A Mathematical Model to Describe Ship Motions Leading to Capsize in Severe Astern Waves

by
Masami HAMAMOTO and **Abdul MUNIF**
Osaka University

ABSTRACT

A reasonable method used in prediction of ship motions leading to capsize in severe waves is developed on the basis of strip method. In this method the variation of metacentric height in waves is taken into account. Several simulations were conducted to predict the stability against capsizing of a container carrier 15000GT in severe waves due to parametric rolling. Finally the stable and unstable areas of the ship running in severe astern seas are computed

1. INTRODUCTION

As well known a linearized dynamic-hydrodynamic analysis of ship motion in waves has been successfully obtained in the strip method. The strip method now provides a workable design tool for predicting the average seakeeping performance of a ship early in the design process. The Performance which has been successfully described by linear procedures are ship motions, structural loads and even occurrence of seemingly nonlinear large amplitude phenomena such as the frequency of slamming and bow immersion. However, the strip method has partially developed for predicting stability against capsizing due to the parametric resonance, pure loss of stability and broaching-to of a ship running through astern seas, because the strip method has been mainly concerned with small amplitude periodic motion in a higher frequency range. By taking into account the variation of metacentric height based on the right arm curve of a ship in waves, it is useful to review some of the features of linear motion theory in hopes that its results may provide some guidance and insight into ship motion leading to capsizing. When a ship is running through waves with a constant forward speed U and encounter angle χ to waves, the linearized equations of motion with respect to heaving displacement ζ_G, pitching angle θ, swaying displacement η_G, yawing angle ψ and rolling angle ϕ, are described by:

$$
\begin{aligned}
m\ddot{\zeta}_G &= Z(Rad) + Z(Dif) + Z(FK) + W \\
I_{yy}\ddot{\theta} &= M(Rad) + M(Dif) + M(FK) \\
m\ddot{\eta}_G &= Y(Rad) + Y(Dif) + Y(FK) \\
I_{zz}\ddot{\psi} &= N(Rad) + N(Dif) + N(FK) \\
I_{xx}\ddot{\phi} &= K(Rad) + K(Dif) + K(FK) - W\overline{GM}\phi
\end{aligned}
\tag{1}
$$

where m is the mass of ship, and, I_{yy}, I_{zz} and I_{xx} the mass moments of inertia of ship about y,z and x axes as shown in Fig.1, $Z(Rad)$ and $Y(Rad)$ the radiation forces of heaving and swaying motions, $M(Rad), N(Rad)$ and $K(Rad)$ the radiation moments of pitching, yawing and rolling motions, $Z(Dif)$ and $Y(Dif)$ the diffraction forces of incident waves, $M(Dif), N(Dif)$ and $K(Dif)$ the diffraction moments of incident waves, $Z(F.K)$ and $Y(F.K)$ the Froude-Krylov forces including the hydrostatic forces, $M(F.K), N(F.K)$ and $K(F.K)$ the Froude-Krylov moments including the hydrostatic moments W the ship weight and $\overline{GM}$ the metacentric height.

In these linearized equations, the radiation, diffraction and Froude-Krylov forces acting on a section of the ship in the equilibrium position, can be computed by the ordinary strip method, but the linearized restoring moment is computed for the equilibrium position can not be used for a ship with metacentric height varies with respect to the relative position of ship to waves and wave steepness. As pointed out by Paulling[1) 2)], the variation of metacentric height is caused by the change of water plane area in the flare of fore and aft parts of ship hull plane with respect to the relative position of a ship to a wave. In order to take into account the effect of the variation, the linearized restoring moment of a ship in astern seas should be described by:

$$W\overline{GM}\left[1+\frac{\Delta\overline{GM}}{\overline{GM}}\cos\left(\omega_e t-k\xi_0\right)\right]\phi \tag{2}$$

instead of $W\overline{GM}\phi$ in the last equation of Eq.(1).

where $\Delta\overline{GM}$ is the variation of metacentric height, ω_e the encounter frequency of ship to waves, k the wave number and ξ_0 the initial position of ship to waves.

The problem here is how to predict the variation of metacentric height consisting of the wave height to length ratio, H/λ, wave length to ship length ratio λ/L, encounter angle of ship to waves χ and the geometry of ship hull. The purpose of this study is to investigate the insight of parametric resonance taking into account the variation of metacentric height of a contain carrier running with constant forward speed U.

2. VARIATION OF METACENTRIC HEIGHT

In general the matacentric height $\overline{GM}$ can be obtained from the righting arm curve which is given by a nonlinear function of rolling angle ϕ. When a ship is displaced in a regular wave with rolling angle ϕ and encounter angle χ of ship to waves, the Froude-Krylov moment $K(F.K)$ including the hydrostatic buoyancy with respect to the rolling about the center of gravity G is described as follows:

$$K(F.K)=-\int_L dx\iint\left[y\left(\frac{\partial p}{\partial z}\right)-(z-\overline{OG})\left(\frac{\partial p}{\partial y}\right)\right]dydz \tag{3}$$

where:

$$\frac{\partial p}{\partial y} = \rho g \sin\phi + \rho g a k e^{-k\sigma d} \sin\phi \cos k\Theta$$
$$- \rho g a k e^{-k\sigma d} \cos\phi \sin\chi \sin k\Theta$$
$$\frac{\partial p}{\partial z} = \rho g \cos\phi + \rho g a k e^{-k\sigma d} \cos\phi \cos k\Theta \qquad (4)$$
$$+ \rho g a k e^{-k\sigma d} \sin\phi \sin\chi \sin k\Theta$$
$$\Theta = \xi_G + x\cos\chi - (y\cos\phi - z\sin\phi)\sin\chi - ct$$

ρ is water density, g the gravitational acceleration, a the amplitude of a regular wave, k wave number, ξ_G the position of ship to wave, c phase velocity of a wave, σ the sectional area ratio of x coordinate, t time, d draft in equilibrium, $\overline{OG}$ the position of center of gravity measured from the origin of body coordinate system $O-x,y,z$ in which the x is directed forward, the z axis directed downward and y axis directed to starboard as shown in Fig.1. In this computation, the integrals are taken over all volume up to the instantaneous submerged surface and the relative position of ship to wave is defined at t =0 by the ratio of ξ_G to the wave length λ.

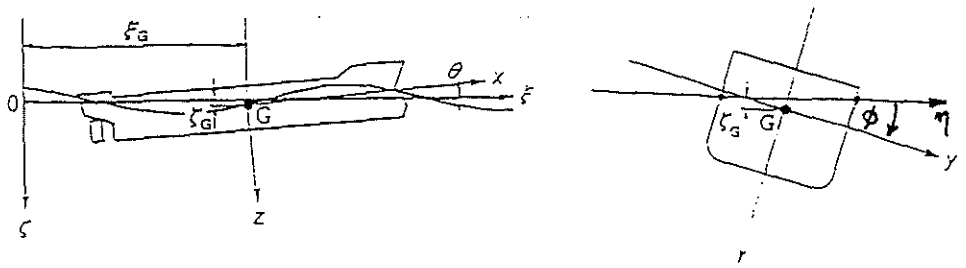

Fig.1. Coordinate system

And the displacement of submerged hull in waves must equals to the ship weight of the equilibrium condition in a still water and the pitching angle θ must be in the balance of pitching moment acting on the submerged hull. The Froude-Krylov moment $K(F.K)$ can be rewritten as:

$$K(F.K) = -\rho g \int_L dx \iint (y\cos\phi - z\sin\phi) dydz$$
$$- \rho g \overline{OG} \sin\phi \int_L dx \iint dydz$$
$$- \rho g a k \int_L e^{-k\sigma d} dx \iint (y\cos\phi - z\sin\phi)\cos k\Theta dydz \qquad (5)$$
$$- \rho g a k \sin\chi \int_L e^{-k\sigma d} dx \iint (y\cos\phi + z\sin\phi)\sin k\Theta dydz$$
$$- \rho g a k OG \sin\chi \int_L e^{-k\sigma d} dx \iint (\sin\phi\cos k\Theta - \sin\chi\cos\phi\sin k\Theta) dydz$$

In this equation, the first and the second terms are righting moments due to the hydrostatic force acting on the submerged volume of ship hull in waves. According to the strip method, the righting arm $\overline{GZ}$ is defined by these two terms as:

$$W\overline{GZ} = \rho g \int_L dx \iint (y\cos\phi - z\sin\phi)dydz + \rho g \overline{OG}\sin\phi \int_L dx \iint (y\cos\phi - z\sin\phi)dydz \tag{6}$$

In studying the large amplitude rolling motion, the method of equivalent linearization has been utilized for describing a dynamic system in which large deviations from linear behavior are not anticipated. A reasonable approximation to the exact behavior of the real system, therefore, would be given by an equivalent linear system having linear coefficient approximately selected. The $\overline{GZ}(wave)$ of container carrier as shown in Fig.2 increases at the wave trough amidship and decreases at the wave crest amidship in comparison with the righting arm $\overline{GZ}(still)$ in still water as shown in Fig.3.

Items		Ship	Model
Length	L(m)	150	2.5
Breadth	B(m)	27.2	0.453
Depth	D(m)	13.5	0.225
Draught	d_f(m)	8.5	0.142
	d_a(m)	8.5	0.142
Block Coefficient	C_b	0.667	0.667
Metacentric Height	$\overline{GM}$(m)	0.3 0.6 0.9	0.051 0.077 0.112
Natural roll Period	T_ϕ	37.95 27.01 21.85	4.9 3.5 2.7
Model scale	---		1/60

Fig. 2. Principal particulars of container carrier

When the ship is rolling in astern seas, the rolling angle develops significantly large. Therefore, the equivalent metacentric height should be determined on the basis of the righting arm curve considered up to an appropriate angle of inclination as follows:

$$\begin{aligned}
\int_0^{\phi_r} \overline{GM}(still)\phi\, d\phi &= \int_0^{\phi_r} \overline{GZ}(still)\, d\phi \\
\int_0^{\phi_r} \overline{GM}(trough)\phi\, d\phi &= \int_0^{\phi_r} \overline{GZ}(trough)\, d\phi \qquad (7) \\
\int_0^{\phi_r} \overline{GM}(crest)\phi\, d\phi &= \int_0^{\phi_r} \overline{GZ}(crest)\, d\phi
\end{aligned}$$

where ϕ_r is the vanishing angle, $\overline{GM}(still)$, $\overline{GM}(trough)$ and $\overline{GM}(crest)$ the equivalent linearized metacentric heights in still water, wave trough and wave crest respectively.

A further consideration is required to specify a reasonable expression of $\overline{GM}(wave)$ leading to a really equivalent solution. For this problem, an assumption is made here that the variation of metacentric height $\overline{GM}(wave)$ is sinusoidal and finally it is given by the following form:

$$\overline{GM}(wave) = \overline{GM}(still)\left[1 + \frac{\Delta\overline{GM}}{\overline{GM}(still)}\cos(\omega_e t - k\xi_0)\right] \tag{8}$$

where

$$\frac{\Delta\overline{GM}}{\overline{GM}(still)} = \frac{\overline{GM}(trough) - \overline{GM}(crest)}{2\overline{GM}(still)} \tag{9}$$

The values of $\overline{GM}(still)$, $\overline{GM}(trough)$ and $\overline{GM}(crest)$ can be obtained by using the energy balance concept, their values depend on the wave steepness H/λ, the wave to ship length ratio λ/L, the encounter angle of ship to waves χ and the metacentric height $\overline{GM}$ in still water. For $\lambda/L=1$, $H/\lambda=1/20$, $\overline{GM}=0.6$m and $\chi=0$ their values are shown in Fig.4, and the values of $\Delta\overline{GM}$ for several wave steepness, $\overline{GM}$ and encounter angle χ are given in Fig.5.

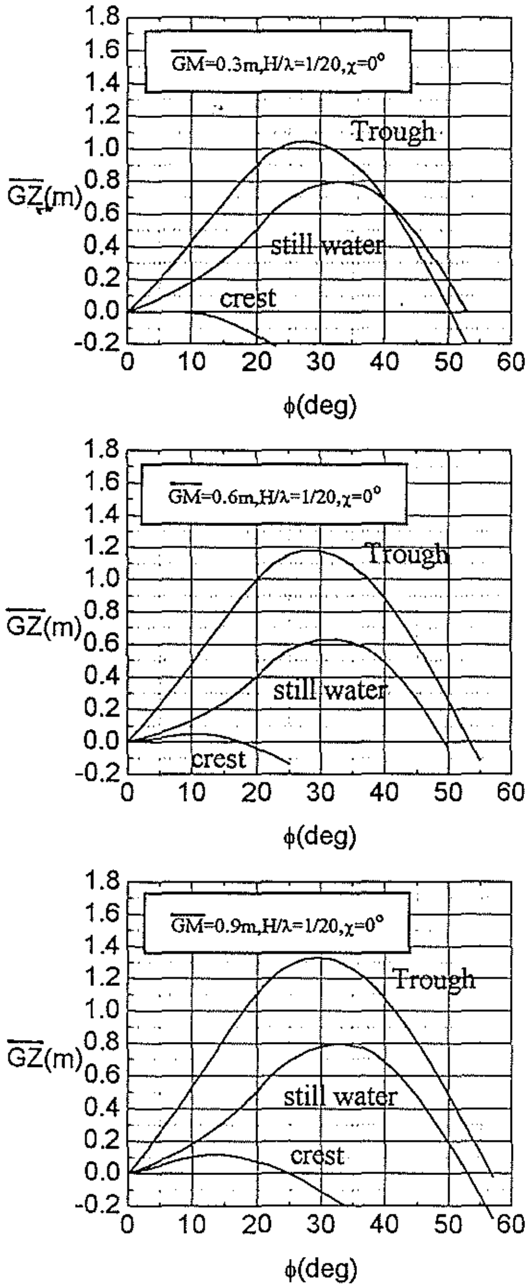

Fig. 3. The righting arm $\overline{GZ}$ curves of container carrier

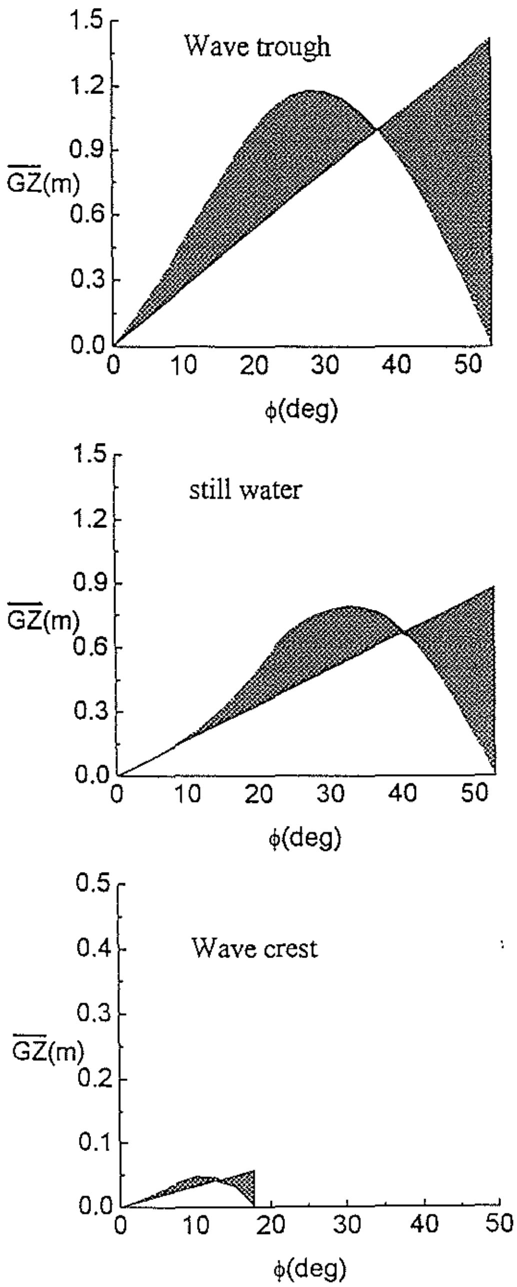

Fig.4. Equivalent linearized metacentric height for $H/\lambda=1/20$, $\overline{GM}=0.6$m and $\chi=0$

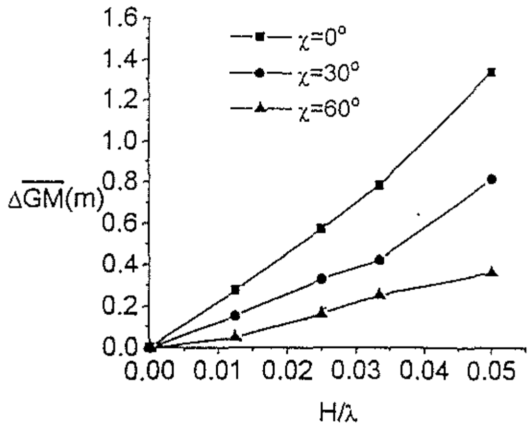

Fig.5. The variation of equivalent linearized metacentric height

3. MATHEMATICAL MODEL AND EXAMPLES OF NUMERICAL SIMULATION

According to the method mentioned in section 2, equivalent linearized equations can be described in the following form:

Combined motions of heave and pitch

$$
\begin{aligned}
&(m+m_z)\ddot{\zeta}_G + Z_{\dot{\zeta}_G}\dot{\zeta}_G + Z_{\zeta_G}\zeta_G + Z_{\ddot{\theta}}\ddot{\theta} + Z_{\dot{\theta}}\dot{\theta} + Z_{\theta}\theta = Z_C\cos\omega_e t + Z_S\sin\omega_e t \\
&(I_{yy}+J_{yy})\ddot{\theta} + M_{\dot{\theta}}\dot{\theta} + M_{\theta}\theta + M_{\ddot{\zeta}_G}\ddot{\zeta}_G + M_{\dot{\zeta}_G}\dot{\zeta}_G + M_{\zeta_G}\zeta_G \\
&\qquad = M_C\cos\omega_e t + M_S\sin\omega_e t
\end{aligned} \tag{10}
$$

Combined motions of sway, yaw and roll

$$
\begin{aligned}
&(m+m_y)\ddot{\eta}_G + Y_{\dot{\eta}_G}\dot{\eta}_G + Y_{\ddot{\phi}}\ddot{\phi} + Y_{\dot{\phi}}\dot{\phi} + Y_{\ddot{\psi}}\ddot{\psi} + Y_{\dot{\psi}}\dot{\psi} + Y_{\psi}\psi \\
&\qquad = Y_C\cos\omega_e t + Y_S\sin\omega_e t \\
&(I_{zz}+J_{zz})\ddot{\psi} + N_{\dot{\psi}}\dot{\psi} + N_{\psi}\psi + N_{\ddot{\eta}_G}\ddot{\eta}_G + N_{\dot{\eta}_G}\dot{\eta}_G + N_{\ddot{\phi}}\ddot{\phi} + N_{\dot{\phi}}\dot{\phi} \\
&\qquad = N_C\cos\omega_e t + N_S\sin\omega_e t \\
&(I_{xx}+J_{xx})\ddot{\phi} + K_{\dot{\phi}}\dot{\phi} + W\overline{GM}\left[1+\frac{\Delta\overline{GM}}{\overline{GM}}\cos(\omega_e t - k\xi_0)\right]\phi \\
&+K_{\ddot{\eta}_G}\ddot{\eta}_G + K_{\dot{\eta}_G}\dot{\eta}_G + K_{\ddot{\psi}}\ddot{\psi} + K_{\dot{\psi}}\dot{\psi} + K_{\psi}\psi = K_C\cos\omega_e t + K_S\sin\omega_e t
\end{aligned} \tag{11}
$$

where the hydrodynamic and hydrostatic coefficients are obtained from the ordinary strip method and the metacentric height taking into account the variation of righting moment in waves is given by the equivalent linearization mention in section 2. It should be noted that the last equation in Eq.(11) is a linear differential equation with respect to the rolling angle ϕ although the unique feature of the equation is the presence of time dependent coefficient of the rolling angle ϕ. Furthermore, this kind of equation has a property of considerable importance in ship rolling problem in that for certain values of the encounter frequency ω_e, the solution is unstable. Physically, this implies that if the roll motion described by Eq.(11) is taking place in unstable region, the amplitude of rolling grows up. The unstable encounter frequency may be found from unstable solution of Mathieu's equation, in which unstable roll occurs when encounter frequency ω_e is equal to twice of the natural frequency ω_ϕ of roll. For this unstable condition $\omega_e = 2\omega_\phi$, the encounter frequency ω_e is given by:

$$\omega_e = \sqrt{\frac{g}{L}}\left|\sqrt{\frac{2\pi L}{\lambda}} - Fn\left(\frac{2\pi L}{\lambda}\right)\cos\chi\right| \tag{12}$$

and the natural frequency ω_ϕ is obtained from the natural rolling period T_ϕ defined by IMO resolution A 562 [8] as follows:

$$\omega_\phi = \frac{2\pi}{T_\phi}$$
$$T_\phi = \frac{2B}{\sqrt{\overline{GM}}}[0.373 + 0.023(B/d) - 0.043(L/100)] \tag{13}$$

where L is the ship length, B the breadth, d the draft, Fn the Froude number and λ the wave length.
By using these relationship, it will be possible to specify the encounter frequency for the ship running with Fn and χ when the parametric resonance occurs.
In general, the parametric resonance keeps a critical rolling of the constant amplitude when the energy due to the roll damping is balanced with the energy due to the variation of metacentric height. The rolling angle grows up when the damping energy is smaller than energy due to the variation of metacentric height and it damps out when the damping energy is bigger than the energy due to the variation of metacentric height.
From the above physical point of view, several numerical simulations were carried out for the container carrier which is running with constant speed U and encounter angle χ in astern seas. Three kinds of metacentric heights, $\overline{GM}$ =0.3m, $\overline{GM}$ =0.6m and $\overline{GM}$ =0.9m of the container are selected to investigate the ship motion leading to capsize, the encounter angle is fixed at $\chi = 0^0, 15^0, 30^0, 45^0$ and 60^0. Figs.6, 7, 8, 9 and 10 are the time history of roll, pitch and yaw motions of the ship with metacentric height $\overline{GM}$ =0.6m in critical and unstable conditions. Finally, from the numerical simulations, it is possible to find out the critical roll of constant amplitude in parametric resonance. Fig.11 shows the waves steepness H/λ for encounter angle χ of the critical rolling motion leading to capsize.

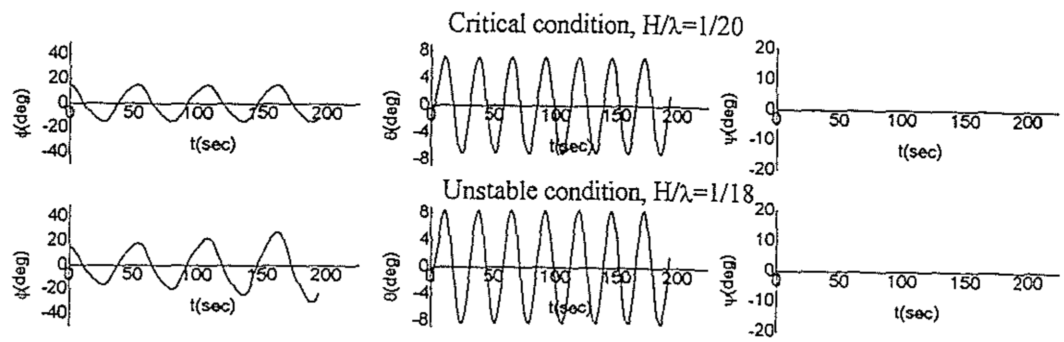

Fig.6. Time history of roll, pitch and yaw in critical and unstable motions $\overline{GM}$ =0.6m, Fn =0.10935 χ =0

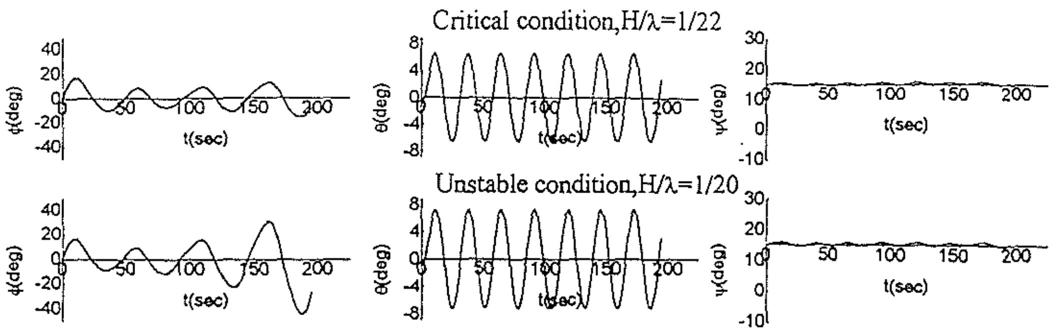

Fig.7. Time history of roll, pitch and yaw in critical and unstable motions $\overline{GM}$ =0.6m, Fn =0.11, χ =15

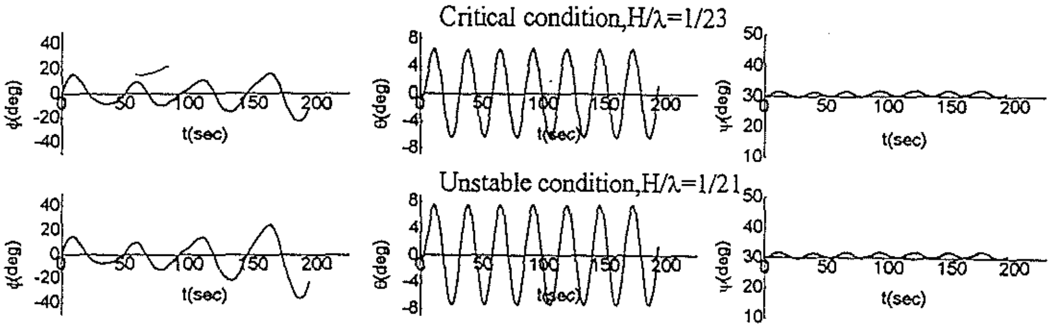

Fig.8. Time history of roll, pitch and yaw in critical and unstable motions $\overline{GM}$ =0.6m, Fn =0.1263, χ =30

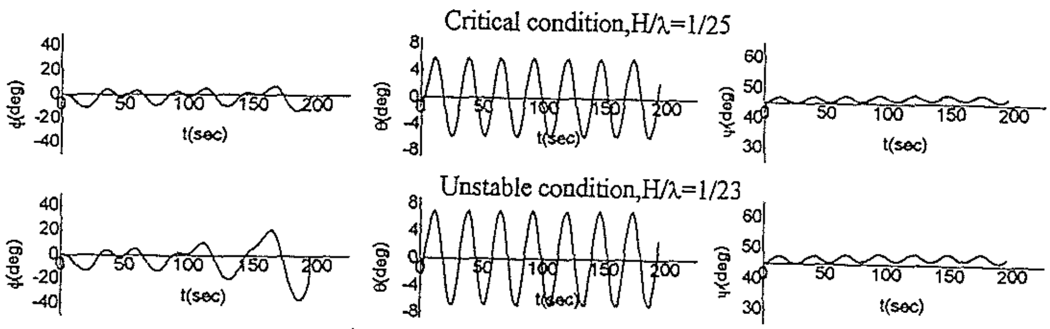

Fig.9. Time history of roll, pitch and yaw in critical and unstable motions $\overline{GM}$ =0.6m, Fn =0.15 χ =45

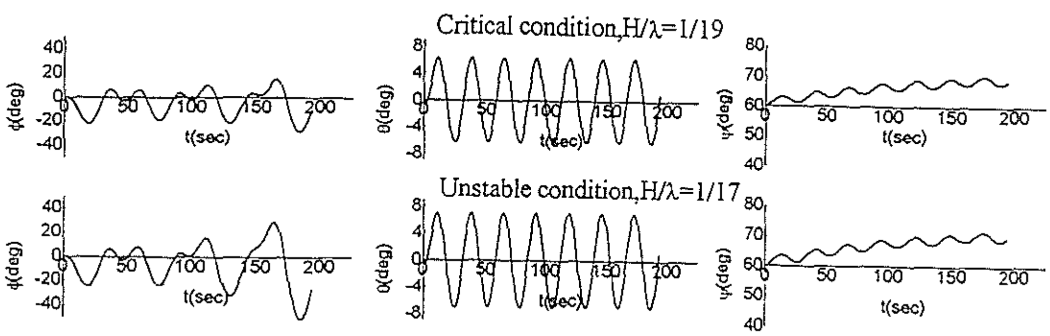

Fig.10. Time history of roll, pitch and yaw in critical and unstable motions $\overline{GM}$ =0.6m, Fn =0.22 χ =60

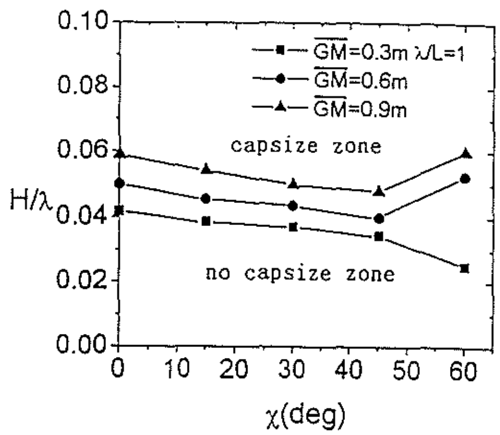

Fig.11. Critical line leading to capsize, H / λ vs χ

4. CONCLUDING REMARKS

An analytical study of ship capsize phenomenon due to parametric resonance is conducted to investigate the occurrence of the critical condition leading to capsize by making use of the ordinary strip method taking into account the variation of metacentric height with respect to relative position of ship to waves. The main conclusions are summarized as follows:

1. The ordinary strip method taking into account the variation of metacentric height in waves is usable for predicting the occurrence of parametric resonance
2. When the ship is running with large encounter angle such as $\chi = 45^0$, the rolling angle deforms due to the wave excitation as shown in Fig.9 although the ship usually rolls with the natural rolling period T_ϕ at the parametric resonance. This rolling motion comes from the combination of the wave induced stability varying with the natural rolling period and wave excitation varying with encounter period T_e
3. For the design $\overline{GM}$ =0.9m and operational $\overline{GM}$ =0.6m, the most dangerous condition leading to capsize is at encounter angle approximately 45^0, but for small metacentric height, $\overline{GM}$ =0.3m, the most dangerous condition is at encounter angle approximately to 60^0, because the Froude number becomes quite large to satisfy the condition of parametric resonance, then the ship is running with the velocity nearly equal to wave celerity. These computational results show a fairly good agreement with the experimental results[4) 5)]

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REFRENCES

1. Kerwin, J.E., "Notes on Rolling in Longitudinal Waves", International Shipbuilding Progress, Vol.2, No.16, 1955,pp.597-614
2. Paulling, J.R., "The tranverse Stability of a Ship in a Longitudinal Seaway", Journal of Ship Research,

SNAME, Vol.4, No.4, 1961, pp.37-49
3. Hamamoto,M., Sera,W., Panjaitan,J.P, "Analysis on Low Cycle Resonance of Ship in Irregular Astern Seas", Journal of the Society of Naval Architects of Japan, Vol.178, 1995, pp.137-145
4. Umeda N.,Hamamoto,M., Takaishi,Y., Chiba, Y., Matsuda A., Sera,W., Suzuki, S., Spyrou, K., Watanabe, K., "Model Experiment of Ship Capsize in Astern Seas", Journal of the Society of Naval Architects of Japan, Vol.177, 1995, pp.207-217
5. Hamamoto,M., Enomoto, T., Sera,W., Panjaitan,J.P, Ito, H., Takaishi,Y., Kan, M., Haraguchi, T., Fujiwara, T., "Model Experiment of Ship Capsize in Astern Seas", second report, Journal of the Society of Naval Architects of Japan, Vol.179, 1996, pp.77-87
6. Hamamoto,M., Panjaitan,J.P, "Analysis on Parametric Resonance of Ships in Astern Seas", Proceeding of Second Workshop on Stability and Operational Safety of Ships, Osaka, November, 1996
7. Paulling,J.R., Oakley, O.H., Wood, P.D.,"Ship Capsizing in Heavy Seas: The Correlation of Theory and Experiments", Proceeding of the International Conference on Stability of Ships and Ocean Vehicles, Glasgow, 1975
8. A.R.J.M.Lloyd, "Seakeeping: Ship Behaviour in Rough Weather", Ellis Horwood Ltd., 1989
9. IMO,: The intact Stability Criteria, Resolution A, 562, 1985