Recent Developments of Theoretical Prediction on Capsizes of Intact Ships in Waves

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Summary

This paper overviews latest developments at Osaka University on theoretical prediction techniques for capsizes of intact ships in waves. First, analytical prediction formulae for predicting the amplitude of parametric rolling in head and following waves are examined for both conventional and modern container ships. Second, an analytical calculation method for capsizing probability is extended to the case of beam-sea capsize with trapped water on deck and the effect of water on deck is remarked with this method. Third, a new prediction technique for threshold of surf-riding in quartering waves as a heteroclinic bifurcation is presented with a successful example. Then the relationship between the threshold and broaching is discussed. These outcomes could directly contribute to development of performance-based or risk-based intact stability criteria, which are under discussion at IMO as its long term task.

Keywords: surf-riding, broaching, parametric roll, dead ship condition, water on deck, heteroclinic bifurcation, capsizing probability, averaging method

1. Introduction

At the International Maritime Organisation (IMO), the revision of Intact Stability Code started in 2002 is now under discussion for realising dynamics-oriented criteria which directly utilise physical and theoretical modelling as an alternative to prescriptive rules. Here three crucial scenarios were selected to be covered by these new criteria, and are 1) restoring arm variation problems such as parametric rolling, 2) dead ship condition and 3) manoeuvring-related problems such as broaching. For this purpose, a splinter group was intersesionally established for each selected phenomenon. (Germany, 2005) Therefore, international research group for intact ship stability, participating in international ship stability conferences and workshops, is really expected to contribute to this regulatory work by executing its theoretical and experimental works in this research area. Responding to this situation, a research team in Osaka University, involving the current authors, continues its research efforts on these three selected phenomena.

For the restoring arm variation problems such as parametric rolling, a series of free-running model experiments were carried out with a model of a 6600 TEU post-Panamax container ship in regular, long-crested irregular, and short-crested irregular waves. Here the dangerous zones of parametric rolling in head and bow seas were identified. (Hashimoto et al., 2005) In parallel, geometrical and analytical studies with realistic restoring modelling were conducted to theoretically predict dangerous zones of parametric rolling. In the last workshop in Shanghai, we presented a hydrodynamic prediction method on restoring arm variation as a function of Froude number, wave height and length. (Umeda et al., 2004b) In this paper, we examine applicability of an analytical method with realistic restoring arm modelling for predicting magnitude of parametric roll in following and head waves. This is because thresholds of dangerous parametric rolling in amplitude for parametric rolling are discussed in recent guidelines from American Bureau of Shipping (ABS) and International Towing Tank Conference (ITTC).

As to the dead ship condition, the final goal could be a risk analysis of capsize with long-term wave and wind statistics taken into account. Thus, a method for accurately and efficiently predicting short-term capsizing probability in irregular beam wind and waves is indispensable. In our opinion, a piece-wise linear approximation of restoring arm, proposed for the case of irregular beam waves by Belenky (1993), is one of the most suitable candidates. This is because this assumption enables us to use analytical solutions without overlooking major nonlinear characters in the phenomena. In the last workshop in Shanghai, we extended this method to the case of beam wind and waves by taking both windward and leeward capsizing into account. (Umeda et al., 2004b) In this paper, for more practical application to smaller ships such as fishing vessels, the method is extended to cover effects due to trapped water on deck.

For the manoeuvring-related problems such as broaching, we clearly reproduced, in a free-running model experiments with an auto-pilot, a series of dynamic phenomena; surf-ring, broaching and capsizing due to centrifugal force. (Umeda & Hamamoto, 2000) Then their numerical modelling were investigated (Umeda & Hashimoto, 2002) and recently a model is proposed to realise even quantitative prediction. (Hashimoto et al., 2004) As a next step, it is essential to directly predict thresholds of surf-riding and broaching without repeating time domain simulation with many different initial and control values. In the last workshop in Shanghai, we presented a numerical method for directly predicting surf-riding threshold in following waves as a heteroclinic bifurcation, a kind of global bifurcation in nonlinear dynamics. (Umeda et al., 2004b) In this paper, this method is extended to the case in quartering waves, in which surging is dynamically coupled with manoeuvring motions, with successful examples.

2. Magnitude of Parametric Rolling

2.1 Mathematical Model

It is pointed out that the Froude-Krylov assumption is not adequate to predict the change of roll restoring moment in waves which is a governing factor for prediction of parametric rolling.(Umeda et al, 2004a) Therefore, the captive model experiments for measuring variation of roll restoring moment in following and head waves for the ITTC Ship A-1 (container ship), which is free in heave and pitch, were conducted, and their results were used to develop a realistic mathematical model for parametric rolling prediction. Principal particulars and body plan of the subject ship are shown in Table 1 and Figure 1, respectively. For realising parametric rolling in following seas and head seas, metacentric heights are selected to be critical to Section 3.1 (general criteria) and Section 4.9 (containership criteria) of the IMO IS code (IMO, 2002), respectively.

A model of uncoupled roll motion based on model experiments is presented in Equation (1). Here, metacentric height (GM) variation in waves was obtained as the difference of the measured roll moments between 0 degrees and 10 degrees of heel, and then its amplitude and mean value were identified by the Fourier expansion of measured time series. Obtained results were fitted with third-order polynomials of Grim's effective wave amplitude, ζ_{ae}. (Grim, 1961)

Table 1 Principal particulars of the ITTC Ship A-1.

Items	
length : L_{pp}	150.0 m
breadth : B	27.2 m
mean draught : T	8.5 m
block coefficient : C_b	0.667
longitudinal position of centre of gravity from the midship : x_{CG}	1.01 m aft
metacentric height : GM	
in follow seas	0.15 m
in head seas	0.739 m
natural roll period : T_ϕ	
in follow seas	43.3 s
in head seas	19.5 s
linear extinction coefficient : a	0.17281
cubic extinction coefficient : c	0.00002

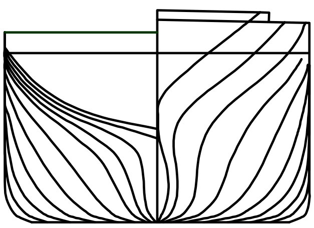

Figure 1 Body plan of the ITTC Ship A-1.

$$\begin{aligned}
&(I_{xx}+J_{xx})\ddot{\phi} = K_P(u)\dot{\phi} + K_{PPP}(u)\dot{\phi}^3 \\
&-mg\{GM\phi + C_3\phi^3 + C_5\phi^5\} \\
&-mg\{(a_1\zeta_{ae} + a_2\zeta_{ae}{}^2 + a_3\zeta_{ae}{}^3) \\
&+(b_1\zeta_{ae} + b_2\zeta_{ae}{}^2 + b_3\zeta_{ae}{}^3)\cos 2\pi(\xi_G/\lambda)\} \\
&\times\{\phi - (1/\pi^2)\phi^3\}
\end{aligned} \tag{1}$$

Here the symbols are defined in the nomenclature.

2.2Geometrical and Analytical Prediction

Poincaré mapping

Based on the above mathematical model, numerical prediction of steady states in following and head seas is carried out. Here the Froude number and wave steepness are selected as control variables. To identify bifurcation structure, we trace steady states from a trivial solution with very small wave steepness by gradually increasing wave steepness. In case the steady state is trivial, an initial roll angle of 0.5 degrees and initial roll angular velocity of 0.0 degrees per second is used for the next step. By utilising the Poincaré mapping in which the Poincaré section is a wave crest amidship condition, a numerically obtained ship roll motion for each wave steepness and Froude number was categorised into four modes: capsizing (capsize), sub-harmonic rolling with twice period as long as the encounter period (period-2), harmonic rolling with the encounter period (period-1), and non-rolling (trivial).

averaging method

Since the above mentioned numerical calculation is time-consuming, it is desirable to use an analytical method. Among various analytical methods in nonlinear dynamics, the authors applied an averaging method to parametric rolling. (Umeda et al., 2004a) and confirmed that the outcomes from the averaging method is almost identical to those from the Poincaré mapping method. (Hashimoto & Umeda, 2004) Here we assume that amplitude and phase of a period-2 solution slowly change. The averaging method provides amplitudes and phases of the solutions and their local stabilities.

Recently maritime regulatory bodies attempt to develop design and operational criteria for preventing parametric rolling. If we set a criterion for excluding parametric rolling regardless of its amplitude, however, it is found that dangerous zone could be very wide. Even with very small wave steepness, parametric rolling could occur. Thus, it is necessary to reduce the dangerous area up to a practical level. Then they are forced to limit the amplitude of parametric rolling. For example, ABS (2004) used in its published guide 15 degrees as a threshold. ITTC (2005) provides analytical formulae of amplitude of parametric rolling for such purpose. Considering the above, effectiveness of such criteria for limiting the dangerous zone and applicability of analytical methods are expected to be examined. Therefore, the authors present dangerous zones with different amplitudes of parametric rolling and compare them with the outcomes from the Poincaré mapping.

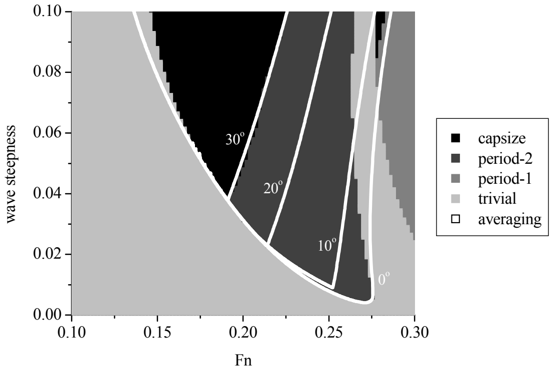

Figure 2 Threshold of parametric rolling calculated by the averaging method and geometrical method with wave length to ship length ratio of 1.0 for the ITTC ship A-1 in following seas.

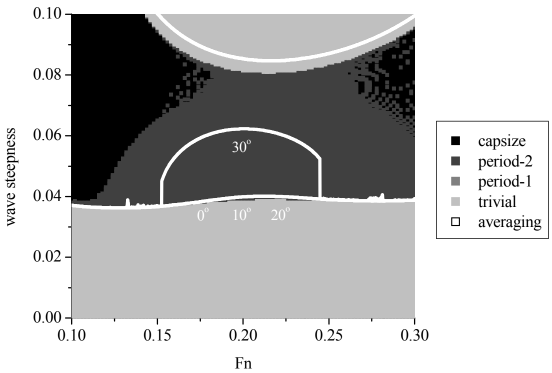

Figure 3 Threshold of parametric rolling calculated by the averaging method and geometrical method with wave length to ship length ratio of 2.0 for the ITTC ship A-1 in head seas.

Figure 2 shows the results obtained by the Poincaré mapping and averaging method in following waves. Here, it is confirmed that the calculated threshold with acceptable roll angle of 0 degrees corresponds to that with Poincaré mapping well. Occurrence area of parametric rolling becomes narrower with increase of acceptable roll amplitude considerably. In case of acceptable roll amplitude of 20 degrees, area of parametric rolling becomes almost a half of that without the limitation of roll amplitude. Figure 3 shows the results in head waves. Because critical wave condition of parametric rolling in head waves can be regarded as a sub-critical bifurcation, parametric rolling suddenly appears with relatively large roll angle. Therefore no significant difference here can be found even if we introduce critical roll amplitude of parametric rolling.

2.3 Effect of hull form

The ITTC ship A-1 discussed above does not have exaggerated flared bow and transom stern, while the C11 class Post-Panamax container ship that suffered an accident due to parametric rolling has. Therefore, to investigate effect of hull form, the authors applied the above procedure to a Post-Panamax container ship as well. The principal particulars and body plan of the post-Panamax container ship used here are shown in Table 2 and Figure 4, respectively. Here the metacentric height used here is designed one, which complies with Section 4.9 of the IMO IS Code with a sufficient margin.

Table.2 Principal particulars of the post-Panamax container ship.

Items	
length : L_{pp}	283.8 m
breadth : B	42.8 m
mean draught : T	14.0 m
block coefficient : C_b	0.629
longitudinal position of centre of gravity from the midship : x_{CG}	5.7 m aft
metacentric height : GM	1.06 m
natural roll period : T_ϕ	30.3 s
linear extinction coefficient : a	0.1926
cubic extinction coefficient : c	0.00038

Numerical results shown in Figure 5 for this ship indicate that threshold of amplitude of 10 degrees is almost identical to that of 0 degrees. Thus, similar conclusion can be obtained on the effectiveness of limitation of amplitude in parametric rolling in head seas also for this ship. Despite larger GM, the critical wave steepness of the post-Panamax container ship is smaller than that of the ITTC Ship A-1. This demonstrates that a container ship having larger flare can be more vulnerable to parametric rolling. In conclusion, it seems to be difficult to realise wider operational zone only with the limitation of amplitude of parametric rolling in head seas. Hull form improvement and/or anti-rolling devices are expected to prevent parametric rolling. (Umeda et al., 2005)

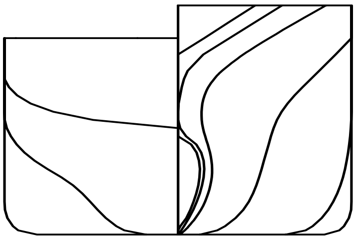

Figure 4 Body plan of the post-Panamax container ship.

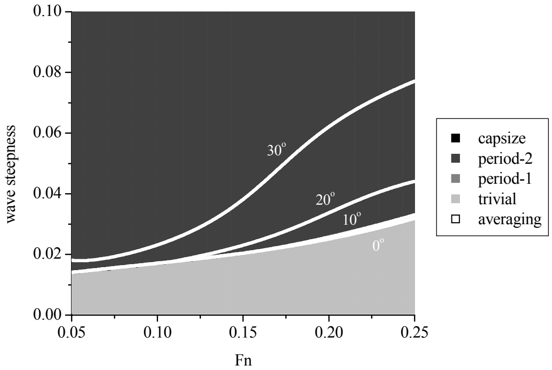

Figure 5 Threshold of parametric rolling calculated by the averaging method and geometrical method with wave length to ship length ratio of 1.0 for the post-Panamax container ship in head seas.

3. Capsizing Probability with trapped water on deck

The authors investigated capsizing probabilities for RoRo ships, a car carrier, a large passenger ship with a piece-wise linear approximation of restoring arms. If we apply this method to smaller ships such as fishing vessels, an additional factor should be taken into account. This is the effect of trapped water on deck. These ships have smaller freeboard but higher bulwark comparing with their ship size. As a result, once shipping water occurs, water can be easily trapped by bulwarks. Although a certain amount of water can flow out through freeing ports, the balance between water ingress and egress can be trapped on deck. Righting arm can deteriorate as a result of such trapped water on deck. Many experimental and numerical works (for example, Adee & Caglayan, 1982, Grochowalski et al., 1998, Belenky et al., 2003) have been published so far on trapped water on deck but few attempts can be found to incorporate it into capsizing probability prediction in short term.

Table 3 Principal particulars of the purse seiner.

Items	
Length Between Perpendiculars: L_{BP}	38.500 m
Breadth: B	8.100 m
Draft: T	2.581 m
Displacement: Δ	474.30 ton
Metacentric Height: GM	1.950 m
Linear Damping Coefficient: μ	0.076 s^{-1}
Quadratic Damping Coefficient: β	1.753 rad^{-1}
Effective Wave Slope: γ	0.613
Lateral Windage Area: A_L	174.67 m^2
Height of Wind Force from the Centre of Hydrodynamic Reaction Force: H	3.860 m

In this paper, we assume that amount of water on deck slowly changes with time while ship motion fast changes. Then the following procedures are proposed. Firstly, mean of amount of water on deck in irregular waves, Q_m, is estimated as a function of significant wave height, $H_{1/3}$ and mean wave period, T_{01}. Secondly, the restoring arm with the mean of amount of water on deck is hydrostatically calculated. Thirdly, the obtained righting arm is approximated with piece-wise linear curve by keeping the angle of vanishing stability, derivatives of righting arm at the angle of vanishing stability and dynamic stability up to the angle of vanishing stability as the same as original ones. Fourthly, the formulae for short-term capsizing probability, P_Q, used by Paroka et el. (2005) are applied to this piece-wise linear system. Finally long term capsizing probability, Pc, as a function of exposure time, T is obtained with the following formula and a joint probability density function of wave height and period, P_{HT}.

$$P_c(T)=\int_0^\infty\int_0^\infty P_{HT}\left(H_{\frac{1}{3}},T_{01}\right)P_Q\left(Q_m\left(H_{\frac{1}{3}},T_{01}\right),H_{\frac{1}{3}},T_{01},T\right)dH_{\frac{1}{3}}dT_{01} \tag{2}$$

Here dynamic effects of water on deck, such as change in damping and added mass, are ignored.

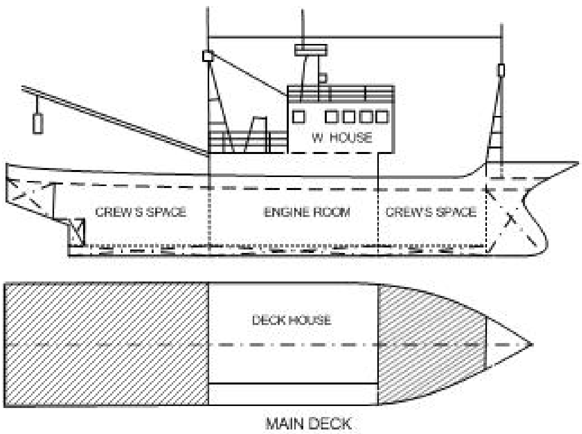

Figure 6. Schematic view of deck well layout.

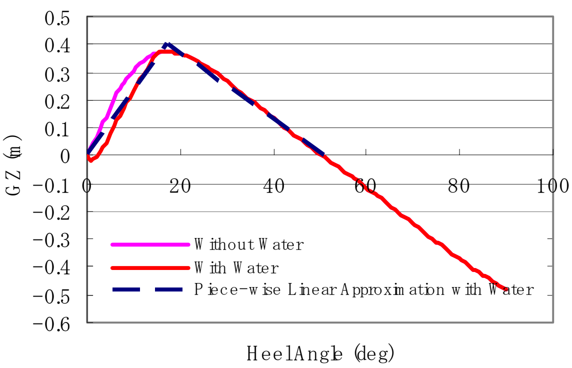

Figure 7 Righting arm curves of the purse seiner with trapped water on deck of 14.68m^3.

The proposed method is applied to a typical purse seiner operating in Japan. Its principal dimensions and schematic view of deck wells are shown in Table 3 and Figure 6, respectively.

The restoring arms with and without trapped water on deck are shown in Figure 7 as well as the piece-wise linear approximation of the latter. Here the metacentric height becomes negative because of trapped water on deck but the effect of trapped water on deck disappears when the external water surface exceeds bulwark top. In this work, the mean of trapped water on deck is estimated with the model experiment by Matsuda et al. (2005) and the wave statistics are done by database from hind-casting for a major fishing area off Kyusyu. (Ma et al., 2004) Final outcomes as capsizing probability with exposure time of 1 hour are shown in Figure 8. This demonstrates that trapped water on deck significantly increases capsizing probability in beam wind and waves except for very poor stability cases. When the metacentric height is very small, natural roll period is much larger than existing wave periods. As a result, roll motion with trapped water on deck can be very small. This could result in smaller capsizing probability in beam waves.

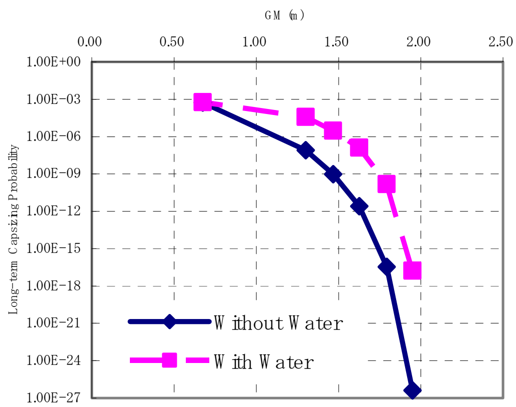

Figure 8 Long-term capsizing probability with and without water on deck for the purse seiner operating off Kyusyu. Here the exposure time is 1 hour.

4. Surf-riding threshold in quartering waves

It is widely accepted that surf-riding is a prerequisite for broaching, which could result in capsizing. This surf-riding corresponds to a equilibrium of a surge-sway-yaw-roll-rudder mathematical model. And the occurrence of surf-riding can be explained as a heteroclinic bifurcation in which a trajectory from unstable surf-riding equilibrium point, i.e. a unstable invariant manifold, connects with a trajectory to another unstable surf-riding equilibrium point, i.e. a stable invariant manifold. (Makov, 1969, Umeda, 1990, Spyrou, 1996) Recently it was confirmed that thresholds of capsizing due to broaching well coincide with heteroclinic bifurcations. (Umeda, 1999) This is the reason why we have to predict surf-riding threshold as a heteroclinic bifurcation point.

In case of pure following seas, uncoupled surge model is sufficient for this purpose and a numerical algorithm for a phase plane spanned by surge displacement and its velocity was presented with a successful example by the authors. (Umeda et al., 2004b)

This method is extended to the case of stern quartering seas. Since this situation requires a surge-sway-yaw-roll-rudder mathematical model, an 8-dimensional phase space should be investigated. The stable invariant manifold here becomes 7-dimensional while the unstable invariant manifold is 1-dimensional. (Umeda, 1999) Thus the additional condition to be satisfied is that the vector normal to the stable invariant manifold should be orthogonal to the unstable invariant manifold.

Numerical examples for the ITTC ship A-2 (Umeda & Hashimoto, 2002) in stern quartering seas are shown in Figures 9-10. Here the results of numerical simulation in time domain starting from a periodic orbit with a sufficiently low forward speed are qualitatively displayed and the heteroclinic bifurcation points obtained by the above method are also plotted. In the case of steeper waves (Figure 9) where a typical broaching was observed in the model experiment (Umeda & Hamamoto, 2000), the heteroclinic bifurcation line almost coincides with the boundary between periodic motion and capsizing due to broaching. In this situation, once surf-riding occurs, broaching and subsequent capsizing are unavoidable. When the heading angle is larger, some discrepancies are found. It is presumed that this could be outcomes of initial value dependence. (Umeda, 1999) In the case of more moderate waves (Figure 10), broaching disappears in the surveyed zone and oscillatory surf-riding (Spyrou, 1995) emerges. The heteroclinic bifurcation line almost coincides with the boundary between periodic motion and stable or oscillatory surf-riding at least for smaller heading angle. Because of smaller wave steepness and wave length, even under a surf-ridded condition, the ship can keep her straight course in average. In the region of stable surf-riding, surf-riding equilibria of the mathematical model are found. Then it is found as a result of calculation that eigenvalues of locally linear system at these equilibria have positive real parts without imaginary parts. Tracing these equilibria towards larger heading angle, Hopf bifurcation in which real part of eigenvalue becomes positive and its imaginary part emerges, is identified. Theses points almost coincide with the boundary between oscillatory surf-riding and “not identified” within simulated time duration of 1000 seconds. This suggests that “not identified” here should be categorised as stable surf-riding if we continue numerical simulation with longer duration. Further investigations of these problems are now under way.

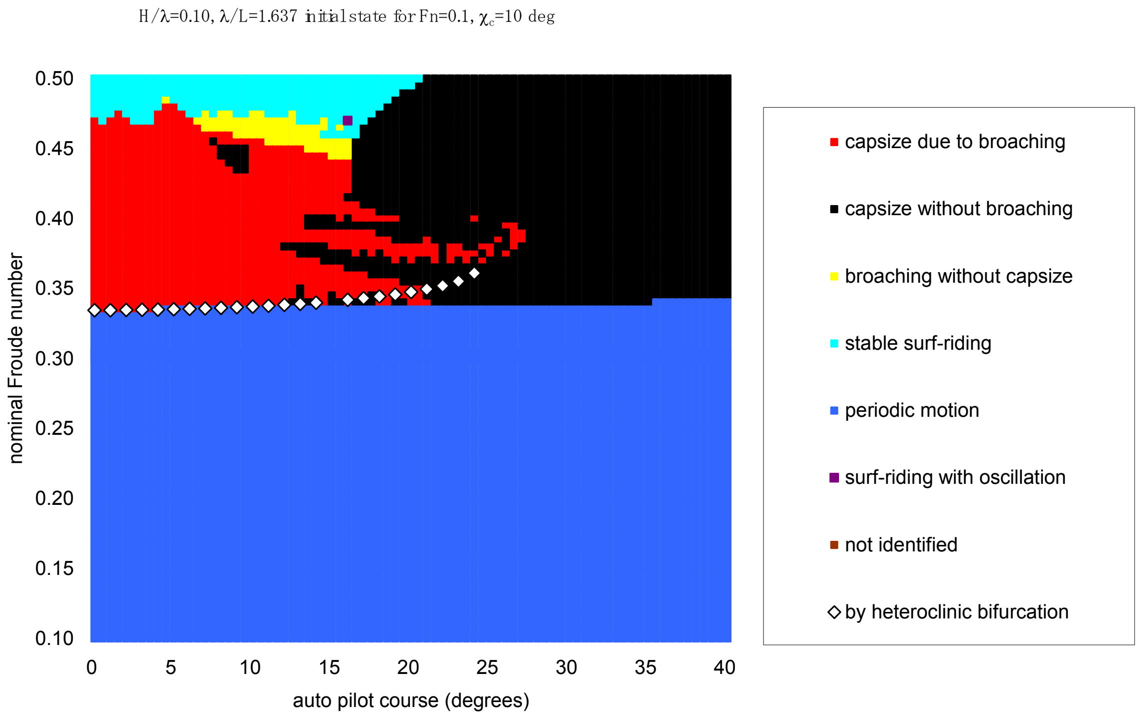

Figure 9 Heteroclinic bifurcation points and motion modes identified with initial value simulations from periodic motions with sufficiently slow forward speed for the ITTC Ship A-2. Here wave steepness is 1/10 and wave length to ship length ratio is 1.637.

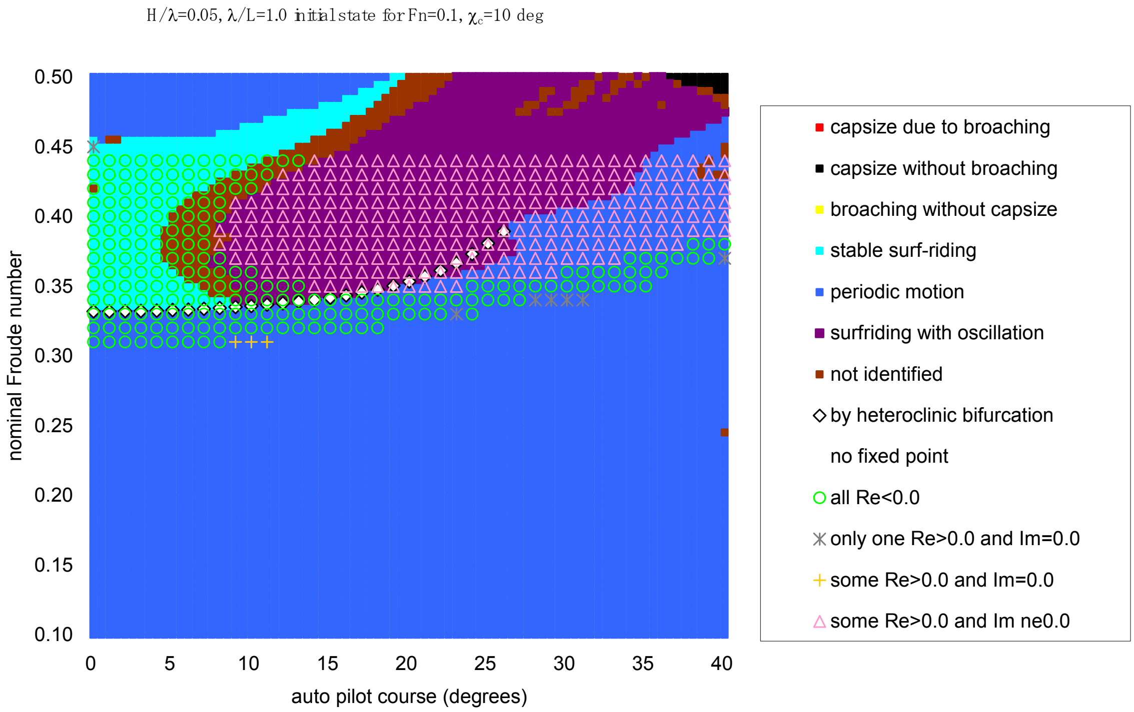

Figure 10 Heteroclinic bifurcation points and motion modes identified with initial value simulations from periodic motions with sufficiently slow forward speed for the ITTC Ship A-2. Here wave steepness is 1/20 and wave length to ship length ratio is 1.0.

5. Conclusions

1) The threshold for parametric rolling with its amplitude of 10 and 20 degrees in head seas almost coincides with that parametric rolling regardless of its amplitude while it is not so in following seas.
2) In irregular beam wind and waves, capsizing probability with water on deck is much larger than that without water on deck except for very small GM cases.
3) A heteroclinic bifurcation point in stern quartering seas can be determined with an iterative numerical algorithm, and well coincides with the boundary between periodic motion and capsizing due to broaching or surf-riding within smaller heading angles

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Nomenclature

g:	gravitational acceleration
I_{xx}:	moment of inertia in roll
J_{xx}:	added moment of inertia in roll
K_P:	linear roll damping coefficient
K_{PPP}:	cubic roll damping coefficient
m:	ship mass
u:	ship velocity in longitudinal direction
ϕ:	roll angle
λ:	wave length
ξ_G:	horizontal distance of ship centre from a wave trough