On the Tail of Nonlinear Roll Motions

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ABSTRACT

The paper describes the qualitative study of the tails of the distribution of large-amplitude roll motions. The nonlinearity of a dynamical system is modeled with piecewise linear stiffness with stable and unstable equilibria. Closed-form formulae were derived for the peak value and its distribution. The tail of the distribution is heavy until in close proximity to the unstable equilibrium and then becomes light with the right bound at the unstable equilibrium. It is shown that the tail structure is related to the shape of the stiffness curve. Physical reasoning for such tail structure is based on the phase plane topology. The tail first becomes heavy due to stretching of the phase plane, which is a result of nonlinearity. The inflection point in the tail (when it becomes light) is related to increased capsizing probability in the vicinity of unstable equilibrium; the position of the inflection point can be evaluated, defining domain of heavy tail applicability.

Keywords: *Nonlinear Roll Motion, Distribution, Extremes*

1. INTRODUCTION

Probabilistic assessment of partial dynamic stability failure is essentially an extreme value problem for nonlinear roll motions. Some progress has been recently reported by Campbell, *et al* (2016) on applying Generalized Pareto distribution (GPD) to model the extreme values of roll peaks, above appropriate threshold (Coles, 2001). Mathematical aspects of the problem are treated in (Glotzer et al 2016). Statistical validation of this method was described by Smith and Zuzick (2015). While, in general, the method has shown satisfactory performance, its accuracy may be improved by applying one-parameter GPD instead of two-parameter GPD. It requires introducing a relation between the GPD parameters based on physical properties of the dynamical system. This relation is the main objective of this paper.

Normally, GPD has two parameters: shape and scale. If the shape parameter equals zero, GPD turns into the exponential distribution. This is the case of a normally distributed quantity; distribution of its extreme values can be approximated by the exponential distribution. If the shape parameter is positive, the tail is usually referred to as “heavy,” as its probability of extreme value is higher compared to normal/exponential case. If the shape parameter is negative, the probability of extreme value is lower compared to exponential and the tail is referred as “light.” One of the specific features of a light tail is a right bound, the upper limit of the distribution; all values exceeding the right bound have zero-probability.

The appearance of right bound in a distribution of roll peaks has a clear physical reason. A peak implies a return after reaching a local maximum. As a ship may capsize, there is a limit for the roll peak, which should be reflected as a right bound by statistics. However, GPD fitting, reported in Campbell, *et al* (2016) resulted in positive shape parameter and no right bound.

The question this paper tries to answer formulates as follows: if a ship can capsize, the tail of roll peak distribution should be light, so why is a heavy tail observed in numerical simulations?

2. PIECEWISE LINEAR SYSTEM

A dynamical system with piecewise linear stiffness is probably the simplest model of capsizing, as it allows recreation of correct phase plane topology, see Figure 1. It also allows a closed form solution for probability of capsizing under some assumptions; see review in (Belenky, *et al* 2016). So consider a dynamical system:

$$\ddot{\phi} + 2\delta\dot{\phi} + \omega_0^2 f_L(\phi) = f_{E\phi}(t) \qquad (1)$$

where δ is a linear damping coefficient and $f_{E\phi}$ is a stochastic process of roll excitation, while the roll stiffness f_L is shown in Figure 1. It is assumed that the excitation is “switched-off” once the roll angle exceeds ϕ_{m0}, reflecting absence of resonance for negative *GM* and limited ability to react on waves.

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Smith T.C., Campbell, B.L. (2013) "On the Validation of Statistical Extrapolation for Stability Failure Rate," *Proc. 13th Intl. Ship Stability Workshop*, Brest, France.

Smith, T. C., and Zuzick, A. (2015) "Validation of Statistical Extrapolation Methods for Large Motion Prediction," *Proc. 12th Intl. Conf. on Stability of Ships and Ocean Vehicles (STAB 2015)*, Glasgow, UK.

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There were two cases when the passing rate fell below 0.9: for headings 55 and 60 degrees at 9 m waves. In general, the variability of the passing rate within the same environment condition is not small. The last column in Table 2 shows averaged passing rate per condition, which is equivalent to 150 extrapolation data sets. The averaging passing rate fell below 0.9 only once, for 55 degree heading, indicating favorable tendency with the increase of sample size.

Finally, if one averages the passing rate over all the conditions tested, the theoretical 0.95 is obtained. This is yet another indication of the statistical correctness of the split-time method.

5. CONCLUSIONS AND FUTURE WORK

The split-time method for estimating probability of capsizing caused by pure loss of stability has been subjected to statistical validation for 14 environmental conditions. The true values were obtained by a very fast volume based numerical simulation with a time of exposure of up to one million hours full-scale. The rare problem solution is based on single degree-of-freedom perturbations. The average passing rate per condition varied from 0.87 to 0.99, falling below 0.90 for a single condition. The passing rate averaged over all the tested condition was 0.95, while the confidence probability was 0.95. These results are encouraging.

At the same time, the described validation campaign shows the necessity to refine the acceptance criteria, in particular what passing rate should be expected depending on how many extrapolation data sets were used. The acceptance criteria are needed for the tier-three validation level which addresses overall acceptance.

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8. REFERENCES

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Table 1 Summary validation conditions and "true" value estimates

Significant wave height, m	Modal Period, s	Heading, degrees	Exposure, hr	Number of Capsizes	Estimate of rate 1/s	Low boundary of rate	Upper boundary of rate
8.5	14	45	200,000	8	1.13E-08	4.24E-09	1.98E-08
8.5	14	60	200,000	31	4.38E-08	2.97E-08	5.93E-08
9	14	35	720,000	12	4.71E-09	2.04E-09	7.37E-09
9	14	40	200,000	12	1.70E-08	8.48E-09	2.68E-08
9	14	45	200,000	51	7.20E-08	5.37E-08	9.18E-08
9	14	50	20,000	7	9.89E-08	2.83E-08	1.84E-07
9	14	55	60,000	69	3.25E-07	2.50E-07	4.05E-07
9	14	60	200,000	176	2.49E-07	2.12E-07	2.85E-07
9	14	65	200,000	80	1.13E-07	8.90E-08	1.38E-07
9	14	70	200,000	6	8.48E-09	2.83E-09	1.55E-08
9	15	45	345,000	10	8.19E-09	3.11E-09	1.33E-08
9	15	60	300,000	11	1.04E-08	4.71E-09	1.70E-08
9.5	15	45	1,000,000	157	4.44E-08	3.74E-08	5.13E-08
9.5	15	60	1,000,000	242	6.84E-08	5.98E-08	7.70E-08

Table 2 Summary of validation results

Significant wave height, m	Modal Period, s	Heading, degrees	Subset duration, hrs	Passing rate Sample 1	Passing rate Sample 2	Passing rate Sample 3	Averaged passing rate
8.5	14	45	2,000	1.00	0.98	0.90	0.96
8.5	14	60	2,000	0.92	0.96	0.94	0.94
9	14	35	2,000	1.00	0.98	0.98	0.99
9	14	40	2,000	1.00	0.98	1.00	0.99
9	14	45	2,000	0.98	0.98	0.96	0.97
9	14	50	2,000	0.98	0.92	0.94	0.95
9	14	55	2,000	0.90	0.80	0.92	0.87
9	14	60	2,000	0.90	0.86	0.94	0.90
9	14	65	2,000	0.94	0.92	0.94	0.93
9	14	70	2,000	0.92	1.00	0.90	0.94
9	15	45	2,000	0.98	0.96	0.96	0.97
9	15	60	2,000	0.96	0.98	0.98	0.97
9.5	15	45	2,000	0.96	0.94	0.96	0.95
9.5	15	60	2,000	0.98	0.94	0.96	0.96