SOME REMARKS ON THE EXCITATION THRESHOLD OF PARAMETRIC ROLLING IN NON-LINEAR MODELLING

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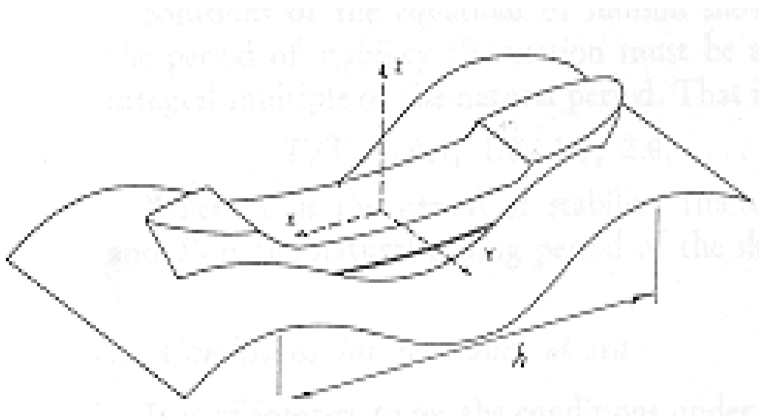
INTRODUCTION I

It is known that the parametric excited rolling in following seas can lead to dramatic roll amplitudes and eventually capsizing.
On the other hand, the parametric rolling in head waves is a less studied phenomenon:
in this case, relative ship velocity is high and consequently the roll damping increases for the effect of forward speed.

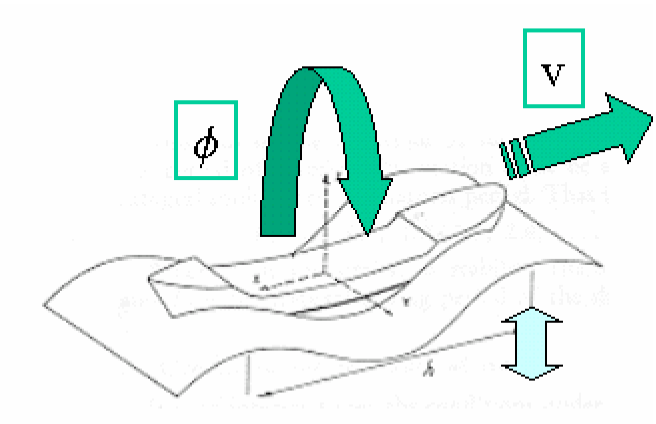


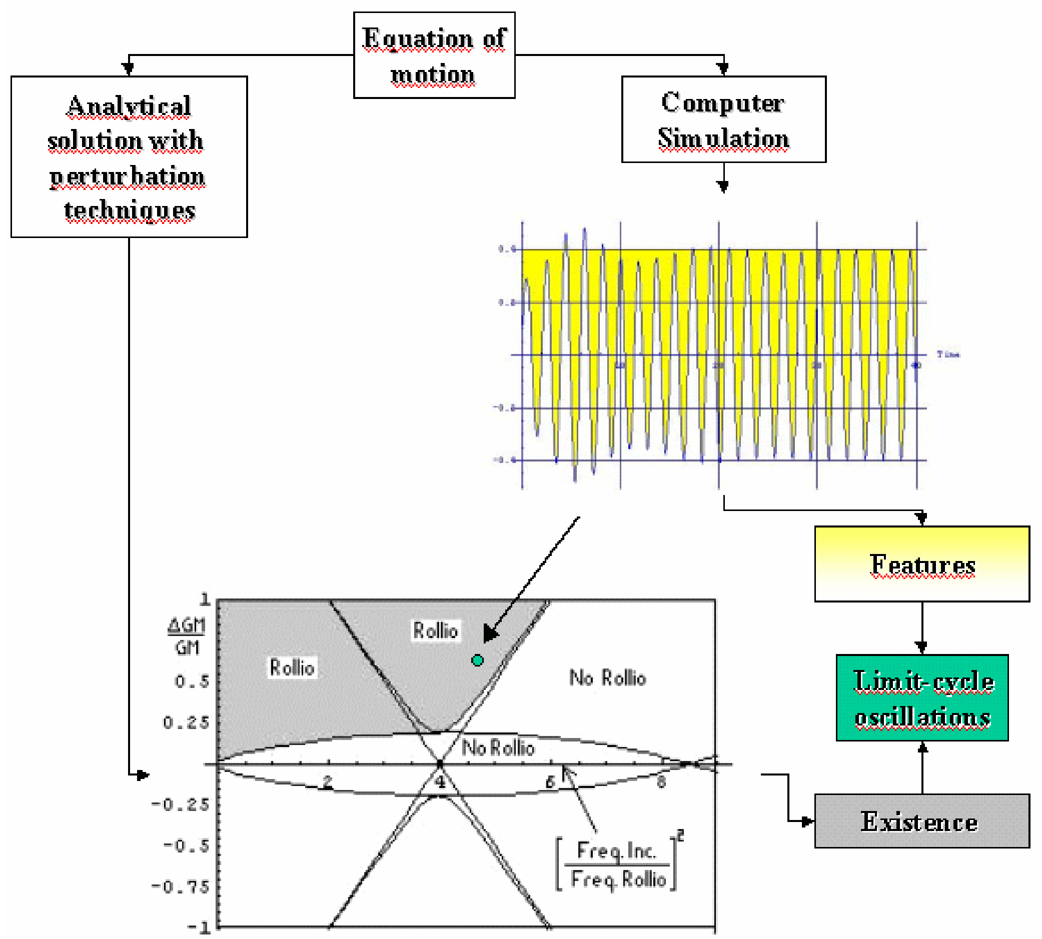
For the onset of parametric rolling:

1. Small deviations from upright position (initial condition on heel angle).

2. **Tuning condition** (with wavelength equal to ship length at waterline) must be $\omega_e / \omega_0 \approx 2$ and may be met **in many operative conditions**.

3. Periodic **fluctuation** of the **transverse stability** sufficiently **high**.





LINEAR APPROACH

Considering a sinusoidal time variation of the transversal metacentric height, the description of ship rolling in a purely longitudinal sea can be obtained by considering the following, linearized mathematical model:

$$\ddot{\phi}+2\mu\dot{\phi}+\omega_0^2\left[1+\frac{\delta\overline{GM}}{\overline{GM}*}\cos(\omega_e t+\varepsilon)\right]\phi=0$$

By changing the independent variable

$$\omega_e t = 2t'$$

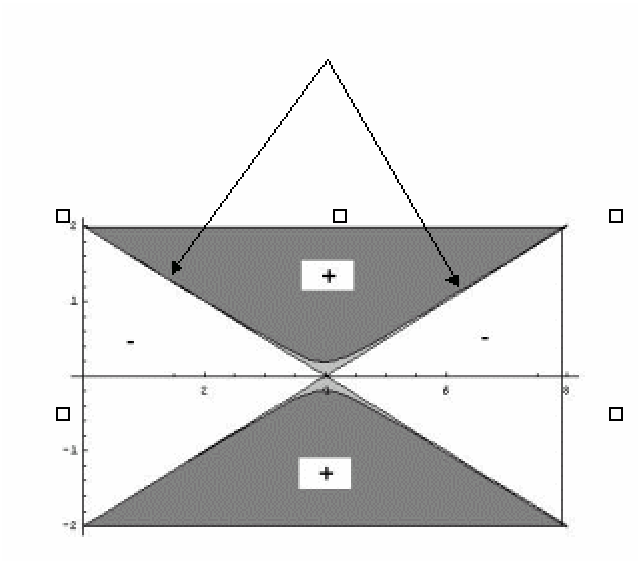
and cancelling the damping by means of a linear transformation $\phi \rightarrow \phi e^{-\mu t}$, we obtain:

$$\ddot{\phi} + 4\frac{\omega_0^2}{\omega_e^2}\left[1 + \frac{\delta\overline{GM}}{\overline{GM}*}\cos(2t)\right]\phi = 0$$

The Floquet theory (σ characteristic exponent) yields that the solutions of damped Mathieu equation will be:

- diverging if both $-\mu^* \pm \sigma > 0$
- stable if $\mu = \sigma = 0$

$$2 - \frac{\omega_e^2}{2\omega_0^2} < \frac{\delta\overline{GM}}{\overline{GM}*} < \frac{\omega_e^2}{2\omega_0^2} - 2$$

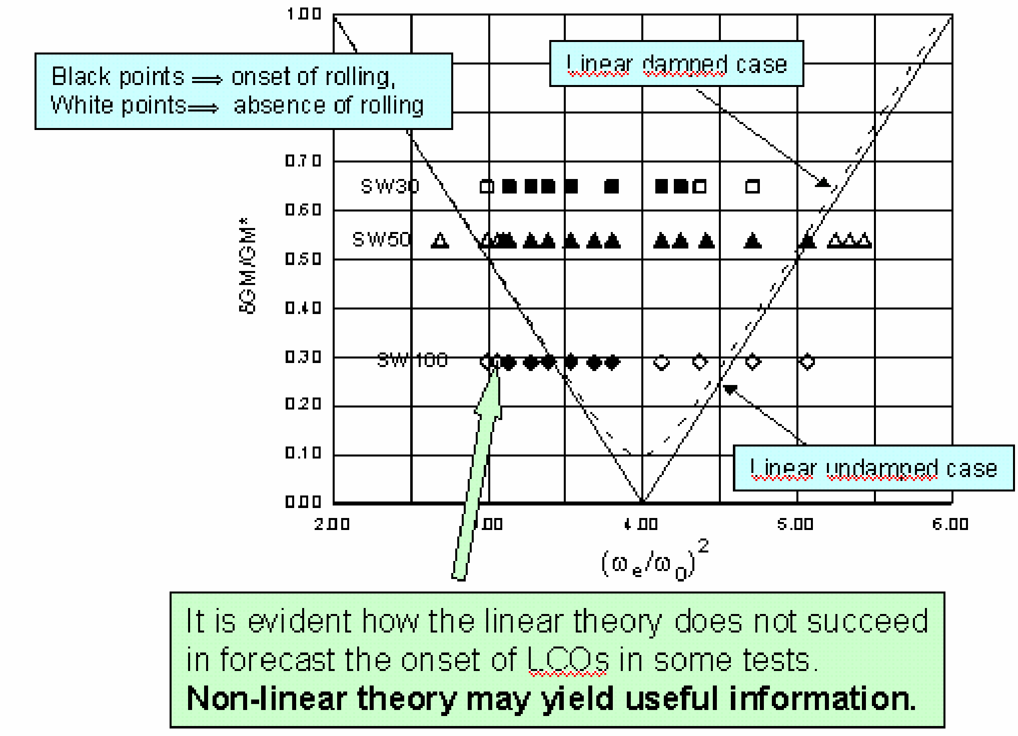


- decaying if both $\mu^* \pm \sigma < 0$.

MODEL TESTING VS. LINEAR APPROACH

Experimental tests on scaled models were performed to determine limit-cycle oscillations (LCO) amplitudes and threshold with respect to ship speed, frequency ratio and metacentric height.

The Figure below is based on data collected from tests in the model basin at DINMA-TRIESTE.



NONLINEAR APPROACH

KBM

$$\dot{C}\cos(t+\theta) - C\dot{\theta}\sin(t+\theta) = 0$$

$$\phi(t) \approx C(\cos(t+\theta))$$

$$\ddot{\phi} + 2\mu^{*}\dot{\phi} + \delta^{*}\dot{\phi}^{3} + \left[1 + \left(p_1 + p_2\phi^{2}\right)\cos 2t\right]\omega_0^{*2}\phi + \alpha_3^{*}\phi^{3} = 0$$

$$\dot{C}\sin(t+\theta) + C\dot{\theta}\cos(t+\theta) = -f\left(C\cos(t+\theta), -C\sin(t+\theta)\right)$$

$$\dot{C} = -\sin(t+\theta) f\left(C\cos(t+\theta), -C\sin(t+\theta)\right)$$

$$\dot{\theta} = -\frac{\cos(t+\theta)}{C} f\left(C\cos(t+\theta), -C\sin(t+\theta)\right)$$

Averaging --> NEXT PAGE

NON-LINEAR APPROACH II

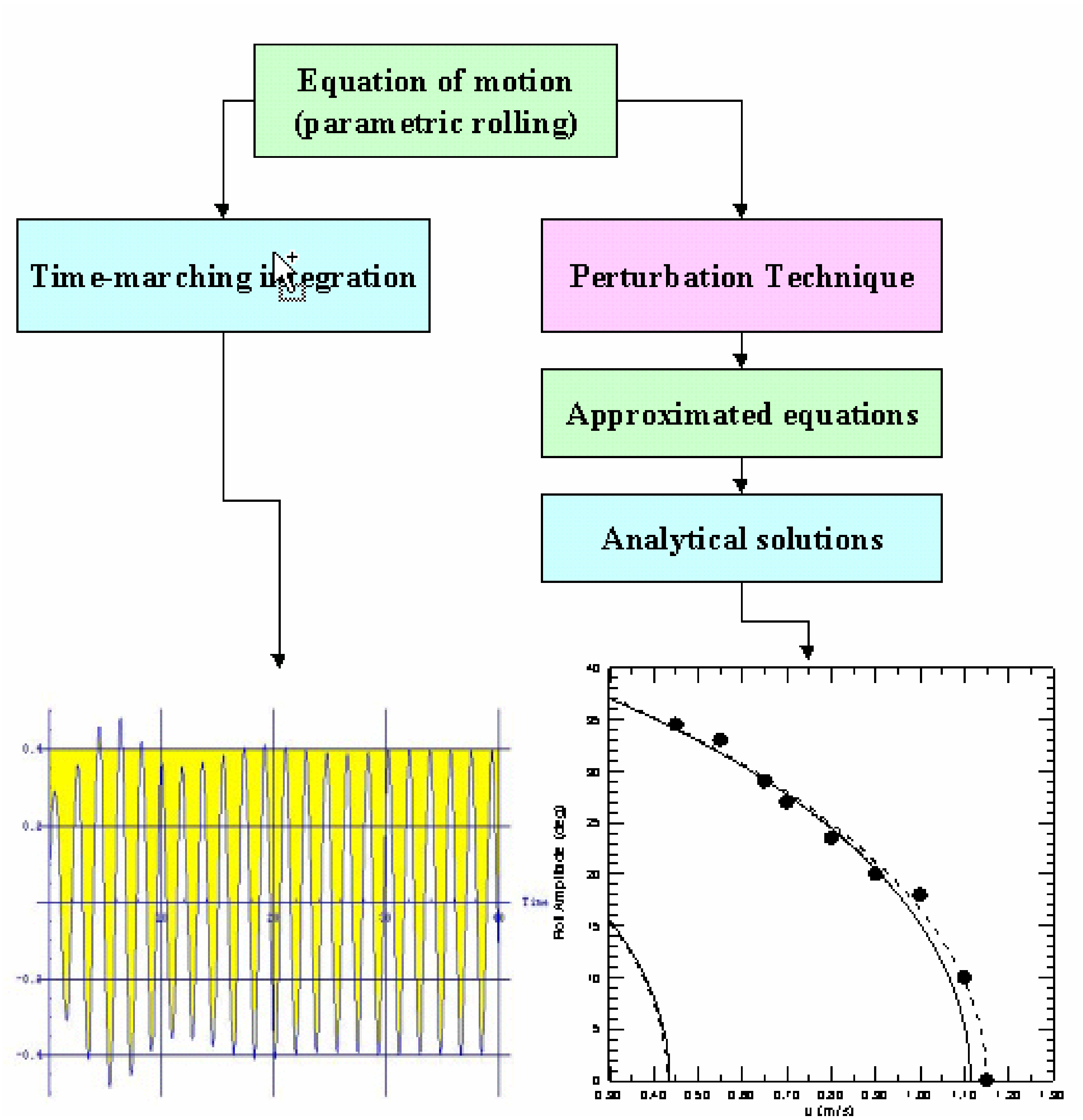
$$\dot{C} = -\sin(t+\theta) f(C\cos(t+\theta), -C\sin(t+\theta))$$
$$\dot{\theta} = -\frac{\cos(t+\theta)}{C} f(C\cos(t+\theta), -C\sin(t+\theta))$$

$$C = \frac{1}{2\pi}\int_0^{2\pi} f(C,\theta)\cos(t)\,dt$$
$$\theta = \frac{1}{2\pi C}\int_0^{2\pi} f(C,\theta)\sin(t)\,dt$$

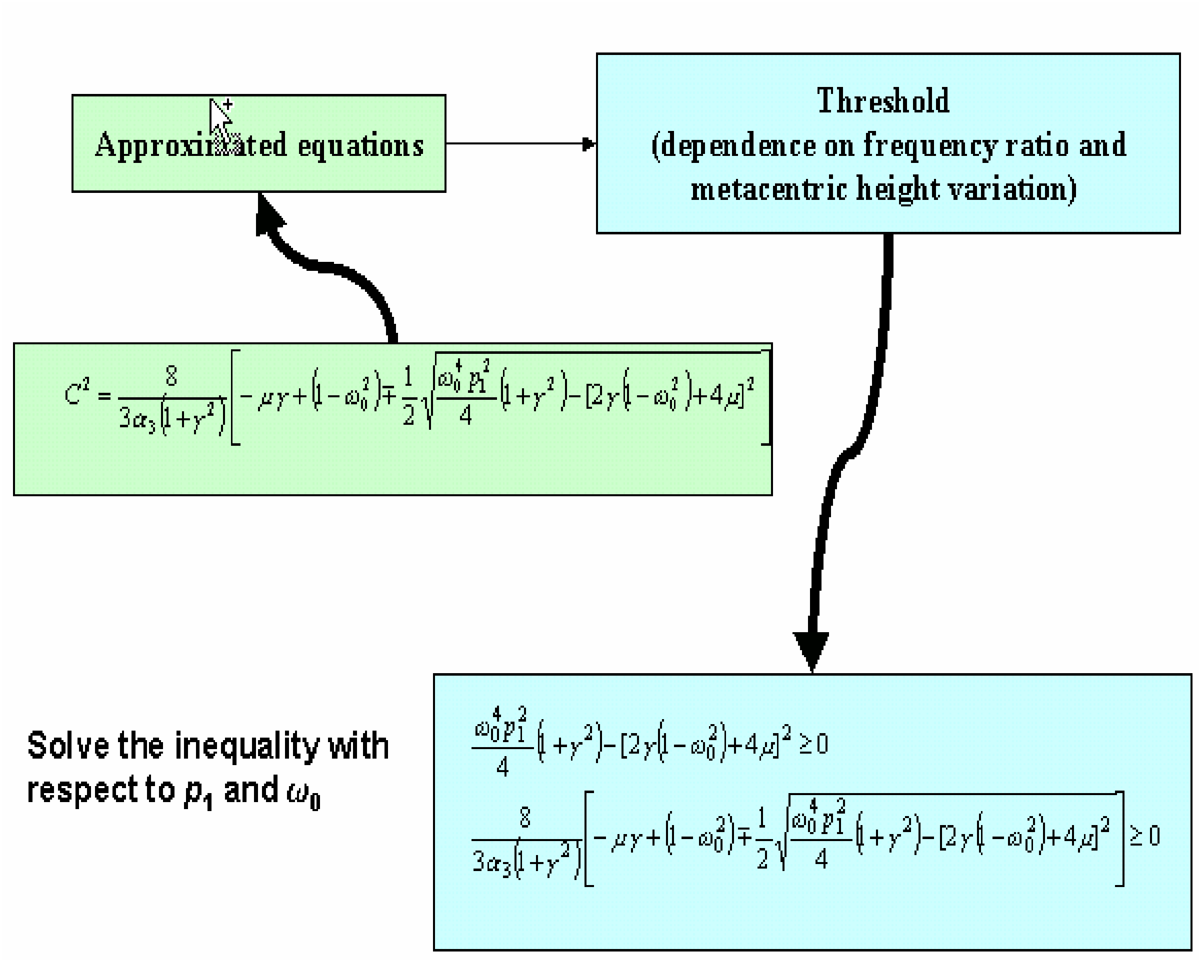
$$\begin{cases} \sin 2\theta\left[2p_1\omega_0^2 + 2p_2C^2\omega_0^2\right] = 3\delta C^2 + 8\mu \\ \cos 2\theta\left[2p_1\omega_0^2 + p_2C^2\omega_0^2\right] = -2(\omega_0^2 - 1 + \frac{3}{4}\alpha_3 C^4) \end{cases}$$

$$\sin^2 2\theta + \cos^2 2\theta = 1$$

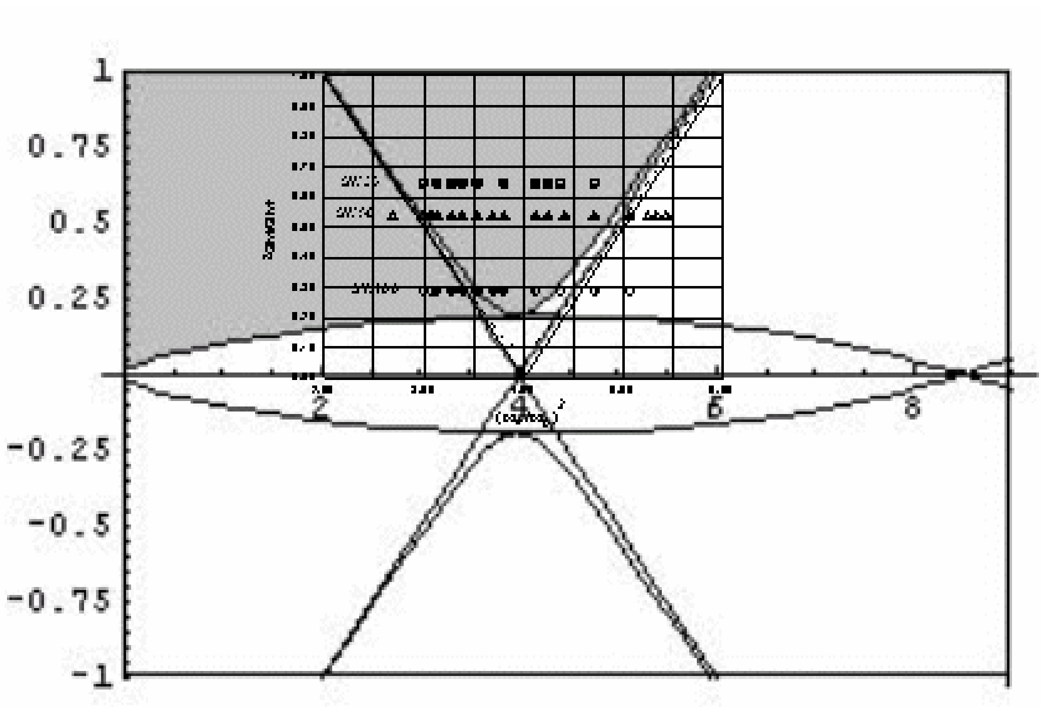
$$C^2 = \frac{8}{3\alpha_3(1+\gamma^2)}\left[-\mu\gamma + (1-\omega_0^2) \mp \frac{1}{2}\sqrt{\frac{\omega_0^4 p_1^2}{4}(1+\gamma^2) - [2\gamma(1-\omega_0^2) + 4\mu]^2}\right]$$



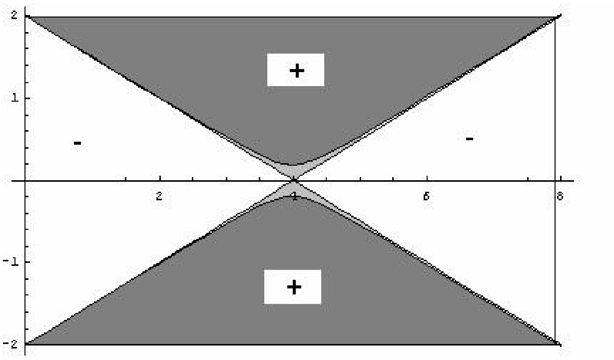
Equation of motion
(parametric rolling)
Time-marching integration
Perturbation Technique
Approximated equations
Analytical solutions
Roll Amplitude (deg)
u (m/s)

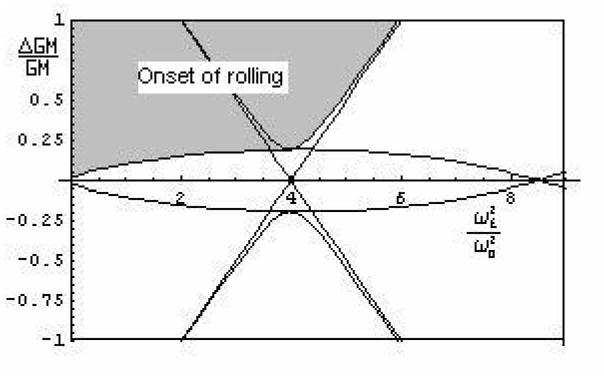


Comparison between theory and experiments



Threshold with Linear and Nonlinear Mathieu Equation





CONCLUDING REMARKS

•The phenomenon of parametric rolling has been investigated from a **mathematical point of view**.

• The **KBM perturbation technique** has been applied to the nonlinear Mathieu equation describing parametric rolling.

• The advantages of analytical solutions via perturbation techniques are:

- "easy" determination of **amplitudes** of periodic motion
- possibility to study the **threshold** for the onset of parametric rolling with respect to some coefficients

• It has been shown that **numerical simulation** and **analytical** investigation of simplified equation of motion provide solutions that are in **good agreement** with model testing.