Estimating Dynamic Stability Event Probabilities from Simulation and Wave Modeling Methods

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ABSTRACT

Predicting the dynamic stability of ships in severe environments is challenging, not only due to the complex non-linear hydrodynamics, but also the need to characterize the rarity of events. The latter involves conducting enough simulations to calculate associated small probabilities or alternate approaches for estimating the rarity of events. This paper presents techniques for calculating probabilities of occurrence of rare dynamic stability events using direct counting, Poisson distribution fitting techniques and estimating the dynamic event probabilities. The latter probability estimate is obtained by defining dangerous wave conditions that produce rare events through hydrodynamic simulations and estimating their probabilities of occurrence through joint probability distributions or simulations of the wave environment. The accuracy of these calculations is discussed. An example application is presented using a U.S. Coast Guard Cutter along with information useful for operator guidance in heavy weather. A recommendation is presented for further work on defining the limiting probabilities one might use for design or operational criteria.

KEYWORDS

Ship stability; Dynamic stability; Simulation probabilities

INTRODUCTION

Intact dynamic stability is an important issue in design and operation particularly of smaller Naval ships and U.S. Coast Guard Cutters operating in demanding weather and wave conditions. Considerable longstanding research effort has been devoted to the estimation of dynamic event probabilities and associated risks for specific vessel types in given operational conditions. This paper discusses techniques for calculating probabilities of dynamic stability events, with emphasis on loss of intact stability and capsize in a dynamic wave environment.

There are varied motivations for risk evaluation (deployment in operations requiring high speeds and maneuverability in heavy seas, operator guidance in escaping storms, longer-term missions in less hostile environments, and so on), as well as the relative importance of determining precise numerical estimates, vis-à-vis reliance on pure operator experience. The direct counting approach for estimation of capsize probability of a vessel in a given operational period is simply to simulate a number of "runs" of the specified duration under conditions of interest and to obtain the proportion of these for which capsize occurs, yielding a natural (binomial) estimate whose statistical properties are known. Moreover, given reliable modeling of vessel motion, such simulation is applicable to capsize from any mode resulting from existing sea conditions in that simulation run, i.e. one does not have to model specific capsize mechanisms but allow the simulation program to determine what if any type of capsize occurs in the given sea. Simulation and direct counting statistics work especially well for severe seas where many runs can be expected to yield capsizes – the relevant estimation statistics is summarized in Part 1. However, for calmer less severe seas it may require excessive numbers of runs to produce even one capsize, resulting in potentially prohibitive simulation, amenable to the use of alternative methods emanating from the seminal work of de Kat (1994) as will be described in Part 2, along with illustrations of their use.

PART 1 SIMULATION ESTIMATES AND THEIR PRECISION

The Probability Estimates

One obvious and time-honored method for estimation of capsize probability of a vessel is to simply simulate a number of “runs” of given duration under conditions of interest and to obtain the proportion of these for which capsize occurs. Given good ship motion simulation, this yields a natural binomial estimate whose statistical properties are known. If n runs each of duration T are involved the capsize probability p_T in the time duration T is estimated by:

$$\hat{p}_T = r / n \tag{1}$$

where r is the number of the n runs for which capsize occurs. In severe sea states this can be accomplished by reasonably few simulations since capsizes will tend to occur relatively frequently, but for calmer less severe seas excessively many simulations may be required to achieve reasonable values of r and hence a meaningful capsize probability estimate.

A natural assumption which we make is that disjoint time intervals are independent as far as capsize is concerned – in the specific sense that the probability that capsize will not occur in either of two disjoint intervals is the product of the probabilities of no capsize for each interval. Based on this the capsize probabilities for intervals of different lengths are simply related to each other. For example it is immediate that if $p=p_1$ the capsize probability in unit time (e.g. 1 hr.) then p_T (the capsize probability for T hours) is related to p by the formula:

$$p_T = 1 - (1-p)^T \approx pT \tag{2}$$

(i.e. $p \approx p_T / T$)

to a close approximation when p is small. When considering lower sea states, we will use the approximation to provide a simple means of calculation of the capsize probability in any period from that in any other period. The same relationships may be used for the *estimates* of the probabilities involved.

Estimation Precision for p_T

ince the number of capsize runs r is a binomial random variable with parameters n, p_T, $\hat{p}_T$ has mean p_T i.e. is unbiased, and variance:

$$\mathrm{var}\left(\hat{p}_T\right) = \frac{p_T(1-p_T)}{n} \approx \frac{p_T}{n} \tag{3}$$

if p_T is small (low sea state conditions)

Hence the precision of the estimator $\hat{p}_T$ of p_T may be summarized by its standard deviation

$$\sigma_T = \sqrt{\frac{p_T}{n}}$$

estimated as:

$$\hat{\sigma}_T = \sqrt{\frac{\hat{p}_T(1-\hat{p}_T)}{n}} \approx \sqrt{\frac{\hat{p}_T}{n}} \tag{4}$$

This of course can be used to gauge how satisfactory $\hat{p}_T$ is as an estimator of p_T. Further, confidence limits and intervals can be simply constructed for p_T by assuming that the binomial distribution for r has reached its limiting normal approximation and hence that $\hat{p}_T$ is approximately normal. These however may be less accurate approximations in view of the small binomial probability, and it may be desirable to use exact intervals, obtained, e.g. from Swogstat.

Table 1 Per hr Capsize probabilities p for different sea states and total simulation time needed for estimating p with rsd=0.5 computed using an improved Poisson Regression model from Leadbetter (1983)

Hs (m)	T_1 (s)	Heading (degrees)	Speed (knots)	Capsize prob p (per hour)	Sim time needed for rsd=0.5 (hrs)
5	8	15	10	$1.35\cdot10^{-3}$	$3.0\ 10^{3}$
5	8	15	15	$8.57\cdot10^{-3}$	$4.7\ 10^{2}$
5	8	45	10	$4.52\cdot10^{-3}$	$8.8\ 10^{2}$
5	8	45	15	$3.51\cdot10^{-3}$	$1.1\ 10^{3}$
5	8	60	10	$2.49\cdot10^{-2}$	$1.6\ 10^{2}$
5	8	60	15	$2.14\cdot10^{-2}$	$1.9\ 10^{2}$
5	10	15	10	$1.69\cdot10^{-5}$	$2.4\ 10^{5}$
5	10	15	15	$1.17\cdot10^{-5}$	$3.4\ 10^{5}$
5	10	45	10	$6.65\cdot10^{-5}$	$6.0\ 10^{4}$
5	10	45	15	$5.37\cdot10^{-5}$	$7.4\ 10^{4}$
5	10	60	10	$4.87\cdot10^{-4}$	$8.2\ 10^{3}$
5	10	60	15	$4.28\cdot10^{-4}$	$9.3\ 10^{3}$
5	12	15	10	$5.57\cdot10^{-8}$	$7.2\ 10^{7}$
5	12	15	15	$4.10\cdot10^{-8}$	$9.8\ 10^{7}$
5	12	45	10	$2.51\cdot10^{-7}$	$1.7\ 10^{7}$
5	12	45	15	$2.09\cdot10^{-7}$	$1.9\ 10^{7}$
5	12	60	10	$2.30\cdot10^{-6}$	$1.7\ 10^{6}$
5	12	60	15	$2.06\cdot10^{-6}$	$1.9\ 10^{6}$
5	14	15	10	$3.56\cdot10^{-11}$	$1.1\ 10^{11}$
5	14	15	15	$2.73\cdot10^{-11}$	$1.5\ 10^{11}$
5	14	45	10	$1.80\cdot10^{-10}$	$2.2\ 10^{10}$
5	14	45	15	$1.53\cdot10^{-10}$	$2.6\ 10^{10}$
5	14	60	10	$2.01\cdot10^{-9}$	$2.0\ 10^{9}$
5	14	60	15	$1.81\cdot10^{-9}$	$2.5\ 10^{9}$

Capsize Probabilities in Standard (e.g. unit time) Period

The above formulae thus enable an appraisal of the accuracy of $\hat{p}_T$ as an estimator of the capsize probability p_T in a run of any length T. The feature of this is that the capsize probability in time T is estimated by simulation of (n) runs of hat same length T. For comparison and mission risk, evaluation it is convenient to have tables for capsize probabilities in standard (e.g. unit – 1 hr., 12 hr etc) periods of time, as a function of sea state. For example, for unit time the above procedure can be carried out exactly by simulating n runs of unit duration, but n may have to be very large to achieve good accuracy (perhaps prohibitively so for low sea states). A further potential disadvantage of short runs is the possibility that part of the time may be required for "startup" before the actual desired ocean conditions for the vessel are reached and initial conditions have an influence on response and related statistics. However, as noted above, the value of p_T for any T determines its value for any other T such as $T = 1$, by (2). In particular for unit duration to estimate $p = p_1$ one may instead estimate p_T for a much larger value of T using an appropriate number n of runs as above and infer the value of p or its estimate via (2). It may then in fact be shown that all such choices lead to the same accuracy in estimation of p, provided p is not too large. For example, when p is small the accuracy depends only on the *total projected simulation time* $M = T n$. That is if one chooses a total simulation time M say, the same estimate of p is obtained by choosing any run length T and taking $n = M / T$ runs of length T , with the same estimation error. This is intuitively reasonable since we might think of a long run of length M formed from M / T consecutive runs of length T. This intuitive reasoning does not apply to large values of p- when numbers of runs have capsizes, when resulting in wasted computer "downtime" following capsize in a run.

Runs Required to Obtain a Given Accuracy in Estimating p

As in the preceding section for estimating a general p_T, the standard deviation σ_1 of $\hat{p}_T$ is an appropriate measure of the accuracy of determination of $p = p_1$. This may be estimated by (5) for a prescribed total run time M. On the other hand if one does not know the value of p one may wish to determine the total run time so that it may be estimated with a given (at least estimated) accuracy $\hat{\sigma}_T$. If $\hat{p}$ were already known this could be obtained simply from (5) with $M = n\ T$ to give:

$$M = \hat{p} / \sigma_1^2 \qquad (5)$$

in which σ_1 is the accuracy (standard deviation) desired. Since the purpose at hand is to obtain a simulation estimate of p with prescribed accuracy, for given sea conditions, a natural procedure will be to use (5) to determine simulation requirements with an estimate $\hat{p}$ obtained in some way other than simulation, such as the Poisson Regression estimate of Part 2. It is not yet known how well such estimates will work for low sea states, but they are available and can be expected to give at least "ballpark" values, hence good guidance in the choice of total simulation time M. Of course once the resulting simulations are run, they can be used to validate the use of alternative and potentially less computer intensive estimation methods for low sea states. To illustrate this consider a series of simulations all with significant wave height 5m, having mean wave period T_1, speeds and headings as in Table 1. These are sea conditions used in simulations run by U.S. Coast Guard. The fifth column contains "Poisson regression estimates" (Part 2) of the 1hr capsize probabilities computed using improvements of the methods in Åberg (2008), and the final column uses these as preliminary estimates to give the total simulation time to give a 50% relative standard deviation (rsd) for the 1 hr simulation estimate.

PART 2 WAVE MODELING – POISSON REGRESSION METHODS

Generalities

We now turn attention to calmer sea conditions, for which an excessive simulation effort may be required, and describe alternative primarily theoretical involving mathematical (stochastic) modeling of sea states and the frequency of occurrence of waves deemed to be threatening to the stability of a vessel. This more theoretical approach requires more detailed understanding of the *particular* capsize mode involved and of wave characteristics which may thus threaten vessel stability. This may be obtained from a combination of theoretical and empirical studies – e.g. simulations for a moderate number of cases. Here we focus primarily on capsize of vessels caused by instability from large waves in following and stern quartering seas – of particular concern for smaller naval vessels. A U.S. Coast Guard cutter is used as an example. Our basic approach emanates from that of de Kat (1994) in which ranges of wave parameters (wavelength, wave-height) prone to cause capsize were suggested and the probabilities of waves with such characteristics evaluated for specific standard sea surface spectra (but without direct consideration of motion of the vessel or its interaction with waves.) Our aims here to obtain closer, much less conservative estimates by development of such ideas in important directions including: (i) characterization of relevant geometrical properties of waves which actually cause capsize when they encounter the vessel, based on U.S. Coast Guard studies of simulation histories of vessel operation in which capsize occurs, and their relationship to the vessel at capsize, and (ii) development of Poisson regression methods for prediction of intensities of critical waves from sea state and vessel parameters.

Dangerous Waves in Following Seas

There is of course an unlimited number of possible capsize modes and as noted, simulation with an appropriate simulation program (e.g. FREDYN deKat 1994) is the best method for estimating probability of capsize and related events under moderately high sea conditions. This approach counts capsizes caused by whatever mode occurs in a given sea and vessel operational conditions, and the event then becomes only statistically dependent to causes of particular modes. For less severe seas capsizes may be rare and one may wish to focus on particular capsize modes of interest, model mathematically the sea – ship motion conditions that are likely to lead to capsize from that mode, and estimate theoretically the probability of such combination for specific sea spectra and vessel operation. In particular we consider vessel capsize in following seas, where larger waves can lift the vessel and cause directional instability, leading to capsize.

Specifically in de Kat (1994) it is observed from simulations that certain specific ranges of wave amplitude and wavelength tended to engender capsize, and the probability that a wave would have parameters in this range is evaluated for given standard sea spectra from known theory by Longuet Higgins (1957). The assumption that such a wave would lead to capsize, thus provided capsize probability estimates. However, the wave calculations were done as would be observed from a ship at rest, and wave-ship encounter interaction was not considered, yielding a very conservative estimate. It is clear that better estimates of capsize probability should be obtainable from a more detailed understanding of what constitutes a "dangerous wave", i.e. what geometrical properties make it more likely to cause capsize when it encounters a vessel from behind. Further, it seems likely that the probability of such a wave causing a capsize will depend on factors such as the position and motion of the vessel relative to the overtaking wave when encounter is initiated. Therefore, U.S. Coast Guard simulations of vessel tracks and parameters were studied and times of capsize recorded along with the shape of the last wave preceding the capsize event, which we refer to as "triggering waves". Typical examples of these are shown in Fig 1 from which it can be seen that a common of the triggering waves is the similar (steep) slope between peak and trough.

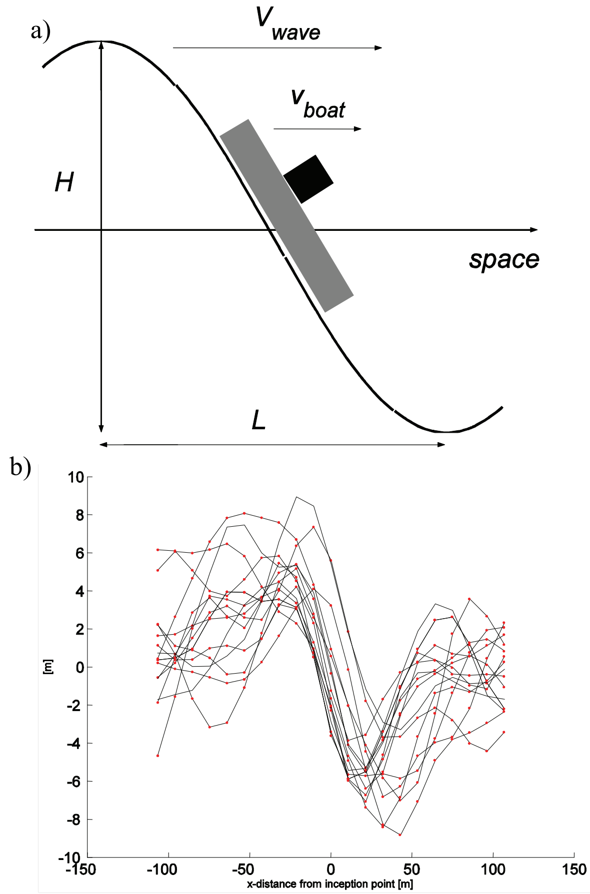

Fig.1: Illustration of an overtaking wave (a).Observed waves triggering capsize (b)

Influenced by this it seems reasonable to define a wave to be "dangerous" if its downward slope lies within some small range (typified by those of Fig 1) as the wave passes the centre of the vessel, in a sense to be made precise below. Influenced by Fig.1, we define a wave to be "dangerous" if its downward slope lies within some small range, viz. between -0.4 and -0.2 as the wave passes the center of the vessel. Our program is to calculate the rate $\mu_D = \mu_D(\theta)$ at which dangerous waves are expected to overtake the vessel, and further adjust this by the estimated probability that a dangerous wave will cause capsize, to obtain the probability of capsize in a given time period. For this we take the mathematically equivalent view point of regarding a vessel as "righted" after capsize and evaluate probability as estimated by the expected number of capsizes, which will depend on the type of ship, and operating conditions: sea state, heading, speed, duration etc.

We summarize the operating conditions in a vector of parameters θ, such as vessel type, heading α; speed v, and sea surface, modeled as a stationary Gaussian field having Bretschneider (1959) power density spectrum. This spectrum is parameterized by the so called significant wave-height H_s and mean wave period T_1, viz. $\theta = (H_s, T_1, \alpha, v)$. Let $\lambda = \lambda(\theta)$ denote the capsize intensity (rate), measured in years^{-1} (hours^{-1}) as convenient, under the operational conditions θ. The probability p_T of capsize in a given time T can be immediately obtained from the capsize rate λ(θ), as p_T=1-exp(-λT).

Estimation of Event Intensity λ(θ)

The proposed approach is to use the estimated event rate intensities λ(θ) in the range about 10^{-2}, which can be accurately estimated using 12 hour mission simulation, to model λ(θ). Then the method is validated, viz. by means of extrapolation to operational conditions θ with smaller risks for capsizing, about 10^{-3}. (Such intensities can be estimated using the simulation program.) Then, if the model satisfactorily predicts such risks, one could expect to use it to predict the annual probabilities and risk. The first step is to calculate $\mu_D = \mu_D(\theta)$ mathematically from the assumed sea state, spectral form, and vessel motion parameters. The calculations are too lengthy and intricate to re-produce here; but, may be found in references Leadbetter (2007), Åberg (2008), Rychlik (2007) (see also the forthcoming work Rychlik (2011).

Two components are involved – first to calculate the rate of occurrence of waves overtaking the vessel, and then adjust this by the probability that the specified steep wave slope will be attained, to give the expected dangerous wave occurrence rate $\mu_D = \mu_D(\theta)$. The second and final step is to adjust this by a factor representing capsize probability from a dangerous wave, requiring estimation from a number of capsize runs. This involves a "Poisson Regression" (or log-linear model, with Poisson distributions which is more natural for counting data than the customary linear regression with normal distributions.)

Specifically we propose the model

$$\lambda(H_s, T_1, \alpha) = b_1 (H_S / T_1)^{b_2} \mu_D(\theta) \qquad (6)$$

This model is then fitted to actual observed capsize rates from a series of simulation runs, fitting the parameters b_1 b_2 by standard Poisson Regression methods. The procedure has been programmed in Excel for these applications, and examples are given in Fig. 2 for a one hour exposure timeframe. Probabilities less than 10^{-8} are negligible and set equal to zero.

Validation of Poisson Regression or Other Estimation Methods

For more severe seas the Poisson Regression estimation (or any proposed alternative estimation method) may be validated by comparison with simulation, which is known to provide reliable results. It is no longer possible to make comprehensive comparisons for low sea states, as noted relative to the required simulation effort evident from Table 1 at least in view of the excessive simulation times which may be needed for a few cases requiring longer simulations that can be done routinely. Such "spot checks" do not all have to be done on the same computer of course, since as shown it is only the total simulation time that affects the value of the (simulation) estimate and its accuracy. Hence, the time needed can be reduced if many computers are available – even if for short periods such as weekends or other normally idle time. Figure 3 presents a comparison of the Direct Counting (Part 1) and the "Dangerous Wave" (Part 2) approaches described in this paper for an example U.S. Coast Guard Cutter.

CONCLUSIONS AND RECOMMENDATIONS

This paper discusses two approaches for estimating the statistics of dynamic events associated with ships operating in diverse wave environments, and their specific application depending on the desired accuracy and simulation time available and practical. In particular these include:

1. Direct counting approaches provide accurate statistical information for estimating the probabilities of dynamic events in severe wave conditions, and enable confidence bounds to be determined for the estimates.
2. Characterization of the wave conditions, wave – vessel interactions, and related probability of occurrence in a given seaway associated with dynamic events is found to be a useful approach to avoid excessive simulation time required to predict rare events, especially for less severe seas.
3. Further work is recommended to compare predicted probabilities to actual experience in severe steep wave conditions in order to benchmark the predictions for operator guidance.

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DISCLAIMER

The opinions expressed herein are those of the authors and do not represent official policy of the U.S. Coast Guard

						EXAMPLE						
						Probability of Capsize Calculation						
						USCG Cutter						
						1 hr Exposure, Stern Quartering Seas, 15 kts						
						Tp (sec)						
		7.5	8.5	9.7	10.9	12.4	13.9	15	16.4	18	20	
	0.5	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	
	1.5	3.669E-08	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	
	2.5	1.560E-08	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	
	3.5		1.670E-05	5.350E-07	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	
			R-16	R-12		Weather Routing Limit						
Hs (m)	4.5			3.100E-03	3.170E-04	9.080E-06	1.060E-07	0.000E+00	0.000E+00	0.000E+00	0.000E+00	
				R-18	R-15	R-12						
	5.5			5.680E-02	1.070E-02	8.250E-04	3.550E-05	2.300E-06	3.860E-08	0.000E+00	0.000E+00	
					R-19	R-15		R-12				
	6.5				9.500E-02	1.340E-02	1.230E-03	1.570E-04	7.500E-06	1.190E-07	0.000E+00	
							R-20	R-15	R-12			
	7.5					8.710E-02	1.320E-02	2.590E-03	2.390E-04	9.510E-06	7.160E-08	
								R-20	R-15		R-12	
	8.5											

Fig 2 – Example of probability of capsize calculations in stern quartering seas using the "Dangerous Wave" approach. R- is significant roll amplitude

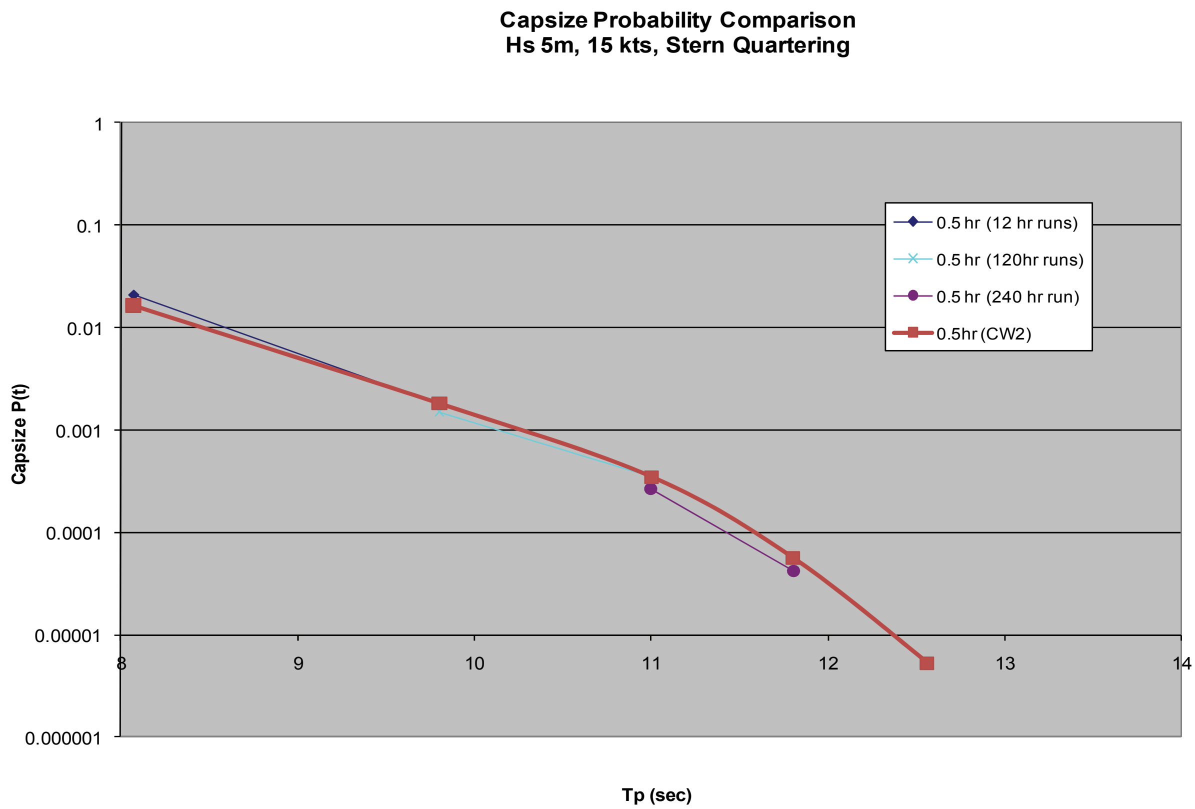

Fig. 3 – Comparison between "direct counting" and "dangerous wave" approaches for predicting probability of capsize