Influence of Surge Motion on the Probability of Parametric Roll in a Stationary Sea State

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ABSTRACT

A typical parametric roll scenario for a ship in head waves implies that the roll motion is coupled with vertical motion of the vessel. The added resistance of the ship is increased when the bow pitches down in a wave crest. As a consequence, the ship speed is slowed down and, hence, the roll resonance condition might be changed. In an attempt to study the influence of this speed variation in waves on parametric roll, the procedure for estimation of probability of parametric roll by Jensen and Pedersen (2006) has been extended to account for the surge motion of the vessel.

KEYWORDS

Parametric roll, FORM, mean out-crossing rate

INTRODUCTION

The roll motion of ships can lead to various types of failures ranging from seasickness over cargo shift and loss of containers to capsize of the vessel. Hence, it is important to minimise the roll motion during a voyage. A special problem is parametric roll as it constitutes a bifurcation problem making it difficult for the ship master to predict the onset of parametric roll as well as to devise procedures to eliminate it if it happens. Large roll angles typically built up over 10 to 20 wave periods. Examples from model test are shown in France et al. (2003).

Linear models based on the Mathieu equation clearly show that parametric roll is a bifurcation problem govern by a sinusoidal variation in the restoring moment during a regular wave passage and that the roll period should be about twice the encounter period to initiate parametric roll.

In real sea states the problem is less obvious. The wave profile along the length of the vessel is now described by random waves, reflecting the wave spectrum of sea. Thereby, also the restoring moment changes randomly. Furthermore, the resonance condition between the roll and encounter periods cannot be satisfied at any point in time.

In the present study the focus is on the resonance condition and, as the encounter period depends on the forward speed, the instantaneous surge velocity is estimated and used in the determination of the encounter period. Thus the previously applied time domain sea keeping model in Jensen and Pedersen (2006) is extended to include the surge mode. Calculations are then carried out for same container vessel in order to investigate the role of this surge motion for the probability of experiencing parametric roll.

The First-Order Reliability Method (FORM) is used to get the probability distributions as it has been shown, Jensen and Pedersen (2006) and Jensen (2007), to be an accurate and efficient procedure for estimation of the extreme value statistics of even very non-linear responses like parametric rolling.

FIRST-ORDER RELIABILITY METHOD APPLIED TO WAVE LOADS

Design point and reliability index

In the First-Order Reliability Method (FORM), the excitation or input process is a stationary stochastic process. Considering in general wave loads on marine structures, the input process is the wave elevation and the associated wave kinematics. For moderate sea states the wave elevation can be considered as Gaussian distributed, whereas for severer wave conditions corrections for non-linearities must be incorporated. Such corrections are discussed and accounted for by using a second-order wave theory in a FORM analysis of a jack-up platform (Jensen and Capul, 2006). In the present paper dealing with the roll motion of a ship, linear, long-crested waves are assumed and hence the normal distributed wave elevation $H(X,t)$ as a function of space X and time t can be written

$$H(X,t)=\sum_{i=1}^{n}\left(u_i c_i(X,t)+\bar{u}_i\bar{c}_i(X,t)\right) \qquad (1.1)$$

where the variables $u_i, \bar{u}_i$ are uncorrelated, standard normal distributed variables to be determined by the stochastic procedure and with the deterministic coefficients given by

$$\begin{aligned} c_i(x,t) &= \sigma_i \cos(\omega_i t - k_i X) \\ \bar{c}_i(x,t) &= -\sigma_i \sin(\omega_i t - k_i X) \\ \sigma_i^2 &= S(\omega_i)d\omega_i \end{aligned} \qquad (1.2)$$

where $\omega_i, k_i=\omega_i^2/g$ are the n discrete frequencies and wave numbers applied. Furthermore, $S(\omega)$ is the wave spectrum and $d\omega_i$ the increment between the discrete frequencies. It is easily seen that the expected value $E[H^2] = \int S(\omega)d\omega$, thus the wave energy in the stationary sea is preserved. Short-crested waves could be incorporated, if needed, but require more unknown variables $u_i, \bar{u}_i$.

From the wave elevation, Eqs. (1.1) - (1.2), and the associated wave kinematics, any non-linear wave-induced response $\phi(t)$ of a marine structure can in principle be determined by a time domain analysis using a proper hydrodynamic model:

$$\phi = \phi(t \,|\, u_1, \bar{u}_1, u_2, \bar{u}_2, ..., u_n, \bar{u}_n, \text{ initial conditions}) \qquad (1.3)$$

Each of these realisations represents the response for a possible wave scenario. The realisation which exceeds a given threshold ϕ_0 at time $t=t_0$ with the highest probability is sought. This problem can be formulated as a limit state problem, well-known within time-invariant reliability theory (Der Kiureghian, 2000):

$$\begin{aligned} & g(u_1, \bar{u}_1, u_2, \bar{u}_2, ..., u_n, \bar{u}_n) \equiv \\ & \phi_0 - \phi(t_0 \,|\, u_1, \bar{u}_1, u_2, \bar{u}_2, ..., u_n, \bar{u}_n) = 0 \end{aligned} \qquad (1.4)$$

The integration in Eq. (1.4) must cover a sufficient time period $\{0,\ t_0\}$ to avoid any influence on $\phi(t_0)$ of the initial conditions at $t=0$, i.e. to be longer than the memory in the system. Proper values of t_0 would usually be 1-3 minutes, depending on the damping in the system. Hence, to avoid repetition in the wave system and for accurate representation of typical wave spectra n = 15-50 would be needed.

An approximate solution can be obtained by use of the First-Order Reliability Method (FORM). The limit state surface g is given in terms of the uncorrelated standard normal distributed variables $\{u_i, \bar{u}_i\}$, and hence determination of the design point $\{u_i^*, \bar{u}_i^*\}$, defined as the point on the failure surface $g=0$ with the shortest distance to the origin, is rather straightforward, see e.g. Jensen (2007). A linearization around this point replaces Eq. (1.4) with a hyperplane in 2n space. The distance β_{FORM}

$$\beta_{FORM} = \min \sqrt{\sum_{i=1}^{n}\left(u_i^2+\bar{u}_i^2\right)} \qquad (1.5)$$

from the hyperplane to the origin is denoted the (FORM) reliability index. The calculation of the design point $\{u_i^*, \bar{u}_i^*\}$ and the associated value of β_{FORM} can be performed by standard reliability codes (e.g. Det Norske Veritas, 2003). Alternatively, standard optimisation codes using Eq. (1.5) as the objective function and Eq. (1.4) as the constraint can be applied.

The deterministic wave profile

$$H^*(X,t) = \sum_{i=1}^{n} \left(u_i^* c_i(X,t) + \bar{u}_i^* \bar{c}_i(X,t) \right) \qquad (1.6)$$

can be considered as a design wave or a critical wave episode. It is the wave scenario with the highest probability of occurrence that leads to the exceedance of the specified response level ϕ_0.

Mean out-crossing rates and exceedance probabilities

The time-invariant peak distribution follows from the mean out-crossing rates. Within a FORM approximation the mean out-crossing rate can be written as follows, Jensen and Capul (2006):

$$\nu(\phi_0) = \frac{1}{2\pi\beta_{FORM}} e^{-\frac{1}{2}\beta_{FORM}^2} \sqrt{\sum_{i=1}^{n} \left(u_i^{*2} + \bar{u}_i^{*2} \right) \omega_i^2}$$

(1.7)

Thus, the mean out-crossing rate is expressed analytically in terms of the design point and the reliability index. For linear processes it reduces to the standard Rayleigh distribution. Finally, on the assumption of statistically independent peaks and, hence, a Poisson distributed process, the probability of exceedance of the level ϕ_0 in a given time T can be calculated from the mean out-crossing rate $\nu(\phi_0)$:

$$P\left[\max_T \phi > \phi_0 \right] = 1 - e^{-\nu(\phi_0)T} \qquad (1.8)$$

The FORM is significantly faster than direct Monte Carlo simulations, but most often very accurate. In Jensen and Pedersen, (2006) also dealing with parametric rolling of ships in head sea the FORM approach was found to be two orders of magnitude faster than direct simulation for realistic exceedance levels and with results deviating less than 0.1 in the reliability index.

PARAMETRIC ROLL IN HEAD SEA

A very comprehensive discussion of intact stability can be found in a recent ITTC report on ship stability in waves, ITTC (2005). The report discusses various modes of failure, i.e. capsize and the prediction procedures available. Some codes, e.g. LAMP, France et al. (2003) and Shin et al. (2005), seem to be very general and can tackle all problems with reasonable accuracy, but are very time-consuming to run, restricting the application to regular waves or very short stochastic realisations.

Other procedures have more limiting capabilities. An example is the ROLLS procedure, Kroeger (1986). In this procedure, the instantaneous value of the righting arm *GZ* is in irregular waves calculated approximately using the so-called Grim's effective wave. The heave w, pitch θ and yaw ψ motions are determined by standard strip theory formulations, whereas the surge motion is calculated from the incident wave pressure distribution. The advantage of this formulation compared to full non-linear calculations is the much faster computational speed, still retaining a coupling between all six- degrees-of-freedom, Krüger et al. (2004).

In the present procedure a simplified version of the ROLLS procedure is applied. The heave motion w is taken to be a linear function of the wave elevation and the closed-form expressions given by Jensen et al. (2004) are used. Pitch is only included through the static balancing of the vessel in waves in the calculation of the *GZ* curve. Furthermore, the sway and yaw motions are ignored as the vertical motions have the largest influence on the instantaneous *GZ* curve. The damping is modelled by a standard combination of a linear, a quadratic and a cubic variation in the roll velocity. Furthermore, the analysis is restricted to head wave. With these simplifications the equilibrium equation for roll ϕ reads, with a dot signifying time derivative,

$$\ddot{\phi} = -2\beta_1\omega_\phi\dot{\phi} - \beta_2\dot{\phi}\left|\dot{\phi}\right| - \frac{\beta_3\dot{\phi}^3}{\omega_\phi} - \frac{(g-\ddot{w})GZ(\phi)}{r_x^2}$$

(2.1)

where r_x is the roll radius of gyration and g the acceleration of gravity. The roll frequency ω_ϕ is given by the metacentric height GM_{sw} in still water:

$$\omega_\phi = \frac{\sqrt{gGM_{sw}}}{r_x} \quad (2.2)$$

The surge motion u is determined from the equilibrium equation:

$$\ddot{u} = \frac{1}{1.05\Delta} \int_{-D}^{h_{cog}+w} p\left(z,t\right) B(z)dz + 10g\left(\frac{\dot{u}}{V}\right)^3 \quad (2.3)$$

The added mass of water in surge is thus taken to be 5 per cent of the displacement Δ and the pressure p is the incident linear wave pressure. Hence, radiation and diffraction effects are ignored. The integral is calculated for the ship cross section at the centre of gravity. The integration is from the draft D of the vessel to the instantaneous position h_{cog} + w of the free surface. Here h_{cog} is the wave elevation at the centre of gravity and $B(z)$ the breadth variation over this section. The second term on the right hand side is an attempt to model the action of the captain to maintain the constant speed V in waves.

It is clear that the model, Eqs. (2.1)-(2.3) is very simplistic, but it is well-suited to illustrate the proposed stochastic procedure as it can model parametric rolling.

The instantaneous GZ curve in irregular waves is estimated from numerical results for a regular wave with a wave length equal to the length L of the vessel and a wave height equal to 0.05L. These numerical results are fitted with analytical approximations, see Jensen and Pedersen (2006).

In a stochastic seaway the following approximation of the instantaneous value of the righting arm $GZ(t)$ is then applied:

$$GZ(\phi,t) = GZ_{sw}(\phi) + \frac{h(t)}{0.05L}\left(GZ\left(\phi, x_c(t)\right) - GZ_{sw}\left(\phi\right)\right) \quad (2.4)$$

This linear relation between GZ and h is clearly an assumption which needs validation. It is used here for the sake of simplicity, but also because the model, Eq.(2.1), by itself only gives an approximate description of reality.

The instantaneous wave height $h(t)$ and the position of the crest x_c are determined by an equivalent wave procedure somewhat similar to the one used by Kroeger (1986):

$$a(t) = \frac{2}{L_e}\int_0^{L_e} H\left(X\left(x,t\right),t\right)\cos\left(\frac{2\pi x}{L_e}\right)dx$$

$$b(t) = \frac{2}{L_e}\int_0^{L_e} H\left(X\left(x,t\right),t\right)\sin\left(\frac{2\pi x}{L_e}\right)dx$$

$$X\left(x,t\right) = \left(x + \left(V + \dot{u}\right)t\right)\cos\psi$$

$$h(t) = 2\sqrt{a^2(t) + b^2(t)}$$

$$x_c(t) = \begin{cases} \dfrac{L_e}{2\pi}\arccos\left(\dfrac{2a(t)}{h(t)}\right) & \text{if } b(t) > 0 \\ L_e - \dfrac{L_e}{2\pi}\arccos\left(\dfrac{2a(t)}{h(t)}\right) & \text{if } b(t) < 0 \end{cases}$$

(2.5)

It is seen that the coupling between roll and surge is solely through the term $X(x,t)$ in Eq.(2.5).

Stationary sea conditions are assumed and specified by a JONSWAP wave spectrum with significant wave height H_s and zero-crossing period T_z. The frequency range is taken to be $\pi \le \omega T_z \le 3\pi$ covering the main part of the JONSWAP spectrum.

Solutions are obtained by embedding the time domain simulation routine, Eqs. (2.1)-(2.3) in a standard FORM code. In the present case, the software PROBAN (Det Norske Veritas, 2003) is used.

NUMERICAL EXAMPLE

A container ship with same main particulars given in Jensen and Pedersen (2006) as Ship #1 is considered. The speed is chosen such that the mean encounter frequency is close to twice the roll natural frequency.

The GZ curves are shown in Figure 1 and 2 and it is clear that a significant reduction in righting lever occurs when the wave crest moves from

AP to 0.25L forward of AP. The lowest value of *GZ* occurs when the wave crest is at amidships. This is quite typical for ships with fine hull forms like container ships.

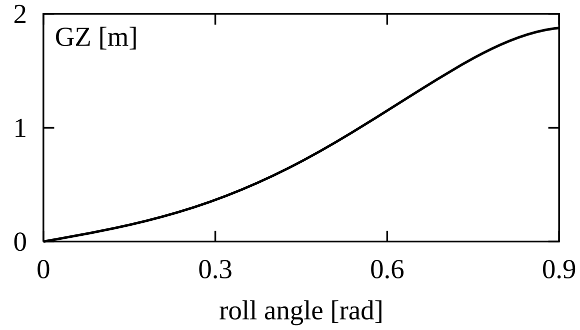

Figure 1: GZ curve in still water.

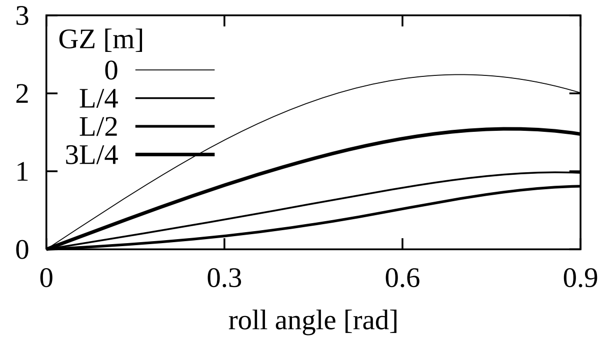

Figure 2: GZ curves in regular waves with wave length equal to the ship length L and a wave height equal to 0.05L. Wave crest positions at x_c= 0, 0.25L, 0.5L, 0.75L and L.

In order to show that Eq. (2.1) can model parametric roll, calculations have been performed with a regular wave with an encounter frequency close to twice the roll frequency, Jensen and Pedersen (2006). Two wave heights are used: one (3.65 m) where parametric roll is not triggered and one slightly higher (3.7 m) where parametric roll develops. The roll motions for the two wave heights are shown in Fig. 3. The onset of parametric roll and its saturation level are clearly noticed.

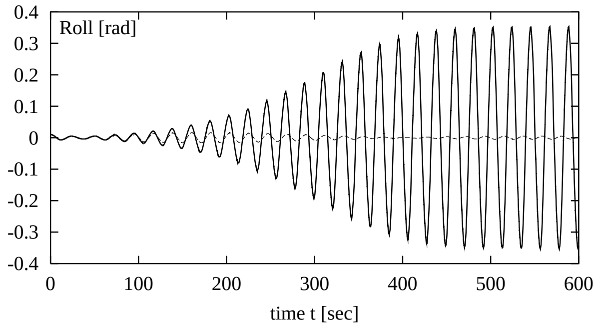

Figure 3: Parametric roll in a regular wave (solid line) and the roll response for a slightly smaller regular wave (dashed line), Jensen and Pedersen (2006).

In the following, results are shown for a sea state with significant wave height H_s = 12 m and zero-crossing wave period T_z = 11.7 s. The zero-crossing period is chosen such that parametric roll can be expected due to occurrence of encounter frequencies in the range of twice the roll frequency. Note, however, that neither the encounter frequency nor the roll frequency is constant in irregular waves. It is also noted that the reliability index β_{FORM} is inversely proportional to the significant wave height H_s, Jensen (2007).

The time domain simulations are carried out from $t = 0$ to $t = t_0$ = 180 s. The effect of the initial condition ($\phi(t = 0) = 0.01$ radians, $\dot{\phi}(t = 0) = 0$) is negligible after about 20 s, but in order to build up parametric roll a longer duration is needed. With n = 25 equidistant frequencies, the wave repetition period relative to the ship is about 200 s depending on the forward speed. In principle the duration to built parametric role could be almost infinity, but the use of longer simulation times than 180 s only changes the mean out-crossing rates, and hence the probability of occurrence, marginally, Jensen (2007).

In the following, results for the design point, i.e. the most probable scenario, corresponding to a roll response of 0.5 rad are shown. The surge acceleration is sown in Fig. 4. The ship velocity is influenced by surge velocity as shown in Fig. 5. The vessel is on average slowed down slightly even with an additional thrust applied.

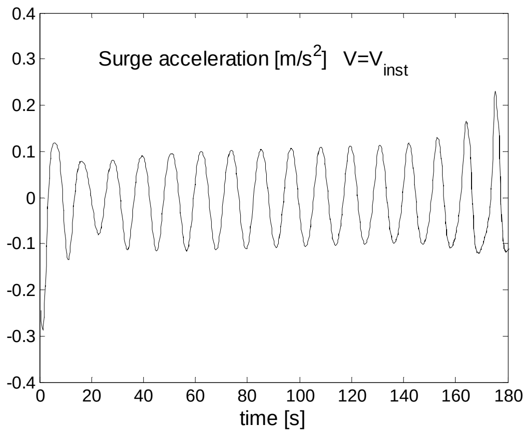

Figure 4: Surge acceleration.

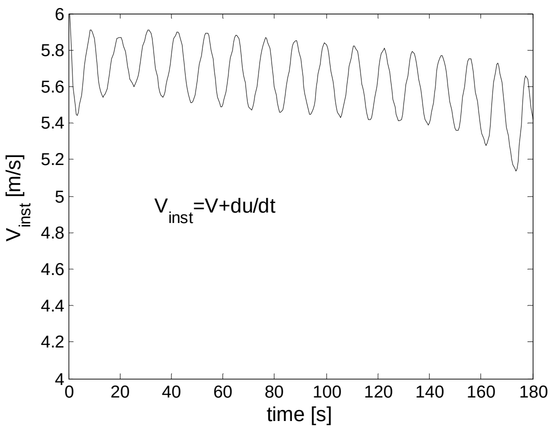

Figure 5: Effect of surge velocity effect on ship velocity.

The critical wave episode, Eq.(1.6), at amidships as function of time for the case with constant ship speed V = 6 m/s is given in Fig. 6. The wave does not change very much from about 20 s to140 s where after the wave elevation changes into a transient wave. An interesting observation is that the critical wave episode then basically turns out to be a sum of two contributions: firstly, a ‘regular’ wave with encounter frequency close to twice the roll frequency and a wave height just triggering parametric roll and, secondly, a ‘transient’ wave with magnitude depending on the prescribed roll response. The reason for the first regular frequency term is the need for a fairly small, but regular wave of sufficient duration and amplitude to trigger the possibility for bifurcation into parametric roll. The second more transient part of the critical wave episode then amplifies this initial parametric roll response to the value prescribed. This behaviour was also found in Jensen and Pedersen (2006).

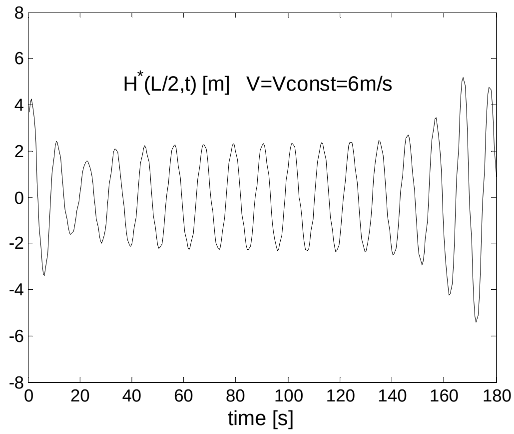

Figure 6 Critical wave episode at amidships, considering the ship speed as constant.

Due to the velocity variations induced by the surge velocity, the critical wave episode changes slightly as shown in Fig.7. The wave shape now has longer crests and sharper troughs due to the slow down when the wave crest is at the fore ship and opposite when amidships.

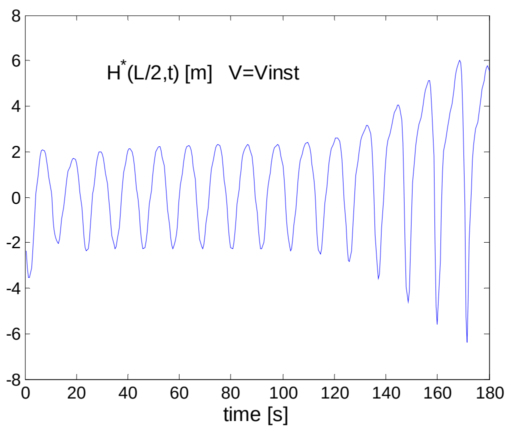

Figure 7: Critical wave episode at amidships, considering the ship speed as varying with the surge velocity.

The associated most probable roll response is given in Fig.8 for the case with varying ship speed. It does not differ very much from the case with constant speed, Jensen and Pedersen (2006)

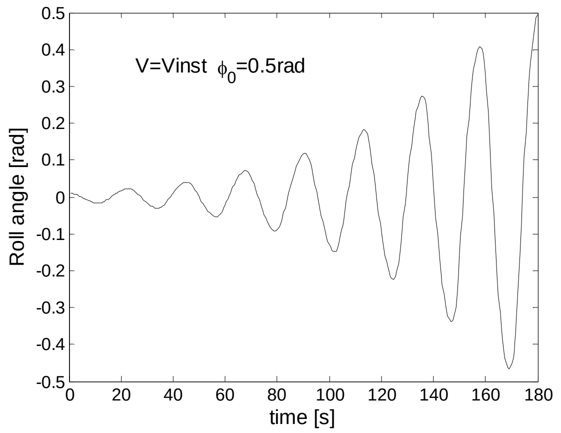

Figure 8: Most probable roll response leading to a roll angle equal to 0.5 rad at *t*=180 s.

The reliability index β_{FORM} as a function of the limiting roll angle ϕ_0 is presented in Fig. 9. The reliability index is seen to increase when surge effect is accounted for. The corresponding mean out-crossing rate, Eq. (1.7), is shown in Fig. 10 and decreases accordingly. This reduced probability of occurrence is explained by the fact that the surge velocity affects the encounter frequency and, hence, tends to violate the parametric roll resonant condition.

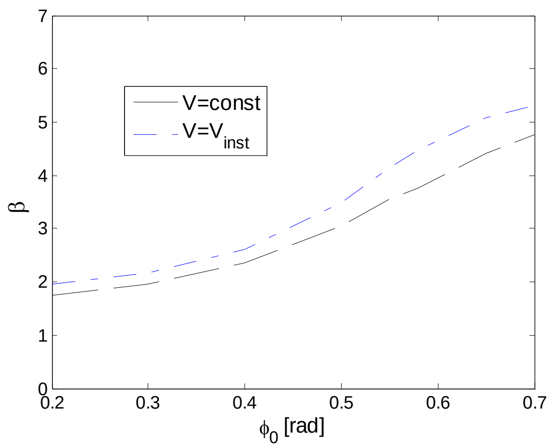

Figure 9: Reliability index as a function of limiting roll angle.

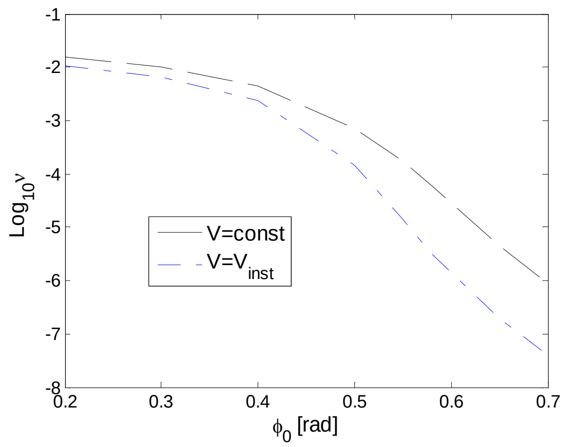

Figure 10: Mean out-crossing rate as a function of limiting roll angle.

CONCLUSIONS

An efficient procedure is presented for calculation of non-linear extreme responses of marine structures subjected to stationary stochastic wave loads. The first step in the procedure requires a formulation of a time domain description of the response as a function of the wave elevation and wave kinematics. This formulation is then implemented in a standard first-order time-invariant reliability (FORM) code, which for given values of the response will solve for the associated design points and reliability indices. An analytical expression, Eq. (1.7), for the mean out-crossing rate in terms of these results is given and the extreme value distribution of the response is then readily obtained by Eq. (1.8). Due to the efficient optimisation procedures implemented in standard FORM codes the calculation is very fast. The ability of the FORM procedure to deal with very low probabilities of occurrence should also be noted. This is a clear advantage over direct simulation methods.

The procedure is illustrated by application to the roll motion of a ship. Surge has been modelled in addition to the roll motion and the effect of surge velocity is accounted for in the calculation of the encounter frequency. For the numerical case considered, the mean out-crossing rate of the limiting roll angle diminishes if surge motion is included. This means that the probability of exceeding a given maximum roll angle decreases.

It should be stressed again that the present hydrodynamic model, Eq.(2.1)-(2.3), is a simplified model. It is chosen for the present study because it can represent parametric roll with physically plausible results. The present formulation, Eqs.(2.1)-(2.5), requires around 10 minutes of CPU time per sea state, heading and speed.

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