Approximation of ship equations of motion from time series data

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ABSTRACT

In this work, ship equations of motion are approximated from numerically simulated test cases. Once approximated, the equations of motion can be used in an effort to predict future motions, to estimate the impact of initial conditions on an experimental data set, and to compare a broad range of experimentally based simulations to those from numerical/analytical models for validation purposes.

KEYWORDS

Capsizing; optimization; equations of motion

INTRODUCTION

Large amplitude ship motions represent complex, even chaotic phenomena. As part of the ongoing pursuit of techniques to evaluate, reconcile, and provide form to experimental data, the authors propose approximating the equations of motion based upon experimental time series. A recent effort towards approximating linearized coefficients as a function of forward speed for coupled heave-pitch ship motions from a numerically generated data series is discussed in Suleiman *et al.* (2000) employing the Eigensystem Realization Algorithm. Additionally, in Suleiman (2000), sway, roll, and yaw motions are considered as well as 5 DOF couplings of heave, pitch, sway, roll, and yaw. While the results using ERA prediction were strong for small amplitude motions, in higher sea states precision was lost. Nonlinear effects are considered separately in the special case when the ship is subjected to parametric resonance (Suleiman, 2000). The emphasis of the work presented herein is on large amplitude motions, primarily in the roll degree of freedom, where nonlinearities are prevalent. The motions are not constrained to any specified type of large amplitude rolling, such as parametric resonance.

Reconstructing equations of motion from data is discussed heavily in the complex systems literature. See for example Breeden and Hübler (1990), Cremers and Hübler (1987), Crutchfield and McNamara (1987), and Eisenhammer *et al.* (1991). While the specific details of different existing methods for approximating equations of motion from an experimental time series vary, all at their most fundamental level are predicated upon ‘guessing’ the form of equations, either composed of a sum of standard functions or some archetypical model, then optimizing the coefficients in the equations to best fit the experimental data.

For the case of vessel motions, a vast body of work exists describing the form of ship equations of motion ranging dramatically in complexity (Lewis, 1989; Bhattacharyya, 1978; Fossen, 1994; Lewandowski, 2004). In this paper a demonstration of concept is presented in which the authors work to approximate equations that fit artificially generated (*i.e.* through numerical simulation) 'experimental' data for single degree of freedom rolling and capsize of the *Edith Terkol* (Soliman & Thompson, 1991). The form of the approximated equations of motion are also taken to be represented by a single degree of freedom roll equation in which linear plus cubic terms are used for damping (Dalzell, 1978) and stiffness. Five unknown parameters are taken to be the damping and stiffness coefficients as well as the magnitude of forcing. The optimization procedure matches the 'experimental' data, accurately reproducing time series for small and large motions in addition to the roll/roll velocity safe basin.

SINGLE DEGREE OF FREEDOM DEMONSTRATION OF CONCEPT

As a numerical demonstration of concept, the single degree of freedom roll/roll velocity time series for the *Edith Terkol* was generated through integration of Equation 1 with parameters $b_1 = 0.0043$, $b_2 = 0.0225$, $c_1 = 0.384$, $c_2 = 0.1296$, $c_3 = 1.0368$, $c_4 = -4.059$, $c_5 = 2.4052$, and $\omega_e = 0.85 \cdot \omega_n = 0.85 \cdot 0.62 = 0.527$ (Soliman & Thompson, 1991).

$$\begin{aligned}&\ddot{\phi} + b_1\dot{\phi} + b_2|\dot{\phi}|\dot{\phi} + c_1\phi + c_2|\phi|\phi + c_3\phi^3 \\ &+ c_4|\phi|\phi^4 + c_5\phi^5 = F\sin(\omega_e t)\end{aligned} \quad (1)$$

The output time series was then treated as though it were an unknown roll/roll-velocity "experimental" data set to which one wishes to approximately fit equations of motion. A simple, standard form of the roll equation of motion was assumed as given in Equation 2 where the subscripted letter u denotes an unknown. The only variable assumed to be explicitly known is the encounter frequency as that is comparatively trivial to approximate from a given data set. If necessary it is feasible to treat encounter frequency as an additional unknown at some computational cost.

$$\begin{aligned}&\ddot{y} + b_{u1}\dot{y} + b_{u3}\dot{y}^3 + c_{u1}y + c_{u3}y^3 \\ &= F_u \sin(\omega_e t)\end{aligned} \quad (2)$$

A Nelder-Mead simplex search is performed to minimize the distance between the phase-space output of the approximated equations of motion $< y, \dot{y} >$ and the "experimental" data $< \phi, \dot{\phi} >$, *i.e.* minimizing the function in Equation 3. The values of $< \phi, \dot{\phi} >_j$ are used as the initial conditions for the simulation to determine $< y, \dot{y} >_{j+1}$. Thus the coefficients for the approximated equations of motion are optimized over the entire time series via comparison of piecewise integrations. To avoid detecting an erroneous local minimum, each case was examined at a number of values for *N*. That is, while the length of a source time series is constant, the number of flow steps, *m* where *m* is the length of the time series divided by *N*, over which each piecewise integration was performed was varied to find a solution balancing local and long-time behavior of the system as well as computation time. For this reason the value deduced for each constant were considered both for ability to reproduce the experimental data, and physical relevance (*ie.* examination of roll damping and restoring arm curves).

$$\sum_{j=1}^{N} \left\| < y, \dot{y} >_j - < \phi, \dot{\phi} >_j \right\| \quad (3)$$

To begin, Equation 1 was used to generate 500 seconds of data with initial conditions $< \phi_0, \dot{\phi}_0 > = < 0,0 >$ and $F = 0.0195$. Using a Nelder-Mead search to optimize values for unknown coefficients in Equation 2 so as to minimize the summation in Equation 3 results in values: $b_{u1} = 0.0095$, $b_{u3} = -0.4445$, $c_{u1} = 0.4063$, $c_{u3} = 0.4209$, and $F_u = 0.0200$. Figure 1 shows the original time series, as generated by integration of Equation 1 and the approximated time series, generated by integration of Equation 2 with the above optimized coefficients. The first 500 seconds of Figure 1 show that the approximated time series accurately reproduces the original source data. Equation 1 is then simulated for an additional 500 seconds and compared to the approximated Equation 2 to demonstrate that the steady state behavior of the system is also captured, that is, that behavior beyond the period used to optimize the unknown coefficients is reproduced by the approximated equations of motion.

Before assigning physical meaning to the approximated coefficients, one must carefully evaluate the limitations of the approximation. For example, the roll stiffness and damping curves bases upon the actual and approximated equations of motion are given in Figure 2. While the curves are fit quite well within the ranges of the incorporated source data, ie. roll from ±0.3 rad and roll velocity from ±0.15 rad/s, outside of these ranges agreement is poor. Indeed the asymptotic trends are distinctly non-physical with, for example, stiffness increasing infinitely with roll angle. By assessing the roll damping and stiffness curves one can glean a greater understanding of the range in which the approximated equations of motion are accurate and physically meaningful thus addressing the “cancellation” effect described by Suleiman (Suleiman, 2000) (in which Suleiman expresses concern that in this type of technique two incorrect parameters may have cancelling detrimental effects thus masking their inaccuracy). Additionally, through this method it is not the intention to state that the optimization has found the damping and stiffness coefficients for the system, rather that a set of coefficients have been found which accurately reproduce the input data series within a viable parameter range.

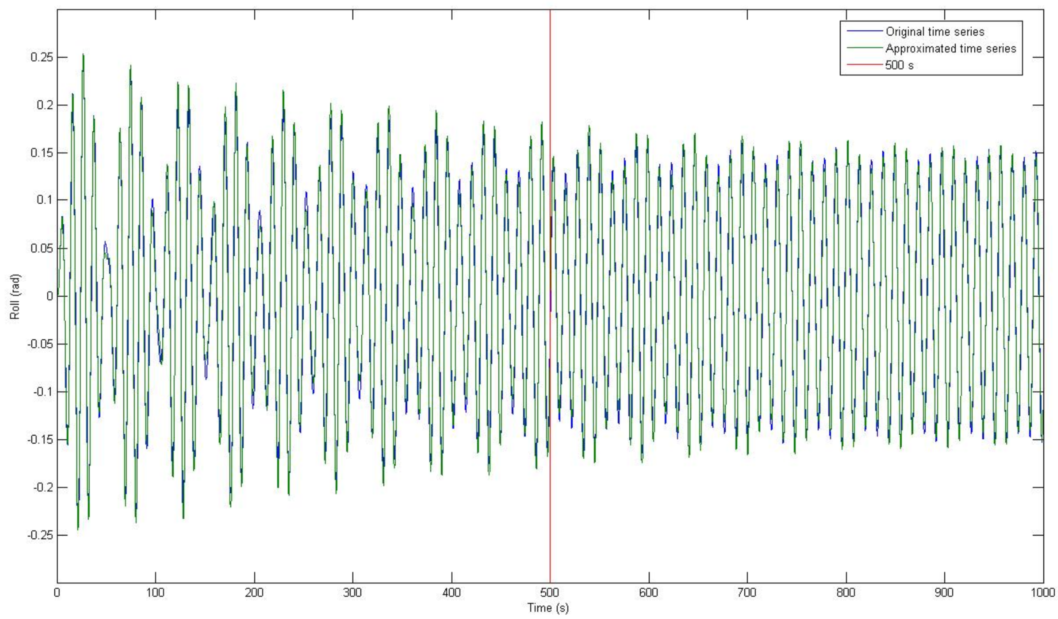

Figure 1: Original and approximated time series for roll/roll velocity initial conditions of < 0, 0 >. Initial 500 seconds of original data used to approximate coefficients of Equation 2 with subsequent 500 seconds demonstrating consistent prediction of steady state behavior.

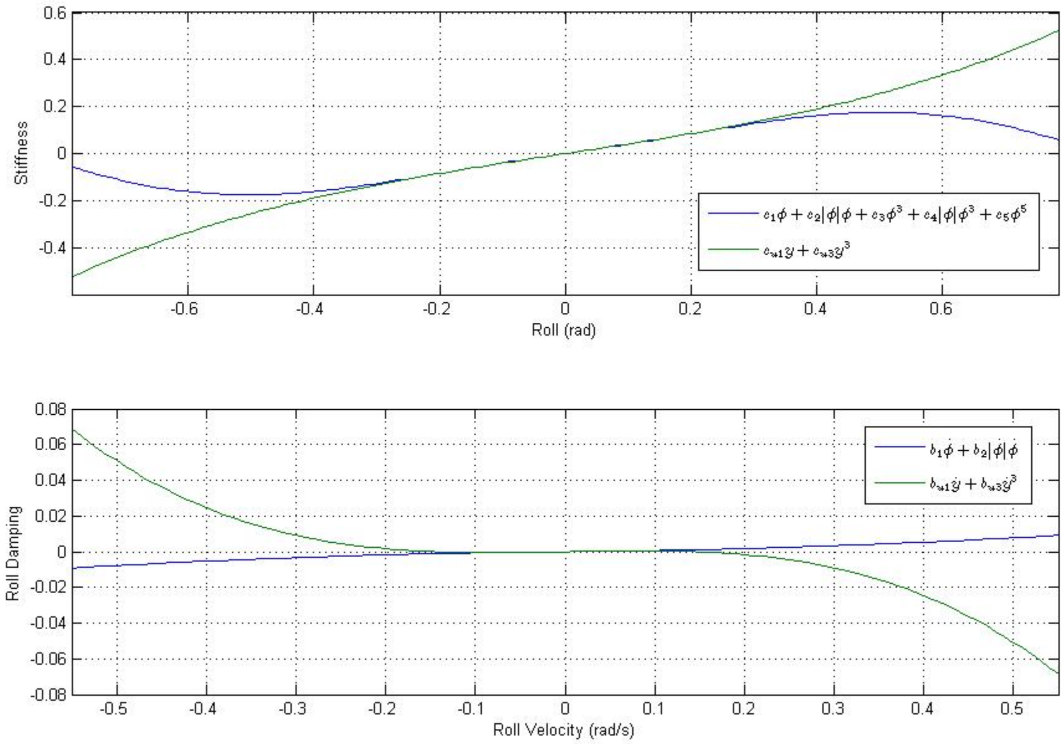

Figure 2: Comparison of original and approximated stiffness and roll damping curves for roll/roll velocity initial conditions of <0,0>.

If one begins at an initial condition leading to larger amplitude motions, the methodology is equally capable of replicating the desired time series while simultaneously capturing the expected roll stiffness and damping behavior over a larger parameter range. The same process described above was conducted for a time series generated from Equation 1 for initial conditions of $< 0.4, 0 >$ for roll and roll velocity. When the optimization routine dictated by Equation 3 is run on Equation 2 the following coefficient values are found: $b_{u1} = 0.0060$, $b_{u3} = 0.0639$, $c_{u1} = 0.4344$, $c_{u3} = -0.2403$, and $F_u = 0.0200$. A comparison of the original and approximated roll time series is given in Figure 3. As with Figure 1 the approximate coefficients are based upon the first 500 seconds of data with the time from 500-1000s presented to show the appropriate capturing of steady state behavior in a quasi-predictive sense. A comparison of the original and approximated stuffiness and roll damping curves for this larger initial condition pairing is given in Figure 4. Intuitively, with the larger range of roll motion, damping and stiffness are accurately represented over a larger roll and roll velocity range than seen in Figure 2 which was based upon roll motions in a smaller region.

With this more accurate model of the restoring damping and stiffness force terms, other initial condition pairs can be simulated. For example, Figure 5 uses the coefficients found from the optimization routine based upon comparison to an original time series with initial roll and roll velocity of $< 0.4, 0 >$ to simulate Equation 2 with initial conditions of $< 0, 0 >$. This simulation is compared to a time series generated from Equation 1 for the same <0, 0> initial condition pair. In general it is noted that roll motion is consistently underpredicted by as much as 4 degrees.

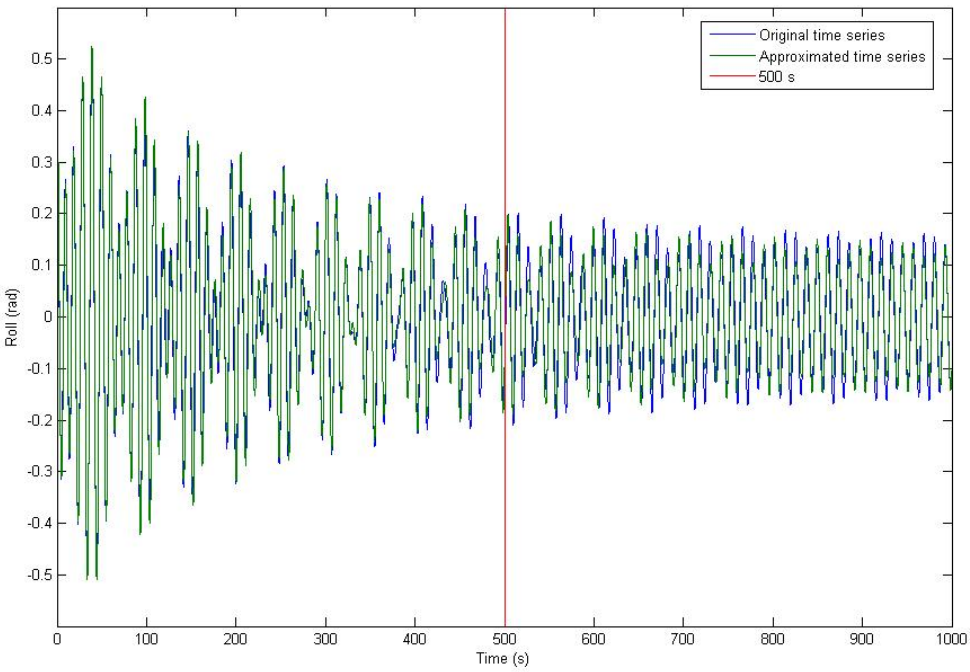

Figure 3: Original and approximated time series for roll/roll velocity initial conditions of < 0.4, 0 >. Initial 500 seconds of original data used to approximate coefficients of Equation 2 with subsequent 500 seconds demonstrating consistent prediction of steady state behavior.

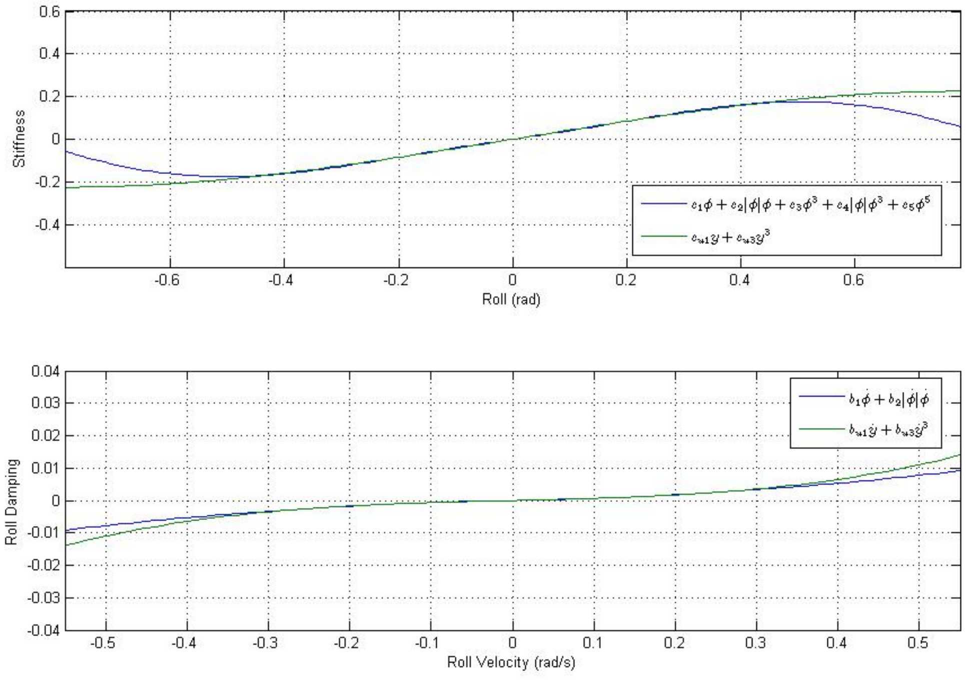

Figure 4: Comparison of original and approximated stiffness and roll damping curves for roll/roll velocity initial conditions of <0.4,0>.

If one uses simulated data near the separatrix between safe and unbounded motions which results in very large amplitude rolling, such as the roll/roll velocity initial conditions of $<-0.4,-0.35>$ a number of interesting findings arise. Specifically, while the estimated solution no longer accurately models small amplitude motions, it captures large amplitude motions as well as the safe basin with strong precision. To begin, consider Figure 6 which shows the original simulated time series for roll/roll velocity initial conditions of $< -0.4,-0.35 >$ (top), $< 0, 0 >$ (middle), $< 0.4, 0 >$ (bottom). The equations of motion are approximated using 500 seconds of data with the initial conditions of $< -0.4,-0.35 >$ to fit Equation 2 resulting in $b_{u1} = 0.0098$, $b_{u3} = 0.0388$, $c_{u1} = 0.5147$, $c_{u3} = -0.7052$, and $F_u = 0.0220$. The largest amplitude case is well replicated and projected for an additional 500 seconds. However, after the large amplitude transients diminish in the smaller amplitude cases, roll motion is consistently underpredicted. By comparing the actual and approximated stiffness and roll damping curves given in Figure 7 it is likely that this underprediction is due to overprediction of stiffness at small amplitudes. However, this large amplitude case does accurately capture the shape of both curves during initial large amplitude transients.

Figure 8 presents a safe basin generated via optimizing the unknown coefficients in Equation 2 to a time series generated from Equation 1 with initial conditions of $<-0.4,-0.35 >$. This non-capsize case on the separatrix between safe and bounded motions reproduces the original safe basin of the system governed by Equation 1 with high fidelity.

CONCLUSIONS

Noteworthy difficulties arise when attempting to apply this form of simplistic, brute-force approach to real data. Specifically, the number of unknowns and reasonable accuracy of the model become cumbersome when applied to realistic conditions. When applying this methodology to David Taylor Model 5514 data, it was quite apparent that idealizing the forcing as being at a single magnitude and frequency would result in inaccurate approximations to the actual case. Adding more forcing amplitude/frequency terms and/or more degrees of freedom results in an excessive number of unknowns for such a brute force optimization based approach.

In an on-board sense, to generate a simplified model for brief predictions of ship behavior, it is possible these difficulties could be overcome by idealizing the system as an oscillator with slowly varying coefficients. As such, a low-order model would be used for an optimal fit over short increments of time and continuously updated. In a similar vein, a neural-network could be used both to select an optimal model of the system and to predict motions. The work of Hess *et al.* has shown great promise for the use of recursive neural networks in maneuvering simulation (Hess *et al.*, 2006).

Clearly for the simplistic case presented above, in which the coefficients of the equation of motion are known, lower and/or different order approximations to the coefficients for Equation 2 could be generated with a least squares regression within some critical range of roll angles or roll velocities (Dalzell, 1978). However, it is a matter of intelligently determining an approximated form of equations of motion, appropriate time scale, and incorporating a sufficiently powerful optimization/determination method to extend this approach to actual experimental data. For example, to overcome the hurdles present with random waves, one might choose to provide an input of wave elevation whilst optimizing on remaining parameters. Alternatively, one might conclude that the truest necessity for predicting large motions is approximating only the next few roll cycles rather than an entire time series thus reducing the difficulty and time cost of an optimization routine.

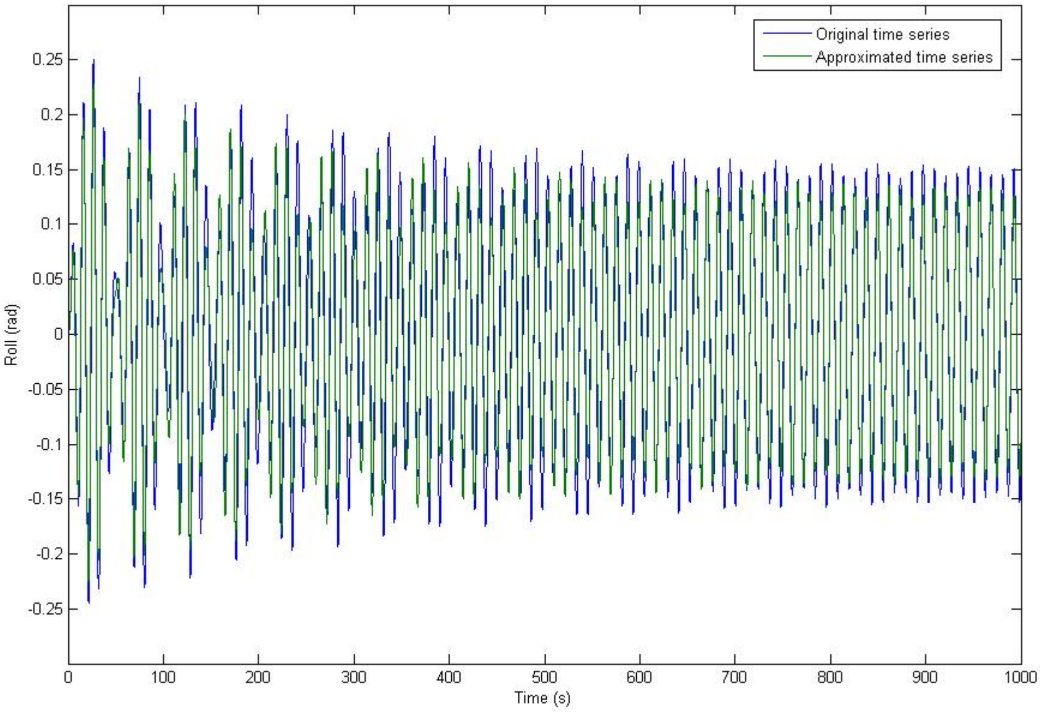

Figure 5: Original and approximated time series for roll/roll velocity initial conditions of < 0, 0 >. Approximated equations of motion generated from optimizing fit to time series data with initial conditions of < 0.4, 0 >.

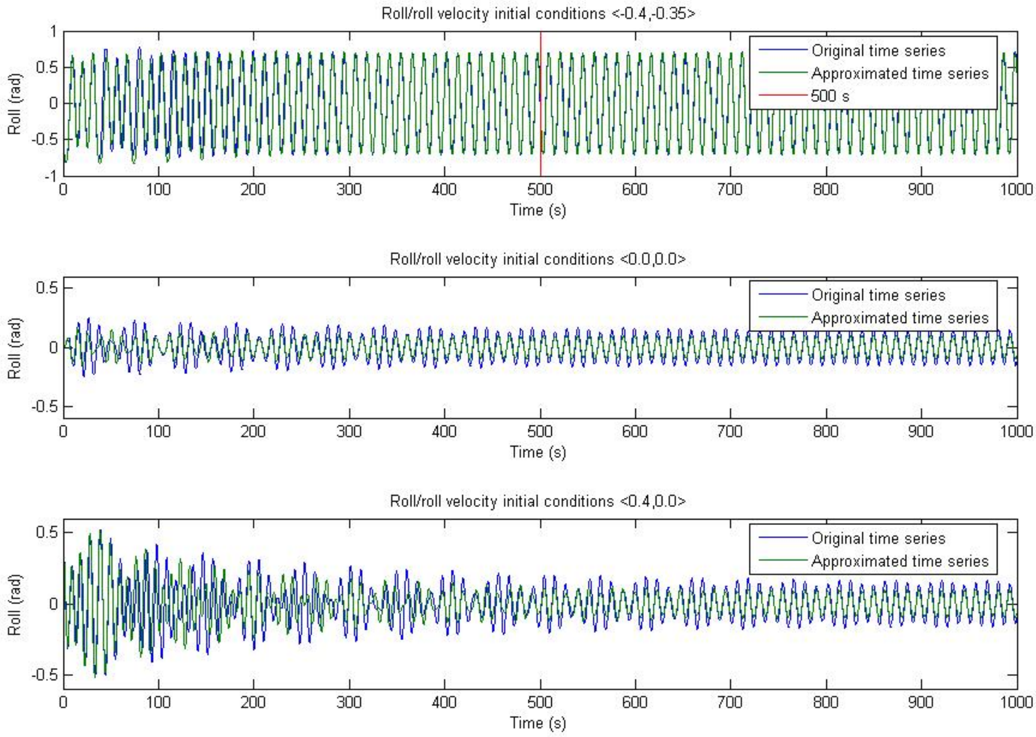

Figure 6: Original and approximated time series for roll/roll velocity initial conditions of < −0.4,−0.35 > (top), < 0, 0 > (middle), <0.4, 0 > (bottom). Approximated equations of motion generated from optimizing fit to timeseries data with initial conditions of <−0.4,−0.35 >.

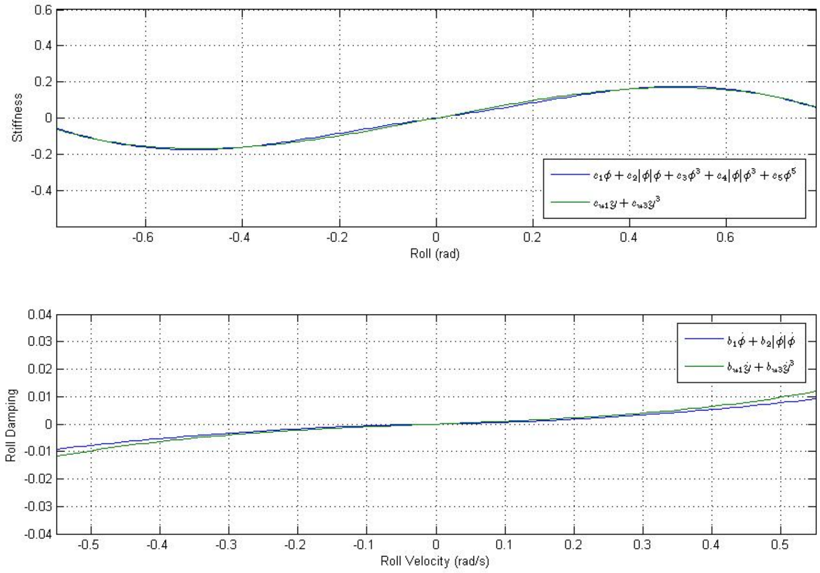

Figure 7: Comparison of original and approximated stiffness and roll damping curves for roll/roll velocity initial conditions of <−0.4,−0.35>.

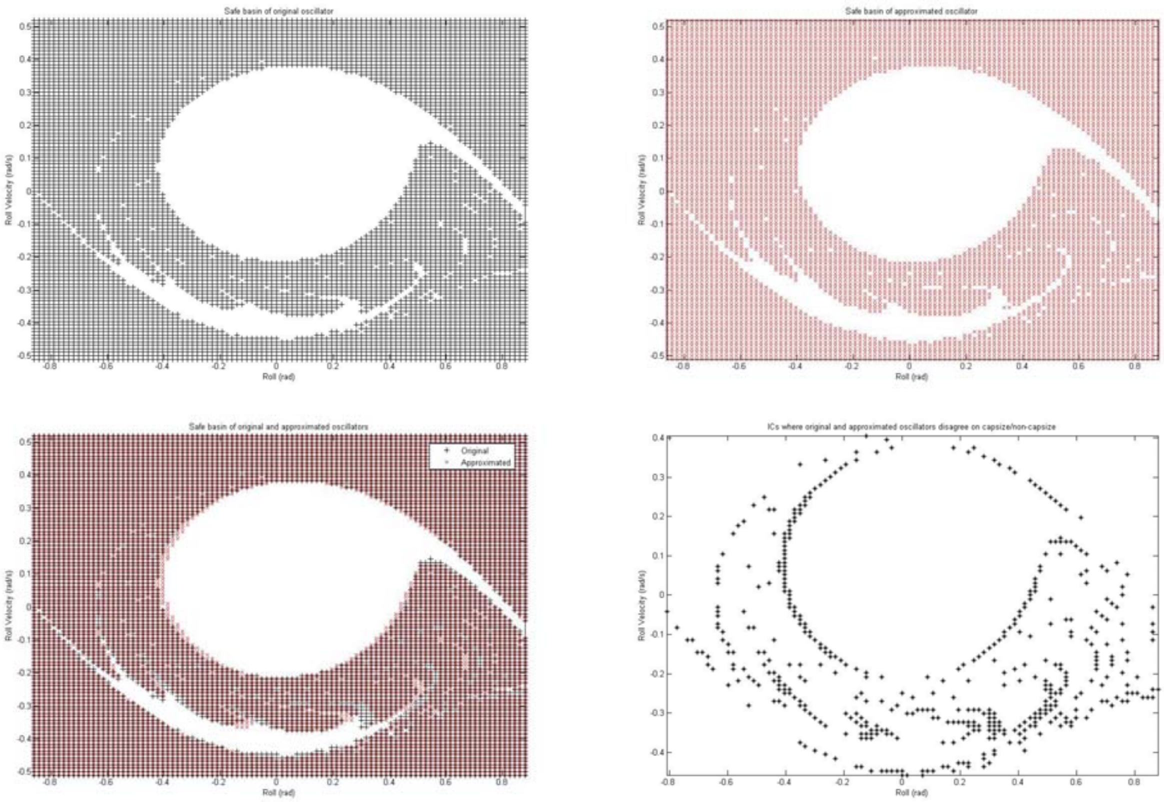

Figure 8: Safe basin for actual equations of motion Eq. 1 (upper left) and approximated equations of motion Eq. 2 (upper right). Both basins plotted together for comparison (lower left) and points in which the approximated and exact solutions disagree highlighted (lower right). Approximated equations of motion generated from optimizing fit to time series data with initial conditions of <−0.4,−0.35>.

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