Critical Distance on a Phase Plane as a Metric for the Likelihood of Surf-Riding in Irregular Waves

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Abstract: The paper addresses the formulation of a metric for the likelihood of surf-riding in irregular waves. This likelihood is a critical element of the split-time method that allows the inclusion of physics in statistical extrapolation. The candidate metric is the distance on the phase plane between the current position and the instantaneous boundary of attraction to the stable surf-riding equilibrium. The distance is measured along the line connecting the position of the dynamical system and the stable surf-riding equilibrium at the initial moment.

Key words: Surf-riding, dynamical system, equilibrium, attraction.

1. Introduction

The split-time method [1] is a procedure for evaluating the probability of a rare stability failure in irregular waves from relatively short samples of time domain data. The application of the method requires the formulation of a metric for the likelihood of stability failure that can be computed at certain, non-rare instances in the time domain. For the case of capsizing due to pure loss of stability, the metric is the difference between the observed and critical roll rate at the instant of upcrossing of an intermediate threshold. The objective of the present study is to formulate such a metric for surf-riding in irregular waves.

The physical mechanism of surf-riding includes the appearance of dynamical equilibria and a ship's attraction to the stable equilibrium [2]. The equilibria appear when the wave surging force becomes large enough to offset the difference between the ship's thrust and its resistance at wave celerity. The equilibrium points are the positions of the ship on the waves where the forces balance exactly. The dynamics of surf-riding in regular waves is fairly well understood [3], but surf-riding in irregular waves is to large extent *terra incognita*. Some advances in the understanding of surf-riding in multi-frequency waves are described in [4]. One of the most significant issues in this area has been the development of an effective definition of wave celerity in irregular waves and practical procedures for calculating it. Unlike the regular wave case where wave celerity is constant, celerity in irregular waves will vary in both space and time and must be considered as a stochastic process of its own. In a similar fashion, the magnitude of the maximum wave surging forces in irregular waves will be varying in space and time.

With this time-dependence of both the wave celerity and the maximum surging forces in irregular waves, a balance of the wave surging, thrust and resistance forces may not always be possible, so the surf-riding equilibrium may exist for only a limited time. Because the time of the existence of equilibrium is not usually limited in mechanics, it would be more

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appropriate to use the term “quasi-equilibrium” in relation to surf-riding in irregular waves.

Indeed, the existence of the quasi-equilibrium is a necessary, but not sufficient, condition for surf-riding, as actual surf-riding includes ship’s attraction. In the case of regular waves, the appearance of the system inside the area of attraction to the equilibrium is the sufficient condition of surf-riding. The formulation of the sufficient conditions for irregular waves is more difficult. Even while the quasi-equilibrium exists, there is no reason to believe that the area of attraction stays the same. It is quite possible also that the topology of phase plane may change back and forth between “coexistence of surging and surf-riding” and “surf-riding only.” To accommodate this, the sufficient condition for surf-riding can be formulated in terms of the distance, in the phase plane, between the instantaneous positions of the quasi-equilibrium and the dynamical system. This formulation may be further extended with a requirement for the dynamical system to spend a certain amount of time in the vicinity of the quasi-equilibrium, thus allowing time for the ship to reach surf-riding condition. This condition is especially important when considering broaching-to following surf-riding, as it may take some time for the yaw instability (if it exists) to develop into broaching-to.

If the quasi-equilibrium does exist at an arbitrary instant of time, there is a neighborhood around the quasi-equilibrium that corresponds to surf-riding and that will exist while the quasi-equilibrium exists. Consider the position of the dynamical system on the phase plane at this instant. If this position is located within the neighborhood, then surf-riding occurs. The distance to the boundary of such a neighborhood can therefore be considered as a possible candidate for the metric of the likelihood of surf-riding. The distance can be measured by the line between the ship’s position and the quasi-equilibrium, but must account for the time dependence of the equilibrium and the neighborhood.

2. Mathematical Model

Consider a simple model for one-degree-of-freedom nonlinear surging:

$$(M + A_{11})\ddot{\xi}_G + R(\dot{\xi}_G) - T(\dot{\xi}_G, n) + F_X(t, \xi_G) = 0 \qquad (1)$$

Here M is mass of the ship, A_{11} is longitudinal added mass, R is resistance in calm water, T is the thrust in calm water, n is the number of propeller revolutions, F_X is the Froude-Krylov wave surging force, and ξ_G is longitudinal position of the center of gravity in the Earth-fixed coordinate system; the dot above the symbol stands for temporal derivative. Polynomial presentations are used for the resistance and thrust:

$$\begin{aligned} R(U) &= r_1 U + r_2 U^2 + r_3 U^3 \\ T(U,n) &= \tau_1 n^2 + \tau_2 nU + \tau_3 U^2 \end{aligned} \qquad (2)$$

The coefficients r and τ are meant to be fit to the appropriate calm water curves [5].

As the model is meant at this stage to be qualitative, a linear wave-body formulation is appropriate. Therefore,

$$F_X(t, \xi_G) = \sum_{i=1}^{N} A_{Xi} \cos(k_i \xi - \omega_i t + \varphi_i + \gamma_i) \qquad (3)$$

Here A_{Xi} is the amplitude of the surging force for each component frequency of the incident wave, while γ_i is the phase shift between the wave elevation and the force components. Details of the surging force calculation can be found in [1].

3. Candidate Metric – Distance to the Manifold

First, consider the case of regular waves. The boundary for the domain of attraction to a stable equilibrium is the unstable invariant manifold. It can be computed by integration in inverse time from unstable equilibrium, as illustrated in Fig. 1.

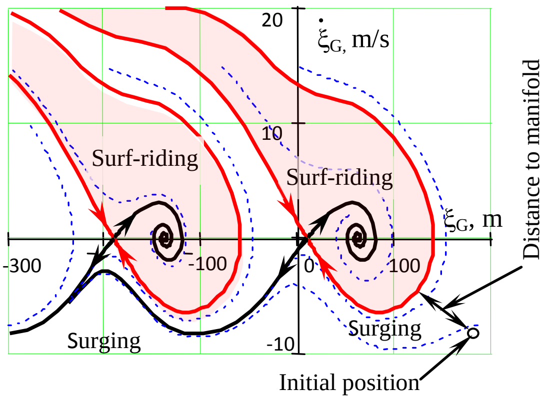

Fig. 1 – Phase plane for co-existence of surging and surf-riding in regular waves

However, the direct calculation of the invariant manifold may be not trivial in numerical sense. It requires careful management of the step of integration. In principle, such an approach can be extended to irregular waves [6], but the calculation cost renders such an approach impracticable.

However, it is not necessary to know the entire manifold in order to evaluate the distance. To find the single point on the manifold that characterizes the distance, one may consider a perturbation algorithm, similar to [1]. Fig. 2 illustrates such a calculation that consists of short simulations. Initial conditions for these simulations lie on the line connecting the dynamical system's position and the stable equilibrium at an arbitrary instant of time.

The initial position in Fig. 2 corresponds to surging. The variation of the initial conditions along the line (between the initial position and the stable equilibrium) defines an iterative process that converges to the critical point, at which the difference between initial conditions leading to surging and surf-riding falls below a pre-defined tolerance. These calculations converge after 9~12 iterations with the relative tolerance at 0.1%, and they take about a second on a single processor of a standard laptop computer.

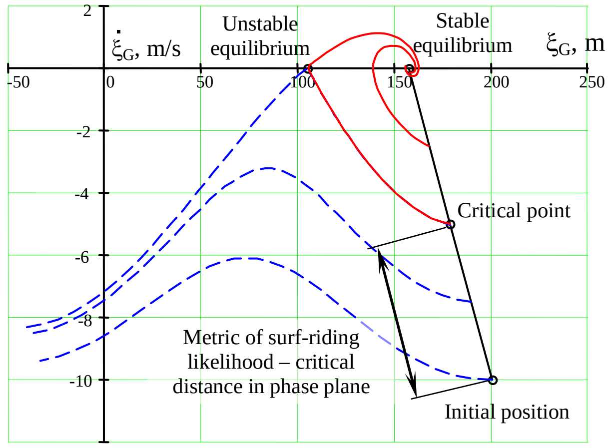

Fig. 2 – Perturbation algorithm to find a "distance to manifold"

5. Metric in the Time Dependent Dynamical System

The introduction of the irregular waves into the dynamical system defined by equation (1) essentially makes it time dependent [7]. Prior to the full implementation of irregular waves, the concept can be tested by considering an artificial time dependence consisting of simultaneous changes of wave frequency and amplitude, as illustrated in Fig.3. These changes alter the balance between thrust and resistance (see Fig.4). As a result, the surf-riding equilibria cease to exist around t=280 seconds, and surf-riding becomes impossible after that time. Fig.5 shows the evolution of the surf-riding equilibria caused by these changes to the waves.

The introduction of time dependence into the dynamical system changes the situation significantly. The surf-riding equilibria move, the domain of attraction changes, and the boundaries of the attraction move and are no longer invariant. However, the calculation result for the metric with perturbations looks very similar to the regular wave case, as shown in Fig. 6.

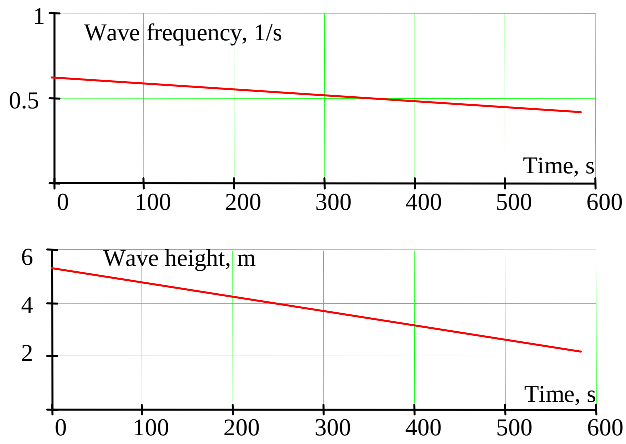

Fig. 3 – Wave parameters for artificial time-dependence

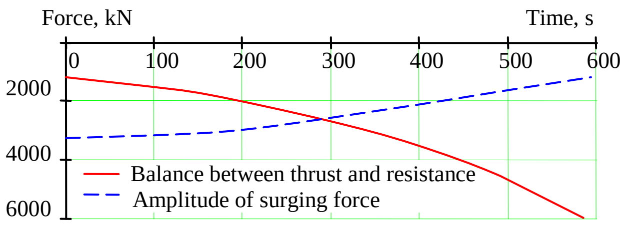

Fig. 4 – Changes in forces caused by time dependence

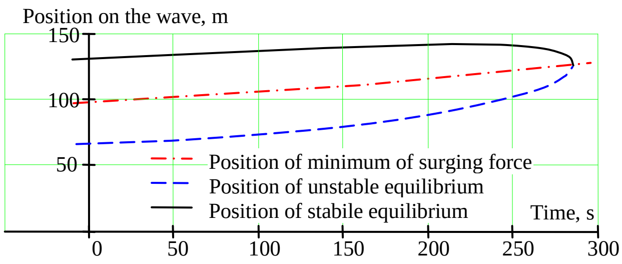

Fig. 5 – Evolution of surf-riding equilibria

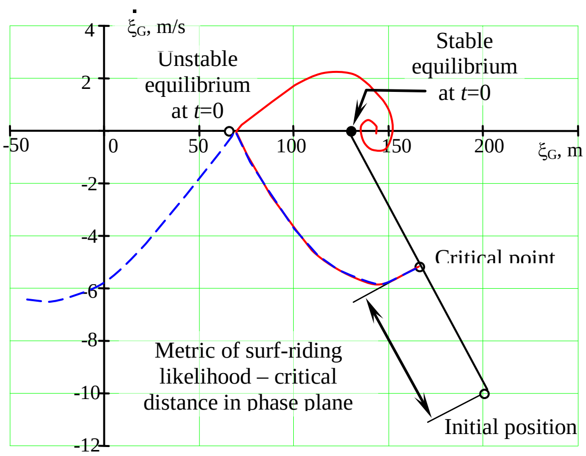

Fig. 6 – Perturbation algorithm to find a "distance to manifold"

However, some changes can be seen. The initial positions of the equilibria are no longer in the centers of the saddle and focus points. Indeed, the saddle point is located where the unstable equilibrium will be when the dynamical system will reach that position in phase plane. The same statement can be made with regard to the position of the stable equilibrium and the focus point.

6. Conclusions and Future Work

The present study addresses the formulation of a metric for the likelihood of surf-riding that could be applied to the case of irregular waves. Indeed this formulation implies existence of surf-riding equilibria at the time instance when the metric is evaluated.

The candidate metric is a distance between a given position of the dynamical system in the phase plane and the boundary of attraction to the stable surf-riding equilibrium. The metric is measured along the line between the position of the dynamical system and stable equilibrium at the same instant of time. It has been demonstrated that the candidate metric can be computed for a model of the dynamical system incorporating a time-dependence of the wave parameters.

The next step is to determine if the metric can be computed for the dynamical system under stochastic excitation and then whether the occurrence of surf-riding in a long series of simulations can be predicted by extrapolation of this metric from a short series of simulations.

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