Application of Wave Groups to Assess Ship Response in Irregular Seas

Christopher C. Bassler, Vadim Belenky, Martin J. Dipper, Jr.

David Taylor Model Basin (DTMB), Naval Surface Warfare Center, Carderock Division

ABSTRACT

A method using wave groups to evaluate ship response in heavy seas is presented. A ship sailing in a stochastic environment is difficult to model because of both the rarity and significant nonlinearity of the large motion responses. In the proposed method, wave groups which are critical to ship response are defined, separating the complexity of the nonlinear dynamics of ship response from the complexities of a probabilistic description for the response. In this formulation, wave groups may be considered as a possible method to solve the problem of rarity in a deterministic manner. Details of the procedure to obtain ship-specific thresholds and time-between wave groups are discussed. A procedure using wave groups to evaluate the probability of a rare event, the undesirable response, is also presented.

KEYWORDS

wave groups, dynamic stability, seaway loads, problem of rarity, fold bifurcation

INTRODUCTION

Severe wave conditions present increased risk to ships and other ocean-going vessels. These large waves, in particular sequences, or groups, may cause structural damage or stability failure for a ship operating in these conditions. Because of its significance as a sequence of excitation events, a wave group may present a higher probability of severe ship structural or stability response than a single large wave. Therefore, they must be considered when modeling severe wave environments and when identifying operational conditions where there is increased risk to the vessel. However, these wave groups, which are most critical to the ship dynamics performance, may differ from common oceanographic definitions of wave groups. A critical, or dynamically significant event, is based on a combination of initial conditions, sequence of excitations, and the duration of excitation. For ship designers, operators, and researchers, the important practical matter remains: which waves or wave groups will result in a significant, or undesirable, ship response.

BACKGROUND

Differences between wave groups, as considered in oceanography and in nonlinear ship response, are briefly discussed. A more detailed review of these differences can be found in Bassler, *et al.* (2010). The use of wave groups as a method to solve the problem rarity, with the possibility of experimental validation, is also discussed.

Wave Groups in Oceanography

A wave group is defined as a series of waves, with wave heights larger than a specified threshold, and with approximately equal periods (Masson & Chandler, 1993; Ochi, 1998). Large-amplitude wave groups are often formed in developing seaways or by intersecting storms (Buckley, 1983; Toffoli, *et* *al.*, 2004; Onorato, *et al.*, 2006), or due to the interaction effects of waves and currents.

Spectral shape can also significantly influence grouping; wave grouping increases as the wave energy spectrum becomes narrower (Goda, 1970; Goda, 1976). This spectral narrowing often occurs in a fetch-limited growing sea (Longuet-Higgins, 1976). Bimodal sea states, formed by wind generated waves and swell, are also much more likely to contain groups of large-amplitude waves (Rodriguez & Guedes Soares, 2001).

Wave Groups and Nonlinear Ship Motions

Accounts of ships experiencing groups of large waves, such as the "Three Sisters," have been reported (Buckley, 1983, 2005). Because they may present a more serious risk to a vessel than single large-amplitude waves (Kjeldsen, 1984), groups of waves must also be considered in models of ship response to severe wave environments.

Su (1986) suggested that a wave group, with one or more extremely large waves, would provide a better environmental design scenario than a single extreme wave or a group of regular waves. Philips (1994) also expressed the need to develop a combined, spatially-temporally-defined extreme wave group for ship design

Tools have been developed for ship design where wave groups are used to induce a specific ship motion response. This approach was discussed by Blocki (1980) and Tikka & Paulling (1990) to study parametric roll, using wave groups to induce parametric excitation. Additional studies of the applications of wave groups to parametric roll response have been made by Boukhanovsky & Degtyarev (1996) and Spyrou (2004). Alford has used a design wave train method to produce a desired motion response (Alford, 2008). An assessment procedure for parametric roll in early-stage ship design was developed by Belenky & Bassler (2009), which consists of determining the response to a "typical" wave group. This paper also attempts to address some of the issues related to the definition of a "typical" wave group.

Wave Groups and the Problem of Rarity

Dangerous ship behaviors are caused by either extremely high or extremely steep waves, or a sequence of waves with particular frequencies. These waves, or their combinations, are rare and assessing their probability of occurrence remains a difficult problem.

Once these waves generate large excitation, a large-amplitude response may be expected. For a dynamical system that describes ship motions, this means that nonlinearities are significant for the response. If a dynamical system has significant nonlinearities, it becomes very sensitive to initial conditions. Depending on the initial conditions, very different responses may result: from merely tracking the contour of a large wave to catastrophic motions, including capsizing.

The main difficulty with the assessment of dynamically-related undesirable events, or dynamic "failures," is both their rarity and significant nonlinearity, which need to be addressed simultaneously. Assessing the dynamical response to these wave sequences constitutes the general problem of rarity– when the time between events is long, compared to a relative time-scale (Belenky, *et al.*, 2008). The problem of rarity may be solved by separating the ship response into sub-problems, according to their time scale. The simplest example of implementation using this approach is the piecewise-linear method for calculating capsizing probability (Belenky, 1993; Paroka & Umeda, 2006; Paroka, *et al.*, 2006; Belenky, *et al.*, 2009). The same principle was also applied for nonlinear response using numerical simulations (Belenky, *et al.*, 2008a).

Consideration of groups of large waves is another way to separate the time scales, using the time between groups and the duration of a group. It is assumed that all important dynamic behavior occurs at the time while the group of waves passes the ship. This time is relatively short, and the group can be taken as a sequence of deterministic waves, which induce instability for a ship. Then the probability of encountering one of these critical wave groups was computed for a given route and duration (Themelis & Spyrou, 2007; 2008). This approach was also used by Umeda, *et al.* (2007) for broaching assessment.

As a result of separating of the time scales, there are two problems. The first problem is evaluating the response of a nonlinear dynamical system to a group of large deterministic waves (the "rare" problem). The initial conditions of the dynamical system at the moment of encounter with this group are random. The probabilistic characteristics of these initial conditions must come from the solution of the second problem, which considers ship motions in less severe waves, during the time between the groups (the "non-rare" problem). Then the probability of encounter for the ship with this critical wave group must be calculated.

Model Experiments

One of the obvious additional advantages of the wave group approach over other methods to address the problem of rarity is that it can be used in model experiments, as well as numerical simulations. Because of this, some of the inherent difficulties with validation of ship response in random seas, which more closely approximate the ocean environment, can be addressed.

Completely random wave testing can be difficult because very long run times are needed to ensure extreme events with low probability of occurrence are realized, including large waves or wave groups. Realizations of the most severe wave conditions in a random seaway require long time durations and are generally not repeatable. Also, because of the temporal and spatial limitations of a basin, it is impractical to ensure the critical excitation events are realized with standard irregular wave model experiments.

A review of previous and existing techniques for ship motions and structural testing methods is given in Bassler, *et al.* (2009; 2010). An experiment was previously conducted to generate large-amplitude deterministic wave groups, with characteristics similar to those observed in ocean measurements (Bassler, *et al.*, 2009).

DEFINITION OF A WAVE GROUP

Groups of large waves present a sequence of environmental conditions which may result in severe dynamic responses of a ship, either for the resulting ship motions, structurally, or both. However, not all wave groups will be significant in causing a severe response. Therefore, the definition of a wave group must be formulated from the perspective of ship dynamics.

Large-amplitude response, caused by the wave group, is likely to be nonlinear. However, methods with linear approximations are only applicable to relatively small-amplitude motions. Therefore, the wave elevation or wave slope angle resulting in significantly nonlinear response may be used as a threshold for the "ship dynamics" definition of a wave group.

One of the effects of nonlinearity is the dependence of the response on initial conditions. In order to consider the response to a wave group encounter as a single random event, the response to the current wave group should be independent from the response to the previous wave group. As a result, there should be enough time between these groups for the autocorrelation function of the response to effectively die out. Therefore, large waves that are close to each other in sequence should be considered as part of the same group, even if they are actually separated by a few small waves.

A sample wave group is shown in Fig. 1. As observed, the first group has three waves and all of them are above the threshold. The second group has six waves, of which four waves are above the threshold, and two waves are below the threshold. This example illustrates the difference between the "oceanographic" and "ship dynamics" definitions of the wave group. From the "oceanographic" point of view, the second group has only two waves (III) and (IV). The group is preceded by a single large wave (I) and is followed by a single large wave (VI).

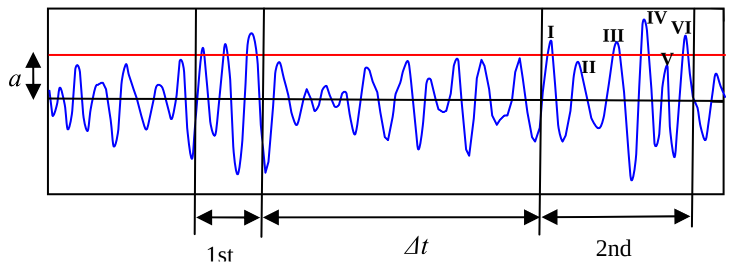

Fig. 1: Wave groups from a sample wave time-series realization, with specified amplitude threshold, *a*, and the time between wave groups, *Δt*.

However, from the point of view of the ship dynamics, all six waves must be considered together. Even if the wave (II) is small when the large wave (III) is encountered, the influence of the first large wave (I) still affects the motions. As a result, the "ship dynamics" wave group may have a more complex shape, but the encounter with a wave group becomes a Poisson flow event and time between them is expected to be distributed exponentially.

This definition of a "ship dynamics" wave group provides a generalized sequence of waves, resulting in nonlinear ship response. In the example shown for this paper, the threshold is only specified for wave crests. However, the same formulation may also be extended to wave troughs, and both the crests and troughs should be considered for a practical assessment.

SPECIFICATION OF A THRESHOLD

The threshold, *a*, or minimum level resulting in significant response, may be different depending on which problem of dynamics is being evaluated and also depends on the relative size of the ship and the waves and operational conditions for the ship. Below this threshold, the ship response may be considered small, and modeled with linear methods.

As a simple example to examine this possible definition, a 1-DOF roll equation with linear damping and single-harmonic excitation is considered,

$$\ddot{\phi} + \delta 2\dot{\phi} + \omega_0^2 f(\phi) = \alpha_e \cos(\omega_e t) \qquad (1)$$

where δ is the damping ratio, ω_e is the frequency of excitation, and α_e is amplitude of excitation. The nonlinear stiffness may be considered in a form of a cubic parabola, which makes the system, (1), the Duffing oscillator.

Consider three different amplitudes of excitation: α_1, α_2 and α_3. Below a critical response level, α_1, the ship response is considered linear. Above this level, the system may exhibit some indication of nonlinear behavior, such as a fold bifurcation (Fig. 2). The Duffing oscillator is the simplest dynamical system capable of producing a fold bifurcation (Guckenheimer & Holmes, 1983; Thompson & Stewart, 1986; Spyrou, 1997). One of the justifications for such a definition is that the fold bifurcation for roll motion has been observed experimentally (Francescutto, *et al.*, 1994).

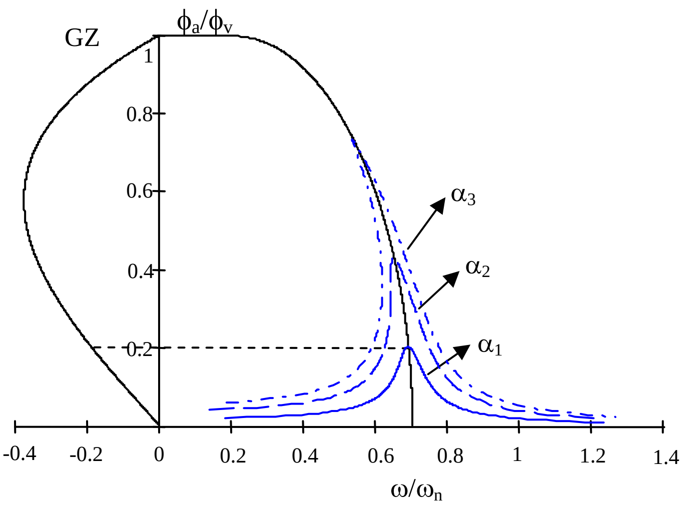

Fig. 2: Backbone curve and response curve for roll motion, with the ship-specific "*GZ* curve", modeled using the Duffing oscillator. The transition between linear response, α_1, and nonlinear response, α_2 and α_3, where fold bifurcation is observed, is identified.

In this formulation, the amplitude of the wave slope that enables fold bifurcation to occur can be considered as the threshold, *a*, in the definition of the "ship dynamics" wave group. However, within the conceptual framework of this approach, other definitions for significant events should be considered and examined as well.

TIME BETWEEN WAVE GROUPS

The time between wave groups, Δt, which can result in significant response events should be long enough so that these events can be considered independent. There are two reasons for this definition. First, by allowing enough time to pass between wave groups, the initial conditions for the wave group response become an objective of the "non-rare" problem and can be evaluated using, for example, frequency domain techniques. Second, it allows for the application of Poisson flow to the large response event caused by excitation from the wave group. The latter is very important, because it allows an explicit relation between the probability of failure and time of exposure.

To determine Δt, an autocorrelation function of roll response may be used. Because the time between groups is associated with small-amplitude response, the autocorrelation function, $r(\tau)$, can be easily computed from the response spectrum, available from the frequency domain calculations.

$$r(\tau) = \frac{1}{V}\int_0^\infty S_\phi \cos(\omega t) d\omega \qquad (2)$$

where S_ϕ is the roll response spectrum and V is the variance. The cross-correlation function, $c(\tau)$, is defined as:

$$c(\tau) = \frac{1}{V}\int_0^\infty S_\phi \sin(\omega t) d\omega \qquad (3)$$

and used to obtain the envelope, $e(\tau)$, of the autocorrelation function of roll.

$$e(\tau) = \sqrt{r(\tau)^2 + c(\tau)^2} \qquad (4)$$

Using the envelope of the autocorrelation function, a time can be identified when the autocorrelation function has decreased below a specified value, such as 5%. For the notional example presented (Fig. 3), the autocorrelation function for roll response from a Bretschneider sea state 8 spectrum will decrease to 5% after 94 seconds.

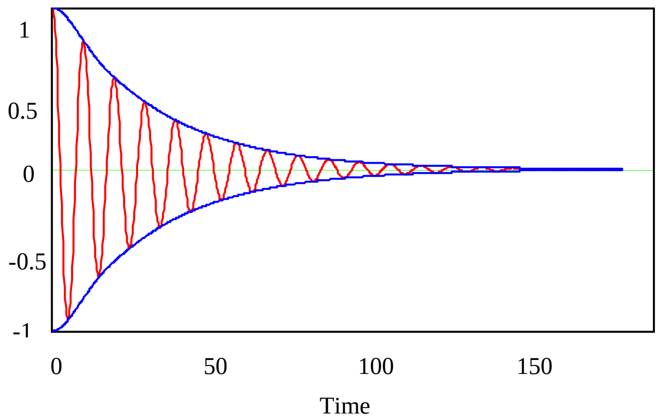

Fig. 3: Autocorrelation function of ship response, with envelope, which can be used to determine Δt.

PROBABILITY OF FAILURE DUE TO ENCOUNTER WITH A WAVE GROUP

Assuming that Poisson flow is applicable to wave group encounters, then the probability of failure during exposure time, t_e, can be determined.

$$P_F(t_e) = 1 - \exp[(-\lambda_{GS} P_{FE}) \cdot t_e] \qquad (5)$$

Here λ_{GS} is the rate of encounter of a critical wave event, either a single wave or a group, and P_{FE} is the probability of failure, once such critical wave event is encountered. As the mechanism of failure may be different when encountering a single wave or a group, it makes sense to express these quantities separately

$$P_F(t_e) = 1 - \exp[(-\lambda_G \cdot P_{FEG} - \lambda_S \cdot P_{FES}) \cdot t_e] \qquad (6)$$

where P_F is the probability of failure, t_e is the time of exposure, λ_G is the rate of encounter of a wave group, and λ_S is the rate of encounter of a single wave. P_{FEG} is the probability of failure if a wave group is encountered and P_{FES} is the probability of failure if a single wave is encountered.

The rate of encounter of a wave group or single wave, λ_{GS}, may be estimated from a time series as:

$$\lambda_{GS}^{\ *} = \frac{m^*(N_{GS})}{t_e} = \lambda_G^{\ *} + \lambda_S^{\ *} \tag{7}$$

An asterisk is used to distinguish the statistical estimate from the theoretical value. N_{GS} is the total number of waves above the threshold, both groups and single waves, observed during a window of the duration, t_e, and $m^*(N_{GS})$ is the estimate of the mean value of the total number of waves. The total number of waves above the specified threshold is given by

$$N_{GS} = N_G + N_S \tag{8}$$

where N_G is the total number of wave groups and N_S is the total number of single waves.

The total number of wave groups is given by

$$N_G = N_{GS} \cdot P_{EG} = N_{GS} \cdot \sum_{i=2}^{\infty} pmf(n_i) \tag{9}$$

where P_{EG} is the conditional probability of encountering a wave group, and $pmf\ (n_i)$ is the probability mass function of the ith wave in a group.

The total number of single waves, N_S, above the specified threshold is given by

$$\begin{aligned} N_S &= N_{GS} \cdot pmf(n=1) = N_{GS} \cdot (1 - P_{EG}) \\ &= N_{GS} \cdot P_{ES} \end{aligned} \tag{10}$$

where P_{ES} is the conditional probability of encountering a single wave, and $pmf(n=1)$ is the probability mass function of the number of single waves above the specified threshold.

In (7), $\lambda_G^{\ *}$ is the rate of encounter for a wave group and $\lambda_S^{\ *}$ is the rate of encounter for a single wave.

$$\lambda_G^{\ *} = P_{EG} \cdot \lambda_{GS}^{\ *} \tag{11}$$

$$\lambda_S^{\ *} = P_{ES} \cdot \lambda_{GS}^{\ *} = (1 - P_{EG}) \cdot \lambda_{GS}^{\ *} \tag{12}$$

CHARACTERISTICS OF WAVE GROUPS

In order to examine the robustness of the method to characterize wave sequences of dynamical significance to ship response, or wave groups, a sample Sea State 8 wave data set and arbitrary wave amplitude threshold and time between groups, were used to examine the distributions of these wave characteristics.

A sample set of 200 realizations, each 2600 seconds long, from a Bretschneider sea state 8 spectrum (H_s=11.5 m, T_m= 16.4 s) was used. As an example to illustrate the method, the wave amplitude threshold was specified to be a= 5 m and the time between wave groups Δt= 50 seconds.

The encounter with a wave group is assumed to follow a Poisson flow event; therefore, the time between them is expected to be distributed exponentially. This is confirmed by the results of a Pearson chi-square goodness-of-fit test for the sample wave data set (Fig. 4).

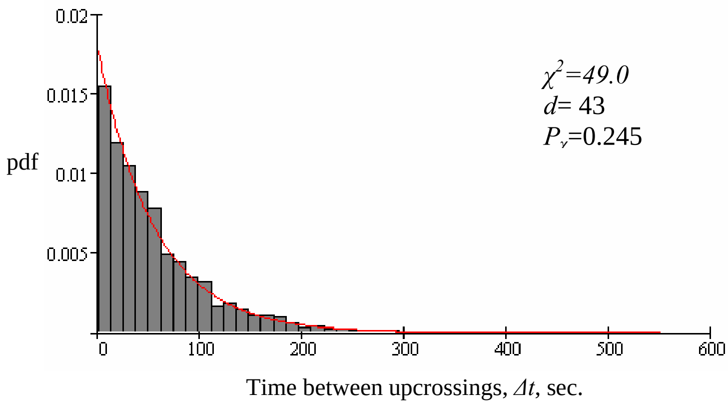

Fig. 4: Distribution of the time between wave groups, Δt, with an amplitude threshold of a= 5m, and results from a Pearson chi-square goodness-of-fit-test for the sample Bretschneider sea state 8 data set.

Number of Waves in a Group

The procedure for counting of number of waves in a group is straight forward, once the wave groups have been identified as described above. The histogram of the number of waves in a group is shown for the example data set

(Fig. 5). The most outstanding feature of this histogram is a very tall first bin, which corresponds to the case where a “group” has only one wave. These are single large waves, and from the ship dynamics perspective may be considered separately. For actual wave groups with two or more waves, the distribution appears similar to exponential.

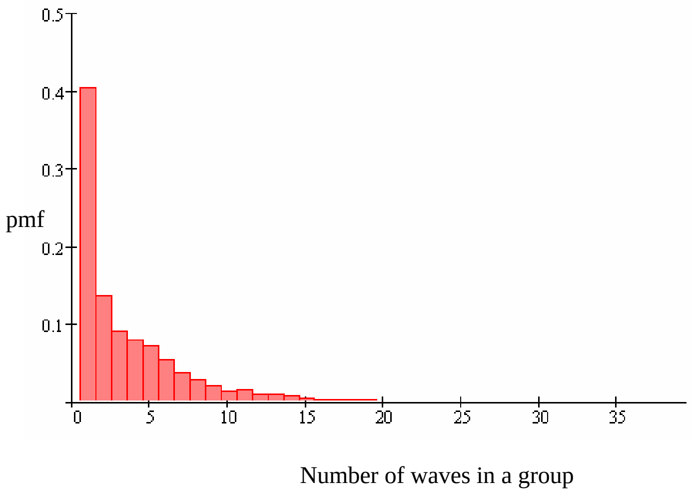

Fig. 5: Distribution of the number of waves in a group for the sample Bretschneider sea state 8 data set, with a= 5m and Δt= 50 seconds.

The following additional parameters were obtained from the simulated wave data set: amplitude for the nth wave in the group, period for the nth wave in the group, and wave steepness of the nth wave in the group. Additional discussion of these characteristics is given in Bassler, *et al.* (2010).

CONCLUSIONS

In this paper, a method to evaluate ship response in heavy seas using wave groups was discussed. The response for these events may be characterized by a high degree of nonlinearity. Modeling a significantly nonlinear system in a stochastic environment is difficult. Because of the rarity and significant nonlinearity for the large response, either numerical simulations and/or model tests must be used.

The principle idea behind using wave groups is to enable separation of the complexity of nonlinear dynamics of ship response from the complexities of a probabilistic description for the response. This separation may be achieved by considering irregular waves as a series of wave groups, which are capable of producing undesirable response, interlaced with intervals of relatively benign waves. Then the nonlinearity of the response only becomes important during the duration of the groups, while the intervals of benign waves are only “responsible” for providing the initial conditions when encountering the wave group.

The wave group can be considered as deterministic sequence of waves exciting a nonlinear dynamical system. With this formulation, wave groups may be considered as a possible method to solve the problem of rarity and, with the wave group characteristics related to ship-specific properties, can be solved in a deterministic manner.

A wave group is defined as beginning with the first upcrossing of the specified threshold, a, and ending with a downcrossing, of the threshold, where the next upcrossing of the threshold occurs at a time greater than the specified minimum duration between groups, Δt. Both the threshold and duration can be specified based on the given ship type and seaway information. This method enables wave group characteristics to be obtained from time-series information, or from merely spectral information, which may be available from wave buoys in the area of operation for a ship. Using this method, a procedure to evaluate the probability of a rare event, the undesirable response, using wave groups is also presented.

For future work, a probabilistic model of wave groups will be obtained, by fitting distributions to the characteristics. Then realizations of wave groups with the representative probabilistic characteristics must be realized in the time-domain, using either numerical simulations or experiments, or both.

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