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Monday, August 23, 2021 12:04 PM

1. $x_1 = (13(-5)+7) \bmod 12$
$= (-58) \bmod 12$
$= 2$

$x_2 = (13(2)+7) \bmod 12$
$= 33 \bmod 12$
$= 9$

$x_3 = (13(9)+7) \bmod 12$
$= (124) \bmod 12$
$= 4$

$x_4 = (13(4)+7) \bmod 12$
$= (59) \bmod 12$
$= 11$

$x_5 = (13(11)+7) \bmod 12$
$= (150) \bmod 12$
$= 6$

2. $\frac{100}{5} + \frac{100}{5^2} = 20 + 4 = 24$

3. base case: $n=0$ $\quad 0-0+0=0$ divisible by 5 ✓

Assume $n^5 - 5n^3 + 4n$ is divisible by 5 for all $k \geq 0$

$(k+1)^5 - 5(k+1)^3 + 4(k+1) = (k+1)((k+1)^4 - 5(k+1)^2 + 4) = (k+1)((k+1)^2 - 4)((k+1)^2 - 1)$

$= (k+1)(k+1-2)(k+1+2)(k+1-1)(k+1+1)$

$= \underbrace{(k+1)(k-1)(k+3)(k)(k+2)}$

5 consecutive integers therefore there is always a number divisible by 5 in the product therefore k+1 is divisible by 5, so proven by induction

4. same as $2^{42} \bmod 11$

$= (2^{21})^2 \bmod 11 = (2 \cdot (2^{10})^2)^2 \bmod 11 = (2 \cdot ((2^5)^2)^2)^2 \bmod 11$

$= (2 \cdot ((32)^2)^2)^2 \bmod 11$

$= (2 \cdot ((10^2)^2))^2 \bmod 11$

$= (2 \cdot (100)^2)^2 \bmod 11$

$= (2 \cdot (1^2))^2 \bmod 11$

$= 4$

5. $309 = 112 \cdot 2 + 85$

$112 = 85 + 27$

$85 = 27(3) + 4$

$27 = 4(6) + 3$

$4 = 3 + 1$

$3 = 1(3) + 0$

$\gcd = 1$ so 309 and 112 are relatively prime

6. $54 = 16(3) + 6 \rightarrow 6 = \underset{r_0}{54} - \underset{r_1}{16}(3)$

$16 = 6(2) + 4 \rightarrow 4 = r_1 - 2(r_0 - 3r_1) = 7r_1 - 2r_0$

$6 = 4(1) + 2 \rightarrow 2 = r_0 - 3r_1 - (7r_1 - 2r_0) = 3r_0 - 10r_1$

$4 = 2(2) + 0$

$\gcd(54, 16) = 2$

$x = 3 \quad y = -10$

7. $33 \cdot 17 = 561 \equiv 1 \bmod (112)$