A pragmatic approach to roll damping

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ABSTRACT

Roll damping is probably the most intriguing of the components of hydrodynamic reaction in ship dynamics. It is also a problematic one - small, nonlinear, difficult to predict or measure and key determinant of ship stability. Without question, some of the problems faced in calculating or measuring roll damping are intrinsic. It can be argued, however, that most of the difficulties do not originate from physical anomalies of energy dissipation in roll but are due to fundamental flaws in the approach to roll damping estimation or measurement. The root causes of these flaws stem from three concepts, central to analysis of hydrodynamic reaction in roll: decomposition of the hydrodynamic reaction moment to added moment of inertia and roll damping moment, the assumption of small-amplitude motions and the inevitable coupling to other modes of motion. In this paper, the authors present a pragmatic approach to these fundamental concepts and discuss the implication of wrong assumptions, pertaining to definition, measurement, calculation and use of roll damping in intact and damaged ship dynamics.

Keywords: *roll motion, damping, hydrodynamics.*

1. MOTIVATION

The motivation and content for this paper derives from some of the journal and conference articles on roll damping published in recent years. Focusing only at the STAB papers and the most recent research projects, it is apparent that roll damping, as a research topic, attracts considerable attention. The problems addressed by researchers vary from uncertainty assessment in deriving critical damping from roll decay tests, estimation of damping from roll decay or forced roll (Wasserman). Both numerical and physical experiments are often conducted to the highest of standards with the help of sophisticated hardware and the most advanced analytical techniques. Unfortunately, it appears that many of the experiments on hydrodynamics of roll motion put emphasis on technicalities rather than the actual physics of the problem. Consequently, in spite of the perfect execution, the experiments per se are ill conditioned. Hence, whilst numbers are produced with remarkable efficiency and accuracy understanding of the nature of the problem is not being advanced. In the pursuit for finding a perfect solution, the fact that that effort has been expended on solving the wrong problem has been overlooked. In this respect, it is a good opportunity to have a more pragmatic view at the problem in hand.

2. THE EXPERIMENT

The following discussion is based on the physical experiments conducted in 2009/2010 at the Kelvin Hydrodynamic Laboratory of the Department of Naval Architecture, Ocean and Marine Engineering of the University of Strathclyde. The main objective of the experiments involved determining the hydrodynamic reaction in harmonic roll motion of an unconstrained cylindrical body forced to oscillate in calm water by an internal gyroscopic apparatus. The measurements, conducted in intact and damaged conditions were reported in (Cichowicz, 2012)

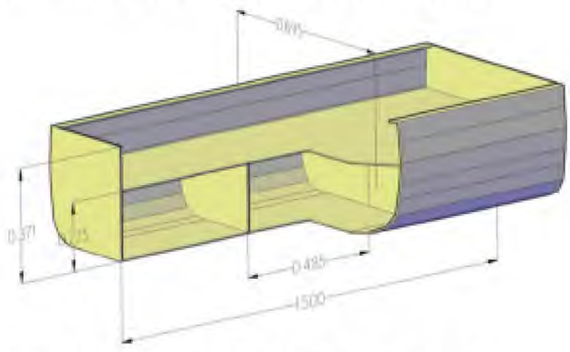

Figure 1: Main particulars of the tested cylinder

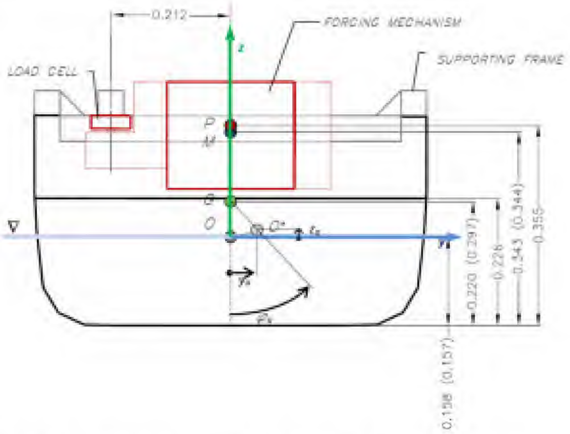

Figure 2: Schematic view of the model configuration

Figure 3: Photograph of the model taken during the test in intact condition

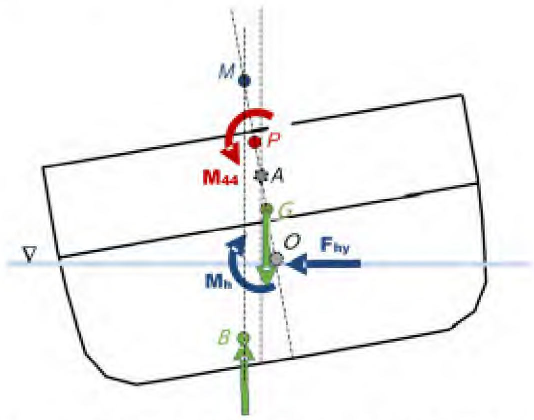

Figure 4: Free body diagram of the system under consideration

3. MATHEMATICAL MODELLING

Figure 4 shows a free-body diagram of the system under consideration. Given its cylindrical shape, the body (symmetrical with respect to centre-plane and midship-section) the system is represented as a 3DoF harmonic oscillator with the sway and heave motions resulting from coupling with roll (i.e. sway and heave are roll-induced). It is noteworthy, however, that due to the shape of the body and relatively small amplitudes of motions, the contributions from have were considered insignificant and for that reason the system could be simplified to 2DoF.

The moment to sustain motion, M_{44} ,was generated by an internal gyroscopic device pivoted about the point P. A single axis load-cell afforded the coupling between the forcing apparatus and the hull. The hydrodynamic reaction was expressed as a hydrodynamic moment, M_h and the force F_{hy} (introduced to capture the reaction due to the roll-into-sway coupled motion). The coupled motion was accounted for by the following condition

$$\begin{aligned} y_A(t) &= y(t) - \overline{OA} \cdot \varphi(t) = \\ &= y_a \sin(\omega t + \varepsilon_y) \\ &- \overline{OA} \cdot \varphi_a \sin(\omega t + \varepsilon_\varphi) \end{aligned} \tag{1}$$

The term $y_A(t)$ in (1) represents the lateral displacement of the instantaneous axis of rotation, $\overline{OA}$ is the elevation of the instantaneous axis of rotation above the waterplane, φ_a and y_a denote amplitudes of roll and roll-induced-sway, respectively, while ε_φ and ε_y stand for phase lags of the related motions (with respect to M_{44}), ω is circular frequency of oscillations and t, is time. It is noteworthy that the above expression would vanish if the roll and roll-induced-sway were in phase, i.e. if$\varepsilon_\varphi = \varepsilon_y$.

Given that the external moment (moment to sustain motion) was measured about the point P, it was convenient to express the equations of motion about this point as well. The system of two scalar equations of motions corresponding to the free-body diagram from Figure 4 is given below

$$\begin{aligned} &a_{22}\ddot{y} + a_{24}\ddot{\varphi} + b_{22}\dot{y} + b_{24}\dot{\varphi} \\ &= -m(\ddot{y} - \overline{OA} \cdot \ddot{\varphi}) \end{aligned}$$

$$\begin{aligned} &a_{22}\ddot{y}\,\overline{OP} + a_{24}\ddot{\varphi}\,\overline{OP} + a_{42}\ddot{y} + a_{44}\ddot{\varphi} \\ &+ b_{22}\dot{y}\,\overline{OP} + b_{24}\dot{\varphi}\,\overline{OP} + b_{42}\dot{y} + b_{44}\dot{\varphi} \\ &= M_{44} - (I_{44} + m\,\overline{AG}^2)\ddot{\varphi} - c_{44}\varphi \\ &- m(\ddot{y} - \overline{OA} \cdot \ddot{\varphi})\,\overline{AP} \end{aligned} \tag{2}$$

4. ANALYSIS

The system of equations given by (2) is constructed without any specific simplifications or assumptions (e.g., with respect to symmetry of the coefficients) and contains eight unknown hydrodynamic coefficients. Since the condition (1) may be interpreted as kinematic constraint, Lagrange's multipliers were chosen as the method to derive the hydrodynamic coefficients from the underdetermined system of equations of motion. The results of the analysis are presented in more detail in the following paragraphs.

Phase difference between roll and roll-induced sway

The results of measurements in both intact and damaged conditions, show clearly a measurable phase difference between roll and roll-into-sway motions (i.e. difference in phase angles measured with respect to moment M_{44}). The phase difference is particularily large in case of damaged hull at the sloshing resonance frequency (around 6.5 rad/s in this experiment) where it indicates strong damping effect in the roll-into-sway coupled mode of motion (Figure 5).

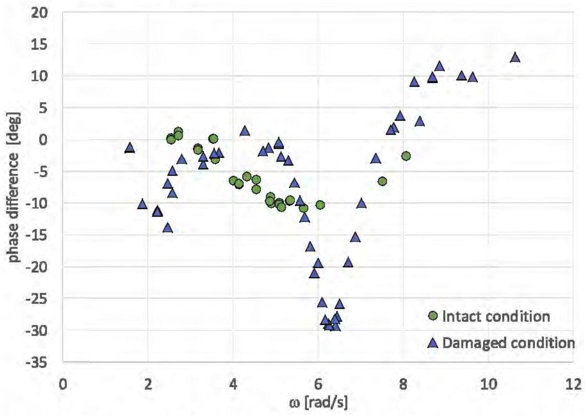

Figure 5: Phase difference between roll-into-sway and roll motion.

An immediate consequence of the relatively strong damping in roll-into-sway coupled mode of motion is that the inertia term of the sway equation in (2) does not vanish and the following paragraphs will show that this has some other, more significant, implications.

Hydrodynamic coefficients

Application of Langrange's multipliers method to the underdetermined system (2) gave rather interesting results, namely that:

- the sway coefficients a_{22} and b_{22}vanish in intact condition
- in damaged condition a_{22} and b_{22} vanish in the entire frequency range except the relatively narrow band around and beyond sloshing resonance
- the sway-into-roll coefficients a_{42} and b_{42}vanish in intact condition
- in damaged condition the coefficients a_{42} and b_{42} practically vanish outside the narrow bad around the sloshing frequency
- the roll (a_{44} and b_{44}) and roll-into-sway (a_{24} and b_{24}) coefficients are well determined across the entire frequency range

It is noteworthy that in the case of intact hull all the coefficients that vanish are those associated with the pure sway and sway-into-roll modes of motions, i.e. the motions that were not induced by the forcing device. The same holds for the damaged hull but only in the regions outside the sloshing resonance. Based on the above observations it can be concluded that the mathematical model given by (2) adequately describes motions of intact hull in the entire range of frequencies and the damaged hull outside the range of sloshing resonance. Furthermore, during the oscillations within the range of frequencies, close to sloshing resonance, the damaged ship experienced significant, constant velocity, drift. For this reason, the lateral displacement of the flooded hull was described as a linear combination of the translation in direction of the y-axis and harmonic oscillations i.e.$y(t) = v_d t + y_a \sin(\omega t + \varepsilon_y)$, where v_d stands for the constant drift velocity.

Axis of rotation

Un unconstrained body forced to roll in calm water-plane will oscillate about the so-called natural axis of rotation. This natural axis is instantaneous but herein, due to relatively small amplitudes and negligible heave motion, its eleveation is assumed constant throughout the entire cycle (at a given frequency). What is important, however, is that the elevation changes substancially across the frequency range, which is particularly well noticeable in the case of the damaged ship.

An analysis presented in (Balcer, 2004) shows that the location (i.e., elevation above the water plane) of the ship natural axis of rotation is a function of mass distribution within the oscillating system, comprising hull and the fluid domain (i.e. it is passing through the centre of mass of the entire system)

$$\overline{OA} = \frac{m\,\overline{OG} - a_{24}}{m + \delta m} \quad (3)$$

In the original paper, sway added mass, a_{22}, was shown as parameter δm. Considering, however, that the sway added mass vanish everywhere except the range of sloshing resonance of the damage ship it was necessary to replace it with a more suitable parameter in order to balance the equation. It was achieved by taking the measured elevation $\overline{OA}$ together with the a_{24} term, determined from the measurements and solved for δm. The results of this exercise show that the mass of the hull can reasonably well approximate the parameter δm across the entire frequency range except the range of sloshing frequency of the flooded hull (Figure 7). Obviously, it can be clearly seen that neither (3) nor (4) contain all the parameters needed to describe the elevation of the axis of rotation, which must depend on other coefficients as well.

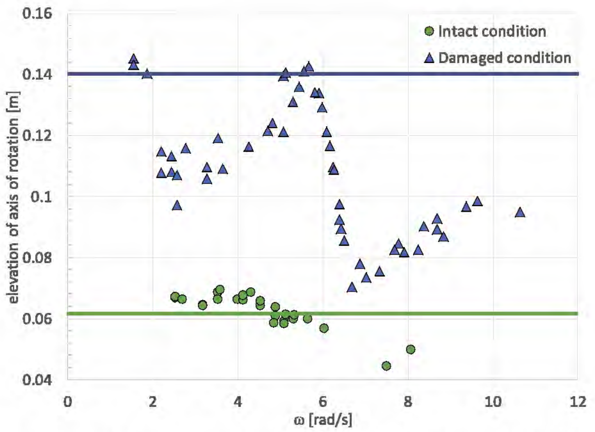

Figure 6: Elevation of axis of rotation above the calm water-plane

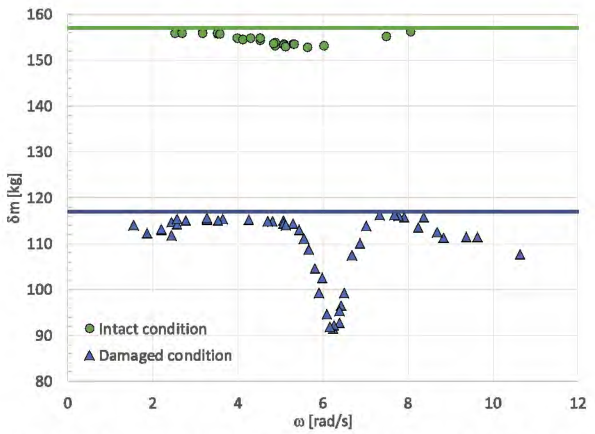

Figure 7: Comparison of the parameter δm as in (3) and the mass of the hull in intact and damaged conditions (solid lines)

Nevertheless, Figure 8 shows that outside of the sloshing resonance, the predicted elevation matches the measurements well. Hence, for the purpose of the following discussion it can be assumed that (4) describes the elevation of axis of rotation with satisfactory accuracy.

$$\overline{OA} = \frac{m\,\overline{OG} - a_{24}}{2m} \quad (4)$$

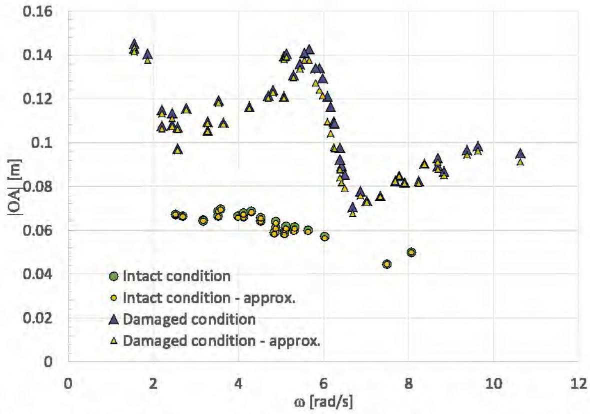

Figure 8: Comparison of measured and approximated by (4) elevation of the axis of ration.

5. SYNTHESIS

Whilst the discussion thus far has been centred about rather fragmented observations. These clearly indicated that

- Roll-induced sway is not exactly in phase with roll because of damping of in the coupling of roll-into-sway. As a result, the body is undergoing sideways motions (and the hydrodynamic reaction F_{hy} does not vanish) even if the external excitation has a form of pure moment.
- Elevation of the natural axis of rotation is determined by the mass of the body, its vertical centre of gravity and the added mass of roll-into-sway. Consequently, the elevation is a function of frequency of oscillation.
- The simple mathematical model of (2) remains valid even in the case of damaged hull, provided that the drift velocity is properly accounted for.

In order to synthesise this evidence, it is most convenient to look at the intact hull first and to subsequently attempt to extrapolate the findings for the damaged hull.

Firstly, it can be recalled that, according to Lagrange's multipliers method, all sway and sway-into-roll coefficients vanish from the intact ship equations of motions. Thus, the system (2) assumes the following, simplified form:

$$a_{24}\ddot{\varphi} + b_{24}\dot{\varphi} = -m(\ddot{y} - \overline{OA} \cdot \ddot{\varphi})$$

$$a_{24}\ddot{\varphi}\,\overline{OP} + a_{44}\ddot{\varphi} + b_{24}\dot{\varphi}\overline{OP} + b_{44}\dot{\varphi} = M_{44} - (I_{44} + m\,\overline{AG}^2)\ddot{\varphi} - c_{44}\varphi - m(\ddot{y} - \overline{OA} \cdot \ddot{\varphi})\,\overline{AP} \quad (5)$$

Taking advantage of the orthogonality, the equations can be expanded at the instant where $\omega t + \varepsilon_\varphi = 0$ and consequently $\ddot{\varphi}$ and φ vanish. At this instant it is implied that $M_{44} = M_{44a}\sin(\varepsilon_\varphi)$, $\dot{\varphi} = \varphi_a\omega$ and $\ddot{y} = -y_a\omega^2\sin(\varepsilon_y - \varepsilon_\varphi)$. Consequently, the equations of motion take the following form:

$$b_{24}\varphi_a\omega = my_a\omega^2\sin(\varepsilon_y - \varepsilon_\varphi)$$

$$(b_{24}\overline{OP} + b_{44})\varphi_a\omega = -M_{44a}\sin(\varepsilon_\varphi) + my_a\omega^2\sin(\varepsilon_y - \varepsilon_\varphi)\,\overline{AP} \quad (6)$$

Considering that roll amplitudes are small or moderate, it is implied that $y_a = \overline{OA}\sin\varphi_a \cong \overline{OA}\varphi_a$ and the sway equation ca be expressed as:

$$b_{24} = m\,\overline{OA}\,\omega\sin(\varepsilon_y - \varepsilon_\varphi) \quad (7)$$

Following similar procedure, allows for expressing the second (moment) equation as follows:

$$(b_{24}\overline{OP} + b_{44})\varphi_a\omega = -M_{44a}\sin(\varepsilon_\varphi) + m\overline{OA}\varphi_a\omega^2\sin(\varepsilon_y - \varepsilon_\varphi)\,\overline{AP} \quad (8)$$

However, it can be noted that the second term on the RHS of the above equation is simply $b_{24}\varphi_a\omega(\overline{AP})$. Hence, after a simple rearrangement, the moment equation can be given as:

$$b_{44} = \frac{-M_{44a}\sin(\varepsilon_\varphi)}{\varphi_a\omega} + b_{24}(\overline{AP} - \overline{OP}) \quad (9)$$

However, since $\overline{AP} = \overline{OP} - \overline{OA}$, the above equation can be further simplified, as shown next.

$$b_{44} = \frac{-M_{44a}\sin(\varepsilon_\varphi)}{\varphi_a\omega} - b_{24}\overline{OA} \quad (10)$$

At a first glance, there is nothing particularly remarkable about this equation. However, when combined with (7), it yields

$$b_{44} = \frac{-M_{44a}\sin(\varepsilon_\varphi)}{\varphi_a\omega} - m\,\overline{OA}^2\omega\sin(\varepsilon_y - \varepsilon_\varphi) \quad (11)$$

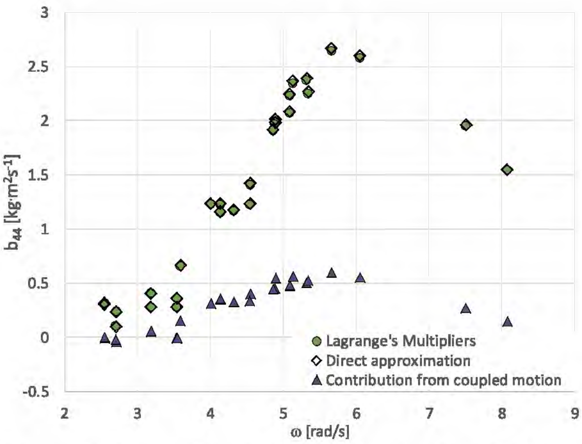

Figure 9 Comparison between b_{44} estimated by the Langrange's multipliers method as approximated by (11) for intact hull. The contribution from coupled roll-into-sway is represented by triangles.

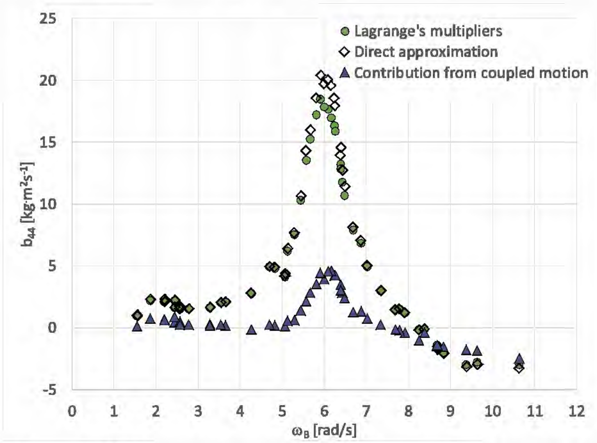

Figure 10 Comparison between b_{44} estimated by the Langrange's multipliers method and approximated by (11) for flooded hull. The contribution from coupled roll-into-sway is represented by triangles.

The results presented in Figure 9 and Figure 10 show very good agreement between the roll damping coefficient derived by Lagrange's multipliers method and approximated by (11) for both intact and flooded hull. In case of the flooded ship, direct approximation overestimates the damping in the range of flooding resonance.

6. ERRORS AND UNCERTAINTY

With roll damping being such a small quantity and of such complex composition, any inaccuracies in measurements in model experiments, particularly linked to the restoring/inertia moments will have a large impact on the value of the hydrodynamic coefficient being derived. Specifically, the uncertainty study reported in

(Cichowicz, Jasionowski, & Vassalos, 2011) and elaborated further in (Cichowicz, 2012) shows clearly that restoring coefficient, amplitude of external moment and hull inertia are dominant contributors to the uncertainty in estimates of roll added inertia (Figure 11). In the case of roll damping coefficient the key contribution comes from the phase angle between the excitation and response with some measurable impact from the magnitude of the forcing moment (Figure 12)

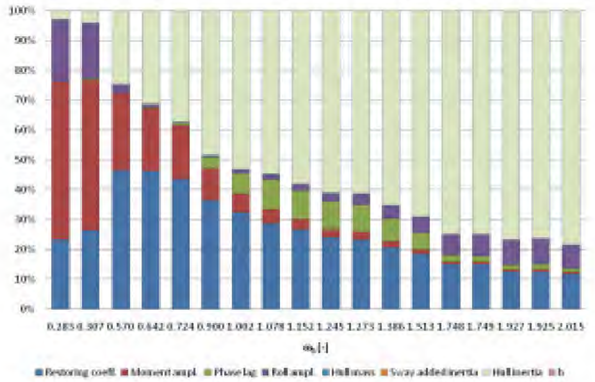

Figure 11 Relative contributions to the total error in a_{44} coefficient

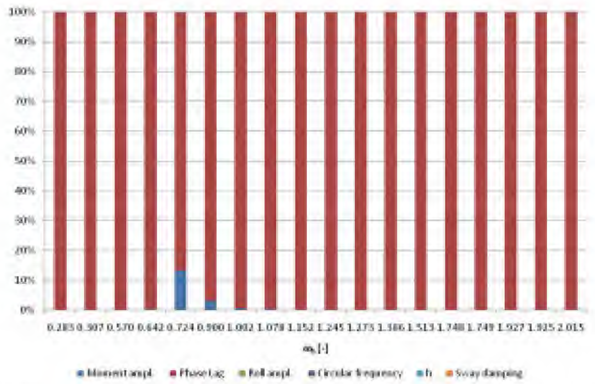

Figure 12 Relative contributions to the total error in b_{44} coefficient

7. CONCLUDING REMARKS

- Coupling of roll into sway affects roll damping through the square of the elevation of natural axis of rotation (eq. (11)).
- The elevation of axis of rotation depends on the added mass coefficient a_{24}, thus any hull fitting changing substantially the pressure distribution around the hull (i.e. added moment of inertia) such as bilge keels, will change the elevation of natural axis of rotation.
- Forcing roll motion about an arbitrary axis $\overline{OR}$, will have a strong impact on the dynamic equilibrium of the system, thus introducing additional forces necessary to maintain the constraints $\overline{OR} \neq \overline{OA}$ and $\varepsilon_y = \varepsilon_\varphi$. Since both constraints are dependent on a_{24} the forces to maintain this constraint will be affected by bilge keels, thus leading to nonlinear changes in roll damping.
- All roll and roll-into-sway coefficients can be derived from the forced-roll experiments on a floating body through (5), (7) and (11).

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