Probabilistic Assessment of Parametric Rolling: Comparative Study of Detailed and Simplified Models

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ABSTRACT

Current capability in predicting the occurrence of parametric rolling is explored by comparing results obtained by a well-known panel code against predictions obtained by a simple analytical criterion of parametric roll growth in the principal region, deriving from a classical Mathieu-type model. A modern post - panamax containership has been selected as basis of this investigation, that was known to exhibit head-seas parametric rolling at low speed. Recorded discrepancies in stability boundary predictions were converted into quantitative differences in the probability of realising parametric rolling in a realistic seaway.

KEYWORDS

Ship; stability; probability; risk; parametric; roll; capsize.

INTRODUCTION

In their bare essence or as backbone of reliability and risk analysis, probabilistic methods receive positive regard in engineering. In the field of ship stability, the debate on probabilistic methods of assessment has been running for long; yet the current regulatory framework, and notwithstanding recent amendments, reflects little of this. Current international activity makes one feel however that developments are forthcoming (see for example IMO 2007).

The probabilistic properties of a Gaussian sea can be accounted by a number of parametric models of sea waves. However, the serious setback towards the quantitative prediction of the probabilistic properties of extreme ship responses comes from the intricacies of mathematical models of ship motions, especially at condition far from the upright equilibrium, combined with the deficiency of current probabilistic methods to handle such complex models of ship dynamics. In the domain between rudimentary idealized modeling that is amenable to some analytical processing and brute force simulation, several approaches can be (and have been) devised, each with distinctive strengths and weaknesses.

A new method of probabilistic assessment of intact stability was developed recently, described in two deliverables of the Safedor integrated project of the European Commission (see Spyrou and Themelis 2006 for the theory; and Themelis et al. 2007 for the implementation). The method targets capsize modes in an individual basis and thus their probability of occurrence can be distinguished. Moreover, spatial variation of the probability along a ship route or in an area of operation can be produced, a feature that should constitute a useful aid to weather routing. The method exploits the tendency for groupiness of high waves which implies approximate periodicity within the run length of the group that produces the excitation to the ship.

As is obvious, a critical contributor to any probabilistic assessment that exploits numerical predictions of ship dynamics is the quality of these predictions and although we are still far from being content with the state-of-the-art in this field it would be useful to know at least how different tools perform against each other. In the current paper the focus is on a comparison of calculated capsize probabilities using, on the one hand, a simplified model of ship dynamics; and on the other, a detailed panel code that, to our knowledge, has not been used before for the prediction of ship instability. The phenomenon of parametric rolling has been set as the focal point of interest and the investigated ship was a modern post - panamax containership.

The paper is structured as follows: In the next section are presented some basic facts about parametric rolling and a simple analytical criterion is summarised for the growth of roll in the so-called principal region of parametric instability. In the ensuing section is described briefly the panel code that has been used in the current comparative study. Moreover, simulations are presented that verify the capability of this software to capture parametric rolling. Comparisons of critical wave group predictions then follow between the simplified and the detail model. Lastly, the reflections of the findings by these two methods, in terms of the probability of exhibiting parametric instability, are determined.

BOUNDARIES OF PARAMETRIC ROLLING

The region that is customarily identified with principal parametric resonance is in the hollow part of curves (a) and (b) in Fig. 1. This boundary is in fact a bifurcation locus (types shown on the figure) where loss of stability of the upright position equilibrium is generated. Depending on the initially hardening (case shown) or softening character of the restoring curve, the marked bifurcation types are interchanged. Whilst analytical or numerical prediction methods usually target this linear boundary, in reality roll oscillations can be realized also to its exterior. Underneath (b) exists the locus of folding of the oscillation amplitude [curve (c)] that is a boundary resulting purely from the nonlinearity system. Notably, between (b) and (c) the upright equilibrium coexists with the oscillatory response; as a matter of fact, parametric rolling could arise there only by a discontinuous transition; for example invoked by a sudden gust or impulsive wave effect.

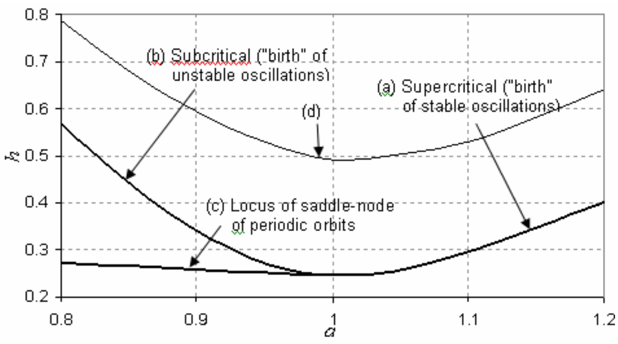

Fig. 1: Region of principal parametric instability of damped nonlinear Mathieu-type system ($a = 4\omega_0^2 / \omega_e^2$ and h is representative of the intensity of fluctuation of the restoring).

These boundaries refer to long-term dynamic behaviour which is unlikely to be reached in a realistic seaway. Thus they are not a convenient interface with the probabilistic wave group analysis. The true technical question is not whether parametric instability can be exhibited 'after' an infinite number of wave encounters, but rather whether roll oscillations can grow to a dangerous level within a few waves. Therefore, transient response boundaries should be sought, like the curve (d) which has been drawn in a qualitative sense in Fig. 1, and that normally lie higher than the boundary curves obtained through the customary stability analysis of steady-states. To determine such 'transient' boundaries entails some understanding of the mechanism of parametric roll growth, i.e. of the law of transient response. A closed form expression could be sought if the permissible roll angle is not set too high (which however leaves out the nonlinear oscillations that exist in the high amplitude region), as follows:

Assume a Mathieu-type equation as a fundamental level model for parametric rolling:

$$\ddot{\varphi} + 2k\,\dot{\varphi} + \omega_0^2\left[1 - h\cos\left(\omega_e\,t\right)\right]\varphi = 0 \qquad (1)$$

k is the linear 'half-dimensional' roll damping, h is the parametric excitation amplitude and ω_e, ω_0 are respectively the encounter frequency and the natural roll frequency (undamped). Then, it can be shown that at exact principal resonance the unstable motion builds-up according to the following approximate general rule (Spyrou 2005):

$$\varphi\left(pT_0\right) \approx e^{-\frac{2p\pi k}{\omega_0}}\left(\frac{e^{\frac{p\pi h}{2}} + e^{-\frac{p\pi h}{2}}}{2}\right)\varphi \qquad (2)$$

To realize a q-fold increase in roll amplitude within p roll cycles from some initial roll angle φ_0 the following condition should hold according to our approximate theory:

$$\ln q = \ln\frac{\varphi}{\varphi_0} \approx -\frac{2p\pi k}{\omega_0} + \ln\left(\frac{e^{\frac{p\pi h}{2}} + e^{-\frac{p\pi h}{2}}}{2}\right) \qquad (3)$$

As the dominant exponential term is the one with positive sign, the following expression delineates the necessary parametric excitation h for achieving q-fold growth:

$$h \approx \frac{4k}{\omega_0} + \frac{0.693 + \ln q}{1.571\,p} \qquad (4)$$

The above expression may be generalised for non-perfect tuning. This extended version will be used in fact in the later sections of this paper but, for the sake of brevity, its exact form is omitted.

SWAN2: A NUMERICAL CODE FOR TIME DOMAIN SIMULATIONS

SWAN2 is a numerical time-domain code for the analysis of the steady and unsteady free surface flows past ships which are stationary or cruising in water of infinite or finite depth or in a channel (SWAN2 2002). This software will be used for predicting parametric rolling in the time domain. *SWAN2* solves the steady and unsteady free-surface potential flow problems around ships using a three-dimensional Rankine Panel Method in the time domain using a distribution of quadrilateral panels over the ship hull and the free surface. The developers of the software argue that the numerical solution algorithms lead to convergent, accurate and efficient wave flow simulations without problems related with numerical dissipation. The ship hull input to *SWAN2* is in the form of standard Computer Aided Design (CAD) generated offsets. Moreover, the creation of the panel mesh over the ship hull and the free surface is performed internally by suitable routines of the program.

Water depth may be either infinite or finite with uniform depth. Mono-hulls, catamarans, trimarans and SES are supported. A mono-hull is supposed to be longitudinally symmetric with cruiser or transom stern. The free-surface flows are solved about the reference coordinate system fixed at the ship's mean position while a uniform stream with velocity U flows in the negative x-direction. A selection between linear and nonlinear Froude-Krylov forces should be made.

Two input files are necessary; a hull offset file and a job control parameters file that contains mainly sea wave characteristics (wave height and period) and operational information (ship mass, speed and etc). In terms of the output files, the grid information on the computational mesh, the principal ship hydrostatic particulars and inertial properties are included. Other ouput files concern the time history of the motions as well as of the forces and moments acting on the body.

Numerical constraints and stability problems have been identified as in the case of large panel aspect ratio where the discrete solution of the wave flow may become unstable which affects negatively the accuracy of the solution. Numerical convergence studies have been performed by the developers of the software in order to ensure that such errors are negligible.

THE INVESATIGATED CONTAINERSHIP

A modern post panamax containership has been selected as a basis for the investigation. This ship had been known to exhibit parametric instability in head seas and at low speed. Her basic particulars are collected in Table 1.

Table 1: Ship particulars

L_{BP} *(length)*	288.87 m	Δ (disp/ment)	11395 t
B *(beam)*	42.80 m	T_0 (natural roll period)	30.26 s
D *(depth to upper deck)*	24.40 m	*KG* (vertical position of centre of gravity above keel)	18.83 m
T_d *(mean draught)*	14.00 m	*GM* (metacentric height)	1.08 m

The hull geometry of the ship was introduced into *Maxsurf* 11.03 which is a well-known commercial ship design software. In Figure 2 is shown a rendered view of the modelled hull. This software has been utilised in the context of application of the analytical criterion, in order to determine the amplitude of parametric excitation incurred by restoring's variation.

Moreover, to predict her motions in the time domain the ship has been modelled also with the program *SWAN2*. In Figure 3 can be seen a characteristic 3D plot mesh generation of the containership, as obtained with *SWAN2*.

Fig.2: Rendered hull of containership exported from Maxsurf.

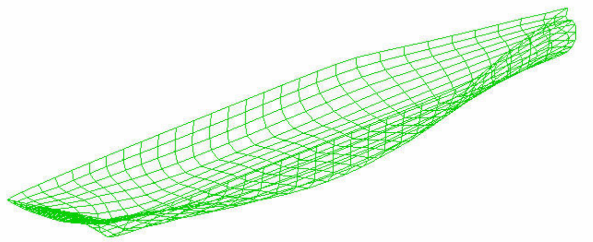

Fig.3: 3D plot mesh generation of containership by SWAN2.

CONTAINERSHIP DYNAMICS ACCORDING TO ALTERNATIVE TOOLS

At first, the critical wave groups that are predicted by each tool to be capable of generating parametric rolling will be contrasted against each other. The calculation of the respective probabilities of encountering these critical wave groups will follow in the final stage as a case study.

Speeds hosting tendency for parametric rolling

It was confirmed by both methods that the ship is liable to head seas parametric rolling at the low speed range. In Figure 4 are shown the critical combinations of ship speed and wavelength where head-sea parametric rolling should be observed when an exact tuning condition has been realised (in mathematical terms when $\alpha \doteq 4\omega_0^2 / \omega_e^2 = 1$). However, as said earlier, critical combinations of speed and wavelength covering a broad range around $\alpha = 1$ will be taken into account.

Estimation of damping

Roll damping plays an important role for the growth of parametric rolling and, as a matter of fact, in the specification of critical wave groups. In the analytical criterion the damping has been calculated by a method described in Themelis and Spyrou (2006).

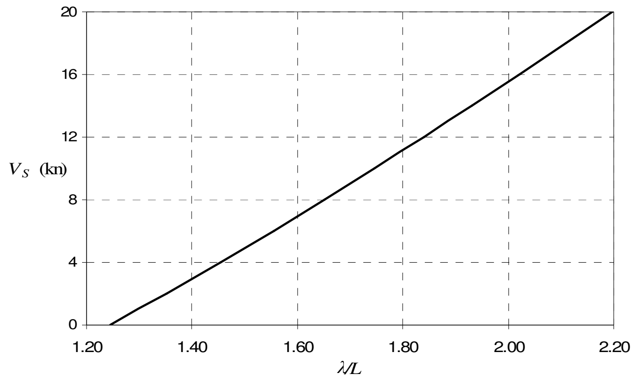

Fig. 4: Critical combinations of speed and wavelength for head seas paramedic rolling ($a = 1$).

Specifically, the viscous part of the damping is estimated by taking into account the skin friction, eddy making and the drag force from appendages (e.g. bilge keel) due to rolling motion. An equivalent linear damping coefficient has been estimated around natural roll frequency, with no forward speed, which is subsequently introduced into the analytical criterion (Eq. 4). The ship has been examined without bilge keels and the value of equivalent linear 'half-dimensional' damping k was determined as 0.0129 s^{-1}. As the speed effect was not taken into account in this calculation method, the roll damping coefficient that appears in the application of the analytical criterion is essentially a constant.

On the other hand, numerical decay tests at various speeds have been carried out with SWAN2. Roll damping calculation in SWAN2 follows the well-known method of Ikeda (1978) and Himeno (1981). Corrections due to forward ship speed are taken into account as lifting effects influence significantly roll damping (Sclavounos 1996). In Figures 5 and 6 are shown two decay tests of the investigated containership; the first with no forward speed and the second with moderate speed. In Figure 7 is compared the calculated linear damping coefficient as function of speed, against the constant damping coefficient obtained from the alternative "in-house" method. The two methods produce identical values at a speed around $V_S = 13$ kn.

Specification of critical wave groups

To ensure that the analytical criterion is applicable with reasonable accuracy, a roll angle of 15^0 has been set as the limiting inclination whose exceedance should be avoided. An uncertain heel disturbance that is necessary for the growth of roll amplitude up to the specified limit is considered, distributed in the range $\varphi_0 = 0^0 - 6^0$. Similarly, a probabilistic speed distributed in the range $V_S = 0 - 20$ kn has been considered.

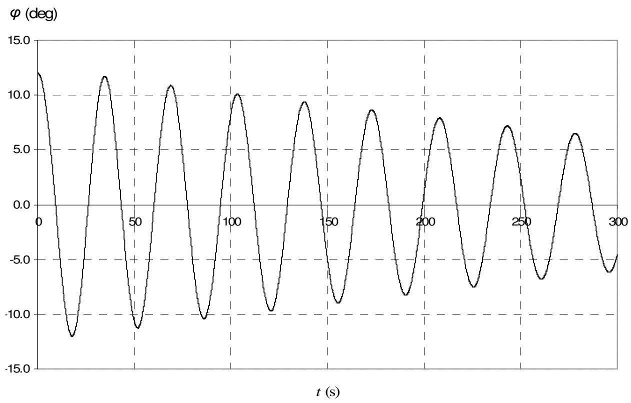

Fig. 5: Roll decay with no forward speed.

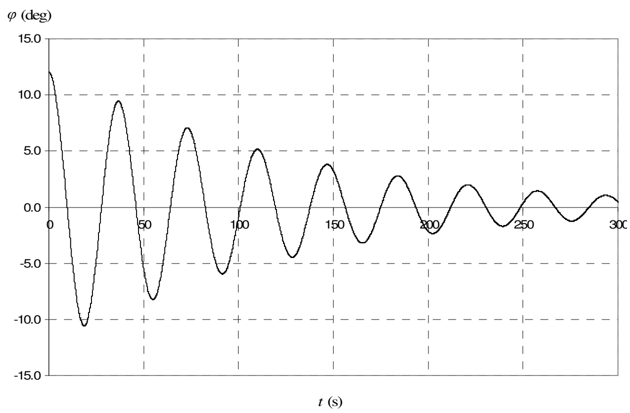

Fig. 6: Roll decay with forward speed $V_S = 18$ kn.

Analytical criterion

As already explained, the criterion targets the critical magnitude of parametric excitation h_{cr} that is necessary for realising a q-fold increase from some initial roll disturbance, within a limited number (say p) of roll cycles. At condition of exact principal resonance, $2p$

wave encounters are required for p parametric roll cycles. In Figure 8 one observes the critical parametric amplitudes which correspond to $a=1$, for three different initial conditions, each assumed to be the representative of a narrow range of initial roll angles. Up to 8 waves in a group have been considered because a larger number corresponds to trivially low probability of encounter.

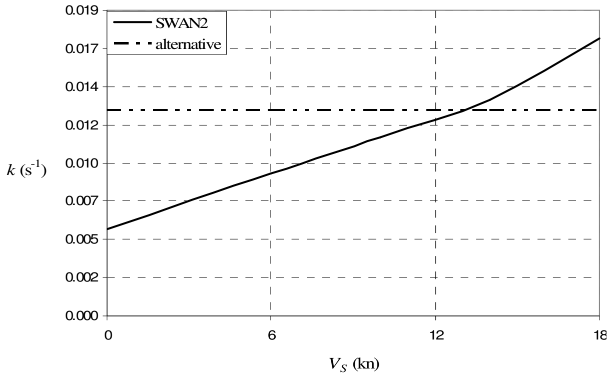

Fig. 7: Comparison of damping coefficients at various speeds.

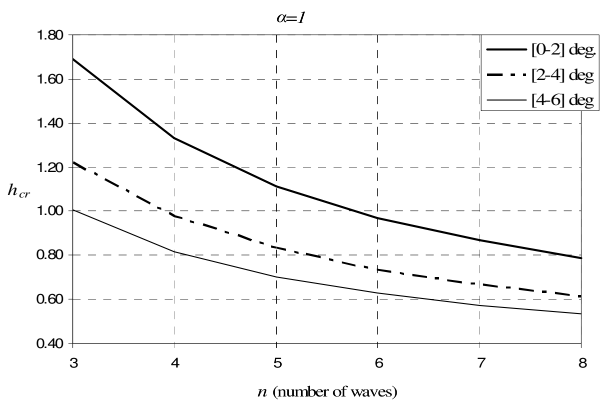

Fig. 8: Critical parametric amplitudes for $a=1$.

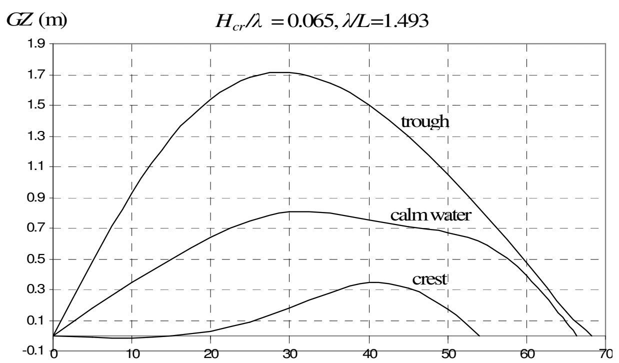

Fig. 9: Characteristic GZ variation.

Critical parametric excitation amplitudes should be transformed into critical wave heights by taking into account the geometry of the submerged hull. Thus the variation of restoring between wave crests and troughs needs to be calculated for a range of wavelengths that are near to satisfying the condition $a=1$. In Figure 9 is shown a characteristic GZ variation that corresponds to one of the identified critical wave group specifications.

Numerical simulations

Critical wave group characteristics are identified on the basis of a campaign of numerical simulations, with occasional recording of the occurrence of growth of the roll amplitude to 15^0 within the allowed number of wave encounters. Nonlinear Froude-Krylov force calculation was selected. As manifested by the characteristic time-histories shown in Figures 10 to 12 referring to the principal instability regime, SWAN2 can capture parametric rolling.

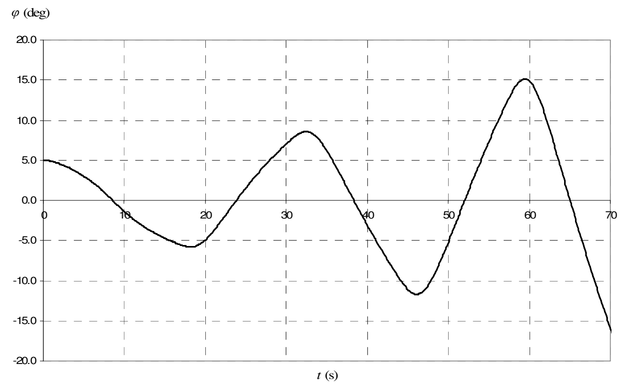

Fig. 10: Roll response in head waves ($a=0.8$, $H=9.9$ m).

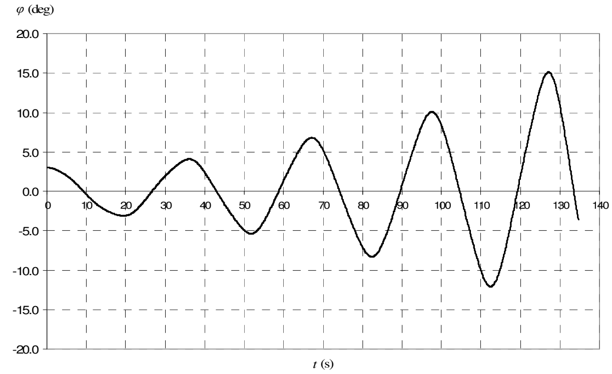

Fig. 11: Roll response in head waves ($a=1$, $H=7$ m).

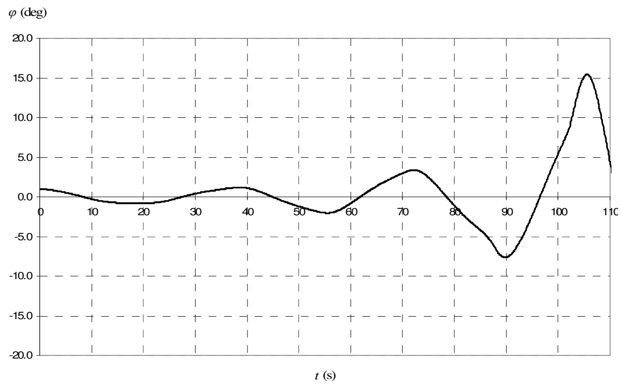

Fig. 12: Roll response in head waves ($a = 1.2$, $H = 10.4$ m).

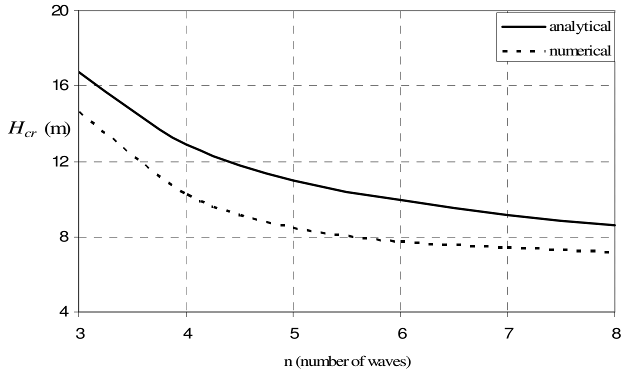

Fig. 13: Critical wave height as function of run length ($a = 1$, $V_S = 14$ kn , $\varphi_0 = 5^0$).

COMPARISON OF CRITICAL WAVES

The comparison between analytical and numerical predictions was focused on differences of wave height with all other parameters kept identical. The critical wave heights as calculated by the two prediction techniques are illustrated in Figure 13. Despite the fact that, according to Figure 7, for the selected speed of 14 kn higher roll damping was present in the numerical simulation model than the one inserted in the analytical criterion, the critical wave heights predicted by the numerical tool turned out to be lower. This could possibly be attributed to effects incurred by the heave and pitch motions.

Next, the critical wave heights that generate 5-fold increase in roll amplitude within $n = 3, 4, ..., 8$ wave encounters have been compared for various speeds. The results are shown in Figures 14 to 19, varying from one figure to the next the wave group run length n. A trend of wave height increase is noted as the speed is raised, that follows a similar pattern for both prediction methods. In the case of the *SWAN2* simulations one could also notice that, despite its increase with speed, roll damping does not seem to affect substantially the critical wave heights. Therefore, the augmentative trend in wave height should be attributed mainly to an increase of the required wavelength; in its turn owed to the higher speed required for achieving frequency tuning at $a = 1$.

An alternative option is to fix the speed, determine those wave lengths that are eligible to incur head-seas parametric rolling when a obtains a value around 1, and then determine the critical wave heights according to the wave group run length. Such diagrams are collected in the set of Figures 20 to 24, covering a wide range of speeds.

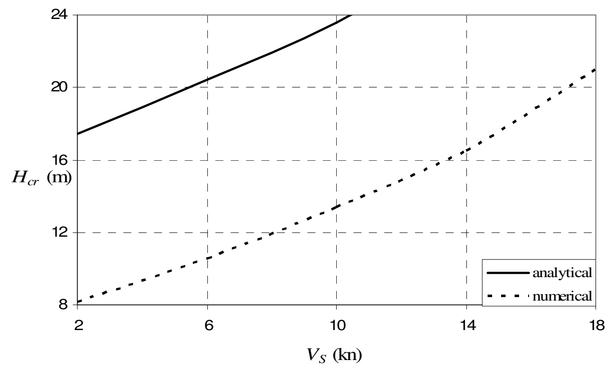

Fig. 14: Critical wave height as function of speed ($a = 1$, $q = 5$ and $n = 3$ wave encounters).

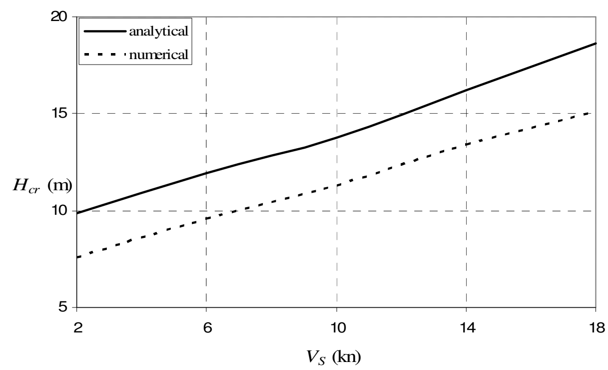

Fig. 15: Critical wave heights ($a = 1$, $q = 5$ and $n = 4$).

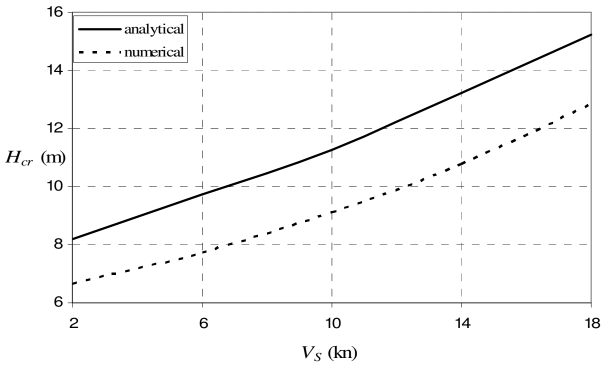

Fig. 16: Critical wave heights ($a = 1$, $q = 5$ and $n = 5$).

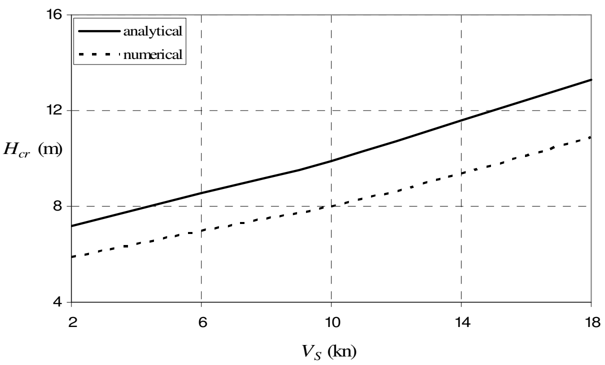

Fig. 17: Critical wave heights ($a = 1$, $q = 5$ and $n = 6$).

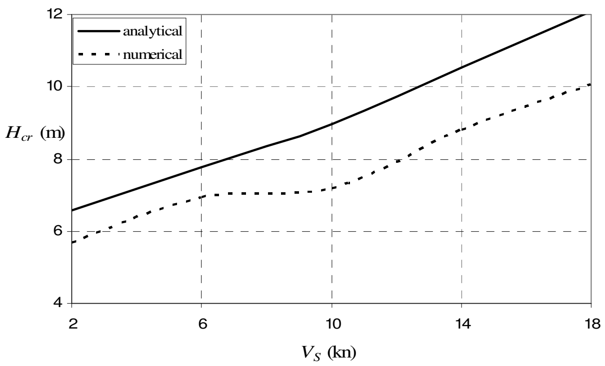

Fig. 18: Critical wave heights ($a = 1$, $q = 5$ and $n = 7$).

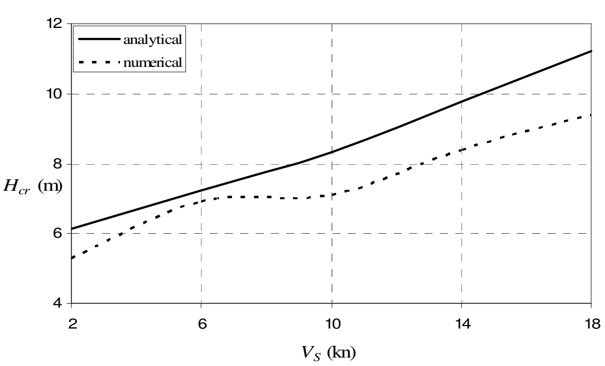

Fig. 19: Critical wave heights for $a = 1$, $q = 5$ and $n = 8$).

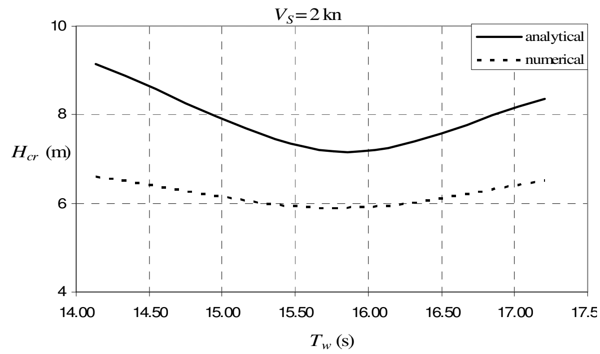

Fig. 20: Critical wave height as function of wave period ($q = 5$, $V_S = 2$ kn and $n = 6$).

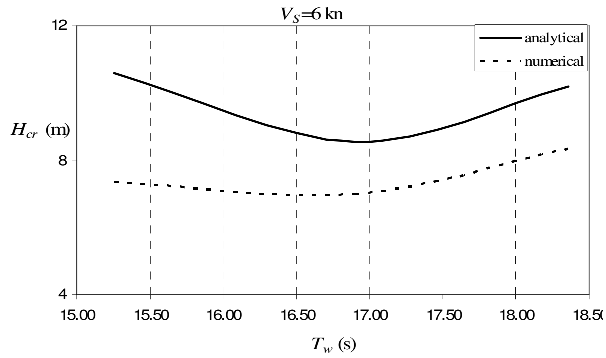

Fig. 21: Critical wave heights ($q = 5$, $V_S = 6$ kn and $n = 6$).

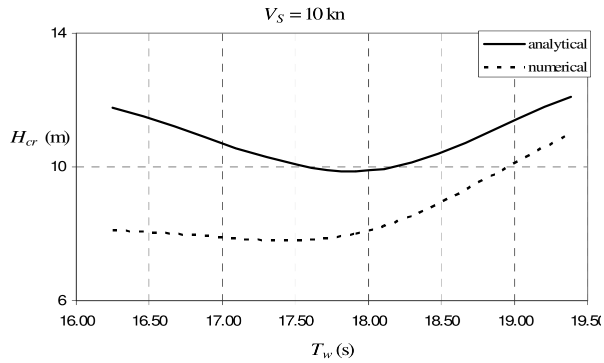

Fig. 22: Critical wave heights ($q = 5$, $V_S = 10$ kn and $n = 6$).

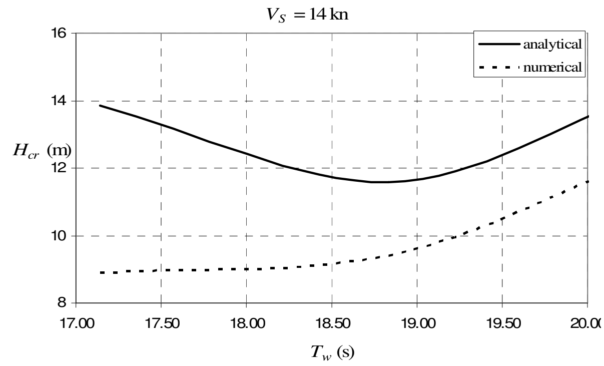

Fig. 23: Critical wave heights ($q = 5$, $V_S = 14$ kn and $n = 6$).

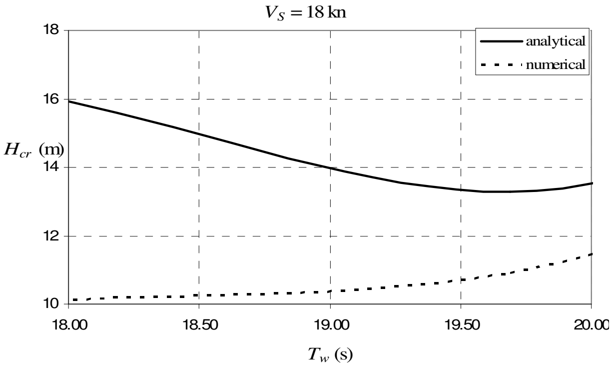

Fig. 24: Critical wave heights ($q = 5$, $V_S = 18$ kn and $n = 6$).

CALCULATION OF PROBABILITIES

Probabilities to encounter the wave groups specified in the previous paragraphs, given a selection of sea state, have been calculated. A comprehensive discussion on the joint and marginal probability density functions that are necessary for such a calculation are given in Themelis and Spyrou (2006). Very briefly, the underlying theory of the current probabilistic wave group analysis is based on a modification of Kimura's (1980) approach, developed by Battjes & Van Vledder (1984). Assumption of the Markov chain property is made for the successive waves in the group. Adding to the versatility of the methodology, necessary probability calculations exploit spectral information of the wave field; i.e. there is no need of using direct time-series results.

A sea state with $H_S = 7.6\ m$ and $T_P = 16.4$ s has been picked out of the set of results of a hindcast study which addressed the North Atlantic region (Behrens 2006). Such weather could have prevailed at location 59.5^{0}N - 7^{0}W, with a few hours duration on January 14th, 1991. The JONSWAP spectrum was assumed in order to expedite the calculation procedure (Hasselmann 1973). The peakness parameter of this spectrum was determined as recommended in DnV (2002).

The obtained probability values are illustrated in Figure 25 as functions of speed. Concerning initial condition, the assumption was made of some uncertain heel disturbance with the following distribution: 50% to lie within $0^0 - 2^0$, 30% in $2^0 - 4^0$ and 20% in $4^0 - 6^0$. Lastly, the overall probability values are shown in Table 2. To this, the speed was treated as a probabilistic parameter obtaining values in the range 0 – 20 kn. Five successive sub-ranges with length 4 kn were then considered and probabilities were attributed as follows: 30% for the middle sub range and 20% for each one of the others.

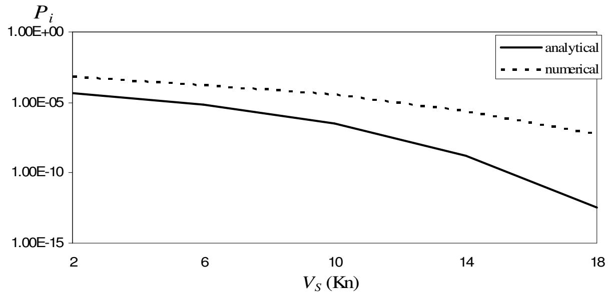

Fig. 25: Probabilities of encountering the critical wave groups in terms of ship speed.

Table 2: Total probabilities

Analytical criterion	1.047×10^{-5}
Numerical simulations	1.663×10^{-5}

CONCLUDING REMARKS

The results of a comparative study between a well-known panel code and a simple analytical criterion have been presented, in terms of the boundary of instability of parametric rolling of a post-panamax containership. Transient growth at the initial phase of the occurrence of the phenomenon has been targeted by the two methods. The critical wave heights obtained by the analytical criterion were in every case higher than those obtained by the panel code, even when the damping was higher. This could reflect the effect of pitch and heave motions in a way aggravating the effect of restoring variation. Reasonable coincidence of predictions was found at frequency tuning about $a = 1.2$. Despite the grave differences in the modelled physics between the two models, one should regard these predictions, as well as others that are presented in the open literature from time to time, with caution and should not rush to consider as more correct those produced by the detailed tool.

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