Stochastic Wave Inputs for Extreme Roll in Near Head Seas

Dae-Hyun Kim and Armin W. Troesch
Department of Naval Architecture and Marine Engineering
The University of Michigan, Ann Arbor

ABSTRACT

An approach to generate the extreme value distribution of parameteric roll in near head sea conditions is presented using a Design Load Generator (DLG), a process to approximate the extreme value distribution of a Gaussian random variable. Statistics of the roll amplitudes of a Joint High Speed Sealift (JHSS) concept hull calculated from the DLG and the Large Amplitude Motion Program (LAMP) are compared in Weibull space with the results from limited Monte Carlo simulations. The interpretation of the return period of the DLG results is included.

KEYWORDS

Parametric Roll, Extreme Value Distribution, Design Load Generator, Linear Oscillator

INTRODUCTION

The underlying physics of parametric roll has been studied extensively by researchers. As a result, many aspects of this nonlinear phenomenon can now be explained reasonably clearly (e.g., France *et al.*, 2003; Shin *et al.*, 2004). However, the prediction of parametric roll in a stochastic seaway is a question to be answered with further research. One recent effort uses the First Order Reliability Method (FORM), a popular approach in the field of structural reliability, to calculate the most probable time evolution of parametric roll yielding a known roll angle at a known time (Jensen & Pedersen, 2006). Although the nonlinearity associated with parametric roll is partially recovered with a closed-form roll response model used to approximately find the design point, the hydrodynamic model for the roll response is a simplified one and the wave model is still linear. Moreover, the wave episode leading to the most probable roll response time series is essentially a regular wave except the neighborhood of the maximum roll. Whether this near regular wave behavior is due to the small number of wave frequencies (i.e., 15 ~ 50) in the wave model or the intrinsic nature of the FORM solution, which is just a point on the failure surface with the shortest distance from the origin, is not clear. One of the limitations inherent with this method is that the closed-form roll response model will play a crucial role in the calculation of the reliability index, which may result in significant deviations in the exceedance probability of the most probable time evolution of the known response (see Vidic-Perunovic, 2011).

A more desirable approach would be to use the time domain seakeeping tools that can capture the onset of parametric roll with higher fidelity. For example, a 3D time domain body nonlinear code, Large Amplitude Motion Program (LAMP) has been successfully applied to the prediction of parametric roll (France *et al.*, 2003; Shin *et al.*, 2003). Then, Monte Carlo simulations in theory should be able to generate the statistics of parametric roll in irregular seaways. However, several questions still persist. For example, what kind of probability distribution would parametric roll follow? Is the roll response with parametric roll an ergodic process? (Shin *et al.*, 2004) In addition, Monte Carlo simulations in general are very expensive. The purpose of the current paper is to present an approach to the extreme value distribution of parametric roll as practically as possible using a Design Load Generator (DLG). The DLG has been previously applied to estimate the statistics of not only Gaussian or slightly non-Gaussian processes but also of a highly non-Gaussian process (Kim & Troesch, 2010; Kim *et al.*, 2010; Alford *et al.*, 2011).

METHODS

Design Load Generator

The DLG is a process to approximate the extreme value distribution of a Gaussian random variable without expensive Monte Carlo simulations. The extreme value distribution of a Gaussian random variable, of course, can be calculated theoretically (see, e.g., Ochi, 1990). For example, a random variable X sampled from Eqn. (1) follows the Gaussian distribution as N goes to infinity due to the central limit theorem.

$$x(t) = \sum_{j=1}^{N} a_j \cos(\omega_j t + \varepsilon_j) \tag{1}$$

Here ε_j is a uniformly distributed random variable between $-\pi$ and π. The amplitude a_j is obtained from a single-sided spectrum $S(\omega_j)$ as

$$a_j = \sqrt{2S(\omega_j)\Delta\omega_j} \tag{2}$$

The standard deviation of the random process, σ, may be calculated from the area under the spectrum $S(\omega_j)$ as

$$\sigma^2 = \sum_{j=1}^{N} S(\omega_j)\Delta\omega_j = \sum_{j=1}^{N} \frac{1}{2} a_j^2 \tag{3}$$

The probability density function (PDF) of the random variable X is then

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/(2\sigma^2)} \tag{4}$$

The largest value out of m zero-mean Gaussian random samples is also a random variable and designated as X_m. The PDF of X_m can be theoretically derived as

$$f_{X_m}(x) = m\left(\frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/(2\sigma^2)}\right)\left(\Phi\left(\frac{x}{\sigma}\right)\right)^{m-1} \tag{5}$$

By differentiating Eqn. (5) with respect to x, the most probable (or the peak value) $\hat{x}$ can be obtained: the number of observations m is related to the peak extreme value of the random variable by

$$\frac{1}{m} \sim 1 - \Phi\left(\frac{\hat{x}}{\sigma}\right) \quad \text{as} \quad m \to \infty \tag{6}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Eqn. (6) leads to the definition of the target extreme event (TEV), which is

$$TEV = \frac{\hat{x}}{\sigma} = \frac{\text{design response}}{\sigma} \tag{7}$$

It is evident that TEV is closely related to the return periods of the extreme events: a higher TEV represents a rarer event.

Let X'_m be a new random variable of the extreme value in the DLG and modeled by

$$\begin{aligned} x'_m(0) &= \sum_{j=1}^{N} a_j \cos\left(\omega_j \cdot 0 + \varepsilon'_{m,j}\right) \\ &= \sum_{j=1}^{N} a_j \cos\left(\varepsilon'_{m,j}\right) \end{aligned} \tag{8}$$

where the extreme is assumed to occur at $t = 0$ without loss of generality. Once σ and m (or TEV) are fixed, the DLG calculates the PDF of $\varepsilon'_{m,j}$ such that X'_m approximates X_m (Alford *et al*., 2011). Since the PDF of $\varepsilon'_{m,j}$ is given, the shape of $x'_m(t)$ around $t = 0$ is readily available. Assuming X is the Gaussian response from a linear system, the corresponding Gaussian input wave time series is readily available, from which any nonlinear time domain simulation can further be conducted. When the correlation coefficient between $x'_m(t)$ and the corresponding nonlinear response is 1, the two responses have the same return period (i.e., the same m). As the correlation weakens, the DLG-generated nonlinear response becomes the lower bound of the real nonlinear response of the same return period. Then, the question becomes what linear response is closely correlated to the parametric roll response of a vessel. In this paper, an imaginary damped mass spring oscillator of which forcing is the relative motion of ship's stern with respect to the input wave is considered. In addition, it is hypothesized that the response of the oscillator would is closely related to the parametric roll. Since the output response of the oscillator is still linear, the return period of the DLG-based extreme response of the oscillator can easily be calculated. Choosing the parameters of the oscillator will be discussed in the next section.

Requirements for the Onset of Parametric Roll

In order to devise the parameters of the oscillator, the onset of parametric roll is first considered. Parametric roll is said to occur when a few critical requirements are satisfied. The conditions summarized in (France *et al*., 2003) are repeated in Table 1.

Table 1: Typical Requirements for Parametric Roll

Requirements
$\omega_{roll} \sim 0.5\omega_e$
$\lambda \sim O(L)$ (or $0.8L \le \lambda \le 1.2L$)
$\eta_o \ge \eta_c$
Roll damping is low.

In Table 1, ω_{roll} is the roll natural frequency, ω_e is the wave encounter frequency, λ is the wave length, L is the ship length, η_o is the input wave height, and η_c is a threshold value. In addition, a ship is said to be more vulnerable to head sea parametric roll when its stern area is wide and flat and bow flare is pronounced. In this sense, a significant amount of parametric roll is not expected for the Joint High Speed Sealift (JHSS), of which numerical model and principal dimension are shown in Fig. 1 and Table 2, respectively. This suggests that the simulation of extreme parametric roll for the JHSS through Monte Carlo simulations is even less feasible.

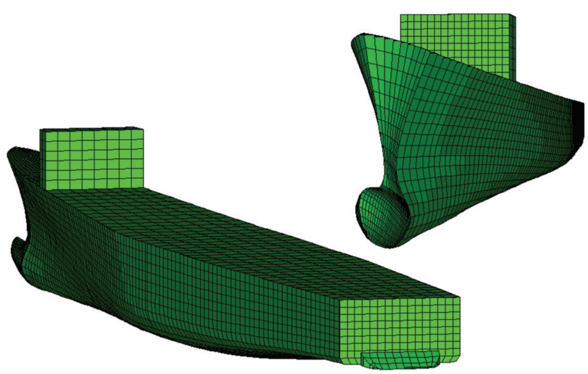

Fig. 1: Joint High Speed Sealift

Table2: Principal dimensions of JHSS

Parameter	Value
LOA	303 m
Beam	32.0 m
Draft	8.65 m
Displacement	35122 tonnes
Model Number	5663

In the current research, the roll response of the JHSS is simulated using the LAMP. Depending on the level of approximations, four solver options (LAMP1 ~ LAMP4) are available in the LAMP. LAMP2 is an approximate non-linear model with body-exact Froude-Krylov and hydrostatic forces and the perturbation potential solved over the mean wetted hull surface. LAMP4 is the most realistic solver option because all three forces (i.e., hydrostatic force, Froude-Krylov force, and the forces due to the perturbation potential) are calculated over the instantaneous free surface. However, previous research has shown that LAMP2 can capture parametric response (e.g., France *et al*., 2003; Shin *et al*., 2004) and LAMP2 is significantly faster than LAMP4. Consequently, LAMP2 is used in this paper unless otherwise specified. It should be mentioned that the roll damping model in the LAMP simulations is a crucial system parameter to be studied carefully. Since there is no roll experimental data available for this concept hull, attempts to find a better roll damping model have not been made. Therefore, the default Kato roll damping model embedded in the LAMP is utilized. The comparison between the default Kato model and a tuned damping model for a container vessel is presented in (France *et al*., 2003).

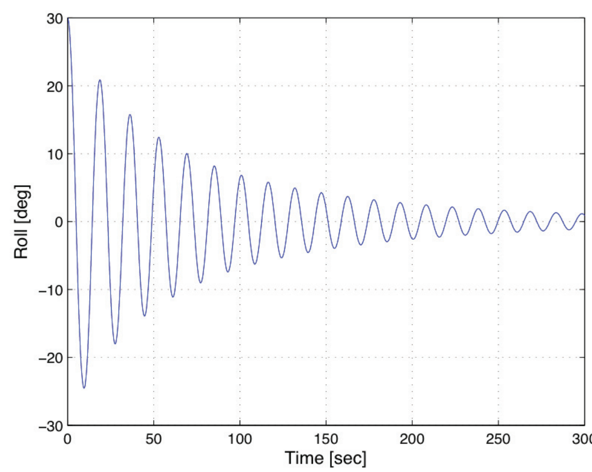

Fig. 2: Roll Decay Test with LAMP2 at V=9.8517 m/s

The test condition chosen for this paper is given in Table 3, which is based on Table 1 and a numerical roll decay test. The roll decay test using LAMP2 is shown in Fig. 2. The significant wave height and modal period correspond to a sea state 8; a condition that JHSS can reasonably be expected to encounter several times during its operational lifetime.

Table 2: Example Test Case

Parameter	Value
H_{sig}	11.5 m
T_{modal}	14.0 sec
S(ω)	Bretschneider
Ship Speed V	9.8517 m/s
Heading Angle β	170 deg
TEV	4.5

Linear Oscillator

An imaginary damped mass spring oscillator can be expressed as

$$m\ddot{x} + c\dot{x} + kx = \alpha m z_p(t) \qquad (9)$$

$z_p(t)$ is the relative motion of a point p on ship's stern with respect to the incident wave and α is an arbitrary constant. The linear relative motion $z_p(t)$ may be expressed as

$$z_p(t) = \eta_3(t) + L_p \eta_5(t) - \eta_p(t) \qquad (10)$$

where $\eta_3(t)$ is the heave displacement, L_p is the distance from ship's center to the location p, $\eta_5(t)$ is the pitch angle in radians (positive bow down), and $\eta_p(x,t)$ is the incident wave height at the location p. Assuming a harmonic input forcing with a frequency ω,

$$\ddot{x} + 2\varsigma\omega_n \dot{x} + \omega_n^2 x = \alpha \tilde{z}_p e^{i\omega t} \qquad (11)$$

where the natural frequency ω_n and the damping ratio ζ are simply

$$\omega_n^2 = \frac{k}{m} \quad \text{and} \quad \varsigma = \frac{c}{2m\omega_n} \qquad (12)$$

Introducing the complex frequency function $H_l(i\omega)$ defined as the response of the oscillator divided by the input relative motion,

$$H_1(i\omega) = \frac{\omega_n^2 \tilde{x}}{\alpha \tilde{z}_o} = \frac{1}{1 - \left(\omega/\omega_n\right)^2 + i\left(2\varsigma\, \omega/\omega_n\right)} \qquad (13)$$

The complex transfer function $H(i\omega)$ is then defined as the response divided by the input incident wave becomes:

$$H(i\omega) = H_0(i\omega) H_1(i\omega) \qquad (14)$$

where $H_0(i\omega)$ is an usual relative motion transfer function. $H(i\omega)$ is the transfer function to be applied in the DLG. Since *TEV* is the ratio of maximum response to the standard deviation σ of the process, α is set equal to ω_n^2 without loss of generality. In this paper, ω_n is set as twice the roll natural frequency from the roll decay test, ζ is 0.04, and L_p is 128.51 m

RESULTS AND DISCUSSION

To assess the likelihood of the JHSS experiencing large roll in near-head seas (i.e., β = 170 deg), completed in this research are LAMP simulations based on an ensemble of the DLG wave sequences and comparable Monte Carlo simulations based on 300 uniformly distributed random wave phase components. The DLG/LAMP simulations are based on an ensemble of short time series (200 seconds in length) identified by the DLG process. The number of wave frequencies *N* is 101. The Monte Carlo simulations consist of 500 fifteen-minute time histories (minus 20 seconds at the beginning of each record to remove transients) spanning an effective exposure period of approximately 125 hours. For the Monte Carlo simulations, *N* is 301. This strategy is adopted to avoid repetition in the incident wave profiles (see, e.g., Belenky, 2005). By taking the average of 500 sequences of the Monte Carlo simulations, the approximate Root Mean Square (RMS) values for the heave, pitch, and roll responses are calculated to be approximately 1.57 m, 1.70 deg, and 1.14 deg, respectively. The roll mean period, based on zero-upcrossings, is about 13.5 sec.

The DLG/LAMP simulations are conducted for three degrees of freedom - heave, roll, and pitch - vessel motions responding to 999 DLG identified incident wave profiles configured to produce a *TEV* of 4.5 for the linear oscillator given in Eqn. (11). A 4.5 *TEV* maximum amplitude for the oscillator has an approximate Rayleigh exceedance probability of 1/25,000. The corresponding mean return period is approximately 85 hours, when the mean frequency, ω_{mean} = 0.5145 rad/sec, calculated from the response spectrum of the oscillator is converted to actual time. An example DLG/LAMP realization is shown in Fig. 3. Note that in Fig. 3, the maximum incident wave (η_0) elevation at midship was approximately 4.8 times the wave RMS, the maximum heave (η_3) elevation was approximately 6.3 times the heave RMS, and the maximum pitch (η_5) elevation was approximately 4.7 times the pitch RMS, while the maximum roll (η_4) elevation was approximately 34.4 times the roll RMS!

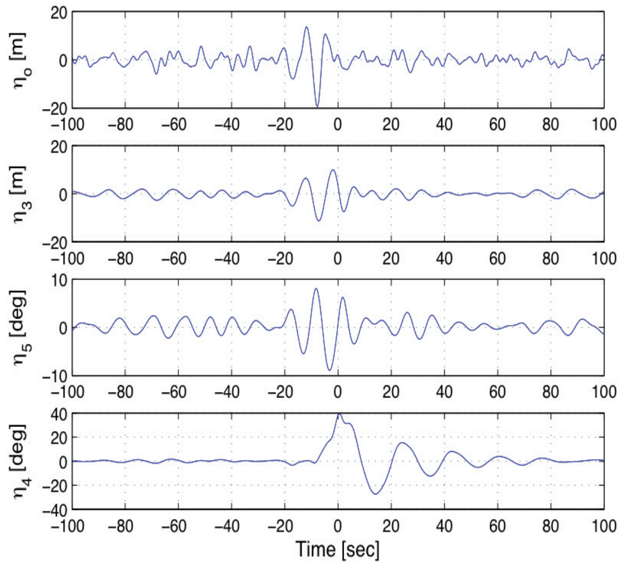

Fig. 3: Example DLG/LAMP Realization (*TEV*=4.5)

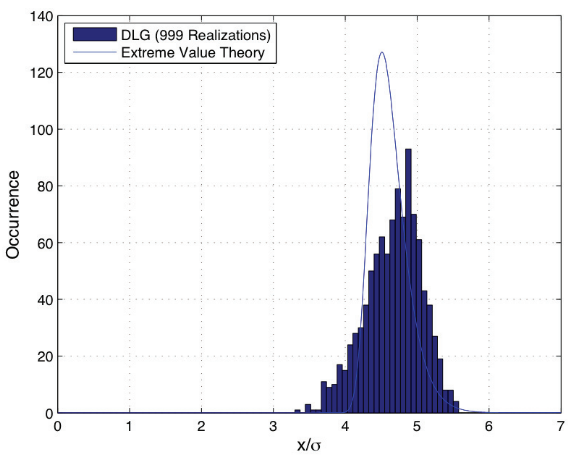

Fig. 4: Histograms of 999 DLG Generated Extreme Response X of Linear Oscillator and Corresponding Roll Response X at TEV = 4.5

As mentioned, the DLG approximates the extreme value distribution of the response *X* of the oscillator. To show the level of approximation by the current DLG, the histogram of 999 realizations and the expected *histogram* from Eqn. (5) are compared in Fig. 4 The maximum roll amplitudes in the DLG ensemble (LAMP2) are shown in Fig. 5 Note that the most probable roll maximum in Fig. 5is approximately 9 deg. Also note that there are 4 of the 999 realizations that exceed 39 deg, including three capsize sequences. A snapshot of one of the LAMP2 capsize sequences is shown in Fig. 6

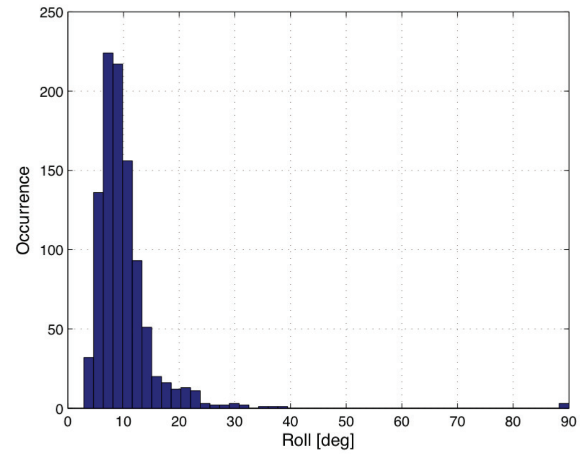

Fig. 5: Histograms of 999 DLG Generated Extreme Response X of Linear Oscillator and Corresponding Roll Response Maximum Roll Response ζ_4 at TEV = 4.5

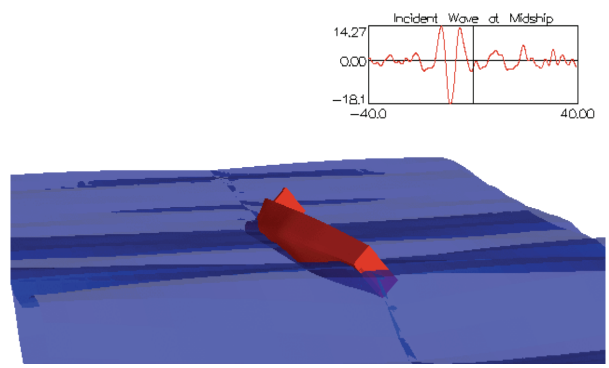

Fig. 6: Example LAMP Simulation (One of Three Capsize Cases)

Fig. 7 combines the result of the Monte Carlo simulations and the DLG in Weibull space. It should be noted that the extreme value PDF of the roll is exaggerated. This figure supports, to a degree, the hypothesis upon which the linear oscillator in Eqn. (5) is based. That is, large, nonlinear, non-Gaussian roll in near head seas is correlated with the response of a linear oscillator with a natural frequency of twice the roll natural frequency and excited by large stern relative motion. In addition, while the most expected roll maximum shown in Fig. 5 lies on the very proximity of the curve traced by the Monte Carlo simulations in Weibull space, there is a finite probability that the JHSS will experience conditions with the same average return period that lead to capsize. The probability can be summarized as follows:

For the operational condition given in Table 3, the most probable roll amplitude is approximately 9 deg, which represents a lower bound for the return period of 85 hours. In addition, when considering all the roll motions in the 999 member ensemble, extreme values greater than 39 deg, including three capsize realizations, occur approximately 0.4 % of the time.

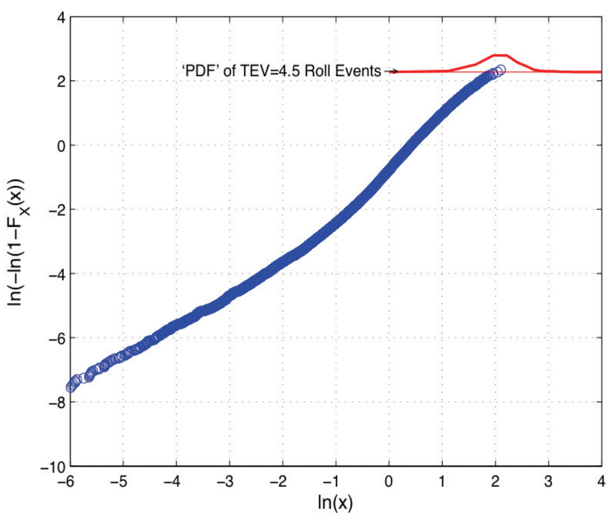

Fig. 7: Weibull Plot for Roll Monte Carlo Simulations

CONCLUSION

This study calculates the extreme value distribution of parametric roll for a given return period. Since the DLG is based on a Gaussian process, the extreme value distribution of the extreme roll is a lower bound of the *exact* extreme value distribution. The DLG can generate an ensemble of short design wave time series, from which the extreme value distribution of the corresponding nonlinear responses can be calculated relatively efficiently. For example, about 85,000 hours worth of Monte Carlo simulations are required to obtain a comparable histogram presented in this paper. As such, the DLG method can provide a valuable tool for designers in establishing safe operating envelopes.

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