Autonomously Searching for Algal Blooms Using Optimization Techniques

MATH 4553 - Introduction to Optimization

Max DeSantis

1 Abstract

Uncontrolled large growths of algae, often dubbed harmful algal blooms (HABs), can result in the deterioration of ocean, lake, and river ecosystems due to the production of hazardous toxins and the depletion of water resources. The Oklahoma State University Unmanned Systems Research Institute (USRI), in conjunction with the OSU Mercury Robotics engineering design club, is developing an unmanned surface vehicle (USV) meant to autonomously seek out and relay the position of an algal bloom's most-concentrated point. My objective for this project, adapted for the curriculum of MATH 4553, was to develop an autonomous search algorithm to locate the algal bloom's "source" through the application of gradient descent optimization methods. The algorithm is tested though the development of a simple kinematic simulator and algal distribution model.

2 Introduction

The number of harmful algal blooms (HABs) has increased in recent history, caused by a combination of natural environmental effects and human-caused eutrophication commonly attributed to fertilizer runoff [1]. HABs are responsible for the deaths of many plants and animals living in or around water, as they often produce toxic byproducts that cause harm when ingested. These toxins can be transmitted to humans and animals through contaminated drinking water and the consumption of other affected creatures, such as shellfish. Algae overgrowth is also harmful to underwater life, as less sunlight penetrates the depths and slows or stops photosynthesis. Furthermore, a rapid algae growth due to heightened nutrient levels (commonly phosphorous and nitrogen) will quickly die once nutrients are depleted; this results in a large scale bacterial decomposition that consumes the water's dissolved oxygen. Overall, HABs pose a threat to aquatic life and provide motivation to detect, track, and mitigate their harmful presence and effects.

Recently, robotic platforms have been used in the detection and monitoring of HABs. In one example, found in [2], a cooperative robotic system is used to detect algal blooms using aerial photography from an unmanned aerial vehicle (UAV) and subsequently remove the algae using devices attached to an unmanned surface vehicle (USV). Alternatively, satellite imagery can be used to detect large blooms [3]. A project supported by the Unmanned Systems Research Institute (USRI) aims to develop a USV to detect and seek out locations of high algae concentration. My role in this project is to develop the control system to search for the algal bloom's center. The vehicle is capable of measuring the water's dissolved oxygen in real time; dissolved oxygen should be inversely related to the algal concentration after an overgrowth. This can be interpreted as a zeroth-order minimization problem, where the USV must locate the global minimum of dissolved oxygen using only samples collected at its position.

3 Methodology

The problem can be formulated as a form of black-box optimization constrained to the physical borders of the body of water in question.

$$\text{minimize } f(\mathbf{x}) \tag{1}$$
$$\text{subject to } \mathbf{x} \in \Omega \tag{2}$$

where $f : \mathbb{R}^2 \to \mathbb{R}$ is the unknown function defining the algae concentration, Ω is the set of points defining the water's surface, and $\mathbf{x} \in \mathbb{R}^2$ is the USV's position on the water's surface. This problem is approached with the understanding that any algorithm or technique must be implemented on a real-world autonomous vehicle that can only measure f at the vehicle's location. To facilitate testing and iterative development, the problem is simplified using a number of assumptions.

- Water currents and wind effects on algae distribution are ignored
- The effects of mixing water (due to propellers, currents, wildlife, etc.) on dissolved oxygen concentration are ignored
- The concentration is initially assumed to be unimodal and Gaussian in nature
- There is assumed to be no noise in the concentration

The technique of gradient descent was used to handle the optimization. Traditionally, this requires the computation of the objective function's gradient at a given point to determine the direction of travel. Due to the nature of the problem, the gradient is unknown and unable to be directly computed. It is instead approximated by measuring discrete points surrounding a center location, and computing the measured differences between each point and the center. This is comparable to the one-dimensional secant method that approximates a first derivative using a secant line. The technique is shown in Figure 1, where blue indicates the line of travel of the vehicle, red are points where samples are taken, and green are the computed gradients. As can be seen, the vehicle samples the center then travels outwards counter clockwise, sampling at each vertex, until returning to the center. Gradients are then computed and the steepest gradient is selected as the best direction to travel.

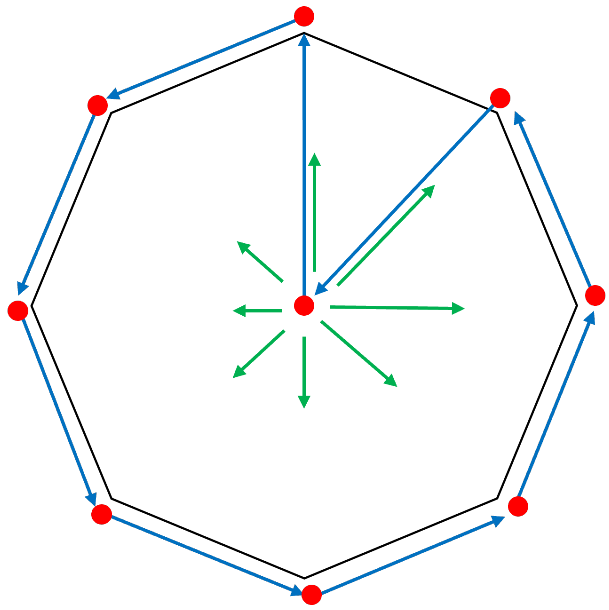


Figure 1: Gradient approximation technique

Using the measured gradient, a gradient search technique can be implemented. Two approaches are analyzed: a fixed-step descent method and a steepest descent method. Generally, gradient descents can be modelled as:

$$x_{k+1} = x_k - \alpha_k \nabla f(\mathbf{x}) \tag{3}$$

In the case of fixed-step descent, $\alpha_k = \alpha$ for all k, resulting in the algorithm travelling a fixed distance in the direction of the gradient before recomputing. The steepest descent method would instead select α_k such that the descent continues until a local minimum is reached along the one-dimensional direction of travel. In the case of the autonomous vehicle, it would instead travel in one direction until it measures the concentration begins to increase. At that point, it would recompute the gradient and begin its line search again.

A simple kinematic simulator is developed to test the algorithm, which requires modelling of the algal distribution and boat behaviors. The algae is modelled using *scipy*'s multivariate normal function, with mean and covariance as parameters. The probability density function (PDF) is directly used as the algae concentration. The simulator also displays the vehicle's search progress over time overtop the distribution's contour plot, allowing comparisons between techniques. The simulator allows the vehicle to turn in place instantaneously.

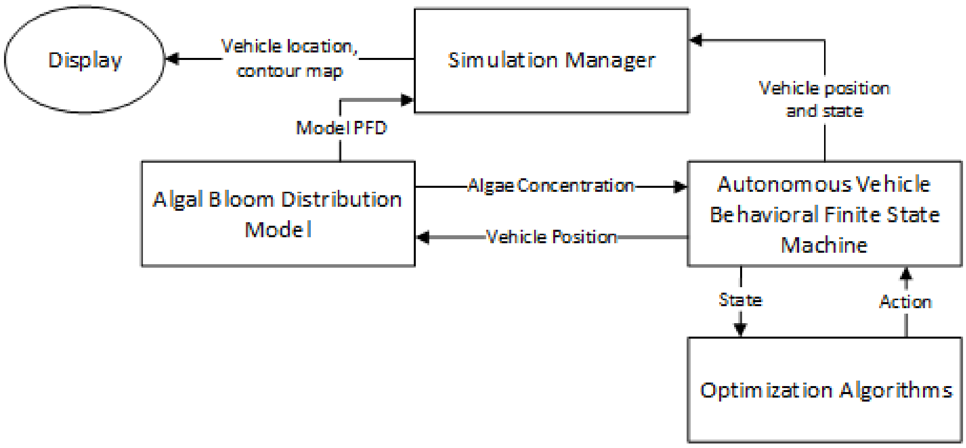


Figure 2: Kinematic simulator architectural block diagram

4 Conclusion

Performance was measured by comparing the results of the simulation with varying parameters. The path of the boat is shown in red. The red rings around points in the path are when it computed the gradient at that location. The contour plot shows the location of the local minimum, with cooler colors being smaller values.

Both techniques have pros and cons. The steepest descent tends to fail to converge precisely on the minimum, although it does get very close to it. The fixed-step will successfully converge given a small enough step size, but it takes much longer to do so. Unless otherwise noted, there were 32 samples taken to compute the gradient (ie samples were taken on each vertex of a 32-sided regular polygon centered at the vehicle's sample start location) and the distance from a vertex to the center was 0.5 meters. The vehicle speed was taken at 1 meter per second for all tests.

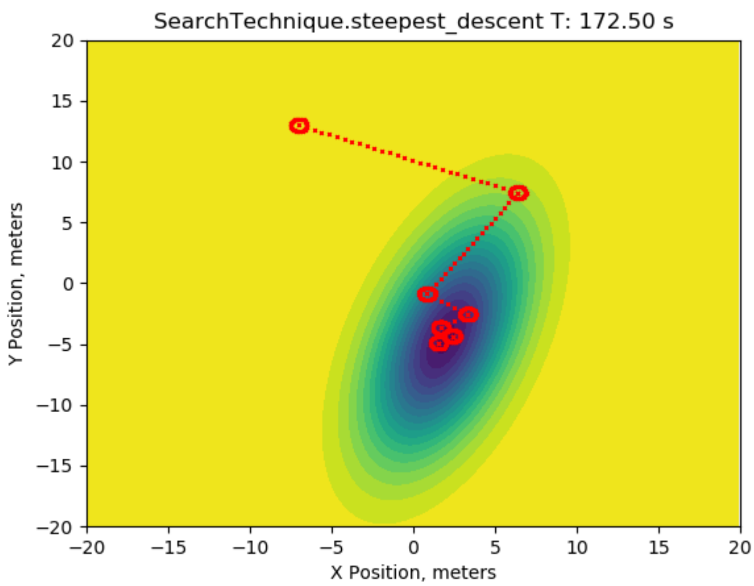
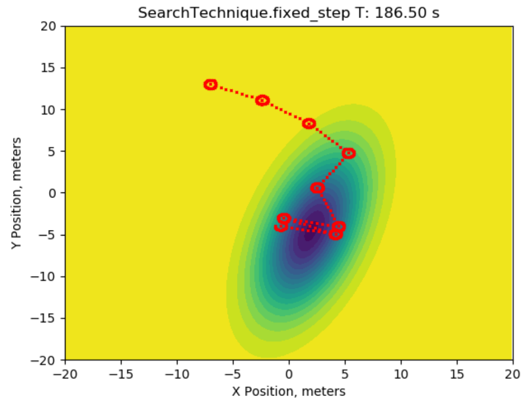
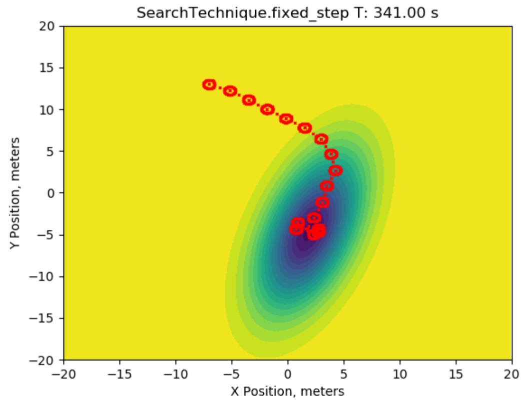


Figure 3: Steepest descent reaching, but not converging on, minimum within 120 seconds

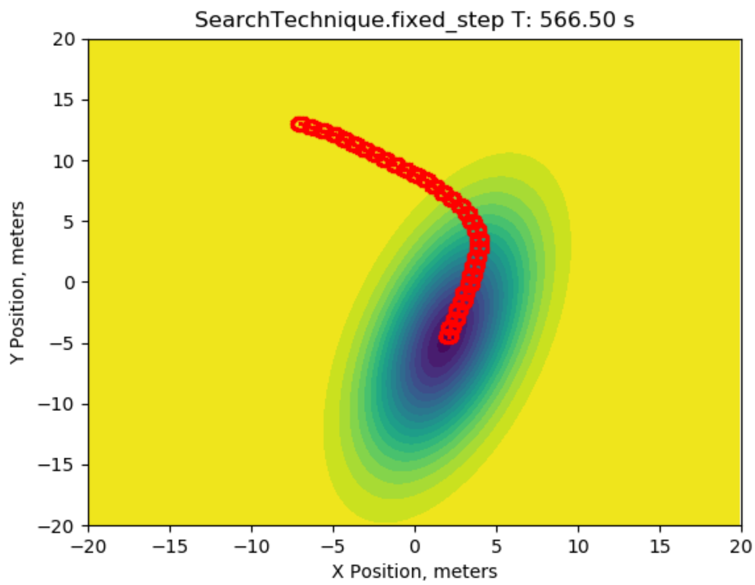
Figure 3 shows a steepest descent on a unimodal distribution. The algorithm approaches the minimum but begins oscillating; this is likely because of the discontinuous sampling used to compute the gradient or the distance at which samples are taken. Figure 4 shows multiple attempts of fixed-step descents, with varying results. Generally, the shorter the step size the more likely the algorithm will converge. On the other hand, shorter step sizes involve many time-consuming gradient measurements.



(a) 5 Meter step size - large oscillations, approaches within 120 seconds



(b) 2 Meter step size - smaller oscillations, approaches within 220 seconds



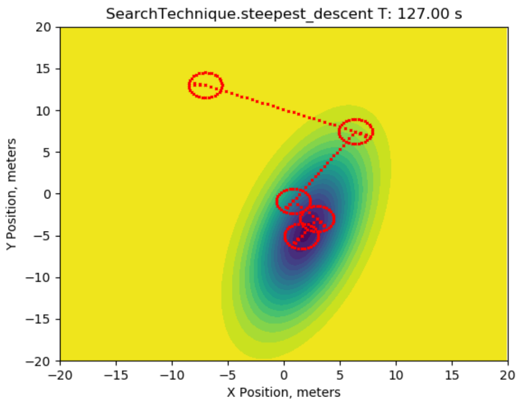
(c) 0.75 meter step size - converges on minimum, takes 566 seconds

Figure 4: Fixed-step-size performance with varying step sizes

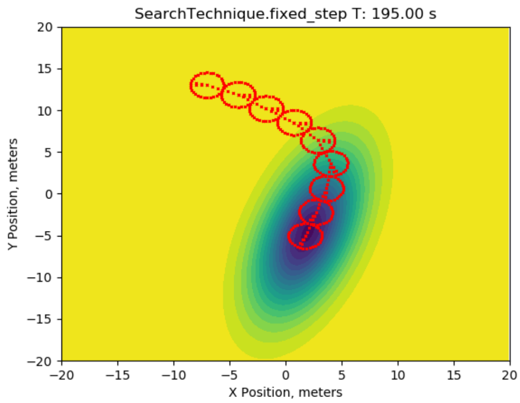
A comparison between steepest descent and fixed-step is shown in Figure 5. In this example, the sampling radius (radius of polygon in which gradient is measured) is increased to 1.5 meters. Both algorithms converge, although the steepest descent does so much faster.

An attempt on a bimodal algae distribution is shown in Figure 6. Both methods approach a local minimum, but not the global minimum. This is to be expected given the nature of gradient descent algorithms, and can likely be handled by marking local minimums before continuing the search.

Overall, the algorithm is able to successfully reach local minimums using the limited information. Although many times the vehicle was unable to converge on the minimum, it still oscillated nearby. This problem would be addressable on the real platform with additional logic. Furthermore, the vehicle need not converge exactly, as a relatively precise estimate may be sufficient. In light of this, steepest descent performs the best - it minimizes time spent computing gradients unnecessarily, therefore reaching the objective fastest. Of course, many assumptions were made in the algae model that introduce inaccuracies

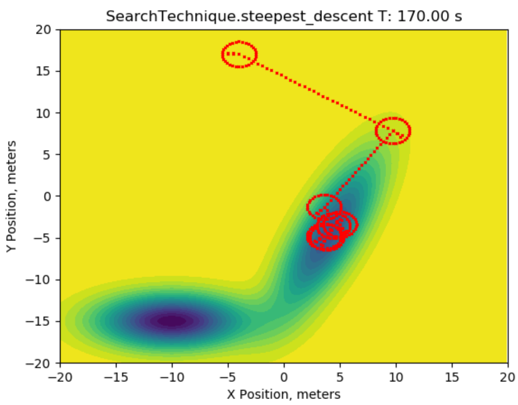


(a) Steepest descent converging in 127 seconds

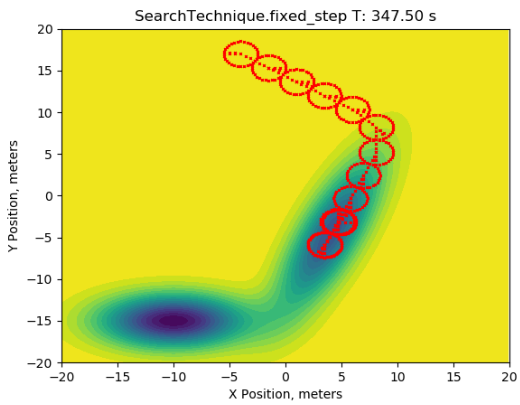


(b) Fixed step converging in 195 seconds

Figure 5: Steepest vs fixed-step descent with sampling radius of 1.5 meters, fixed-step of 3 meters



(a) Steepest descent on bimodal distribution converging in 170 seconds



(b) Fixed step on bimodal distribution failing to converge after 347 seconds

Figure 6: Steepest descent and fixed-step approach on bimodal distributions

into the results in terms of the real-world problem. Improvements may be possible by modelling algal diffusion, movement over time, and measurement noise. The algorithm successfully finds a local minimum with bimodal distributions, but may not find the global minimum.

Traditional gradient descent was selected due to its relative simplicity to implement. An alternative approach, using the concept of stochastic gradient descent, might perform differently according to the specified algae model. Regardless, the algorithm developed proved successful and a strong start towards the objective of locating concentrated algae in a bloom.

References

[1] K. G. Sellner, G. J. Doucette, and G. J. Kirkpatrick, "Harmful algal blooms: causes, impacts and detection," vol. 30, no. 7, pp. 383–406.

[2] S. Jung, H. Cho, D. Kim, K. Kim, J.-I. Han, and H. Myung, "Development of algal bloom removal system using unmanned aerial vehicle and surface vehicle," vol. 5, pp. 22166–22176. Conference Name: IEEE Access.

[3] D. M. Anderson, "Approaches to monitoring, control and management of harmful algal blooms (HABs)," vol. 52, no. 7, pp. 342–347.