Estimation of the metacentric height by using onboard monitoring roll data based on time series analysis

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ABSTRACT

In this paper, a novel procedure to estimate the metacentric height (GM) is proposed based on an autoregressive modeling procedure and a general state space modeling as to an onboard monitoring roll data. Firstly, the autoregressive modeling procedure is applied to estimate a natural frequency on the roll motion. After that, the general state space modeling procedure is applied to estimate the GM by using the estimated natural frequency. In order to verify the proposed procedure, model and onboard experiments were carried out. From these results, it can be confirmed that the proposed procedure can achieve the good estimation in which the estimated results are good agreement with the given one in model experiments and the derived one from stability manual corresponding to the ship condition in onboard experiments.

Keywords: *General state space modeling procedure, Monte Carlo Filter, Nonlinear observation*

1. INTRODUCTION

It is very important for a captain, officers and crews of a ship to understand the value of metacentric height (GM) under navigation. On the other hand, technique of onboard measurement on ship motions, vibration and so on has been improved in recent years. From this background, the onboard monitoring data concerning ship motions can be used to develop a safe navigation support system for heavy weather operation. In the fact, Bradley and Macfarlane (1986), Brown and Witz (1996), Ohtsu (2008) and so on had developed the system to estimate the GM. And also, Iseki et al. (2013) and Hirayama (2015) have developed the navigation support system to remain the safe navigation in heavy weather operation.

In these research, as to the way to estimate the GM dynamically, there are Brown and Witz (1996) and Ohtsu (2008). These methods use the natural frequency on the roll motion. However, it seems that the way to estimate the natural frequency has some problems. That is, in these methods an autoregressive model is used to estimate the natural frequency. In Brown and Witz (1996) the model order of the autoregressive model is fixed with 2nd order. And also in Ohtsu (2008) the natural frequency is approximated by a peak frequency of a spectrum on the roll motion, although the model order of the autoregressive model can be automatically determined by Akaike Information Criterion (AIC) [Akaike, 1973]. As the pointed out by Yamanouchi (1956), in general the roll motion cannot be approximated by 2nd order autoregressive model, since the roll motion in waves is driven by a colored noise sequence. And the natural frequency cannot be approximated by the peak frequency of the spectrum, since the peak frequency on the roll motion slightly varies with an encounter angle relationship between the ship and waves. Therefore, to estimate the natural frequency needs to use the way like Yamanouchi (1956). In this paper we focused on the way of Yamanouchi (1956) from the viewpoint of the convenience of calculation algorithm, although as such way there are Ohtsu and Kitagawa (1989), Iseki and Ohtsu (1999), Terada and Kitagawa (2009) and Terada et al. (2016).

On the other hand, even if we can estimate the natural frequency, we must also estimate a radius of gyration (k). This is big problem with respect to accurate estimation of the GM. In order to treat this problem, in general an empirical formula is used. However, according to knowledge of recent statistical science, we can also estimate the k with the GM. That is, we can apply the way called a general state space modeling procedure [Kitagawa, 1996] which is a class of time series analysis. This way is especially effective to solve the nonlinear problem in time series analysis, since these is the powerful tool to achieve the state estimation of the state space model which is called the Particle Filter (Monte Carlo Filter).

From these background, in this paper, we introduce a novel procedure to estimate the GM based on time series analysis concerning the onboard monitoring roll data. This procedure is constructed by the combination of two different statistical methods. First one is an estimation of the natural frequency of roll motion based on the way of Yamanouchi (1956), and other one is an estimation of the GM by using the estimated natural frequency at the previous step based on the general state space modeling procedure. In the second step, as mentioned before, the k is also estimated with the GM at same time. In this case, the influence of the estimation error of the natural frequency can be absorbed in the process of the general state space modeling procedure. This point is the most different point from other method to estimate the GM, and is novelty. In order to verify the accuracy of the proposed procedure, model and onboard experiments were carried out. From there results, we can confirm that the proposed procedure can achieve the good estimation in which the estimated results are good agreement with the given one in model experiments and the derived one from stability manual corresponding to the ship condition in onboard experiments. Obtained findings are reported in detail.

2. ESTIMATION OF THE NATURAL ROLL FREQUENCY

As the amplitude of the roll motion is enough small, consider the following roll motion equation:

$$\ddot{x}(t) + 2\alpha\dot{x}(t) + \omega^2 x(t) = u(t) \tag{1}$$

where, $x(t)$ is a roll angle of the ship, α is a damping coefficient, ω (=$2\pi f$) is a natural angular frequency, f is a natural frequency and $u(t)$ is an external disturbance, respectively. Here, as mentioned before, $u(t)$ is treated as the stochastic process and does not satisfy the assumption of the white noise sequence, since the characteristics of the roll motion change with the frequency characteristic of the external disturbances such as waves and winds.

According to Yamanouchi (1956), Equation 1 can be approximated by the following 2nd order autoregressive model.

$$x(n) + a_1 x(n-1) + a_2 x(n-2) = u(n) \tag{2}$$

Where,

$$\begin{cases} \alpha = -\dfrac{1}{2}\log a_1 \\ f = \cos^{-1}\left(-\dfrac{a_1}{2\sqrt{a_2}}\right) \end{cases} \tag{3}$$

On the other hand, $u(n)$ can be also approximated by the following M-th order autoregressive process.

$$u(n) = \sum_{i=1}^{M} b_i u(n-i) + v(n) \tag{4}$$

Where $v(n)$ is the Gaussian white noise sequence with mean 0 and variance σ^2. By substituting Equation 2 into Equation 4, then the following autoregressive model can be obtained.

$$x(n) = \sum_{i=1}^{M+2} c_i x(n-i) + v(n) \tag{5}$$

Here, for example, if $M = 2$, then the relationship between coefficients c_* and a_*, b_* can be written as follows:

$$\begin{cases} c_1 = a_1 + b_1 \\ c_2 = a_2 + a_1 b_1 + b_2 \\ c_3 = a_2 b_1 + a_1 b_2 \\ c_4 = a_2 b_2 \end{cases} \tag{6}$$

Therefore, to estimate the natural frequency, we firstly perform the determination of the best autoregressive model based on the minimum AIC estimation method [Akaike, 1973]. And then, the coefficients a_1 and a_2 can be obtained by solving the algebraic equation like Equation 6. And finally, the natural frequency can be calculated by using the relation of Equation 3. It should be noted that the solution of Equation 6 can be calculated by using the Newton-Raphson method.

3. ESTIMATION OF THE METACENTRIC HEIGHT (GM)

To estimate the GM and the k, consider the following equation:

$$f = \frac{\sqrt{g\,\mathrm{GM}}}{2\pi k} \tag{7}$$

where g is the gravitational acceleration. Furthermore, allow that the f, the GM and the k in Equation 7 gradually change with the time n. Then, Equation 7 can be expressed as follows:

$$f(n) = \frac{\sqrt{g\,\mathrm{GM}(n)}}{2\pi k(n)} \tag{8}$$

In this case, we add an observation noise in Equation 8, and consider that model the observation model in the general state space model. Moreover, we replace the $f(n)$ with the $y(n)$ according to the general expression of the state space modeling procedure and we consider that the $y(n)$ (= $f(n)$) is given as the observation data. As a results, Equation 8 can be written as follows:

$$y(n) = h(\mathrm{GM}(n), k(n)) + \varepsilon(n) \tag{9}$$

where $h(*)$ is the nonlinear mapping function corresponding to Equation 8 and $\varepsilon(n)$ is the observation noise according to the Gaussian white noise sequence with mean 0 and variance τ^2.

Now, we introduce the following vector:

$$\boldsymbol{x}(n) = [\mathrm{GM}(n), k(n)]^T . \tag{10}$$

Where, the notation T means the transpose of the vector. And, suppose that the time evolution of the GM(n) and the $k(n)$ can be achieved by a random walk model shown in Equation 11.

$$\boldsymbol{x}(n) = \boldsymbol{x}(n-1) + \boldsymbol{w}(n) , \tag{11}$$

where $\boldsymbol{w}(n)$ is the 2-dimensional Gaussian white noise sequence with mean vector $\mathbf{0}$ and variance-covariance matrix $\mathbf{\Sigma}$. Here, we consider Equation 11 the system model in the general state space model.

By simultaneously considering the Equation 9 and 11, we can obtain the following general state space modeling procedure.

$$\begin{cases} \boldsymbol{x}(n) = \boldsymbol{x}(n-1) + \boldsymbol{w}(n) \\ y(n) = h(\boldsymbol{x}(n)) + \varepsilon(n) \end{cases} . \tag{12}$$

Note that $h(\boldsymbol{x}(n))$ in Equation 12 is same with $h(\mathrm{GM}(n), k(n))$ in Equation 9.

To implement the state estimation of Equation 12, we apply the Monte Carlo Filter (MCF), which is a type of the particle filter, proposed by Kitagawa (1996). The MCF is powerful tool for the nonlinear and non-Gaussian state space modeling such as the general state space modeling, and can be expected as the way to estimates the Eq. 12, since we use the nonlinear observation model shown in Eq. 9. Concretely, the estimation of the probability distribution can be done by the repeat of the one-ahead prediction and the filtering based on an idea of sequential Bayesian inference. This significant merit is that the estimates gradually converge the true value. It should be noted that we show the detail of the MCF in APPENDIX-I.

Note that this procedure is called “A Self-organizing state space modeling procedure” [Kitagawa, 1998], since the procedure includes the completely unknown parameter $k(n)$.

4. RESULTS AND DISCUSSIONS

Model experiments

In order to verify the proposed procedure, we firstly carried out the free running model experiments concerning a container ship at the marine dynamics basin belonging to Japan Fisheries Research and Education Agency. The principal perpendiculars and the photo are shown in Table 1 and Fig. 1, respectively.

Table 1: Principal particulars of the sample ship.

L_{pp}	85.0 m	GM	0.828 m
B	14.0 m	T_ϕ	13.3 sec
d_m	3.54 m	k'_{yy}	0.264
W	2993.21ton		

Note: Scale ratio = 1/33

Figure 1: Photo of the sample ship.

We show the one of the results of the model experiments. The conditions are as follows:

- The model ship speed is corresponding to 10[knots] in actual ship.
- The encounter angle relationship between the ship course and the wave direction is 0[degrees], that is, the model ship ran under the following seas.
- The measurement device is the Fiber Optic Gyro (FOG) sensor made by Tamagawa seiki Co., Ltd., and its sampling rate is 20[Hz].
- The waves are the long-crested irregular waves, are reproduced by the conditions in which the significant wave height $h_{1/3}$ is 1[m] and the mean period T_{01} is 6[sec].
- Note that the results of the model scale have been transformed in to the value of the actual ship.

As preparation of the GM estimation, as shown in Fig. 2 we made the 100 data set from one record of the measured time series data such that the number of analysis data always becomes 300 samples, because the measurement time in the model experiment has the constraint. It should be noted that to use 300 samples is decided by the viewpoint of the calculation time.

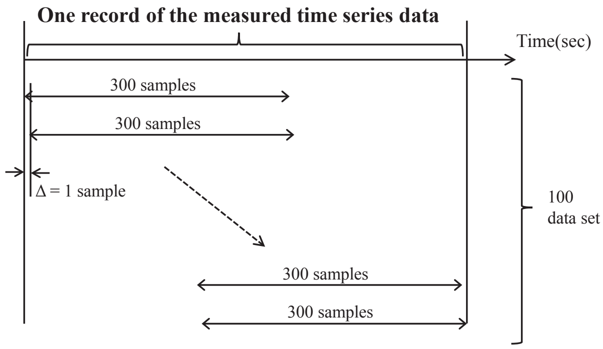

Figure 2: Schematic diagramconcerning the contraction of the data set.

The estimation of the GM was performed against these data. Fig. 3 and Fig. 4 show the results of the natural frequency and the GM, respectively. In these figures, the horizon axis indicates the number of the data set. In Fig. 4, the vertical axis indicates the expectation of the filter distribution estimated by the MCF. As mentioned above, the estimated GM depends on the accuracy of the estimated natural frequency, since the GM is calculated by using the estimated natural frequency. From these figures, it can be seen that the estimated GM is bigger than the given one, when the estimated natural frequency is bigger than the given one. However, it can be seen that the estimated GM gradually coincides with the given one, when the estimated natural frequency approaches to the given one. Therefore, it can be considered that the proposed procedure is the powerful tool concerning the GM estimation.

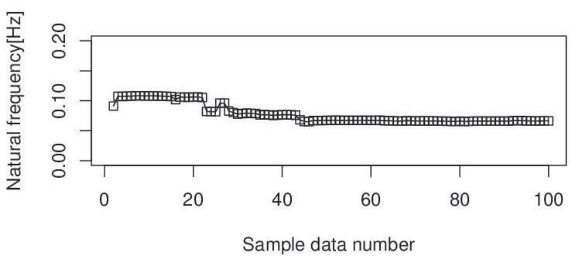

Figure 3: Results of the estimated natural frequency.

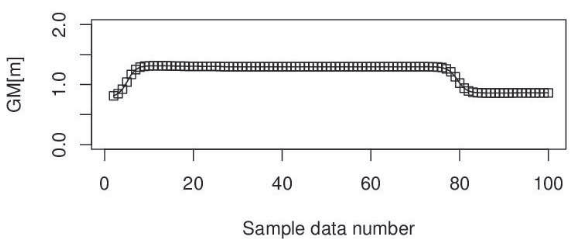

Figure 4: Results of the estimated GM.

Onboard experiments

Secondly, we carried out the onboard experiments concerning the container ship shown in Table 1 and Fig. 1 in order to verify the proposed procedure. In this case, we used the data measured under the navigation from Tokyo to Sendai in Feb. 23, 2015. This was a voyage of one and a half days. The GM recorded in the abstract log of the sample ship at that time is 2.23[m]. This value was calculated by a loading calculator based on stability manual. The measurement of the roll motion was done by the satellite compass “SC-30” made by FURUNO ELECTRIC CO., LTD. The stationary time series without the influence of an altering course and a speed change were used in order to evaluate the estimated GM. Fig. 5 and Fig. 6 show the results of the natural frequency and the GM, respectively. In these figures, the horizon axis indicates the number of the data set. In Fig. 6, the vertical axis indicates the expectation of the filter distribution estimated by the MCF. From Fig. 5, it can be seen that the results of the natural frequency of each data set have large dispersion comparison with the results of the model experiments. As to this, it may be considered that there is the limitation of the way of Yamanouchi (1956), since the actual seas confirmed by the weather map at that time was quite complex. Under the influence of this result, as shown in Fig. 6, it can be seen that the results of the GM of each data set have slight dispersion. However, the average of these is 2.04[m], we consider that the estimated GM can be competently canceled the influence of the fluctuation of the natural frequency. This fact is most important point, and is the evidence that modeling succeeds. Therefore, as well as the discussion of the model experiments, it can be considered that the proposed procedure is the powerful tool concerning the GM estimation, even if the case of actual seas.

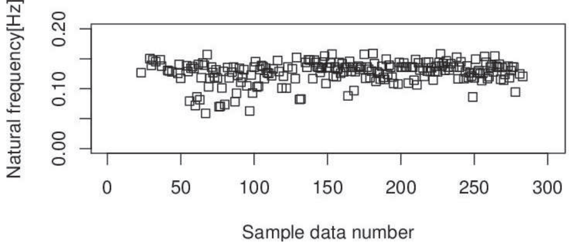

Figure 5: Results of the estimated natural frequency.

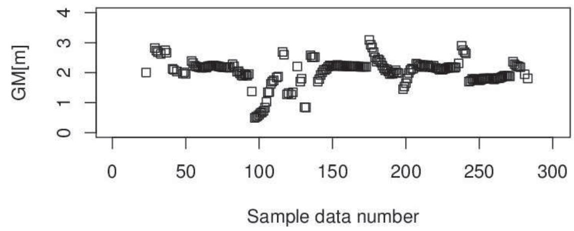

Figure 6: Results of the estimated GM.

5. CONCLUSIONS

In this paper, we introduce the novel procedure to estimate the metacentric height (GM) based on time series analysis concerning the onboard monitoring roll data. This procedure is constructed by the combination of two different statistical methods. First one is an estimation of the natural frequency of roll motion based on the way of Yamanouchi (1956), and other one is the simultaneous estimation of the GM and the radius of gyration by using the estimated natural frequency at the previous step based on the general state space modeling procedure. And this procedure also has the characteristics in which the influence of the estimation error of the natural frequency can be absorbed. This point is the most different point from other method to estimate the GM, and is novelty. In order to verify the accuracy of the proposed procedure, model and onboard experiments were carried out. The main conclusions are summarized as follows.

I. From the free running model experiments, we can confirm that the proposed procedure can achieve the good estimation in which the estimated results are good agreement with the given one in model experiments.
II. From the onboard experiments, we can confirm that the proposed procedure can achieve the good estimation in which the estimated results are good agreement with the derived one from stability manual corresponding to the ship condition in onboard experiments, even if the case of actual seas.

Therefore, we can conclude that the proposed procedure for the GM estimation is the powerful tool to remain the safe navigation, because the GM estimation can achieve with only the time series data of roll motion.

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8. APPENDIX-I

Here, as mentioned before, we show the detail of the Monte Carlo Filter (MCF).

Here, it should be noted that in this part, the symbol (n) that is a meaning of variable for the time used in Eq. (12) is expressed by subscript symbol, for simple expression of equations. In this method, each probability density function that is the predictor $p(\boldsymbol{x}_n|\boldsymbol{Y}_{n-1})$ and the filter $p(\boldsymbol{x}_n|\boldsymbol{Y}_n)$; where $\boldsymbol{Y}_n$ is the set of observations $(y_1,\ldots,y_N)$, is approximated by J particles, which can be regarded as independent realizations from that distribution. According to Kitagawa (1996), it can be shown that these particles can be recursively given by the following Monte Carlo Filter algorithm:

[Step 1]

Generate the 2 dimensional random number $\boldsymbol{f}^{(j)}_0 \sim p_0(\boldsymbol{x})$ for $j = 1 \sim J$.

Here, the $\boldsymbol{f}^{(j)}_0$ are the initial values of the state variables for the j-th which are sampled from the initial filter distribution $p_0(\boldsymbol{x})$ of the state variables. It should be noted that the assumption in which the radius of gyration (k) exists within from 0.3B to 0.5B was used, and the realizations were sampled from a uniform distribution U[0.3B, 0.5B]. On the other hand, as to the metacentric height (GM), the realizations were sampled from a normal (Gaussian) distribution $N(\mu, (\mu/4)^2)$. Where, μ was given by the following equation:

$$\mu = (0.8Bf)^2 \quad \text{(A-1)}$$

[Step 2]

Repeat the following steps for $n = 1 \sim N$.

1. Generate the 2 dimensional random number $\boldsymbol{w}^{(j)}{}_n \sim q(\boldsymbol{w})$ for j=1 ~ J.

Here, the $\boldsymbol{w}^{(j)}{}_n$ are the realizations of the system noise for the j-th which are sampled from the given system noise distribution $q(\boldsymbol{w}) \sim N(0, \boldsymbol{\Sigma})$.

2. Compute the following equation:

$$\boldsymbol{p}^{(j)}{}_n = \boldsymbol{f}^{(j)}{}_{n-1} + \boldsymbol{w}^{(j)}{}_n \tag{A-2}$$

Here, the $\boldsymbol{p}^{(j)}{}_n$ are the realizations of the predictive distribution, and this equation correspond with the random walk model for the one ahead prediction shown in Eq. 11.

3. Compute the likelihood function $\alpha^{(j)}{}_n$ as follows:

$$\alpha^{(j)}{}_n = \frac{1}{\sqrt{2\pi\tau^2}} \times \exp\left(-\frac{1}{2\tau^2} g(y_n, \boldsymbol{p}^{(j)}{}_n)^2 \right) \tag{A-3}$$

Note that $g(*,*)$ is the following inverse function concerning the observation noise $\varepsilon(n)$.

$$g(y_n, \boldsymbol{x}_n) = y_n - \frac{\sqrt{g\,\mathrm{GM}_n}}{2\pi k_n} \tag{A-4}$$

Here, the likelihood function $\alpha^{(j)}{}_n$ expresses the good fit to the data of the realizations concerning the predictive distribution of the state variables, and have a role of a weight function. The realizations for the GM and the k have infinite combination in this stage. Therefore, the following sampling with replacement can be done in order to obtain the filter distribution of the state variables.

4. Generate $\boldsymbol{f}^{(j)}{}_n$ according the following probability for j=1 ~ J by the resampling of $\boldsymbol{p}^{(1)}{}_n \sim \boldsymbol{p}^{(J)}{}_n$.

$$\Pr\left(\boldsymbol{f}^{(j)}{}_n = \boldsymbol{p}^{(j)}{}_n\right) = \frac{\alpha^{(j)}{}_n}{\alpha^{(1)}{}_n + \cdots + \alpha^{(J)}{}_n} \tag{A-5}$$

In this stage, as to the realizations of the GM and the k sampled from the predictive distribution, the realizations in which the fit to the data is wrong are disappeared and the realizations in which the fit to the data is good are copied.

As mentioned above, the separation of the GM and the k can be achieved appropriately by the repeat of the one ahead prediction process (1. and 2.) shown in the [Step 2] and the filtering process (3. and 4.) shown in the [Step 2]. Anyway, it is very important point to use the assumption in which the GM and the k vary with the time, and the separation of them can be achieved appropriately by this effect.

Note that in this study the number of particles is 1,000,000 from the view point of the calculation time. The accuracy of the state estimation, namely the estimation of the GM and the k, depends on the number of realizations.