On Alignment of the Second Generation IMO Intact Stability Criteria with the Goal Based Standards: Evaluation of the Safety Level

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ABSTRACT

Alignment of IMO's Goal-Based Standards (GBS) to the second generation intact stability criteria can be achieved two separate ways: one is through direct stability assessment and the other through vulnerability criteria. The alignment of direct stability assessment is straight-forward; it yields probability of failure that is a part of a formal safety assessment. The alignment of the vulnerability criteria can be done through the safety level. It is more challenging because of the limited amount of information available for vulnerability criteria (especially on the first level) and requirements for simplicity of mathematical models. Nevertheless, the use of the wave group theory approach and previous work of one the authors along with upcrossing theory allow formulation of such a method.

KEYWORDS

IMO, Intact Stability, Safety level, Goal-based Standards

INTRODUCTION

Goal-based standards (GBS) represent a significant paradigm-shift in regulation philosophy and practice; instead of prescribing the means of achieving safety, GBS formulates the objective, leaving the freedom of achieving this objective to a designer (see, *e.g.* Hoppe, 2005, Kobylinski, 2007). For intact stability regulation, GBS may be considered as a synonym of the risk-based or probabilistic approach. Indeed, the probability of stability failure as a universal indicator of danger is a natural metric of the goal of safety. While this term was not traditionally used in relation with probabilistic approach, it is naturally aligned with the GBS.

For example, the document MSC 92/WP.9 (Annex 1), which contains draft guidelines for the approval of alternatives and equivalents as provided for in various IMO instruments, acknowledges that approval risks assessment and reliability analysis by Administrations is an increasingly acceptable practice, especially for novel designs. Also, risk analysis is an important part of a formal safety assessment (FSA), which is considered for use in the IMO rule-making process.

A comprehensive (and still up-to-date) review of risk-based approach to intact stability can be found in (Kobylinski & Kastner, 2003). The most difficult problem is the calculation of probability of stability failure in absolute sense. In other words, what does the term "probability of stability failure" mean?

Stability failures on realistic sea conditions are rare and cannot be assessed by direct numerical simulation of reasonable fidelity. Thus, the problem of rarity (as defined in the framework of the second generation of IMO intact stability criteria, see Annex 1 to SLF 54/3/1) inevitably leads to the problem of statistical extrapolation, see also Annex 21 to SLF 54/INF.12 and Peters, *et al.* (2012). The ability to determine the probability of stability failure in an absolute sense means that an extrapolation method is capable of recovering the "true" value for the probability of stability failure that would be observed from a data set of very long duration. Recent results (Smith & Campbell, 2013) indicate that these capabilities may become available in the near future.

The probability of stability failure in an absolute sense allows consideration of intact-stability hazards together with other hazards, like fires, machinery failures etc, making intact stability fully assessable with risk analysis and FSA.

The next question is how the alignment with GBS propagates through the multi-tiered structure of the second generation of IMO stability criteria. Since probabilistic criteria are expected to be used for direct stability assessment, the alignment would be trivial on tier 3. Probability of stability failure produced by the direct stability assessment directly "plugs-in" into FSA and risk analysis. It is more difficult for vulnerability criteria as there is less information available and the calculation methods are much simpler than the direct assessment. Indeed, it is especially difficult for vulnerability criteria level 1. Addressing this challenge is the primary objective of this paper.

SAFETY LEVEL & PROBABILITY

The framework of the second generation of intact stability criteria requires evaluation of the safety level of all proposed criteria (document SLF 54/3/1, annex 1, paragraph 3.2). The safety level is "understood to be a level of safety from stability failure" (*ibid*., paragraph 3.1)

Wind and wave environment are inherently random. Thus, there is always a possibility of stability failure. The safety level of the stability criterion is a measure of how remote the possibility of stability failure is if a ship meets the standard used with the criterion. Hence, the safety level of a vulnerability criterion is measured as a probability of stability failure of a ship that passes that criterion.

Three methods of calculation of probability of stability failure were reviewed in (Belenky, *et al.* 2012). Among these, the method of critical wave groups (Themelis & Spyrou, 2007) seems to be the most promising in the context of the safety level because it allows separation of the problem into probabilistic and dynamical parts. The environmental information is present in the vulnerability criteria as a reference wave height that can be related to a wave scatter diagram (see Appendixes 1, 2 and 3 of SLF 55/INF.15 as well as Appendix 1 of SLF 55/WP.3). Thus, the probabilistic part of the problem can be solved without particular ship information. The dynamical part of the problem has to be considered in a simplistic way, which is consistent with complexity level of vulnerability criteria.

This paper focuses on safety level for vulnerability criteria level one for stability failures related to the variation of righting lever in waves failure modes (parametric roll and pure loss of stability).

PARAMETRIC ROLL

The Criterion

The level 1 vulnerability criterion and standard for parametric roll is formulated as (Annex 1 SLF 55/WP.3)

$$\frac{\Delta GM}{GM} \geq S \tag{1}$$

Where GM is calculated in calm water, ΔGM is the variation of the GM value caused by the passing wave and S is a standard. This criterion can be interpreted as transitional solution of the Mathieu equation (Peters, *et al.* 2011) and considered as an amplification factor applied to initial roll angle:

$$Af = \frac{\phi_1}{\phi_0} = \frac{\sqrt{2}}{2}\exp\left(\frac{\pi N_W S}{4} - \mu\pi N_W\right) \tag{2}$$

where ϕ_1 is the roll angle associated with the stability failure that occurs after the action (passing) of N_W waves; while ϕ_0 is the initial roll angle; μ is roll damping coefficient expressed as a fraction of critical damping.

To evaluate the safety level (i.e. the probability of stability failure if the criterion is satisfied), the criterion (1) should be exactly equal to the standard S and the wave height (used to calculate ΔGM) should be equal to the reference wave height H_{ref}. Then, the stability failure caused by parametric roll can be associated with encountering a wave group of N_W waves or more, with the height not less than H_{ref}, each of which is capable to generate parametric resonance.

As it is well known, two conditions need to be satisfied to generate parametric resonance in roll. The first condition is the presence of parametric excitation with a magnitude that is above the threshold. This condition is only partially accounted in the criterion (1), because not all waves are capable to cause significant stability variations. If the waves are too short or too long compared to the ship length, stability variations while the wave passes will not be significant. The second condition is related to the encounter frequency (i.e. speed & heading dependence), which should be about twice the natural frequency of roll. For the sake of conservativeness, this condition is assumed to always be met when of the first-level vulnerability check is done. This assumption is also used for the safety level evaluation. Thus, speed and heading are not considered.

Wave Groups

The use of probabilistic properties of wave group to ship dynamics was first proposed by Spyrou & Themelis (2005) followed by application to long-term probabilistic assessment of stability during the voyage (Themelis & Spyrou, 2007) as well as application to broaching (Umeda, *et al* 2007) and parametric roll (Maki, *et al* 2011). The original approach from (Spyrou & Themelis, 2005) is mostly followed here using theory of upcrossing for more robust relation between the time and probability.

Since the wave group is defined as N_W waves exceeding certain threshold a_G, the event of encounter of a group can be considered as upcrossing of the threshold followed by N_W-1 waves with peaks exceeding the threshold. If the threshold is set high enough, the event of encounter can be considered to follow Poisson flow with the rate

$$\xi_G(N_W) = \xi(a_G) \cdot P\left(\bigcap_{i=2}^{Nw}(a_i > a_G)\right) \qquad (3)$$

Where the $\xi(a_G)$ is a rate of upcrossing of the threshold a_G by the water surface. Assuming normal distribution for wave elevations, the rate is expressed as:

$$\xi(a_G) = \frac{1}{2\pi}\sqrt{\frac{V_D}{V_W}}\exp\left(-\frac{a_G^2}{2V_W}\right) \qquad (4)$$

where V_W is the variance of wave elevation and V_D is the variance of the derivative of wave elevations. Further modeling of the wave group follows (Themelis & Spyrou, 2007).

The properties of wave amplitudes are described using envelope theory. It is assumed that only amplitudes of two consecutive waves are dependent because the autocorrelation function of the wave envelope usually dies out within two mean periods. Then, the set of amplitudes of consecutive waves is represented by a Markov chain and the rate of encounter of a group with N_W waves can be written as

$$\xi_G(N_W) = \xi(a_G) \cdot \left(P(a_2 > a_G \mid a_1 > a_G)\right)^{N_W - 1} \qquad (5)$$

The conditional probability that the second wave exceeds the threshold a_G as the first wave exceeds it as well is calculated from the joint distribution of two consecutive amplitudes available from envelope theory:

$$f(a_1,a_2)=\frac{a_1a_2}{V_W^2p^2}\exp\left(-\frac{a_1^2+a_2^2}{2V_Wp^2}\right)\times \mathbf{I_0}\left(\frac{a_1a_2\sqrt{1-p^2}}{V_Wp^2}\right) \quad (6)$$

Here $\mathbf{I}_0$ is the modified Bessel function of the first kind and zero order (standard function included in most mathematical handbooks and software packages), p is the parameter derived from the spectrum and the mean period T_1. It is calculated as:

$$p(T_1)=\sqrt{1-k(T_1)^2-r(T_1)^2} \quad (7)$$

Where $k()$ and $r()$ are the autocorrelation function and the result of sine transformation of wave spectrum density s(ω):

$$r(\tau)=\frac{1}{V_W}\int_0^\infty s(\omega)\cos(\omega\tau)d\omega \quad (8)$$

$$k(\tau)=\frac{1}{V_W}\int_0^\infty s(\omega)\sin(\omega\tau)d\omega \quad (9)$$

Account for Wave Length

To be "dangerous", a wave group must contain waves of certain length and be capable of causing significant stability variations. That means that the frequency of i-th wave in a group is assumed to satisfy:

$$\omega_i\in[\omega_{low};\omega_{up}] \quad (10)$$

The limits of the frequency of the wave that can cause significant stability variation can be easily expressed through using the dispersion relation:

$$\omega_{low}=\sqrt{\frac{2\pi g}{\lambda_{high}}}\ ;\quad \omega_{high}=\sqrt{\frac{2\pi g}{\lambda_{low}}} \quad (11)$$

The probabilistic formulation of a group is based on a Markov chain assumption that the current wave amplitude only depends on the amplitude of the previous wave, but does not depend on the amplitude of a wave before the previous one. It is logical to make a similar assumption for the wave length. However, a formula of joint distribution of two consecutive wave amplitudes and frequencies is not readily available from the classic envelope theory. In lieu of this formula, it is assumed that there is no direct dependence between frequencies of two consecutive waves

$$P(\omega_1\in[\omega_{low};\omega_{up}]\cap\omega_2\in[\omega_{low};\omega_{up}])= P(\omega_1\in[\omega_{low};\omega_{up}])P(\omega_2\in[\omega_{low};\omega_{up}]) \quad (12)$$

That does not mean, however that there is no possibility to account for the dependence between the frequencies of two consecutive waves at all. The frequency depends on the amplitude and the amplitudes of two consecutive waves are related, see formula (6). The rate of encountering groups capable to cause parametric roll is expressed as

$$\xi_G(N_W)=\xi(a_G)\times P(\omega_1\in[\omega_{low};\omega_{up}]\,|\,a_1>a_G)\times \left(P(a_2>a_G\,|\,a_1>a_G)\times P(\omega_2\in[\omega_{low};\omega_{up}]\,|\,a_2>a_G)\right)^{N_W-1} \quad (13)$$

The relation between amplitude and frequency of a single wave is available from (Longuet-Higgins, 1983) or directly from classic envelope theory using conditional distribution of amplitude and derivative of phase Φ':

$$f_{\Phi'|a}(\Phi',a)=\frac{a}{\sqrt{\omega_{m2}^2-\omega_{m1}^2}\sqrt{2\pi V_W}}\times \exp\left(-\frac{a^2}{2V}\frac{(\Phi'-\omega_{m1})^2}{(\omega_{m2}^2-\omega_{m1}^2)}\right) \quad (14)$$

Where ω_{m1} is the mean frequency and ω_{m2} is the mean spectrum bandwidth.

$$\omega_{m1}=\frac{2\pi}{T_1};\quad \omega_{m2}^2=\frac{V_D}{V_W} \quad (15)$$

The frequency of the wave is associated with the absolute value. Since the distribution (14) is normal, the conditional distribution of frequency is a folded normal distribution:

$$f_{\omega|a}(\omega,a)=f_{\Phi'|a}(\omega,a)+f_{\Phi'|a}(-\omega,a) \quad (16)$$

Safety Level

Since an encounter of a group with N_W waves capable of causing parametric roll follows Poisson flow, the probability that no such group will be encountered during time T is:

$$P_G(T, N_W) = \exp\left(-\xi_G(N_{NW}) \cdot T\right) \qquad (17)$$

For a given standard S and amplification factor associated with stability failure, formula (2) yields the number of waves in a group that causes parametric roll N_{PR}. So, all the wave groups that contains more than N_{PR} waves lead to stability failure. Thus, the probability of encountering no such groups during time T can be expressed as:

$$P_{nf}(T) = \prod_{i=N_{PR}+1}^{\infty} P_G(T, i) = \exp\left(-\xi_{AG} T\right) \qquad (18)$$

where ξ_{AG} is defined as:

$$\xi_{AG} = \sum_{i=N_W+1}^{\infty} \xi_G(i) \qquad (19)$$

The rate of encounter of "dangerous" wave groups depends on a wave spectrum and its parameters. If a spectrum can be defined by significant wave height Hs and the mean zero-crossing period Tz, one can write:

$$\xi_{AG} = \xi_{AG}\left(Hs, Tz\right) \qquad (20)$$

Equation (20) relates the rate of encounter of the "dangerous" wave group with a sea state, *i.e.* a cell in a wave scatter diagram. The long-term rate that is independent on a particular sea state can be expressed as:

$$\xi_{LTG} = \sum_i \sum_j W_{ij} \xi_{AG}\left(Hs_i, Tz_j\right) \qquad (21)$$

where W_{ij} is a statistical weight of the cell of a scatter diagram with significant wave height Hs_i and Tz_j.

The long-term rate of encounter is meant to be a numerical representation of the safety level for parametric roll.

PURE LOSS OF STABILITY

The Criterion

The level 1 vulnerability criterion and standard for pure loss of stability is formulated as (Annex 1 to SLF 55/WP.3)

$$GM_{\min} \geq S \qquad (22)$$

Where $GM_{\min}$ is the minimal value of the metacentric height achieved while a wave passes the ship with the reference wave height and length equal to the ship length.

Stability failure caused by pure loss of stability is manifested as a large roll angle caused by significant degradation of stability on the wave crest. Similar to parametric roll, degradation of stability on the wave crest may be seen as an amplification factor for an initial roll angle. However, even the simplest model will require for the mechanism of amplification a significant amount of data making it impractical as the method for the level 1 vulnerability criteria.

Development of a simple model of pure loss of stability requires calculation of the criterion value for waves with the height different than reference height and the length different from ship length. These calculations cannot be done without information of the hull form. Thus, the evaluation of safety level requires a set of sample ship geometries.

The standard S in the condition (22) is a positive value. To achieve significant roll angle an external excitation must be modeled. The result may depend on timing of the excitation and duration while the stability is decreased. If the height of an encountered wave exceeds significantly the reference height, the value of GM for some ships may become negative. Then dynamics of the ship is described by a repeller (dynamical system with the repelling term instead of attracting term – the case of negative stability). The achieved angle of the repeller strongly depends on the duration while stability is decreased in the wave crest. The evaluation of the time interval while stability is decreased in the wave crest requires additional data such as speed and heading. It may be influenced by surging motion and affected by the existence of surf-riding equilibrium. All these factors make modeling of dynamics too complex for the evaluation of safety level for the level 1 vulnerability criterion.

Wave Representation

Pure loss of stability may be caused by encounter of a single wave capable of causing significant variation of the righting arm, so

$$\omega \in [\omega_{low}; \omega_{up}] \tag{23}$$

where boundaries of the frequency range are defined by equation (11). The height of this wave has to exceed the reference wave height used in the criterion (22):

$$a \geq 0.5H_{ref} \tag{24}$$

The rate of encounter such a wave ξ_{PL} can be calculated with equation (13) for N_W=1:

$$\xi_{PL} = \xi(0.5H_{ref}) \times P(\omega \in [\omega_{low}; \omega_{up}] \mid a > 0.5H_{ref}) \tag{25}$$

where the first term is defined by equation (4) and the second term is calculated using equation (16).

Safety Level

Instead of attempting to evaluate an amplification factor using a dynamical model for pure loss of stability, the stability failure event can associated with the $GM_{\min}$ value below the standard. This approach still requires computation of the $GM_{\min}$ value for a series of waves for a set of representative hull geometries. The value $GM_{\min}$ is presented as a function of wave frequency and amplitude.

$$GM_{\min} \leftarrow (\omega, a) \quad : \quad \omega \in [\omega_{low}; \omega_{up}]; \quad a \geq 0.5H_{ref} \tag{26}$$

Calculations are performed for the KG value satisfying condition (22), such as $GM_{\min}$ equals to the standard value exactly. The failure event indicator variable is defined as:

$$E(\omega, a) = \begin{cases} 1 & if \quad GM_{\min}(\omega, a) < S \\ 0 & if \quad GM_{\min}(\omega, a) \geq S \end{cases} \tag{27}$$

The conditional probability of pure loss of stability for a particular value of wave amplitude is:

$$P(GM_{\min} < S \mid a) = \int_{\omega_{low}}^{\omega_{up}} f_{\omega|a}(\omega, a) E(\omega, a) d\omega \tag{28}$$

The probability of pure loss of stability for the amplitude exceeding the specified value (the half of reference wave height) is:

$$P(a \geq 0.5H_{ref} \cap GM_{\min} < S) = \int_{0.5H_{ref}}^{\infty} f_a(a) \cdot P(GM_{\min} < S \mid a) da \tag{29}$$

The conditional probability of pure loss of stability if the wave amplitude exceeds the specified value is:

$$P(GM_{\min} < S \mid 0.5H_{ref} > a) = \frac{P(0.5H_{ref} > a \cap GM_{\min} < S)}{P(0.5H_{ref} > a)} \tag{30}$$

Then, the safety level of the criterion (22) is associated with the probability that no stability failure occur during time T if the condition (22) is met for the reference wave height. This probability can be expressed as an upcrossing of the reference level by the wave elevation with the reduced rate

$$\xi_{GM} = \xi_{PL} P(GM_{\min} < S \mid a > 0.5H_{ref}) \tag{31}$$

The long-term safety level is calculated by weighted averaging of the rate of encounter in the same manner as performed in equation (21)

$$\xi_{LTGM} = \sum_i \sum_j W_{ij} \xi_{GM} \tag{32}$$

The long-term rate of encounter is meant to be used as a numerical representation of the safety level for pure loss of stability. Approaches used for evaluation of the safety level for parametric roll and pure loss of stability are similar, but this similarity does not extend beyond the seaway description (i.e., the use of the wave scatter diagram). A simple dynamical model of the failure was used for parametric roll, which was not possible for pure loss of stability. Instead the failure was associated with a substandard value of the criterion, i.e. the value of GM in waves.

CONCLUDING COMMENTS

The alignment of Goal-Based Standards and the second generation of IMO intact criteria is both natural and achievable. Since the goal of stability is to provide safety from capsizing or large roll angle (and/or acceleration), probability is a natural metric for stability in waves. Also probability of stability failure is a part of risk analysis and formal safety assessment.

Thus, there are no issues to align GBS with the second generation intact stability criteria when the criteria have probabilistic nature i.e. for the direct stability assessment. The alignment with vulnerability criteria that are deterministic can be achieved through the safety level. The safety level is a probability that the stability failure will occur for a ship that passes the criteria. The evaluation of the safety level is required by the framework of the second generation intact stability criteria. To align the vulnerability criteria with GBS, the safety level can be used as metric of the goal.

The calculation of the safety level for the first vulnerability check of stability failures related to variation of righting arm in waves can be done through the reference wave height because the information about the ship is very limited. The safety level of the parametric roll criterion is calculated as time-dependent probability of exceedance of the specified roll angle, based on wave group approach. The transitional solution of Mathieu equation is used to compute probability of the stability failure if a group of waves is encountered.

The safety level for pure loss of stability is based on the time-dependent probability of encounter of a wave causing GM value to fall below the standard. Since there is no general way to evaluate the GM variation in waves without hull geometry, representative sample ships are needed for safety level calculations for pure loss of stability.

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