Comparison of Some Analysis Methods for Ship Roll Decay Data

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ABSTRACT

Several alternative analysis methods for ship roll decay data are examined, by applying them first to simulated data with and without "noise", and then to model test data. The methods include the "traditional" analysis of the change in the roll amplitude as a function of the mean value, and a method that uses the envelope of the decay curve. Comparisons of the linear and nonlinear roll damping coefficients computed using the alternative methods are performed.

KEYWORDS
Roll Decay; Roll Damping; Free Vibration Analysis

INTRODUCTION

Roll decay tests are often conducted as part of a seakeeping model test program, to determine the roll damping characteristics of the model. One common way to analyze the data consists of tabulating the peak values, and plotting the change in the roll amplitude divided by the mean of the successive peak values, against the mean of the successive peak values for each cycle. If the roll damping is a quadratic function of roll angular velocity, this plot will be a straight line, the intercept and slope of which are proportional to the linear and second-order damping coefficients, respectively.

Unfortunately in practice this procedure is often less than satisfactory since the plot usually exhibits much scatter, particularly at the lower roll amplitudes. Furthermore, the slope of a line fitted to the data is generally dominated by one or two data points corresponding to the initial oscillation cycles; small changes in these values have large effects on the computed quadratic damping coefficient.

Alternatively, the change in roll angle can be plotted directly against the mean roll angle which ideally results in a quadratic curve passing through the origin. A second-order polynomial fit (with zero intercept) yields the first- and second-order damping coefficients. The scatter of the data about the quadratic curve is generally less than that about the straight line described in the previous paragraph, yielding an apparently better fit; however for "real" data the resulting coefficients are almost always notably different than those produced by the linear fit.

In this paper these two fitting methods are compared, and an alternative analysis method that makes use of the envelope of the roll decay curve is explored. The alternative method and the traditional methods are applied first to some synthetic data and then to some available model test data, and the resulting damping coefficients compared. =

SINGLE DEGREE-OF-FREEDOM ROLLING

It will be assumed that the free roll motion can be adequately represented by the familiar single degree-of-freedom second-order ordinary differential equation:

$$\ddot{\phi} + 2b(\dot{\phi})\dot{\phi} + \omega_0^{\,2}(\phi)\phi = 0 \qquad (1)$$

$$\phi(0) = \phi_0;\ \dot{\phi}(0) = 0 \qquad (2)$$

where ϕ is the roll angle, a function only of time, and ω_0 is the undamped natural roll frequency.

"TRADITIONAL" ANALYSIS METHODS

The traditional method is described by Rawson and Tupper (2001) and is routinely applied by many model test basins. We will assume that the damping has the following form:

$$b = b_1 + b_2|\dot{\phi}| = b_1 + b_2\omega|\phi| \qquad (3)$$

corresponding to quadratic damping. Now let successive roll maxima in a roll decay time history be denoted as ϕ_0, ϕ_1, …,ϕ_n, ϕ_{n+1},…. In the case of linear damping (b_2=0) it is easily shown that

$$\begin{aligned}\phi_n/\phi_{n+1} &= e^{2\pi b/\omega} \text{ or } \ln(\phi_n/\phi_{n+1}) = \frac{2\pi b}{\omega} \\ &= \frac{2\pi\eta\omega_0}{\omega} = \frac{2\pi\eta}{\sqrt{1-\eta^2}}\end{aligned} \qquad (4)$$

From (4) after a little algebra we can obtain an expression for the ratio of the change in roll amplitude divided by the average amplitude:

$$\frac{\phi_{diff}}{\phi_{avg}} = \frac{2(\phi_n - \phi_{n+1})}{\phi_n + \phi_{n+1}} = 2\tanh\frac{\pi b}{\omega} \qquad (5)$$

Thus we have

$$\frac{\phi_{diff}}{\phi_{avg}} \approx \frac{2\pi b}{\omega} = \frac{2\pi\eta}{\sqrt{1-\eta^2}} \qquad (6)$$

provided that $\pi b/\omega$ is small. In the case of nonlinear damping, one can relate the change in roll amplitude to the energy dissipated in each swing (e.g., Meskell (2011)) to obtain

$$\begin{aligned}\phi_{diff} &\approx \frac{2\pi}{\omega} b_1\phi_{avg} + \frac{16}{3} b_2 {\phi_{avg}}^2 \\ &= 2\pi\eta_1\frac{\omega_0}{\omega} + \frac{16}{3} b_2 {\phi_{avg}}^2\end{aligned} \qquad (7)$$

where η_1 is a "linear fraction of critical damping":

$$\eta_1 = \frac{b_1}{b_{CR}} = \frac{b_1}{2\sqrt{I_\phi C_\phi}/2I_\phi} = \frac{b_1}{\omega_0} \qquad (8)$$

Here I_ϕ and C_ϕ are roll inertia and restoring moment coefficients. Equation (7) provides the relationship between the coefficients determined by the curve fits and the first- and second-order damping coefficients. Note that these expressions are approximate, assuming that the damping is "small" (generally true for ships). Note also that the second-order coefficient b_2 is dimensionless (but one must be clear on whether the coefficient is expressed in terms of degrees or radians).

FREEVIB METHOD

A method for analysis and identification of free vibration data has been developed some time ago by Feldman (1994). The method exploits the fact that the damping is related to the rate of change of the envelope of the decay curve, which can be obtained from the analytic signal corresponding to the decay data. Unlike the traditional analyses, this method makes use of the entire time series, not just the maxima, and can accommodate nonlinear restoring moments and any form of amplitude dependence of the damping coefficient. All of the key equations are provided in Feldman (1994, 1997) and so will not be repeated here. Feldman is not the only investigator to exploit the relationship between damping factor and the envelope (eg, Messina et.al. (2007); Karjalainen et.al. (2002)), but he is the only one to provide free software for application of the method, in the form of a protected (non-readable) MATLAB subroutine "FREEVIB" *

FREEVIB has four inputs: the signal in the time domain (sampled at a constant rate); the sampling frequency; the "Hilbert filter order N, 100≤N≤240", and a flag indicating whether the signal represents displacement, velocity or acceleration. The four outputs are the portion of the input signal chosen for analysis (see below); the signal envelope and instantaneous frequency; the computed natural frequency; and the damping coefficient. The outputs are all time series with length equal to that of the selected portion of the signal. Limitations stated in the "readme" file accompanying the software are that the sampling frequency must be 10f0 to 100f0, where f0 is the system natural frequency; and the length of the input vector must be greater than 3 times the filter order N.

When the routine is invoked in MATLAB, a time series plot appears and the user is prompted to select a portion of the signal for analysis using the cursor. Three output plots then appear, displaying the selected portion of the signal along with the envelope and instantaneous frequency; the "backbone" curve along with the estimated frequency response function; and the "elastic force" and "friction force" plotted against

* http://hitech.technion.ac.il/feldman/

displacement and velocity, respectively. Unfortunately the data shown in these latter three plots is not part of the output.

ANALYSIS: SYNTHETIC DATA

Prior to analyzing some actual test data, the three methods described above were first applied to some "roll decay curves" computed using MATLAB's ODE solver. The simulations were conducted using the following parameters:

TABLE 1 Parameters for simulated time series

Period, sec	10
Initial roll angle, deg	10
Linear damping ratio	0.05
Nonlin/lin damping b2/b1	0.1
b1	0.0315
b2	0.00315

The simulated time series is shown on Figure 1. Values of the coefficients b_1 and b_2 determined from the curve-fit coefficients of ϕ_{diff}/ϕ_{avg} vs ϕ_{avg} ("traditional-1") and ϕ_{diff} vs ϕ_{avg} ("traditional-2") using Equation (7) are tabulated below (first two columns):

TABLE 2 Damping coefficients from synthetic data

	No noise		0.01 deg noise		0.1 deg noise	
Method	b1	b2	b1	b2	b1	b2
Simulation (input)	**0.0315**	**0.00315**				
Traditional-1	0.0313	0.00300	0.0307	0.00338	0.0247	0.00503
Traditional-2	0.0315	0.00291	0.0312	0.00304	0.0293	0.00338

It can be seen that there is little difference between the results of the two fitting methods, and both do a good job of determining the actual values.

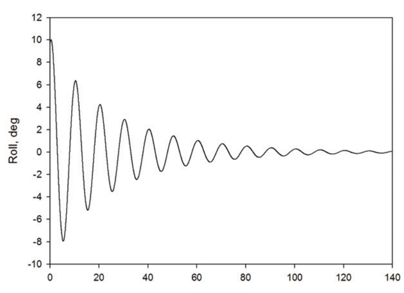

Fig. 1: Simulated roll decay curve with quadratic damping

However, real data seldom looks as "nice" as Figure 1, so the analysis was repeated twice, with random "noise", uniformly distributed between ±0.01 deg and ±0.1 deg respectively, superimposed on the computed decay curve. The resulting plots of ϕ_{diff}/ϕ_{avg} vs ϕ_{avg} and ϕ_{diff} vs ϕ_{avg} for the latter case (±0.1 deg of noise) are shown on Figures 2 and 3. The effect of the noise is most apparent on the "traditional-1" method, which is reflected in the results in Table 2.

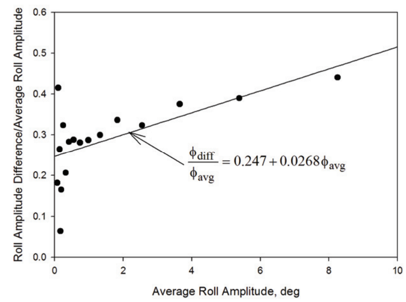

Fig. 2: "Traditional-1" analysis of "noisy" simulated data

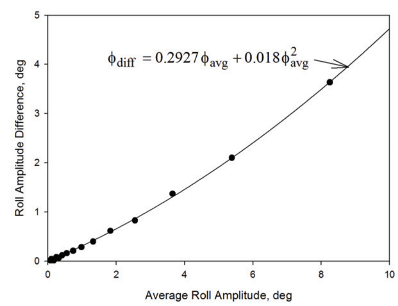

Fig. 3: "Traditional-2" analysis of "noisy" simulated data

Application of FREEVIB to the decay curve immediately reveals a problem due to the finite signal length: the computed envelope does not pass through the initial peak in the time series and is obviously not correct for the first cycle of the motion. Prof. Feldman [private communication] acknowledges this problem and recommends "padding" or "mirroring" the data; the effect is to artificially move the initial point "to the right" in the time series; the "end effect" thus occurs in the "artificial" segment of the time series and so can be ignored. Unfortunately, for real data this extrapolation process is not straightforward, because spurious "wiggles" in the envelope can still appear at the junction of the "real" and "artificial" signals (for synthetic data this difficulty is easily overcome by starting at a larger initial value and discarding the analysis of the first several cycles). This is a serious problem because any nonlinearities are manifested to the largest degree in the first cycle, so this data should not be simply “thrown out”.

It was found that simply extending the time series “backwards in time” with a simple sinusoid, having the same period as the real signal (the initial value if the period is varying) and matching the initial peak, usually produces a satisfactory Hilbert transform and envelope curve (using the MATLAB “hilbert” function). However this trick does NOT seem to work with FREEVIB, evidently due to the filtering and smoothing that is being carried out. For FREEVIB analysis of the real data described below, the first several cycles of data was fit to a (linearly) damped sinusoid, which was used to extrapolate the curves backwards in time for several cycles. For the synthetic signals, the simulations were simply repeated with a larger initial angle as mentioned above; the “analysis window” was started where the roll amplitude was about 10 degrees.

Figure 4 is a plot of the “total” roll damping coefficient b from FREEVIB against the envelope value, for the three simulations (no noise and with two noise amplitudes). On this plot the intercept corresponds to b_1 and the slope to ωb_2. There is an obvious problem at low values of the roll amplitude (another possible “end effect”) which is magnified in the presence of noise. Fitting straight lines to these results, after discarding the low amplitude results that are obviously erroneous[†], yields the coefficients tabulated below:

TABLE 3 Damping coefficients from synthetic data, FREEVIB

	No noise		0.01 deg noise		0.1 deg noise	
Method	b1	b2	b1	b2	b1	b2
Simulation (input)	**0.0315**	**0.00315**				
FREEVIB N=100	0.0315	0.00279	0.0314	0.00280	0.0307	0.00300
FREEVIB N=200	0.0315	0.00270	0.0315	0.00270	0.0311	0.00279

These results show that FREEVIB identifies the linear coefficient quite well even in the presence of significant noise; however, for reasons that are not yet evident, the identified nonlinear coefficient is about 10% too low. Note that increasing the filter order N smoothes the computed damping vs amplitude curve, but reduces the accuracy of the computed nonlinear coefficient.

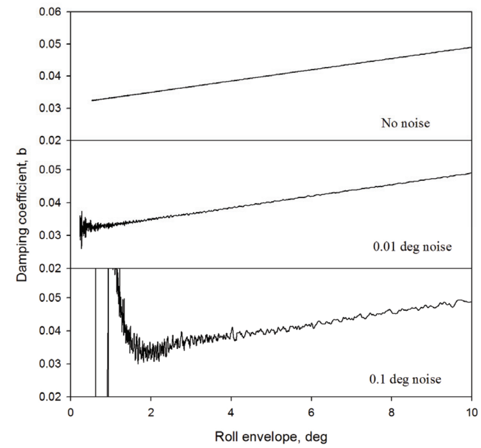

Fig. 4: Damping coefficient from FREEVIB analysis of simulated data

ANALYSIS: MODEL TEST DATA

The analysis methods described above have been applied to a set of available model test data, from the extensive series of tests conducted at the Iowa Institute of Hydraulic Research using “DTMB Model 5512” (Irvine et.al. (2004)), a 46.6-scale model of a DDG-51 class destroyer. This data includes tests with and without bilge keels, at a range of ship speeds.

Roll decay tests were conducted at a range of speeds (Froude numbers Fn of 0.069 to 0.410), with and without bilge keels (these results will be labeled as "BK" and "BH" for "Bilge Keels" and "Bare Hull" respectively; thus "BK410" denotes the case of Fn = 0.410 with bilge keels). Note that the model was held fixed in all other degrees of freedom in these tests, so that the 1-DOF roll equation is appropriate. A series of decay tests was carried out for each condition, with 9 distinct initial roll angles ranging from 2.5 to 20 degrees; time series data is available for the first 5 oscillation periods for each case. Full-scale natural roll period is about 10.5 sec.

For the present study, data from tests at Fn = 0.069, 0.280, 0.340, 0.410 was analyzed. In order to obtain a time series having more than 5 peaks, two or three time series (from tests at different initial angles) were "cut and pasted" together. In all cases the time series with the initial angle of 20 deg was used; the others were selected based on

[†] It could be argued that this unfairly skews the comparison with the "traditional" methods, where nothing was discarded; however, retaining all data here results in nonsensical values that preclude any reasonable evaluation of this method.

the best match to the "tail" of the preceding time series (the series with initial angle of 2.5 or 3 degrees was generally used).

The paper length limitation does not permit showing all of the plots, but some typical results of the "Traditional methods" are shown on Figure 5. The figure shows that the results using the troughs (minima) are consistent with those obtained with the peaks (maxima), so both sets were used in the curve fits.

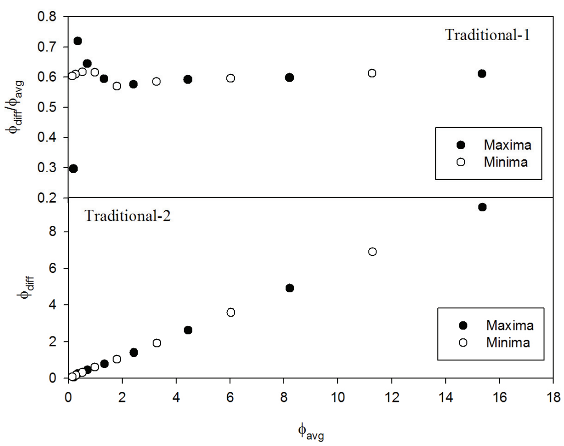

Fig. 5: Typical plots for analysis by "traditional" methods

Prior to the FREEVIB analysis, the data from runs at the highest speed was resampled at a lower rate to satisfy the FREEVIB criterion (sampling rate must be lower than 100 times the oscillation frequency). Figure 6 shows all FREEVIB results for the non-dimensional damping coefficient b',

$$b' = b/2\pi\omega_0 \qquad (9)$$

(where the values of $\omega_0(\phi)$ also obtained from the FREEVIB analysis were used) plotted against the envelope amplitude. Several observations can be made:

- FREEVIB does not seem to produce reasonable results below a roll amplitude of about 2 degrees
- There are apparently spurious "wiggles" in the curves, particularly at the lowest speed and at the smaller roll amplitudes

Unfortunately, no firm quantitative conclusions about the "true" behavior of the damping coefficient with roll amplitude can be drawn from these results. In particular, the quadratic damping model, which would produce straight lines on Figure 6, cannot be confirmed or ruled out. To facilitate comparison with the traditional methods, straight lines were fit to each curve, which yield η_1 and b_2. A summary of all of the coefficients determined from the IIHR data is provided in Table 4.

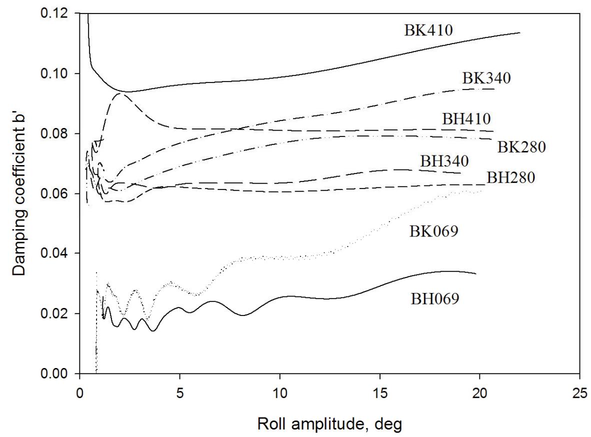

Fig. 6: Damping coefficient from FREEVIB analysis of IIHR model test data

The results in Table 4 are quite consistent although there is much more scatter in the b_2 values; see Figures 7 and 8. The linear coefficient increases almost linearly with speed; the nonlinear coefficient decreases with increasing speed. The additional contribution of the bilge keels to the linear coefficient seems to be essentially constant between Fn=0.069 and Fn=0.280, and subsequently increases with speed. A definitive conclusion as to the speed effect on the bilge keel contribution to the nonlinear coefficient cannot be made on the basis of this data.

Table 4 Roll damping coefficients for IIHR Model 5512

		Traditional-1		Traditional-2		FREEVIB	
Case	Fn	η1	b2	η1	b2	η1	b2
BH069	0.069	0.017	0.055	0.011	0.092	0.014	0.057
BH280	0.280	0.061	0.009	0.057	0.035	0.059	0.011
BH340	0.340	0.063	0.018	0.058	0.055	0.062	0.017
BH410	0.410	0.076	0.039	0.079	0.010	0.082	-0.002
BK069	0.069	0.022	0.110	0.019	0.128	0.017	0.124
BK280	0.280	0.065	0.066	0.064	0.078	0.062	0.064
BK340	0.340	0.066	0.110	0.071	0.078	0.068	0.084
BK410	0.410	0.093	0.024	0.093	0.018	0.091	0.055

To help quantify the scatter among the results of the three analysis methods, the percentage difference of each prediction of η_1 and b_2, as a percentage of the mean of the three methods, is given in Table 5. The table shows that the computed linear coefficients, at Fn ≥ 0.28, agree to within about 5% for the three methods; the highest discrepancy occurs for the nonlinear coefficient for the bare hull, exceeding 100% of the mean value in two cases.

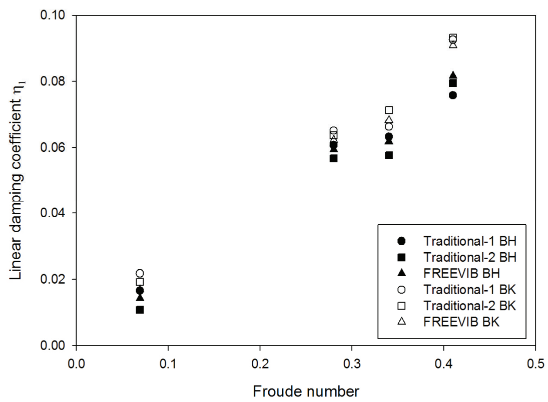

Fig. 7: Comparison of linear damping coefficients, with and without bilge keels

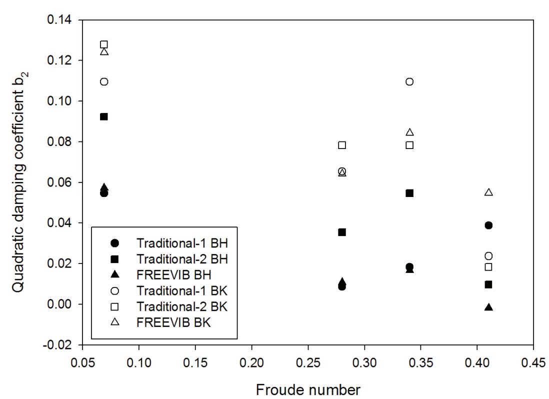

Fig. 8: Comparison of quadratic damping coefficients, with and without bilge keels

CONCLUDING REMARKS

Application of two variants of a commonly-used analysis method for roll decay to synthetic data shows that fitting a quadratic curve to a plot of ϕ_{diff} vs ϕ_{avg} yields coefficients that are less affected by noise than those determined in a linear fit of ϕ_{diff}/ϕ_{avg} vs ϕ_{avg}. An alternative analysis method based on the Hilbert transform shows promise; however the available routine FREEVIB requires further examination and possible modification before it can be judged to be a satisfactory option for roll decay data analysis.

TABLE 5 Difference of computed coefficients relative to the mean of all results, expressed as a percentage of the mean

		η1			b2		
Case	Fn	Trad-1	Trad-2	FREEVIB	Trad-1	Trad-2	FREEVIB
BH069	0.069	19.3	-21.9	2.6	-19.6	35.6	-15.9
BH280	0.280	3.0	-3.8	0.8	-53.0	93.8	-40.8
BH340	0.340	3.9	-5.3	1.4	-38.9	83.2	-44.3
BH410	0.410	-4.0	0.7	3.3	149.2	-37.7	-111.4
BK069	0.069	12.8	-0.4	-12.4	-9.0	6.2	2.9
BK280	0.280	2.1	0.0	-2.2	-5.6	13.0	-7.4
BK340	0.340	-3.3	4.0	-0.7	20.7	-13.6	-7.1
BK410	0.410	0.4	1.1	-1.5	-26.7	-43.4	70.1

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