Extension of Statistical Extrapolation Techniques for a Strongly Non-linear System Using the Split-Time Method

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ABSTRACT

Statistical extrapolation techniques are widely used in the prediction of extreme ship motions and the risk of stability failures. The righting arm curve of a ship represents the stiffness of the dynamical system and shows extreme nonlinearity with increasing roll angle. The split-time method is applied in an attempt to extend the range of valid extrapolation range of the Envelope Peaks Over Threshold (EPOT) method for a beam seas case.

KEYWORDS

Problem of Rarity, Principle of Separation, Partial Stability Failure, Statistical Extrapolation, Peaks Over Threshold

INTRODUCTION

The assessment of partial stability failures through numerical simulation or physical model tests is complicated by the problem of rarity and the highly non-linear nature of the rolling motion of a ship. Statistical extrapolation methods are widely used for the extrapolation of risk for levels of roll that were not observed in the simulated or observed time frame. If formulated correctly, statistical extrapolation techniques have embeded in them some amount of non-linearity and are therefore capable of some amount of extrapolation in mildly non-linear regimes of motion. As the extrapolation approaches and transitions into the highly non-linear region (such as near the peak of a ship's righting arm curve) these extrapolations become highly suspect as the dynamics of the ship's rolling motion changes.

The Envelope Peaks Over Threshold (EPOT) (Campbell and Belenky 2010, 2010a) method was developed as a robust extrapolation method for the mildly non-linear region. In this paper the Split-Time method developed by Belenky, et al (2008) is applied to extend the applicable range of statistical extrapolation. For conditions where the maximum motions are less than the peak of the righting arm curve, the EPOT method is used to obtain the exceedance rate of the peak of the righting arm curve and the Split-Time method is used beyond the peak. A single degree of freedom ship rolling equation is used to produce the stochastic data used in this study.

MODEL OF SHIP ROLL

The model of ship roll is fairly simple, as accurate motion characterization of ship is not the focus. Rather, the focus is on using the actual righting arm curve for a ship as the stiffness term in the equation. Roll motion was modeled using Equation 1.

$$\ddot{\phi} + 2\cdot\left(0.1\cdot\omega_0\right)\cdot\dot{\phi} + \omega_0^{\,2}\cdot\frac{GZ(t)}{GM_{Upright}} = F(t) \quad (1)$$

GZ(t) is interpolated from the righting arm curve (see Fig. 1) of the ONR tumblehome hull with a notional toposide (see Fig. 2). The righing arm curve is shown in Figure 1 below. The primary peak of the righting arm curve is at 35 degrees.

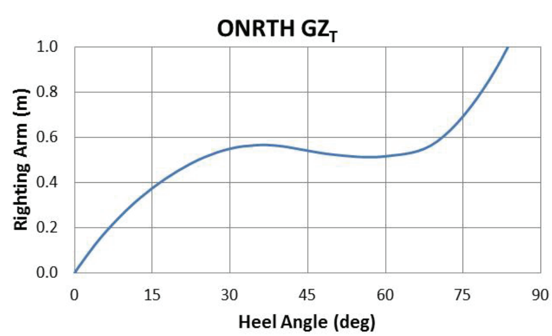

Fig. 1: Righting Arm Curve of the ONRTH Hull form

The forcing function $F(t)$ is a simulated random seaway modeled using a Breschneider Spectrum. The period of the seaway was taken as 16.4 seconds, the most probable period for a Sea State 8 in the North Atlantic Ocean. The significant wave height was adjusted to obtain the desired levels of motions for this study.

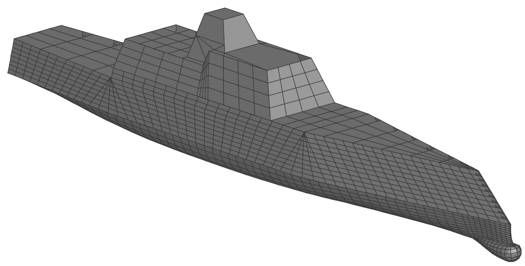

Fig. 2: ONRTH Hull form with Notional Topside

STATISTICAL FORMULATION

The EPOT Statistical Extrapolation Method

The EPOT method is based on the principal of separation. The non-rare problem is a statistical estimate of the exceedance of a roll angle that is small enough so that enough observations are made. The peaks of the signal's envelope are collected. The exceedance rate of some threshold a_1 is then given by:

$$\xi(a_1) = \frac{k_{a_1}}{T} \quad (2)$$

Where k_{a1} is the number of peaks that are above threshold a_1 and T is the total exposure time.

The rare problem is solved by fitting a distribution to the peaks that are above threshold a_1. The probability of rolling to a higher level a_2 once threshold a_1 has been exceeded is given by equation 2.

$$P(E > a_2 \mid E > a_1) = 1 - F_{Em}(a_2) \quad (2)$$

Where F_{Em} is the CDF of the peaks of the envelope that are above threshold a_1. The total exceedance rate for level a_2 is then given by:

$$\xi(a_2) = \xi(a_1) \cdot P(E > a_2 \mid E > a_1) \quad (3)$$

Generally a_2 cannot be any greater than the peak of the righting arm curve, as information about the extreme non-lilnearity of motion in this region is not embeded in the distribution of peaks.

Split-Time Method

The Split-Time method (Belenky et al 2008) is likewise based on the principal of separation. The non-rare problem here is the exceedance rate at the peak of the righting arm curve. This is the point where the system changes from being a repeller to an attractor. The stiffness of the system is proportional to the derivative of the righting arm curve. At the peak of the righting arm curve there is essentially no stiffness. Since there is no stiffness, the system response to forcing functions of any frequency will be greatly diminished.

The rare problem is then solved by doing a series of calm water simulations with the intial roll angle set to the peak of the righting arm curve. A series of simulations is then executed with varying initial roll rates searching for the critical roll angle that will cause the ship to roll to some higher roll angle. The lack of response to the forcing function (sited in the previous paragraph) is the justification for executing these simulations in absence of the seaway.

The probability of the ship rolling to angle a_3 once the peak of the righting arm curve (a_2) has been exceeded is:

$$P(\phi > a_3 \mid E > a_2) = 1 - F_{\dot{\phi} a_2}(\dot{\phi}_{crit(a_3)}) \quad (4)$$

Where $F_{\dot{\phi} a_2}$ is the cumulative distribution function of roll rate at the moment the peak of the righting arm curve is crossed and $\dot{\phi}_{crit(a_3)}$ is the critical roll rate required to exceed level a_3.

Complete Formulation

If a_2 is set the be the peak of the righting arm curve, then we have a compatible boundary between the EPOT and Split-Time methods.

EPOT cannot, in principle, extrapolate above this level and the Split-Time method cannot be used below this level.

The full formulation for exceedance rates past the peak of the righting arm curve marries these two techniques together.

$$P(\phi > a_3) = \frac{k_{a_1}}{T} \cdot (1 - F_{Em}(a_2)) \cdot \left(1 - F_{\dot{\phi}a_2}(\dot{\phi}_{crit(a_3)})\right) \quad (5)$$

BUILDING THE ROLL RATE DISTRIBUTION

We require the distribution of roll rate at the moment level a_2 is crossed. In principle it is not always possible to directly build the conditional roll rate distribution since, in the general case, level a_2 has not been reached. Belenky et al (2008) showed, however, that the distribution of roll rate at the point some roll angle is crossed is independent of the actual value of the roll angle in question. The probability density function for the conditional roll rate distribution is given by:

$$f_{\dot{\phi}a2}(v) = \frac{dF(v)}{dv} = \frac{v \cdot f_{\dot{\phi}}(v)}{\int_0^{\infty} \dot{\phi} \cdot f_{\dot{\phi}}(\dot{\phi}) d\dot{\phi}} \quad (6)$$

Where $f_{\dot{\phi}}$ is the distribution of instantaneous roll rate and $f_{\dot{\phi}a2}$ is the conditional distribution. Belenky also proved that, if the instanteous roll rate is normally distributed, the conditional roll rate distribution will be Rayleigh distributed.

For the present study this means the conditional roll rate distribution can be built at one level of upcrossing and used for any other level. To build the distribution, there are several options. Equation 6 could be used after building the distribution of instanteous roll rate or the distribution could be built directly by finding the upcrossings and and getting the roll rate for those instants in time. A distribution may then be fit to this data directly. As this is the most direct approach, this is what was done.

Since the motion being simulated was non-linear, it was expected that the conditional roll rate distribution, while being close to Rayleigh, may deviate from the assumption of a Rayleigh distribution. While the distributions were "visually" close to fitting the data, they did not always satisfy the χ^2 or Kolmogorov-Smirnov goodness of fit tests. A Weibull distribution, however, generally passed the goodness of fit tests for all levels examined. The shape parameters of the Wiebull sdistribution were generally close to 2.0 (meaning they were close to being a Rayleigh distributions), but deviated enough that there was a discernable difference in shape and passage of the goodness of fit tests. Figure 3 below shows the Weibull distribution fit for one case.

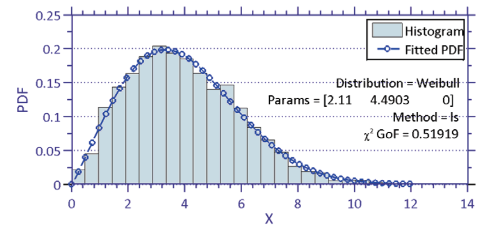

Fig. 3: Weibull Disbribution fit to Conditional Roll Rate Data

CRITICAL ROLL RATE

In order to find the critical roll rate ($\dot{\phi}_{crit(a_3)}$) required to roll to some level a_3, a series of unforced roll decay type simulations were executed with the same system model. The initial roll angle was set to 35 degrees, the boundary between the two methods, and a range of initial roll rates ranging from 0 to 30 degrees/second (in steps of one) were run. The maximum value was recorded from each simulation and plotted against the initial roll rate for that simulation (see Fig. 4).

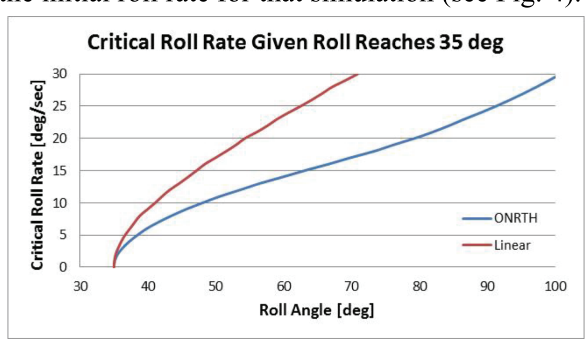

Fig. 4. Critical Roll Rate Vs. Max Roll Angle

From these simulations the relationship between critical roll rate and maximum roll angle is derived. Also presented in Figure 4 is a curve for an equivalent system with linear stiffness. For this model, the initial GM was matached to the

ONRTH roll model. It is clear that in the highly non-linear region of the ONR tumblehome hull's righting arm curve (past it's primary peak), the maximum roll angle achieved is much higher than for the linear model for a given initial roll rate.

RESULTS

Case 1

For the first case examined, the direct counting estimate of exceedance rate is well behaved, that is it appears smooth (see Fig. 5). In Figures 5 and 6 the following notation is used in the legend:

- SPCR: statistical peak crossing rate (direct counting)
- CIL: lower bound of the confidence interval
- CIU: upper bound of the confidence interval
- EPOT: the Envelope Peaks Over Threshold method
- EPOT+ST: The Split-Time method on top the the EPOT Method

The conditional roll rate distribution was built when the roll angle crossed a level of 1.5 times the standard deviation of the roll angle signal (9.29 deg). For this case the Split-Time solution (EPOT+ST) is within the EPOT confidence interval down to exceedance rates of less than 10^{-20}/sec. This corresponds to an hourly probability of about $10^{-16.4}$.

Case 2

Case 2 had a slightly smaller forcing function. The direct counting results for this case were less well behaved. Above 20 degrees the data was sparse and, consequently had a high level of uncertainty (see inset of Figure 6). This portion of data greatly influenced the EPOT extrapolation, however, causing the slope of the exceedance rate curve to flatten out. In this case the roll angle achieved at an exceedance rate of 10^{-20} is greater than for Case 1 where the forcing amplitude was larger. This does not make physical sense.

For Case 2 the Split-Time result diverges from the EPOT result immediately. The behavior of the Split-Time solution beyond 35 degrees is more inline with what would be expected. Another interesting observation is that the slope for the Split-Time estimate matches up well to the direct counting estimate at 20 degrees (where it stops behaving smoothly). If the distribution fits used in the EPOT solution weighted the higher values less, it is conceivable that a much better match would be made between the EPOT and Split-Time solutions.

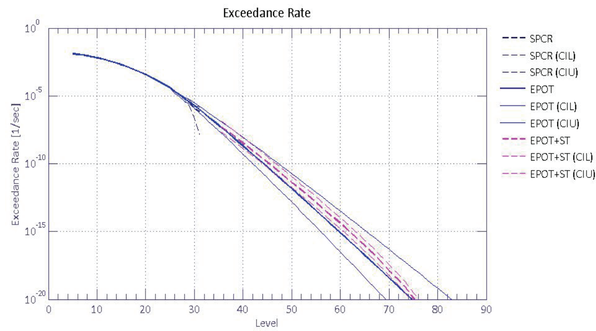

Fig. 5: Exceedance Rate Estimates for Case 1

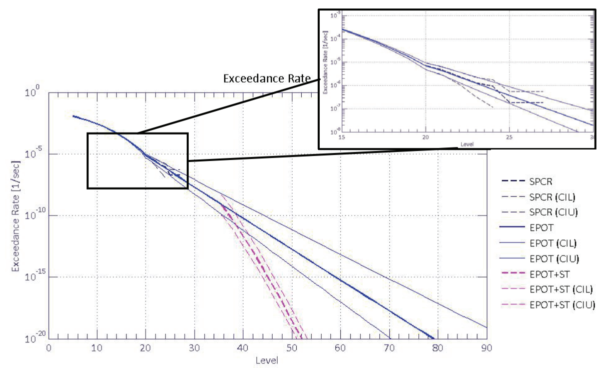

Fig. 6: Exceedance Rate Estimates for Case 2

CONCLUSION

It was demonstrated that the EPOT and Split-Time methods could be used together for the extrapolation of risk across regions of high non-linearity. The general trends of the EPOT + Split-Time (EPOT+ST) method was generally as expected. The EPOT+ST method gives a physical basis for statistical extrapolation across the peak of the righting arm curve for the simplified case that was examined (single DoF, constant righting arm curve).

DISCUSSION

Confidence Interval

The EPOT solution has a full characterization of the confidence interval for all of it's sub parts. The Split-Time method is comprised of two pieces. The critical roll rate piece is deterministic. Given that there are no uncertainties applited to the inputs for the model, the critical roll rate has not uncertainty associated with it. The conditional roll rate distribution does have uncertainty, as it is a distribution fit to a sample of data. This uncertainty has not been captured in this paper, though the process is fairly straightforward.

Validation

No attempt to validate the resulting composite method was made in this study. Future work in this area would involve using a similar system to the one employed with piece-wise linear stiffness where the result could be analytically derived.

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