Description Logic 1: Syntax and Semantics

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Autumn 2015

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Overview

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- Each description logic describes a language, and each language differ in expressibility vs. reasoning complexity, defined by allowing or disallowing different constructs (e.g. conjunction, disjunction, negation, quantifiers, etc.) in their language.

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- Cross-fertilisation of applications and theory
- Today: large impact on Semantic Web (sign up for INF3580/4580!)

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- $\mathcal{T}$ is a set of terminology definitions (i.e. complex descriptions of concepts or roles), called the *TBox* (e.g. $Human \sqsubseteq Mammal$, $Mother \equiv Parent \sqcap Woman$)

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where A is an atomic concept, C and D are concepts, and R is a role. We allow

- ABox assertions: $C(a)$ and $R(a, b)$ for individuals a, b, concepts C and roles R;
- TBox axioms: $C \sqsubseteq D$ for concepts C and D.

$\mathcal{ALC}$: Semantics

A model $\mathcal{M}$ for a knowledge base $\mathcal{K}$ consists of

- a nonempty set Δ, and
- an interpretation function $\_^{\mathcal{M}}$, such that:
  - for every constant c, $c^{\mathcal{M}} \in \Delta$,
  - for every atomic concept A, $A^{\mathcal{M}} \subseteq \Delta$,
  - for every atomic role R, $R^{\mathcal{M}} \subseteq \Delta \times \Delta$,

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\top^{\mathcal{M}} &= \Delta \\
\bot^{\mathcal{M}} &= \emptyset \\
(\neg C)^{\mathcal{M}} &= \Delta \setminus C^{\mathcal{M}} \\
(C \sqcup D)^{\mathcal{M}} &= C^{\mathcal{M}} \cup D^{\mathcal{M}} \\
(C \sqcap D)^{\mathcal{M}} &= C^{\mathcal{M}} \cap D^{\mathcal{M}} \\
(\forall R.C)^{\mathcal{M}} &= \left\{ a \in \Delta \mid \forall b \in \Delta \left(\langle a, b \rangle \in R^{\mathcal{M}} \to b \in C^{\mathcal{M}} \right) \right\} \\
(\exists R.C)^{\mathcal{M}} &= \left\{ a \in \Delta \mid \exists b \in \Delta \left(\langle a, b \rangle \in R^{\mathcal{M}} \wedge b \in C^{\mathcal{M}} \right) \right\}
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As usual, we will write $\mathcal{K} \vDash \psi$ if for any model $\mathcal{M}$ we have that $\mathcal{M} \vDash \mathcal{K} \Rightarrow \mathcal{M} \vDash \psi$.

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We will use the following shorthand notation:

- $C \equiv D$ instead of the two axioms $C \sqsubseteq D$ and $D \sqsubseteq C$;
- $\mathcal{A} \models \psi$ instead of $\langle \emptyset, \mathcal{A} \rangle \models \psi$;
- $\mathcal{T} \models \psi$ instead of $\langle \mathcal{T}, \emptyset \rangle \models \psi$.

Example

TBox:

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\begin{aligned}
Animal &\sqsubseteq LivingThing \\
Donkey &\equiv Animal \sqcap \forall hasParent.Donkey \\
Horse &\equiv Animal \sqcap \forall hasParent.Horse \\
Mule &\equiv Animal \sqcap \exists hasParent.Horse \sqcap \exists hasParent.Donkey \\
\exists hasParent.Mule &\sqsubseteq \bot
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ABox:

$$Horse(Mary) \quad Mule(Peter) \quad Donkey(Sven)$$

$$hasParent(Peter, Mary) \qquad hasParent(Peter, Carl)$$

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The function π map concepts to first-order formulae:

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Theorem

$a^{\mathcal{I}} \in C^{\mathcal{I}}$ iff $\mathcal{I} \models_{FOL} \pi_x(C)[a/x]$, and $\mathcal{I} \vDash C \sqsubseteq D$ iff $\mathcal{I} \models_{FOL} \Pi(C \sqsubseteq D)$.

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E.g.:

$$\begin{aligned}
\pi_x\,(Animal \sqcap \forall hasParent.Donkey) &= Animal(x) \wedge \forall y(hasParent(x,y) \rightarrow Donkey(y))\\
\Pi(Animal \sqsubseteq LivingThing) &= \forall x(Animal(x) \rightarrow LivingThing(x))
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  - $\mathcal{H}$: Role hierarchies;
  - $\mathcal{R}$: Complex role hierarchies;
  - $\mathcal{N}$: Cardinality restrictions;
  - $\mathcal{Q}$: Qualified cardinality restrictions;
  - $\mathcal{O}$: Closed classes;
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- We name the languages by adding the letters of the features to $\mathcal{ALC}$. So e.g. $\mathcal{ALCN}$ is $\mathcal{ALC}$ extended with cardinality restrictions and $\mathcal{ALCHI}$ is $\mathcal{ALC}$ extended with role hierarchies and inverse roles.

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- It is common to shorten $\mathcal{ALC}$ (extended with transitive roles) to just $\mathcal{S}$ for more advanced languages, so e.g. $\mathcal{SHOIN}$ is $\mathcal{ALC} + \mathcal{H} + \mathcal{O} + \mathcal{I} + \mathcal{N}$.

Normal extensions

– $\mathcal{H}$ – Role Hierarchies: We allow TBox axioms on the form $R \sqsubseteq P$ for atomic roles. Semantics:

$$\mathcal{M} \models R \sqsubseteq P \Leftrightarrow R^{\mathcal{M}} \subseteq P^{\mathcal{M}}$$

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  e.g. $hasFather \sqsubseteq hasParent$;

- $\mathcal{R}$ – Complex role hierarchies: We allow roles on the form $R \circ P$ and TBox axioms on the form $R \circ P \sqsubseteq P$ and $R \circ P \sqsubseteq R$ for any two roles. Semantics:

$$(R \circ P)^{\mathcal{M}} := \left\{\langle a, b\rangle \in \Delta^{\mathcal{M}} \times \Delta^{\mathcal{M}} \mid \exists c \in \Delta^{\mathcal{M}} \left(\langle a, c\rangle \in R^{\mathcal{M}} \wedge \langle c, b\rangle \in P^{\mathcal{M}}\right)\right\}$$

  and

$$\mathcal{M} \vDash R \sqsubseteq P \Leftrightarrow R^{\mathcal{M}} \subseteq P^{\mathcal{M}}$$

  e.g. $friendOf \circ enemyOf \sqsubseteq enemyOf$.

Normal extensions

– $\mathcal{N}$ – Cardinality restrictions: We allow concepts on the form $\leq n\, R$ and $\geq n\, R$ for any natural number n. Semantics[1]:

$$(\leq n\, R)^{\mathcal{M}} := \{a \in \Delta^{\mathcal{M}} \mid \#\{b \in \Delta^{\mathcal{M}} \mid \langle a, b\rangle \in R^{\mathcal{M}}\} \leq n\}$$
$$(\geq n\, R)^{\mathcal{M}} := \{a \in \Delta^{\mathcal{M}} \mid \#\{b \in \Delta^{\mathcal{M}} \mid \langle a, b\rangle \in R^{\mathcal{M}}\} \geq n\}$$

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- $\mathcal{Q}$ – Qualified cardinality restrictions: We allow concepts on the form $\leq n\,R.C$ and $\geq n\,R.C$ for any natural number n. Semantics:

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  e.g. $\mathit{RichPeople} \sqsubseteq\ \geq 2\,\mathit{owns}.\mathit{House}$.

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e.g. $Days \sqsubseteq \{monday, tuesday, wednesday, thursday, friday, saturday, sunday\}$;

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  e.g. $\mathit{Days} \sqsubseteq \{\mathit{monday}, \mathit{tuesday}, \mathit{wednesday}, \mathit{thursday}, \mathit{friday}, \mathit{saturday}, \mathit{sunday}\}$;

- $\mathcal{I}$ – Inverse roles: We allow roles on the form R^-. Semantics:

$$(R^-)^{\mathcal{M}} := \{\langle a, b\rangle \in \Delta^{\mathcal{M}} \times \Delta^{\mathcal{M}} \mid \langle b, a\rangle \in R^{\mathcal{M}}\}$$

  e.g. $\mathit{hasParent}^- \sqsubseteq \mathit{isChildOf}$;

Normal extensions

- $\mathcal{O}$ – Closed classes: We allow concepts on the form $\{a_1, a_2, \ldots, a_n\}$ where a_i are individuals. Semantics

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  e.g. $\mathit{hasParent}^- \sqsubseteq \mathit{isChildOf}$;
- $\mathcal{D}$ - Datatypes: We introduce a set of datatypes: *int*,*string*,*float*,*boolean*, and so on. They all have a fixed interpretation, that is, the same for all models.

Examples

$$OnlyChild \quad \sqsubseteq \quad Person \sqcap \neg \exists hasSibling.\top$$

Examples

$$\begin{aligned} \mathit{OnlyChild} &\sqsubseteq \mathit{Person} \sqcap \neg\exists \mathit{hasSibling}.\top \\ \mathit{Animal} &\sqsubseteq \leq 2\ \mathit{hasParent}.\mathit{Animal} \sqcap \geq 2\ \mathit{hasParent}.\mathit{Animal} \end{aligned}$$

Examples

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Examples

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\begin{array}{rcl}
\mathit{OnlyChild} & \sqsubseteq & \mathit{Person} \sqcap \neg\exists \mathit{hasSibling}.\top \\
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$$
\exists R.\top \sqsubseteq C
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& & & \\
\exists R.\top & \sqsubseteq & C & \text{Domain}
\end{array}
$$

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\begin{aligned}
\exists R.\top &\sqsubseteq C \qquad \text{Domain} \\
\top &\sqsubseteq \forall R.C
\end{aligned}
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$$
\begin{array}{rcll}
\exists R.\top & \sqsubseteq & C & \qquad\qquad \text{Domain} \\
\top & \sqsubseteq & \forall R.C & \qquad\qquad \text{Range}
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R & \sqsubseteq & R^{-} & \text{Symmetry}
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R & \sqsubseteq & \neg R^{-} & \text{Asymmetry}
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Complexity results

http://www.cs.man.ac.uk/~ezolin/dl/

Common restricted languages: $\mathcal{EL}$

The description logic $\mathcal{EL}$ allow the following concepts:

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& \bot & \mid & \text{(bottom concept)} \\
& \{a\} & \mid & \text{(\textit{singular} enumeration)} \\
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with the following axioms:

- $C \sqsubseteq D$ and $C \equiv D$ for concept descriptions D and C.
- $P \sqsubseteq Q$ and $P \equiv Q$ for roles P, Q.
- $C(a)$ and $R(a, b)$ for concept C, role R and individuals a, b.

Common restricted languages: $\mathcal{EL}$

Not supported (excerpt):

- negation, (only disjointness of classes: $C \sqcap D \sqsubseteq \bot$),
- disjunction,
- universal quantification,
- cardinalities,
- inverse roles,
- plus some role characteristics.

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- plus some role characteristics.

- Captures language used for many large ontologies.
- Checking ontology consistency, class expression subsumption, and instance checking is in **P**.
- "Good for large ontologies."

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The description logic $\mathit{DL}\text{-}\mathit{Lite}_R$ allows the following concepts:

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C \rightarrow & A & | & \text{(atomic concept)} \\
& \exists R.\top & | & \text{(existential restriction with } \top \text{ only)} \\
D \rightarrow & A & | & \text{(atomic concept)} \\
& \exists R.D & | & \text{(existential restriction)} \\
& \neg D & | & \text{(negation)} \\
& D \sqcap D' & | & \text{(intersection)}
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- disjunction,
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- Captures language for which queries can be translated to SQL.
  - Conjunctive queries over a *DL-Lite* knowledge base can be expanded with the TBox to a conjunctive query that can be answered over the Abox alone. This is called *first order rewritability*.
- "Good for large datasets."

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D \rightarrow & A & \mid & \text{(atomic concept)} \\
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The description logic $\mathcal{RL}$ (also called DLP) allow the following concepts:

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- $C \sqsubseteq D$, $C \equiv C'$, $\top \sqsubseteq \forall P.D$, $\top \sqsubseteq \forall P^{-}.D$ $P \sqsubseteq Q$, $P \equiv Q^{-}$ and $P \equiv Q$ for roles P, Q and concept descriptions D and C.
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Contents

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- OWL provides a concrete syntax for writing axioms, implementations of reasoners over the axioms, and a query language that applies the reasoners for knowledge extraction.

OWL 2 Profiles

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  - OWL 2 RL: Corresponds to $\mathcal{RL}$, and is designed for compatibility with rule-based inference tools.
- OWL Full (not a proper DL): Anything goes: classes, relations, individuals, highly expressive, not decidable. But we want OWL's reasoning capabilities, so stay away if you can—and you almost always can.

Contents

What cannot be expressed in DLs: Brothers

- Given terms

$$hasSibling \qquad Male$$

- …a brother is *defined* to be a sibling who is male
- Best try:

$$\begin{aligned}
&hasBrother \sqsubseteq hasSibling \\
&\top \sqsubseteq \forall hasBrother.Male \\
&\exists hasSibling.Male \sqsubseteq \exists hasBrother.\top
\end{aligned}$$

- Not enough to infer that *all* male siblings are brothers

What cannot be expressed in DLs: Diamond Properties

- A semi-detached house has a left and a right unit
- Each unit has a separating wall
- The separating walls of the left and right units are the same
- "diamond property"
- Try. . .

$$
\begin{aligned}
&\mathit{SemiDetached} \sqsubseteq \exists \mathit{hasLeftUnit}.\mathit{Unit} \sqcap \exists \mathit{hasRightUnit}.\mathit{Unit} \\
&\mathit{Unit} \sqsubseteq \exists \mathit{hasSeparatingWall}.\mathit{Wall}
\end{aligned}
$$

- And now what?

What cannot be expressed in DLs: Connecting Properties

- Given terms

$$Person \qquad hasChild \qquad hasBirthday$$

- A twin parent is defined to be a person who has two children with the same birthday.
- Try...

$$\begin{aligned} TwinParent \equiv Person \quad & \sqcap \ \exists hasChild.\exists hasBirthday[\ldots] \\ & \sqcap \ \exists hasChild.\exists hasBirthday[\ldots] \end{aligned}$$

- No way to connect the two birthdays to say that they're the same.
  - (and no way to say that the children are *not* the same)
- Try...

$$TwinParent \equiv Person \sqcap \geq_2 hasChild.\exists hasBirthday[\ldots]$$

- Still no way of connecting the birthdays

Reasoning about Numbers

- Reasoning about natural numbers is undecidable in general.
- DL Reasoning is decidable
- Therefore, general reasoning about numbers can't be "encoded" in DL
- For instance, *there is no largest prime number*:

$$\forall n.\exists p.(p > n \land \forall k, l.p = k \cdot l \rightarrow (k = 1 \lor l = 1))$$

- Could try...

$$\begin{gathered} Number(zero) \\ Number \sqsubseteq \exists hasSuccessor.Number \\ \top \sqsubseteq \leq 1\, hasSuccessor.\top \end{gathered}$$

- Cannot encode addition, multiplication, etc.
- Note: a lot can be done with other logics, but not with DLs
  - Outside the intended scope of Description Logics

FO-rewritability

Assume $\mathcal{T}_{\mathcal{L}}$ is the set of TBoxes over the language $\mathcal{L}$, and that UCQ is the set of queries that are unions of conjunctive queries, and let

$$\mathcal{K} \vDash q_1 \vee q_2 \Leftrightarrow \mathcal{K} \vDash q_1 \text{ or } \mathcal{K} \vDash q_2$$

$$\mathcal{K} \vDash q_1 \wedge q_2 \Leftrightarrow \mathcal{K} \vDash q_1 \text{ and } \mathcal{K} \vDash q_2$$

A description logic $\mathcal{L}$ enjoys *first order rewritability* if there exists a rewriting function $\rho : \mathcal{T}_{\mathcal{L}} \times UCQ \to UCQ$, such that for any knowledge base $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ over $\mathcal{L}$ and any conjunctive query $q(\vec{x})$ over $\mathcal{K}$ we have that

$$\mathcal{A} \vDash \rho(\mathcal{T}, q(\vec{a})) \Leftrightarrow \mathcal{K} \vDash q(\vec{a})$$

This allows us to divide the querying up into two stages: i) translation of the query, and ii) ABox querying. This is useful for e.g. translating a query from a DL query to an SQL query where the ABox is a relational database.

E.g. let $\mathcal{T} := \{C_1 \sqsubseteq D, C_2 \sqsubseteq D, A \sqsubseteq C_1\}$ and $q(x) := D(x)$ we have that for any Abox $\mathcal{A}$ that

$$\mathcal{A} \vDash D(a) \vee C_1(a) \vee C_2(a) \vee A(a) \Leftrightarrow \langle \mathcal{T}, \mathcal{A} \rangle \vDash D(a)$$