of capsize of ship on such random waves.

The physical meaning of Melnikov function can be understood as the work external forces has done on the orbit. For hetro-clinic orbit, the orbit will become unstable if the Melnikov function is negative, which implies that the work done by the wave excitation is over the energy exhausted by the damping during one oscillation. But it is not the case if the orbit is inside the hetro-clinic orbit.

In order to judge if it will goes to capsize, let us refer to the relationship of energy and work during ship oscillating on wave. In the oscillation, the work done by the external wave excitation forces and the energy exhausted by the damping of the system should be balanced in one oscillation. It means if the energy is excess, then it should be converted into another form as the kinetic or potential energy, which in turn will increase the energy level of the oscillation. The amount of which can be expressed as the increase of amplitude. By calculating such amplitude increase, it maybe possible to judge if the oscillation amplitude of this orbit reaches the stability vanishing angle. Actually, in most cases, ship will capsize if the amplitude reaches to hetro-clinic orbit or to stability vanishing angle.

The determination of the Probability of capsize

As we have mentioned in previous section that on some orbits low than stability vanishing angle, the Melnikov function may have positive value which implies that there is excess of energy during one oscillation. The increase of the amplitude $\Delta\theta$ due to such excess energy ΔE can be determined by solving the following equation

$$\Delta E = \frac{1}{2}GM(\theta+\Delta\theta)^2 - \frac{1}{4}C_3(\theta+\Delta\theta)^4 - \left(\frac{1}{2}GM\theta^2 - \frac{1}{4}C_3\theta^4\right)$$
$$= GM\theta\Delta\theta - C_3\theta^3\Delta\theta + C_3\frac{3}{2}\theta^2\Delta\theta^2 - C_3\theta\Delta\theta^3 \qquad (12)$$

By using this equation and the Melnikov function obtained from above calculation, the increase of amplitude can be determined for all of the amplitude that has an excess energy. Considered that the increase of amplitude is a small quantity less than 1, it is appropriate to neglect the quantities including higher order $\Delta\theta$ The approximate values of amplitude to which it will increase to the edge of stability due to the wave excitation then obtained. Table 2 show those amplitude

Tab.2 The amplitude of capsize for different wave height

Wave Height (m)	1	2	3	4	5	6	7
Amplitude .(rad)	1.265175	1.136075	1.032796	0.9940657	0.8262364	0.7358668	0.6584072

It is believed that for those amplitudes, due to the action of random waves the ship will roll to stability vanishing angle and finally capsize. So we can consider that PDF beyond this amplitude
has no meaning and should be deleted. We have according to the results obtained so far of the instability oscillation amplitude in each wave height, calculated the portion of the Probability distribution function which has to be cut off due to the happening of capsize as shown in Fig 5.

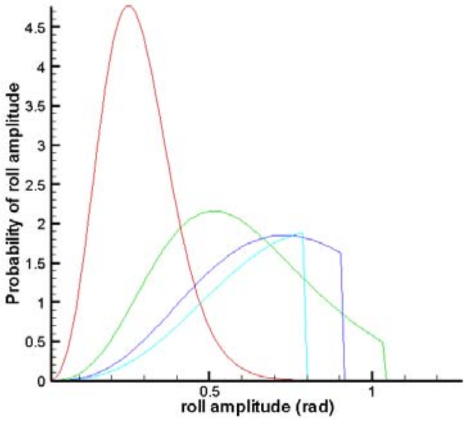

Fig11. The modification of PDF due to capsize

Time domain calculation of the capsize of a ship on random waves:

In order to investigate the probability of capsize of a ship on random beam waves, a series of calculation of safe basin of a ship on given random wave train were performed. The wave train used in the calculation was generated with a random phase angle series, so it is different one by one randomly, which implies that the outcome of the eroded safe basin is also a random event. The results of calculation were expressed as the ratio of the area of the safe basin and the total calculated area of initial conditions. The definition of the safe basin is the area of initial condition for which the ship under the given excitation capsize don't happen. If we consider that one initial condition represents one of the possibilities that ship may meet on a random wave train, the out come of safe basin should be random and have some meaning about the PDF of ship capsize. We have defined in a previous paper the term PS which is the ratio between the calculated area and the area of eroded safe basin, which in our cases is a random variable.

$$PS = \frac{N_{erode}}{N_{total}} \tag{13}$$

where N_{erode} ----- The number of points of not capsize cases, N_{total} ----- The total number of calculated initial conditions

Because of the outcome of PS depends on wave excitation. So it varies randomly for different waves. In other words it is a random variable. We use the average value to represent such quantity. In our calculation a series of wave train (wave height and period) were generated according to given wave spectra and generated occasionally the random phase series. Twenty wave trains were used in the average for one set of wave specification.

If we considered the area of safe basin on very low waves corresponds to the condition of none capsize happen. In other words, the area of safe basin corresponds to very low level of external excitation is considered to be the normal status of ship safety no matter what there are a lot of initial conditions which out of the safe basin will lead ship to capsize. Then the reduction of the area implies the increase of the capsize PDF is

$$P_{capsize} = \frac{PS_{low} - PS_h}{PS_{low}} \quad (14)$$

By using such consideration we converted the results of PS obtained so far into $P_{capsize}$. Fig 12 show the calculated results.

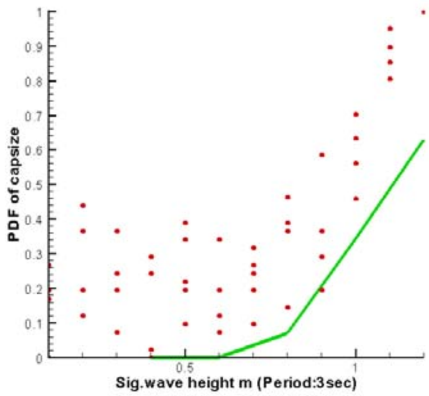

Fig12. Comparison between the estimated capsize probability obtained with those from the calculation of safe basin on a 3sec period waves

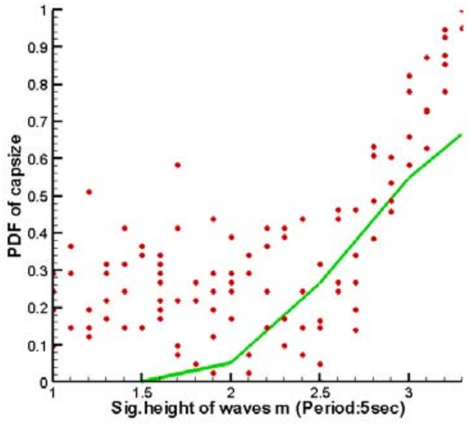

Fig13. Comparison between the estimated capsize probability obtained with those from the calculation of safe basin on a 5 second period waves

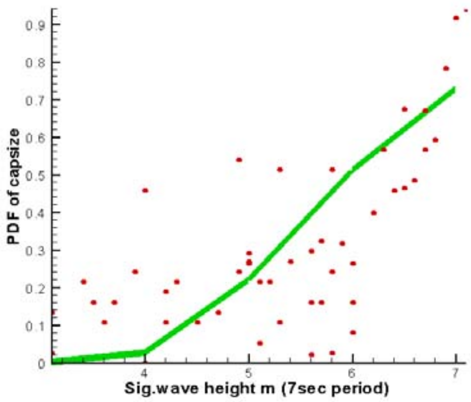

Fig14. Comparison between the estimated capsize probability obtained with those from the calculation of safe basin on 7 second period waves

After comparing the results of the PDF between those obtained from the safe basin calculation and theoretical consideration, several features may have been noticed.

First. It seems the general trend of the PDF of capsize predicted by the theoretical consideration is conformed approximately with the time simulation calculation. But the reality of capsize is more scattering than the theoretical prediction. This is due to the theory is based on the analysis of large amplitude oscillation, which means the oscillation is varied more slowly. But this is not the case in real sea.

The second is that from the graphics of the capsize PDF of ship on waves of different average period, one can seen that the influence of period is clear. It shows that the predicted PDF (green line) is lower than the simulated one (scatter points) in shortest period 3second, while it raises when the period increases. It may be explained by the condition of jumping on different branches of the nonlinear oscillation response function.

Concluding remarks

From above results obtained both theoretically or numerical tests several conclusions may drawn:

1. It seems the comparison between the probability of capsize predicted from the consideration of Melnikov function and the probability obtained from the safe basin calculation is not very bad. The discrepancy between them can be explained by the randomness of waves, since in our theoretical model the narrow band of wave spectra are required, which implies that the wave are closer to regular. The results looks like that the randomness of wave may have some effects on the happening of capsize.
2. Comparison also implies that the concept of phase flux of safe basin is useful in the study of the ship capsize. But in order to obtain the probability of capsize directly, the Melnikov function on sub-harmonics of oscillation have to be investigated.
3. From the graphics of the PDF of the rolling amplitude of ship on random beam waves obtained so far, it shows that the upper end of the rolling amplitude is around the stability vanishing angle. This evidence support our use of the stability angle as the threshold of capsize.

There are still several problems remain. Although we have in our study used the relationship of work and energy in one oscillation to estimate the condition of capsize, but it may be considered as a intuitively guess. The rigorous prove should be found to solidify the foundation of such prediction. Also the influence of the randomness of waves and oscillation have to be studied in detail.

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