# Modelling of compartment connectivity and probabilistic assessment of progressive flooding stages for a damaged ship

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**ABSTRACT**

Large cruise vessels have subdivision and compartment connectivity of unique complexity, making predictions of floodwater propagation a particularly challenging task. Evermore so, the plethora of internal openings leads to a large number of opening status combinations, a well-known problem in identifying flooding paths and assessing progressive flooding stages. This paper presents a novel approach aiming at reducing the problem to manageable size. The method enables a fully probabilistic approach for assessing progressive flooding stages and the examples presented demonstrate that it converges to a practical number of possible realisations even in the case of a realistic model of a large cruise vessel. The result show clearly that the methodology will render overly simplified models for assessment of vulnerability from internal openings obsolete and that it may be further refined for implementation to a range of applications.

**Keywords:** *Damage stability, Compartment connectivity, Progressive flooding, Opening modelling, Progressive flooding stages.*

## 1. INTRODUCTION

Large cruise vessels have an internal subdivision and compartment connectivity of unparalleled complexity. This makes predicting floodwater propagation in damaged condition a particularly challenging task with the number of possible flooding paths growing exponentially with the number of internal openings. This problem was highlighted in the European research project EMSA III (EMSA, 2016), addressing the contribution to risk from watertight doors, for Cruise and RoPax ships in collision flooding emergencies and considering door opening frequencies (from historical data), crew actions and door reliability.

The assessment of the impact of a single open watertight door on stability carried out by the project, led to the observation that the impact of any one single open door was small in comparison with the impact of combinations of multiple open doors. Furthermore, the impact on stability was proved insensitive to the opening's allowance category (as defined in MSC.1/Circ. 1380 (IMO, 2010) and summarised in Table 1) of doors comprising that particular combination (e.g. an opened door of category C would degrade stability, on average, to the similar extent as a door of category A).

Due to the combinatorial character of the problem the opening (doors in particular) statuses result in an immense number of possible combinations, $N$, increasing exponentially with the number of $n$ doors available which is governed by Eq. 1 below.

$$N = 2^n \tag{1}$$

It is clear that the stability assessment involving all possible combinations of doors is infeasible, thus resulting in the necessity for developing simplified models, such as the one proposed within the EMSA project. Notably, such simplified models may neglect potentially critical combinations of doors. However, the number of possible initial damage extents is limited, and for every one of these, there are also a limited number of directly connected compartments. This entails that only the status of doors directly within the boundary of a specific initial damage extent needs to be considered in the first (and all subsequent flooding stages). This view enables to limit the problem and has been the basis for the development of a novel modelling approach presented in this paper.

**Table 1: Opening allowance categories for watertight doors according to MSC.1/Circ. 1380 (IMO, 2010).**

| Categories | Opening allowance |
|---|---|
| Category A | Permitted to remain open during navigation by the Administration according to SOLAS regulation II-1/22.4. |
| Category B | May be opened during navigation when work in the immediate vicinity of the door necessitates it being opened, according to SOLAS regulation II-1/22.3. |
| Category C | May be opened during navigation to permit the passage of passengers or crew, according to SOLAS regulation II-1/22.3. |
| Category D | Shall be closed before the voyage commences and shall be kept closed during navigation according to SOLAS regulation II-1/22.1. |

## 2. PROGRESSIVE FLOODING

### *Deterministic representation*

Traditionally, and in contrast to the overall probabilistic damage stability regulations laid out in Reg. II-1/7 of SOLAS (IMO, 2006) progressive flooding stages are deterministic, determined by the openings watertightness alone rather than the opening frequencies. The underlying assumption is that watertight openings prevent progressive flooding even if they are allowed open in specific circumstances as seen in Table 1, simply because they can be *closed in time* by crew.

Considering separate progressive flooding stages is required only for non-watertight openings seriously restricting equalisation (with the equalising time over 60 seconds) as is laid out in the explanatory notes of SOLAS (IMO, 2017). Instantaneous equalisation (below 60s) assumes immediate flooding and allows including the progressively flooded compartments in the initial damage extent without a separate stage.

The non-watertight structural elements and doors seriously restricting the floodwater ingress are typically represented by A-class fire rated bulkheads and doors. In a single watertight zone, there may be a range of A-class boundaries leading to the exponential combinatorial problem on a local scale. Simplified approaches have been suggested to tackle the problem, such as the *neighbouring approach* as implemented in the stability software NAPA (NAPA, 2018), where the next connections (or stages) are considered as all the neighbouring compartments sharing a limit (bulkhead) with the currently damaged rooms and grouping those in a single combined stage.

### *Probabilistic representation*

Deterministic approach is not suitable to address the risk contribution from watertight doors simply because it does not cater for random statuses of the actual openings. The first-principles probabilistic models, in addition to considering the damage breaches (initial extent) as a statistical variable, need to capture the stochastic behaviour of the internal connectivity of the vessel. The latter involves dynamically changing opening status with the associated opening frequencies as well as the uncertainty inherent in the openings resistance to leak and collapse when closed (progressive extent), as is illustrated in Figure 1. Both, the openings status frequencies and leak/collapse hydrostatic head distributions will influence the probability of progressive flooding through the opening. Similarly, if the stochastic nature of the vessel movements in waves is not accounted for directly, e.g. by time-domain simulations, it may be introduced as a probabilistic model within the traditional static assessment (e.g. as the probability of the internal water-elevation exceeding the vertical opening position or leak/collapse heads). Such governing variables may be represented as a total probability of progressive flooding and they will determine various realisations of progressive flooding stages, related to each initial damage extent, as will be discussed in the next section.

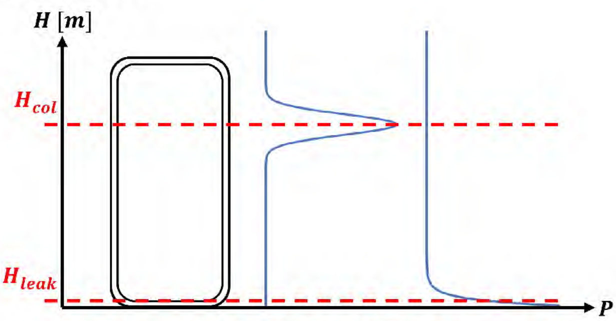


**Figure 1: Example of leak and collapse pressure heights modelled with probability distributions (Norm./Exp.) to account for inherent uncertainty and to enable a fully probabilistic consideration.**

### *Progressive extent realisation*

A simple event (probability) tree, as shown in Figure 2 below, can illustrate the various realisations of initial and corresponding progressive extents. The top event represents any breach resulting from a collision damage (or contact/grounding), which branches out to all the possible initial damage extents.

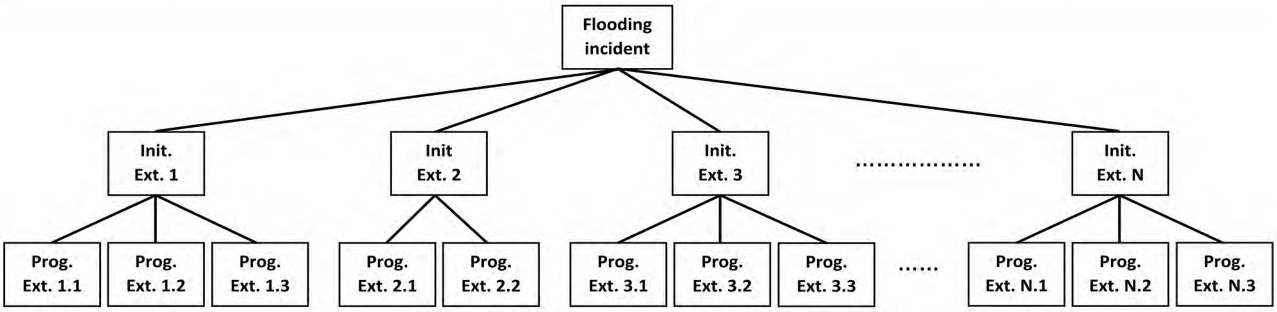


**Figure 2: Event (probability) tree of damage extents (First row: breach resulting from collision damage or contact/grounding, second row: possible initial extents of damage, third row: possible progressive extents of damage).**

A range of respective progressive damage extents may originate from each of these initial extents depending on the openings open/closed state, leak/collapse resistance and the openings position in relation to the floodwater elevation during the flooding evolution. All branches of progressive extents stemming from each of the initial extents, should sum to the initial extent probability according the total probability theorem, as given by Eq. 2, where $x = initial, y = progressive$.

$$P(x) = \int_y P(x|y)P(y)dy \qquad (2)$$

For example, the progressive extents in the leftmost branch are representing all possible progressing extent originating from the initial damage extent number one (if there is no progressive extent the initial extent remains unchanged and considered as total extent which would still be represented with a separate branch in the tree). The actual number of possible realisations of progressive extents will be governed by the number of connections in direct contact with the initial damage extent, and subsequent connections thereafter. An initial damage extent comprising a single compartment with just a couple connections would therefore be expected to have a smaller number of possible progressive extents than an initial damage extent comprising several compartments and multiple connections.

This being said, it would not necessarily be so as the probability of progressive flooding will be governing, e.g. if all the doors leading from the extent with several compartments connected had a progressive flooding probability of 1, the progressive extent where all connected compartments were progressively flooded would in fact be the only realisation possible. The various realisations are highly related to the combinatorics problem as was discussed in the foregoing.

To illustrate the combinatorial problem with multiple permutations, we may consider an example compartmentation shown in Figure 3. The compartmentation comprises six rooms (compartments): *A, B, C, D, E* and *F*, and six watertight doors: *a, b, c, d, e* and *f*. Compartment *F*, marked in yellow, is breached and considered as the initial damage extent. For simplicity, we assume that the probability of progressive flooding is solely governed by the door opening status (frequency), disregarding other variables as was mentioned in the previous section.

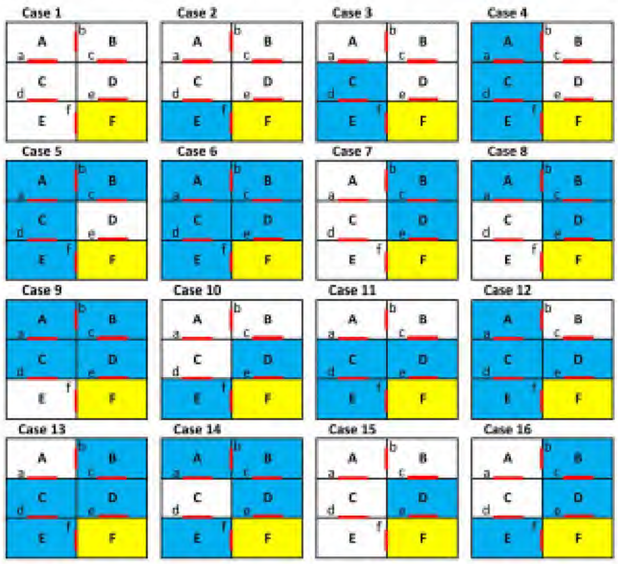

**Figure 3: Example compartmentation with doors and possible flooding realisations (Doors are marked in red, initial flooding is marked in yellow, and progressively flooded compartments are marked in blue).**

A door's opening status may be modelled by a *Bernoulli process* with the opening frequency represented by the parameter $\lambda$, as shown by Eq. 3.

$$P(open) = \lambda, P(closed) = 1 - \lambda \qquad (3)$$

The assumed opening frequency for the example compartmentation is summarised in Table 2.

**Table 2: Assumed opening frequencies for example compartmentation.**

| Door | Opening frequency, $\lambda$ |
|---|---|
| a | 0.90 |
| b | 0.95 |
| c | 0.10 |
| d | 0.30 |
| e | 0.05 |
| f | 0.70 |

To calculate the realisation probability of *Case 1*, all the various door status combinations that are possible needs to be considered. In total, there are $2^6 = 64$ possible permutations (combinations) of door statuses in this specific case. Out of these, 16 permutations result in *Case 1* being realised (i.e. there is 16 progressive flooding scenarios originating in room *F*). Probability of *Case 1* may therefore be calculated by summing all these realisations as shown by Eq. 4, where $n$ is the number of realisations resulting in a specific initial damage extent $x$.

$$P(X = x) = \sum_{i=1}^{n} P_{x_i} \tag{4}$$

It may be shown that this results in a probability of: $P(X = Case\ 1) = 0.285$. Another way to calculate the realisation probability may be illustrated as in the following. For *Case 1* to be realised, doors $e$ and $f$ have to be closed ($open = 1, closed = 0$) whilst the status of the remaining doors status is not affecting the outcome. Hence, the probability of this particular case is simply the joint probability of the two relevant doors being closed (using the probability rule of conditionality governed by Eq. 5 and calculated in Eq. 6).

$$P(X = x) = P(e, f) = P(e)P(f) \tag{5}$$

$$\begin{aligned} P(X = Case\ 1) &= P(e = 0, f = 0) \\ &= (1 - \lambda_e)\,(1 - \lambda_f) \\ &= (1 - 0.05)\,(1 - 0.70) \\ &= 0.285 \end{aligned} \tag{6}$$

The second case, *Case 2* may be calculated by the same method (it is only governed by doors *d*, *e* and *f*). For the case to be realised, doors *d* and *e* have to be closed and door *f* has to be open whilst the status of the remaining doors statuses does not affect the realisation. The probability of *Case 2* may again simply be calculated as the joint status probability of the three relevant doors.

$$\begin{aligned} P(X = Case\ 2) &= P(d = 0, e = 0, f = 1) \\ &= (1 - \lambda_d)\,(1 - \lambda_e)\lambda_f \\ &= (1 - 0.30)(1 - 0.05)70 \\ &= 0.4655 \end{aligned} \tag{7}$$

The process can be repeated for all 16 cases, but we will consider *Case 6*, with all doors part of the progressive boundary, as a final example. This case may result from flooding progression by two routes with multiple door realisations leading to the same case. In fact, seven realisations will result in *Case 6;* summarised below by Eq. 8 to 14 with the total probability as given by Eq. 15.

$$\begin{aligned} &P_1(X_1 = Case\ 6) = P(a, b, c, d, e, f = 1) = \cdots \\ &\cdots = \lambda_a\lambda_b\lambda_c\lambda_d\lambda_e\lambda_f = \cdots \\ &\cdots = 0.90 \cdot 0.95 \cdot 0.10 \cdot 0.30 \cdot 0.05 \cdot 0.70 = \cdots \\ &\cdots = 0.0009 \end{aligned} \tag{8}$$

$$\begin{aligned} &P_2(X_2 = Case\ 6) = P(a, b, c, d, e = 1, f = 0) = \cdots \\ &\cdots = \lambda_a\lambda_b\lambda_c\lambda_d\lambda_e(1 - \lambda_f) = \cdots \\ &\cdots = 0.90 \cdot 0.95 \cdot 0.10 \cdot 0.30 \cdot 0.05 \cdot (1 - 0.70) = \cdots \\ &\cdots = 0.0004 \end{aligned} \tag{9}$$

$$\begin{aligned} &P_3(X_3 = Case\ 6) = P(a, b, c, d, f = 1, e = 0) = \cdots \\ &\cdots = \lambda_a\lambda_b\lambda_c\lambda_d(1 - \lambda_e)\lambda_f = \cdots \\ &\cdots = 0.90 \cdot 0.95 \cdot 0.10 \cdot 0.30 \cdot (1 - 0.05) \cdot 0.70 = \cdots \\ &\cdots = 0.0171 \end{aligned} \tag{10}$$

$$\begin{aligned} &P_4(X_4 = Case\ 6) = P(a, b, c, e, f = 1, d = 0) = \cdots \\ &\cdots = \lambda_a\lambda_b\lambda_c(1 - \lambda_d)\lambda_e\lambda_f = \cdots \\ &\cdots = 0.90 \cdot 0.95 \cdot 0.10 \cdot (1 - 0.30) \cdot 0.05 \cdot 0.70 = \cdots \\ &\cdots = 0.0021 \end{aligned} \tag{11}$$

$$\begin{aligned} &P_5(X_5 = Case\ 6) = P(a, b, d, e, f = 1, c = 0) = \cdots \\ &\cdots = \lambda_a\lambda_b(1 - \lambda_c)\lambda_d\lambda_e\lambda_f = \cdots \\ &\cdots = 0.90 \cdot 0.95 \cdot (1 - 0.10) \cdot 0.30 \cdot 0.05 \cdot 0.70 = \cdots \\ &\cdots = 0.0081 \end{aligned} \tag{12}$$

$$\begin{aligned} &P_6(X_6 = Case\ 6) = P(a, c, d, e, f = 1, b = 0) = \cdots \\ &\cdots = \lambda_a(1 - \lambda_b)\lambda_c\lambda_d\lambda_e\lambda_f = \cdots \\ &\cdots = 0.90 \cdot (1 - 0.95) \cdot 0.10 \cdot 0.30 \cdot 0.05 \cdot 0.70 = \cdots \\ &\cdots = 0.0001 \end{aligned} \tag{13}$$

$$\begin{aligned} &P_7(X_7 = Case\ 6) = P(b, c, d, e, f = 1, a = 0) = \cdots \\ &\cdots = (1 - \lambda_a)\lambda_b\lambda_c\lambda_d\lambda_e\lambda_f = \cdots \\ &\cdots = (1 - 0.90) \cdot 0.95 \cdot 0.10 \cdot 0.30 \cdot 0.05 \cdot 0.70 = \cdots \\ &\cdots = 0.0001 \end{aligned} \tag{14}$$

$$P(X = Case\ 6) = \sum_{i=1}^{n=7} P_{X_i} = \cdots \quad (15)$$

$$\cdots = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 = \cdots$$

$$\cdots = 0.0009 + 0.0004 + 0.0171 + 0.0021 + \cdots$$

$$\cdots + 0.0081 + 0.0001 + 0.0001 = \cdots$$

$$\cdots = 0.0287$$

Table 3 below summarises the probability calculations for all example cases. The second calculation methodology comprises less combinations of doors, as only the doors located within the flooding boundary is of interest. However, *Case 1* and *2* are the simplest of the example cases, and it is relatively easy to calculate their realisation probability by manual calculations, being governed by a few doors. If more doors are governing, such as in *Case 6*, increasing various realisations of doors may result in the same progressive damage extent, which will complicate the problem. Nevertheless, the manual calculations method cannot be applied to a realistic case of a large cruise vessel with thousands of possible initial damage extents and numerous connections, hence an alternative approach is essential.

## 3. GRAPH MODEL OF COMPARTMENT CONNECTIVITY

The problem of opening permutations can be addressed more efficiently than the direct calculations with the help of *Graph Theory*. Graph Theory is a well-known mathematical modelling technique for representing pairwise connections between objects (nodes) with the relationship maintained by edges (lines). The application of graphs ranges from the evacuation modelling software *Evi* (Vassalos et al, 2001) through social networks (Zweig, 2016) to navigational- and road-networks (Thomson et al, 1995). Any exhaustive review of theory and applications of graph theory is outside the scope of this paper, but reference is made to introductory texts such as (Bondy et. al., 1976).

In modelling of compartment connectivity as a graph, the compartments are simply represented by the nodes (points) and openings are represented by edges (lines). For example (Dankowski & Krüger, 2013) represented compartment connectivity by deterministic directed graphs (i.e. without the ability to account for probabilities). Graph model of the example compartmentation from Figure 3 is presented in Figure 4 below. For the purpose of compartment connectivity, we are not interested in distances between locations (as is often used for road networks); instead we may rather use the weights representing the probability of progressive flooding between compartments, or opening frequencies, depending on how we define the problem.

Representing the edges by probabilities turns the graph into an *uncertain graph*, a well-known technique utilised for example in network reliability (Khan, 2018). In the compartment connectivity example, existence of the edge implies possible progressive flooding between the nodes (compartments). However, progressive flooding only occurs if at least one of the edges is connected to the initial damage extent (the source node).

**Table 1: Probability summary of case realisations for example compartmentation.**

| Case, $i$ | Calculation formulae | Result |
|---|---|---|
| $P_{Case\ 1} =$ | $(1.00-0.05)\cdot(1.00-0.70)$ | $= 0.2850$ |
| $P_{Case\ 2} =$ | $(1.00-0.30)\cdot(1.00-0.05)\cdot 0.70$ | $= 0.4655$ |
| $P_{Case\ 3} =$ | $(1.00-0.90)\cdot(1.00-0.05)\cdot 0.30\cdot 0.70$ | $= 0.0200$ |
| $P_{Case\ 4} =$ | $(1.00-0.95)\cdot(1.00-0.05)\cdot 0.90\cdot 0.30\cdot 0.70$ | $= 0.0090$ |
| $P_{Case\ 5} =$ | $(1.00-0.10)\cdot(1.00-0.05)\cdot 0.90\cdot 0.95\cdot 0.30\cdot 0.70$ | $= 0.1535$ |
| $P_{Case\ 6} =$ | $0.0009+0.0004+0.0171+0.0021+0.0081+0.0001+0.0001$ | $= 0.0287$ |
| $P_{Case\ 7} =$ | $(1.00-0.70)\cdot(1.0.-0.95)\cdot 0.10\cdot 0.05$ | $= 0.0001$ |
| $P_{Case\ 8} =$ | $(1.00-0.70)\cdot(1.00-0.90)\cdot 0.95\cdot 0.10\cdot 0.05$ | $= 0.0001$ |
| $P_{Case\ 9} =$ | $(1.00-0.30)\cdot(1.00-0.70)\cdot 0.05\cdot 0.10\cdot 0.95\cdot 0.90$ | $= 0.0009$ |
| $P_{Case\ 10} =$ | $(1.00-0.30)\cdot(1.00-0.10)\cdot 0.05\cdot 0.70$ | $= 0.0220$ |
| $P_{Case\ 11} =$ | $(1.00-0.90)\cdot(1.00-0.10)\cdot 0.30\cdot 0.05\cdot 0.70$ | $= 0.0009$ |
| $P_{Case\ 12} =$ | $(1.00-0.95)\cdot(1.00-0.10)\cdot 0.90\cdot 0.30\cdot 0.70\cdot 0.05$ | $= 0.0004$ |
| $P_{Case\ 13} =$ | $(1.00-0.90)\cdot(1.00-0.95)\cdot 0.30\cdot 0.70\cdot 0.05\cdot 0.10$ | $= 0.0000$ |
| $P_{Case\ 14} =$ | $(1.00-0.30)\cdot(1.00-0.90)\cdot 0.70\cdot 0.05\cdot 0.10\cdot 0.95$ | $= 0.0002$ |
| $P_{Case\ 15} =$ | $(1.00-0.70)\cdot(1.00-0.10)\cdot 0.05$ | $= 0.0135$ |
| $P_{Case\ 16} =$ | $(1.00-0.30)\cdot(1.00-0.95)\cdot 0.70\cdot 0.05\cdot 0.10$ | $= 0.0001$ |
| $Sum =$ | $\sum_{i=1}^{n} P_{Case\ i}$ | $= 1.0000$ |

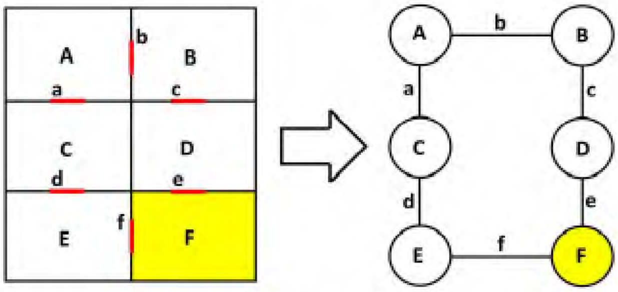


**Figure 4: Compartments mathematical abstraction as graph (compartment or node marked in yellow is initial damage extent, or source node).**

This conditionality can be accounted for readily by implementing search algorithms for traversing the graph structure. Such algorithms comprise Breadth-First-Search (BFS) (Moore, 1959), and Depth-First Search (DFS) (Trémaux, 1859–1882). In the example compartmentation the opening frequencies can be used to sample (create) the connections (edges) between the compartments (nodes) for multiple instances (samples). An example of such sampled realisations is shown in Figure 5 below, where dashed lines represent non-existing edges and continuous lines represent existing edges.

The nodes (compartments), having existing edges and a valid connection to the source node (initial extent) are part of the progressive extent (blue nodes in the figure). The sampling process, if done sufficient number of times, should result in accurate approximation of the realisation probability of the openings, while search algorithms account for the conditionality of the connections (i.e. they return only the relevant progressive stages with connection to the source node, representing the initial extent of damage). The sum of each flooding realisation (initial and progressive combined), divided by the number of samples, represents the estimate of the respective case-realisation probabilities. In order to verify the approach, the example flooding cases are sampled with $N = 100{,}000$ samples. The results shown in Table 4 demonstrate good agreement with the calculated probabilities.

**Table 4: Progressive flooding case (realisation) probability from manual calculation and sampling scheme.**

| Case, *i* | P, calculation | P, sampling |
|---|---|---|
| $P_{Case\ 1}$ = | 0.2850 | 0.2847 |
| $P_{Case\ 2}$ = | 0.4655 | 0.4650 |
| $P_{Case\ 3}$ = | 0.0200 | 0.0201 |
| $P_{Case\ 4}$ = | 0.0090 | 0.0091 |
| $P_{Case\ 5}$ = | 0.1535 | 0.1542 |
| $P_{Case\ 6}$ = | 0.0287 | 0.0287 |
| $P_{Case\ 7}$ = | 0.0001 | 0.0001 |
| $P_{Case\ 8}$ = | 0.0001 | 0.0001 |
| $P_{Case\ 9}$ = | 0.0009 | 0.0009 |
| $P_{Case\ 10}$ = | 0.0220 | 0.0220 |
| $P_{Case\ 11}$ = | 0.0009 | 0.0010 |
| $P_{Case\ 12}$ = | 0.0004 | 0.0004 |
| $P_{Case\ 13}$ = | 0.0000 | 0.0000 |
| $P_{Case\ 14}$ = | 0.0002 | 0.0002 |
| $P_{Case\ 15}$ = | 0.0135 | 0.0135 |
| $P_{Case\ 16}$ = | 0.0001 | 0.0001 |
| Sum = | 1.0000 | 1.0000 |

## 4. REAL-CASE EXAMPLE

The ship model selected for case study is based on a large modern cruise vessel of 100,000 GT, currently in operation. The vessel main particulars are presented in Table 5 below. The vessel internal compartment connectivity comprises a total of 894 openings, covering doors, hatches, etc. The model of the internal arrangement is shown in Figure 6 below.

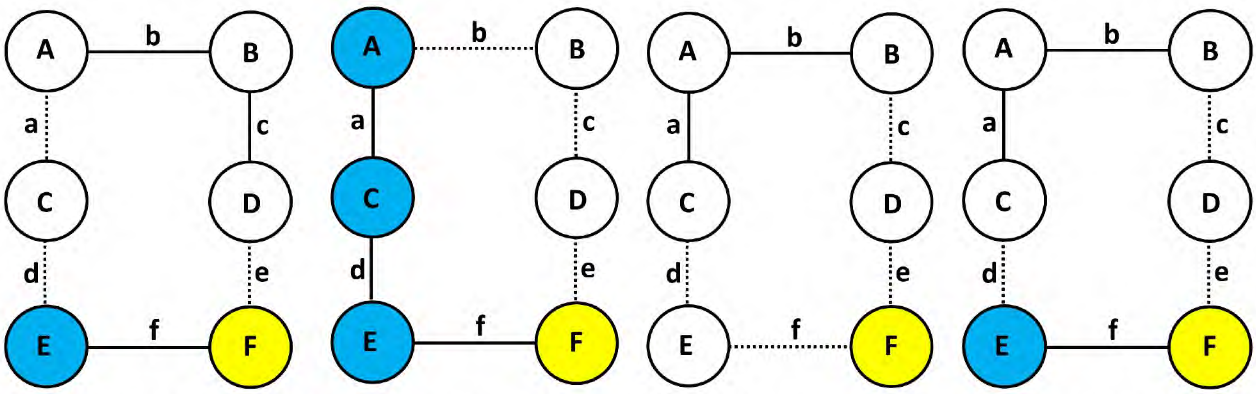

**Figure 5: Sampled edge existence in example compartmentation represented as uncertain graph (Dashed lines represent non-existing edges or no progressive flooding realization, and continuous lines represent existing edges or progressive flooding realization. Initial flooding is marked in yellow, and progressively flooded compartments are marked in blue).**

**Table 1: Particulars of the sample ship**

| Parameter (symbol) | Value [designation] |
|---|---|
| Length between perp. ($L_{BP}$) | 273 [m] |
| Breadth ($B$) | 36 [m] |
| Depth ($D$) | 21 [m] |
| Gross tonnes ($GT$) | 100000 [tonnes] |
| Number of passengers (-) | 2800 [persons] |
| Number of crew (-) | 1050 [persons] |

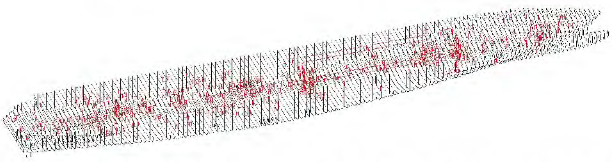

**Figure 6: Test-Vessel stability model with internal openings.**

Due to lack of actual data, the opening frequencies are based on their opening allowance category (supported by data adopted from the EMSA project, which has been derived from onboard records of various vessel types). Protected, non-watertight openings not imposed by any category, has been given an assumed opening frequency of 0.5 for the purpose of illustration. The frequencies are shown in Table 6 for the various opening categories.

In reality, such values would vary with specific doors depending on compartment type and crew/passenger traffic. The probability of doors being closed in time by crew is represented by a correction factor. In the EMSA project, such a correction has been modelled as a function of time, however, for illustration purposes, this has been taken as constant 90% success rate (only for watertight doors). In this specific example the correction factor accounts also for reliability of the doors.

**Table 2: Assumed opening frequencies for test vessel per allowance category.**

| Ope. Allow. category | Ope. Freq. | Corrected |
|---|---|---|
| A | 0.850 | 0.085 |
| B | 0.600 | 0.060 |
| C | 0.100 | 0.010 |
| Protected non-WT | 0.500 | 0.500 |
| Unprotected non-WT | 1.000 | 1.000 |

To limit the result, a single initial damage extent has been chosen to be implemented with the sampling methodology, to produce progressive extent realisations. Furthermore, for the purpose of illustration, we have considered the opening frequencies alone disregarding other variables such as leak/collapse heads and position of openings in relation with the floodwater elevation (this will obviously result in compartments being marked as part of the progressive extent (lost buoyancy), but not necessarily flooded). The initial damage case selected for illustration is a 2-zone damage, comprising 2 compartments and is illustrated in Figure 7 below.

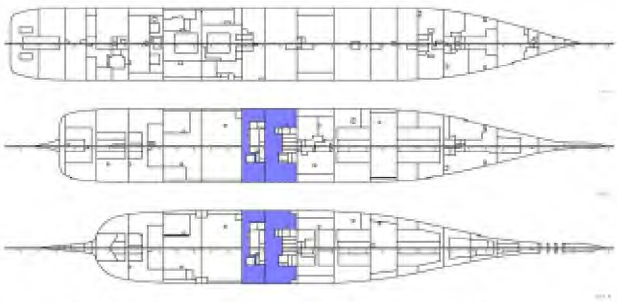

**Figure 7: Initial damage extent of the case study.**

For implementation of the sampling methodology, we generated $N = 1{,}000$ samples using the Bernoulli process, resulting in a corresponding number of graphs representing the state-space. The traversing search algorithm (BFS in this specific example), identified 86 unique progressive extents originating from specific initial extent, stemming from 6 openings with direct connection to the initial extents boundary. In order to rank the cases we make use of traditional statistical methods such as confidence intervals (CI). In the discrete domain, the CI may be represented as the number of cases with the largest probability, that results in a specific proportion of the total probability. The summary results of CI-based ranking are shown in Table 7. For example, the 90% CI simply indicate that there is a 90% probability that following a damage breach comprising the initial extent, the progressive extent would result in one out of nine cases as is seen in Table 8 below.

**Table 7: Confidence Intervals (CI) and corresponding number of related progressive extents for 1,000 samples.**

| Confidence Interval, CI [%] | Number of prog. extents |
|---|---|
| 50 | 3 |
| 80 | 6 |
| 90 | 10 |
| 95 | 36 |
| 99 | 76 |
| 100 | 86 |

**Table 8: Progressive extents representing a 90% Confidence Interval (CI), including initial extent (two leftmost comp.).**

| Case | P | Compartments |
|---|---|---|
| 1 | 0.233 | R070101 R080116 EX070101 R070102 |
| 2 | 0.206 | R070101 R080116 EX070101 EX080101 R070102 |
| 3 | 0.115 | R070101 R080116 |
| 4 | 0.112 | R070101 R080116 EX080101 |
| 5 | 0.072 | R070101 R080116 R070102 |
| 6 | 0.055 | R070101 R080116 EX080101 R070102 |
| 7 | 0.052 | R070101 R080116 EX070101 |
| 8 | 0.046 | R070101 R080116 EX070101 EX080101 |
| 9 | 0.005 | R070101 R080116 EX070101 EX080101 R070102 R080201 |
| SUM | 0.902 | |

All case realisations representing the 90% CI are illustrated in Appendix I, including also realisation No. 36, corresponding to the transition to the 95% CI for illustrating a less probable, but larger progressive extent. Case No. 3 represents the initial stage alone, where no additional compartments are progressively flooded. From the various progressive extent realisations presented in Appendix I, it is seen that the 90% CI are mostly comprising smaller A-class boundary compartments within the watertight boundaries as would be expected, simply due to the assignment of a 50% opening rate. More substantial progressive extents with compromised watertight boundaries are only seen above the 90% CI, as is represented by case realisation 36 in figure I-10.

## 5. CONCLUDING REMARKS

This paper presents a fully probabilistic methodology for modelling compartment connectivity with the help of graph theory. The method utilises state-of-the art search algorithms for maintaining the probabilistic conditionality of connection to source (initial extent). This simplifies the problem, as non-existing connections to the source are disregarded. A simple example has been provided to demonstrate that the method converges to the actual probabilities. The fully probabilistic modelling approach enables the use of traditional statistical methods and probabilistic evidence for quantifying the choices of progressive flooding extents in place of analysis of all possible combinations, which is highly infeasible (impossible in most cases). The realistic case study presented in this paper demonstrates that the method identifies a manageable number of possible progressive flooding extents. The choice of detail, and number of resulting cases are governed by the confidence interval and number of samples used. The methodology is capable of rendering the overly simplified models for assessment of vulnerability from internal openings obsolete. Apart from the survivability assessment the method may also be employed in emergencies to avoid compartments imposed by floodwater, smoke, or fire in a range of emergency situations, and may therefore provide a tool in identifying optimal evacuation routes.

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## APPENDIX I

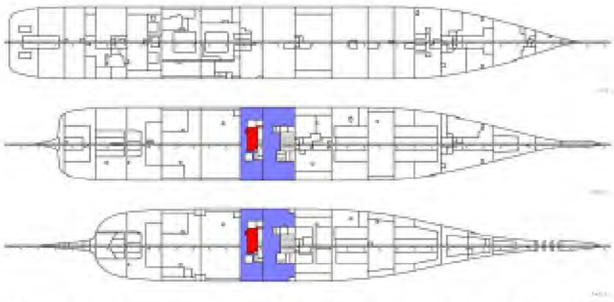

**Figure I-1: Progressive flooding realization 1, P = 0.2219**

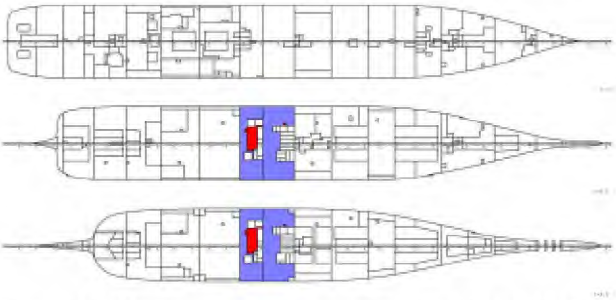

**Figure I-2: Progressive flooding realization 2, P = 0.1964**

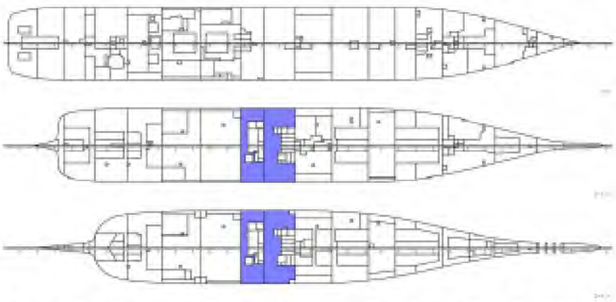

**Figure I-3: Progressive flooding realization 3, P = 0.1212**

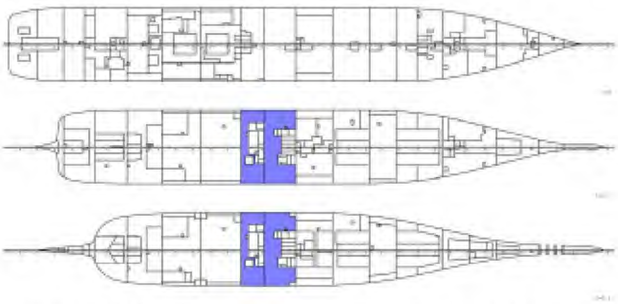

**Figure I-4: Progressive flooding realization 4, P = 0.1152**

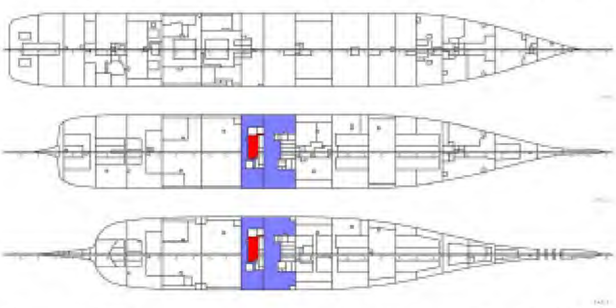

**Figure I-5: Progressive flooding realization 5, P = 0.0603**

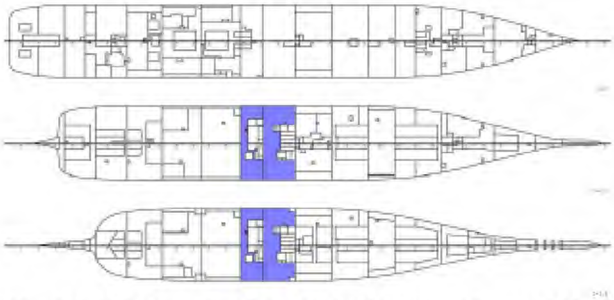

**Figure I-6: Progressive flooding realization 6, P = 0.0552**

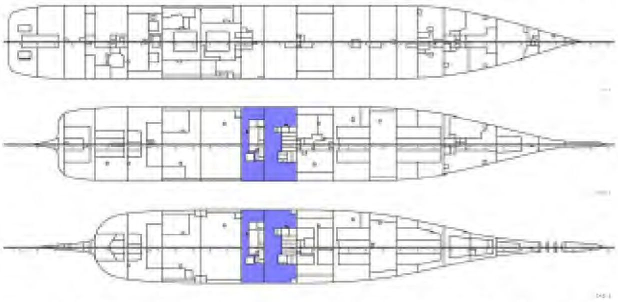

**Figure I-7: Progressive flooding realization 7, P = 0.0534**

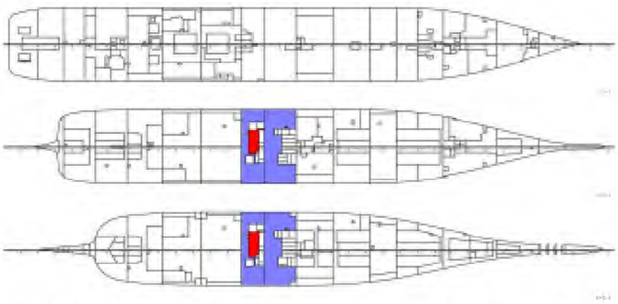

**Figure I-8: Progressive flooding realization 8, P = 0.0058**

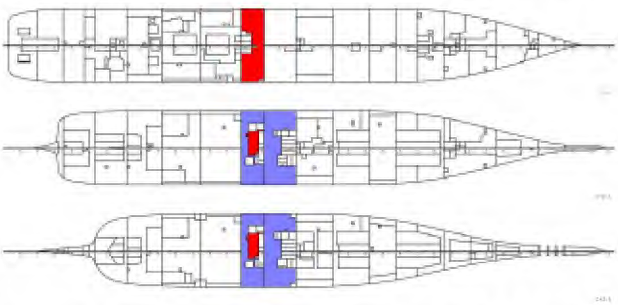

**Figure I-9: Progressive flooding realization 9, P = 0.0046**

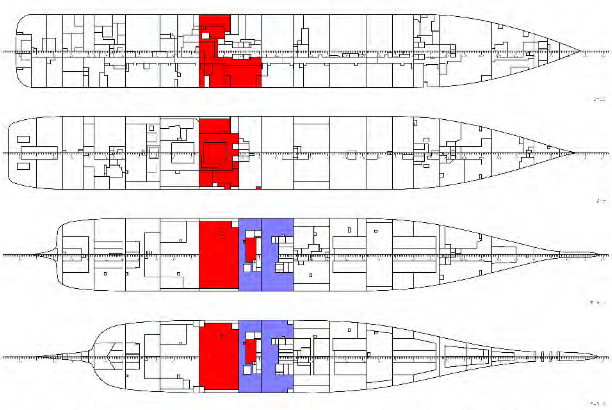

**Figure I-10: Progressive flooding realization 36, P = 0.001**