Split-time Method for Calculation of Probability of Capsizing Due to Pure Loss of Stability

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ABSTRACT

The paper describes the numerical implementation of the Split-Time method with a full account of stability variation in waves. Split-Time method gathers data on the difference between the instantaneous and critical roll rate at the instant of upcrossing of the intermediate threshold. This data appears to be a good metric for predicting capsizing and can be extrapolated with an exponential distribution. The paper also looks into the meaning of a perturbation of the roll rate at the instant of upcrossing, showing its relation with classic definition of ship stability and the general definition of motion stability of a dynamical system.

KEYWORDS

Probability of capsizing, stability variations in waves

INTRODUCTION

The idea of the split-time method is to separate the very complex problem of predicting the probability of capsizing in irregular waves into two simpler problems: non-rare and rare. The non-rare problem is an upcrossing of an intermediate threshold, while the rare problem focuses on the conditions at the instant of upcrossing that would lead to capsizing or a large angle of roll. The non-rare problem is meant to be solved through the direct statistical processing of time-domain motion data so the intermediate threshold is expected to be low enough that upcrossing statistics may be evaluated directly. The rare problem is solved by perturbing the roll rate at the instant of upcrossing in order to find the value that leads to the specified stability failure. The general scheme is illustrated in Figure 1, the inset shows the perturbations.

A comprehensive overview of the development of the method has been recently published by authors (Belenky, *et al.* 2012); the results described in this paper may be seen as a direct continuation of the cited paper.

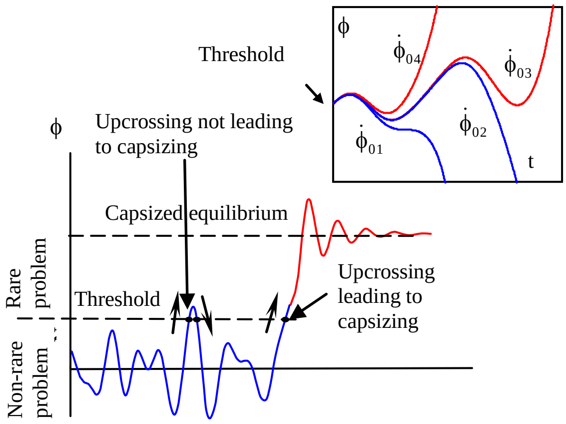

Fig. 1 Concept of Split-Time Method (Belenky, *et al.* 2012)

PROBABILITY OF CAPSIZING

With the separation of the problem described above, the probability of capsizing during time T is expressed as

$$P(T) = 1 - \exp\left(-\xi P_C T\right) \qquad (1)$$

ξ is upcrossing rate through the intermediate threshold (non-rare problem) and P_C is a probability of capsizing after the threshold has been crossed (rare problem). As the intermediate threshold is set low enough to observe upcrossings, the non-rare problem does not present a great challenge and most of the focus is on the rare problem.

Capsizing after upcrossing is associated with an exceedance of a critical roll rate by the roll rate at upcrossing, where the critical roll rate is defined as the minimum roll rate at upcrossing which leads to capsize. In principle, the critical roll rate $\dot{\phi}_{cr}(t)$ can be defined at any instant of time, so the difference between the instantaneous and critical roll rates defines a process:

$$y(t) = \dot{\phi}_{cr}(t) - \dot{\phi}(t) \qquad (2)$$

Where ϕ is an instantaneous roll angle and the dot above the symbol stands for temporal derivative. The probability of capsizing after upcrossing is expressed as:

$$P_C = \int_{-\infty}^{0} f_c(y)dy \qquad (3)$$

The probability density distribution f_c refers to the values of the process $y(t)$ taken at the instant when the process $\phi(t)$ up-crosses the intermediate threshold ϕ_{m0}. The distribution of the process y at upcrossing can be expressed as (Belenky et al. 2011):

$$f_c(y) = \frac{\int_0^{\infty} \dot{\phi} f(\phi = \phi_{m0}, \dot{\phi}, y)\, d\dot{\phi}}{\int_0^{\infty} \dot{\phi} f(\phi = \phi_{m0}, \dot{\phi})\, d\dot{\phi}} \qquad (4)$$

The evaluation of the integral (4) numerically represents significant difficulties as it would require fitting a joint distribution of roll and the instantaneous and critical roll rates. The fact that roll angles and rates are not correlated does not provide any noticeable simplification, as for non-Gaussian processes the absence of correlation may not mean independence. In particular, the dependence of roll and roll rate may be significant in stern quartering seas (Belenky, *et al.* 2012).

To avoid these numerical difficulties, the critical roll rate should be calculated at the instant of upcrossing only. Thus, only the distribution $f_c(y_c)$ needs to be modeled.

PERTURBATION AND DEFINITION OF STABILITY

The critical roll rate is calculated by repeating the numerical integration of the equations of motion starting from the instant of upcrossing. The roll rate at upcrossing is perturbed and used as part of the initial conditions while all other initial conditions remain the same.

The numerical procedure is expected to be used with an advanced hydrodynamic code and be code-independent. The only requirement is restart capability. For numerical codes that model ship motions using integro-differential equations with hydrodynamic memory effects, the complete potential solution must be saved at the instant of upcrossing. However, simpler models without memory effects can be used for the development of the procedure; provided that they capture the stability variation in waves, which is essential for modeling phenomena such as pure loss of stability.

Weems & Wundrow (2013) describe a 3-DOF hybrid model that uses a volume-based calculation for the nonlinear Froude-Krylov and hydrostatic forces. Diffraction and radiation are modeled with coefficients, so there are no hydrodynamic memory terms, but the inseparability of excitation and restoring are included. Thus, this model is “half-way” between a hydrodynamic code and a dynamical system described with ordinary differential equations (ODEs).

Sample calculations were performed for ONR Topsides Series tumblehome configuration (Bishop et al. 2005) at zero-speed in stern-quartering long-crested seas, modeled using a Bretschneider spectrum with a significant wave height of 11.5 m and modal period of 14 s.

Figure 2 shows the final three iterations for the calculation of the critical roll rate at one time instant of upcrossing. It is important to note that all of the perturbed solutions with an initial roll rate below critical converge to the unperturbed solution.

One may pose the question of why the perturbations are applied only to roll rate, while ship motions are modeled with three degrees of freedom (heave, roll and pitch). According to d'Alembert's principle (see, *e.g.*, Synge & Griffith 1959 or McCuskey 1962), the sum of the differences between the total forces acting on a system of mass particles and the time derivative of the momentum of the system is zero along any virtual displacement:

$$\sum_i \left(\mathbf{F}_i - m_i \mathbf{w}_i\right)\cdot \delta\mathbf{r}_i = 0 \qquad (5)$$

Here i is the index of a particle of the system, $\mathbf{F}_i$ is the total force on the particle (excluding constraints), m_i is the mass of the particle, $\mathbf{w}_i$ is the acceleration of the particle, and $\delta\mathbf{r}_i$ is the virtual displacement of the i-th particle, consistent with constraints. In particular, for a rigid body, d'Alembert's principle allows interpretation of a dynamical problem as a static problem by including inertial forces in the equations of equilibrium. Thus, at every time instant, a ship may be considered in a dynamical equilibrium if inertia forces are included.

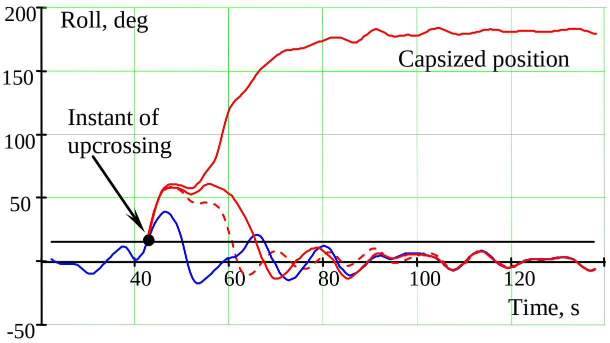

Fig. 2: Final iterations of calculation of critical roll rate

Euler (1749) defined a stable ship as "being inclined by external forces returning back to its original position, once the forces ceased to exist." A perturbation on roll rate can be considered as the application of a force for a very short time, and stability can be claimed if the ship subsequently returns to its original equilibrium, which in this context means the original, unperturbed time history.

At the same time, response of a dynamical system is defined as (asymptotically) stable if, after a perturbation, the perturbed response tends to the unperturbed response.

Thus, both Euler's definition and the general definition of motion stability are equivalent and consistent with the perturbation approach and critical roll rate represents a stability boundary at the given instant of time

RARE PROBLEM

General Approach

The essence of the rare problem is modeling the distribution of the difference between the critical and the instantaneous roll rate y_c at the instant of upcrossing. A histogram of this value is shown in Figure 3 based on upcrossing data from 20 records, each of half-hour duration. The upcrossing threshold was set to 13 degrees, which provides a sample with N_Y=207 data points. There were no capsizings observed during this particular dataset, so all the available data points are positive.

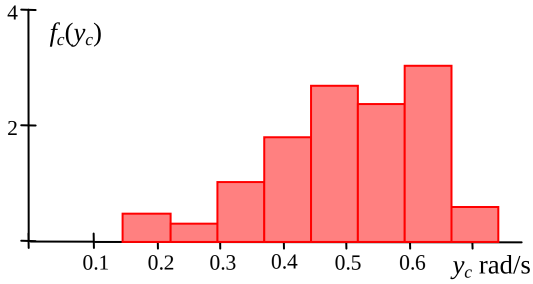

Fig. 3: Histogram of the difference between the critical and the instantaneous roll rate y_c at the instant of upcrossing.

Actually, the modeling of the entire distribution of y_c is not necessary to be able to calculate probability of capsizing after upcrossing; only the tail of the distribution is needed. There is a proof (Leadbetter *et al.* 1983, see also Coles 2001) that the tail (*i.e.* the part of the distribution beyond a certain value *a*) can be approximated with generalized Pareto distribution (GPD):

$$f(x) = \begin{cases} \dfrac{1}{\sigma}\left(1 + k\,\dfrac{x-a}{\sigma}\right)^{-\left(1+\frac{1}{k}\right)} & k \neq 0 \\ \lambda \exp\left(-\lambda(x-a)\right) & k = 0 \end{cases} \qquad (6)$$

Where k is a shape parameter, while σ and λ are scale parameters.

Properties of the Tail

It can be seen from (6) that the exponential distribution is a particular case of GPD. It is possible to show that the distribution (4) actually has an exponential tail when used with the closed-form expression available for a piecewise linear system (Belenky *et al.* 2012):

$$f_c(y_c) = \frac{f_{xy}(\phi_{m0}, y_c)}{\xi_x(\phi_{m0})} \times \left(\frac{\sigma_{d|xy}}{\sqrt{2\pi}} \exp\left(-\frac{(m_{d|xy}(y_c))^2}{2\sigma^2_{d|xy}} \right) + \frac{m_{d|xy}(y_c)}{2}\left(1-\operatorname{erf}\left(-\frac{m_{d|xy}(y_c)}{\sqrt{2}\sigma_{d|xy}}\right)\right)\right) \quad (7)$$

where f_{xy} is the joint normal distribution of the "carrier" process x and the difference between the critical and instantaneous roll rates y. The carrier process $x(t)$ is defined as:

$$x(t) = \phi(t) - \phi_m(t) + \phi_{m0} \quad (8)$$

Were $\phi_m(t)$ is a random component of maximum of the piecewise linear stiffness, while ϕ_{m0} is its mean value corresponding to "calm water" case. The derivative of the carrier process is identified with the symbol $d(t)$. The symbols $m_{d|xy}$ and $\sigma_{d|xy}$ are the conditional mean value and conditional standard deviation of the derivative of the carrier process evaluated for particular values of the processes x and y, based on normality of all three processes: $x(t)$, $y(t)$ and $d(t)$:

$$m_{d|xy}(y) = \frac{\sigma_d}{1-r^2_{xy}} \times \left(\frac{(y-m_y)r_{yd}}{\sigma_y} - \frac{\phi_{m0} r_{yd} r_{xy}}{\sigma_x} \right) \quad (9)$$

$$\sigma_{d|xy} = \sigma_d \frac{\sqrt{1-r^2_{xy}-r^2_{yd}}}{\sqrt{1-r^2_{xy}}} \quad (10)$$

σ with a subscript stands for standard deviation of the corresponding process while r with two subscripts stands for a correlation coefficient for these two processes; m_y is a mean value of the process of differences between the instantaneous and critical roll rate taken at any time instant (which is different from the time instant of upcrossing).

Finally $\xi_x(\phi_{m0})$ is the upcrossing rate for the carrier process over the level of ϕ_{m0}.

The error function erf is defined here as:

$$\operatorname{erf}(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp\left(-\frac{u^2}{2}\right) du \quad (11)$$

Formula (7) is a product of normal distribution f_{xy} and the following expression:

$$f_1(y_c) = \frac{\sigma_{d|xy}}{\sqrt{2\pi}} \exp\left(-\frac{(m_{d|xy}(y_c))^2}{2\sigma^2_{d|xy}} \right) + \frac{m_{d|xy}(y_c)}{2}\left(1-\operatorname{erf}\left(-\frac{m_{d|xy}(y_c)}{\sqrt{2}\sigma_{d|xy}}\right)\right) \quad (12)$$

The mean value of the process $y(t)$ (calculated at any point) must be a positive value, in particular it equals 0.396 in the example considered in (Belenky *et al.* 2011). That means that the negative values of y_c are relatively far from the mean value and formula (12) is dominated by the linear function (9); this gives the distribution (7) its negative skew, see Figure 4. All numerical parameters are taken from (Belenky *et al.* 2011). That means that the negative tail distribution (7) is made up of the tail of the normal distribution and a linear function with negative slope. Indeed, the tail is exponential.

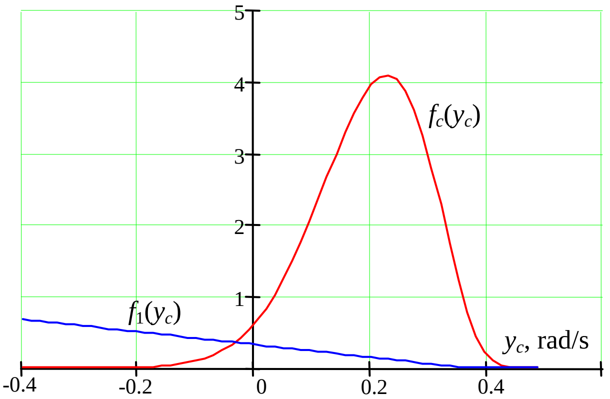

Fig. 4: Theoretical distribution of the difference between instantaneous and critical roll rate at the instant of upcrossing

Fitting Exponential Tail

Prior to fitting the exponential tail to the sample of differences between critical and instantaneous roll rates taken at the instant of upcrossing, it is convenient to convert the data to positive skew and tail:

$$\begin{aligned} u_c &= Lf - y_c \\ Lf &= \max(y_c) \end{aligned} \tag{13}$$

The probability of capsize after upcrossing is:

$$P_C = \int_{Lf}^{\infty} f(u_c)du_c \tag{14}$$

The next step is to find where the tail starts, which is the value a in formula (6). Following the procedure described in Coles (2001), a series of thresholds for u_c are introduced. For this numerical example, 12 thresholds are chosen to have 10 points above the highest threshold and 30 points below the lowest threshold. Fitting the tail means calculating parameter λ in formula (6). It is carried out with the method of maximum likelihood estimation (MLE). The log likelihood function is expressed as:

$$l(\hat{\lambda}) = \sum_{i=1}^{N(a)} \ln\left(f(u_i, \hat{\lambda})\right) \tag{15}$$

Where the f is the exponential distribution with the parameter λ; $N(a)$ is the number of data points u_i above the threshold a. The ^ or “hat” symbol above the symbol means “estimate”. The maximum likelihood estimate for the parameter l is computed from the condition:

$$\frac{\partial l(\hat{\lambda})}{\partial \hat{\lambda}} = \frac{\partial}{\partial \hat{\lambda}}\left(\sum_{i=1}^{N(a)} \ln\left(f(u_i, \hat{\lambda})\right)\right) = 0 \tag{16}$$

This leads to the following expression for the estimate of the parameter, which is a function of the threshold a.

$$\hat{\lambda}(a) = \left(\frac{1}{N(a)} \sum_{i=1}^{N(a)} (u_i - a)\right)^{-1} = \frac{1}{\hat{m}_u(a)} \tag{17}$$

$m_u(a)$ is the mean value of the data above the threshold a.

As any other estimator, the parameter in (17) is a random variable and has to come with a confidence interval. Since the number of data points is rather large, it seems to be logical to assume normal distribution; thus the mean value and the variance of the estimate need to be found.

Formula (17) can be interpreted as the deterministic function of the random argument of the mean value of the data points:

$$\hat{\lambda} = g(\hat{m}_u) = \hat{m}_u^{-1} \tag{18}$$

As any other function, it can be expanded into a Taylor series in the neighbourhood of the mean value of the estimate; only the first order terms are kept:

$$g(m_x) \approx g(\hat{m}_u) + g'(\hat{m}_u)(m_x - \hat{m}_u) \tag{19}$$

The function (19) is a linear function of the random argument; it is the mean value estimate of the data above the threshold. The mean value and the variance of the argument are known:

$$\begin{aligned} E(\hat{m}_u(a)) &= \hat{m}_u(a) \\ Var(\hat{m}_u(a)) &= \frac{\hat{V}_u(a)}{N(a)} \end{aligned} \tag{20}$$

Here $V_u(a)$ is the variance of the data points above the threshold a. Since an exponential distribution is assumed, the variance is equal to the square of the mean value:

$$\hat{V}_u(a) = (\hat{m}_u(a))^2 \tag{21}$$

The mean value and the variance of the linear function (19) can then be expressed as:

$$\begin{aligned} E(g(\hat{m}_u)) &= g(\hat{m}_u) = \hat{\lambda} \\ Var(g(\hat{m}_u)) &= (g'(\hat{m}_u))^2 Var(\hat{m}_u) = \frac{\hat{\lambda}^2}{N(a)} \end{aligned} \tag{22}$$

Figure 5 shows these estimates with 95% confidence interval as a function of the thresholds. This plot allows the threshold where tail starts to be found. Since equation (6) is a good approximation of the tail, once it is legitimate it does not depend where the threshold is set. That mean that once the tail starts, the parameter estimates are statistically identical and a horizontal line plotted from the lowest “eligible” threshold will cross through the confidence intervals of all thresholds above it. According to Figure 6, the tail starts around a=0.25 rad/s with $N(a)$=101 data points remaining above the threshold. Figure 6 shows a histogram of the data above that threshold and the fitted exponential tail (please note the change of the bin size of the histograms in Figures 3 and 6 due to change in the volume of data and its variance).

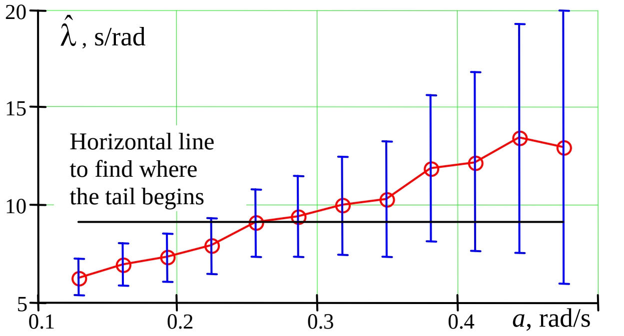

Fig. 5: Parameters of the exponential fit of the tail

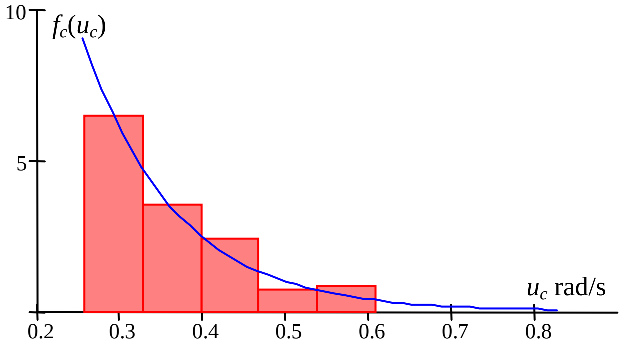

Fig. 6: Exponential fit for the tail

Probability of Capsizing after Upcrossing

Once the tail has been fit, the estimation of the conditional probability of exceeding the level Lf, associated with stability failure, is straightforward:

$$\hat{P}_{Lf|a} = \hat{P}(u > Lf \mid u > a) = \exp\left(-\hat{\lambda}(Lf - a)\right) \quad (23)$$

Calculation of confidence of interval of the estimate (23) is easy; the distribution of the estimate of the parameter λ was assumed normal and its variance is already known, see equation (22). Taking a logarithm from both parts of equation (23) leads to:

$$\ln\left(\hat{P}_{Lf|a}\right) = -\hat{\lambda}(Lf - a) \quad (24)$$

The logarithm of the estimate is a deterministic function of the random variable and it is linear; thus the logarithm of the estimate also has normal distribution with the mean value equal to the logarithm of the probability estimate itself and the variance calculated as:

$$Var\left(\hat{P}_{Lf|a}\right) = (Lf - a)^2 \frac{\hat{\lambda}^2}{N(a)} \quad (25)$$

The second component of the probability of capsizing is the probability to exceed the threshold a. Its estimation does not encounter any principle difficulties:

$$\hat{P}_a = P(u > a) = \frac{N_Y - N(a)}{N_Y} \quad (26)$$

Confidence interval for the estimate of probability P_a can be evaluated using binomial distribution for the estimate:

$$low\left(\hat{P}_a\right) = \frac{Q_B\left(0.5\left(1 - P_{\beta 3}\right), N_Y, p = \hat{P}_a\right)}{N_Y}$$
$$up\left(\hat{P}_a\right) = \frac{Q_B\left(0.5\left(1 + P_{\beta 3}\right), N_Y, p = \hat{P}_a\right)}{N_Y} \quad (27)$$

Where Q_B is a quantile (inverse of the cumulative distribution function) of the binomial distribution; the first argument is the probability corresponding to the quantile, the second argument is a number of Bernoulli trials and the third argument is a probability of “success” in the individual trial. The confidence probability $P_{\beta 3}$ is a confidence probability for a component. Since the formula (1) contains three estimates, the confidence probability for each of them is calculated as:

$$P_{\beta 3} = \sqrt[3]{P_\beta} \quad (28)$$

Where P_β is desired confidence interval for the estimate of probability of capsizing 95% used in this numerical example

The complete rare problem, i.e. probability of capsizing after the upcrossing, is calculated as

$$\hat{P}_C = \hat{P}_a \cdot \hat{P}_{Lf|a} \quad (29)$$

Boundaries of the confidence interval are calculated using the product of corresponding boundaries of the estimates in equation (29).

$$low\left(\hat{P}_C\right) = low\left(\hat{P}_a\right) \cdot low\left(\hat{P}_{Lf|a}\right)$$
$$up\left(\hat{P}_C\right) = up\left(\hat{P}_a\right) \cdot up\left(\hat{P}_{Lf|a}\right) \quad (30)$$

COMPLETE SOLUTION

Non-Rare Problem

The solution of the non-rare problem is relatively simple within the framework of the adopted approach. The rate of upcrossing of the intermediate threshold ϕ_m needs to be estimated (not to be confused with the threshold for the differences between critical and instantaneous roll rate at the instant of upcrossing). By this definition, the threshed must be chosen in a way that the number of upcrossings becomes representative. For the threshold ϕ_m=13 deg in the considered numerical example, there were N_Y=207 upcrossings. The upcrossing rate can be estimated as

$$\hat{\xi} = \frac{N_Y}{N_t N_r \Delta t} \tag{31}$$

Where N_t=3600 is the number of points in each record, N_r=20 is the number of records and Δt=0.5 s is the time increment.

Boundaries of the confidence interval can be evaluated using binomial distribution, in a similar fashion to a probability estimate:

$$low\left(\hat{\xi}\right) = \frac{Q_B\left(0.5\left(1-P_{\beta 3}\right), N_t \cdot N_r, \hat{p}_\xi\right)}{N_t \cdot N_r \cdot \Delta t}$$

$$up\left(\hat{\xi}\right) = \frac{Q_B\left(0.5\left(1+P_{\beta 3}\right), N_t \cdot N_r, \hat{p}_\xi\right)}{N_t \cdot N_r \cdot \Delta t} \tag{32}$$

$$\hat{p}_\xi = \frac{N_Y}{N_t N_r}$$

The order of arguments for the quantile of the binomial distribution Q_B is the same as in equation (27).

Rate of Failures

The rate of failures is a more practical probabilistic metric for capsizing than probability, since the latter depends on the time of exposure:

$$\xi_C = \xi P_C \tag{33}$$

Thus, the extrapolated estimate of the rate of capsizes is expressed as:

$$\hat{\xi}_C^{Ex} = \hat{\xi} \cdot \hat{P}_a \cdot \hat{P}_{Lf|a} \tag{34}$$

The boundaries of the confidence interval are products of the respective boundaries of each component:

$$\begin{aligned} low\left(\hat{\xi}_C^{Ex}\right) &= low\left(\hat{\xi}\right) \cdot low\left(\hat{P}_a\right) \cdot low\left(\hat{P}_{Lf|a}\right) \\ up\left(\hat{\xi}_C^{Ex}\right) &= up\left(\hat{\xi}\right) \cdot up\left(\hat{P}_a\right) \cdot up\left(\hat{P}_{Lf|a}\right) \end{aligned} \tag{35}$$

INITIAL VALIDATION

To test the concept, an initial validation was carried out. The validation of the split-time method was understood in terms of the procedure for the validation of extrapolation methods developed and followed by Smith & Campbell (2013). Tier 1 of such validation procedure requires the comparison of the "true" value (estimated for a relatively large data set) with a series of extrapolations performed using much smaller data sets.

As this effort was primarily focused on the testing the concept, the size of the "large" dataset was only 500 hours. The conditions for this data and its source were described earlier in the paper. There were 21 capsizings in this data set – sufficient to estimate the true value:

$$\hat{\xi}_C^T = \frac{N_C}{N_t N_R \Delta t} \tag{36}$$

Where NC is a number of capsizings observed, N_R=1000 is the number of records in the "large" dataset (not to be confused with number of records in the extrapolation data set N_r=20). Evaluation of confidence interval for the "true" value, in principle, can be done similar to the non-rare problem (see Equation 31). However, the total number of points in the large data set can be too large to calculate the binomial quantiles. In this case, a normal distribution can be used instead of binomial (Smith & Campbell 2013). The results of the initial validation are shown in Figure 7. As one can see, 9 out of 10 extrapolations provided overlap of the confidence intervals and thus are considered "passed".

Following recommendations from Smith & Campbell (2013), the passing rate of 90% is accepted here, leaving 5% for non-passing due to random reasons (since P_β=0.95) and another 5% for other reasons, such as imperfections of tail fit. Then, for 10 trials, one can expect at least 7 extrapolations passed:

$$Q_B\left(0.5\left(1-P_\beta\right), N_E = 10, p = 0.9\right) = 7 \quad (37)$$

where N_E=10 is number of extrapolations and p=0.9 is required passing rate. Indeed the conclusion is favourable for the split-time method.

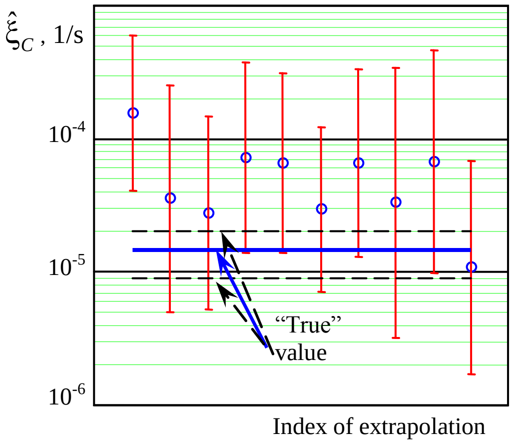

Fig. 7: Results of initial validation of the Split-Time method

CONCLUSIONS AND FUTURE WORK

The most important result of the present work is the first indication that the Split-Time method is capable of the evaluation of the probability of capsizing for the case of variation of righting arm in waves (i.e. for pure loss of stability).

The Split-Time method for righting arm variation has finally been made computable. This was achieved by fixing the threshold in roll angle and calculating the critical roll rate at the instant of upcrossing only. The tail was then fitted for the sample of the differences between the critical and instantaneous roll rates at the instant of upcrossings.

Also, the insight into perturbation of the roll rate at upcrossing has revealed the equivalency between the classical definition of ship stability and the general definition of motion stability of a dynamical system.

This has made the difference between the instantaneous and critical roll rates taken at the instant of upcrossing an effective metric for capsize event. Unlike the peak of the roll angle, this quantity remains smooth when capsizing occurs, so standard statistical extrapolation procedures become applicable.

A few items remain for the future work. Sample calculations carried out for a limited number of cases show some conservative bias, hinting that the exponential tail may be too heavy. It makes sense that the Generalized Pareto Distribution for non-zero shape parameter will do better.

Also, validation should be performed with cases significantly more rarity, like the one shown in Smith & Campbell (2013).

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