Fast time domain evaluation of Anti-Roll Tank and ship coupling using non-linear retardation functions

Nicolas F.A.J. Carette, MARIN, n.carette@marin.nl

ABSTRACT

Anti-Roll Tanks (ART) have been used for more than a century to damp the roll motion of ships. These devices exist in various configurations, passively and actively controlled. All versions rely on resonant water motions in a chamber which, by essence, is a very non-linear process. To account for these non-linearities, several approaches have been proposed, where the most recent and complete one is the direct coupling of time domain seakeeping codes with a CFD models of the ART. However, this approach comes at the price of relatively high computation effort. This is in contradiction with the need for long simulations to establish the effects of the non-linearities in the ART reaction forces on extreme events. To reduce the computation costs of a direct simulation, a new technique is proposed which uses retardation functions based on harmonic ART response data. The technique proposed here uses a family of retardation functions with a Hilbert transform method for time dependent interpolations to capture the non-linearity in the response of the tank as a function of excitation amplitude.

Keywords: *Time domain; seakeeping; Anti-roll tank; free surface tank; U-tank*

1. INTRODUCTION

In the early design phase, numerical methods provide an efficient method to predict the motions of a ship. However, it is well known that, due to its underlying resonance principle, the response of an ART is strongly non-linear. This is already known for a long time from observations on board ships (see Watts 1883; Lewison 1975)and is confirmed by numerical studies (see Chu et al. 1968; Verhagen, van Wijngaarden 1965) and experimental campaigns (see van den Bosch, Vugts 1966; Stigter 1966). Therefore, the numerical model that predicts the merits of an ART should take these non-linear effects into account.

Time domain seakeeping codes are widely used to study the behaviour of a ship in a seaway when non-linearities, in either the excitation or the reaction forces, are expected. Therefore, a method to include also the effect of an ART in such a simulation seems of great value. The most straightforward approach is to couple such seakeeping code to a CFD model of the ART (see van Daalen et al. 2001; Cercos-Pita et al. 2015). However, CFD calculations of an ART take typically in the order of several hours per hour of simulation on multi-CPU clusters, whereas time domain seakeeping codes usually runs faster than real time on a simple single-core desktop PC. Early approaches attempted to simplify response of an ART by considering an that of an equivalent pendulum. However, this is considered too simplistic to capture the non-linearity of the response (see Abramson, Silverman 1966). Therefore, because of the absence of another analytical time domain model, both for either free surface or U-type ART, another approach is proposed here.

The approach developed here is based on the use of so-called retardation functions, or more commonly named impulse response functions, for damping and added mass of floating oscillating bodies as proposed by Cummins (see Cummins 1962; Ogilvie 1964; Journée 2001). Such an approach is very fast and light regarding computational effort, and can be used for any ART if its reaction forces (damping, restoring or added mass) are available. However, this method assumes a linear damping. This problem is addressed by means of an interpolation based on the instantaneous excitation envelope. Following earlier work (Carette 2015), the effective gravity angle (EGA), which is determined by the local transverse accelerations and the local vertical accelerations, is adopted as the measure for the excitation of the ART.

2. METHOD

ART response

The response of the ART at each time step can be written in the form of a convolution of its retardation function and history of excitation velocity $\dot{\phi}$. Because the response of a tank is easily known at zero-frequency, rather than at infinite frequency, the infinite added mass is here replaced by the zero-frequency restoring term, and leads thus to the following equation for the roll reaction moment at time t:

$$M_x(t)=\int_0^\infty K(\tau)\dot{\phi}(t-\tau)d\tau+C_0\phi(t) \qquad (1)$$

where K is the retardation function obtained from equation (2), and the damping b is derived from harmonic oscillation tests (see van den Bosch, Vugts 1966), CFD calculations (Kerkvliet et al. 2014) or frequency domain ART models (see Verhagen, van Wijngaarden 1965; Stigter 1966). The restoring term C_0 is of course the free surface effect of the ART, and can be easily estimated based on the tank geometry.

$$K(\tau)=\frac{2}{\pi}\int_0^\infty b(\omega)\cos(\omega\tau)d\omega \qquad (2)$$

To cope with the non-linearity of the response due to the excitation amplitude, a linear interpolation is used. Prior to the time domain calculations, N retardation functions are computed for a range of amplitudes of the excitation ϕ_a, rather than only one like in the case of a perfectly linear damping. At each time step during the simulation, the current amplitude is estimated from the envelope of the excitation amplitude which is computed using a Hilbert transform. The history of the excitation envelope is stored along with the history of the excitation amplitude and velocity. The history of the envelope is used to obtain time dependent linear interpolation coefficients c_i for each time step in the past. The retardation function at the current time step is obtained by summation of the coefficients and the retardation functions along the amplitude axis. In this way, each motion sample will be convoluted with a retardation function obtained from linear interpolation based on the amplitude envelope at that time.

$$K(t)=\sum_{\phi_a}^{N}c_i(t-\tau)K_i(t-\tau) \qquad (3)$$

For every step in the simulation the local envelope of the excitation amplitude is obtained through a Hilbert transform of the history of the preceding time steps. The window of the envelope has the a time span equal to the one of the retardations. However, such a transform has large deviations at the fore and aft ends of the window, thus leading to incorrect prediction of the envelope at the current time step. Various techniques have been developed to reduce those effects in signal analysis, with the easiest being a simple mirroring of the data. However, mirroring the data can introduce discontinuities that reduce its benefits. An alternative method uses motion prediction based on the current position, velocity and acceleration. The quality of this method is however limited in the case of non-linear simulations. In the present work a hybrid method is used. The method detects different cases and applies either central symmetry, axial symmetry, time shifts or motion prediction. Afterwards, to smoothen the mirroring, a slope correction of the mirrored part of the data is applied by using the instantaneous acceleration compared to the slope at the mirroring junction. The different mirroring cases are:

- Immediately before a zero crossing: a central symmetry around the zero crossing is done (Figure 1).
- Close to a peak: a y-axis symmetry around the peak is used.
- After a peak: y-axis symmetry around the back-face of the peak is done (Figure 2).
- After a zero crossing:
  - If the sample is lower than a peak in the past: a y-axis symmetry around the back face of a lower peak (Figure 3).
  - If the sample exceeds all available peaks in the past: no symmetry is used, the two next samples are predicted using the current position, speed and acceleration.

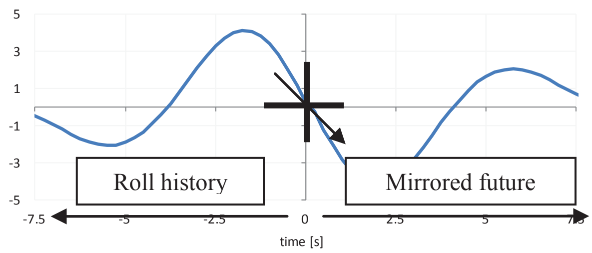

Figure 1: Central symmetry at zero crossing

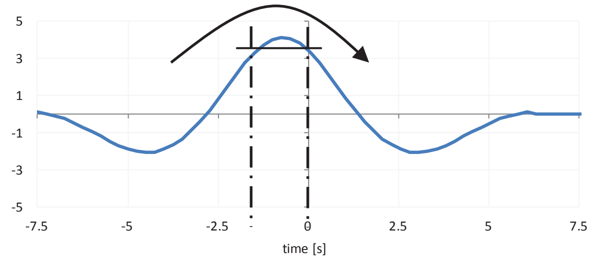

Figure 2: Back face symmetry after peak

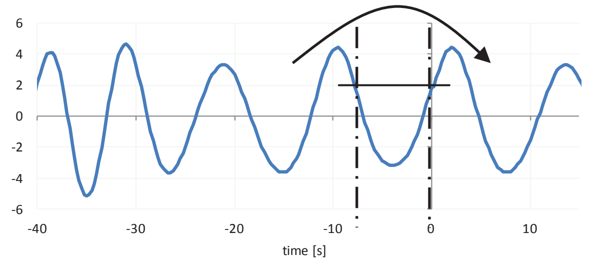

Figure 3: y-axis symmetry around a previous peak

Coupled response

The ship motions are computed using a time domain solver where the response of the tank is added as an external force computed at the beginning of each time step before the integration. The motion excitation applied to the ART is based on the EGA, rather than the roll, which is computed at the centre of the ART, and is defined as the angle to the vertical of the acceleration in the transverse plane. This angle thus includes the roll angle, but also the sway and heave accelerations at the tank location. Use of the EGA, which introduces a strong sway-roll coupling which was not accounted for in the older roll-based methods.

Initially, for verification purposes, a simple one degree of freedom solver using added mass, potential damping and wave excitation from a potential code was used. This solver uses a 5th order Runge-Kutta integrator available in the scipy library (see Hairer et al. 1993). A linear and quadratic damping can also be included. The integrated function is given in equation (4):

$$\ddot{\phi} = \frac{F_{inc} + F_{diff} + F_{ret} + F_{ART} - B_L\dot{\phi} - B_q\dot{\phi}\overline{\dot{\phi}} + C_{xx}\phi}{I_{xx} + A_{xx}} \quad (4)$$

The excitation force at each integration time step i is based on the average between the current time step and the previous time step. The retardation forces, including those of the ART, are based on the previous time step and kept constant during the integration.

The time domain, six degrees of freedom code FREDYN was used (see de Kat, Paulling 1989; de Kat, Paulling 2001). This code uses a linear added mass, wave damping and diffraction from 2D strip theory calculations. The Froude-Krilov component in the wave excitation is non-linear, taking into account the instantaneous underwater geometry. The code includes various semi-empirical models for control surfaces and appendages. As in the 1 DoF model, the ART forces are computed at each time step from the motion history up to that step, and kept constant during the integration.

3. RESULTS

To verify the non-linear retardation function technique, a stepwise approach was used. Firstly, the use of impulse response functions to capture the damping and restoring effects of an ART was verified using forward and backward convolutions. Secondly, the envelope capturing technique was evaluated on its own by means of spectral analysis. Thirdly, the time domain response of a tank tested under irregular roll motion was computed. Finally, the computed coupled motions of a ship with an ART were compared to experiments.

Retardation function of an ART

Due to its relatively narrow peak, the damping of an ART will lead to relatively longer retardation functions than a typical wave damping operator. Moreover, for an ART, the added mass is not used, but the restoring term. The shape of the restoring coefficient of an ART is however not optimal for a Fourier transform , that is required in the derivation of the retardation function, as it has an offset between the value at zero and at infinite frequencies, due to the free surface effect. A Fourier transform works better in the case of a signal starting and finishing at the mean value.

To verify the adopted approach, the response of a reference U-tank was generated using Stigter's model (see Stigter 1966). The use of this analytical model is to ensure that the frequencies can be freely chosen to ensure the highest quality of the retardation functions. The chosen tank has a natural period of 8.3 seconds and a mass of water of about 134 tonnes. This tank has some internal damping due to limited ventilation, although it has rounded duct edges, such that its damping peak at low amplitude is relatively narrow. At larger amplitudes, the width of the peak increases rapidly. Figure 4 shows that the damping from the analytical model is in very good agreement with experimental data, both in the frequency and in the amplitude directions. Figure 5 presents the derived retardation functions based on the damping at various excitation amplitudes. Due to the width of the damping peak at small amplitudes, the retardation function is much longer than at larger amplitudes.

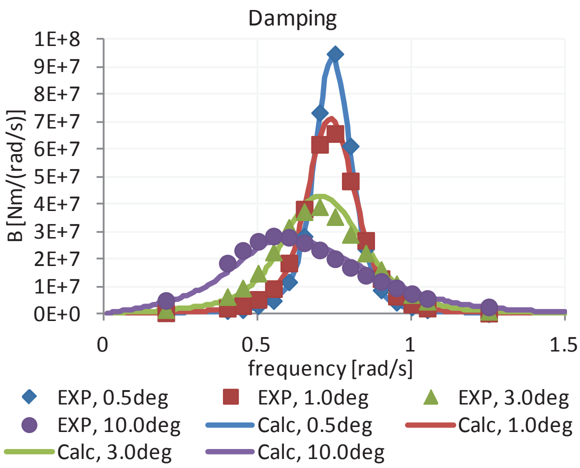

Figure 4: Damping of U-tank using Stigter's model

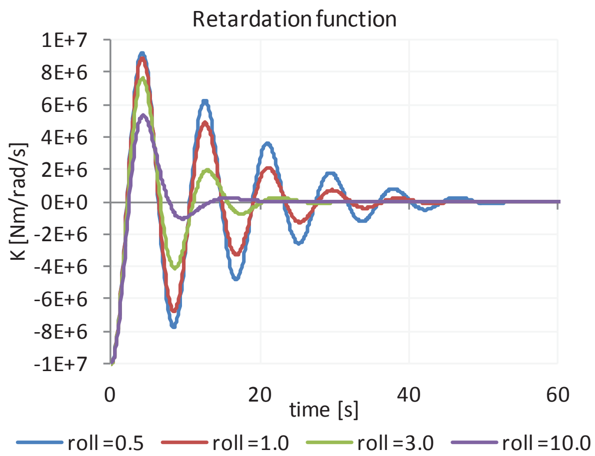

Figure 5: Retardation functions for ART

To check the representation of the restoring term, it was reconstructed from the retardation functions using the inverse convolution given in equation (5).

$$c(\omega)=\omega\int_{0}^{t}K(t)\sin(\omega t)\,dt \qquad (5)$$

Figure 6 shows that the obtained restoring term is good at the lower frequencies and around the resonance area, but deviates from the frequency domain values for increasing frequency and roll amplitude. The deviation seems to be driven by the amplitude of the damping at very low frequencies. The error in the restoring term should not be too important around the natural period of the tank, otherwise the resonance of the coupled ship and ART system will be affected by this method as the restoring term has a direct influence on the resonance frequency. If the resonance conditions are of importance for the ship performance study, it would be advised to correct the free surface effect C_0 such that the restoring term after convolution is zero at the natural period of the tank. The motions at low frequencies will then be affected by the artificially reduced free surface effect.

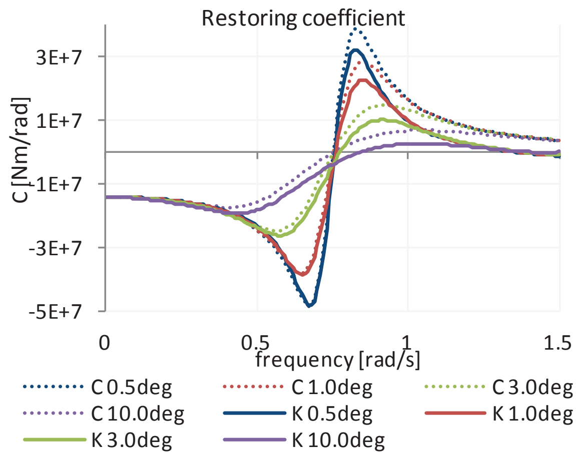

Figure 6: Restoring term of ART before and after convolution

Overall, the shape of the response of f an ART in terms of its damping and restoring moments seem well represented by the used retardation functions, although it may lead to relatively long convolution time spans in the case of small amplitudes and low internal damping.

Estimation of the excitation envelope

The linear interpolation technique between the retardation functions relies on the evaluation of the current motion amplitude. Due to the end effects of the Hilbert transform, this evaluation is subject to some error depending on the current sample being around a peak, around a zero crossing or in-between. To evaluate the quality of the hybrid mirroring technique, some tests were carried out with synthetic time traces generated from different types of spectra, and the envelope was compared with various parameters such as time span, time step, spectrum width and peak frequency of the spectrum. The time trace was generated for 1800 s. The spectrum was based on a simple Hanning window centred around the peak frequency, and with a given width.

Firstly, the effect of the time span of the window used for the envelope was studied. A time trace was generated with an irregular spectrum with a peak frequency of 1 rad/s and a width of 0.5 rad/s. The time step used was 0.1 s. A window with a given time span was then ran across the signal, and the envelope at the end of the window was compared to the envelope of the complete signal. Figure 7 shows that the length of the time span does not have much effect on the quality of the envelope using the hybrid mirroring, and is considerably better than a direct Hilbert transform of the window. The direct Hilbert transform shows strong oscillations around the true envelope at twice the peak frequency of the spectrum. The envelope with the mirrored data shows much smaller deviations, however it is somewhat discontinuous. The discontinuities are due to the discrete logic in the mirroring technique.

Figure 8 shows the spectrum of the envelope. The direct Hilbert transform of the window typically shows a peak at twice the peak frequency of the spectrum of the signal. The envelope with mirrored data has much lower deviations at those frequencies. The discontinuities due to the discrete logic introduce local peaks in the spectrum, but at frequencies way above the region of interest.

Secondly, the peak frequency of the signal spectrum was varied, keeping the width of 0.5 rad/s and the window span to 60 s. Figure 9 shows that the hybrid mirroring technique yields a very good estimate of the envelope around the peak frequency of the spectrum for a range of peak frequencies. It also clearly shows the peak in the direct Hilbert at twice the peak frequency of the spectrum. This peak could be problematic as it might affect the ART response in a frequency region where it already increases the ship motions; however, the hybrid technique solves this issue.

Finally, the width of the signal spectrum was varied from very narrow (0.25 rad/s) to very wide (2 rad/s), keeping the peak frequency at 1 rad/s. Figure 10 shows that the width of the spectrum does not have much influence on the quality of the envelope with mirroring, with a slight improvement as the width is reduced, although at the cost of peaks in the envelope spectrum at the harmonics of the incoming spectrum.

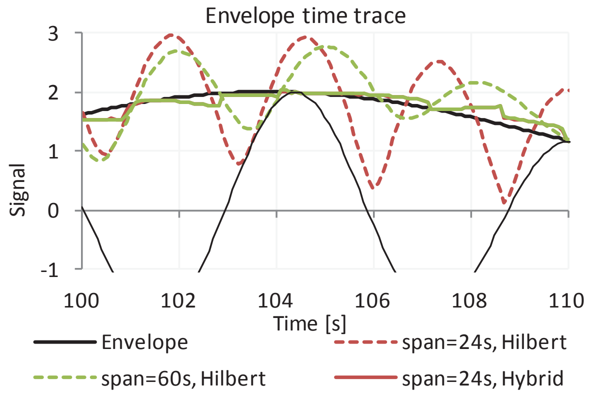

Figure 7: Envelope with various window sizes

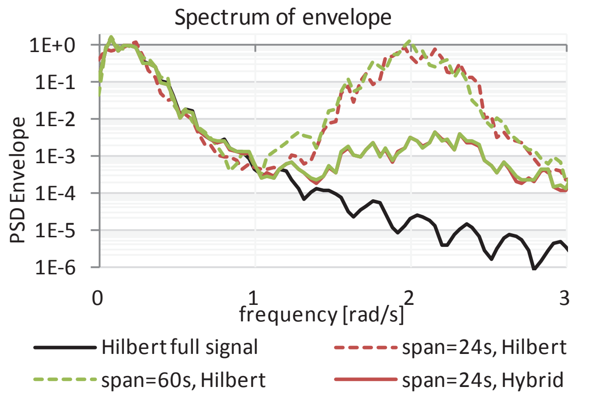

Figure 8: Spectrum of the envelope with various time spans

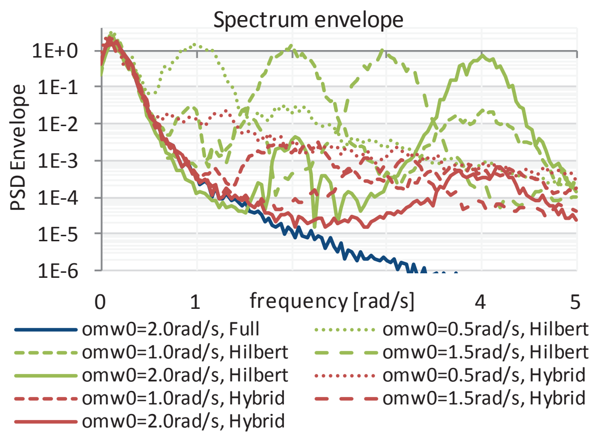

Figure 9: Spectrum envelope with various peak frequencies

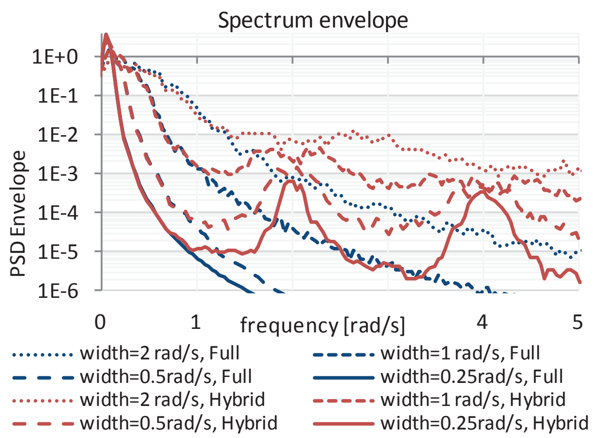

Figure 10: Spectrum of the envelope with various signal spectrum width

Non-linear retardation functions

The reference U-tank used up to now was also tested with irregular roll excitation with a significant amplitude of 2 degrees on an oscillation table. The test was carried out for 30 minutes full scale. The tank was tested with rounded and sharp duct edges to vary the internal damping. A flume type free surface tank of similar natural period and weight was also tested with the same motion time traces. In both cases, the peak of the motion spectrum was centred around the natural period of the tank. To validate the non-linear retardation functions, the response of both tanks was computed for a range of amplitudes using Stigter's model for the U-tank and with Verhagen's model for the free surface tank. The range of amplitudes was chosen such that it would overlap the irregular roll motions during the test. The non-linear retardation functions based on these operators were then used to reconstruct the irregular reaction forces of the tank using a time step of 0.25 s, which was sufficiently small to have no influence on the calculation. Figure 11 presents the result of the calculations compared to the experiments in the form of distributions of the amplitudes of the reaction moment. The frequency of exceedance is plotted on a Raleigh scale, on this scale the amplitude distribution of a narrow-banded perfectly linear process would show as a straight line (see Ochi, Bolton 1973). The amplitude has been divided by the RMS of the linear solution. The results show a clear improvement with the non-linear solution that now follows a non-linear distribution with a bias towards lower extremes. This distribution of the amplitudes of the response moment shows that the tank is, as expected, less efficient at large amplitudes than it is at small ones. Therefore, the response of the ship may be biased towards larger extremes if the tank is the significant source of damping.

As an example, a one degree of freedom simulation was carried out with the DDG51 equipped with the tested U-tank ART. The loading condition of the vessel was chosen to be tuned with the ART, and in such way that the ART would represent about 2% of the displacement. The calculations were performed with and without the ART for 10 h with a time step of 0.1 s. The calculations without ART were done with additional damping such that the RMS motions would be similar to those with the ART. This was done with either a purely linear damping, or with non-linear damping. Figure 12 presents the distribution of the amplitudes of roll from the different solutions. As expected, the roll amplitude distribution with the linear damping follows the (straight-line) Rayleigh distribution. The result with the non-linear damping shows considerably lower extreme values. What was less expected is that the ship with ART presents an almost linear distribution. This means that an ART reduces typical values of the response (for instance the mean amplitude, or the RMS) much better than the extreme values.

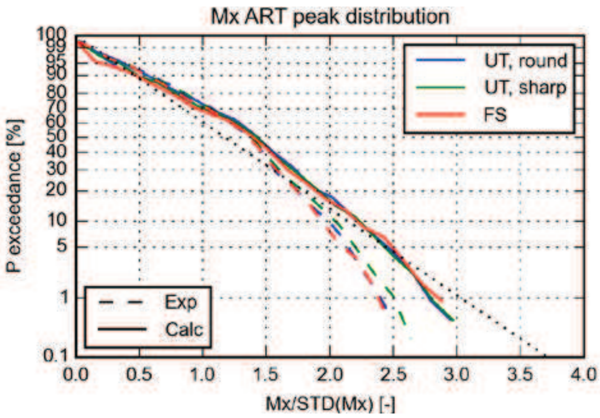

Figure 11: ART response moment distribution with irregular motions of 2 deg SSA

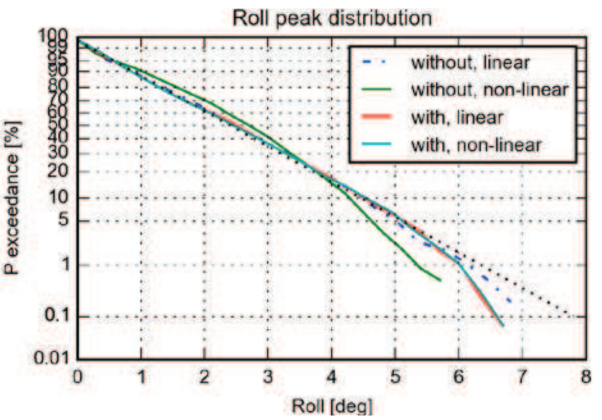

Figure 12: Roll distribution, 1DoF time domain, Hs=0.5m, Tp=8.7s, with and without ART

The fact that the roll with ART ends up more linear than one would expect based on the RAO of the tank moment is partly due to the width of the wave spectrum and tank size. Indeed, the ART response decreasing with increasing amplitude does not have the same effect at the roll resonance as at other frequencies. The ART damps the motions at resonance, but increases them at lower and higher frequencies, the non-linearities partly cancelling each other. This also explains why the solution with the linear retardation functions gives very similar results to the one with non-linear retardation functions, in this case with the linearization around the significant roll amplitude. However, using non-linear retardation functions saves the trouble of having to find the right linearization amplitude. Moreover, in the case of tanks with larger dimensions in the longitudinal direction of the ship, the frequencies where the tank increases the motions are further apart. This, combined with a narrow wave spectrum, might even increase the larger roll amplitudes.

Coupled motions

Finally, the coupling of non-linear retardation functions for an ART with ship motions were verified by comparison with experimental data. An 18000 tonnes heavy lift vessel equipped with a 210 tonne free surface ART was tested in beam seas at zero speed. The waves were generated with a JONSWAP spectrum with a peak period equal to the ship's natural roll period and with two different heights, 0.75 and 1.5 metres. The tests and calculations were carried out for 30 minutes full scale. The ship model was restrained in surge, sway and yaw by means of a soft spring setup with low natural frequencies to avoid interaction with the roll response . Prior to the tests, roll and sway decay tests were performed.

The calculations were carried out with FREDYN without surge, yaw and pitch motions. The sway motions were restrained with a spring coefficient corresponding to the experimental soft spring. The roll damping parameters were based on a linear and a quadratic coefficient derived from the roll decay tests. The response of the ART was derived using Verhagen's model, and checked by means of oscillation tests for the ART. The excitation of the ART was the EGA at the tank's location. Figure 13 shows the roll distribution with and without tank from the experiments and calculations. It shows that the calculation model captures quite well the damping due to the tank. The distribution of roll with ART appears also much more linear than with only bilge keels. Figure 14 presents the RAO of roll, where the double peaked character of the response with ART is clearly visible. The predicted RMS of roll was within 1% from the result of the experiment for the lower wave height and within 7% for the higher wave height. On a single core 2.1GHz PC the calculations without ART were running at 15 times faster than real time, and those with ART at 3 to 7 times real time.

The use of the EGA rather than the roll is in this case quite important as the roll period of the ship is very long. In such a case, the sway motions are not small compared to the roll, especially of the damped ship, such that the EGA deviates substantially from the roll. Figure 15 presents the roll distribution with the free surface tank using either roll or EGA as excitation parameter during the calculations.

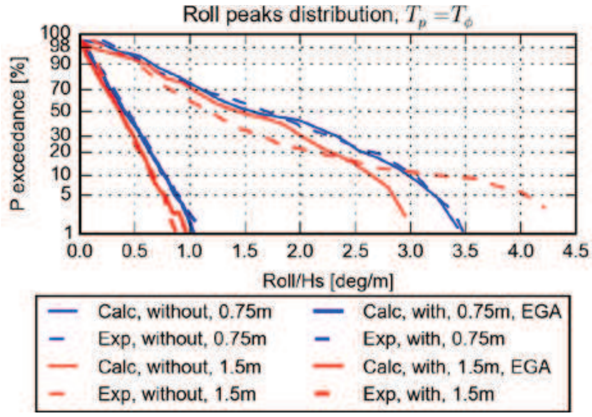

Figure 13: Roll distribution with and without free surface tank

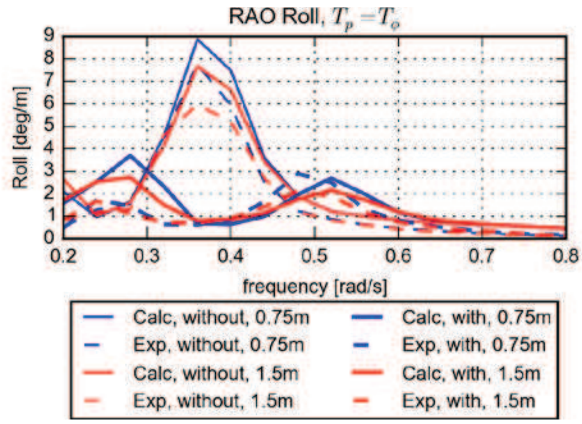

Figure 14: Roll RAO in irregular waves, with and without free surface tank

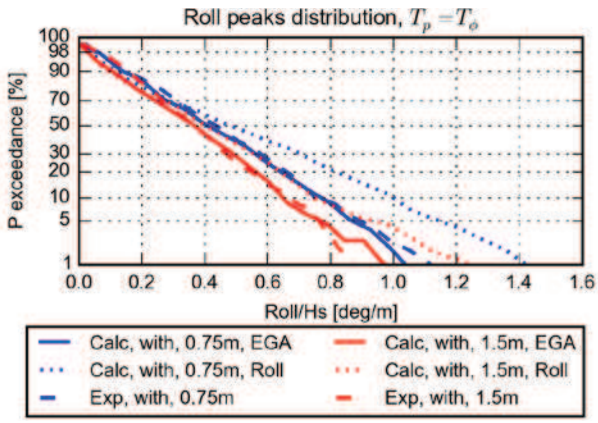

Figure 15: Roll distribution, with tank, roll vs. EGA

4. CONCLUSIONS

A new technique to include the non-linear effect of an ART in time domain calculations has been successfully developed. It uses an estimate of the envelope of the EGA and an interpolation in a set of amplitude dependent retardation functions..

The adopted use of retardation functions to capture the response of the ART as a function of excitation frequency and amplitude works with good accuracy. The use of envelope based interpolation at each time step offers a fast and efficient technique to capture the excitation amplitude dependence of the tank response.

The hybrid mirroring technique offers an accurate envelope prediction at the end of the time window. This technique greatly improves the quality of the Hilbert transform at the ends, but at the cost of small discontinuities at high frequency

Finally, the use of the EGA as excitation parameter for the tank greatly improves the prediction of the tank-ship coupling in conditions where the sway is non-negligible, which should be the case if the tank is properly designed.

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