

# Optimisation for fuel efficiency in hybrid vehicles

## Introduction

The problem handled in this report is one concerning the optimisation of fuel efficiency in hybrid vehicles. In certain types of hybrid vehicles, it is possible to combine the power provided by the internal combustion engine and the electric motor to provide a higher overall power and torque output, without increasing the fuel consumption. This can be especially interesting for public transport applications, like busses. This is because they drive fixed, limited distance trajectories inside city centres. In these circumstances, only using internal combustion engines generally results in very low efficiency rates, because of the frequent acceleration and deceleration in slow traffic and at bus stops. Using the electric engine to provide the torque during those acceleration phases can significantly reduce the fuel consumption, and during the deceleration phases the energy normally dissipated in the brakes can be recaptured using regenerative braking.

In this report, simplified models will be constructed of a hybrid electric vehicle, which will be used to optimise the use of the electric energy with regards to fuel consumption, in a specific city drive cycle.

## Objective

The aim of this project is to investigate how to optimally combine the workings of the electric drive and internal combustion engine in a parallel hybrid vehicle, to minimise the fuel consumption in a specific drive cycle. To do this, a bus driving cycle is loaded in to the simulation. This drive cycle consists of a speed profile, from which the acceleration profile is derived via a first order method, and finally the required power can be obtained by calculating the associated forces, as will be explained later in the report. The assumption is made that the vehicle can always exactly provide the required power (no dynamics and uncertainties are regarded), as implementing dynamics would go beyond the scope of the project, and make little difference in the obtained results, as the variations in the produced power would likely be small with regards to the total power and energy requirements.

At every time instant, a fraction  $\lambda$  of the required power will be produced by the internal combustion engine, and a fraction  $(1 - \lambda)$  will be produced by the electric drive. Via the simulation, the value of  $\lambda$  will be optimised at every time instant, in order to minimise the final fuel consumption.

## Vehicle model

The vehicle modelled will be a parallel hybrid electric vehicle. As mentioned before, in this vehicle, an electric drive and internal combustion engine can work together, both providing a portion of the power required by the vehicle at any time instant. The power of both drives is combined via a mechanical coupler.

The share of the portion they provide can be varied throughout time. The vehicle model therefore consists of two main models: the internal combustion engine and the electrical drive. Finally, the summing of the powers in the mechanical coupler will just be modeled as a simple summation of both powers, multiplied by a transmission efficiency factor. As well as the models of the hybrid vehicle, a calculation of the required power at any time instant, depending on the imposed speed profile, is also required.

### Power calculation

The required power is calculated via the formula:  $P = F \cdot v$ , where  $F$  is the required force at any time instant. This force consists of three components: acceleration force, aerodynamic drag force and friction force between the tires and the road. Notice that in this case the assumption of a flat road is made, so there is no gravitational component in the formula, also, it is assumed there is no wind. The formula for the total required force becomes:

$$F_{req} = m \cdot \mu \cdot g + 1/2 \cdot \rho_{air} \cdot A_{eq} \cdot C_{drag} \cdot v^2 + m \cdot C_{mass\ factor} \cdot a.$$

The mass factor coefficient in the acceleration part of the formula accounts for the rotational inertia of the motor and transmission parts, which can be modelled as an equivalent increase in the mass, depending on the current gear ratio. For simplicity, the assumption is made that gear shifts always occur at the same velocity, so one gear ratio is assigned to every speed.

Finally, the total required power by the engine and electric drive equals the power calculated via the previous formula, multiplied by a transmission efficiency factor to account for the mechanical losses:

$$P_{drivetrain} = F_{req} \cdot v \cdot \eta_{trans}.$$

As mentioned earlier, the power to be delivered by the internal combustion engine will then be  $P_{ICE} = \lambda * P_{drivetrain}$ , and the power required by the electrical motor will be  $P_{el} = (1 - \lambda) * P_{drivetrain}$ .

### Internal combustion engine model

From the internal combustion engine model, the main parameter that is desirable to track is the fuel consumption, which will in the end be part of the cost function. One way to calculate it, is to simply start from the maximum energy content in the fuel used, which can be either gasoline or diesel. To derive the instantaneous fuel consumption from this, it is sufficient to know the total efficiency of the engine and transmission, and the total required power to drive the vehicle. The fuel consumption,  $\dot{m}_f$  [kg/s], then becomes:

$$\dot{m}_f = P_{req} / (\eta_{engine} \cdot H_f)$$

Here  $H_f$  is the energy content of the fuel [J/kg].

Figure 1 displays the engine efficiency for a gasoline and diesel type engine used in the vehicle models, with respect the ratio of the produced power and the maximum attainable power, and the ratio of the current rpm and the rpm at which maximum attainable power is reached. As can be seen on the images, the efficiency stays below 40% at all times for diesel engines, and below 30% for gasoline engines. The engines reach their maximum efficiencies as the power and rpm get close to the values of the maximum attainable power and corresponding rpm.

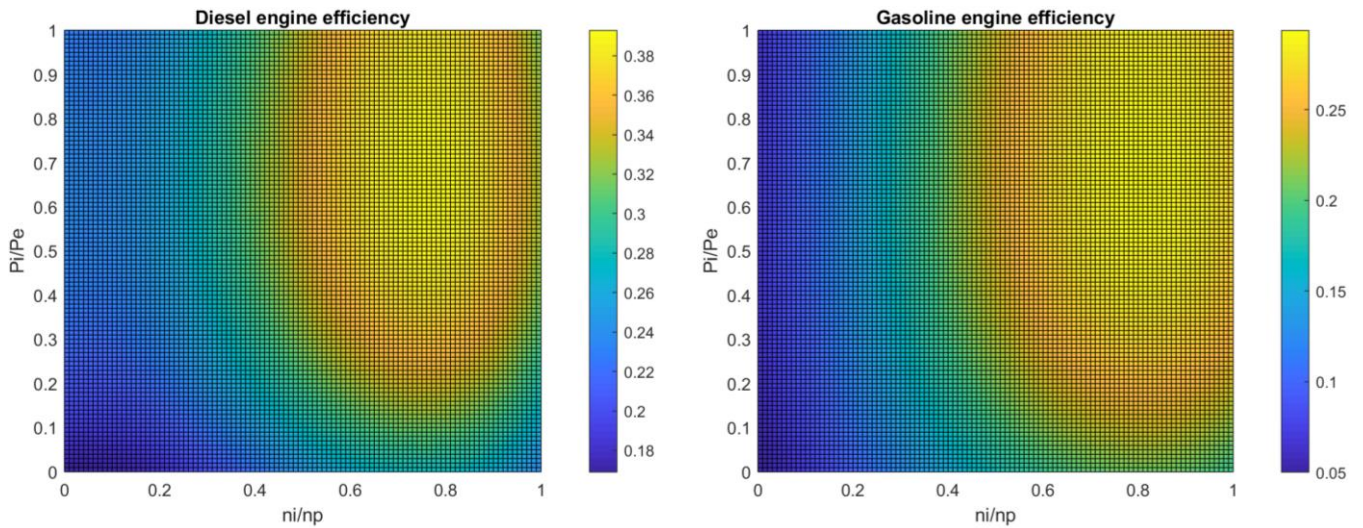


Figure 1: Diesel and gasoline engine efficiency charts

Both to calculate this maximum attainable power and to calculate the engine efficiency from this, analytical curve fit models are used, which give an accuracy between 2% and 7%, according to the type of vehicle chosen and the drive cycle, which is sufficient for this project, as no very high accuracy is required.

It should be noted that this model greatly simplifies the actual workings of the motor, as it disregards influences such as road and fuel conditions, temperature, lag, ... This results in a statical model, where the fuel consumption only depends on the current power need and speed, and does not depend on the previous state of the engine.

### Battery model

For simplicity, the model of the battery that will be used in the simulation consists of a simple voltage source, represented the built in voltage by the battery, and the resistor, representing the battery's internal resistance. This is a statical model, meaning dynamic effects, commonly modelled by a resistor - capacitor pair, are ignored. This assumption is valid, as the vehicle is large, with a big inertia, and we assume the required power can always be delivered.

The voltage from this source is also called the open circuit voltage. Both the resistance and the voltage generated are dependent on the amount of charge still remaining in the battery, or state of charge (SOC)

of the battery. Concerning the voltage, it can be seen that it experiences a 20% drop with regards to the initial voltage as the SOC decreases. At very low SOC, when the battery is almost out of charge, the voltage very rapidly decreases to zero. For the internal resistance, an opposite effect can be noticed. It remains fairly constant throughout most of the charge depletion, but experiences a rapid increase in value when the battery reaches a low SOC. If the constraint is added that the SOC has to remain above 10%, which is beneficial for the batteries health and vehicle drivability, the internal resistances can be modelled as a constant.

Furthermore, these values also vary according to temperature, but in this project, for simplicity, the assumption is made that temperature of the battery remains more or less constant, at room temperature conditions. Next, also the effects of the battery age is neglected. It is well known that the effective capacity of a battery decreases as it runs through more charge-discharge cycles. In this simulation, the assumption of a brand new battery is made, meaning that the full capacity is still available.

For the estimation of  $V_{oc}$  at any time, a curve-fitting model is used that was fitted to match the characteristics of Lithium-manganese + Lithium-NMC blend batteries, which are commonly used in electric vehicles, such as the popular Nissan Leaf.

Next, the depletion of the battery charge as the electric motor is running must be modelled. Because the equation of the state of charge is given by

$$SOC = Q_{remaining}/Q_{capacity},$$

its decrease in time can be written as

$$\dot{SOC} = I_{battery}/Q_{capacity},$$

with  $I_{battery}$  the current being drawn from the battery. This current can be calculated by means of the conservation of energy principle, that says that the power drawn from the battery equals the power consumed by the electric motor plus some losses, easily modelled by an efficiency coefficient. The power provided by the battery to the system, accounting for the losses in the internal resistance, is

$$P_{bat} = V_{oc} \cdot I_{bat} - I_{bat}^2 \cdot R_{int}$$

Finally, from this equation, the current can be calculated as

$$I_{bat} = (V_{oc} - \sqrt{(V_{oc}^2 - 4 R_{int} P_{bat})}) / (2 R_{int})$$

Since the open circuit voltage is known from its relation to the SOC, the battery current can be easily calculated, allowing to constantly update the SOC in the simulation.

Similarly, a model can be made for the regenerative braking of the battery. When the vehicle is decelerating at a high enough rate, the braking force can become high enough to overcome the friction and aerodynamic drag force. In normal circumstances, this power would just be dissipated, but with battery systems, the energy can be partially recovered. This is possible by using the electric drive which normally powers the wheels as generator, powered by the rotation of the wheels, and producing a current back to the battery.

For this, the same basic battery model can be used. The main difference is here that the battery current flows in the opposite direction. Because their difference is negligible with respect to the other battery parameters, the assumption is made that the internal resistance is equal for charging and discharging. The power equation then becomes:

$$P_{rec} = V_{oc} \cdot I_{rec} + I_{rec}^2 \cdot R_{int},$$

where  $P_{rec}$  is the (positive) power coming back from the generator, and  $I_{rec}$ , the battery current, taken positive. This  $P_{rec}$  equals the total vehicle power, calculated from the force equations, multiplied by the transmission efficiency and the generator conversion efficiency. The final expression for the recharge current becomes:

$$I_{rec} = (-V_{oc} + \sqrt{V_{oc}^2 + 4 R_{int} P_{rec}}) / (2R_{int})$$

Analogous to the discharging case, the SOC can then be calculated using:  $\dot{SOC} = I_{rec} / Q_{capacity}$ .

## Constrained optimization

A constrained optimisation strategy was implemented. This is done by writing a solver that handles nonlinear constrained problems using the **SQP method**. The solver can compute the Hessians of the quadratic programs both via the BFGS or the Gauss-Newton method, depending on specification provided by the user. For the use of the GN method, however, the cost function has to be formulated in such a way that it can be written as  $f(x) = F(x)^T F(x)$ .

The aim of the optimisation is to minimise the fuel consumed by the internal combustion engine. Initially, the cost function could therefore be:  $\min_{\lambda} CO_{2tot}$ , or the total fuel consumption. There, however, some other factors that should be taken into account in the optimization. Because it is likely beneficial for the flow of power through the drivetrain as smoothly as possible, a second term can be added that penalizes large differences in lambda between two time steps. Finally, a term was also added to penalize big discharge currents from the battery, as large discharges may be harmful to the battery lifetime and range. This last term might, however, counteract the minimization of the fuel consumption. The final cost function then becomes:

$$\min_{\lambda} \alpha_1 * CO_{2tot} + \alpha_2 * \sum I_{discharge} + \alpha_3 * \sum \lambda_i - \lambda_{i-1}.$$

The constraints applied to the optimization follow from the physical reality of the problem itself. In real life, the maximum attainable power output of the motor and electrical drive are always limited. In the case of the motor, this limit is calculated using an analytical approximation, as explained earlier. For the electrical part, the maximum discharge power was assumed to be a constant value, although in reality this value depends on the duration of the discharge. Constraints were added so that these limits are not exceeded. Furthermore, a secondary constraint was added for the battery, setting a lower bound for its SOC. Depleting a battery too much is very harmful for its lifetime, and the battery might start working improperly when it's depleted too much. A limit on the minimum SOC for the battery was therefore set to 20%. Finally, three more constraints were added for limiting the values of lambda. For one, the value of lambda must remain between 0 and 1, and secondly, a constraint was also added making sure that lambda is equal to zero when the battery is recharging during braking. The final problem constrained optimization problem then becomes:

$$\min_{\lambda} \alpha_1 * CO_{2tot}^2 + \alpha_2 * \left( \sum I_{discharge} \right)^2 + \alpha_3 * \left( \sum \lambda_i - \lambda_{i-1} \right)^2$$

$$s. t. P_{eng max} - P_{engine} \geq 0$$

$$SOC - 0.2 \geq 0$$

$$P_{el max} - P_{el drive} \geq 0$$

$$P_{drivetrain} * \lambda \geq 0$$

$$\lambda \geq 0$$

$$1 - \lambda \geq 0$$

Notice that all terms in the cost function are squared. This is to punish larger values for the different factors in the cost function. This notation also allows to easily convert the cost function into a form that can be used for the Gauss-Newton optimization.

One problem encountered when testing the SQP constrained optimiser is that, especially for low initial values of the batter SOC, it proved difficult to intuitively find an initial value within the feasible set for lambda to initialise the optimisation with. To solve this issue, the optimisation is preceded by another optimisation algorithm that attempts to find a feasible initial value for lambda. This optimization goes as follows:

$$\begin{aligned} & \min_{\lambda, \mathbf{s}} \mathbf{s} \\ & \mathbf{s} + \begin{cases} P_{eng\ max} - P_{engine} \geq 0 \\ SOC - 0.2 \geq 0 \\ P_{el\ max} - P_{el\ drive} \geq 0 \\ P_{drivetrain} * \lambda \geq 0 \\ \lambda \geq 0 \\ 1 - \lambda \geq 0 \end{cases} \\ & \mathbf{s} \geq 0 \end{aligned}$$

Where the vector  $\mathbf{s}$  is an initialized with sufficiently high, positive values. The idea here is that, if a feasible point exists,  $\mathbf{s}$  will go to zero. The optimization algorithm displays a message indicating that a feasible starting point is found when the values of  $\mathbf{s}$  are below a certain threshold once the optimization has finished.

## Optimization results

The speed trajectory taken for this optimization is a segment out of a bus trajectory through an urban neighborhood, with several accelerations and decelerations. However, due to virtually increase the energy consumption, the time step between data points has been increased. This way, a clear depletion of the battery is visible, and the optimization algorithm can be properly tested. The speed trajectory is given in image 2.

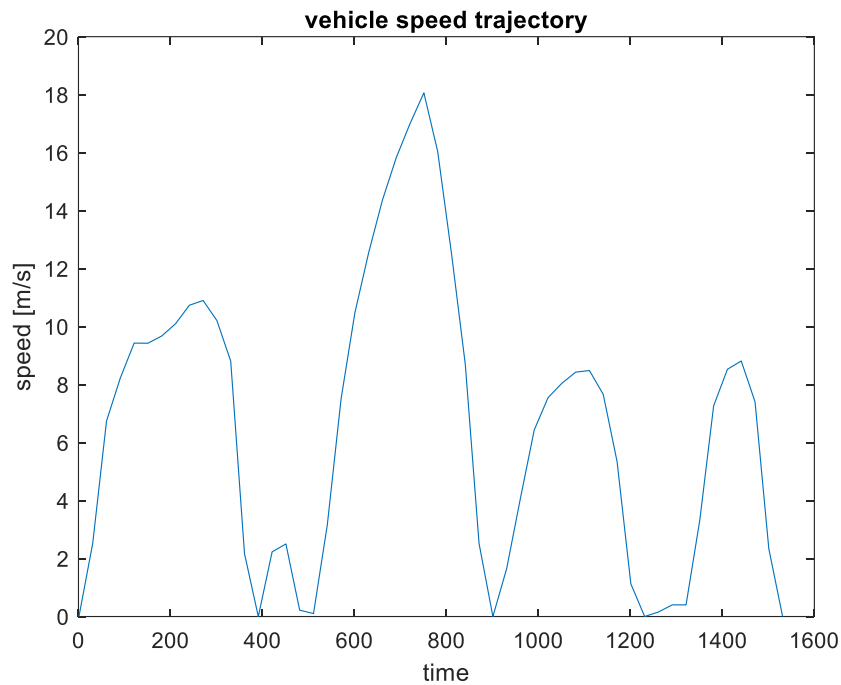


Figure 2: Speed trajectory used in the optimisation

As illustration, the total fuel consumption and SOC plots are given for a simulation where lambda is kept constant to a value of 0.5 in figure 3.

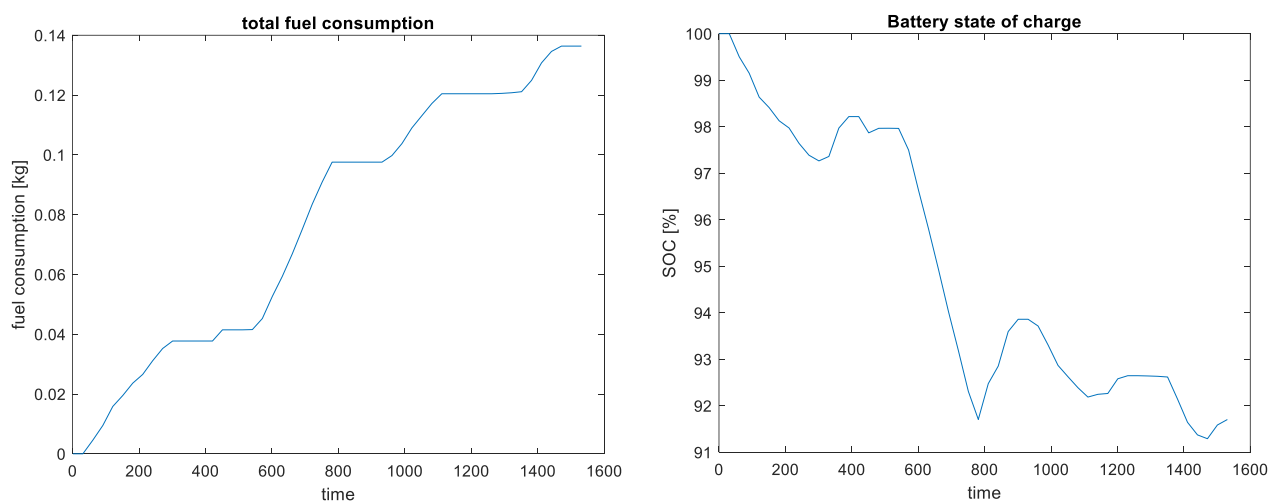


Figure 3: Fuel consumption and SOC with lambda constant at 0.5



The total fuel consumption comes to around 0.14kg, but as can be seen on the graph of the SOC, the battery is clearly not being used to its full potential. In image 4, the same parameters are plotted, together with the computed lambda, after optimizing only for optimal fuel consumption using the BFGS algorithm, where the initial SOC of the battery was only 40%. As can be seen on the graphs, there is a clear improvement, with the total consumption dropping from 0.14kg to about 0.035kg. From the graph of the SOC, it is also

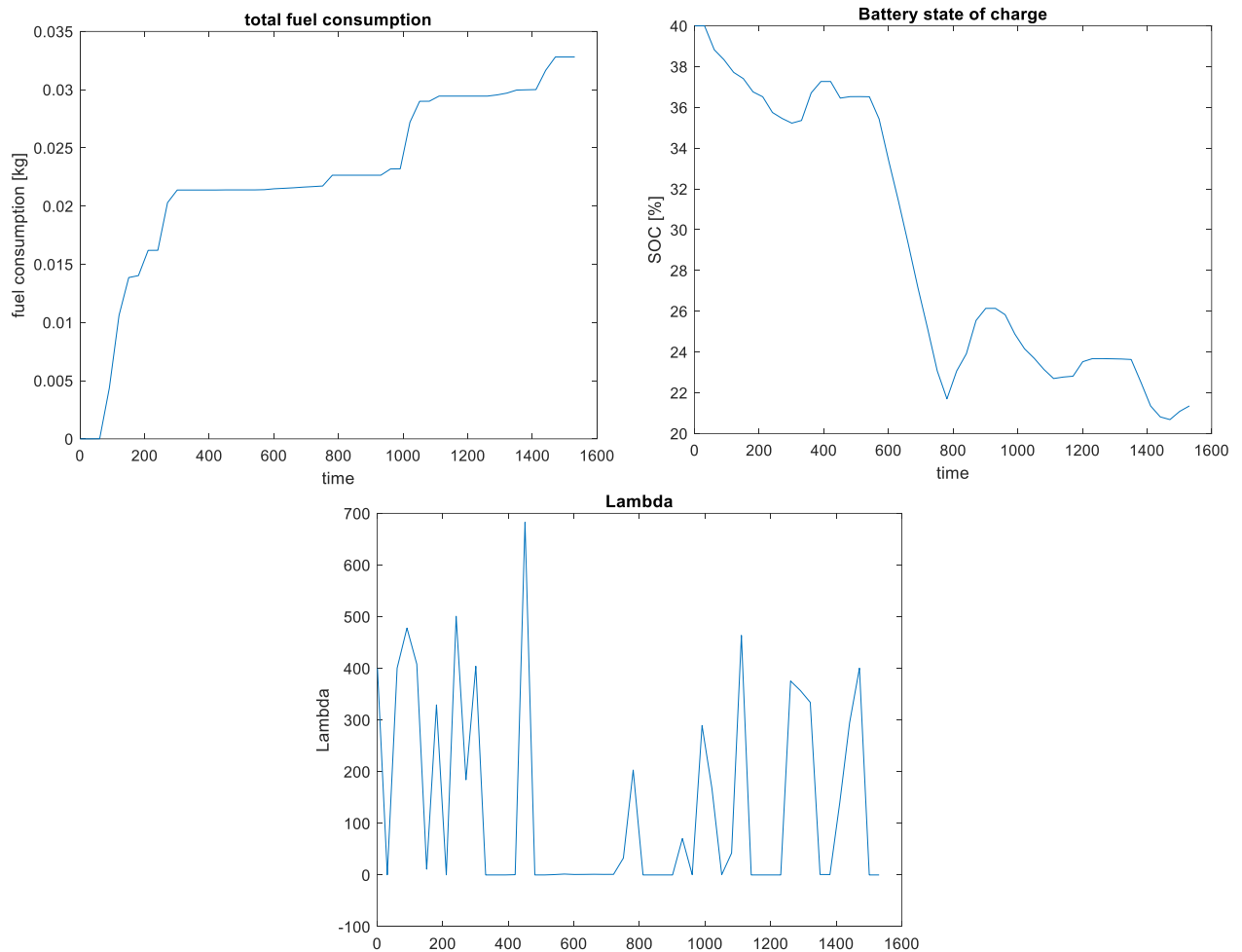


Figure 4: Optimisation for fuel efficiency with initial SOC = 40%

clear that the battery has been more efficiently used, with the SOC dropping to its bottom limit first, then recuperating a little, and then dropping again. Finally, however, since the other two factors of the optimization were set to zero, no care was taken for sudden changes in lambda, or peaks in discharge current. Note that the lambda has been scaled from values

from 0 to 1000 instead of 0 to 1.

When extra terms are added to the cost function to account for the other two parameters, the results look like the graphs displayed in figure 5. As can be seen on the graphs, a tradeoff has been made. The fuel consumption here is slightly higher than in the case where only an optimization for fuel efficiency was done. This is because of the influence of the other two factors. Lambda is greatly smoothed out, resulting in a smaller difference in lambda between time steps, and its peaks are far smaller, resulting in smaller contributions from the discharge current to the total cost. Notice that the general shape of the SOC curve

is roughly the same in all cases. This is because the intervals during which acceleration and deceleration happens stay the same in all cases. Only the slope varies slightly in the different cases.

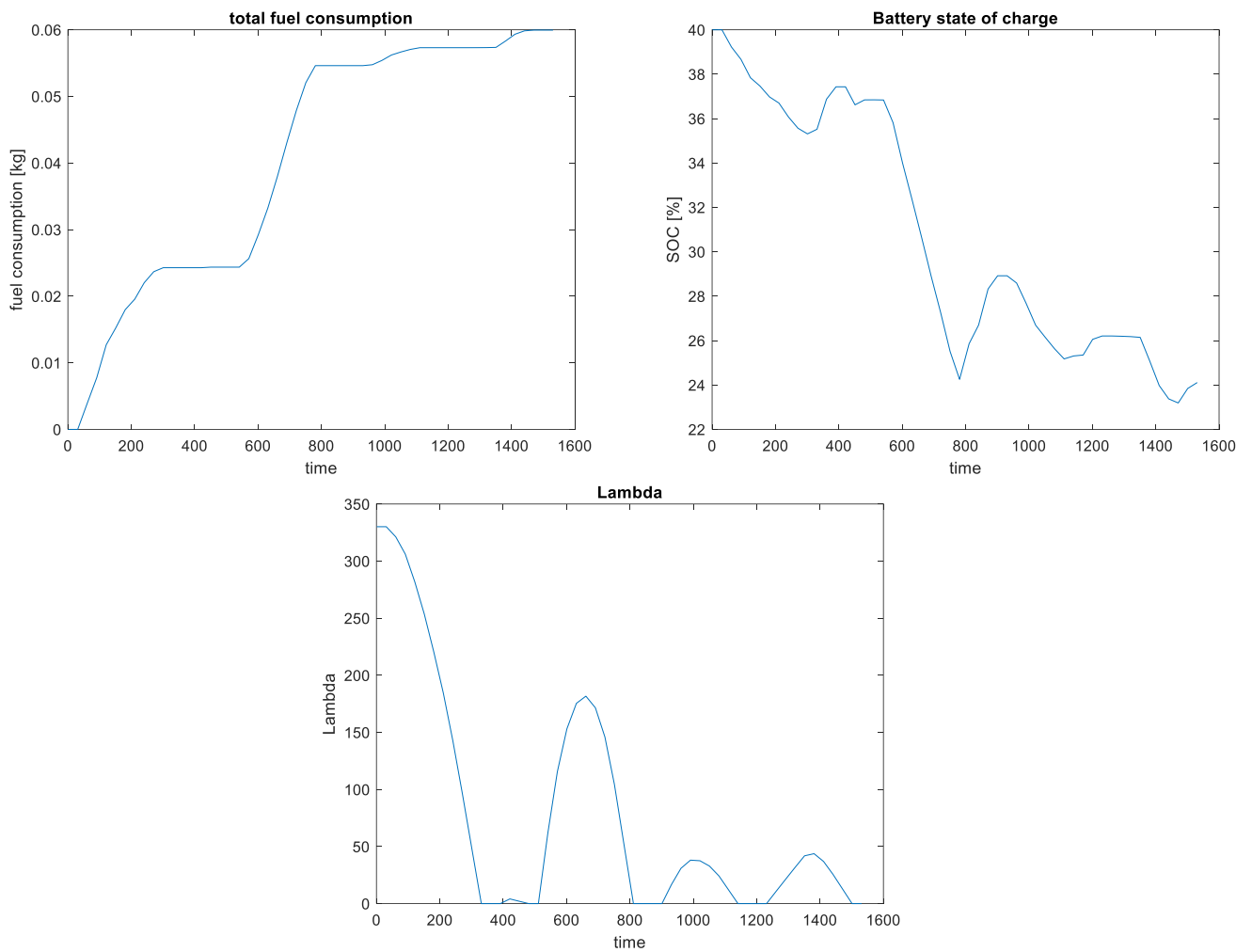


Figure 5: Full optimisation with initial SOC = 40%

The results produced in these optimizations seem rather feasible, as it looks like the battery is always used more or less to its full potential. The general shape of the 'lambda' curves, however, will always be more or less the same for this velocity profile, with lambda climbing up to some optimal value before decreasing again, because the values of lambda always have to go back to zero when the vehicle goes into recharge mode. If more time were available, solutions could be explored where this last property is slightly altered, as to not necessarily produce parabola shapes.

Also when the algorithm is initialized with an SOC value close to the lower limit, the algorithms perform fairly well, with lambda values being high initially, so the battery can recharge, before going to lower values. Once again, it can be seen on figures X that the batteries are used to a large extend of their allowed capacity. Of course, the fuel consumption is significantly higher in this case, but it is still within reason compared to the fuel consumption when lambda was set equal to 0.5 for the entirety of the trajectory.

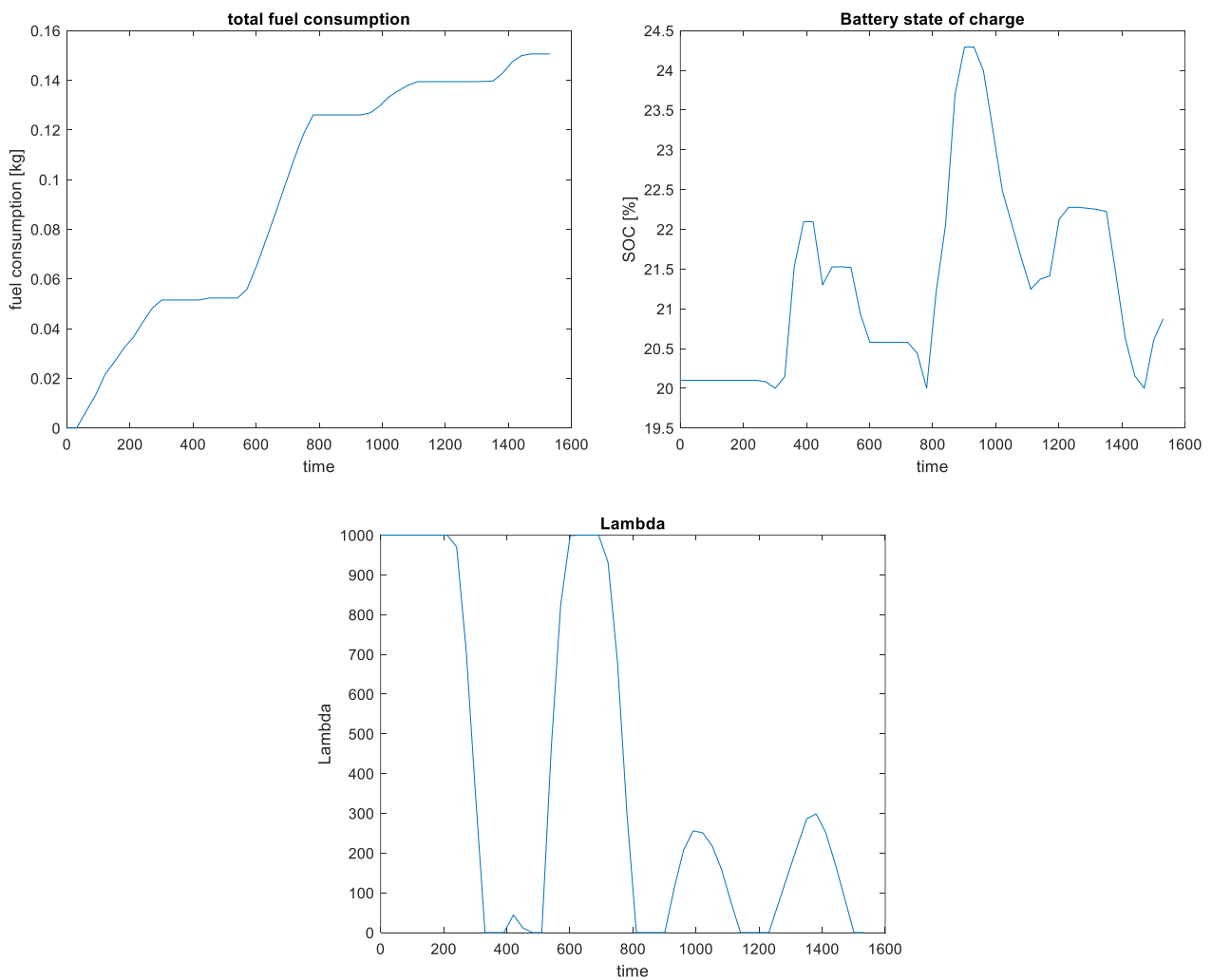


Figure 6: Full optimisation with initial SOC = 20.1%

When initializing the ‘starting point optimization’ with different initial values for  $\lambda$  produces more or less the same results. In this case, this was tested by initializing the velocity profile used above with  $\lambda$  values ranging from 0 to 1, and roughly the same result was produced every time. This is likely because the algorithm goes through two optimizations – the starting point and the actual optimization – resulting in it gradually converging to the same solution every time. The initial feasible  $\lambda$ s for initial values of 1, 0.5 and 0 are given in figure 7. Even though they differ quite substantially from each other, they still converge to the same value after the second optimization, indicating that the found solution is close to the absolute maximum.

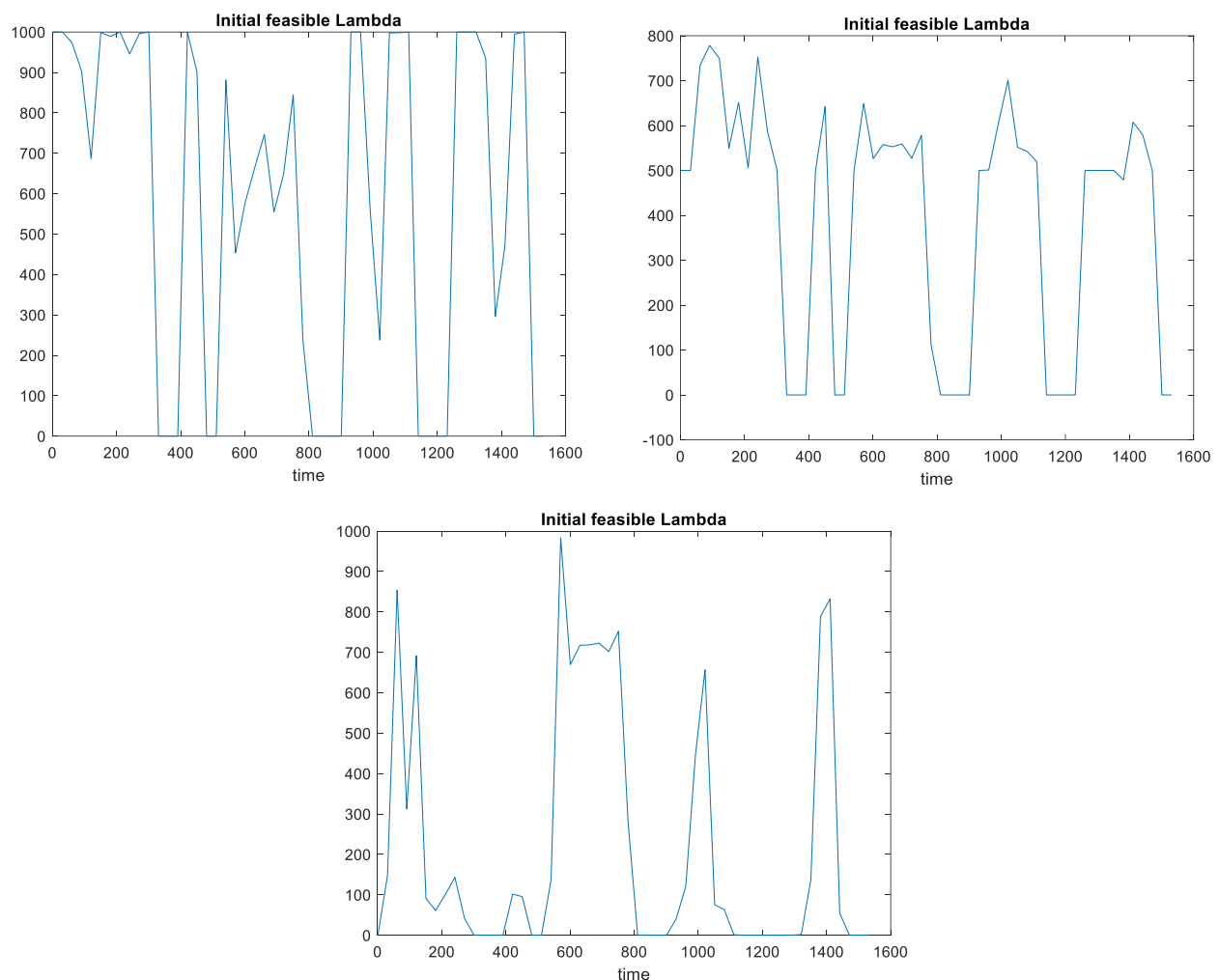


Figure 7: Initial feasible lambdas for initial lambda = 1 (top left), 0,5 (top right), 0 (bottom)

## Outlook

Overall, it can be concluded that the algorithms produce rather satisfactory results. The optimized values for  $\lambda$  appear to be realistic, as they utilize the battery to a large extent of its potential, and produce significantly lower fuel consumption values than when just random values for  $\lambda$  are used. By varying the weights of the different factors in the cost function, it is also shown that different aspects, such as smooth  $\lambda$  transitioning, or lower current peaks, can be prioritized.

When more time would be available, however, several things could still be explored in order to improve the algorithm and the realism of the used models. First of all, more constraints could have been added to make the vehicle model realistic, such as a demand for minimum engine efficiency, or a minimum speed at which the engine would start to work (this would be an approximation of an electrical start-stop motor). Additions could also have been made on the battery side, such as improving the model to be a second order model of a battery, so transitional phases can also be included. As mentioned earlier, it could also have been interesting to alter the constraint pushing  $\lambda$  to zero when the vehicle goes into recharge mode, as this limits the ways in which  $\lambda$  can be optimized.

Next, different ways for optimizing could have been experimented with. One other possible strategy was to solve the problem as an unconstrained problem, and adding the constraints to the cost function, making them into 'soft constraints'. These would then still be allowed, but punished by an increased cost. This would likely increase the speed of the optimization algorithm, at a cost of results that might be less realistic and violate one or more constraints.

Finally, it should be noted that the optimization used here is not without flaws. The program still fails in some instances. Oftentimes, this is due to the step size in the line search part being too large, causing the algorithm to go back into the unfeasible region, and can be fixed by decreasing the maximum size. At other times, the algorithm gets stuck in an eternal loop due to the step size being too small. This mostly occurs when the current solution is just outside the feasible region. Therefore, when the program returns the exit statement that the maximum number of iterations are reached, the found solution is often still rather feasible, but with a small violation of one or more constraints. The program exit can also be forced to occur earlier by decreasing the exit tolerance for the constraints. This goes, of course, at a cost of minor constraint violations.

## Sources

### Engine model

<https://pdfs.semanticscholar.org/4614/5b07ccc742c29305f3dfcb6094ee673ad7ea.pdf>

<http://dergipark.gov.tr/download/article-file/351374>

### Battery and hybrid model

A Generalized SOC-OCV Model for Lithium-Ion Batteries and the SOC Estimation for LNMCO Battery -  
CAIPING ZHANG

Modeling and Control of a Power-Split Hybrid Vehicle - JINMING LIU AND HUEI PENG

Online Lithium-Ion Battery Internal Resistance Measurement Application in State-of-Charge Estimation  
Using the Extended Kalman Filter - DIAN WANG, YUN BAO AND JIANJUN SHI

<https://pdfs.semanticscholar.org/5c4d/caf8741c28f055c9b182f624e77141c62d13.pdf>

<https://power.eecs.utk.edu/pubs/BaskarThesisFinal2.pdf>

### optimization algorithm

Constrained Numerical Optimization for Estimation and Control, Lecture Notes - LORENZO FAGIANO - 2019