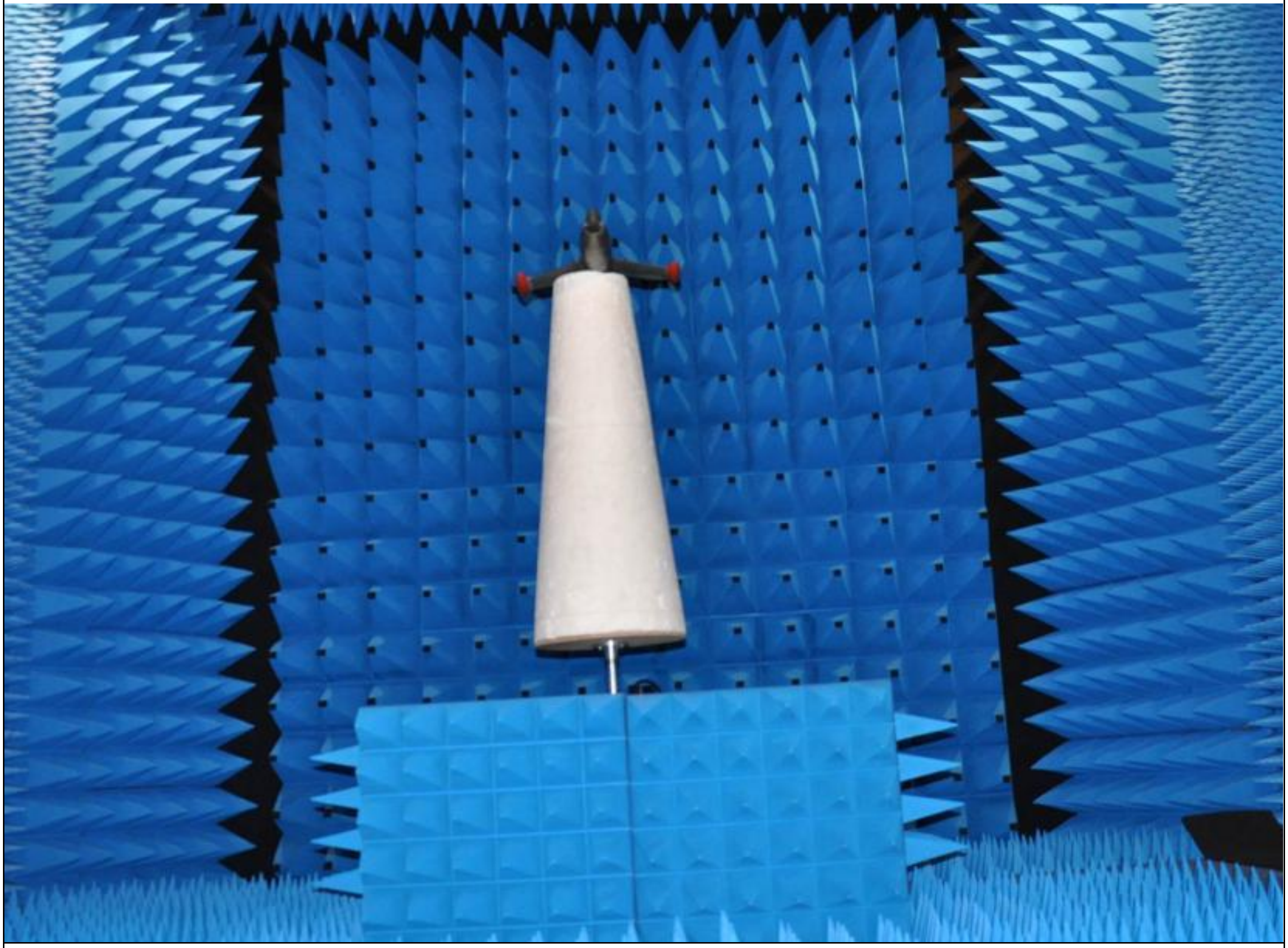

UE 6.1 – Status Report
Sparse Radar Imaging Technique
FISE - 2021

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Introduction

The RADAR Cross Section is a measure that indicates how detectable an object is by a RADAR, and acts as an intrinsic electromagnetic signature to said object. The greater the RCS[1], the easier to detect hence the constant consideration whether the object should be detected or should be stealthy. The RCS bears more information than just a surface, and given the right data, it can be used to classify and identify aircrafts, ships, and so on.

With an anechoic chamber, we have optimal conditions to fully quantify the RSC of an object according to the RADAR's operating frequency, and the angle at which the RADAR's beam hits the object. Given the data given by the Vector Network Analyser, and using classical operations such as Fourier's Transform, we can create a 3D representation of the object. Such techniques are known and used since the 1980's. [1]

Yet, the aforementioned techniques can provide only moderate quality results. This project aims at implementing a version of the Sparse Radar Imaging Technique (SPRITE) that should give us better results. The sparse approach gives us access to new algorithms, and allows us to take less measurements, which is a plus considering how tedious acquiring them is.

In the first chapter, I will mention a few notions that are essential in order to comprehend the methods and results. Then, I will emphasize on the data acquisition process, to conclude with the introduction of SPRITE[2] and comparison to existing methods.

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[1] Radar Cross Section
[2] SParse Radar Imaging TEchnique

I. Required notions

Before dwelling into how to generates images from RCS measurements, we must remind ourselves of a few basic principles and results.

1. RADAR

1.1. History of Radar

A radar is a device designed to emit an electromagnetic signal and “listen” for an echo, to determine distance, location, and other characteristics of a target in its beam.
No scientist can actually claim to be the inventor of the Radar, as it is accumulation of many technologies and scientific advancements in which several countries took a part in. [2]
It all began with the work of James Clerk Maxwell, in which he demonstrated that electric and magnetic fields travel through space like waves, at the speed of light. Electromagnetic waves were discovered years later, by Heinrich Hertz, confirming Maxwell’s theories. At the beginning of the 20th century, Nikola Tesla hinted that EM[3] waves could be used to detect moving metallic objects. As time went by, numerous application came to be, and inventions such as the magnetron and the klystron allowed nations such as the United States, the United Kingdom, Germany, Japan and France to have Radars during World War II.

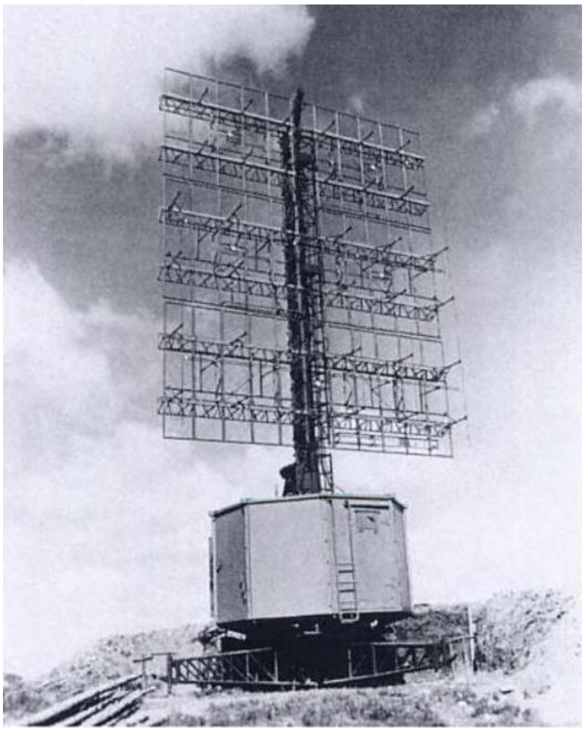

Figure 1 - Early german Warning Radar, Freya (1940)

Compared to visual observation, and optical systems, the radar has many advantages: can be unmanned, being operational day or night, in all weather, and through different technologies, can provide more than just detection.

1.2. Basic principle

Even though the calculations and equation can be complex, the basic principle of a Radar is quite simple to grasp. An antenna sends an EM wave in a certain direction: If the wave encounters an object, it will scatter and eventually bounce

[3] Electromagnetic

back for the antenna to receive. Knowing that an EM wave propagates at the speed of light and being able to compute the delay between emission and reception ensures we can compute the distance travelled by the wave. Finally, we have:

$$R = \frac{c\tau}{2}$$

Equation 1 - Basic Radar Principle

Where

- R is the distance between the Radar and the target (m)
- c is the speed of light ($m.s^{-1}$)
- τ is the delay between emission and reception of the wave. (s)

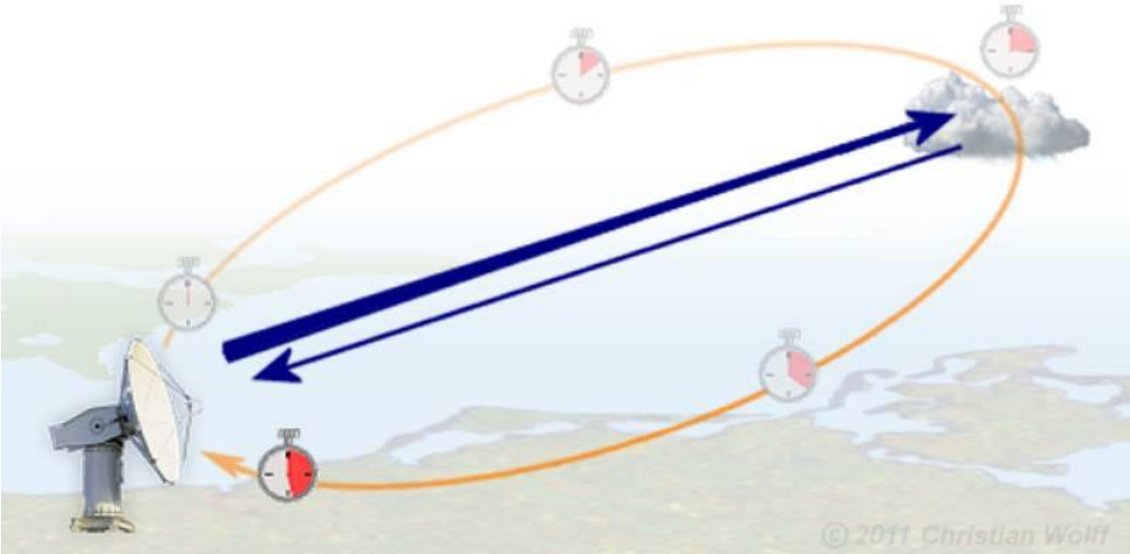

Figure 2 - Illustration of the wave trajectory [3]

Furthermore, if the direction of the emitted EM wave is perfectly known, in azimuth and elevation, we can deduce a position of our target, with a certain degree of resolution depending on intrinsic characteristic of our radar. [3]

1.3. Radar Equations

The Radar range equation provides the most useful mathematical relationship for engineers and technicians to dimension a Radar. It accounts for

- Radar system parameters (Antenna, frequency, power…)
- Target Parameters (Radar Cross Section)
- Background effect (Clutter, noise, interference, and jamming)
- Propagation medium (absorption and scattering)

In a general case, the radar range equation is:

$$\mathrm{R_{max}} = \left(\frac{G^2\lambda^2\,\sigma}{(4\pi)^3\,\mathrm{P_{min}}}\frac{1}{L}\right)^{1/4}$$

Equation 2 - Radar Range Equation

Where

- $\mathrm{R_{max}}$ is the maximum range of the radar (m)
- $\mathrm{P_{min}}$ is the minimum power that can be detected by the Radar (W)
- G is the Gain of the antenna used by the Radar (W)
- λ is the Radar's operating frequency (m)
- σ is the Radar Cross Section of the target (m²)
- L characterise the loss due to background effect and the propagation medium

Rearranged, in a monostatic case[4], we can state:

$$\mathrm{P_r} = \mathrm{P_e} \frac{G^{2\lambda^2}}{(4\pi)^3 R^4} \frac{1}{L} \sigma$$

Equation 3 - Radar Equation (Monostatic)

Where

- $\mathrm{P_r}$ is the power received by the Radar (W)
- $\mathrm{P_e}$ is the power emitted by the Radar (W)
- R is the distance at which the target is (m)

In a bistatic case[5],

$$\mathrm{P_r} = \mathrm{P_e} \frac{G_e G_r \lambda^2}{(4\pi)^3 R_1^2 R_2^2} \frac{1}{L} \sigma$$

Equation 4 - Radar Equation (Bistatic)

Where

- G_e is the Gain of the antenna used by the emitting Radar (W)
- G_r is the Gain of the antenna used by the receiving Radar (W)
- R_1 is the distance between the target and one radar (m)
- R_2 is the distance between the target and the other radar (m)

2. Radar Cross Section

In the previous equations, the Radar Cross Section was used, but was not defined properly. Expressed in m², the RCS[6] is a measure that indicates how detectable an object is by a Radar. It is mathematically defined by:

$$\sigma = 4\pi^2 \frac{|{E_s}^2|}{|{E_i}^2|}$$

Equation 5 - Radar Cross Section definition

Where

- E_s is the backscattered electric field (V/m)
- E_i is the incident electric field (V/m)

As indicated in Equation 3, the greater the RCS, the greater the power received by the antenna and thus the probability of detection. This raises concern as to whether the object is stealthy or is not. A greater RCS can be useful for an airplane or sailboat far at sea, granting them more chances to be detected. On the contrary, a smaller RCS grants airplanes/ships and other systems brings stealth, and a potential head start in an operational situation.

[4] Only one radar emitting and receiving
[5] One radar emitting, another one receiving
[6] Radar Cross Section

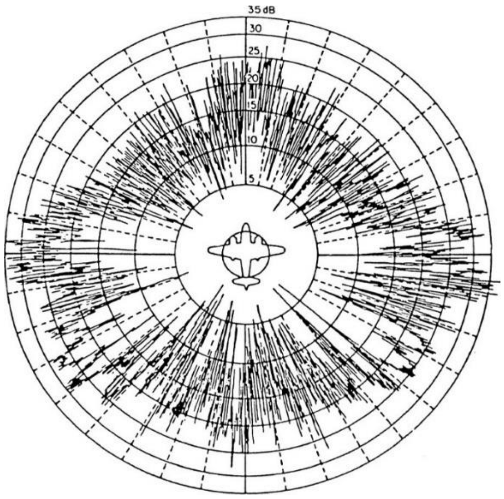

Figure 3 - Radar Cross Section diagram of a B-26 Bombardier – f = 3 GHz

The RCS of an object depends on an extensive number of parameters, ranging from its geometric shape, the material it is made of, as well as the EM wave polarisation. All these parameters are thoroughly chosen to ensure stealth or better detection right off the bat. [4]

3. Sparsity

The sparse representation of signal is a representation with a few significant parameters or values, the rest of them being equal to zero, or close to be.

When confronted to linear problem that can be put into the following form such as $x = D\alpha$, where D is a m by n matrix called the dictionary, x is a vector of length m, α is a vector of length p, the core sparse representation problem is defined as the quest for the sparsest possible representation α satisfying $x = D\alpha$. For instance, a Matching Pursuit algorithm will look for a sparse representation or solution to the linear problem, one non-zero coefficient at a time. By doing so, a MP[7] algorithm will quickly give a good approximation of the solution, which will be sparse. [5]

4. Radar Imaging techniques

As mentioned in RADAR, we can extrapolate more than just the presence of a target, or its distance. We are able to use Radars to create 2D, or even 3D images of objects.

4.1. Doppler Effect

When asked about the doppler effect, which we can experience in our daily lives, people often use the example of an ambulance passing by. The pitch of the siren varies during time, because of the ambulance’s speed. The same thing happens with EM waves.

Radars transmit a signal to a target and receives an echo signal from it. Based on the time delay of the received signal, the radar can compute the target’s distance. If the target is moving, the frequency of the received signal will be shifted from the frequency of the transmitted signal: the Doppler effect. The Doppler frequency shift is determined by the radial velocity of the target. This shift is often measured in the frequency domain, by using the Fourier Transform.

[7] Matching Pursuit

4.2. SAR

Synthetic-Aperture Radars are a type of radar used to create 2D images or 3D reconstructions of objects. Here, object is a broad term as landscapes are often represented thanks to SAR[8]. SAR also stands for the airborne and spaceborne technique to create images remotely. It uses the motion of the radar antenna over a target region to provide finer spatial resolution than conventional beam-scanning radars. The first SAR images were formed when a C46 aircraft was to map a section of Key West in Florida. [6]
A SAR works just like a regular radar except that it is moving but given that its trajectory is most likely known[9], we can account for the doppler shift induced and correct it after the fact: this allows us to recreate images of the scanned scene. SAR imagery is based on successive signal processing algorithms called range compression and azimuth compression. [7]

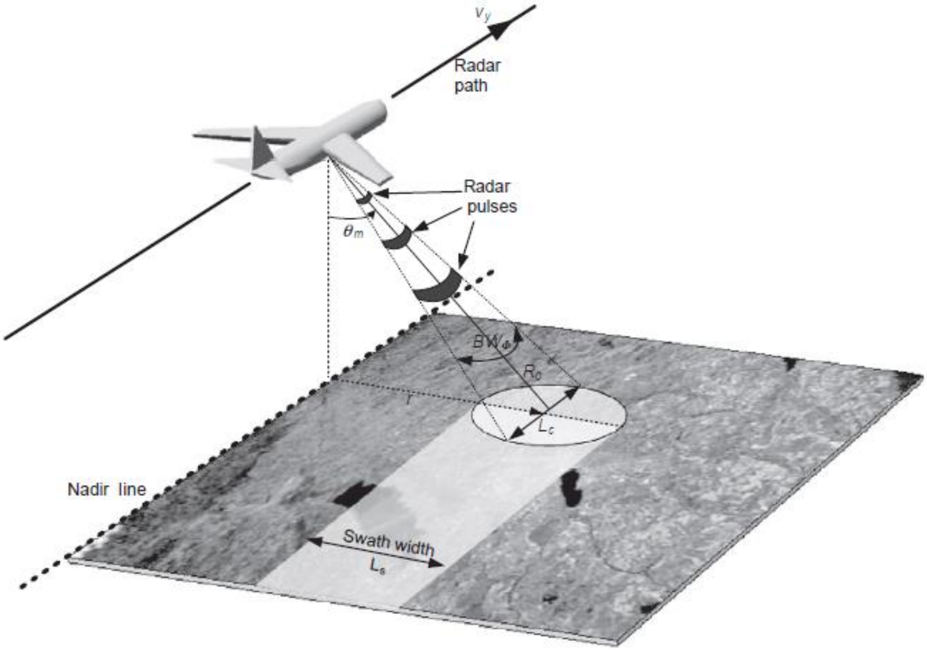

Figure 4 - Basic SAR operation

4.3. ISAR

Inverse Synthetic Aperture Radar is a signal processing technique for imaging moving targets. An ISAR image has the ability to display the scattering 'hotspots' of our target. Unlike SAR imaging, the radar is stationary, and the target is moving. As indicated by the names, SAR and ISAR can be similar on some aspects, and a spotlight SAR geometry with a circular path is analogous to ISAR geometry. [8]

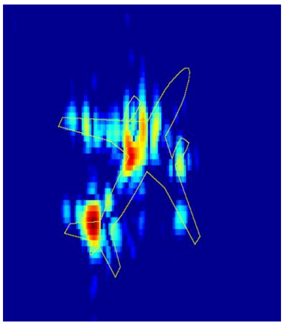

Figure 5 - ISAR image of an airplane

[8] Synthetic Aperture Radars
[9] Global Navigation Satellite Systems and Inertial Measurement Units used together provide tremendous precisions

II. Measurements acquisition

For us to reconstruct an image, we will have to acquire data. We can begin to work with simulated data, but the model used may not account for natural phenomena occurring that potentially bring the quality of the reconstruction down. To get a better look at the RCS of an object, we will have to measure it in an anechoic chamber.

1. Anechoic chamber

An anechoic chamber is a room where we try to get only the electric field scattered by the object of interest. No anechoic chamber is alike. Indeed, each one is defined by a certain number of parameters that will determine the characteristics of the chamber. For instance, depending on what we ought to measure, the size will be different. *Solange,* located in Bruz, can house 1:1 representation of combat aircrafts or drones, whereas ENSTA Bretagne's chamber is limited to small objects.

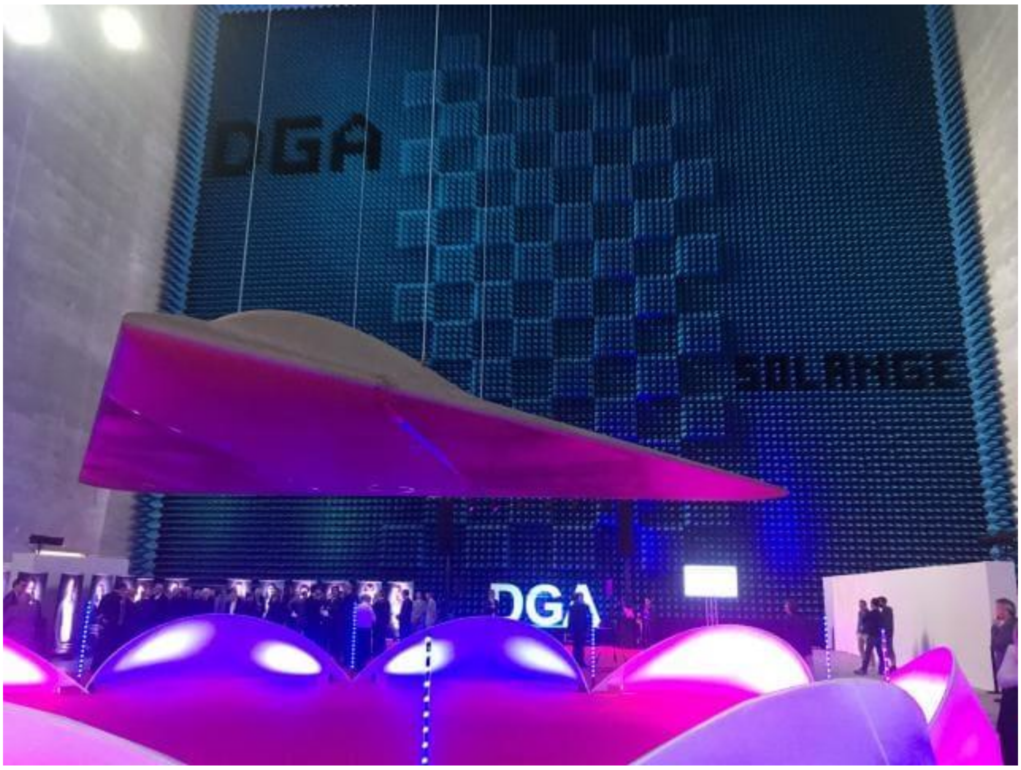

Figure 6 - SOLANGE anechoic chamber - DGA Maitrise de l'Information - Bruz

Other parameters are:

- Measurement configuration: monostatic, bistatic, quasi monostatic
- Polarisation of the incident wave
- Measurement Frequency range
- The target's mass
- The maximum expected RCS [9]

The wavelength of our incident wave will also be decisive on the shape and size of the foam used on the wall. The purpose of this specially coated foam is to limit as much as possible reflection of the walls and mounting mechanism that will pollute the acquisition. With the help of a carbon-rich paint and its pyramidal shape, the foam acts as a wave trap and dissipate waves through Joule heating. [10]

2. Antennas and frequency range

Having designed our chamber as intended for our purposes, we now have to choose the antennas and layout that will be relevant for our measurements. We will have to keep in mind that to get co-polarisation and cross-polarisation, a particular antenna must be used. As for the layout, we have the choice between monostatic, quasi monostatic, and bistatic. Here in ENSTA Bretagne, we can choose from bistatic and quasi monostatic:
Two horn antennas are disposed on a rail, facing the mounting mechanism for the measured object.
Horn antennas can operate over a wide range of frequencies, which is critical since the anechoic chamber in ENSTA Bretagne must operate from 2 GHz[10] to 18 GHz. [11]

The resolution at which our measurement chain will be able to operate is directly linked to this frequency range, such that:

$$\delta\mathrm{r} = \frac{c}{2B}$$

Equation 6 - Spatial Resolution

Where

- δr is distance resolution (m)
- c is the speed of light ($m.s^{-1}$)
- B is the frequency range (Hz)

3. Vector Network Analyser

The VNA[11] compares the incident signal, which it generates, to the received signal, which was scattered by the measured object. The value resulting from this comparison is complex and called the S-parameter. The S-parameter is complex because, unlike a Scalar Network Analyser, the VNA not only measure the amplitude, but also the phase of the received signal.

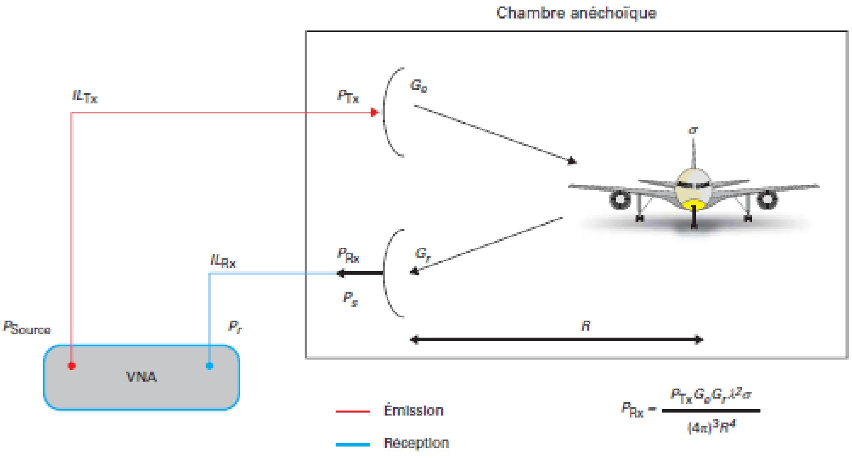

Figure 7 - Simplified VNA measurement process

To link the VNA and the antennas, coaxial cords are often used. Every connection between components must be considered as each link of the chain begets power loss depending on the frequency.
Just as with the anechoic chamber, we must choose our VNA considering certain parameters such as:

- Frequency range
- Maximum power delivery for the incident wave
- Maximum received power
- The receiver's Johnson–Nyquist noise (or thermal noise)

[10] ENSTA Bretagne's chamber is NOT a Faraday cage, Wi-Fi and Bluetooth from nearby offices can and will ruin measurements
[11] Vector Network Analyser

4. Protocol

4.1. Measurement without target and calibration

Even though an anechoic chamber aims to minimise the reflections of EM waves on the walls, it is impossible to guarantee that parasites signals will not be included in our measurements. Furthermore, the measurement chain will most certainly bring its own noise: hence the need to evaluate the levels of noise beforehand.
By measuring the electromagnetic signature of the room, without any target, we can then subtract those values in order to minimise the noise levels and get more accurate readings.

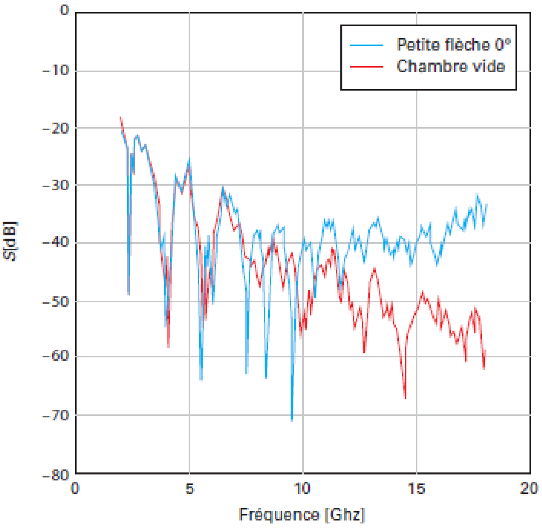

Figure 8 - Comparison of the measured electric field in an empty anechoic room and with a small metallic arrow [9]

According to Figure 8, at some frequencies, electric field levels can be very similar with or without our target. The electric field measured in the chamber is subtracted in a vectorial way, such that the phase of the electric field is also subtracted.

In addition to measurement without a target, we also have to calibrate our measurement equipment thanks to a perfectly known target according to the Geometrical Theory of Diffraction. For instance, we can use a metal plate or a metal sphere, precisely positioned on the pedestal. For common and simple geometric shape, the GTD[12] gives us a standard to go by, and most importantly a correction coefficient to apply to the measurement following this calibration phase.
Every measurement campaign starts with this phase, only then can we place our target on the pedestal.

4.2. Target placement and measurements

The placement of the target is of the essence because its backscattered field and thus its RCS greatly depends on the angle of the incident wave. Misaligning our target could mean having poor results. The first step is the alignment of the object with the beam of emission of our antenna, and make sure its angle is relevant to what we ought to measure.

When in place, the measurement campaign can finally begin. In ENSTA Bretagne's anechoic chamber, thanks to the 3 motors included in the pedestal, we can make our target spin on two axes (list and bearing) and translate on another (vertical). At a given angle, the antenna will emit as many times as necessary at various frequencies, and the scattered electric field will be measured by the receiving antenna. All of this is monitored and handled by a GUI[13] on a PC, which is connected to the VNA.

At the end of the measurement campaign, we end up with a matrix containing the amplitude and phase of the backscattered electric field. By using techniques such as ISAR, we will be able to reconstruct a 2D representation of our target.

[12] Geometrical Theory of Diffraction
[13] Graphical User Interface

III. Sparse Radar Imaging TEchnique

Radar imaging is often an ill-posed problem: the number of unknows is larger than the number of data. Conventional and common methods use data reformatting in the form of zero-padding to compensate and be able to apply inverse Fast Fourier Transform, giving adequate results. Yet, by using sparse representation algorithm, and extending the concept of scatter points, researchers were able to produce high resolution RCS imaging. [12]

1. Problem statement

The acquisition method is the same as depicted in *Protocol,* we end up with a RCS matrix, that we will call σ.
We have:

$$\sigma = \mathbf{Ha} + \mathbf{n}$$

Where

- $\mathbf{H}$ is the model matrix
- n is a noise vector
- a is scattering map, which is the RCS

Given the fact that the problem is ill-posed, H is rank deficient and thus non-invertible: it is mandatory to regularize the problem that means consider prior information on the scattering map. *l1-regularisation* methods with the scattering map l1 norm a sparse promoting penalty, Orthogonal Matching Pursuit, and MUSIC[14] spectral estimation can be used, and give consistent results: the scattering map is composed of scattering hotspots.

In a high frequency context, specular reflections are the main scattering mechanisms, and not diffusion: scattering hotspot do not particularly account for those phenomena, which is why SPRITE introduced scattering segments and facets.

2. Applying SPRITE

SPRITE comes with 5 priors that are crucial:

- The vertical projection of the scattering map a is sparse.
- Specular facets are topologically connected, and the scattering coefficient is constant over each facet
- The associated penalties are of l1-norms of a denoised by anisotropic total variations.
- The criterion is strictly convex
- The EM extent of the target is of finite support

According to those priors, we can write:

$$\hat{\boldsymbol{a}} = \underset{\boldsymbol{a}\in\mathbb{C}^N}{\operatorname{argmin}} \begin{cases} \mathcal{J}(\boldsymbol{a}) \\ \text{s.t. } \boldsymbol{a} \in \mathcal{C} \end{cases}$$

Equation 7 - Optimisation problem for estimated scattering map

with

$$\mathcal{J}(\boldsymbol{a}) = \frac{1}{2}\|\boldsymbol{\sigma} - \boldsymbol{H}\boldsymbol{a}\|_2^2 + \mu\|\boldsymbol{P}\boldsymbol{a}\|_1 + \lambda\|\boldsymbol{D}_x\boldsymbol{a}\|_1 + \lambda\|\boldsymbol{D}_y\boldsymbol{a}\|_1 + \frac{\nu}{2}\|\boldsymbol{a}\|_2^2$$

Equation 8 - Optimisation Criterion

To solve this optimisation problem, SPRITE uses Alternating Direction Method of Multiplier (ADMM). This method bodes well with strictly convex optimisation criteria: combined with the priors, ADMM mathematically ensures that our optimisation will converge. During the ADMM loop, nullifying the gradient is a matter of linear algebra, and considering

[14] Multiple Signal Classification

a lot of the matrices involved are either shift matrices, diagonal, or circulant, the calculations are quicker. Furthermore, the scattering map update is computed very efficiently in the frequency domain by FFT and IFFT[15].

3. Results

The previous method ensured convergence thanks to the algorithms and criterions chosen. Here is how researchers compared SPRITE to other methods. In an anechoic chamber, a metallic Perfectly Electrically Conducting (PEC) right circular cone is placed, pointing towards the antenna, in a monostatic configuration. On this cone are placed 6 rounded patches, that will be the scatterers to detect and map.

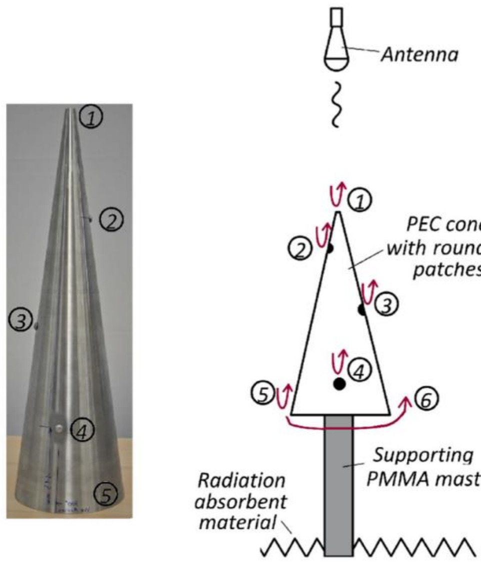

Figure 9 - Picture of the cone / Simplified Schematics of the measurement acquisition

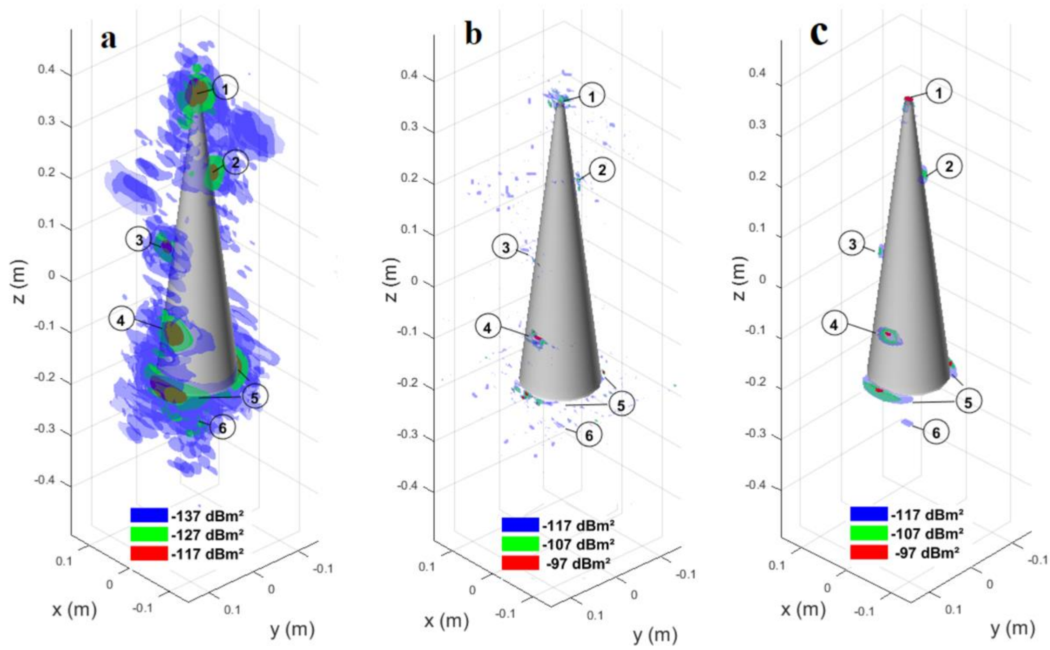

Figure 10 - (a) Classical Approach / (b) l1-regularisation / (c) SPRITE

15 Fast Fourier Transform and its inverse transformation.

A good result would be a 3D mapping of the scatter points and only those. According to *Figure 10*, the classical approach does not bode well in this particular situation, with low power levels and noise everywhere. When looking at the l1-regularisation method, we can already see the benefits of a sparse approach: better power levels and more localised hotspots. Finally, SPRITE gives us a sparse map that perfectly reflects the patches position.

Conclusion

The purpose of this status report is to introduce the notions that are crucial to understanding the underlying principles of Radar, RCS, and imaging. After explaining how the Sparse Radar Imaging Technique operates, we had the opportunity to see that the results were as good as advertised by the researchers, even compared to other sparse representations.
We will implement this technique on MATLAB®, replicate the results and assess its performance compared to more common approaches, on simulated data as well as real data.

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