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Monday, August 23, 2021 12:04 PM

1. $x_1 = (13(-5)+7) \bmod 12$
$= (-58) \bmod 12$
$= 2$

$x_2 = (13(2)+7) \bmod 12$
$= 33 \bmod 12$
$= 9$

$x_3 = (13(9)+7) \bmod 12$
$= (124) \bmod 12$
$= 4$

$x_4 = (13(4)+7) \bmod 12$
$= (59) \bmod 12$
$= 11$

$x_5 = (13(11)+7) \bmod 12$
$= (150) \bmod 12$
$= 6$

2. $\frac{100}{5} + \frac{100}{5^2} = 20 + 4 = 24$

3. $n^5 - 5n^3 + 4n = n(n^4 - 5n^2 + 4) = n(n^2-4)(n^2-1)$
$= \underbrace{n(n-2)(n+2)(n-1)(n+1)}$

5 consecutive integers therefore there is always a number divisible by 5 (aka a number that ends in 5 or 0) in the product, therefore the number is always divisible by 5

4. Same as $2^{42} \bmod 11$
$= (2^{21})^2 \bmod 11 = (2 \cdot (2^{10})^2)^2 \bmod 11 = (2 \cdot ((2^5)^2)^2)^2 \bmod 11$
$= (2 \cdot ((32)^2)^2)^2 \bmod 11$
$= (2 \cdot ((10^2)^2))^2 \bmod 11$
$= (2 \cdot (100)^2)^2 \bmod 11$
$= (2 \cdot (1^2))^2 \bmod 11$
$= 4$

5. $309 = 112 \cdot 2 + 85$
$112 = 85 + 27$
$85 = 27(3) + 4$
$27 = 4(6) + 3$
$4 = 3 + 1$
$3 = 1(3) + 0$
gcd = 1 so 309 and 112 are relatively prime

6. $54 = 16(3) + 6 \rightarrow 6 = \underset{r_0}{54} - \underset{r_1}{16}(3)$
$16 = 6(2) + 4 \rightarrow 4 = r_1 - 2(r_0 - 3r_1) = 7r_1 - 2r_0$
$6 = 4(1) + 2 \rightarrow 2 = r_0 - 3r_1 - (7r_1 - 2r_0) = 3r_0 - 10r_1$
$4 = 2(2) + 0$
$\gcd(54, 16) = 2$

$x = 3 \quad y = -10$

7. $x = 33 \bmod 112 \qquad r_0 = 112 \quad r_1 = 33$

$\gcd(112, 33)$

$112 = 33(3) + 13 \qquad 13 = r_0 - 3r_1$
$33 = 13(2) + 7 \qquad 7 = r_1 - 2(r_0 - 3r_1) = 7r_1 - 2r_0$
$13 = 7(1) + 6 \qquad 6 = r_0 - 3r_1 - 7r_1 + 2r_0 = 3r_0 - 10r_1$
$7 = 6(1) + 1 \qquad 1 = 7r_1 - 2r_0 - 3r_0 + 10r_1 = 17r_1 - 5r_0$
$6 = 1(6) + 0$

$1 = 17r_1 - 5r_0$ 17