Some discussions about the probability of capsize Of a Ship in Random beam waves

Xianglu Huang Xinying Zhu
Shanghai Jiao Tong University

Abstract

This paper deal with the problem of probability of a ship capsize in random beam waves. By using the method of nonlinear oscillation together with the theory of stochastic differential equation, some properties of the probability distribution of ship rolling were investigated. Several methods about this problem published recently were commented. Some concluding remarks were presented.

Introduction

The problem of ship capsize on random waves is a complicated problem ,which is hardly to be solved due to the nonlinear property of the problem and the randomness of the motion. Because of the capsize of a ship on waves always link to the large amplitude rolling, so the large amplitude nonlinear rolling has to be investigated. In such case, the motion equation is a **Duffing** equation with soft spring. Ordinarily we adopt a polynomial expression to fit the stability curve of a ship. Due to its asymmetry property, only the odd order terms are kept in the expression. Also considering that the stability will vanish after the rolling angle reached several point, the sign of the terms should fulfil such requirement.

The motion equation can be simply expressed as

$$\ddot{\theta} + 2\xi\omega_0\dot{\theta} + \omega_0^2\theta(1 + \varepsilon\theta^2) = \gamma N(t) \tag{1}$$

in which

θ Ship roll angle

$2\xi\omega_0$ Linear damping coefficient

ω_0 Natural frequency of roll

ε The coefficient of third order term of stability curve

$N(t) = \dfrac{dW(t)}{dt}$ white noise $W(t)$ Wiener Function

Actually, the wave excitation in(1)should be a time trace with color spectra. But it require large amount of numerical effort to handle such color excitation. Because of we only want to survey qualitatively the effect of randomness on the probability of large amplitude rolling. It should be appropriate to replace the color excitation by the white noise $N(t)$. Converted the original equation into state equations, we obtained the following stochastic differential equations

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 2\xi\omega_0 x_2 - \omega_0^2 x_1(1 + \varepsilon x_1^2) + \gamma N(t) \end{cases} \tag{2}$$

in which

$$x_1 = \theta$$

$$x_2 = \dot{\theta}$$

This equation can simply written in the form of vector Ito's stochastic equation

$$d\vec{X} = m(\vec{X},t)dt + \vec{Q}(\vec{X},t)dW(t)$$

in which the drift coefficient is

$$m[\vec{X},t] = \begin{bmatrix} m_1[\vec{X}(t)] \\ m_2[\vec{X}(t)] \end{bmatrix} = \begin{bmatrix} x_2 \\ -2\xi\omega_0 x_2 - \omega_0^2 x_1(1+\varepsilon x_1^2) \end{bmatrix} \tag{3}$$

while the matrix of diffusion coefficients is

$$Q[\vec{X},t] = \begin{bmatrix} Q_1[\vec{X}(t)] \\ Q_2[\vec{X}(t)] \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \gamma \end{bmatrix} \tag{4}$$

Before we start to deal with this stochastic equations, we survey the property of the **Hamilton** system relate to this equation. This system actually is the equation ignore the damping and external excitation. The solution of which is the free oscillation without damping.

$$x_0(t) = \sqrt{\frac{1}{\alpha}}\tanh(\frac{\tau}{\sqrt{2}}) \tag{5}$$

$$y_0^{\pm}(t) = \sqrt{\frac{1}{2\alpha}}\sec h^2(\frac{\tau}{\sqrt{2}}) = \pm(\sqrt{\frac{1}{2\alpha}} - \sqrt{\frac{\alpha}{2}}x_0^2) \tag{6}$$

in which $x_0 = \theta$, $y_0 = \dot{\theta}$, $\alpha = \frac{\varepsilon}{\omega_0^2}$, $\tau = \omega_0 t$, '+' denote the upper half branch of the orbit, while '-' denote the lower part. The different orbit corresponds to the initial rolling angle.

The corresponding phase portrait is shown in following picture.

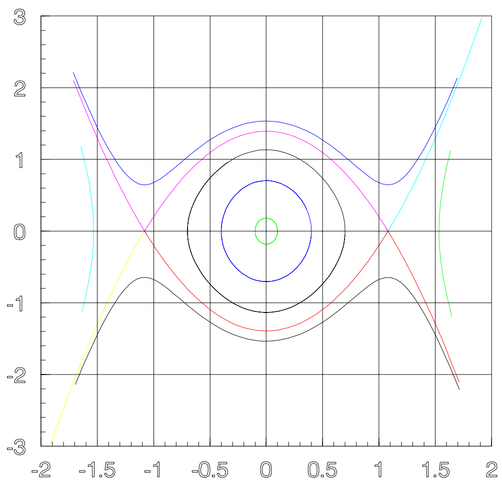
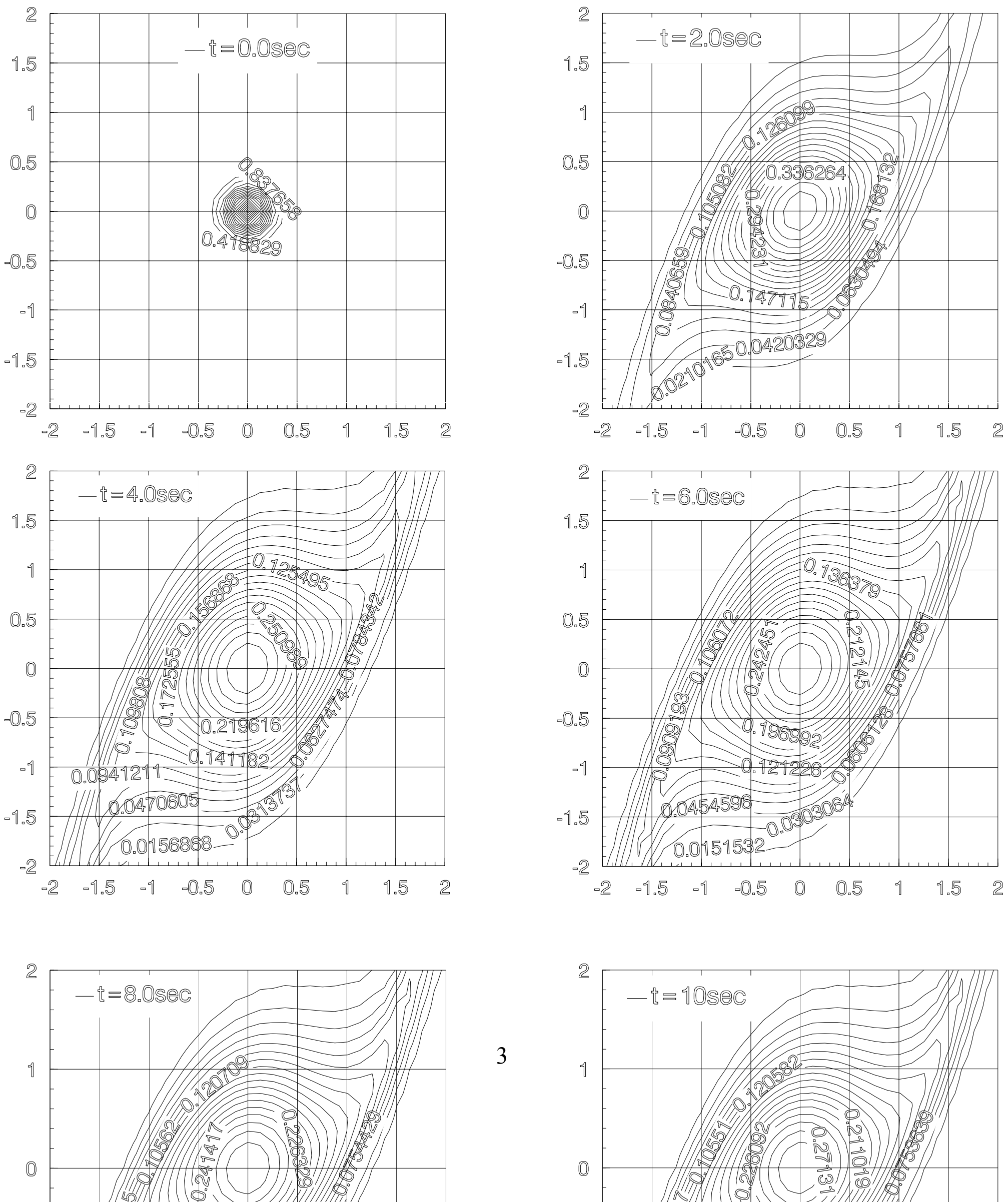


Fig 1. Phase portrait

It is shown on such phase portrait that when the motion reaches the point of the stability vanishing angle, the close orbit will begin to broken. The phase trace will go away to another attractor. So the phase trace which pass through the point corresponds to the stability vanishing angle formed the boundary between stability and instability flow. **Thompson** has suggested the idea of safe basin in which he linked the initial condition with the happening of the capsize. He has found that if there is some external excitation which result in the happening of capsize, the form of safe basin will have the property of fractal, and the area which enclosed by the boundary of stability will be reduced. It is called the erosion of safe basin. This is a very good idea which can be used to survey the possibility of capsize. But, the difficulty involved is large amount of computation effort . Another method is proposed by **Troesch** et,al . They suggested to used a quantity called phase flux, which is determined by the **Melnikov** function to quantify the area exported outside from the safe basin. Although these two method have made this very complicated problem clear, but they all have the short come as to solve the problem quantitatively. The reason is the method involved in these two method are not statistical. The method of phase flux partly considered the input to have some kind of spectrum. But it is plausible that since capsize is a phenomenen which linked to the large amplitude rolling, how can it be calculated only by the integral of input on the hetro-clinic orbit in a linear manner. For this reason we start to investigate the probability behavior of the rolling motion in time domain, and try to find the relationship between these probability behavior and capsize of a ship.

Determination of PDF of ship rolling on random wave by solving FKP equations

It is obvious that to have the information of ship large amplitude rolling on a random wave train,



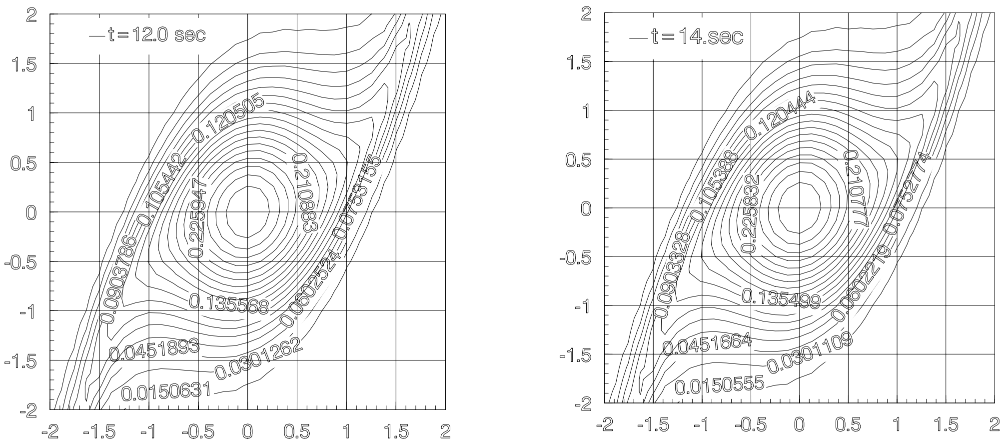


Fig2 The variation of the PDF with time

It should be noted that the PDF is largely depend on the sign of its higher order restoring term. Fig 5 shows the PDF with the same parameter combination as in Fig2 except the sign of third order restoring term is positive.

the appropriate way is to solve the problem in time domain. That is to solve the corresponding Ito's (stochastic differential equations)SDE, which turns to find the solution of corresponding FKP equations under several given initial conditions. For our purpose the SDE has the form of (2). It's FPK equation has the form as

$$\frac{\partial p}{\partial t} = -\frac{\partial m_1 p}{\partial x_1} - \frac{\partial m_2 p}{\partial x_2} + \frac{\partial^2 g_{22} p}{\partial x_2^2} \tag{7}$$

in which

$p(x_1, x_2; t)$ is the probability density distribution function(PDF)

$$G(\vec{X}) = (g_{i,j}(\vec{X})) = \vec{Q}(\vec{X})\vec{Q}(\vec{X})^T = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \gamma^2 \end{bmatrix} \tag{8}$$

This two dimensional second order partial equation was solved by a numerical method using Finite Difference Scheme in time domain. The detail of such method can refer to another paper. The calculated result for a combination of parameters as: Ship displacement 480 ton, mass coefficient 1089 ton m^2, damping coefficient 100 ton m and restoring 0.7m with cubic coefficient –0.7. The level of excitation white noise is 0.8 are shown in fig 2

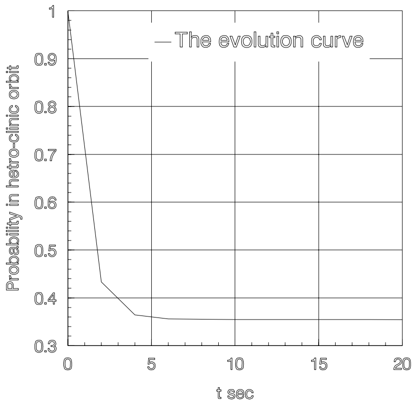
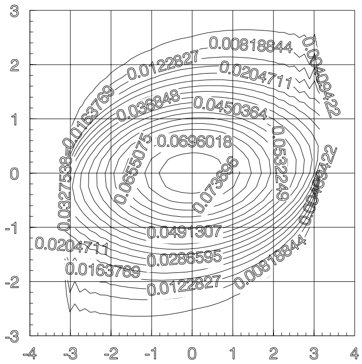


Fig3 The variation of probability inside hetro-clinic orbit with time



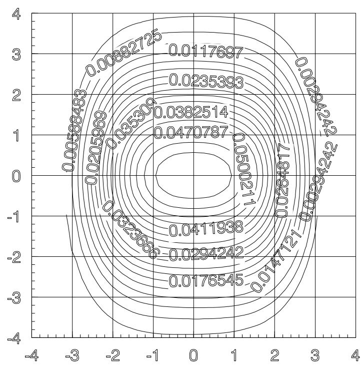


Fig 4 The PDF of a linear system

Fig 5 The PDF of a system with positive third order term

From the figs it can be seen that the variation of the PDF of the rolling process in time is diffuse from the center, at which a δ function is located. But is shows that the PDF reaches the stationary status quickly. Fig 3 is the variation cumulated probability enclosed by the hetro-clinic orbit with the time. It showed that at first the value reduce rapidly and then become slow after 2-3 second. Although the rate of reduction become insignificant after 10 seconds, it still has some small value.

The condition of capsize of a ship on random waves

Although the solution of the FPK equations can be used to show the variation of the PDF with time, it is no use for the determination of the capsize of a ship in a random sea. In order to determine the capsize, we have to at first look further into the process of the happening of the capsize of a ship. The capsize of a ship implies it will goes away from its up righting position to another balance position as 180 degree, in other words turn to up side down. It is known from the first paragraph that for a Hamiltonian system with the soft spring like the ship rolling. The phase portrait has the form of close orbit before it reaches the stability vanishing angle or saddle points. The orbit passing through the point corresponds to the stability vanishing angle is the boundary between the solution of stability flow and instability flow. The instability flow outside the boundary broken into four branches and all goes out to the next attractor. This orbit is called hetroclininc orbit It is clear that in a non-external excitation and no damping oscillation situation, the ship will capsize if it's rolling amplitude large over the stability vanishing angle. But when there is excitation and damping, things will become complicated. In order to make sure if the hetroclinic orbit is stability or instability in forced oscillation case, one can use the **Melnikov** function technique, which is defined as follow:

If there is a system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_0(\boldsymbol{x}) + \varepsilon \boldsymbol{f}_1(\boldsymbol{x}, \boldsymbol{t})$$

Then the **Melnikov** function is defined as

$$\boldsymbol{M}(\boldsymbol{t}) = -\int_{-\infty}^{\infty} \{\boldsymbol{f}_0[\boldsymbol{x}_0(\tau)], \boldsymbol{f}_1[\boldsymbol{x}_0(\tau), \tau - \boldsymbol{t}]\} \times \exp[-\int_0^{\tau} \boldsymbol{T}_r \frac{\partial \boldsymbol{f}_0}{\partial \mathrm{x}}(\boldsymbol{x}_0(\boldsymbol{u}))\boldsymbol{du}]\boldsymbol{d}\tau \tag{9}$$

in which

$\boldsymbol{f}_0$	External excitation
$\boldsymbol{x}_0$	The offset of stability/instability flow
$\boldsymbol{T}_r \frac{\partial \boldsymbol{f}_0}{\partial \boldsymbol{x}}$	is the trace of $\frac{\partial \boldsymbol{f}_0}{\partial \boldsymbol{x}}$ matrix
{,}	Poisson bracket defined as {a, b}=$\{a_1 b_2 - a_2 b_1\}$, $a = \{\begin{matrix} a_1 \\ a_2 \end{matrix}\}$, $b = \{\begin{matrix} b_1 \\ b_2 \end{matrix}\}$

Melnikov function measures the distance between the stability and instability flow of the same sandal point. For a system with damping but without external excitation, its **Melnikov** function is a negative constant. Under the action of oscillatory external excitation, the value of **Melnikov** function will oscillate around its mean value ,the amplitude of which is proportional to the external excitation. When the intensity of the external excitation over several threshold, the **Melnikov** function will have zero point. The stability flow will across to the instability flow. The boundary of safe basin will have the fractal character. So the zero value of **Melnikov** function equivalent to the condition of the safe basin erosion.

The **Melnikov** function of a ship without bias can be expressed explicitly as (Jiang 1996)

$$M_\delta(t_0,\theta_0) = \widetilde{M}_\delta(\boldsymbol{t}_0, \boldsymbol{\theta}_0) - \overline{M}_\delta \tag{10}$$

in which

$$\overline{M}_0 = \frac{2\sqrt{2}}{3\alpha}\delta_1 + \frac{8}{15}\left(\frac{1}{\sqrt{\alpha}}\right)^3 \delta_2 \tag{11}$$

$$\widetilde{M}_0(t_0,\theta_0)=\sqrt{\frac{2}{\alpha}}\gamma\pi\Omega\frac{\cos(\Omega t_0+\theta_0+\psi)}{\sinh\left(\frac{\Omega\pi}{\sqrt{2}}\right)} \tag{12}$$

$$\delta_1=\frac{B_{44}\omega_n}{C_1\Delta}\quad \delta_2=\frac{B_{44q}}{I_{44}+A_{44}}\quad \alpha=\frac{-C_3}{C_1}\quad \omega_n=\sqrt{\frac{C_1\Delta}{I_{44}+A_{44}}}\quad \Omega=\frac{\omega}{\omega_n}$$

$$\varepsilon\gamma=HF_{roll}\,/\,C_1\Delta$$

$\overline{M}_\delta$ and $\widetilde{M}_\delta(t_0,\theta_0)$ are the average part and time varied part of **Melnikov** function. Then, the condition of the happening of safe basin should be

$$\frac{2\sqrt{2}}{3\alpha}\delta_1+\frac{8}{15}\left(\frac{1}{\sqrt{\alpha}}\right)^3\delta_2=\frac{\sqrt{\frac{2}{\alpha}}\gamma\pi\Omega}{\sinh\left(\frac{\Omega\pi}{\sqrt{2}}\right)} \tag{13}$$

But, in real sea the wave are random. The above expression is not available. In order to survey the effect of random waves on the ship safe basin. We consider that the random waves can be modeled as superposition of a series of regular waves, and at first investigate the effect of a wave train formed by two different sinusoidal waves $\zeta(t)=a_1\cos\omega_1 t+a_2\cos\omega_2 t$ on the safe basin of ship rolling motion.

The zero point of **Melnikov** function of such a wave group is as follows

$$\frac{2\sqrt{2}}{3\alpha}\delta_1+\frac{8}{15}\left(\frac{1}{\sqrt{\alpha}}\right)^3\delta_2=\sqrt{\frac{2}{\alpha}}H\pi\left(\frac{F_{roll1}\Omega_1}{\sinh\left(\frac{\Omega_1\pi}{\sqrt{2}}\right)}+\frac{F_{roll2}\Omega_2}{\sinh\left(\frac{\Omega_2\pi}{\sqrt{2}}\right)}\right) \tag{14}$$

From this expression, it seems the capsize is unlikely to happen in this two regular wave combination case, which implies that the randomness of the wave will reduce the possibility of the capsize.

For a ship rolling on a random waves, the determination of its **Melnikov** function is differ from the determinate cases in two aspects. First, the motion of a ship is random, so the motion will be random too, the variables of motion should attach with a probability and the **Melnikov** function will be a statistical average value. We define that it has the form as

$$\overline{M}_0(t)=\delta_1\int_{-\infty}^{\infty}\int_{y_h^- -\delta y_h^-}^{y_h^- +\delta y_h^-}\int_{x_h^- -\delta x_h^-}^{x_h^- +\delta x_h^-}y_h^2(t)p(x,y,|x',y')dx'dy'dt+\delta_1\int_{-\infty}^{\infty}\int_{y_h^+ -\delta y_h^+}^{y_h^+ +\delta y_h^+}\int_{x_h^+ -\delta x_h^+}^{x_h^+ +\delta x_h^+}y_h^2(t)p(x,y,|x',y')dx'dy'dt$$

$$+\delta_2\int_{-\infty}^{\infty}\int_{y_h^- -\delta y_h^-}^{y_h^- +\delta y_h^-}\int_{x_h^- -\delta x_h^-}^{x_h^- +\delta x_h^-}y_h^2(t)|y_h(t)|p(x,y,|x',y')dx'dy'dt$$

$$+\delta_2\int_{-\infty}^{\infty}\int_{y_h^+ -\delta y_h^+}^{y_h^+ +\delta y_h^+}\int_{x_h^+ -\delta x_h^+}^{x_h^+ +\delta x_h^+}y_h^2(t)|y_h(t)|p(x,y,|x',y')dx'dy'dt \tag{15}$$

$$\widetilde{M}_\delta(t)=\int_{-\infty}^{\infty}\int_{y_h^+ -\delta y_h^+}^{y_h^+ +\delta y_h^+}\int_{x_h^+ -\delta x_h^+}^{x_h^+ +\delta x_h^+}y_h(t)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\varsigma(t+t')p(x,y,\varsigma,\dot{\varsigma}|x',y',\varsigma',\dot{\varsigma}')dx'dy'd\varsigma'd\dot{\varsigma}'dt \tag{16}$$

It is known that the **Melnikov** function is the distance between the stability flow and instability flow of the disturbed system. The zero value of **Melnikov** function means the hetroclinic orbit is the stability boundary. The hetroclinic orbit will be stability if

$$\widetilde{M}_{\delta}(t) \le \overline{M}_{0}(t)$$

otherwise it is in-stability. Obviously, this condition determines the erosion of safe basin. These two quantities $\widetilde{M}_{\delta}(t)$ and $\overline{M}_{0}(t)$ are depending on the level of external excitation and are function of time **Troesch** has used the method of **Melnikov** function to calculate the so called phase flux.

By using(13)(14)we have for the ship with the combination of parameters as in section 1.calculate $\widetilde{M}_{\delta}(t)$ 、 $\overline{M}_{0}(t)$ also the cumulated probability enclosed by the hetroclinic orbit was calculated by integrating the PDF inside the hetroclinic orbit. The intensity of the external excitation is from 0.2 to 0.8. The PDF of each case together with the hetroclinic orbit were shown in the following graphics.

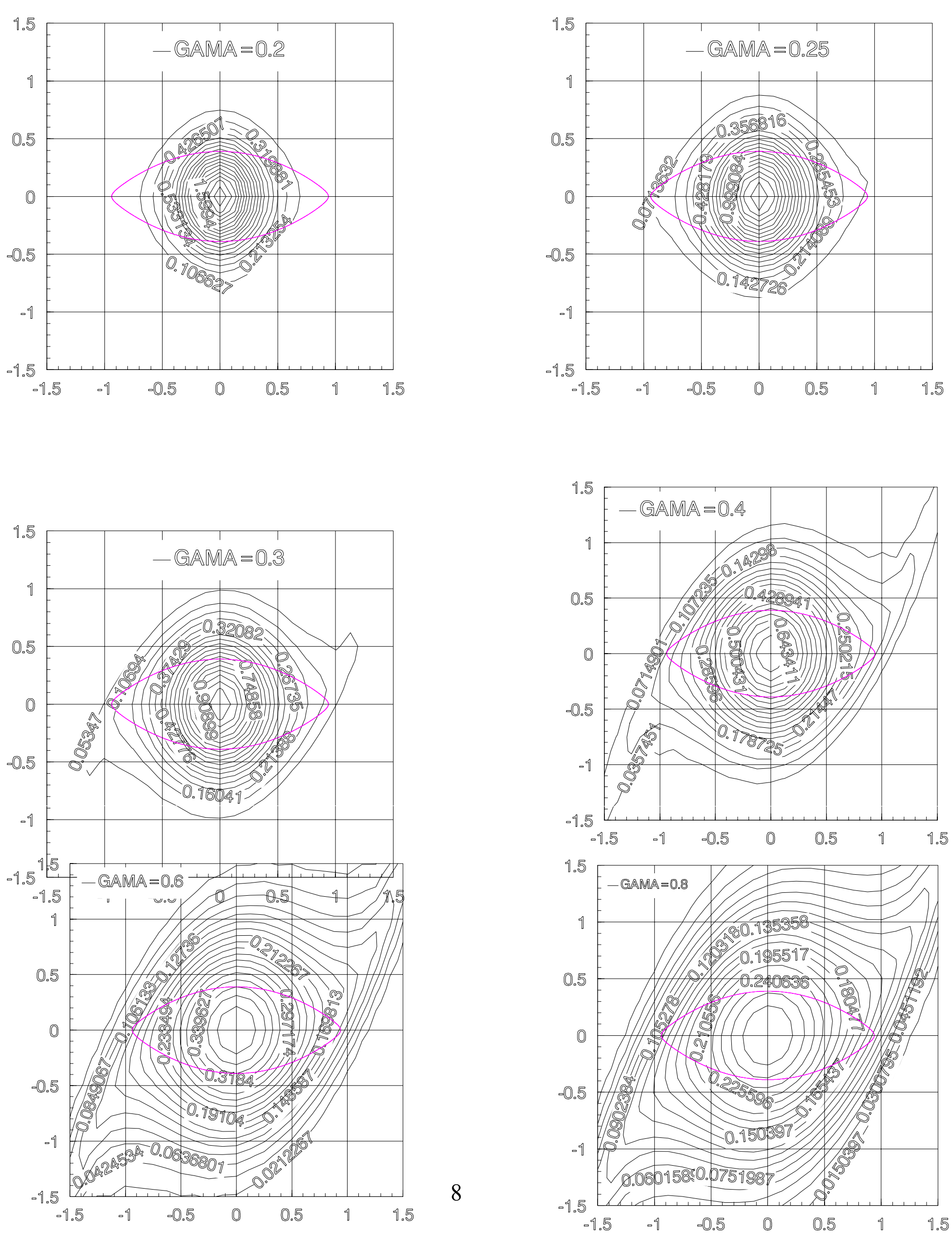


Fig6 The PDF of varies external excitation level

The calculated results of these series of excitation level are included in the following table.

Calculated **Melnikov** function and probability inside the hetroclinic orbit

Level	0.8	0.6	0.4	0.3	0.25	0.2
$\overline{M}_0$	0.3962453	0.4972652	0.6504055	0.7308647	0.7482907	0.6842267
$\widetilde{M}_0$	0.4837243	0.5257167	0.5614390	0.5463681	0.5106571	0.4176435
$\overline{p}$	0.3544042	0.4455447	0.5999293	0.7179332	0.7974694	0.8886672
Status	instability	Instability	stability	stability	stability	Stability

Some discussions

There are some remarks which was illuminated from the above discussions.

At first, we can see that the PDF of the ship rolling has reflected some motion characteristics which may link to the ship capsize. The difference in the appearance between Fig 4.5 and Fig3, show that the form of the stability curves has very strong influence on the probability property of rolling motion especially in large amplitude range. But, it also shows that only PDF itself, can not be used to identify the happening of capsize, and its probability.

The happening of capsize of a ship on a wave train is the transition of ship from its upright position (zero position) to another balance position (180deg), or in other words up side down. In the language of nonlinear mechanics, It means from one attractor goes to another. While such transition is passing through the saddle point, or the stability vanishing angle. This is very important to investigation the role of such stability vanishing angle, since we need in the determination of the happening of ship capsize, the condition at which capsize will happen. Not like in another first passage problem, the condition of capsize is defined by the motion equation itself. In the phase portrait of the corresponding Hamilitonian system, there is a close orbit passing through the saddle point, which is the boundary between the stability flow and instability flow, which is clearly a threshold of capsize in non-external cases. But for a system with external excitation to determine the stability boundary is very complicated. Nevertheless, the hetroclinic orbit is still an important role to be used in the capsize prediction. To judge if this orbit still can be used as a threshold in forced oscillation case , the **Melnikov** function has to be used. The stability of the hetroclinic orbit can be judged from the sign of the **Melnikov** function. It was indicated in the table that the stability character of the orbit is depends on the excitation level. In low excitation intensity, the orbit is stable. Only when γ is over the level 0.6, the boundary become unstable.

Another importance feature is the probability inside the hetroclinic orbit. Calculation shows that the probability will become less and less when the excitation level increase. It means the motion at first remain in the orbit, then will escape from the orbit as the excitation become intense. So, it is clear for the capsize there are two conditions which should be full filled. First the oscillation much reach the orbit then the orbit become unstable. These two conditions all depend on the excitation level, but in principle there are not the same. We have to consider them separately in determine the condition of capsize.

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