On Vulnerability Criteria for Parametric Roll and Surf-riding

William S. Peters
Office of Design and Engineering Standards, Unites States Coast Guard

Vadim Belenky, Christopher C. Bassler
David Taylor Model Basin (NSWC/CD)

Kostas Spyrou
National Technical University of Athens

ABSTRACT

This paper describes the technical background and justification for proposed vulnerability criteria for parametric roll and surf-riding. The presented level one parametric roll criterion contains two conditions. The frequency condition is based on the Ince-Strutt diagram and the change of stability condition is based on the transient solution of the Mathieu equation. The level 1 vulnerability criterion for surf-riding includes consideration for ship length. The level 2 criterion is based on the probability of encounter with a wave that is capable of causing surf-riding.

KEYWORDS

Dynamic stability, parametric roll, surf-riding

INTRODUCTION

At this time, the development of the second generation of IMO intact stability criteria is focused on the development and testing of vulnerability criteria (SLF 53/WP.4).

The world's commercial fleet is rapidly changing to include typologies for which long experience does not exist. Therefore, the second generation of intact stability criteria is intended to be based on fundamental physical principles. To combine the robustness of mathematical models and to keep these new criteria practical, a multi-tiered structure was envisioned (Belenky, et al 2008). The role of vulnerability criteria is to provide a quick check to determine if the vessel under consideration may be vulnerable to certain stability failures, and establish if the need for further dynamic stability analysis is justified.

The bulk of information on the development of vulnerability criteria is reflected in the document SLF 53/INF.10, with a series of comments submitted to SLF 53, including SLF 53/3/7 and SLF 53/3/8. This paper examines the background of these two documents Also it can be considered as a further development of Peters, et al (2010), which was presented at the 11th International Ship Stability Workshop.

PARAMETRIC ROLL

In order to give rise to parametric roll, parametric excitation (the change of stability in waves) must satisfy two conditions: its frequency (the wave encounter frequency) must be within a certain range and its magnitude must be above the threshold (resulting from damping). The Mathieu equation (and Ince-Strutt diagram) is the simplest mathematical model that can be used to check if these conditions are satisfied. The ABS Susceptibility Criteria are based on this approach (ABS 2004, Shin et al, 2004).

Spyrou (2005) proposed a more practical version of the criteria, also based on the Mathieu equation. This idea was further used as a background for SLF 53/3/7 and is briefly reviewed below.

Boundaries of the 1st instability zone of an Ince-Strutt diagram have a known approximation:

$$p_{B1,B2} = \frac{1}{4} \pm \frac{q}{2} \qquad (1)$$

Here p and q are parameters of the Mathieu equation

Assuming $q = 1$ (this is a conservative assumption as most container ships have q = 0.25~0.4), the first requirement of the criterion was formulated, in terms of speed range (in knots)

$$V_{S1} = \frac{\sqrt{gL}}{0.5144}\left(\frac{1}{\sqrt{2\pi}} - \frac{\sqrt{2 \cdot L}}{T_\phi\sqrt{g}}\right)$$

$$V_{S2} = \frac{\sqrt{gL}}{0.5144}\left(\frac{1}{\sqrt{2\pi}} - \frac{\sqrt{6 \cdot L}}{T_\phi\sqrt{g}}\right) \quad (2)$$

Here L is length of a ship, and T_ϕ is the natural period of roll.

This criterion was used in SLF 53/3/7 as a preliminary condition related to speed. If the service speed of a ship does not fall into the range defined by equations (2), the ship is not susceptible to parametric roll and a further vulnerability check is unnecessary. This preliminary condition makes the criteria easy to use, as the calculation of stability in waves is not required if the ship speed is outside of the range (2).

The second condition is based on the transient solution of the Mathieu equation, as the steady state solution may be too conservative (SLF 48/4/12). It was proposed to use the approximate solution according to Hayashi (1985):

$$\phi(t) = e^{-\delta t}\left(C_1 e^{\kappa\omega_m t}\sin(\omega_m t - \varepsilon) + C_2 e^{-\kappa\omega_m t}\sin(\omega_m t + \varepsilon)\right) \quad (3)$$

Here C_1 and C_2 are arbitrary constants determined through the initial conditions, κ is a parameter controlling the growth or decay of oscillations, ω_m is mean frequency of roll (accounting for changing stability in waves), δ is a linear roll damping coefficient, and ε is a phase shift depending on the magnitude of stability change and the ration of encounter and mean roll frequencies.

Assuming the middle-range of principal parametric resonance ($\omega_e = 2\,\omega_m$, ω_e being encounter frequency) and zero initial roll rate, it is possible to find such a magnitude of stability changes, h, that leads to the f–fold increase of the amplitude after n cycles of oscillations:

$$h = 2\frac{\ln f + \ln 2}{\pi n} + \frac{4\delta}{\omega_m} \quad (4)$$

As a first expansion, assuming $f = 5$ while $n = 4$ and using the ABS (2004) recommendation for the damping coefficient, $\delta = 0.03\omega_0$ (ω_0 – natural roll frequency in calm water), the second condition can be formulated as:

$$\frac{GM_a}{GM_m} = h \geq 0.49 \quad (5)$$

Here GM_a is a magnitude of GM change in waves and GM_m is the mean value of the GM changes in waves. Calculations of the stability change in waves are meant to be performed as recommended by Shin et al (2004), where the length of wave is assumed to be equal to ship length. The height of wave is taken as:

$$H = \begin{cases} 0.05L & if \quad L < 100 \\ \frac{L}{3}(0.2 - 0.0005L) & if \quad 100 \leq L < 300 \\ 0.01667L & L \geq 300 \end{cases} \quad (6)$$

Here L is the length of the ship.

Spyrou proposed also to relate the wave height with the wave length by using the principle of equal probability in order to establish a fair basis for the safety assessment of ships of all sizes. This entails a decrease in wave steepness for a longer ship, thus considering the fact that a high value of wave steepness is less probable to occur for long waves. It is noteworthy that a similar approach to steepness was implicitly used in the formulation of the IMO weather criterion.

Parametric roll is caused by a consecutive excitation from a series of waves. Therefore, it is logical to evaluate the probability of encounter of a group of waves, rather than for a single wave. This can be done using wave group representations described in Themelis and Spyrou, (2007; 2008) and Themelis (2008).

Sequential wave heights are presented using a Markov chain. This means that the wave height is assumed to be dependent only on the height of the previous wave, but is independent of the waves prior to the previous wave. This assumption seems to be quite logical, since the wave envelope (a curve that contains all of the wave heights) has an autocorrelation function with relatively fast decay; so the correlation is practically zero after two wave periods.

Because a joint distribution of wave heights and wave lengths is known, this approach allows for the calculation of the probability of encounter of a number of waves of a given length (actually the length within a given range) and height. Spyrou calculated how the steepness depends on wave length, keeping probability of encounter of a group of four waves constant ($P = 6.3 \times 10^{-6}$). The calculations were carried out for a significant wave height of 5 m and modal period of 12 s. There were two series of calculations, using a different range for wave length. The results of the calculations are shown in Fig. 1
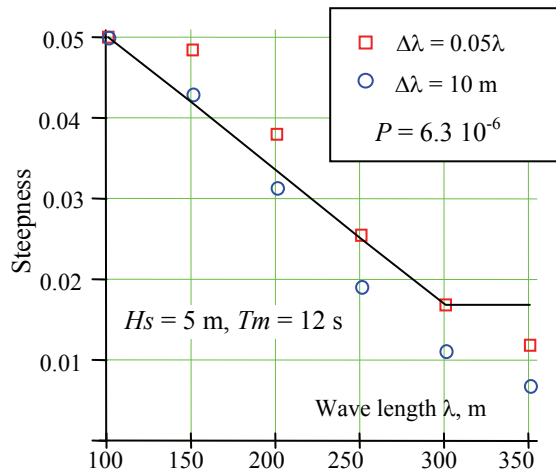

Fig. 1 Dependence of wave steepness on length, based on equal probability of encountering a group of four waves

This analysis, although preliminary, still allows for capturing the dependence of steepness on length, and avoiding unnecessary penalization of large ships

Results of the application of these criteria are shown in Table A1 (in the Appendix), along with the Level 2 vulnerability check described by Peters et al (2010) and in Annex 5 of SLF 53/INF.10.

SURF-RIDING

The document SLF 53/3/8 proposes that the vulnerability criteria for broaching-to are to be based on the second threshold of surf-riding in following seas. ("Second threshold" means the speed in following waves for which surf-riding will occur regardless of the ship's position on the wave and instantaneous speed.) Using the surf-riding phenomenon as an indicator for broaching

seems to be logical, as the surf-riding is a pre-cursor of broaching and the model of surf-riding in following waves is relatively simple.

Using the second threshold seems to be logical as well, as the first threshold may appear for a relatively slow speed, when surf-riding is possible only theoretically (i.e., only at a single point on a following wave).

IMO operational recommendations (MSC.1/Circ. 1228) also reportedly used the second threshold to establish a recommendation on speed reduction in following seas. If the Froude number exceeds 0.3, surf-riding and resulting broaching is considered likely.

However, surf-riding and broaching are caused by a steep wave with the wave length comparable to the length of the ship. It should be intuitively clear then, that an encounter with a steep wave (say 1/10) of the 30 m length is much more likely than for a wave that is 300 m long. A wave height of 3 m is quite common, while a wave height of 30 m would be considered rather exceptional.

Therefore, the criterion should consider at least the length of the ship (Umeda 2010). Belenky and Spyrou (2011) developed the following formula based on systematic calculations of the Froude number corresponding to the second threshold :

$$Fn > 0.28 \quad if \quad L \le 80 \ m$$
$$Fn > 0.0000181 \cdot L + 0.282 \quad if \quad L > 80 \ m \quad (7)$$

This criterion was developed based on a sample population of 17 ships (listed in Table A1 in the Appendix). For each of these ships, the second threshold was calculated using Melnikov's method (Spyrou 2006, also described briefly below) for the ranges of wave lengths ($0.75L \sim 2L$) and steepness values ($1/8 \sim 1/40$). Results are shown in Fig. 2.

The averaged curve shown in Fig. 2 gives a relation between the Froude number corresponding to the second threshold and the wave steepness.

$$H/\lambda = 0.0310 \, Fn^{-3} + 0.06226 \qquad (8)$$

Using formula (8), together with a joint distribution of the wave amplitudes and wave number, an approximate probability of encounter

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with a wave of a given length, which is capable of causing the surf-riding, can be found:

$$P(Fn, \lambda, H_S, T_Z) = \int_{a_{cr}(Fn,\lambda)}^{a_{lim}} f\left(a \mid k = \frac{2\pi}{\lambda}\right) da \quad (9)$$

Where $a_{cr}(Fn,\lambda)$ is a critical amplitude of the wave. which can be found from formula (8), and a_{lim} is a hydrodynamic limit for the wave amplitude. Formula (9) is specific for a given sea state, as is defined by the significant wave height, H_S, and the mean zero-crossing period, T_Z, if a Bretschneider spectrum is used.
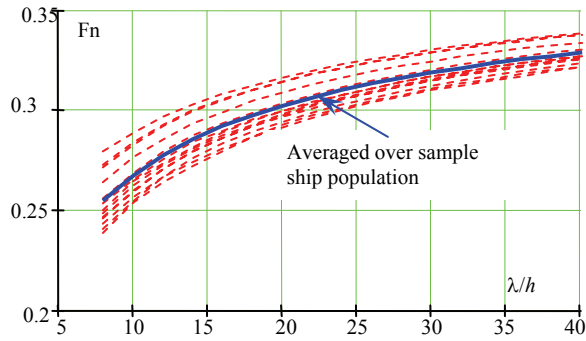

Fig.2 Threshold Froude number, as a function of wave steepness

For the purpose of developing a criterion, the wave length was assumed to be equal to ship length. By doing so, the probability becomes only a function of the Froude number and by averaging it over the wave scatter data enables the sea–state specific data to be eliminated (Umeda 2010). The scatter data from IACS Recommendation 34 was used here (IACS 2001).

To derive a benchmark probability level, a reference ship length of 80 m and Froude number 0.28 have been assumed. This yields the reference probability $7.8 \cdot 10^{-3}$ and formula (7) – see Fig. 3.

Formula (7) was derived from a specific ship population; because of this, it may change if the derivation is tried on another, or larger, population of ships. At the same time, the difference between these ships, however noticeable, is not dramatic (Fig. 2). As a result, there are two advantages of this approach: (a) a simple formula, and (b) a safety level that may be related to the reference probability.

Alternatively, the document SLF-53/3/8 proposes even simpler formula:

$$Fn \geq 0.3 \quad if \quad L \leq 200 \, m \quad (10)$$

When shown together with formula (7), one can see that both these formula somewhat agree (see Fig. 3).
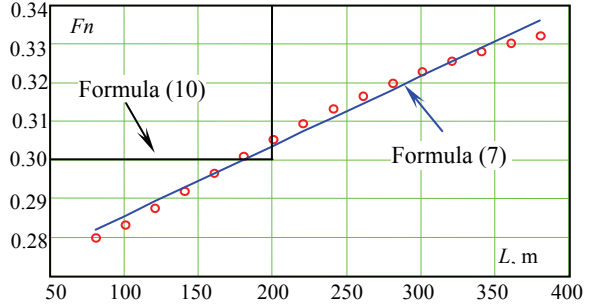

Fig. 3 Froude number as a function of ship length, under the condition of the equivalent probability of encountering a wave capable of causing surf-riding

The second level vulnerability criterion proposed in SLF 53/3/8 is based on the second threshold calculated using the ship hull form. One of the ways of performing these calculations is to use the Melnikov method. Spyrou (2006) derived the Melnikov function in the following form:

$$M(n) = -\frac{r(n)}{q} - \frac{4}{\pi} p_1(n) + 2p_2 - \frac{32}{3\pi} p_3 \quad (11)$$

Here, coefficients p_i and r are derived from a cubic approximation of the curves for thrust and resistance of the propulsor, while q is a coefficient depending on the amplitude of the wave surging force.

The Melnikov function (11) is written for non-dimensional nonlinear surging equation:

$$x'' + p_1(n)x' + p_2 x'^2 + p_3 x'^3 + \sin x = r(n)$$
$$x = k\xi_G \quad (12)$$

Here, k is the wave number and ξ_G is a position of the center of gravity of the ship relative to the wave trough. Details of the application of the method are also available from Appendix 1 of Belenky and Spyrou (2011). An example of the Melnikov function is shown in Fig. 4.

The geometrical meaning of Fig. 4 is the distance between the boundaries of the domains of attraction to periodic surging and the surf-riding equilibrium. The zero-value of the Melnikov function corresponds to the situation when the

boundaries touch each other. As it is known, this situation is characterized by disappearance of periodic surging, i.e. the second threshold has been reached. Therefore, the numerical solution (11) yields the number of shaft revolutions corresponding to the calm water speed settings of the second threshold.
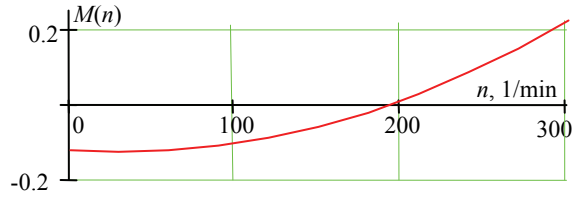

Fig. 4 Melnikov function of the ONR Tumble-home topside ship, wave length 150 m, wave steepness 1/15

Because surf-riding is a single wave event, it makes sense to present an irregular seaway as a sequence of waves, each of which is characterized by random length λ_i and height $2a_j$, associated with a certain discretization of the joint distribution of the length and height. This allows for the formulation the criterion in probabilistic form:

$$C2_{ij} = \begin{cases} 1 & if \quad Fn > Fn_{TR}(\lambda_i, a_j) \\ 0 & if \quad Fn \le Fn_{TR}(\lambda_i, a_j) \end{cases}$$

(13)

$$C2(H_S, T_Z) = \sum_{i=1}^{N_\lambda} \sum_{j=1}^{N_a} W_{ij} C2_{ij}$$

Here Fn_{TR} is the Froude number corresponding to the second threshold for the wave length λ_i and height $2a_j$; W_{ij} is the statistical weight associated with the wave with the length λ_i and height $2a_j$.

Results of the sample calculation of the criterion (13) are shown in Table A1 in the Appendix.

To avoid the necessity of choosing a particular sea state, it is proposed in SLF 53/3/8 to average the criterion (13) over a wave scatter diagram:

$$C2_L = \frac{1}{N_{Tot}} \sum_{H_S} \sum_{T_Z} C2(H_S, T_Z) N(H_S, T_Z)$$

(14)

SUMMARY

The paper describes the background and results of sample calculations for vulnerability criteria for parametric roll and surf-riding.

The level 1 criterion for parametric roll contains two conditions; the first one is based on

the frequency range for parametric resonance. The second is based on a transient solution of the Mathieu equation and expresses the magnitude of stability change in waves as sufficient to cause a specified increase of the roll response amplitude.

The vulnerability criteria for surf-riding are based on the second threshold. Level 1 criterion takes into account the length of the ship, as the likelihood of encounter of a steep wave diminishes with the increase of the wave length. The level 2 criterion is also based on the probability of encountering a wave capable of causing surf-riding, which is associated with exceeding the second threshold. The Froude number corresponding to the second threshold is calculated using Melnikov's method.

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APPENDIX: SAMPLE SHIPS AND RESULTS OF SAMPLE CALUCLATIONS

Table A1 Results of sample calculations

Type	L, m	Vs, kn	Fn	Parametric Roll						Surf-Riding				
				V_{s1}, kn	V_{s2}, kn	Vulnerable Eq (2)	GM_a/GM_m	Vulnerable, Eq (5)	PR L2 CrN, deg	Vulnerable PR L2	Vulnerable, Eq(10)	Eq (7)	Vulnerable Eq (7)	Eq (13)	Vulnerable
Bulk Carrier 1	275	16	0.16	3	-24	Y	0.09	N	10	N	N	0.31	N	0.00	N
Bulk Carrier 2	145	14	0.19	13	1	Y	0.4	N	10	N	N	0.29	N	0.00	N
Containership 1	323	25	0.23	16	-4	Y	1.42	Y	25	Y	N	0.32	N	0.00	N
Containership 2	376	25	0.21	17	-5	Y	0.9	Y	25	Y	N	0.33	N	0.00	N
Containership 3	330	25	0.23	10	-14	Y	0.58	Y	25	Y	N	0.32	N	0.00	N
Containership 4	283	25	0.24	8	-15	Y	0.75	Y	25	Y	N	0.32	N	0.00	N
Containership 5 (C11)	262	25	0.25	7	-16	Y	0.77	Y	25	Y	N	0.31	N	0.00	N
Fishing Vessel 1 (Purse Seiner)	35	17	0.48	-6	-21	Y	0.16	N	10	N	Y	0.28	Y	0.56	Y
Fishing Vessel 2	22	14	0.50	2	-5	Y	0.09	N	10	N	Y	0.28	Y	0.34	Y
General Cargo 1 (S60)	122	18	0.27	14	5	Y	1.03	Y	10	N	N	0.29	N	0.00	N
General Cargo 2 (C4)	161	18	0.23	8	-9	Y	0.57	Y	15	Y	N	0.29	N	0.00	N
LNG Carrier	268	18	0.18	-1	-31	Y	0.18	N	10	N	N	0.31	N	0.00	N
Naval Combatant 1 (ONRFL)	150	30	0.40	2	-19	Y	0.58	Y	10	N	Y	0.29	Y	0.20	Y
Naval Combatant 2 (ONRTH)	150	30	0.40	-18	-53	Y	0.26	N	10	N	Y	0.29	Y	0.20	Y
Passenger Ship	276	25	0.25	-17	-59	Y	0.57	Y	25	Y	N	0.32	N	0.00	N
RoPax	137	19	0.27	-3	-25	Y	0.82	Y	25	Y	N	0.29	N	0.00	N
Tanker	320	14	0.13	-16	-60	N	0.03	N	10	N	N	0.32	N	0.00	N