MELNIKOV'S METHOD FOR NON-LINEAR ROLLING MOTIONS OF FLOODED SHIP

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Abstract

To investigate the rolling motions of flooded ship, non-linear coupled dynamics of roll and flooded water are derived using Lagrange's principle. Decoupling the coupled equations gives a single-freedom ordinary derivation equation, which can describe the rolling motion of the flooded ship. Melinkov's method is used to predict the chaos behavior of the ship. Numerical computations are performed for three statuses of a ship model. Comparison shows that the more cabins flooded, the more likely a chaos phenomenon will appear.

Key word： radiaton，finite element method，boundary element method，stochastic analysis

1. Introduction

A ship in waves can exhibit nonlinear response, which may lead to undesirable motion, including capsizing. Many researchers have considered nonlinear ship motion in waters[1][2].

Nonlinear motion of a flooded ship in water has been investigated during these years. In the past years, the stability of flooded ship is estimated by modifying that of the original ship in the view of stability lost. However, the flooded water has some critical effects on ship motion in waves. And it was indicated that the nonlinearly coupled dynamics of roll and flooded water is the key to understanding the problem.

For the nonlinear motions of coupled system, it was studied in the wide range of science and engineering fields. It is not easy to elucidate the intricate mechanism of these systems, but application of the dynamical system theory is one of a number of promising approaches. Many researchers have considered nonlinear ship motion in waves theoretically. Lots of nonlinear phenomena, such as bifurcation and chaotic, are revealed using nonlinear mathematical models for ship motions[3]. On the other hand, not only theoretical, but also experimental, works are indispensable, because it is hard to get an exact mathematical model for this complex problem[4].

In this paper, a mathematical model for nonlinearly coupled motions of roll of the ship and flooded water in regular waves are derived using Lagrange's principle. To focus on the motion of the ship, the coupled equations are reduced to a single-freedom nonlinear ordinary derivation equation, where the effect of the flooded water is considered. Melnikov method[5] is used to analyze the conditions of chaos rolling motion. A ship model is used as the object and three statuses are considered. Numerical results are compared and shows that, with the same amount of water, the more cabins flooded, the more likely a chaos phenomenon will appear.

2. Mathematical model of the nonlinear motions of flooded ship

To simplify the model, we assumpt: (i) Only roll motion of the ship and flooded water are considered and sway and heave modes are neglected. (ii) The surface of flooded water is flat with slope χ. (iii)The motion of flooded water can be approximated by that of a materical particle located at the center of gravity Gw. (iv) The wave-forcing moment varies sinusoidally with the same angular frequency Ω as incident waves. (v) The damping moments of the ship and the flooded water vary linearly with $\dot{\phi}$ and $\dot{\chi}$, respectively. Two coordinates are set as figure 1.

Gs is the center of gravity. Bs is the location of the center of buoyancy. Gw is the location of the center of gravity of the flooded water. φ is the roll angle of the ship. χ is the angle between the surface of flooded water and coordinate η. θ is the angle between the surface of flooded water and coordinate y.

In this model, the kinetic energy of the ship (K_s) and water (K_w), the potential energy of the ship (P_s) and water (P_w), the rate of energy dissipation(D_v) and the work done by wave (P_f) can be expressed as formula (1).

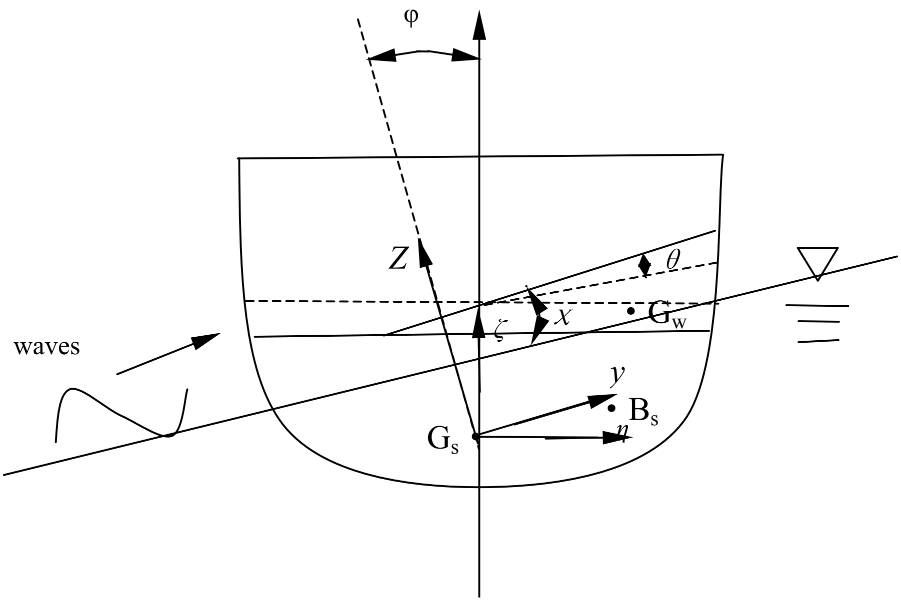

Figure 1 coordinates definition

$$\begin{cases} P_s = \int_0^{\varphi} M_f d\varphi = C_{-1}(\varphi) \quad P_w = mgz_w \\ K_s = \frac{1}{2} I_0 \dot{\varphi}^2 \quad K_w = \frac{1}{2} m(\dot{y}_w^2 + \dot{z}_w^2) = \frac{1}{2}\rho D_0[(\dot{\chi}E_1 + \dot{\varphi}F_0)^2 + (\dot{\chi}F_1 - \dot{\varphi}E_0)^2] \\ D_v = \frac{1}{2}\nu_s \dot{\varphi}^2 + \frac{1}{2}\nu_w \dot{\chi}^2 \quad P_f = M_w \varphi \end{cases} \tag{1}$$

Let L denote the Lagrange function, the Lagrange's principle gives:

$$\begin{cases} L = K_s + K_w + P_s + P_w + P_f \\ \frac{d}{dt}(\frac{\partial L}{\partial \dot{\varphi}}) - \frac{\partial L}{\partial \varphi} + \frac{\partial D_v}{\partial \dot{\varphi}} = 0 \\ \frac{d}{dt}(\frac{\partial L}{\partial \dot{\chi}}) - \frac{\partial L}{\partial \chi} + \frac{\partial D_v}{\partial \dot{\chi}} = 0 \end{cases} \tag{2}$$

Substitute formula (1) into formula (2), a two-freedom coupling equation group is get as formula (3), which can describe the motion of the ship and flooded water.

$$\begin{aligned} C_{11}\ddot{\varphi} + C_{12}\ddot{\chi} + C_{13}\dot{\varphi}^2 + C_{14}\dot{\chi}^2 + C_{15}\dot{\varphi}\dot{\chi} + C_{16}\dot{\varphi} + C_{17}\dot{\chi} + C_{18} = 0 \\ C_{21}\ddot{\varphi} + C_{22}\ddot{\chi} + C_{23}\dot{\varphi}^2 + C_{24}\dot{\chi}^2 + C_{25}\dot{\varphi}\dot{\chi} + C_{26}\dot{\varphi} + C_{27}\dot{\chi} + C_{28} = 0 \end{aligned} \tag{3}$$

Commonly speaking, C_{ij} in (3) is a nonlinear function of φ and χ. Thus (3) is nonlinear ordinary derivation equations in essence. To reduce the two-freedom equations into a one-freedom equation, two steps are performed. Firstly, only linear part of (3) are considered. And it is easy to decoupled the coupling system as:

$$\begin{cases} \varphi = \Phi \sin \omega t \\ \chi = \mathrm{X} \sin \omega t + \mathrm{H} \end{cases} \tag{4}$$

Secondly, substitute the χ in the first equation using formula (4). A single-freedom nonlinear ordinary derivation equation is get as formula (5), which can be used to describe the motion of the ship, including the effects of the flooded water.

$$\begin{aligned}&\left[M+G_1(\omega t)+I_1\varphi^2\right]\ddot{\varphi}+\left[D+G_2(\omega t)\right]\dot{\varphi}+\left[C+G_3(\omega t)\right]\varphi+\\&\left[Q+G_5(\omega t)\right]\varphi^2+\left[N+G_4(\omega t)\right]\varphi^3=F(\omega t)\end{aligned}\tag{5}$$

where: M, I_1, D, C, Q and N are constant, which are related to the ship parameters. $G_i(\omega t)$ and $F(\omega t)$ are functions of time and frequency.

Choosing non-dimensional parameters as:

$$\begin{cases}\Omega_0=\sqrt{\dfrac{|C|}{M}}\\ \varphi=\theta\sqrt{\dfrac{|C|}{N}}\\ t=\Omega_0\tau\end{cases}\tag{6}$$

(5) can be non-dimensionized as:

$$\theta''-\theta'+\theta^3=f(\theta'',\theta',\theta,\tau)\tag{7}$$

where: "’" denote derivation to "τ" ,and

$$\begin{aligned}f=&F(\omega\Omega_0\tau)\frac{\sqrt{N}}{|C|\sqrt{|C|}}-\left(\frac{G_1(\omega\Omega_0\tau)}{M}+\frac{I_1|C|}{MN}\theta^2\right)\theta''-\sqrt{\frac{1}{M|C|}}[D+G_2(\omega\Omega_0\tau)]\theta'-\\&\frac{D_{11}}{M}\sqrt{\frac{|C|}{N}}(\theta')^2-\frac{G_3(\omega\Omega_0\tau)}{|C|}\theta-\frac{G_4(\omega\Omega_0\tau)}{N}\theta^3-\sqrt{\frac{1}{N|C|}}(Q+G_5(\omega\Omega_0\tau))\theta^2\end{aligned}\tag{8}$$

If only the zero and first quantity are kept, (7) gives:

$$\theta''+\varepsilon\cdot d_0\theta''+(-\theta+\theta^3)+\varepsilon\cdot i_1\sin\omega t\theta^3+\varepsilon\cdot q_1\sin\omega t\theta^2=\varepsilon\cdot l_1\sin\omega t\tag{9}$$

where: ε is a small perturbation.

Formula (9) is a well-known Duffing-Holmes oscillator, chaos analysis of which is studied in simple mechanical systems.

3. Melnikov method

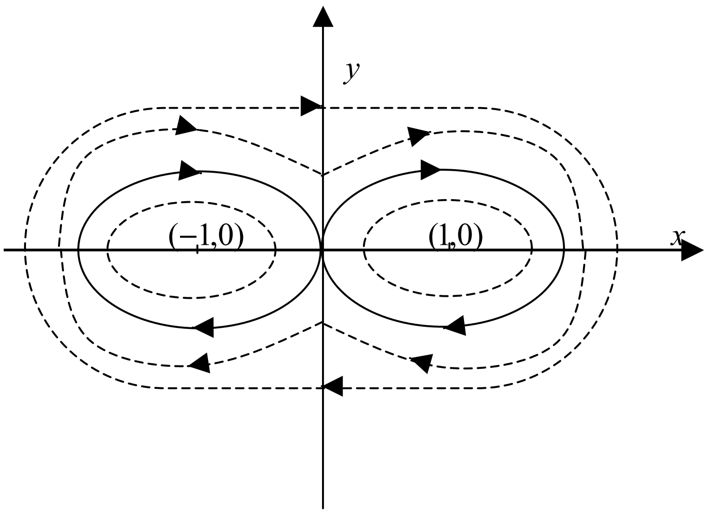

Figure 2 Phase diagram of the undisturbed system

Set $x=\theta$， $y=\theta'$， (9) gives:

$$\begin{cases}x'=y\\ y'=x-x^3-\varepsilon(d_0y+i_1\sin\omega tx^3+q_1\sin\omega tx^2)+\varepsilon\cdot l_1\sin\omega t\end{cases}\tag{10}$$

If $\varepsilon = 0$, (10) becomes a Hamliton system. And Hamliton function gives:

$$H(x,y) = \frac{1}{2}y^2 - \frac{1}{2}x^2 + \frac{1}{4}x^4 = h \tag{11}$$

The non-perturbed system has two centers at $(\pm 1,0)$ and a hyperbolic saddle point at $(0,0)$. The phase diagram is shown in figure 2. The separatrix consists of two homoclinic orbits. The generalized orbits can be written using elliptic function as:

$$\begin{cases} x_{\pm}^0(t) = \pm\sqrt{2}\sec h(t) \\ y_{\pm}^0(t) = \mp\sqrt{2}\tanh(t)\sec h(t) \end{cases} \tag{12}$$

When $\varepsilon \neq 0$, to test for transverse homoclinic intersections, use is made of the well-known Melnikov function[5][6], which is a measure of the distance between the perturbed stable and unstabled manifolds in the Poincare map. The generalized Melnikov function for the homoclinic orbits can be constituted as:

$$\begin{aligned} M_{\pm}(t_0) &= \int_{-\infty}^{+\infty} y_{\pm}^0(t)\{-d_0 y_{\pm}^0(t) - [i_1 \sin\omega(t+t_0)](x_{\pm}^0(t))^3 \\ &\quad -[q_1 \sin\omega(t+t_0)](x_{\pm}^0(t))^2 + l_1 \sin\omega(t+t_0)\}dt \\ &= \int_{-\infty}^{+\infty} y_{\pm}^0(t)\{-d_0 y_{\pm}^0(t) - [(i_1 \cos\omega t_0)\sin(\omega t)](x_{\pm}^0(t))^3 \\ &\quad -[(q_1 \cos\omega t_0)\sin(\omega t)](x_{\pm}^0(t))^2 + (l_1 \cos\omega t_0)\sin(\omega t)\}dt \\ &= -d_0 I_1 - i_1 I_2 \cos\omega t_0 - q_1 I_3 \cos\omega t_0 + l_1 I_4 \cos\omega t_0 \end{aligned} \tag{13}$$

From (13), it is easy to see that if

$$\left| l_1 I_4 - i_1 I_2 - q_1 I_3 \right| > \left| d_0 I_1 \right| \tag{14}$$

Melnikov function has simple zeros and is independent of ε. That means for ε sufficiently small, the stable and unstable trajectories intersect transversely, which implies a chaos phenomenon.

(13) can also be expressed in the view of work. It is easy to find that

$$\int_{-\infty}^{+\infty} y_{\pm}^0(t)[-d_0 y_{\pm}^0(t)]dt = -d_0 I_1$$

is work done by damping moment, while

$$\int_{-\infty}^{+\infty} y_{\pm}^0(t)[i_1 \cos\omega t_0 \sin\omega t (x_{\pm}^0(t))^3 - q_1 \cos\omega t_0 \sin\omega t (x_{\pm}^0(t))^2]dt = -q_1 I_3 \sin\omega t_0$$

$$\int_{-\infty}^{+\infty} y_{\pm}^0(t)[(l_1 \sin\omega t_0)\cos(\omega t)]dt = l_1 I_4 \sin\omega t_0$$

are work done by wave moment and parameter excitation, respectively. And (14) means that if work done by wave moment and parameter excitation greater than that done by damping moment, a chaos phenomenon appears.

Numerical results

To predict the condition of chaos motion of the ship, numerical computations are performed. In this paper, a ship model, designed according to certain true ship on the base of similitude theory, is used as the numerical object. A Fortran code is developed for computation. The Wave moment is increased from zero and (14) is used as a judgment to get the critical wave moment when the chaos motions appear. To verify the chaos motions, numerical simulations are performed on the ship rolling motions. Frequency spectrum is used to analysis the motion characteristic of the rolling motions. The ship model parameters are listed in table 1. Three statuses are computed.

Figure 3 is the results of Status I. Figure 4 is the results of Status II. And Figure 5 is the results of Status III. For each status, four figures are presented. (a) gives the critical wave moments at different frequencies. Chaos motions will appear if wave moment is beyond the critical curve. A wave moment value beyond the critical curve is selected and the ship rolling motions are simulated using fourth-order Runge-Kutta method. The phase diagram of the rolling orbit and the time series are presented in figure (b) and (c). (d) gives the frequency spectrum of the rolling motions. It can be seen that rolling motions excited by wave moment greater than the critical value shows non-periodical characteristic and chaos phenomenon appears.

Table I Ship model parameters

Length	3.11 meter
Width	0.32 meter
Draft	0.092 meter
Weight	42.24 kilogram
Status I	3 cabins flooded with water 8.4 kilogram
Status II	2 cabins flooded with water 8.4 kilogram
Status III	5 cabins flooded with water 8.36 kilogram

Comparison of these three status shows, that for Status III, critical wave moment is the lowest , which implies that chaos phenomenon is the most likely to happen for Status III, and then Status I. The result implies that with the same amount of water, chaos will appear more likely with more cabins flooded. Because chaos motion, which may be some relation to the capsizing motion of the ship motion, it is more dangerous for ships with more cabins flooded. This conclusion may be meaningful for ship design and manipulation.

5. Conclusions

In this paper, Melnikov method is used to predict the conditions of chaos rolling motion of a flooded ship. Firstly, nonlinearly coupled motions of roll and flooded water in regular waves are derived using Lagrange's principle. To simplify the model, the equations are decoupled and reduced to a single-freedom nonlinear ordinary derivation equation, which can describe the rolling motions of the flooded ship. Secondly, Melinkov function, which is a measure of the distance between the perturbed stable and unstable manifolds in the Poincare map, is constituted. And the condition of chaos appearance is derived by setting the condition when Melnikov function has simple zeros and is independent of ε. Numerical computations are performed to predict the critical wave moment for a ship model under three statuses and simulations show the chaos phenomena. Comparison of the three status shows that more cabins flooded is more dangerous.

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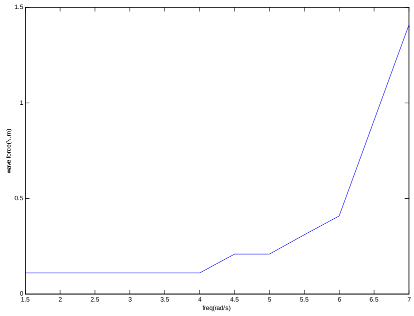
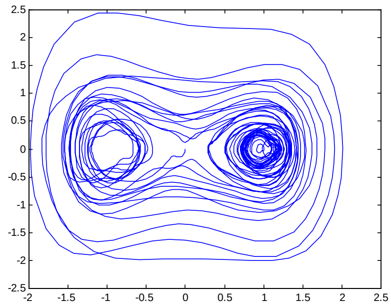
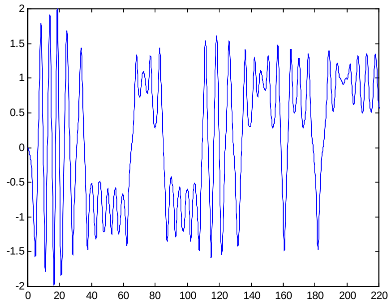


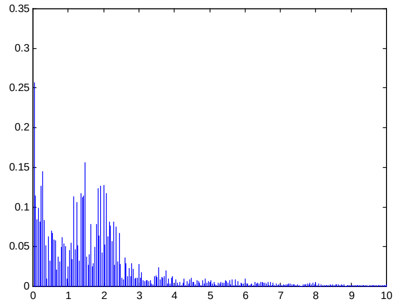
(a) Critical wave moment curve



(b) Phase diagram with wave moment 2.72N.m and frequency 4.5rad/s

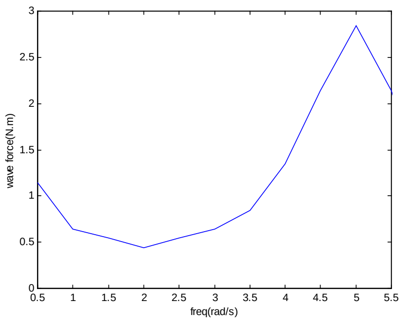


(c) Time series of (b)

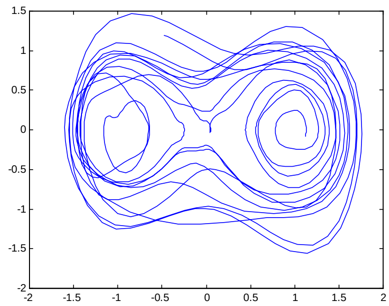


(d) Frequency spectrum of the motion in (b)

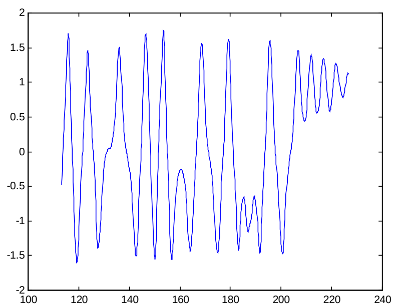
Figure 3 Results of Status I



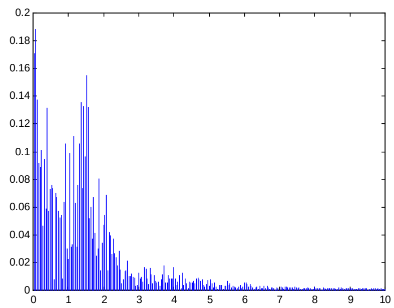
(a) Critical wave moment curve



(b) Phase diagram with wave moment 6.14N.m and frequency 5.5rad/s

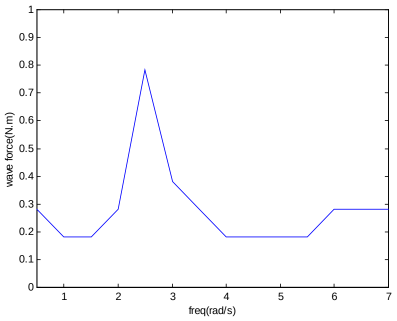


(c) Time series of (b)

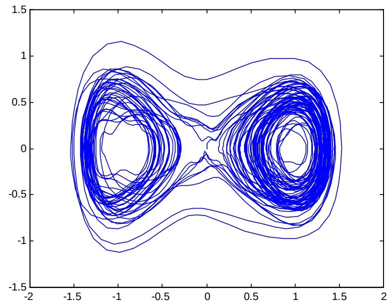


(d) Frequency spectrum of the motion in (b)

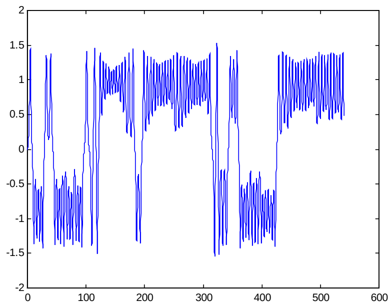
Figure 4 Results of Status II



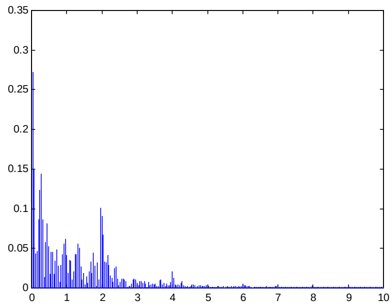
(a) Critical wave moment curve



(b) Phase diagram with wave moment 3.98N.m and frequency 4.2rad/s



(c) Time series of (b)



(d) Frequency spectrum of the motion in (b)

Figure 5 Results of Status III