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NONLINEAR AND STOCHASTIC ASPECTS OF PARAMETRIC ROLLING MODELLING

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SUMMARY

This paper addresses, starting from an extensive series of tests in longitudinal regular waves (already done) and irregular waves (in progress), the problems connected with the threshold formulation for parametric rolling and its amplitude modelling above threshold. Both head and following waves have been considered, also in view of the greater attention to head sea conditions called during IMO/SLF discussion on the revision of the Intact Stability Code. Particular attention is given in the regular wave case to the nonlinear damping, nonlinear restoring and nonlinear parametric excitation terms. The mathematical models so developed are "compared" with experimental results by means of an ad hoc parameter estimation technique.

It is on the other hand well known that several different thresholds can be proposed in the case of irregular waves and that the nonlinear modelling of roll motion variance above threshold is at present not properly addressed. Here too, a series of experiments will be conducted in the presence of narrow band irregular waves having the bandwidth as parameter. The use of approximate analytical techniques will allow to obtain a mathematical description of the nonlinear parametric rolling.

NOMENCLATURE

SLF is the Stability, Load Lines and Fishing Vessels Sub-committee of MSC
MSC is the IMO Maritime Safety Committee
IMO is the International Maritime Organisation
ϕ instantaneous roll angle, transversal inclination
A, B, C, roll motion amplitudes
T ship draught
B ship beam
s_w wave steepness
T_0 natural roll period
x_G longitudinal centre of gravity from mid perpendicular
KG height of centre of gravity on keel
GM initial metacentric height
μ coefficient of linear roll damping
ν nondimensional coefficient of linear roll damping
μ_{eq} coefficient of the equivalent linear roll damping
ν_{eff} effective linear roll damping
β coefficient of quadratic roll damping
δ coefficient of cubic roll damping
Δ ship displacement
GZ righting arm
ω_0 natural roll frequency
ω_e encounter wave frequency
ω_d natural roll frequency in the presence of damping
p_1, p_2, p_{ave} coefficients of parametric excitation
α_3 coefficient of cubic arm nonlinearity of righting arm
L ship length at waterline
L_{bp} ship length between perpendiculars
v ship speed
m_0, m_1, m_2 statistical moments of spectrum
sbw spectral bandwidth
h(t) stochastic process representing GM fluctuation in random waves
$\alpha(t)$ phase
S_0 white noise level
S_z spectral density
ω_m modal frequency of filter
γ damping of filter

1. INTRODUCTION

A research project was recently ended in collaboration with INSEAN – Italian Ship Model Basin on the parametric rolling in regular longitudinal waves [1] and a new one is starting on parametric rolling in irregular longitudinal waves. The first was aimed at a better understanding of the nonlinear modelling, the effect of damping on the threshold and on the rolling amplitude above threshold. The second one is aimed at a better understanding of the nonlinear modelling in the presence of irregular sea, of the threshold definition and of the probability of exceedence of "dangerous" roll amplitude in realistic sea conditions. In both cases the head sea condition will be considered due to the easier checking in short facilities like the towing tank of Trieste University and in view of the entailed dangerous phenomena whose presence has been dramatically confirmed recently. It is indeed a matter of fact that, apart the addressing of head sea condition as a potentially dangerous one made by one of the authors [2-5] this subject was previously present very seldom in the literature [6,7,14,15]. Several written contributions to recent IMO/SLF call for greater attention to the parametric rolling in head waves [8,9] in the existing regulation concerning sailing in following/quartering waves [10].

1.1 RULES ARE CHANGING - ANALYTICAL RESULTS VERSUS PERFORMANCE RULES

It is again a matter of fact that most part of international regulations as regards safety from ship capsizing or sinking, is written in the form of prescriptive rules and usually in quite simplified way. There is at present an opening towards alternate methods of assessing equivalent levels of safety and a call for a change from prescriptions to performance. This last approach will take several years to be developed and it is not clear if prescriptive rules will survive along it and beyond in a parallel course or not. This is better discussed in the companion paper [11]. Here we just want to present some contribution made by our research group towards the development of nonlinear and stochastic approach to the problem of parametric rolling, made in analytical terms, eventually using the well known perturbative approximate methods. It is the opinion of the authors, indeed, that at least in the short term a contribution to develop tools for the evaluation of the expected maximum roll amplitude in longitudinal waves will be valuable in the frame of increasing the safety and seakindliness of transportation at sea.
It is indeed again a matter of fact that a great effort was devoted in the last 25 years, since the first Stability Conference, to parametric rolling. Nevertheless no simple rule regarding motion amplitude/acceleration exceedance was developed to be used at design stage as it was done on the contrary in IMO Res. A.167 and A.562. The only safety measure approved was the MSC707 regarding ship handling [10]. Many of the interested parties simply don't believe that parametric rolling is a real danger to avoid which it is necessary to incorporate "additional" safety at design level. Others think parametric rolling is a matter of seakeeping and not of stability, like in the concluding remarks of 23rd ITTC.

1.2 CONTENTS OF THIS PAPER

Some results from the first research projects will be firstly presented; in the following the preliminary analysis made on the problems, partly open, entailed by the presence of a stochastic sea will be given.
Several ship models were tested. While full results can be found in the project report [1], here the case of a RoRo passenger ship, called TR2 (ship model hull # C73-97), will be presented. The body plan and the main data are reported in Fig. 1 and Table. 1 below.

2. PARAMETRIC ROLLING IN REGULAR WAVES

2.1 MATHEMATICAL MODELLING

As it is often done in the analytical studies devoted to parametric rolling, the problem is split in two parts:
- the evaluation of the wave-ship interaction to obtain the parametric excitation to be included in the roll motion equation to modify the restoring term;
- the analysis of the solutions of the roll motion equation, their stability and the amplitude as a function of ship speed.

Table. 1. Model and full scale data of RoRo pax TR2 used in the experiments.

RoRo pax - C73-97
Model data (scale 1:50): Δ=61.720 kg_f GM=0.0173 m KG=0.1732 m T=0.1175 m x_G=-0.072 m T_0=2.30 s ω_0=2.73 rad/s (average value assumed 2.7 rad/s) L_{bp}=2.644m
Full scale data: Δ=7715 t_f GM=0.865 m KG=8.660 T=5.875 m x_G=-3.599 T_0=16.26 s ω_0=0.386 rad/s (average value assumed 0.382 rad/s) L_{bp}=132.2 m

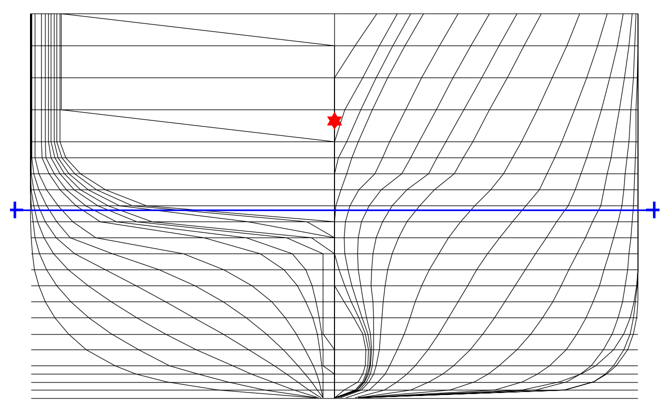

Fig. 1. Schematic body plan of the ship TR2. The floatation line (full loading condition) and the vertical position of the centre of gravity is also indicated.

The problem is thus modelled as a 1.5 degrees of freedom, since the first action above includes, in principle the vertical motions of the ship (eventually in the equilibrium case assumed in the fixed or free trim hydrostatic computations in the wave) in addition to the wave. As a result, the following model was considered:

$$\ddot{\phi} + 2\mu\dot{\phi} + \beta\left|\dot{\phi}\right|\dot{\phi} + \delta\dot{\phi}^3 + \left[1 + \left(p_1 + p_2\phi^2\right)\cos\omega_e t\right]\cdot \cdot\omega_0^2\phi + \alpha_3\phi^3 + \alpha_5\phi^5 + \ldots = 0 \quad (1)$$

Here only the cubic term $\alpha_3\phi^3$ was retained in the righting moment. Posing $\omega_e t = 2t'$ and preserving the initial name for the independent variable, one has:

$$\ddot{\phi} + 2\mu^* \dot{\phi} + \delta^* \dot{\phi}^3 + \left[1 + \left(p_1 + p_2 \phi^2\right)\cos 2t\right] \omega_0^{*2} \phi + \alpha_3^* \phi^3 = 0 \qquad (2)$$

with:

$$\mu^* = \frac{2\mu}{\omega_e} \quad \delta^* = \delta \frac{\omega_e}{2} \quad \alpha_3^* = \alpha_3 \frac{4}{\omega_e^2} \quad \omega_0^{*2} = \frac{4\omega_0^2}{\omega_e^2}$$

Several alternate forms of Eq. 1 and 2 were proposed by different authors (for a review of the extensive literature see [1]). In the following the mathematical modelling based on Eq. 2 will be called "uncoupled", to distinguish from other where the time dependent and the nonlinear angle dependent terms in the righting arm are "coupled", as in:

$$\ddot{\phi} + 2\mu^* \dot{\phi} + \delta^* \dot{\phi}^3 + \left[1 + \left(p_1 + p_2 \phi^2\right)\cos 2t\right] \left(\omega_0^{*2} \phi + \alpha_3^* \phi^3\right) = 0 \qquad (3)$$

Anyway, the main features are:

- nonlinear damping. This can be made by a linear plus cubic or by a linear plus quadratic model, provided that they are adjusted to dissipate the same energy per cycle (of course this makes the coefficients amplitude-dependent);
- nonlinear restoring, in this case limited to cubic degree to avoid excessive analytical complications (this is a non-essential choice if use is made of codes for algebraic equation handling);
- nonlinear effect of vertical motions and waves on righting arm modulation (factor p_2). This is an assumption specific to this mathematical model and is based on the observation that the difference between the two extreme values of righting arm as a function of inclination (roughly wave crest and wave through amidships) grows to a maximum value and then goes to zero in a way that the parametric forcing p can roughly be approximated by a quadratic function. In Fig. 2 this behaviour is reported for the examined ship.

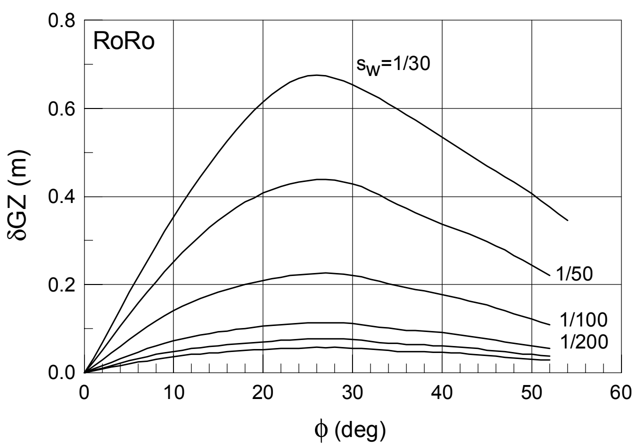

Fig. 2. Difference between the righting arm corresponding to wave through amidships and wave crest amidships as a function of the transversal inclination. The curves refer to wave steepness s_w=1/30, 1/50, 1/100, 1/200, 1/300.

2.2 THE THRESHOLD FOR PARAMETRIC ROLLING

Eq. 2 is a homogeneous one, so that the upright position is always an equilibrium solution (trivial solution $\phi(t) \equiv 0$). Its stability depends on the actual values of the linear damping coefficient μ, parametric forcing (both the linear coefficient $p_1 = \frac{\delta \overline{GM}}{\overline{GM}^*}$ and the nonlinear one p_2) and on the range of encounter frequency ω_e, which in turn depends on selected wave length, ship speed V and heading. The threshold for instability of upright position, i.e. for the onset of parametric rolling, is usually obtained starting with the linear version of Eq. 2. In spite of the apparent simplicity, linear Mathieu equation is very difficult to solve in finite terms. Anyway, the threshold values for $\frac{\delta \overline{GM}}{\overline{GM}^*}$ for the excitation of parametric rolling in different instability zones, corresponding to the frequency ratios:

$$\omega_e \approx \frac{2}{n}\omega_0 \qquad (4)$$

with n integer have been computed by several authors. In Fig. 3 the regions of instability in the absence of damping corresponding to n=1, 2, 3 are reported. As one can see, the width of the regions is sharply decreasing by increasing n. In the absence of damping, the borders of the instability region cross on the x axis, so indicating that for encounters frequencies given by the exact synchronism condition:

$$\omega_e = \frac{2}{n}\omega_0 \qquad (5)$$

the upright position is unstable even in the presence of a negligibly small perturbation. The presence of damping changes quantitatively the picture giving a minimum value for the threshold, in proximity to the exact synchronism. This minimum value of the instability threshold depends on linear damping in the following way:

$$\frac{\delta\overline{GM}}{\overline{GM}^*} \approx \mu^{\frac{1}{n}} \tag{6}$$

so that it grows fast with the order of the instability region. The combination of this aspect, with the narrowing evident from Fig. 3, explains why only the first region (and sometimes the second) can be expected to play a relevant role in parametric rolling in actual seaways.

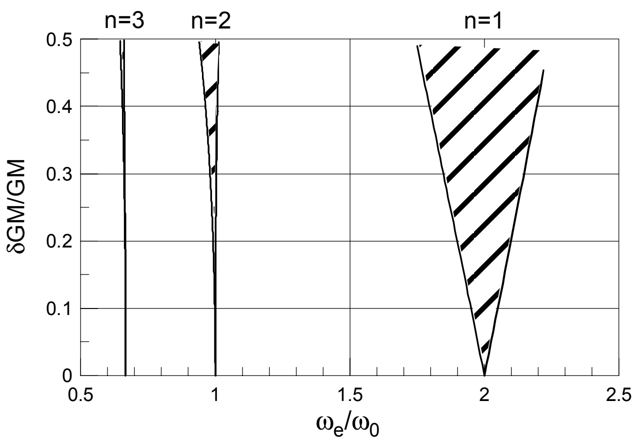

Fig. 3. Threshold boundaries of the first three instability zones for the linear undamped Mathieu equation. The diagram has been adapted to variables relevant to parametric rolling.

Considering thus in particular the first instability region, one has the following expression for the curve separating stability and instability regions in the presence of linear damping:

$$\frac{\delta\overline{GM}}{\overline{GM}^*} = \sqrt{\left(2 - \frac{\omega_e^2}{2\,\omega_0^2}\right)^2 + \frac{\omega_e^4}{2\omega_0^4}\left(\frac{4\omega_0^2}{\omega_e^2} + 1\right)(\mu^*)^2} \tag{7}$$

with a minimum value

$$\frac{\delta\overline{GM}}{\overline{GM}^*} = 4\mu^* = \frac{8\mu}{\omega_e} = \frac{4\mu}{\omega_0} \tag{8}$$

which is a well known result.

2.2 (a) The experimental results

The tests were conducted in the towing tank of Trieste University. Several series of tests were done, aimed at investigating the effect of wave steepness, wave speed, ratio λ_w/L. The stability characteristics in waves have been calculated with a standard code allowing fixed and free trim isocarenic transversal inclinations in the Froude-Krylov hypothesis. The following values of the wave steepness were used for the first series $\lambda_w/L=1$:

s_w=1/30, 1/50, 1/100, 1/200, 1/300, 1/400.

The second series was conducted with s_w=1/50 and λ_w/L=0.75, 0.932, 1.0, 1.25.

Finally, the third series was conducted at the same wave height, corresponding to the wave with s_w=1/50 and λ_w/L=1, at the ratios λ_w/L=0.75, 1.0, 1.25.

Due to basin limitations, most tests were conducted in head waves, but the 1/50 series was conducted in the full range included in the zone of instability.

Prior to tests in waves, the roll damping as a function of speed was determined in calm water. In Fig. 4 the equivalent linear damping μ_{eq} is reported as a function of forward speed together with the adopted analytical fit.
The experimental results have been compared in Fig. 5 and 6 with the threshold for onset of parametric rolling in the $\left(\left(\frac{\omega_e}{\omega_0}\right)^2, \frac{\delta\overline{GM}}{\overline{GM}^*}\right)$. As a general rule in the figures:

- the square points represent cases where the parametric roll was present;
- empty diamond points indicated absence of parametric rolling, i.e. roll decay was observed after perturbation of the upright position;
- empty squares indicate “uncertain” cases, most likely belonging to the decay, but very close to the border (not always).

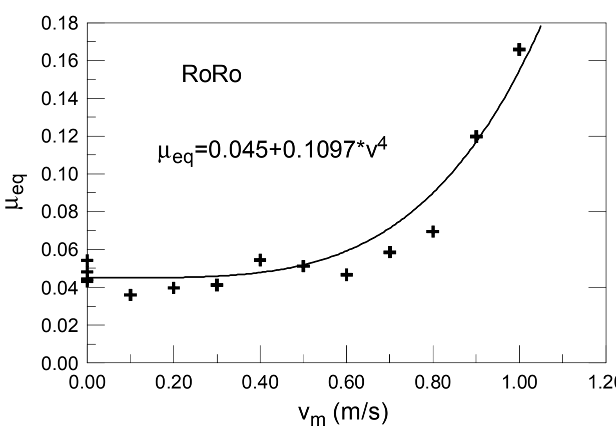

Fig. 4. Ship roll damping in calm water as a function of forward speed together with its analytical fit used in the calculations.

The factor $p_1 = \frac{\delta\overline{GM}}{\overline{GM}^*}$ has been computed both in the fix and in the free trim condition.
It is interesting to observe that even in the mildest environmental condition (s_w=1/400 – waves can hardly be seen in towing tank), the parametric rolling was present, provided the encounter frequency was in the appropriate range.

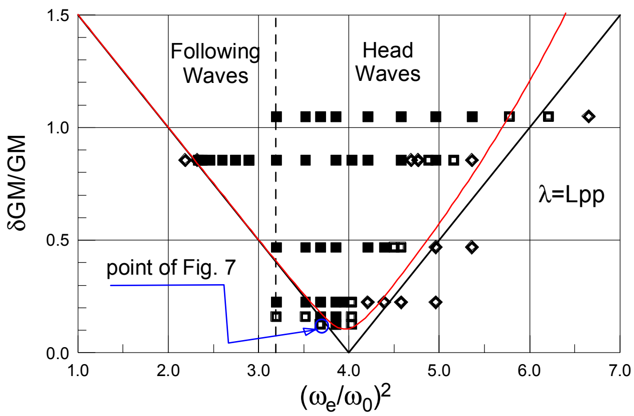

Fig. 5. Comparison between the experimental results and the analytical threshold curves obtained by Eq. 7 with the damping as a function of forward speed reported in Fig. 4. The stability calculations to obtain the factor p_1 were made with fixed trim.

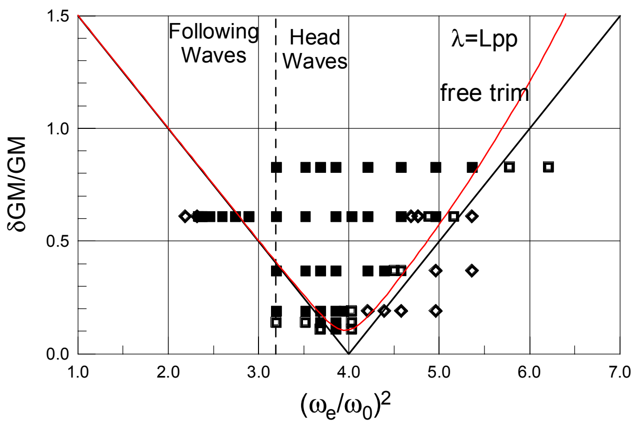

Fig. 6. Comparison between the experimental results and the analytical threshold curves obtained by Eq. 7 with the damping as a function of forward speed reported in Fig. 4. The stability calculations to obtain the factor p_1 were made with free trim.

As one can see from Fig. 5 and 6, the agreement in terms of threshold for onset of parametric rolling is quite good. This is especially evident from the case s_w=1/50 which was done in the full range of ship speeds, including the following sea condition. The agreement with the fixed trim calculation is nevertheless better than the other as less points leading to sustained parametric rolling "escape" the instability region. This poses the question, often debated (see for example [12-15]), of which is the way to compute the parametric forcing due to the waves. In this case it appears that simple hydrostatic equilibrium is sufficient to give reliable results. Alternatively, the effects of the nonlinear restoring on the threshold for the onset of parametric rolling could be advocated [5].

2.2 (b) The effect of damping

The parametric "forcing" term can also be seen as a reduction in the damping, so leading to the consideration of an "effective" damping.

The solution of the linear Mathieu equation is indeed unstable if $-\mu^*+\sigma>0$ where σ is the characteristic exponent obtained by the application of the Floquet theory.

Here unstable means diverging. On the contrary, the solution is converging to zero (i.e. the effect of a perturbation to the upright position is reduced to zero as time increases) if $-\mu^*+\sigma<0$. The intermediate condition corresponds to the borders of the instability regions of Fig. .

When the representative point in the $\left(\dfrac{\omega_e}{\omega_0},\dfrac{\delta\overline{GM}}{\overline{GM}^*}\right)$ is close to the border of the instability zone, the effective damping is very small and the decay of a perturbation is very slow. In Fig. 7, the decay of a perturbation in very small waves is compared with the decay in calm water at an advance speed close to that in waves.

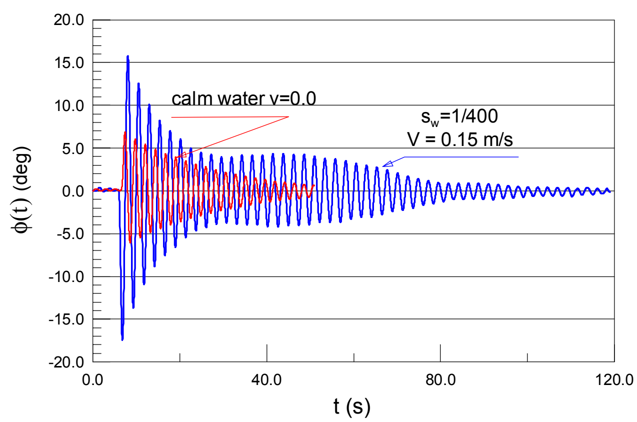

Fig. 7. Roll decay in waves for s_w=1/400 compared with the calm water decay. The point is very close to the border of the instability region (see Fig. 5).

2.3 THE ROLL MOTION AMPLITUDE ABOVE THRESHOLD

2.3 (a) The analytical solution

A perturbation method (here the Van der Pol one) can be applied to the nonlinear Eq. 2 to obtain an approximate analytical solution.

In the first instability zone, one has n=1 and $\omega_e \approx 2\omega_0$, and the stationary solution can be guessed in the form:

$$\phi(t) \approx A \sin t + B \cos t \qquad (9)$$

with A and B slowly varying amplitudes. Deriving, substituting in Eq.2 and using the auxiliary condition:

$$\dot{A}\cos t + \dot{B}\sin t = 0 \qquad (10)$$

the following algebraic system of equations in the unknown amplitudes A and B is obtained:

$$2\langle\dot{A}\rangle = -\left(\mu^* + \frac{3}{4}\delta^*\left(A^2+B^2\right)\right)A$$
$$+B\left(\left(4\frac{\omega_0^2}{\omega_e^2}-1\right)+\frac{3}{4}\alpha_3^*\left(A^2+B^2\right)\right) \tag{11a}$$
$$-2p_1\frac{\omega_0^2}{\omega_e^2}B+2p_2\frac{\omega_0^2}{\omega_e^2}B^3$$

$$2\langle\dot{B}\rangle = -\left(\mu^* + \frac{3}{4}\delta^*\left(A^2+B^2\right)\right)B$$
$$-A\left(\left(4\frac{\omega_0^2}{\omega_e^2}-1\right)+\frac{3}{4}\alpha_3^*\left(A^2+B^2\right)\right) \tag{11b}$$
$$-2p_1\frac{\omega_0^2}{\omega_e^2}A+2p_2\frac{\omega_0^2}{\omega_e^2}A^3$$

The steady solution $C=\sqrt{A^2+B^2}$ can now be obtained by solving the system 11 with respect to A and B by posing $\langle\dot{A}\rangle=\langle\dot{B}\rangle=0$.

In the following, a Parameter Identification Technique (PIT), based on the regression of the analytical solution to the experimental data will be used to obtain an estimate of some relevant parameter, in particular the coefficients expressing the parametric forcing and the nonlinear term in the righting arm. To this end, the use of the system of algebraic equations obtained for steady A and B is not simply viable. We tried thus a simplified approach based on the use of an "average" value for the nonlinear term in the parametric excitation:

$$p_{ave}=p_1+\frac{p_2}{3}\phi^2 \tag{12}$$

in the generic iteration of the zero searching procedure used to obtain the roll motion amplitude C. In this way, the system of algebraic equations reduces to a single second degree one.

Also the coupled mathematical model can be solved in similar way, resulting in a somewhat more complex system of algebraic equations for C. In this case, other perturbation methods may be more appropriate [4,5].

2.3 (b) The experimental results

The results of the tests in terms of steady roll motion amplitude above threshold are reported in Figs. 8 to 11 for the different series. The series at constant s_w and λ_w/L were fitted by means of the numerical solution of Eq. 11 above with the assumption of Eq. 12.

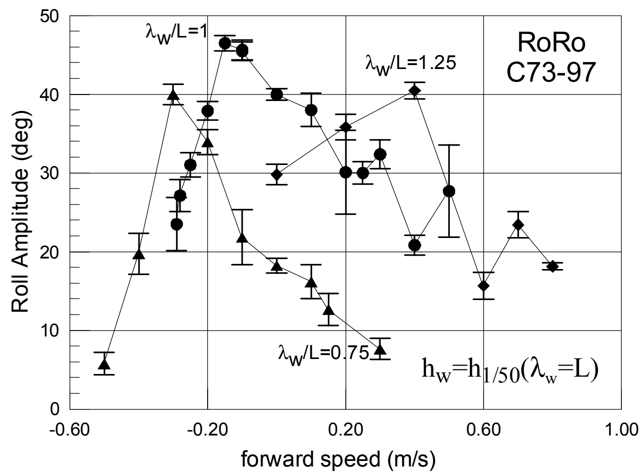

Fig. 8. Steady roll motion amplitude of parametric rolling as a function of ship forward speed (given at model scale) at constant wave height.

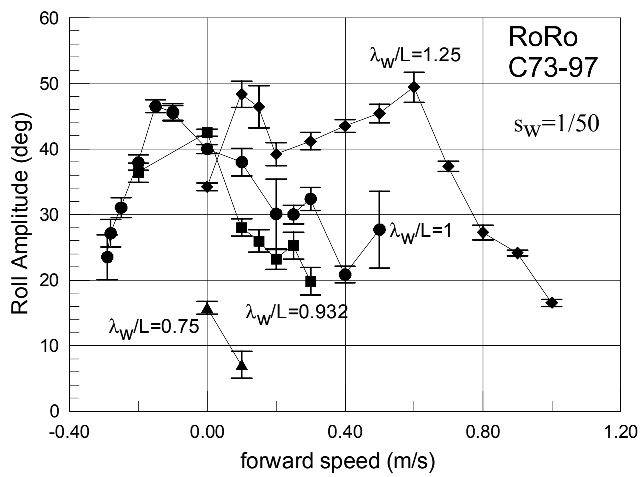

Fig. 9. Steady roll motion amplitude of parametric rolling as a function of ship forward speed (given at model scale) at constant wave steepness.

This gives two branches of the curve $C = function(v)$ originating at the two extreme forward speed values corresponding to the encounter frequencies defining the interval intersection of the threshold (Fig. 5,6) with the horizontal line drawn at the $p_1=\delta\overline{GM}/\overline{GM}^*$ value relevant to the investigated s_w. The upper branch (blue in the figures) corresponds to the stable parametric sub-resonant rolling , while the lower one corresponds to the unstable one (red in the figures).

Again the fitting is quite good. It was obtained with the use of the PIT, i.e. the parameters α_3 and p_1 were estimated by means of the least square fitting of the experimental results. The estimated results for p_1 are reported in Table. 2 together with the values obtained with fixed trim computations.

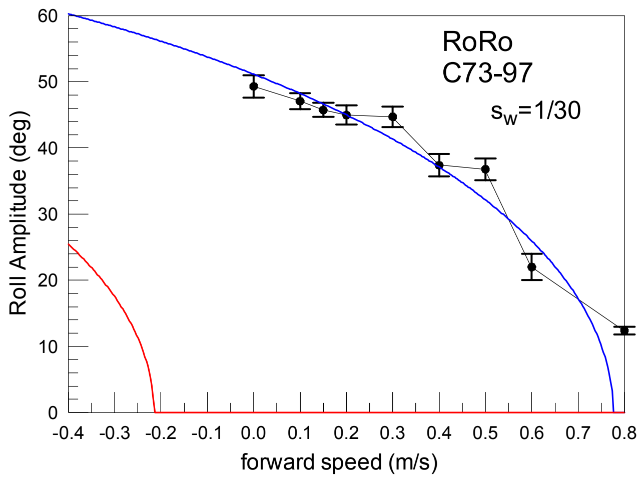

Fig. 10. Steady roll motion amplitude of parametric rolling as a function of ship forward speed (at model scale). The tests refer to λ_w/L =1 and s_w=1/30. The experimental uncertainty is reported in standard way. The continuous curve represent the simulation obtained by assuming Eq. 12.

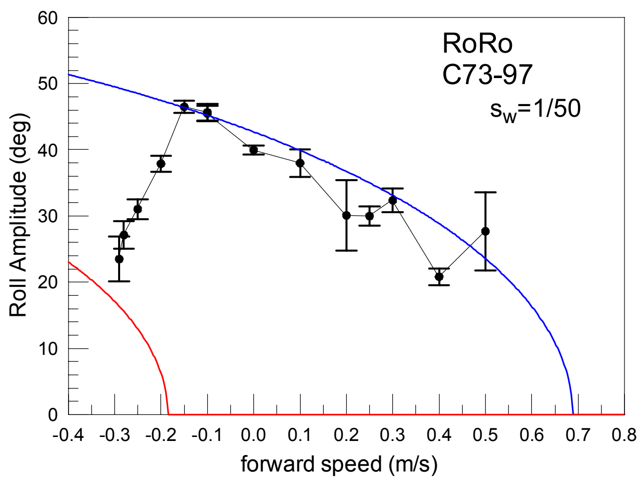

Fig. 11. Steady roll motion amplitude of parametric rolling as a function of ship forward speed (at model scale). The tests refer to λ_w/L =1 and s_w=1/50. The experimental uncertainty is reported in standard way. The continuous curve represent the simulation obtained by assuming Eq. 12.

The comparison is quite satisfactory and confirms that previously obtained analysing the threshold. On the contrary, the cubic term of righting arm is in qualitative agreement, i.e. the trend as a function of the wave steepness is correct, but the values "required" to fit reasonably the experiments are much greater in absolute value than the hydrostatic ones. This problem was encountered earlier [2]. Probably the "coupled" mathematical modelling would account for a greater excursion of the restoring curve as a whole.

Table 2. Estimated and computed (hydrostatic) values of the righting arm to fit experimental results.

s_w	1/200	1/100	1/50	1/30
α_3 (PIT)	-16.77	-17.38	-9.76	-7.23
p_1 (PIT)	0.097	0.287	0.875	1.064
p_1 hydrostatic	0.224	0.468	0.854	1.048
p_2 hydrostatic	0.224	0.468	0.854	1.048

2.3 (c) The role of damping and righting arm nonlinearities

The exceedance of the threshold (both in terms of parametric forcing and encounter frequency) entails the onset of parametric rolling in the presence of any perturbation of the upright position. The linearity of the system does not allow to individuate any upper bound to the oscillation amplitude [4,16] which is expected to grow to divergence or system failure. Only the presence of nonlinear terms in the damping and particularly in the restoring can account for the practically observed reaching of a maximum steady amplitude in the parametric rolling (provided, of course, that the vanishing stability angle or some other non-return condition is not exceeded). The role of the two sources of nonlinearity is of course different. The damping provides extra energy dissipation capability, while the restoring leads the system out of synchronism by increasing the roll motion amplitude, as is particularly evident in the reported results.

3. PARAMETRIC ROLLING IN IRREGULAR WAVES

A research project was undertaken aimed to a better understanding of the threshold and roll motion amplitude above threshold in the presence of parametric rolling generated by a train of irregular waves in longitudinal directions. The research is in progress [17]. We just mention here some aspects which worth further attention:

3.1 DOPPLER EFFECT

As generally observed, the waves spectrum undergoes an "hardening" in following sea, whereas it flattens in head sea conditions. This factor is sometimes overlooked since the spectral bandwidth:

$$sbw = \sqrt{\frac{m_0 \cdot m_2}{(m_1)^2} - 1} \qquad (13)$$

as a function of ship speed goes through a minimum value [18,19] at low speed in following waves, as shown in Table. 3 for a Bretschneider spectrum.

Table. 3. spectral bandwidth as a function of ship speed

	Head sea			Following sea		
U*cos(ψ) [kn]	-20	-10	0	5	7	10
m_0 [m^2]	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625
m_1 [m^2/s]	0.133	0.097	0.061	0.043	0.037	0.029
m_2 [m^2/s^2]	0.546	0.242	0.069	0.031	0.025	0.026
ω_z [rad/s]	2.956	1.968	1.051	0.704	0.632	0.645
T_z [s]	2.126	3.193	5.980	8.922	9.935	9.742
$\overline{\omega}$ [rad/s]	2.128	1.552	0.976	0.688	0.592	0.464
sbw	0.964	0.779	0.399	0.219	0.376	0.966

Since the parametric rolling, like all resonance phenomena, is tied to the presence of wave grouping, i.e. to a train of waves with characteristics very similar, the effective wave groupiness as a function of bandwidth should be carefully investigated.

3.2 THE MODELLING OF THE GM VARIATIONS

This is a very important point since the transfer function between wave spectrum and GM variations needs to be known for a range of ratios λ_w/L. We have tried to use the formulas proposed by Dunwoody [12], but the result, although qualitatively correct, is not completely satisfying. Of course, this is a problem more connected with the threshold exceeding than with the amplitude above threshold. In this last case, which is one of the goals of the ongoing research, also information concerning the nonlinear restoring are of great importance (§2.3c) to identify upper bounds and hence probability of exceedance above threshold (see also [19]).

3.3 THE MODELLING OF THE THRESHOLD

This problem was identified by several authors [20, 21, 17]. Different types of stability requirements can be imposed in the case of stochastic parametric excitation:

- stability of the mean and asymptotic stability of the mean;
- stability of mean square and asymptotic stability of mean square;
- almost sure asymptotic stability.

Of course, they lead to quite different estimates of the threshold.

3.4 THE EFFECTIVE LINEAR DAMPING

Starting from a linear differential equation with parametric forcing term described by a stochastic process *h(t)*:

$$\ddot{\phi}+2\cdot\mu\cdot\dot{\phi}+\omega_0^2\cdot(1+h(t))\cdot\phi=0 \tag{14}$$

and making the substitution:

$$\begin{cases}\phi(t)=A(t)e^{-\mu\cdot t}\cdot\cos(\omega_d\cdot t+\alpha(t))\\ \dot{\phi}(t)=-A(t)e^{-\mu\cdot t}\cdot\\ \qquad\cdot[\nu\cdot\omega_0\cdot\cos(\omega_d\cdot t+\alpha(t))+\omega_d\cdot\sin(\omega_d\cdot t+\alpha(t))]\\ \text{with}\\ \omega_d=\sqrt{\omega_0^2-\mu^2}\end{cases} \tag{15}$$

one can introduce an “effective” linear damping:

$$\mu_{eff}(t)=\mu-\frac{\omega_0^2}{2\omega_d}\cdot h(t)\cdot\sin(2\cdot(\omega_d\cdot t+\alpha(t))) \tag{16}$$

so that the instantaneous roll motion amplitude C(t) is expressed by:

$$\frac{\dot{C}}{C}=-\mu_{eff}(t) \tag{17}$$

The effective linear damping possesses a stochastic component

$$\mu_{eff}^{*}(t)=-\frac{\omega_0^2}{2\omega_d}\cdot h(t)\cdot\sin(2\cdot(\omega_d\cdot t+\alpha(t))) \tag{18}$$

which is to be subtracted to the hydrodynamic damping in the same way as it was done in the case of parametric roll in a regular sea by subtracting the characteristic exponent σ (§ 2.2 b). This proves again to be a very “effective” concept. The parametric roll in a stochastic environment was simulated in time domain to clarify the *local* (in time) departure from stability along a realisation. The following idealised spectrum was employed:

$$S_z(\omega)=\frac{2\cdot\gamma\cdot\left(\omega_m^2+\frac{\gamma^2}{2}\right)\cdot S_0}{\left(\omega_m^2+\frac{\gamma^2}{2}-\omega^2\right)^2+\omega^2\cdot\gamma^2} \tag{19}$$

as it allows simpler control of parameters, moments, bandwidth, etc., which are obtainable in analytical form. The time evolution of the roll motion amplitude (envelope) obtained with a GM fluctuation spectrum given by Eq. 19 with following parameters values is reported in Fig. 12:

$$\omega_m=2\cdot\omega_0$$
$$\omega_0=2.73\ rad/s$$
$$\nu=\mu/\omega_0=0.0165$$
$$sbw=0.1$$
$$\sigma_h=0.065$$

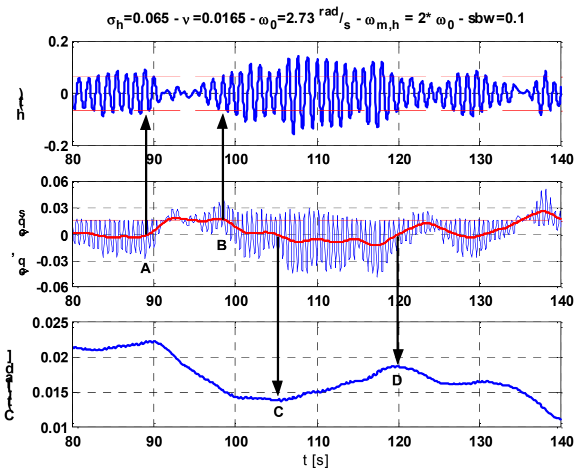

Fig. 12. Effect of the up-crossing of the "deterministic" threshold (Eq. 8) on the motion amplitude (envelope) highlighting the role of the slowly varying component of the effective damping (red line).

For comparison, the time history of the GM fluctuation and of the random part of effective damping are also reported. It appears that the effective damping is rapidly fluctuating in a quite irregular way. If we turn our attention to the slowly varying component of μ_{eff}, obtained by means of a low pass filter from the time history of damping, we can observe that only when the envelope amplitude of the GM fluctuation exceeds the deterministic threshold (Eq. 8), as for example at point B, the effective damping starts decreasing and at its zero down-crossing at time C the roll amplitude starts increasing. Viceversa at time D. In the time interval A-B, h(t) is very small. As a consequence the effective damping is high (close to the calm water value) and the perturbation to the upright position is rapidly quenched. It appears thus that Eq. 8 plays an important role in the case of stochastic parametric excitation too.

As regards the almost sure stability condition (in the large), this depends on the bandwidth of the process representing the GM fluctuations, which in turn is related to that of the wave spectrum and the forward speed of the ship. In the case of narrow banded h spectrum, the threshold for the variance of h(t) is approximately $\sqrt{2/\pi}$ times that given by Eq. 8.

4. CONCLUSIONS

The problems connected with the threshold formulation for parametric rolling and its amplitude modelling above threshold have been discussed in detail on an experimental basis. It appears that the most serious problem in simulating experimental results in regular waves consists in the accurate modelling of the nonlinear restoring terms. Both head and following waves have been considered, also in view of the greater attention to head sea conditions called during IMO/SLF discussion on the revision of the Intact Stability Code.

Passing to the case of parametric rolling in the presence of stochastic excitation, several problems have been identified as regards the mathematical modelling and the identification of an "appropriate" threshold. It is well known indeed that several different thresholds can be proposed in the case of irregular waves. The introduction of the so called "effective" damping was identified as a useful tool both for regular and irregular waves. It constitutes a bond between the "deterministic" threshold for parametric rolling and the local stochastic one.

Finally, the nonlinear modelling of roll motion variance above threshold is at present not properly addressed (with some notable exception like [19]).

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