

Correlations of GZ Curve Parameters

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ABSTRACT

Over the decades of the last few centuries the stability of ships has moved from the art of the shipbuilder and master to the realm of regulatory agencies. In that time several concepts for assessing stability have emerged, all rooted in the *GZ* curve; the curve that defines the relationship between the angle of heel and the moment arm of the righting couple that would return the ship to the angle of static equilibrium, which is usually 0° . Within each concept there are usually several parameters suggested as stability criteria including righting arms, areas under the curve and moments of areas under the curve. Criteria were developed out of expert knowledge and have been supported by good service, but the basis is not clearly documented. Many of these criteria have been observed to be correlated so as to fail to provide additional information or, conversely, to give a different perspective on the same information. This study looks at the correlations between the parameters in the standards used by many navies, including those based on the seminal work by Sarchin and Goldberg and those used by the German and Dutch navies (among others). The study looks not only within each set, but looks for correlations between the parameter sets as well. The intent is to gain insight into the parameters and the phenomena they represent, and to identify the optimal parameter set for regression against probabilistic results of simulations.

Keywords: *GZ curve, Correlation of Stability Indicators.*

1. INTRODUCTION

The Cooperative Research Navies (CRNav) Dynamic Stability Project has developed tools for assessing dynamic stability of intact ships. The Naval Stability Standards Working Group (NSSWG) has overseen the use of the tools to investigate the relationship between risk of capsizing and various geometry and stability parameters. The risk of capsizing was characterized by the probability of exceeding a critical roll angle (PECRA), although the “critical roll angle” could also take on a number of other important connotations, such as machinery or weapon limits.

The probability of exceeding a critical roll angle (PECRA) is determined by running multiple, time-domain simulations of a ship in a specific loading condition at a set speed and heading (the operating point of the vessel) in waves of a given significant height and modal period (the environmental condition). The time series of roll responses are used to determine the PECRA. The probability outcomes are later used as the regressands (response variables) in analysis

investigating relationships with parameters associated with ship stability.

A former paper [1] describes the study of how the PECRA vary with the input control variables of ship speed (V), ship heading relative to the wave system (β), significant wave height (H), and modal wave period (τ). The study looked into the variations between ships and between loading conditions, and investigated the issue of the range and resolution of the sets of input control variables that will fully characterize the total probability of exceeding a critical roll angle (TPECRA) across all input variables for each load condition of each ship.

The objective of the present study is to look at those *GZ* parameters that may be indicators of risk. While the PECRA in the former study are the regressands, the parameters in focus here are regressors. The set of regressors starts with a selection of parameters that form criteria in many naval standards, broadening the selection of parameters, essentially by using each of the parameters across all of the methods. The study then seeks to reduce the number of parameters to those that are not linearly correlated, and should, therefore, provide additional information. The goal

of the work is first to find the smallest set of parameters that can still represent the likely set of regressors, and second to identify the groups of parameters that are linearly correlated.

The next section will discuss the choice of parameters. Following that will be a brief description of how the data was validated prior to correlation analysis. The section after that will discuss the reduction of the parameter set based on the correlation analysis. Finally conclusions will be presented.

2. SELECTION OF PARAMETERS AS REGRESSORS

Although work is on-going to improve capabilities for assessing stability in real environments, many of the current criteria in both merchant and military standards are based on the *GZ* curve. In particular, many naval stability standards are based on work by Sarchin and Goldberg [2], and by Wendel [3] and influenced by the work of Rahola [4]. The principal tool has been the *GZ* curve, a locus of righting arms as the ship is inclined to various angles of heel. Various naval standards use very similar criteria but often have differences too. The seminal paper by Sarchin and Goldberg [2] formed the basis or greatly influenced the standards of the US and its allies, while the foundational work of Wendel [3] provided the basis for the German and Dutch naval standards (as well as other nations). The former work was based on US experience during World War 2, including the tragic (intact) loss of several vessels during a typhoon in 1944. It works with the Calm-Water (Still-Water) *GZ* Curve and heeling levers corresponding to winds of up to 100 knots. The latter work also applied the concept of balancing the ship on a wave.

A set of parameters were selected to represent the majority of those used to evaluate stability performance in the various naval standards.

Basic Parameters

Some of these parameters significantly pre-date Sarchin and Goldberg [2]. As such they have been applied by some naval organizations for a very significant period of time and are the framework upon which such standards as NES109 [5] were built (see Figure 1 and Table 1).

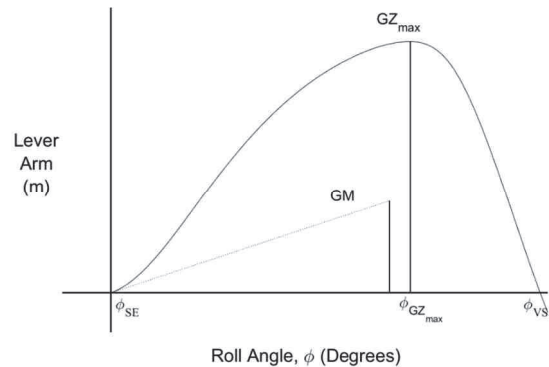


Figure 1: Basic Righting Arm Parameters - Fully Static Angles and Lever Arms.

Sarchin and Goldberg

Other measures were derived from an energy balance approach. These assess the relationship between the shape and area characteristics of the calm water righting curve against an assumed environmentally induced heeling curve. The energy balance assessment parameters selected are given in Figure 2 and Figure 3. These measures were proposed by Sarchin and Goldberg [2] and form the core of many of the current naval stability standards (e.g., [5][6][7][8]).

Table 1: Basic Righting Arm Parameters - Fully Static Angles and Lever Arms.

Parameter	Description	Source
GM	The metacentric height (fluid) for the ship at the given loading condition. Assessed for $n000$, $c000$, $t000$, and $s000$ only.	Bouguer
ϕ_{SE} (ϕ_{SE})	The angle of <u>Static Equilibrium</u> for the ship at the given loading condition, in a particular <i>balance state</i> . This angle is typically, but not necessarily, 0° for a ship with no heeling lever (e.g. wind). When a beam wind is applied, it is the angle at which the wind heeling lever arm curve first intersects the <i>balance state</i> GZ curve.	RN c. 1900 S & G [2]
ϕ_{VS} (ϕ_{VS})	The angle of <u>Vanishing Stability</u> for the ship at the given loading condition, in a particular <i>balance state</i> . When a beam wind is applied, it is still the angle of vanishing stability, but it may occur at the angle where the wind heeling lever arm curve intersects the <i>balance state</i> GZ curve a second time, if the intersection is above the $GZ = 0$ axis.	
RPS $RRPS$	Range of positive stability for the ship at the given loading condition, in a particular <i>balance state</i> . If there is no down-flooding or other influences, this will be $\phi_{VS} - \phi_{SE}$. The residual range of positive stability for the ship at the given loading condition, in a particular <i>balance state</i> , with a beam wind applied. (See also ϕ_{VS})	RN c. 1900 vH [10] BV [9]
ϕ_{GZmax} (ϕ_{GZmax})	The angle at which the maximum righting lever arm occurs for the ship at the given loading condition, in a particular <i>balance state</i> . The angle at which the maximum residual righting lever arm occurs for the ship at the given loading condition, in a particular <i>balance state</i> , with a beam wind applied. The residual righting lever is the righting lever remaining above the wind lever curve.	RN c. 1900
$GZmax$ ($GZmax$)	The maximum righting lever arm of the ship at the given loading condition, in a particular <i>balance state</i> . The maximum residual righting lever arm of the ship at the given loading condition, in a particular <i>balance state</i> , with a beam wind applied.	RN c. 1900 vH [10]
ϕ_{REF} (ϕ_{REF})	The reference angle for the ship at the given loading condition, in a particular <i>balance state</i> , with a beam: $\phi_{REF} = \begin{cases} 35^\circ & \text{if } \phi_{SE} \leq 15^\circ \\ 5^\circ + 2 \times \phi_{SE} & \text{otherwise} \end{cases}$	vH [10] BV [9]
$GZ\phi_{REF}$ (GZ'_{REF})	The residual righting lever arm at ϕ_{REF} for the ship at the given loading condition, in a particular <i>balance state</i> , with a beam wind.	BV [9]
A_{ratio}	The ratio of areas A_1 / A_2 for the ship at the given loading condition, in a particular <i>balance state</i> , with a beam wind. The area under the <i>balance state</i> GZ curve, above the $GZ = 0$ axis and the wind heeling lever arm curve, between ϕ_{SE} and ϕ_{VS} ($A_{\phi_{SE}-\phi_{VS}}$ assuming no down-flooding). The area above the <i>balance state</i> GZ curve, and under the wind heeling lever arm curve, between ϕ_{SE} and the <i>roll-back</i> angle, ϕ_{RB} , where the difference, $\phi_{SE} - \phi_{RB}$, is typically 25° .	S & G [2]

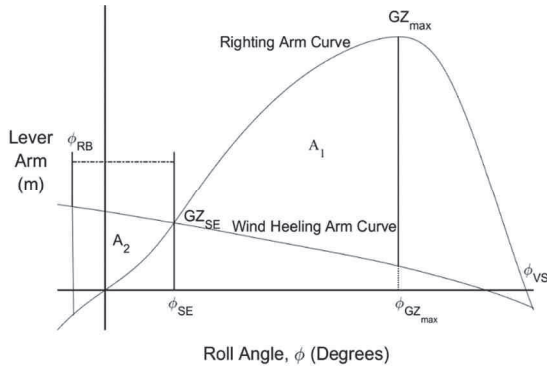


Figure 2: Illustration of the Sarchin and Goldberg [2] Criteria.

In the original Sarchin and Goldberg [2] criteria and therefore the US Navy standard, DDS 079 1 [5], these parameters are related to the application of a beam wind heeling arm as detailed in Table 2.

Wendel

A different approach is achieved by employing righting curves that have been determined with the vessel being balanced on a crest or in a trough of a wave of an assumed proportion to the vessel. Figure 3 and Table 1 illustrate the wave adjusted GZ assessment parameters selected from those embodied in van Harpen [10] (the RNLN navy standard) based on BV1030-1 [9], the German Federal Navy standard, which originates in the work of Wendel [3].

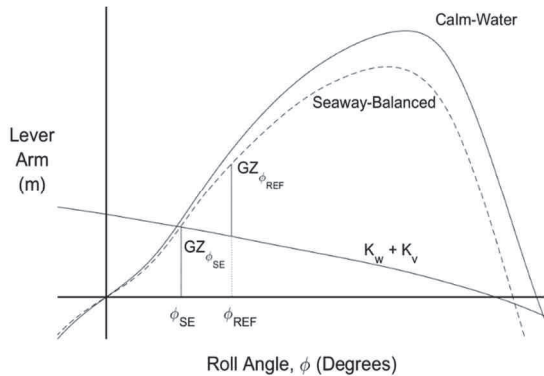


Figure 3: Illustration of the van Harpen [10] (Wendel, see [3]) Criteria.

These measures take the effect of waves on the transverse stability into account by calculating the righting arms with the vessel balanced on a sinusoidal wave of a height H (m) which is determined according to:

$$H = \frac{\lambda}{10 + 0.05\lambda} \quad (1)$$

where the wavelength, λ is set equivalent to the design waterline length of the vessel.

The wave-balanced GZ curves are determined for the cases where the vessel is balanced with the crest amidships and with the trough amidships and also for what is termed the seaway-balanced righting arm which is the mean of the former curves:

$$GZ_{seaway} = \frac{GZ_{trough} + GZ_{crest}}{2} \quad (2)$$

As part of the van Harpen criteria, an additional GZ parameter, the residual righting arm, GZ'_{REF} , is determined at a reference angle, ϕ_{REF} (see [10]).

As applied in van Harpen [10] and BV1030-1 [9], these measures are related to the application of a heeling arm that is a combination of the beam wind heeling and a free surface heeling arms, $K_w + K_v$, as detailed in Table 2. Note that the beam wind heeling arm, K_w , differs from that used for the Sarchin and Goldberg criteria, in that the former employs a $\cos^3(\cdot)$ relationship and the latter a $\cos^2(\cdot)$. Because the question of how to model the wind is not settled, for the sake of simplicity only the Sarchin and Goldberg beam heeling arm is considered in this investigation.

All standards suggest the use of various wind speeds for different vessels and operational environments. The full set of wind speeds examined herein is: 50, 60, 70, 80, 90, and 100 knots.

Form Parameters

In order to aid the subsequent analysis and allow some degree of discrimination between traditional and more modern hull forms a number of form parameters have also been selected for analysis. These are listed in Table 4.

3. EXPANSION OF PARAMETER SET

The parameters that are normally used only with a particular GZ curve and wind lever curve were extended for use with all four wave balance curves and all wind conditions, except for GM which was only evaluated for the curves without wind heeling levers applied.

Areas between major angles (see Table 3) were included in the parameter set. Note that the areas at higher angles do not attempt to account for down-flooding as this would make comparing results between ships more difficult. Also included is the determination of the 1st moment of area of the

righting arms again with, and without, the application of the various heeling arms.

Each parameter is prefixed by a code ($bwww$) which defines the wave balance and the wind speed used. The first letter designates the wave balance condition and the following three digits define the wind speed applied:

$b \in \{n, c, t, s\}$ corresponding to the balance state $\in \{\text{'calm-water' (no wave), 'crest-balanced', 'trough-balanced', 'seaway-balanced'}\}$

$www \in \{050, 060, 070, 080, 090, 100\}$ corresponding to the wind speed $\in \{50, 60, 70, 80, 90, 100\}$ knots.

MATLAB functions were used to investigate the calm water GZ curve and the wave adjusted curves with and without a wind lever applied. This results in 28 cases altogether for each loading condition of each ship (see Figure 4).

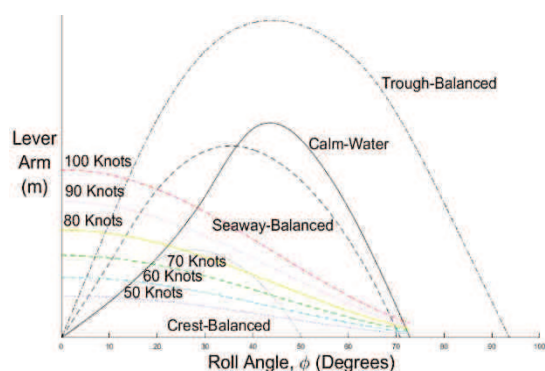


Figure 4: Range of Righting Arm and Wind Heeling Arm Curves.

4. SHIPS

Eight frigate-type ships were used in this study, with volume displacements from 2400 to 5060 cubic meters and GM values between 0.267 and 1.645. The ships were defined as watertight up to and including the weatherdeck. No account was taken of the presence of superstructure for buoyancy, but the lateral and frontal areas of the superstructure were used to calculate the wind heeling curves. All load conditions were at zero trim.

It is important to note that:

- Some of the loading conditions may not reflect practice as they were originally chosen to accomplish a study different from the current one.

- Most of the ships in this study were not designed against the wave-balance methodology.
- The methodologies – whether based on Sarchin and Goldberg or on Wendel – do not apply the wind speeds as indiscriminately as they are applied in this study.

Tqble 2: Heeling Terms for Energy Balance and Wave Adjusted Analysis.

Parameter	Definition	Origin	Naval Standard
l_w	The wind heeling arm $l_w = \frac{0.0195 V^2 A_w h \cos^2 \phi}{\Delta \times 1000}$ V = nominal wind speed (kts) A_w = lateral sail area (m ²) h = height of center of area above half draft (m) Δ = displacement (tonnes)	S & G [2]	DDS079 [5] CFTO [7] RAN [8] NES109 [6]
K_w	The wind heeling arm $K_w = \frac{p_w A_w h}{\Delta} \times (0.25 + 0.75 \cos^3 \phi)$ A_w = lateral sail area (m ²) h = height of center of area above half draft (m) Δ = displacement (tonnes) $p_w = C_w \frac{\rho_a}{2} V_a^2$ C_w = lateral windage coefficient (s ² ·m ⁻¹) ρ_a = air density (tonnes·m ⁻³) V_a = wind speed (m·s ⁻¹)	BV [9]	vH [10]
KL_v	he free surface heeling arm $K_v = \frac{\sum_{j=1}^n \rho_j i_j}{\Delta} \sin \phi$ ρ_j = density of contents of each slack tank (tonnes·m ⁻³) i_j = moment of inertia of each free surface (m ⁴) Δ = displacement (tonnes)	BV [9]	vH [10]

Tqble 3: Stability Assessment Parameters from GZ Curve – Areas under the GZ Curve.

$A_{\phi 1 \phi 2}$	The area under the <i>balance state</i> GZ curve between two specific roll angles. The residual area under the <i>balance state</i> GZ curve between two specific roll angles, above the $GZ = 0$ axis and the wind heeling lever arm curve.	
$M1x_{\phi 1 \phi 2}$	The 1 st moment (about the $GZ = 0$ axis) of the area under the <i>balance state</i> GZ curve between two specific roll angles. The 1 st moment (about the $GZ = 0$ axis) of the residual area under the <i>balance state</i> GZ curve between two specific roll angles, above the $GZ = 0$ axis and the wind heeling lever arm curve.	
$M1y_{\phi 1 \phi 2}$	The 1 st moment (about the $\phi = 0$ axis) of the area under the <i>balance state</i> GZ curve between two specific roll angles. The 1 st moment (about the $\phi = 0$ axis) of the residual area under the <i>balance state</i> GZ curve between two specific roll angles, above the $GZ = 0$ axis and the wind heeling lever arm curve.	
Case 1: $\phi 1 = \phi iSE$ Case 2: $\phi 1 = \phi iSE$ Case 3: $\phi 1 = \phi iGZmax$ Case 4: $\phi 1 = \phi iSE$ Case 5: $\phi 1 = \phi iREF$	$\phi 2 = \phi iVS$ $\phi 2 = \phi iGZmax$ $\phi 2 = \phi iVS$ $\phi 2 = \phi iREF$ $\phi 2 = \phi iVS$	CRN [1] (<i>calm water areas</i>) BV1030-1 [9] (<i>wave balance areas</i>)

Table 3: Form Assessment Parameters.

Parameter	Description
L	Length on waterline (m)
L_{aft}	Length on waterline aft of midship (m)
L_{fwd}	Length on waterline forward of midship (m)
B	Breadth on waterline (m)
T_{Mean}	Mean draft (m)
F_{Mean}	Mean freeboard (m)
A_{MS}	Midship area (m ²)
A_{WP}	Waterplane area (m ²)
A_{WPaft}	Waterplane area aft of midship (m ²)
A_{WPfwd}	Waterplane area forward of midship (m ²)
∇	Volume of displacement in loading condition (m ³)
∇_{aft}	Volume of displacement aft of midship (m ³)
∇_{fwd}	Volume of displacement forward of midship (m ³)
RoB	Reserve of Buoyancy (m ³)
VCB	Vertical Center of Buoyancy (m)
LCG	Longitudinal Center of Gravity (m)
KG	Vertical centre of gravity (fluid) (m)
A_{RR}	Relative rudder area (%)

5. GZ CURVE AND FORM PARAMETER DATA VALIDITY

As can be seen in Figure 4, sometimes the wind heeling curve passes over top of the righting arm curve. This happens mostly with the crest-balanced curve, but in a few instances with the seaway-balanced curve.

The MATLAB code used in this study will return “NaN” for the *GZ* parameters associated with these load conditions. When, for a given ship, the number of loading conditions with valid data drops to 2 the correlation function will also return “NaN”, avoiding the false linear correlation based on only 2 data points (linear by default).

In addition to checking for those cases where data is not available due to the wind curve exceeding the *GZ* curve, the values of the parameters as read/calculated from the *GZ* curves were checked to be sure that they were real numbers and that they varied with the load conditions; i.e., were not constant. Additionally, the robustness of the data was checked by counting how many of the ships had valid data. This was intended to give some confidence that the results are more widely applicable, at least within the set of frigate-like hull forms.

The data was confirmed to be valid over all 8 ships with 2 groups of exceptions. The first group includes all the *GZ* parameters for c080, c090, c100, and s100, which are each reduced by the number of load conditions where the wind curve exceeds the *GZ* curve as mentioned above. The second group is made up of the areas and moments of areas under the *GZ* curve associated with ϕ_{REF} at higher wind speeds and/or lower *GZ* curves; i.e., for n080, n090, n100, c050, c060, c070, c080, c090, c100, t070, t100, s080, and s090. The two groups overlap for c080, c090, and c100, but not for s100, or any *GZ* parameters not associated with ϕ_{REF} . The only other data that was valid for less than all 8 ships was the *IMO A1A2* ratio for the crest-balanced curve, which could be related to the low *GZ* curve.

6. REDUCTION OF PARAMETER SET

Within the large set of parameters several parameters are correlated. This would cause problems for the multi-parameter regression.

If the correlated parameters were grouped together, a single representative could be chosen for the regression analysis. The question becomes: Which parameter is the optimal representative of the group? Two options are immediately apparent. The first option is to “let the data decide”; the parameter that is most strongly correlated with the others is the best representative; this would seem to indicate it is in a sense “central” in the group. The second option is to choose a parameter based on additional, user-supplied requirements. For example, ease of calculation could be an additional criterion. Alternatively, the most physically meaningful parameter the selection condition. This suggests that there is a ranking of the parameters based on computational ease or other considerations, and that the ranking could be used to choose the “optimal” representative of the group. Analysis was performed with several ranking schemes, but the groupings based on linear correlation were quite consistent for all of them.

Correlation results

The valid data for the candidate parameters were checked for linear correlation using the built-in function in MATLAB. The correlation results were also filtered such that only correlation coefficients with a p-value less than 0.05 were kept. This means that there is less than 5% risk that the correlation coefficient is in error in predicting the linear correlation between the parameters.

Correlation analysis can be thought of as analogous to finding the relative projection of a vector on a plane, where the percent of the vector that falls in the plane is a function of the angle the vector makes out of the plane. Indeed, the correlation coefficient is analogous to the cosine squared of that angle. The cosine squared of 45° is 0.5 and represents a vector that is as much in-plane as out-of-plane. At 30° (0.75), the vector is more aligned with the plane, and at 15° (0.933), the vector is strongly aligned with the plane.

The correlation coefficients were evaluated to give a pass-fail matrix for each of the three thresholds. The sum of the matrices was taken across all 8 ships as a measure of robust correlation. The sums for each threshold were compared to investigate the strength of the correlations. The difference in the number of correlations exceeding 0.5 and the number exceeding 0.75 was only 0.25% of the total possible correlations, while the difference in the number exceeding 0.933 and the number exceeding 0.75 was 12.3% of the total possible (0.933 vs. 0.5 was 12.45%). It is clear that

most of the change in robustness occurs between the 0.75 and 0.933 thresholds, meaning that most of the linear correlations found are reasonably strong and robust across the ship set, and 87.7% of the correlations are very strong and robust.

Partitions

The correlation results for all three thresholds showed a clear partitioning of the parameters into groups as follows:

The relative rudder area, A_{RR} , is robustly correlated with the mean freeboard and the reserve of buoyancy in all wave balances and wind conditions. The relation between the latter 2 variables is understandable, as they are both measures of the hull form above the water. The link to the relative rudder area may be due to design “rules of thumb”. The consistency across wave and wind states is to be expected, since these parameters are associated with ship form and are independent of the environmental conditions for any given waterline.

In a similar manner, the other form parameters are robustly correlated; i.e., the vertical center of buoyancy with the mean draft, the midship cross-sectional area, the waterplane area as a whole and split into fore and aft areas, as well as the volume displacement as a whole and in fore and aft volumes. The after waterplane area can be less robustly correlated to the others at the highest threshold. All these measures are related to the immersed hull geometry, and all are independent of environmental conditions for a specified waterline.

The longitudinal center of gravity is correlated to itself across all wind speeds and wave balances, as expected. It is also correlated to the A_{RR} – freeboard – reserve of buoyancy group for half of the ships. One might have expected it to be more related to underwater form than above-water form.

The vertical center of gravity, KG , is correlated strongly with ϕ_{iSE} and ϕ_{iREF} up to the 0.75 threshold, but separates at the 0.933 threshold.

The areas and moments related to the GZ curve between ϕ_{iSE} and ϕ_{iREF} do not show robust correlations. This would indicate they should be independent regressors.

The remaining GZ parameters, $AIA2$, GM , ϕ_{iVS} , ϕ_{iGZmax} , RPS , $GZmax$, $GZ\phi_{iREF}$, and the areas and moments between ϕ_{iSE} and ϕ_{iVS} ,

ϕ_{iSE} and ϕ_{iGZmax} , ϕ_{iGZmax} and ϕ_{iVS} , and ϕ_{iREF} and ϕ_{iVS} , are correlated for all ships at some wind-wave states, and for fewer ships at others.

The groups above are independent of each other for most ships and wind-wave cases examined, and therefore represent a partitioning of the parameters into an above-water-geometry group that could be represented by the reserve of buoyancy or mean freeboard; a below-water-geometry group that could be represented by the mean draft; the LCG ; a small group of GZ parameters that are correlated to KG ; a larger set of GZ parameters that are correlated to $GZmax$, and, finally, a number of independent parameters that are either related to the area between ϕ_{iSE} and ϕ_{iREF} or are less robustly correlated to $GZmax$ at certain wave balances and wind speeds.

7. CONCLUSIONS

Very few of the parameters investigated resulted in invalid data. In only one case was the data unavailable over all load conditions. Only a few cases were found where the data was constant over the load conditions and therefore the parameters could not be used as regressors.

Form parameters were consistently partitioned into an above-water set and an underwater set. $GZmax$ and many other GZ parameters showed strong correlations robustly over the set of ships. Parameters associated with the REF angle from the German and Dutch standards showed mixed correlation results; i.e., not robust over the ship set for all wind-wave cases. They were, however, not always available for all wind-wave cases.

The following groups of regressors are suggested:

Independent of wave balance or wind speed:

- Mean freeboard – representative of the group including relative rudder area and reserve of buoyancy.
- Mean draft – representing the group containing VCB, AMS, AWP, AWPaft, AWPfwd, VolDisp, VolDispaft, and VolDisp fwd.
- KG .

Wind and wave influenced:

- GZmax – representing most of the other GZ parameters.

Independent regressors:

- Parameters associated with the REF angle from the German and Dutch standards. With these it is clear that the wave balance and wind speeds influence the data.

8. FUTURE WORK

Future work could include non-dimensional ratios of parameters.

Linearity in correlations can also be described as linearity in the coefficients; that is, the data itself could be acted upon by a function such as $\sin(x)$ or $\exp(x)$, or it could be raised to a power (e.g., x^2). These functions could be used to reduce the parameter set further if “linear” correlations can be found.

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