Dynamic Peculiarities of an Unstable Ship Rolling in Waves

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Despite classification societies requirement for any ship to have an initial metacentric height being positive, in practice however the accidents when a ship loses her initial stability are quite possible. Among the reasons of that it can be non-correct loading, stock consumption, flooding of the upper compartments, for example, due to extinguish the fire, heavy icing.

Analytical methods accessing ship rolling allow with acceptable accuracy to calculate roll kinematic characteristics always supposing a ship possesses positive initial transversal stability. In situation when a ship looses her initial stability well known methods do not work properly due to strong non-linearity of the ship stability curve in vicinity of static list angle [1], [2].

In principle, as analysis of published works shows [4] - [8], very popular now time-domain simulation technique is able to solve the problem but it will be very time-consuming and tiresome procedure. For avoiding such infinite routine calculations we have tried to investigate behavior of a **s**hip with a **n**egative **i**nitial **s**tability (SNIS) by means of traditional analytical so-called "**v**aried **s**cale method" (VS-method) [3] applied to analysis of non-linear differential equations.

Hereinafter we present the basic results of unstable ship dynamics investigation based on mentioned approach.

As the ship motion mathematical model two "loll type" equations are suggested. The first one is Duffing equation which simulates SNIS rolling in full frequency domain (θ denotes a roll angle)

$$\ddot{\theta} + 2\nu_\theta \dot{\theta} - \alpha \cdot \theta + \beta \cdot \theta^3 = m_0 \cos(\omega t - \rho_0)\,; \tag{1}$$

the second one is Mathieu equation reflecting the features of a ship rolling at encounter frequency $\omega \approx 2\omega_\theta$ ($\omega_\theta = \omega_\theta(\theta_a)$ - roll natural frequency), where, as known, the ship can undergo the parametric swinging

$$\ddot{\theta} + 2\nu_\theta \dot{\theta} - \alpha(1 + a \cdot \cos \omega t) \cdot \theta + \beta \cdot \theta^3 = 0\,. \tag{2}$$

In equation (2) $a = \dfrac{\Delta\alpha}{\alpha}$ is a relative amplitude of initial stability coefficient α alteration, caused by waves and ship motions. The rest of symbols in both the suggested equations are common ones in ship dynamics.

At representation of restoring moment in the chosen equations we restrict ourselves for simplicity by cubic polynomial because our attention in the work focuses mainly on steady state rolling regimes with amplitudes far from vanishing angle of a stability curve when the extra terms in GZ curve expansion can not affect strongly the principle dynamic qualities of a ship. As concerns investigation of SNIS behavior in vicinity of the vanish angle this situation has been simulated in details by M. Kan [5], [6].

Applying of cubic polynomial $r(\theta) = -\alpha \cdot \theta + \beta \cdot \theta^3$ allows to obtain the simple relationships which describe main static and dynamic parameters of an initially unstable ship. For instance, static list angle θ_{st} will be determined by simple expression $\theta_{st} = \pm\sqrt{\frac{\alpha}{\beta}}$; a natural small amplitude roll frequency around stable equilibrium angle is calculated by formula $\omega_\theta \approx \sqrt{2\alpha}$; a border between the ship small oscillations around static list angle and large ones around upright angle $\theta_{unst} = 0$ is defined by relationship $\theta_{cr} = \sqrt{2} \cdot \theta_{st}$.

A phase portrait of autonomous SNIS oscillations presented in Fig.1 demonstrates all the regions of the possible steady-state roll regimes.

As to forced oscillations in accordance with VS-method idea a non-linear roll equation

$$\ddot{\theta} + 2\nu_\theta \dot{\theta} + r(\theta) = m_0 \cos(\omega t - \rho_0) \tag{3}$$

can be transformed to an equation below

$$Z''(\varepsilon) + \frac{2\nu_\theta}{\dot{\varphi}} Z'(\varepsilon) + Z(\varepsilon) = H(\varepsilon)\,. \tag{4}$$

Equation (4) is linear with $z(\varepsilon) = f[\theta(t)]$, and $\varepsilon = \varphi(t)$, where ε is a new independent variable.

Unknown function $\varphi(t)$ characterizes the variable scale of real time *t* transformation into new one ε. Amplitude function $f[\theta(t)]$ is determined by expression:

$$f(\theta) = \sqrt{2\int r(\theta) d\theta + C}\,; \tag{5}$$

phase function $\eta(t) = \varphi(t) - \varphi(0)$ can be obtained from a solution of more complicate equation

$$t = \int_0^\eta \frac{d\eta}{\dot{\eta}(t)}\,. \tag{6}$$

For restoring moment approximated by the curve $r(\theta) = \pm\alpha \cdot \theta \pm \beta \cdot \theta^3$ right hand side of expression (6) is transformed into the first kind elliptic integral $F(\eta, k)$, and the solution of equation (6) is getting exact but, unfortunately, in an implicit form.

For determination of function $\eta(t)$ in explicit form elliptic integral $F(\eta,k)$ is expanded into trigonometric series. Accuracy of the roll equation solution is determined in dependence on the number of saved terms in the expansion.

Keeping in the expansion two terms the solution of non-linear equation (3) will be structurally the following:

- for small amplitude oscillations around the stable equilibrium positions, such that maximum roll angle $\theta_{max} << \theta_{cr}$, the rolling is characterized by following expression:

$$\theta_{max}^2\sqrt{\frac{\beta}{2}}-\frac{\alpha}{\sqrt{2\beta}}=A_{har}\cos(\omega t-\rho_{har})+A_{sub}\cos[(\omega-\omega_{\theta S})t-\rho_{sub}]+A_{ult}\cos[(\omega+\omega_{\theta S})t-\rho_{ult}]. \quad (7)$$

- for large amplitude oscillations around the unstable equilibrium position (assuming roll amplitude $\theta_0 >> \theta_{cr}$) the roll amplitudes can be determined from equation:

$$\theta\sqrt{\frac{\beta}{2}\theta^2-\alpha}=A_{har}\cos(\omega t-\rho_{har})+A_{sub}\cos[(\omega-2\omega_{\theta L})t-\rho_{sub}]+A_{ult}\cos[(\omega+2\omega_{\theta L})t-\rho_{ult}]. \quad (8)$$

herein:

$$\begin{cases} A_{har}=\dfrac{m_0\omega_{\theta L}}{\sqrt{(\omega_{\theta L}{}^2-\omega^2)^2+4\nu_\theta{}^2\omega^2}};\rho_{har}=\operatorname{arctg}\dfrac{2\nu_\theta\omega}{\omega_{\theta L}^2-\omega^2}; \quad B_L=\dfrac{1}{2}-\dfrac{\pi}{4K(k_L)}; \\ A_{sub}=\dfrac{-m_0 B_L\omega_{\theta L}}{\sqrt{\left[\omega_{\theta L}^2-(\omega-2\omega_{\theta L})^2\right]^2+4\nu_\theta{}^2(\omega-2\omega_{\theta L})^2}};\rho_{sub}=\operatorname{arctg}\dfrac{2\nu_\theta(\omega-2\omega_{\theta L})}{\omega_{\theta L}^2-(\omega-2\omega_{\theta L})^2}; \\ A_{ult}=\dfrac{-m_0 B_L\omega_{\theta L}}{\sqrt{\left[\omega_{\theta L}^2-(\omega+2\omega_{\theta L})^2\right]^2+4\nu_\theta{}^2(\omega+2\omega_{\theta L})^2}};\rho_{ult}=\operatorname{arctg}\dfrac{2\nu_\theta(\omega+2\omega_{\theta L})}{\omega_{\theta L}^2-(\omega+2\omega_{\theta L})^2}; \end{cases}$$

where $\omega_{\theta L}$ - natural roll frequency of large oscillations around the unstable upright equilibrium position that is by means of VS-method defined by formula:

$$\omega_{\theta L}=\frac{\pi\sqrt{\beta\theta_0^2-\alpha}}{2K(k_L)}. \quad (9)$$

The natural roll frequency of small amplitude oscillations around the stable equilibrium positions is defined by another relationship:

$$\omega_{\theta S}=\frac{\pi\sqrt{\alpha-0.5\beta\theta_0^2}}{\sqrt{2}K(k_S)}, \quad (10)$$

where K(k) is a complete elliptic integral of the first kind.

A frequency curve of SNIS rolling in calm water according to formulae (9), (10) is represented in Fig.2.

It is seen from the analysis of expression (8) that for the unstable ship rolling in waves additionally ultra-harmonic oscillations with frequency $(\omega + 2\omega_{\theta L})$ and sub-harmonic ones with frequency $(\omega - 2\omega_{\theta L})$ can exist apart from harmonic oscillations with exciting frequency ω. As the calculating results evidence, amplitudes A_{ult} of higher harmonics in heavy rolling are significantly less than amplitudes of harmonic oscillations, and they can be neglected in practice. As to sub-harmonic regime, amplitudes of such type oscillations can be essential. It is worth noting herein that sub-harmonic oscillations are able to be excited only at exceeding of a certain threshold value of roll damping moment. The latter value is determined using VS-method as well.

Results of calculations driven by expressions (7) and (8) are given in Figs.3, 4.

Examining the SNIS large amplitude roll oscillations close to separatrix (curve 2 in Fig.1), i.e. $\theta_0 > \theta_{cr}$, for specifying of the solution it is necessary to take into consideration at least three terms in the expansion of integral $F(\eta, k)$. As a result, one may find extra overtones with frequencies $(\omega \pm 4\omega_{\theta L})$ near critical amplitude $\theta_{cr} = \sqrt{2}\theta_{st}$ in addition to oscillations with frequencies $(\omega \pm 2\omega_{\theta L})$.Thus the higher number of terms is taken into account in the expansion of the elliptic integral the more complicated forms one can see in the roll oscillations when roll amplitudes are approaching the homoclinic seperatrix. Finally, the ship turns up in the region of stochastic instability characterized by fractal structure in the phase space and by wide spectrum in frequency domain (Figs. 5, 6).

Analyzing Mathieu equation (2) we find approximate relation between amplitude θ_{par} and modulation frequency ω:

$$\theta_{par} = \sqrt{\frac{4}{3\beta}\left[\left(\frac{\omega}{2}\right)^2 + \alpha \pm \frac{1}{2}\sqrt{\alpha^2 a^2 - 4\nu^2\omega^2}\right]}. \tag{11}$$

The resonance zone of parametric excitation is determined in the case by:

$$\frac{\alpha}{2}(1 - \sqrt{a^2 - 8\nu_\theta^{\ 2}/\alpha}) < \left(\frac{\omega^2}{2}\right)^2 < \frac{\alpha}{2}(1 + \sqrt{a^2 - 8\nu_\theta^{\ 2}/\alpha}). \tag{12}$$

Obviously, that the inequality (12) will be correct if the condition $a > \frac{4\nu_\theta}{\omega_{\theta S}^0}$. Here $\omega_{\theta S}^0 = \sqrt{2\alpha}$ is a frequency of extremely small natural oscillations around stable equilibrium position θ_{st}.

The latter inequality defines the minimum modulation amplitude of SNIS stability diagram required for parametric resonance excitation at specified roll damping.

According to equation (11) the calculated parametric rolling in head and beam seas of the ship (series 60) with negative initial GM are plotted in Figs.3, 7.

Conclusions

Summarizing all the results obtained we can conclude the following:

The applied mathematical model reflects quantitatively and qualitatively all the principle features of an unstable ship rolling in waves.

It has been shown that the unstable ship rolls differs from the roll of a ship with positive initial metacentric height. With the help of analytical varied scale method of roll equation's analysis the specific oscillation regimes for an unstable ship have been found: depending on frequency and intensity of waves the unstable ship is rolling or with small amplitudes around stable equilibrium positions or with large amplitudes around unstable upright equilibrium position. Crossing the border between the above rolling regimes with small and large amplitudes the ship can be drawn in chaotic roll motion.

It has been determined that in seaway an unstable ship can undergo resonance motions in wide frequency range: in the case of coincidence of an encountering wave frequency and a ship roll natural frequency we have the principle resonance; in the case of an encountering frequency exceeds the natural roll frequency in two times we may have either sub-harmonic resonance of the second kind or parametric resonance; in the case of a wave frequency is three times higher than natural roll one we may observe the sub-harmonic resonance of the third kind.

The analytical expressions are found to determine the maximum roll amplitudes of the ship with negative initial stability for the cases of roll motions in all the above indicated resonance modes. The conditions of their excitation have been obtained as well. The calculations performed in accordance with the above mentioned expessions have been validated by the results of numerical and physical modeling.

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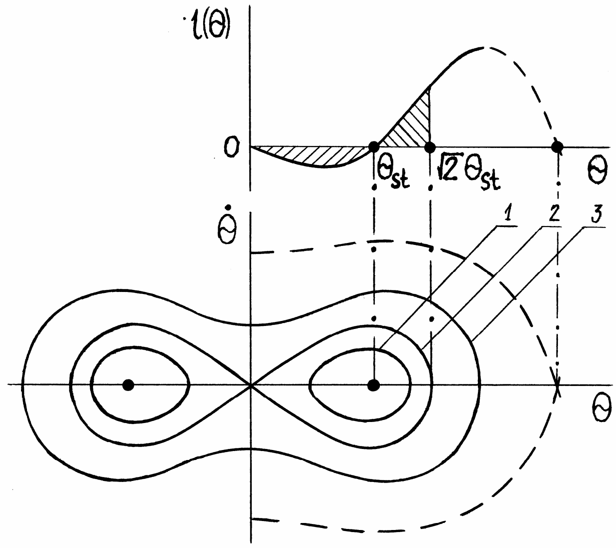

Fig.1 Phase portrait of autonomous roll of the ship with negative initial GM (1 – oscillations around stable equilibrium position; 2 – homoclinic orbit; 3 - oscillations around unstable equilibrium position; - - - - heteroclinic orbit).

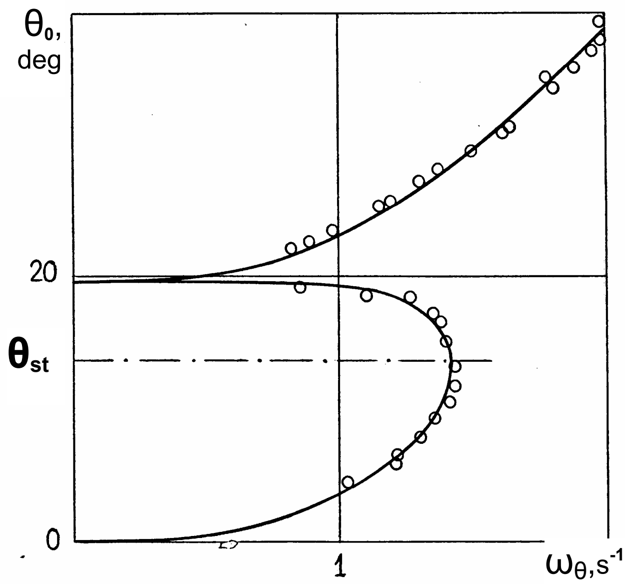

Fig.2 Frequency roll diagram for the unstable ship model of 60th series.

——— Calculations in accordance with (9), (10); ○ - model tests.

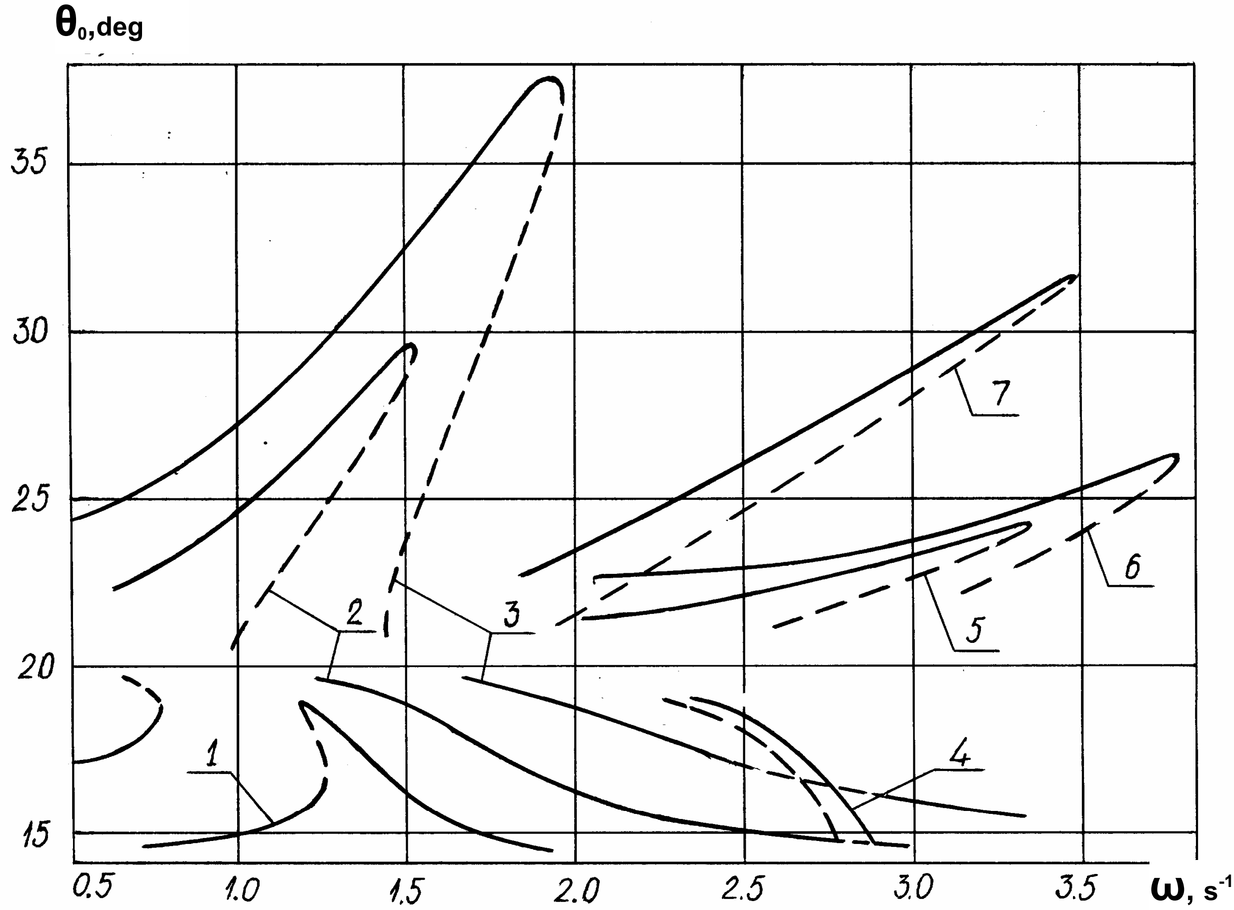

Fig.3 Roll amplitude – frequency diagram for the unstable ship model of 60th series in beam regular waves. Analytic solution.

1 – $m_o = 0.02\ c^{-2}$; 2 - $m_o = 0.1\ c^{-2}$; 3 - $m_o = 0.3\ c^{-2}$: harmonic mode;

4 - $m_o = 0.1\ c^{-2}$: sub-harmonic mode of 2nd kind;

5 - $m_o = 0.1\ c^{-2}$ sub-harmonic mode of 3rd kind;

6 - $m_o = 0.3\ c^{-2}$: sub-harmonic mode of 3rd kind;

7 - $m_o = 0$: parametric oscillations.

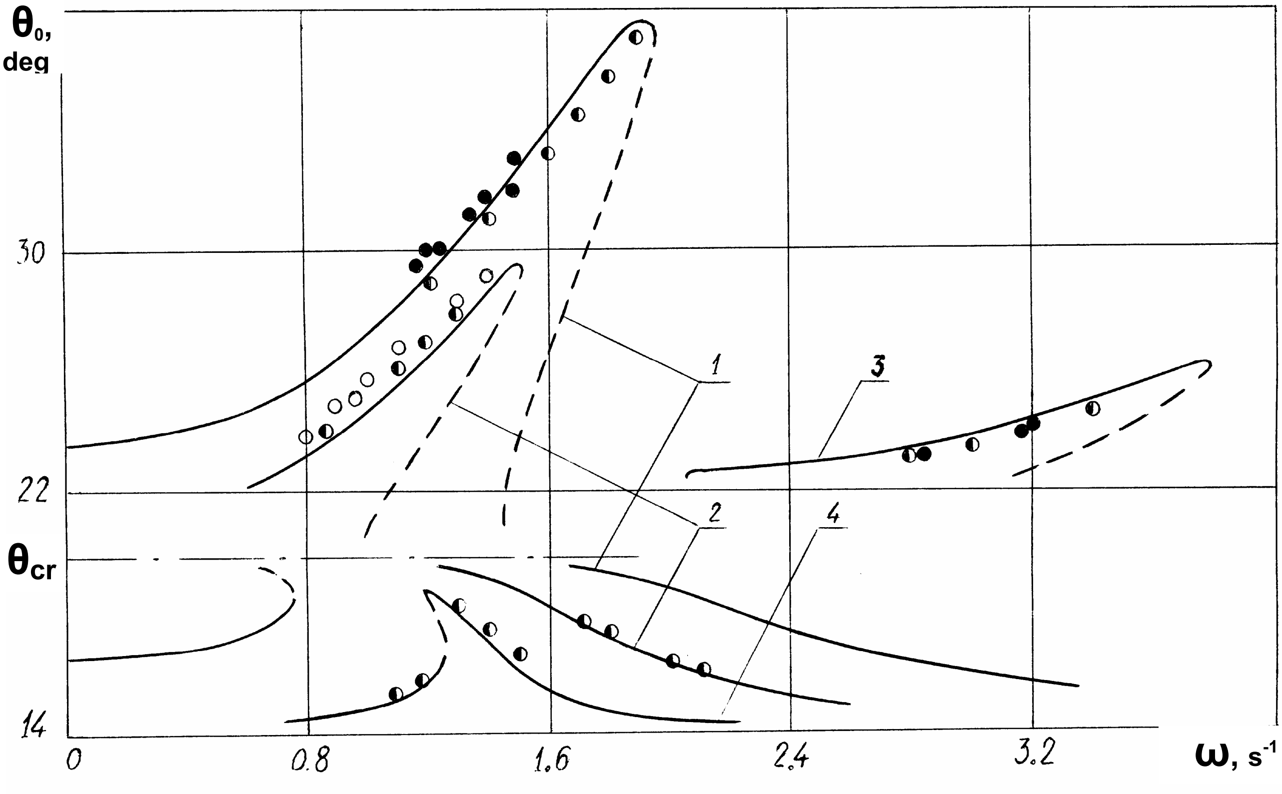

Fig.4 Roll amplitude – frequency diagram for the unstable ship model of 60th series in calm water under harmonic moment excitation.

1 – m_o = 0.3 c^{-2}; 2 - m_o = 0.1 c^{-2}; 4 - m_o = 0.02 c^{-2} : harmonic mode;

3 - m_o = 0.3 c^{-2}: sub-harmonic mode;

$m_o \approx 0.1$ c^{-2} : model test.

○ - numerical solution of Eq. (1)

● - $m_o \approx 0.3$ c^{-2};

○ - $m_o \approx 0.1$ c^{-2} : model test.

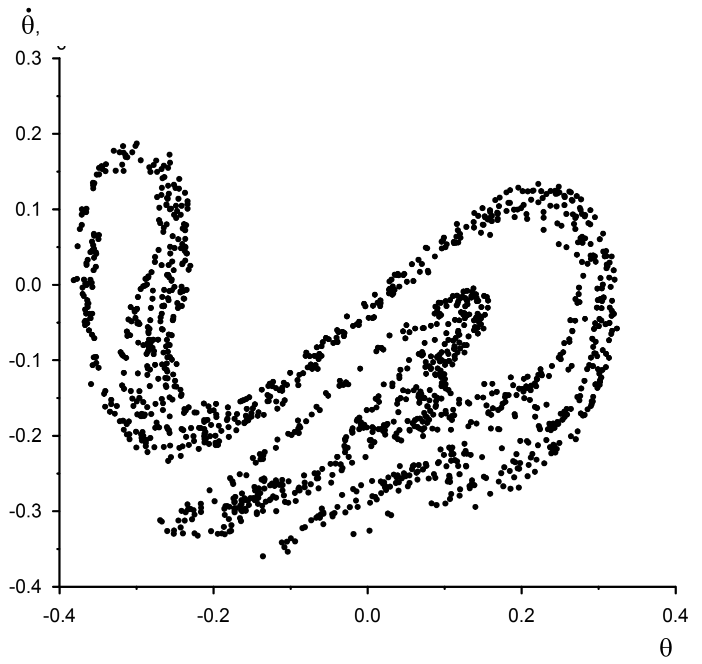

Fig.5 Poincare map corresponding to chaotic motions of SNIS (results of the numerical simulation).

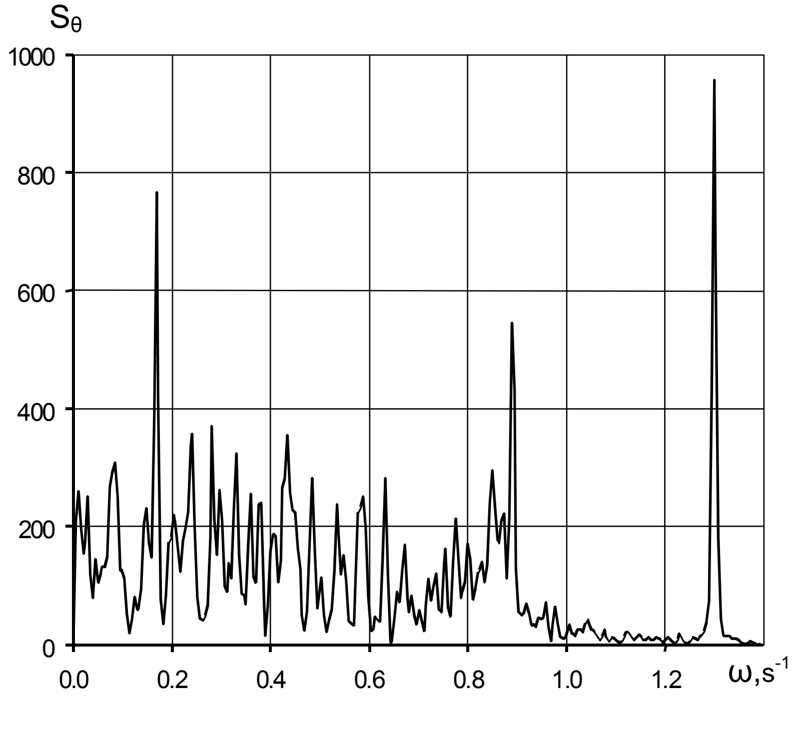

Fig.6 Frequency spectrum of SNIS rolling in chaotic regime (results of the numerical simulation; exciting frequency ω=1.3 rad/s).

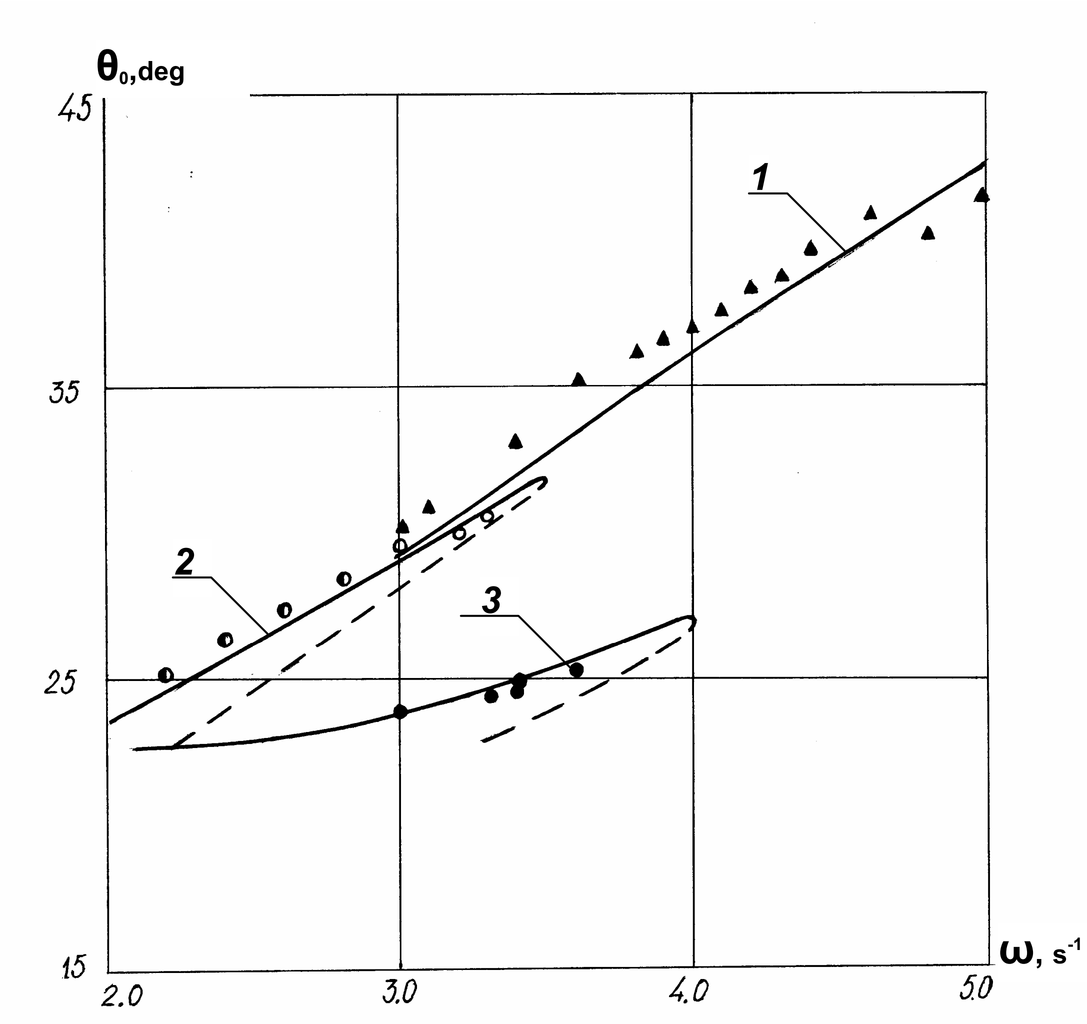

Fig.7 Roll amplitude – frequency diagram for the unstable ship model of 60th series in beam / head regular waves.

1	- parametric oscillations in head sea ;
2	- parametric oscillations in beam sea;
3	- sub-harmonic mode of 3rd kind in beam sea;
○,●,▲	- model tests;
○	- numerical simulation.