Description Logic

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Overview

- trade off : expressibility vs. reasoning complexity, defined by allowing or disallowing different constructs (e.g. conjunction, disjunction, negation, quantiers, etc.) in their language.
- Elements:
  - individuals/ constants(e.g. julia, john)
  - concepts/ unary prediction(e.g. Parents, Male)
  - roles/ binary relations(e.g. ParentOf, SonOf)
- Knowledge Base $\mathcal{K} =< \mathcal{A}, \mathcal{T} >$:
  - $\mathcal{A}$, 也叫$ABox(Axioms)$: is a set of assertions about named individuals (e.g. Person(james), isFatherOf (james,peter))
  - $\mathcal{T}$,也叫$TBox(Axioms)$: is a set of terminology denitions (i.e. complex descriptions of concepts or roles), called the TBox (e.g. $Human \sqsubseteq Mammal$, $Mother \equiv Parent \sqcap Woman$)
  - $\mathcal{R}$,也叫$RBox(Axioms)$:一般不于$\mathcal{T}$区分，是所有复杂role的集合，（e.g. $parentOf \sqsubseteq ancestorOf$, $brotherOf \circ parentOf \sqsubseteq uncleOf$)

Syntax and Semantics

- Constructors: 构建更加复杂的Concept 和 Role

构造算子	语法	语义	例子
原子概念	A	$A^I \subseteq \Delta^I$	Human
原子关系	R	$R^I \subseteq \Delta^I \times \Delta^I$	has-child
对概念C,D和关系(role)R			
合取	$C \sqcap D$	$C^I \cap D^I$	Human ⊓ Male
析取	$C \sqcup D$	$C^I \cup D^I$	Doctor ⊔ Lawyer
非	$\neg C$	$\Delta^I \backslash C$	¬ Male
存在量词	$\exists R.C$	$\{x \mid \exists y. <x,y> \in R^I \wedge y \in C^I\}$	∃ has-child.Male
全称量词	$\forall R.C$	$\{x \mid \forall y. <x,y> \in R^I \Rightarrow y \in C^I\}$	∀ has-child.Doctor

构造算子	语法	语义	例子
数量约束	$\geq n\ R\ .\ C$	$\{x \mid \lvert\{y \mid \langle x,y\rangle \in R^I, y \in C^I\}\rvert \geq n\}$	$\geq$3 has-child .Male
	$\leq n\ R\ .\ C$	$\{x \mid \lvert\{y \mid \langle x,y\rangle \in R^I, y \in C^I\}\rvert \leq n\}$	$\leq$3 has-child .Male
逆	R^-	$\{\langle y,x\rangle \mid \langle x,y\rangle \in R^I\}$	has-child$^-$
传递闭包	R^*	$(R^I)^*$	has-child*

另外，有两个类似于FOL中的全集(true)和空集(false)的算子

top	T	$\triangle^I$	Male $\sqcup$ ¬ Male
Bottom	$\perp$	$\varnothing$	Man $\sqcap$ ¬ Man

- e.g.:
  - $\exists SonOf.\top \sqsubseteq Male$, 限制了SonOf的domain必须是Male的实例
  - $\top \sqsubseteq \forall sonOf.Parent$,限制了SonOf的Range必须是Parent的实例

- Description Logic Profile：描述逻辑语言有很多分支，它们的主要区别就在于支持的构造子不同，导致表达能力和推理复杂度各不相同
  - $\mathcal{ALC}$:

$\mathcal{ALC}$: Syntax

The description logic $\mathcal{ALC}$ (Attribute Language with general Complement) allows the following concepts:

$$
\begin{array}{lll}
C, D \rightarrow & A & \mid \text{(atomic concept)} \\
& \top & \mid \text{(universal concept)} \\
& \bot & \mid \text{(bottom concept)} \\
& \neg C & \mid \text{(negation)} \\
& C \sqcup D & \mid \text{(union)} \\
& C \sqcap D & \mid \text{(intersection)} \\
& \exists R.C & \mid \text{(existential restriction)} \\
& \forall R.C & \mid \text{(universal restriction)}
\end{array}
$$

where A is an atomic concept, C and D are concepts, and R is a role. We allow

- ABox assertions: $C(a)$ and $R(a,b)$ for individuals a, b, concepts C and roles R;
- TBox axioms: $C \sqsubseteq D$ for concepts C and D.

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- Naming Conventions

Naming conventions

- As we have seen $\mathcal{ALC}$ is the *Attribute Language with general Complement*.
- The $\mathcal{C}$ actually denotes an extension of a more restrictive language $\mathcal{AL}$.
- In a similar way, we have the following possible extensions of our logic:
  - $\mathcal{H}$: Role hierarchies;
  - $\mathcal{R}$: Complex role hierarchies;
  - $\mathcal{N}$: Cardinality restrictions;
  - $\mathcal{Q}$: Qualified cardinality restrictions;
  - $\mathcal{O}$: Closed classes;
  - $\mathcal{I}$: Inverse roles;
  - $\mathcal{D}$: Datatypes;
  - ...
- We name the languages by adding the letters of the features to $\mathcal{ALC}$. So e.g. $\mathcal{ALCN}$ is $\mathcal{ALC}$ extended with cardinality restrictions and $\mathcal{ALCHI}$ is $\mathcal{ALC}$ extended with role hierarchies and inverse roles.
- It is common to shorten $\mathcal{ALC}$ (extended with transitive roles) to just $\mathcal{S}$ for more advanced languages, so e.g. $\mathcal{SHOIN}$ is $\mathcal{ALC} + \mathcal{H} + \mathcal{O} + \mathcal{I} + \mathcal{N}$.

Normal extensions

- $\mathcal{H}$ – Role Hierarchies: We allow TBox axioms on the form $R \sqsubseteq P$ for atomic roles. Semantics:

$$\mathcal{M} \models R \sqsubseteq P \Leftrightarrow R^{\mathcal{M}} \subseteq P^{\mathcal{M}}$$

  e.g. *hasFather* $\sqsubseteq$ *hasParent*;
- $\mathcal{R}$ – Complex role hierarchies: We allow roles on the form $R \circ P$ and TBox axioms on the form $R \circ P \sqsubseteq P$ and $R \circ P \sqsubseteq R$ for any two roles. Semantics:

$$(R \circ P)^{\mathcal{M}} := \{\langle a, b\rangle \in \Delta^{\mathcal{M}} \times \Delta^{\mathcal{M}} \mid \exists c \in \Delta^{\mathcal{M}} \left(\langle a, c\rangle \in R^{\mathcal{M}} \wedge \langle c, b\rangle \in P^{\mathcal{M}}\right)\}$$

  and

$$\mathcal{M} \models R \sqsubseteq P \Leftrightarrow R^{\mathcal{M}} \subseteq P^{\mathcal{M}}$$

  e.g. *friendOf* $\circ$ *enemyOf* $\sqsubseteq$ *enemyOf*.

Normal extensions

- $\mathcal{N}$ – Cardinality restrictions: We allow concepts on the form $\leq n\,R$ and $\geq n\,R$ for any natural number n. Semantics[1]:

$$(\leq n\,R)^{\mathcal{M}} := \{a \in \Delta^{\mathcal{M}} \mid \#\{b \in \Delta^{\mathcal{M}} \mid \langle a, b\rangle \in R^{\mathcal{M}}\} \leq n\}$$
$$(\geq n\,R)^{\mathcal{M}} := \{a \in \Delta^{\mathcal{M}} \mid \#\{b \in \Delta^{\mathcal{M}} \mid \langle a, b\rangle \in R^{\mathcal{M}}\} \geq n\}$$

  e.g. $Mammal \sqsubseteq\ \leq 2\, hasParent$;

- $\mathcal{Q}$ – Qualified cardinality restrictions: We allow concepts on the form $\leq n\,R.C$ and $\geq n\,R.C$ for any natural number n. Semantics:

$$(\leq n\,R.C)^{\mathcal{M}} := \{a \in \Delta^{\mathcal{M}} \mid \#\{b \in \Delta^{\mathcal{M}} \mid \langle a, b\rangle \in R^{\mathcal{M}} \wedge b \in C^{\mathcal{M}}\} \leq n\}$$
$$(\geq n\,R.C)^{\mathcal{M}} := \{a \in \Delta^{\mathcal{M}} \mid \#\{b \in \Delta^{\mathcal{M}} \mid \langle a, b\rangle \in R^{\mathcal{M}} \wedge b \in C^{\mathcal{M}}\} \geq n\}$$

  e.g. $RichPeople \sqsubseteq\ \geq 2\, owns.House$.

[1] We let $\#S$ be the cardinality of the set S

Normal extensions

- $\mathcal{O}$ – Closed classes: We allow concepts on the form $\{a_1, a_2, \ldots, a_n\}$ where a_i are individuals. Semantics

$$(\{a_1, a_2, \ldots, a_n\})^{\mathcal{M}} := \{a_1^{\mathcal{M}}, a_2^{\mathcal{M}}, \ldots, a_n^{\mathcal{M}}\}$$

  e.g. $Days \sqsubseteq \{monday, tuesday, wednesday, thursday, friday, saturday, sunday\}$;

- $\mathcal{I}$ – Inverse roles: We allow roles on the form R^{-}. Semantics:

$$(R^{-})^{\mathcal{M}} := \{\langle a, b\rangle \in \Delta^{\mathcal{M}} \times \Delta^{\mathcal{M}} \mid \langle b, a\rangle \in R^{\mathcal{M}}\}$$

  e.g. $hasParent^{-} \sqsubseteq isChildOf$;

- $\mathcal{D}$ - Datatypes: We introduce a set of datatypes: *int*,*string*,*float*,*boolean*, and so on. They all have a fixed interpretation, that is, the same for all models.

- Examples:

Examples

$$
\begin{array}{rcl}
\mathit{OnlyChild} & \sqsubseteq & \mathit{Person} \sqcap \neg \exists \mathit{hasSibling}.\top \\
\mathit{Animal} & \sqsubseteq & \leq 2\ \mathit{hasParent}.\mathit{Animal} \sqcap \geq 2\ \mathit{hasParent}.\mathit{Animal} \\
\mathit{Pet} \sqcap \mathit{Person} & \sqsubseteq & \bot \\
\mathit{Person} & \sqsubseteq & \exists \mathit{loves}.\{\mathit{mary}\} \\
\mathit{Norwegian} & \sqsubseteq & \exists \mathit{comesFrom}.\{\mathit{norway}\} \\
\{\mathit{adam}\} & \sqsubseteq & \neg\{\mathit{eve}\} \\
\mathit{hasFather} \circ \mathit{hasBrother} & \sqsubseteq & \mathit{hasUncle}
\end{array}
$$

$$
\begin{array}{rcll}
\exists R.\top & \sqsubseteq & C & \text{Domain} \\
\top & \sqsubseteq & \forall R.C & \text{Range} \\
R \circ R & \sqsubseteq & R & \text{Transitivity} \\
\top & \sqsubseteq & \leq 1\, R.\top & \text{Functionality} \\
R & \sqsubseteq & R^{-} & \text{Symmetry} \\
R & \sqsubseteq & \neg R^{-} & \text{Asymmetry}
\end{array}
$$