Motion Perturbation Metric for Broaching-to

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ABSTRACT

The paper describes the formulation and calculation of a Motion Perturbation Metric for estimating the probability of broaching-to within the framework of the split-time method. The probability estimation procedure within the split-time framework is divided into two steps or problems. The non-rare problem is focused on statistically observable events and is intended to be solved with a set of relatively high-fidelity numerical simulations in random irregular seas. It is usually related to the statistical estimation of an upcrossing of an intermediate level. The rare problem is formulated for the time instant of upcrossing and is focused on the conditional probability of broaching-to when the upcrossing of the intermediate level has occurred. It is solved by evaluating an instantaneous metric of the likelihood of broaching-to that is extrapolated to the level of broaching-to using a Generalized Pareto Distribution. The motion perturbation method calculates the metric by perturbing the dynamical system toward a dangerous state in phase space. The dangerous state is defined as a set of initial conditions leading to broaching-to, defined here as a deviation from the commanded heading exceeding a given value. The distance in phase space towards the closest dangerous state is the value of metric at the given instant of time.

Keywords: *Broaching-to, Surf-riding, Split-time method, Motion Perturbation Method, MPM*

1. INTRDUCTION

The estimation of a probability of broaching-to in irregular waves from a limited set of high–fidelity numerical simulations has been one of the objectives of the long-term ONR (the US Office of Naval Research) project "A Probabilistic Procedure for Evaluating the Dynamic Stability and Capsizing of Naval Vessels." An overview of the general status and recent progress of the project can be found in Belenky, et al. (2016).

Broaching-to is a violent, uncontrollable turn which occurs despite maximum steering effort. It occurs in following and quartering seas and is, in general, infrequently encountered by a normally controlled ship. Broaching-to may occur in two different scenarios, the most frequent of which is the development of directional instability in yaw during surf-riding (Spyrou, 1996, 1997).

As broaching-to is a strongly nonlinear phenomenon, the split-time framework may be well-suited for its probabilistic characterization. The main idea of the split-time method is to separate the very complex problem of the probabilistic evaluation of rare events in a complex nonlinear dynamical system into two less complex problems. An intermediate threshold for one of the state variables is introduced. The value for the threshold is chosen such that the upcrossings can be observed at a statistically significant rate with high-fidelity time-domain numerical simulation. The rate of upcrossing can then be estimated from the time series – this is the "non-rare" problem. The second part of the split-time method is the "rare" problem, which is focused on calculating a "metric" value which quantifies the risk of the rare event at the instant of each upcrossing. The "metric" must include information on physics that goes beyond what was observed within the simulation. For example, surf-riding can co-exist with periodic surging, and even if only periodic surging was observed in the "non-rare" simulations, the metric should reveal that surf-riding was possible at this time instant for different initial conditions.

The numerical value of the metric is meant to express the "distance to failure" at the instant of upcrossing. Each upcrossing yields a single number, but as the upcrossings were observed in statistically significant quantities, the metric values may be fitted with a Generalized Pareto Distribution (GPD) to produce an extrapolated estimate for the probability of failure.

2. INITIAL DEFINITION OF METRIC

Belenky, et al. (2016) considered a metric for the likelihood of surf-riding that was defined as a distance between the current state and the state where ship would be captured into surf-riding, measured along the line between the current state and the stable surf-riding equilibrium (pseudo-equilibrium in case of excitation with more than one frequency). The practical implementation of this metric encountered difficulties due to the complexity of the phase space of surf-riding in the multi-frequency environment (Spyrou, et al. 2016).

At the same time, the deviation of heading due to broaching-to can be easily detected and measured from a relatively short numerical simulation, see Figure 1. The simulation uses a simplified 3-DOF (surge-sway-yaw) mathematical model that is described in Spyrou, et al. (2015).

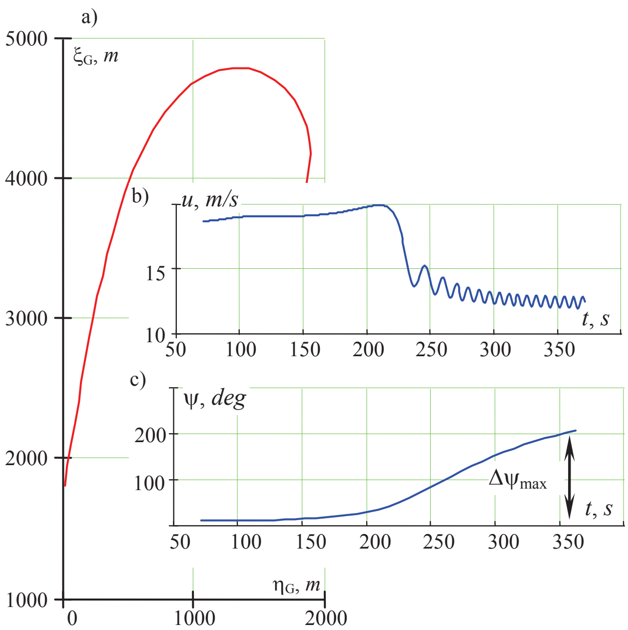

Figure 1 Broaching-to after surf-riding in regular waves: a) trajectory; b) time history of horizontal speed; c) time history of heading

Small deviations in heading, however, will be frequently encountered in oblique waves and do not represent any real danger as they can be easily corrected without adverse consequences. A minimum heading deviation corresponding to broaching-to is therefore defined, somewhat arbitrarily, to be 10 degrees. The initial formulation of the metric is then defined as a distance, in phase space, between the initial state and a critical state leading to a deviation of 10 degrees from the commanded heading, measured along the line between the initial state and a "dangerous" point. The dangerous point leads to broaching-to with a heading deviation which significantly exceeds 10 degrees. The definition of the dangerous point includes, but is not limited to, the stable surf-riding equilibrium/pseudo-equilibrium (Spyrou, et al. 2016; Belenky, et al. 2016a).

3. MOTION PERTURBATION METHOD

The idea of the motion perturbation method (MPM) is to look into alternative variants of the behavior of the dynamical system if the current state is perturbed. It is similar to the motion stability concept: the current state is given a perturbation and the perturbed solution is followed into the future. The difference is that the perturbation is meant to be large.

The perturbations are carried out in multi-dimensional phase space, starting from the vector of initial condition X_0 toward the "dangerous" vector (or point) X_d:

$$\vec{X}_S(\varepsilon) = \vec{X}_0 - \varepsilon \cdot \left(\vec{X}_d - \vec{X}_0\right); \quad \varepsilon \in [0; 1] \qquad (1)$$

A set of sample heading time histories from these perturbations is shown in Figure 2.

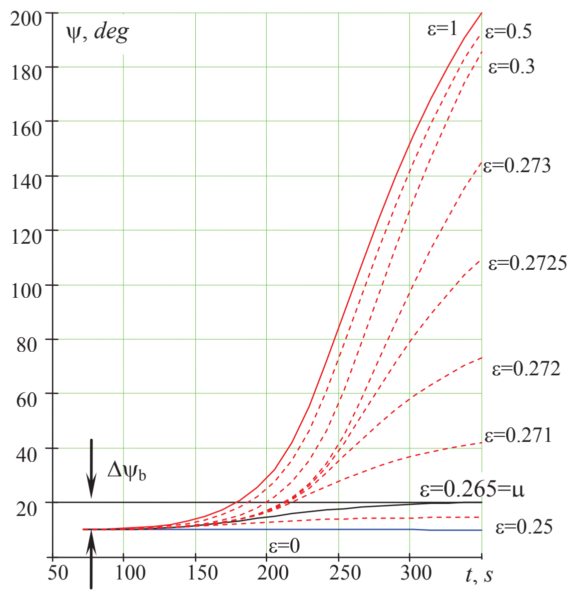

Figure 2 Heading time histories corresponding to perturbations in phase space, for the case of regular wave with a coexistence of periodic surging and surf-riding

The wave in this case is a regular wave for which both periodic surging and surf-riding can result for the same propeller rate. The heading time history which results in a maximum heading deviation of exactly 10 degrees yields the value of metric for the considered case.

4. FURTHER REFINEMENTS OF METRIC FORMULATION

The testing of the initial metric formulation is described in Belenky, et al. (2016). It includes surging/surf-riding coexistence mode in regular waves, bi-chromatic, tri-chromatic and full-band irregular waves. One conclusion was that the stable surf-riding pseudo-equilibrium is not necessarily the most dangerous point. The actual domain of broaching-to in full-band irregular waves may be shifted in comparison to the coexistence case in regular waves, see Figure 3.

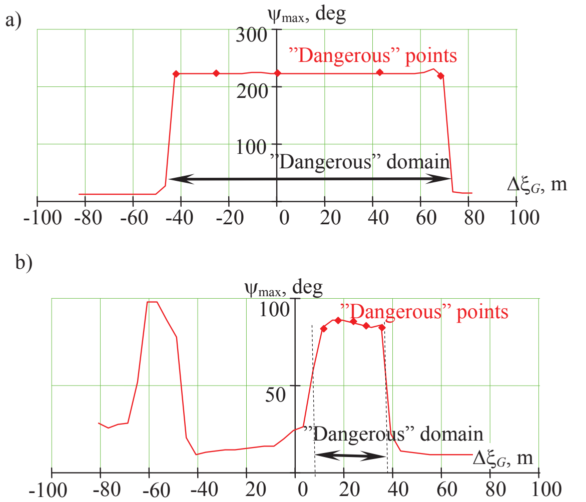

Figure 3 Maximum yaw angle as a function of the initial position of the wave relative to the position of the stable surf-riding equilibrium / pseudo-equilibrium (a) regular waves: surging / surf-riding coexistence mode (b) full-band irregular waves

As a result, an additional step has been added to the metric calculation procedure – a search for dangerous points. This information allows a refinement of the metric calculation. The value of metric actually determines a single point on the boundary of "dangerous broaching" domain in phase space. Several "dangerous" points yield several points on the boundary. The metric can therefore be reformulated as a distance to the boundary in a more strict geometric sense.

The metric also needs to be reformulated to be comparable between different upcrossings, because the critical value ε is defined in terms of relative distance.

Figure 4 shows a projection of the phase space for the coexistence case into the surging phase plane: the distance is measured in ship lengths and the surging speed is expressed in terms of Froude number. The "dangerous" domain is presented with five points. Each of them is used to get a direction for perturbations. Five values of ε corresponding to a heading deviation of 10 degrees have then been obtained.

Figure 4 shows the projection of these boundary points onto the surging phase plane. Three of these points (shown as solid circles) were used to fit the arc of a circle and find its center. It is no surprise that the line between the initial position and the center of the fitted circle comes from the stable surf-riding equilibrium.

The distance between the initial point and the fitted circle on the surge phase plane is measured on the line towards the center of the circle.

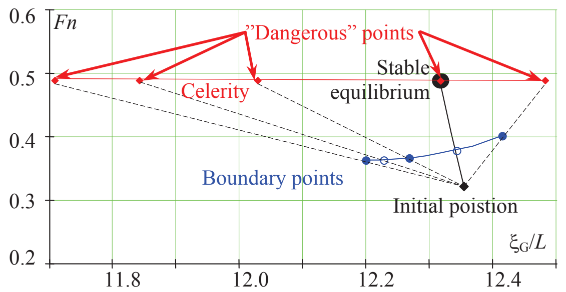

Figure 4 Projection of the phase space on the plane distance vs. surging speed: regular wave, surging / surf-riding coexistence mode

Figure 5 shows this projection for the case of full band irregular waves. This case is more complex. The line between the initial point and the center of the fitted circle does not cross the arc; as the dangerous domain is too narrow. The direction is defined then by the shortest distance shown with red line.

The updated calculation scheme of the metric assumes that the boundary of the "dangerous" domain is smooth. However, Spyrou, et al. (2016) shows that the boundary of the surf-riding domain in the bi-chromatic case can be fractal. These fractal boundaries present difficulties in getting a numerical solution efficiently, as most iteration methods may fail. The fractal boundary has to be approached from one side only and may require development of special computational techniques. However, the considered case seems to have a smooth boundary, as it can be seen from Figure 6.

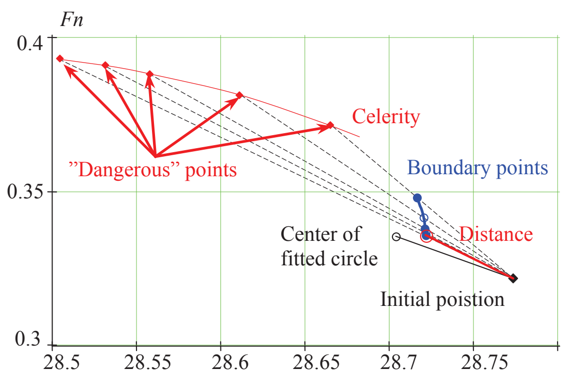

Figure 5 Projection of the phase space on the plane distance vs. surging speed: regular wave, full-band irregular waves

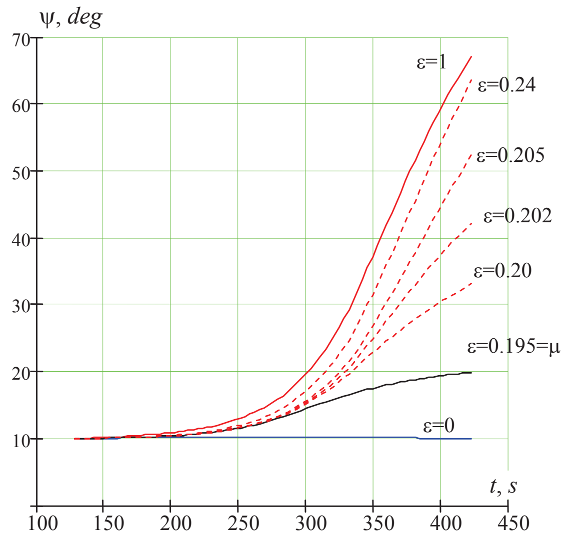

Figure 6 Heading time histories corresponding to perturbations in phase space: full-band irregular case

5. SUMMARY AND CONCLUSIONS

The paper describes the refinement of a MPM metric of likelihood of broaching-to, which is intended to be using within the split-time framework for evaluating a probability of broaching-to in irregular waves.

As the "dangerous" domain for broaching-to in irregular waves does not necessarily contain the stable surf-riding pseudo-equilibrium, a search for dangerous points needs to be carried out. These dangerous points are used to set the direction of MPM perturbations to find points on a broaching domain boundary. These boundary points are projected on the surging phase plane and fitted with a circle; the distance to the curve is the value of the MPM metric.

6. ASKNOWLEGEMENTS

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