A BASIS FOR A DEVELOPING A RATIONAL ALTERNATIVE TO THE WEATHER CRITERION: PROBLEMS AND CAPABILITIES

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SUMMARY

The feasibility of developing a practical ship dynamic stability criterion based on nonlinear dynamical systems' theory is explored. The concept of "engineering integrity" and Melnikov's method are the pillars of the current effort which can provide a rational connection between the critical for capsize wind/wave environment, the damping and the restoring characteristics of a ship. Some discussion about the dynamical basis of the weather criterion is given first, before describing the basic theory of the new method. Fundamental studies are then carried out in order to ensure the validity, the potential and the practicality of the proposed approach for the stability assessment of ships on a wider scale.

1. INTRODUCTION

The weather criterion, adopted as Resolution A.562 by IMO's Assembly in 1985, was a leap beyond the "statistical approach" of the earlier Rahola-type general intact ship stability criteria of 1969 where the safety limits were based empirically on *GZ* characteristics of vessels lost even a 100 years ago (IMO 1995). Undoubtedly, the weather criterion has a rational basis in the sense that there is some account of ship roll dynamics integrated within the stability assessment process. Nonetheless, this analysis has a simplified character and, in the light of recent research advances on the one hand and new trends in design on the other, it seems that the time has come for looking more seriously into the potential of alternative approaches. Incidentally, at IMO we have, as of this year, the discussion about the weather criterion re-opened.

In the present paper we are reporting a few steps taken as part of an attempt to clarify whether the concept of "engineering integrity" of dynamical systems could be used for setting up an improved stability criterion suitable for general application. This corresponds basically to a "geometrical approach" which looks at global behaviour, thus going beyond the ordinary simulation, and aims to characterise system robustness to the critical environmental excitations. In practice this corresponds to determining the limiting wave slope or the significant wave height which, combined with the wind excitation can be sustained consistently during the critical transient stage of ship response to the oncoming waves (which may be of deterministic or stochastic type). In the last decade these ideas have found quite extensive application in research studies of ship stability (see for example the review article of Thompson 1997 and also Falzarano et al 1992); in most cases however they relied on generic forms of roll restoring and hence the question whether this approach can be used effectively for practical stability assessment remains still unanswered. Motivated by this fact we have recently embarked on an investigation aiming to determine the potential and the limitations of this novel approach. Some recent results of this effort are presented in the Sections that follow; firstly however we are setting the scene with a brief review of the weather criterion.

2. BASICS OF THE WEATHER CRITERION

The IMO version of the weather criterion follows closely Yamagata's "*Standard of stability adopted in Japan*" which was enacted as far back as 1957 (Yamagata 1959) while the basic idea had appeared in Watanabe (1938) or perhaps even earlier. The ship is assumed to obtain a stationary angle of heel θ_0 due to side wind loading represented by a lever lw_1 which is not dependent on the heel angle and is the result of a 26 m/s wind. "Around" this angle the ship is assumed to perform, due to side wave action, resonant rolling motion as a result of which it reaches on the weather side momentarily a maximum angle θ_1 (Fig. 1). As at this position the ship is most vulnerable in terms of excitations from the weather-side, it is further assumed that it is acted upon by a gust wind represented by a lever $lw_2 = 1.5 lw_1$. This is translated into a $\sqrt{1.5} = 1.2247$ increase of the wind velocity, assumed to affect the ship for a short period of time but at least equal with half natural period under the assumption that we are at resonance. The choice of wind corresponds to extreme storm situation. In fact, the criterion corresponds to a kind of average between the centre of a typhoon where the wind is very strong but the rolling is assumed that it is not so violent due to the highly irregular nature of the sea, and the ensuing situation which prevails right after the centre of the typhoon has moved away, where the wind velocity is lower but the rolling becomes more intense as the waves obtain a more regular form. As pointed out by Rachmanin in his discussion of Vassalos (1986) the criterion is based on a roll amplitude with 2% probability of exceedance which perhaps corresponds to Yamagata's choice of roll amplitude θ_1 as 70% of the

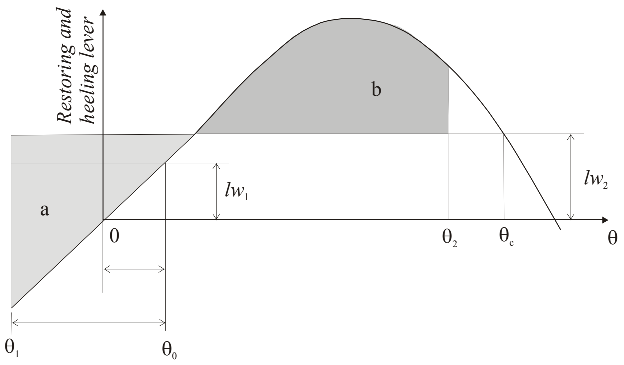

Fig. 1: The IMO weather criterion

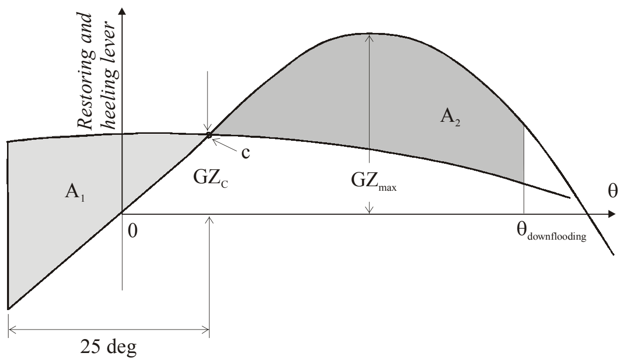

Fig. 3: The naval version of the weather criterion

resonant roll amplitude in regular waves. The requirement for stability is formulated as follows: should the ship roll freely from the off-equilibrium position θ_1 with zero angular velocity, the limiting angle θ_2 to the lee-side should not be exceeded during the ensuing half-cycle. This limiting angle is either the one where significant openings are down-flooded, the vanishing angle θ_v, or the angle of 50^0 which can be assumed as an explicit safety limit, whichever of the three is the lowest. Expressed as an energy balance, the work done by the wind excitation as the ship rolls from the wind-side to the lee-side should not exceed the potential energy at the limiting angle θ_2.

A number of extra comments pertain to the modelling of system dynamics:

Although the input of energy from the waves is taken into account for the calculation of the attained angle θ_1, it is not considered during the final half-cycle and the ship is like being released in still-water from the angle θ_1 (Fig. 2). The only energy balance performed concerns potential energies in the initial and final positions. An advantage of the criterion is that the damping is somehow present in the calculation of the resonant roll angle θ_1, both in terms of hull dimensions, form and fittings. Also, the fact that the energy dissipated through damping during the half-roll is not accounted is not so important, because it does not reduce seriously the maximum attained angle and in any case the result is conservative.

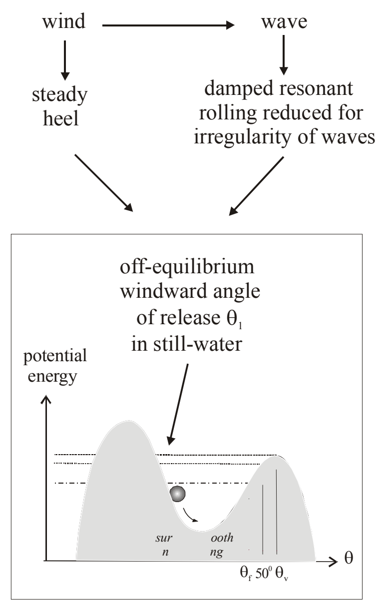

Fig. 2: The basic "structure" of the weather criterion.

However, as pointed out in the recent review of the weather criterion for RO/RO vessels by Francescutto et al. (2000), the damping function is taken as a pure quadratic which is today an unnecessary simplification. Finally, as in most critical cases the nonlinear part of the restoring curve "participates" in the dynamics, it appears unreasonable this effect to be left unaccounted in the calculation of θ_1 (the only nonlinearity considered concerns the damping function). Francescutto et al. (2000) offer also a critique about the selected values for the various parameters of the weather criterion, pointing out that many of these appear to be inappropriate for modern RO-RO ferries.

In 1962 Sarchin & Goldberg presented the "naval version" of the weather criterion which, whilst more stringent than the above, it adheres to the same principle with only few minor differences. This work is the basis for the stability standards applied from most western Navies nowadays, as evidenced by documents such as N.E.S 109 (2000) of the British Navy, DTS 079-1 of the U.S. Navy, NAV-04-A013 of the Italian etc. For ocean-going naval vessels the wind speed is assumed to be 90 knots and it is varying with the square of the heel angle (seemingly an improvement over the straight-line shown in Fig. 1, yet perhaps an equally crude approximation of reality, see the discussion of Prohaska in Yamagata 1959). The amplitude of resonant roll due to beam waves is prescribed to 25 degrees, which means that the important connection with the roll damping characteristics of the ship that is found in IMO's and Yamagata's criterion is lost. Here however is required that the equilibrium angle (point c in Fig.3) does not exceed 20 degrees and also that the GZ at that point is less than 60% of the maximum GZ. The energy requirement of the criterion prescribes that a substantial margin of potential energy should be available at the limiting angle position, in excess of the overturning energy. This is expressed through the well-known relationship $A_2 \geq 1.4 A_1$.

3. THE CONCEPT OF "ENGINEERING INTEGRITY"

Several articles have been published in the recent past about the concept of "loss of engineering integrity" of dynamical systems. The reader who is not acquainted with the basic theory and terminology is referred for example to Thompson (1997). The basic idea is that the safety robustness of an engineering system can be represented by the integrity of its "safe basin", comprised by the set of initial conditions (in our case pairs of roll angle and velocity) which lead to a bounded motion pattern. It is known that the basin is likely to undergo a serious reduction of area ("basin erosion") once some critical level of excitation is exceeded, given the inertial, damping and restoring characteristics of the ship. The reason for this is the complex intersection of "manifolds", i.e. those special surfaces which originate from, or end on, the unstable periodic orbits corresponding to the vanishing angles (it is reminded that the vanishing angles become time-dependent when there is wave-forcing). Detailed analysis about these phenomena can be found in various texts, see for example Guckenheimer & Holmes (1997). The initiation of basin erosion can be used as a rational criterion of system integrity which in our case and for a naval architectural context would be translated as sufficient dynamic stability in a global sense.

The methods that exist for predicting the beginning of basin erosion are the following:

a) Use of the so-called *Melnikov analysis* which provides a measure of the "closeness" of the critical pair of manifolds and hence can produce the condition under which this distance becomes zero. For capsize prediction the method is workable when the damping is relatively low (strictly speaking it is accurate for infinitesimal damping - however, several studies have shown that the prediction is satisfactory for the usual range of ship roll damping values). The method may be applied either fully analytically, or it may be combined with a numerical part when the required integrations cannot be performed analytically.

b) Direct numerical identification of the critical combination of excitation, damping and restoring where the manifolds begin to touch each other. This method is more accurate but obviously requires the use of some specialised software, thus making it less suitable for use in a legislative framework where it is customary the calculation procedure to be expressed in terms of mathematical formulae and Tables so that it can be fully integrated within the text of the standard.

c) Indirect identification of the critical condition by using repetitive safe basin plots until the initiation of basin erosion is shown. The same comment as in (b) applies here.

Melnikov's analysis has been repetitively used in the past in ship capsize research studies and for this reason we shall explore it here further: In the following Section we offer an outline of the method and then we apply it for a family of restoring curves that are parameterised with respect to the strength of the bias. On this basis we determine the critical wave slope for capsize as a function of the wind-induced heel and the damping. While this Section can serve as an example of how Melnikov's method could be applied for assessing the stability of ships subjected to a combination of wind and wave excitations, we should note that this is in fact part of a deeper study aiming to clarify whether the derived Melnikov formula can predict reliably basin erosion for arbitrary bias.

In the remaining Sections we are targeting the concept of engineering integrity itself, addressing the following two directions:

- The value of the concept for higher order restoring curves: Here the longer term objective is to find whether it is workable for arbitrary types of restoring. As a first step we are analysing the family of 5^{th} order restoring functions. In addition, we are applying the method for an existing ship thus providing an example of "real-life" stability assessment.

- The use of the new concept for design optimisation: We have taken as a basis a simple parameterised family of ship-like hull forms with main objective to determine whether this method can discriminate meaningfully between good and poor designs.

4. OUTLINE OF MELNIKOV'S METHOD

Assume the dynamical system

$$\dot{\mathbf{x}}(\tau) = \mathbf{f}(\mathbf{x}) + \varepsilon\, g(\mathbf{x},\tau) \tag{1}$$

where:

$\mathbf{x}$ is in our case the vector $[x, dx/d\tau]^{\mathrm{T}}$ of roll displacement, scaled with respect to the vanishing angle, and roll velocity,

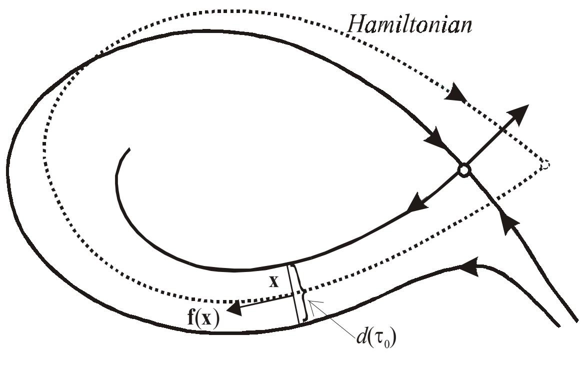

Fig. 4: The safe basin and the distance of manifolds.

$\mathbf{f}(\mathbf{x})$ represents the "unperturbed" (Hamiltonian) part of the dynamical system, and

$\mathbf{g}(\mathbf{x},\tau)$ is the "perturbation" which includes the damping and the explicitly time-dependent wave-forcing.

ε is parameter which represents the smallness of $\mathbf{g}(\mathbf{x},\tau)$

It can be shown that the closeness of manifolds is expressed as (Fig. 4):

$$d(\tau_0) = \frac{\varepsilon\, M(\tau_0)}{|\mathbf{f}(\mathbf{x})|} + O(\varepsilon^2) \tag{2}$$

where τ_0 is a phase angle in the range $0 < \tau_0 < 2\pi/\Omega$. Since the denominator is of the order of 1, the function $M(\tau_0)$ which is called *Melnikov function*, is to first order a good measure of the distance of manifolds:

$$M(\tau_0) = \int_{-\infty}^{+\infty} \mathbf{f}[\mathbf{x}(\tau)] \wedge \mathbf{g}[\mathbf{x}(\tau), \tau + \tau_0] d\tau \tag{3}$$

The wedge symbol $\wedge$ means to take the cross product of the vectors $\mathbf{f}$ and $\mathbf{g}$. The main intention when this method is applied, is to identify the critical combinations of design/operational parameters where the Melnikov function admits real zeros, which means that the manifolds begin to touch each other (it is essential the crossing of manifolds to be transversal but we do not discuss this here further for the sake of brevity).

5. MELNIKOV'S METHOD FOR SHIP ROLLING WITH A WIND-INDUCED BIAS

We have applied the method for the following family of restoring curves which have the bias as a free parameter:

$$R(x) = x(1-x)(1+ax) = x - (1-a)x^2 - ax^3 \tag{4}$$

The parameter a indicates the strength of the bias and in the present context it can be assumed to be due to beam wind loading. Letting $x = x_1$ and $\dot{x} = x_2$ the roll motion can be described by the pair:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 + (1-a)x_1^2 + ax_1^3 + [F \sin\Omega(\tau + \tau_0) - \beta\, x_2] \tag{5}$$

Assuming that the bias is relatively strong, the unperturbed part is represented by the vector:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} x_2 \\ -x_1 + (1-a)x_1^2 + ax_1^3 \end{bmatrix} \tag{6}$$

where

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{7}$$

while the perturbation is:

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} 0 \\ F\sin\Omega\tau - \beta\, x_2 \end{bmatrix} \tag{8}$$

After substitution into (3) the corresponding Melnikov function is given by the following integral:

$$M(\tau_0) = \int_{-\infty}^{+\infty} x_2 (F \sin\Omega(\tau + \tau_0) - \beta\, x_2) d\tau = M_E(\tau_0) - M_D \tag{9}$$

where $M_E(\tau_0)$ is the part due to weave forcing that is explicitly time-dependent and M_D is the part due to the damping.

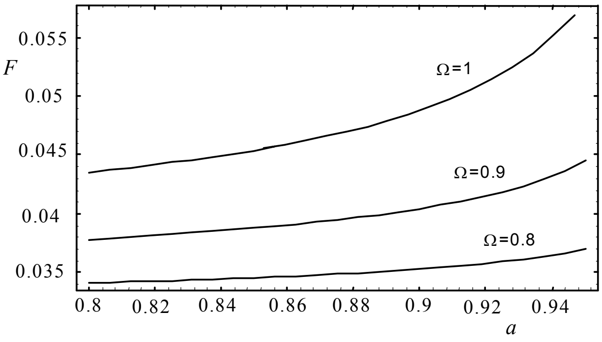

Fig. 5: Critical F as function of the bias. To be noted the initial "sharp" fall of the critical F as the bias a departs from 1 which corresponds to a symmetric system.

For the calculation of (9) we need to have available an explicit expression in terms of time of the roll velocity x_2 corresponding to the unperturbed system. This is determined as follows: The homoclinic orbit goes through the corresponding saddle point at x=1. With the coordinate change $s = x - 1$, the equation of the unperturbed system becomes:

$$\frac{d^2 s}{d\tau^2} = a s^3 + (2a+1)s^2 + (a+1)s \tag{10}$$

With some manipulation the above can be written as:

$$\frac{ds}{d\tau} = \pm h\, s \sqrt{(s+p)^2 - q^2} \tag{11}$$

The parameters that appear in (11) are defined as follows:

$$h = \sqrt{\frac{a}{2}}, \quad p = \frac{2(2a+1)}{3a} \text{ and } q = \frac{\sqrt{2(1-a)(2+a)}}{3a} \tag{12}$$

The solution of (11) is:

$$s(\tau) = x(\tau) - 1 = \mp \frac{\left(p^2 - q^2\right)}{p + q\cosh\sqrt{1+a}\ \tau} \quad (13)$$

Differentiation of (13) yields:

$$\frac{ds}{d\tau} = \frac{dx}{d\tau} = x_2 = \pm \frac{q\sqrt{1+a}\ \left(p^2 - q^2\right)\sinh\left(\sqrt{1+a}\,\tau\right)}{\left(p + q\cosh\sqrt{1+a}\,\tau\right)^2} \quad (14)$$

With substitution of (14) into (9) we can calculate the integral (9) with the method of residues. After some algebra, we obtain that the Melnikov function becomes zero when:

$$F = \beta \frac{h\left[\sqrt{p^2 - q^2}\left(p^2 + 2q^2\right) - 3pq^2 \operatorname{arccosh}\frac{p}{q}\right]}{6\pi\mu\sqrt{p^2 - q^2}} \frac{\sinh \mu\pi}{\sin\left(\mu \operatorname{arccosh}\frac{p}{q}\right)} \quad (15)$$

The sustainable wave slope is:

$$(Ak)_{crit.} = \frac{F\,\varphi_v}{\nu\,\Omega^2} \quad (16)$$

where φ_v is the vanishing angle and $\nu = \dfrac{I_x}{I_x + \delta I_x}$.

The critical wave slope as a function of the frequency ratio (at different values of the bias) that is obtained with application of the above method is shown in Fig. 5. It is very interesting to note that the Melnikov analysis produces an identical result with an energy balance, where the energy influx due to the forcing is equated with the energy dissipated through damping around the remotest orbit of bounded roll of the corresponding "unperturbed" system. In This formulation based on energy balance is intuitively quite appealing and for this reason it is discussed in more detail in Appendix I.

Another noteworthy development is that Melnikov's method has already been applied also for irregular wave excitation, by Hsieh et al. (1994). As a matter of fact, ship stability assessment from such a perspective is also viable and worth considering. For completeness, in Appendix II we outline the main issues involved in the formulation of Melnikov's method for a stochastic wave environment.

6. CONCEPT EVALUATION FOR REALISTIC GZ

Here our objective was twofold: firstly, we wanted to clarify whether the engineering integrity concept is meaningful when applied to higher-order, and thus more practical, GZ curves. Secondly, we endeavoured to apply the new method for an existing ship which satisfies the weather criterion. The complete analysis can be found in Papagiannopoulos (2001).

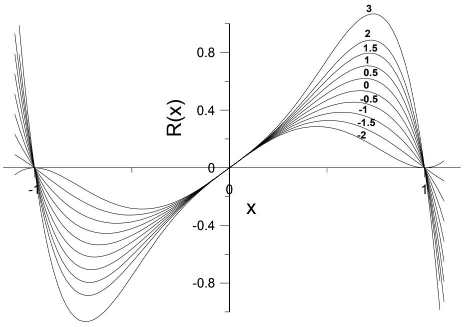

Fig. 6: Scaled "quintic" restoring curves

a) *Higher-order GZ*

In the first instance we considered the family of fifth-order restoring polynomials,

$$R(x) = x + cx^3 - (c+1)x^5 \quad (17)$$

Characteristic GZ shapes produced by (17) as c is varied are shown in Fig. 6. A feature of this family is that it covers hardening ($c > 0$) as well as softening ($c < 0$) type levers. It must be noted that the curves shown in Fig. 5

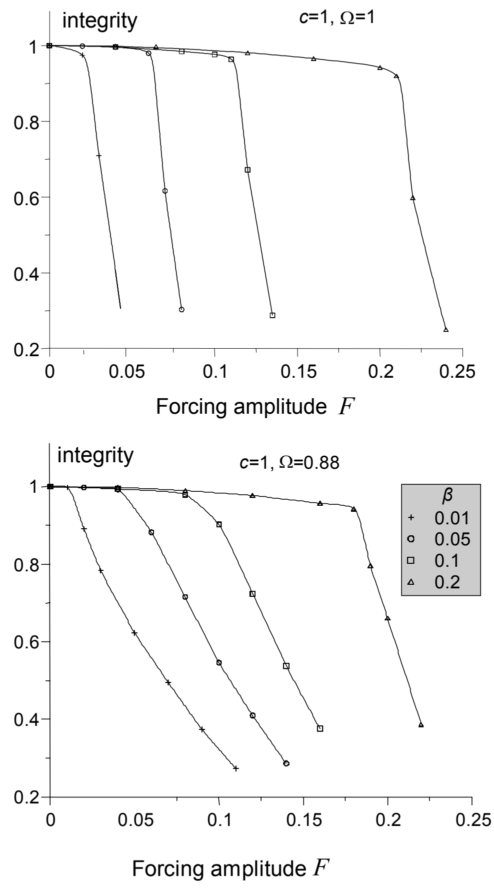

Fig. 7: Integrity diagrams

are scaled which means that there is no need the increase of c to lead to a higher true $GZ_{\max}$. The main purpose of the study was to confirm that there is indeed sudden reduction of basin area beyond some critical wave forcing for all the members of this family,i.e. that the concept is “robust” with respect to the type of the restoring function.

Although there was no problem in applying Melnikov’s method (this could be done even analytically, despite the very involved algebra), we have preferred to carry out basin plots using the software *Dynamics* in order to have a direct view of the process of basin erosion since our objective is the testing of the concept of engineering integrity itself.

In Fig. 7 are shown some typical results of this investigation for an initially hardening GZ at four different levels of damping. The points on the curves are identified with measurement of the basin area and scaling against the area of the corresponding unforced system. The curves confirm that for the fifth-order levers the concept of engineering integrity is applicable without any problem, i.e., there is indeed a quick fall of the curve beyond some critical forcing. This critical forcing should be approximately equal to the forcing determined from the Melnikov method. An advantage however of the direct basin plot is that it allows to determine fractional integrities also, like 90%, 80% etc., as function of damping and excitation (90% integrity means that the remaining basin area is 90% of the initial). This is quite important for criteria development since it provides the necessary flexibility for setting the boundary line; for example one could base the maximum sustainable wave slope at 90% integrity rather than at 100% which might be too stringent.

b) *Application for an existing ship*

We have selected a ferry which operated until recently in Greek waters, with $L_{BP} = 153$ m, B=22.8 m, T=6.4 m, C_b=0.548 and $KG = 10.004$ m. The GZ was known while the damping as a function of frequency was determined using the well-known method of Himeno (1981), while

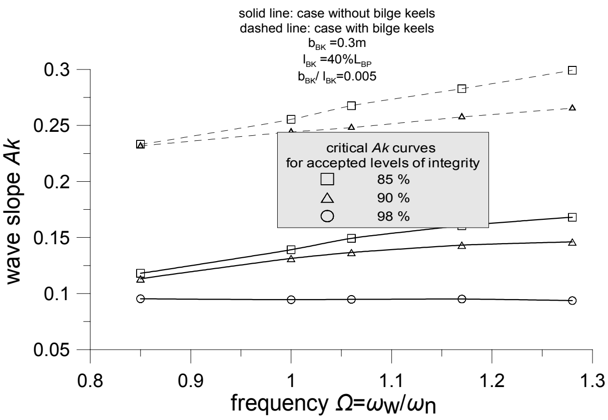

Fig. 8: Critical wave environment for an existing ship, on the basis of the new concept.

for the wave-making part we used the panel code *Newdrift* which is available at the Ship Design Laboratory at NTUA. In Fig. 8 is shown the sustainable wave slope for a range of frequencies around resonance on the basis of repetitive basin plotting. It is obvious that a diagram like this is a very valuable aide for designers as well as operators since it determines the range where survivability in a beam sea environment is ensured. This graph could be contrasted against Fig. 11 of Yamagata (1959) which gives the wave steepness considered in the weather criterion as function of natural period assuming resonance condition.

7. USE OF THE CONCEPT OF ENGINEERING INTEGRITY FOR DESIGN OPTIMISATION

The final study reported here is based on Sakkas (2001) and concerns application of the method for a family of simplified hull forms whose offsets are given by the following equation:

$$y = \pm f(\bar{x},\bar{z}) = \pm X(\bar{x}) \cdot Z(\bar{x},\bar{z}) \tag{18}$$

The same family was used recently by Min & Kang (1998) of Hyundai Heavy Industries for optimisation in terms of resistance for high-speed craft.

The shape of the waterlines is described by the function $X(\bar{x})$ which depends only on the nondimensionalised position $\bar{x} = \frac{x}{l}$ where x is the longitudinal position measured from the middle of the ship and l is the entrance length, with $-1 \le \bar{x} \le 1$. The function $X(\bar{x})$ is expressed as the 4th order polynomial,

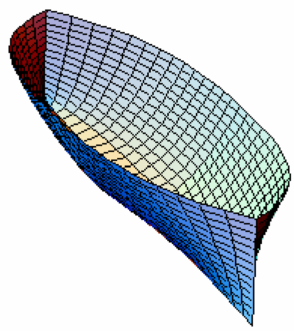

Fig. 9: Characteristic hull-shape of the considered family

$$X(\bar{x}) = \frac{B}{2}\left(1 + a_2\bar{x}^2 + a_3|\bar{x}|^3 + a_4\bar{x}^4\right) \tag{19}$$

The transverse hull sections depend on both longitudinal position and height. They are expressed by the simple function,

$$Z(\bar{x},\bar{z}) = (1+\bar{z})^{n(\bar{x})} \tag{20}$$

where

$$n(\bar{x}) = s + t \cdot |\bar{x}| \tag{21}$$

with s, t free parameters.

After some experimentation we found that for somehow realistic hull shapes the parameters a_2, a_3 and a_4 should take values between –1 and 1, while the parameters s and t should be somewhere between 0 and 0.4. As s and t increase the hull becomes more slender. A characteristic hull-shape can be seen in Fig. 9.

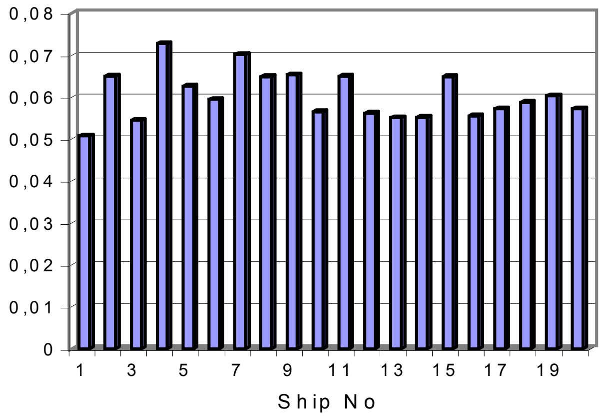

Fig. 10: Roll damping for each examined hull

Although originally the height z is non-dimensionalised with respect to the draught, in our case this is done with respect to the depth because the hull-shape above the design waterline is very important in stability calculations ($-1 \le \bar{z} \le 0$). To narrow further on the free parameters of this investigation we have fixed the length, the beam and the depth respectively to 150m, 27.2m and 13.5m. The displacement was also kept constant at 23,710 t which means that only the block coefficient and the draught were variable. In addition we have considered only hulls fore-aft symmetric. KG was linked to the depth while the windage area was set to be 4.1 L x T. The free parameters were then varied in a systematic way so that $a_2 + a_3 + a_4 = -1$ and $s = t$. In total 20 ships were collected for further stability investigation. For each one of these ships the corresponding GZ curve was determined by using the commercial programme *Autoship* while the damping was calculated as described in the previous Section. The roll radius of gyration was calculated with a well-established empirical formula. The nondimensional damping value β at roll resonance for each ship is shown in Fig. 10.

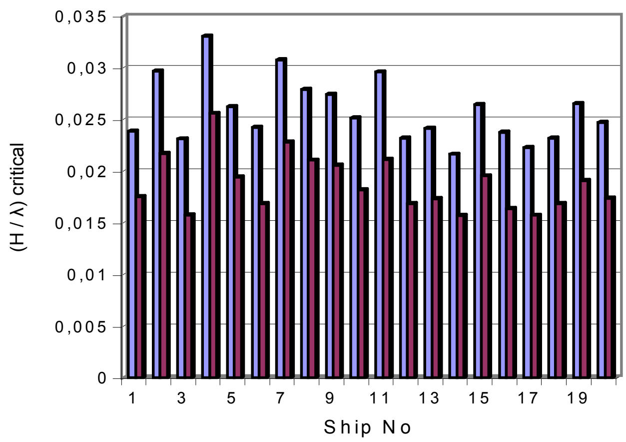

Fig. 11: Dynamic stability performance: Left bars are for waves only; while the right bars are for wind and waves.

For all these simplified ships we applied the weather criterion and we verified that it is fulfilled. Thereafter, knowing the restoring, damping and inertial characteristics for each hull, we run the programme *Dynamics* of Nusse & Yorke (1998) in order to determine the critical wave slope Ak at which basin erosion is initiated. The wind loading was calculated as prescribed in the weather criterion.

The performance of each ship as determined from this procedure is shown in the bar chart of Fig. 11. In the same figure is shown the stability performance of the ships when they were subjected to beam wind loading of the same type and intensity as the considered in the IMO weather criterion. The best hull determined is shown in Fig. 12.

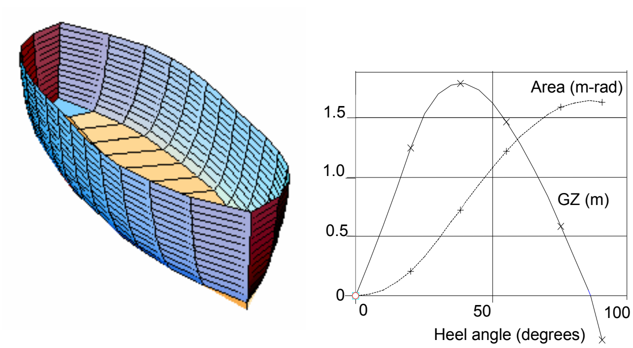

Fig. 12: Best hull and the corresponding restoring curve

8. CONCLUDING REMARKS

We have presented an investigation on the concept of engineering integrity and whether it can be used for developing a practical ship stability criterion. In the spirit of the weather criterion we have confined ourselves to a study of system robustness to combined wind and wave excitations coming from abeam and under the assumption that there are no phenomena such as wave breaking on the ship side, or accumulation of water on the deck. Also, we have left out of the present context any discussion about stability in following seas, a matter which deserves a separate approach due to the different nature of the dynamics involved. We ought to stress out here that although the following-sea is the most dangerous environment of ship operation, it is not seriously addressed in the IMO Regulations, beyond a non-ship-specific operational recommendation (IMO, 1995) and a wording in Res. A.749 asking Administrations to pay attention to this problem without telling them how. As a matter of fact, ships are designed with no requirement for checking their stability at the most critical condition that they may encounter, although the primary modes of ship capsize in a following sea environment are now well understood.

Of course such a matter is not raised for the first time and in the academic world several approaches have been discussed in the past: some are based on the area under the time-varying GZ curve (butterfly diagram, see e.g. Vassalos 1986) while more recently there is a trent for taking into account "more fully" system dynamics. For parametric instability already exist general criteria that have been developed in the field of mechanics. However it is doubted whether these are known to ship designers (for an attempt to summarize these criteria see for example Spyrou 2000). Furthermore, the clarification of the dynamics of broaching has opened up new possibilities for the development of a simple criterion for avoiding the occurrence of this phenomenon too.

It is emphasized that the proposed methodology is a platform for developing also formulations targeting the following-sea environment. Furthermore, since this approach is a global method and is not based on ordinary simulation, it presents also distinctive advantages over the so-called "performance-based methods", a term which, in an IMO-style vocabulary is assumed to mean the running of a limited number of experimental or simulation runs.

Our analysis up to this stage shows that although the proposed method can produce meaningful results there are a few practical problems whose solution is uncertain and would require further research. A deeper look into the Melnikov-based approach is presented in a recently published companion paper (Spyrou et al 2002). Some questions that immediately come to mind are:

- How to deal with down-flooding angles, or more generally limits of stability that are lower than the vanishing angle? The problem arises from the fact that the method targets the phenomena affecting the basin boundary while the amplitude of rolling may be well below that level if there is flooding of non-watertight compartments. The formulation of Melnikov's method as an energy balance might give the idea for a practical solution.

- How to combine the deterministic and the stochastic analysis? This happened quite meaningfully in the weather criterion and some comparable, yet not obvious in terms of formulation, approach is required.

- How to adjust the level of stringency of the criterion? This could be achieved perhaps by setting the acceptable level of integrity to a fractional value, such as 90%, rather than the 100% that is currently considered in research studies.

On the other hand, the method seems that it can easily fulfil the prerequisites of a successful stability criterion which in our own opinion are the following:

- It should have an unambiguous direction of stability improvement, i.e. should be formulated in such a way that the designer can maximise the stability margin of a ship under consideration (not in isolation of course, but in parallel with other performance and safety matters) and not simply check for conformance to limiting values.

- The criterion should be representative of stability in a global sense, i.e. should not be based on a prescribed, and narrowly defined condition where stability should be ensured.

- It should always make clear to the designer the connection with the limiting environmental conditions.

- The method should take into account the transient nature of the ship capsize process and it should account with sufficient accuracy for system dynamics,

- It should be flexible in order to accommodate changes in design trends, i.e. should not be overly dependent on existing ship characteristics which means that the degree "empiricism" intrinsic to the method should be minimal.

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APPENDIX I: Melnikov's method as energy balance

Consider again the roll equation with bias:

$$\ddot{x} + \beta\,\dot{x} + x(1-x)(1+ax) = F\cos\Omega\tau \qquad \text{(I1)}$$

The corresponding Hamiltonian system is:

$$\ddot{x} + x(1-x)(1+ax) = 0 \qquad \text{(I2)}$$

with kinetic energy

$$\text{KE}=\frac{1}{2}\dot{x}^2 \qquad \text{(I3)}$$

and potential energy,

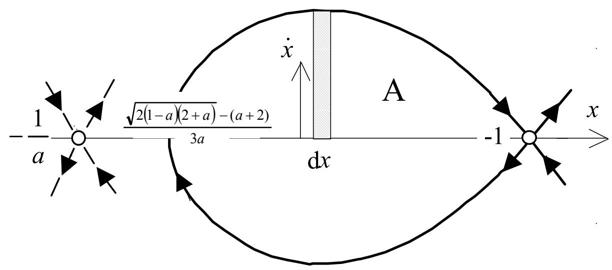

Fig. I1: Energy balance around the homoclinic orbit.

$$\text{PE}=\int x(1-x)(1+ax) = c + \frac{1}{2}x^2 - \frac{(1-a)}{3}x^3 - \frac{a}{4}x^4 \qquad \text{(I4)}$$

It is obvious that if we take as initial point the saddle (x=1, $\dot{x}=0$) and as final the same point after going over the homoclinic orbit ("saddle loop"), the difference in potential energy will be zero. Zero will be also the difference in kinetic energy (at the saddle point the roll velocity is obviously zero).

The energy lost due to the damping "around" the homoclinic orbit is represented by the area A inside the loop times the damping coefficient β which is expressed as (Fig. I1):

$$\text{DE}=\oint \beta\,\dot{x}\,dx = \beta \oint \dot{x}^2\,d\tau \qquad \text{(I5)}$$

The input of energy on the other hand due to the wave excitation is the integral (in terms of the roll angle) of the external roll moment taken around the homoclinic orbit. Mathermatically this is expressed as,

$$\text{WE}=\oint F\sin\Omega(\tau+\tau_0)\,dx = \oint F\sin\Omega(\tau+\tau_0)\,\dot{x}\,d\tau \qquad \text{(I6)}$$

The time constant τ_0 allows to vary the phase between forcing and velocity $\dot{x}$.

The balance of energies requires therefore that, i.e.

$$\mathrm{DE} = \mathrm{WE} \rightarrow \beta \int_{-\infty}^{+\infty} \dot{x}^2 \, d\tau = F \int_{-\infty}^{+\infty} \cos \Omega(\tau + \tau_0) \; \dot{x} \; d\tau \quad \text{(I7)}$$

which is identical with (9) when set equal to zero, i.e. we have obtained a condition identical to the condition $M(\tau_0) = 0$ described by (3).

Despite the clarity of the above viewpoint, a note of caution is essential: The above energy balance "works" also for other nonlinear global bifurcation phenomena, (e.g. a saddle connection) and application should be attempted only when it is already ensured that the underlying phenomenon is an incipient transverse intersection of manifolds which indeed generates basin erosion.

APPENDIX II: Formulation for irregular seas

In the "stochastic" version of Melnikov's method the objective is to determine the time-average of the rate of phase-space flux that leaves the safe basin. The basic theory can be found in Wiggins (1990) and Frey & Simiu (1993). However the specific formulation and adaptation for the ship capsize problem is owed to Hsieh et al (1994). The key idea is that the probability of capsize in a certain wave environment is linked to the rate of phase-flux. The formulation is laid out as follows:

Let's consider once more the roll equation, this time however with a stochastic wave forcing $F(\tau)$ at the right-hand-side. It is noted that the equation should be written in relation to the absolute roll angle because the presentation based on angle relatively to the wave in this case is not practical. It can be shown that the flux rate, i.e. the rate at which the dynamical system loses safe basin area, scaled by the area A of the basin of the unperturbed system, is given by the following expression (Hsieh et al 1994):

$$\frac{\Phi}{A} = \frac{\varepsilon}{A}\left[H_s \, \sigma \; p\left(\frac{M_D}{H_s \, \sigma}\right) + M_D \; P\left(\frac{M_D}{H_s \, \sigma}\right) - M_D \right] + O\left(\varepsilon^2\right) \quad \text{(II1)}$$

where, H_s is the significant wave height, σ is the RMS value of the Melnikov function $M(\tau) = M_E(\tau_0) - M_D$ for unity H_s, and M_D is the damping part of the Melnikov function which is commonly taken as non explicitly time-dependent. For wave forcing with zero mean, the mean value of the wave part M_E of the Melnikov function is zero, thus the mean $E[M(\tau_0)]$ is $-M_D$. Also, *p*, *P* are respectively the standard Gaussian probability density and distribution functions.

The assumption can be made, supported however by simulation studies (Hsieh et al 1994, Jiang et al. 2000), that the flux rate reflects reliably the capsize probability. Conceptually this appears as a direct extension for a probabilistic environment of the connection between basin area and capsizability of a ship under regular wave excitation. The "behaviour" of equation (II1) is that the flux rate increases quickly once some critical significant wave height is exceeded, gradually approaching an asymptote as H_s tends to infinity. Therefore, someone could set a desirable environment of ship operation in terms of a certain wave spectrum, significant wave height etc., and then through proper selection of the damping and restoring characteristics he could try ensure that his design survives in this environment. This is translated in keeping the flux rate as determined by (A1) below a certain limit. Hsieh et al (1994) have suggested to use the point where the asymptote intersects the H_s axis as the critical one. The various quantities that appear in (II1) are calculated as follows:

For the damping term M_D of the Melnikov function is entailed to determine the integral of some power of the time expression of roll velocity at the homoclinic orbit. In some cases it is possible this to be done analytically. For example, for the system described by equations (5) it becomes after calculation:

$$M_D = \frac{1}{3} \beta \, h \left[\sqrt{p^2 - q^2} \, (p^2 + 2q^2) - 3pq^2 \operatorname{arccosh} \frac{q}{p} \right] \quad \text{(II2)}$$

where *h, p, q* are function of the bias parameter *a* and they are defined in Section 5.

σ connects with the spectrum of wave elevation as follows:

$$\sigma^2 = E\left[M_E^2(\tau_0)\right] = \int_0^{+\infty} S_M^+(\Omega) \, d\Omega = 2\pi \int_0^{+\infty} S_{\dot{x}}(\Omega) S_F^+(\Omega) \, d\Omega \quad \text{(II3)}$$

The spectral density function of the wave forcing is linked to the wave spectrum $S_F^+(\Omega) = \left|F(\Omega)\right|^2 S^+(\Omega)$ (this is perhaps the main weakness of the method as this relationship is valid only for a linear process).

The final quantity that needs to be calculated is the power spectrum of the scaled roll velocity

$$S_{\dot{x}}(\Omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \dot{x} \, e^{-i\Omega\tau} \, d\tau \quad \text{(II4)}$$

For the homoclinic loop shown in Fig. I1 this is easily calculated numerically although it can be proven that it receives also an exact analytical solution in terms of hypergeometric functions.

With the above, we can plot the significant wave height versus, for example, the characteristic wave period which gives a straightforward platform for setting a level of acceptability (e.g. survival up to a certain H_s).