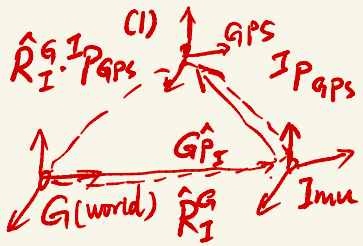
imu_gps_localization eskf

1. 利用GPS位置数据更新状态

$$ {}^G\hat{P}_{GPS} = {}^G\hat{P}_I + \hat{R}_I^G \cdot {}^I P_{GPS} \quad (1) $$



1.1 GPS位置数据的残差

$$ {}^G\tilde{P}_{GPS} = \underbrace{{}^G P_{GPS}}_{\text{测量}} - \underbrace{{}^G\hat{P}_{GPS}}_{\text{估计}} $$

$$ = {}^G P_{GPS} - ({}^G\hat{P}_I + \hat{R}_I^G {}^I P_{GPS}) \quad (2) $$

1.2 GPS位置数据残差关于误差状态的雅各比

因为eskf的状态都是误差状态，$({}^G\tilde{P}_I, {}^G\tilde{V}_I, {}^G\tilde{\theta}_I \cdots)$

(a) 残差 ${}^G\tilde{P}_{GPS}$ 关于 ${}^G\tilde{P}_I$ 的雅各比，对式(1)中 ${}^G\hat{P}_{GPS}$ 加扰动 ${}^G\tilde{P}_{GPS}$：

$$ {}^G\hat{P}_{GPS} + {}^G\tilde{P}_{GPS} = {}^G\hat{P}_I + {}^G\tilde{P}_I + \hat{R}_I^G {}^I P_{GPS} $$

$$ {}^G\hat{P}_{GPS} - {}^G\hat{P}_I + {}^G\tilde{P}_{GPS} = {}^G\tilde{P}_I + \hat{R}_I^G {}^I P_{GPS} $$

$$ {}^I\hat{P}_{GPS} + {}^G\tilde{P}_{GPS} = {}^G\tilde{P}_I + \hat{R}_I^G {}^I P_{GPS} $$

^{G}名不同　两者相等.

$$ \therefore {}^G\hat{P}_{GPS} = {}^G\tilde{P}_I $$

$$ \Rightarrow \frac{\partial {}^G\tilde{P}_{GPS}}{\partial {}^G\tilde{P}_I} = I_{3\times 3} \quad (3) $$

(b) 残差 ${}^G\tilde{P}_{GPS}$ 关于 ${}^G\tilde{\theta}_I$ 的雅各比，对式(1)中旋转 $\hat{R}_I^G$ 加扰动 ${}^G\tilde{\theta}_I$：

$$ {}^G\hat{P}_{GPS} + {}^G\tilde{P}_{GPS} = {}^G P_I + \hat{R}_I^G (I + [{}^G\tilde{\theta}_I]_\times) {}^I P_{GPS} $$

$$ = {}^G P_I + \underbrace{\hat{R}_I^G {}^I P_{GPS}}_{\text{无关项}} + \hat{R}_I^G [{}^G\tilde{\theta}_I]_\times {}^I P_{GPS} $$

省略无关项，

$$ {}^G\tilde{P}_{GPS} = -\hat{R}_I^G [{}^I P_{GPS}]_\times {}^G\tilde{\theta}_I $$

$$\therefore \frac{\partial\, {}^G\tilde{p}_{GPS}}{\partial\, {}^G\tilde{\theta}_I} = -\hat{R}_I^G \left[{}^I p_{GPS}\right]_\times \qquad (4)$$

$$H_{3\times15} = \left[\frac{{}^G\tilde{p}_{GPS}}{{}^G\tilde{p}_I},\ 0_{3\times3},\ \frac{{}^G\tilde{p}_{GPS}}{{}^G\tilde{\theta}_I},\ 0_{3\times3},\ 0_{3\times3} \right] \qquad (5)$$

2、利用GPS速度数据更新状态

残差：如右图，参考系G，IMU坐标系，GPS坐标系三者之间的位置矢量关系为：

$${}^G\hat{p}_{GPS} = {}^G\hat{p}_I + \hat{R}_I^G\, {}^I p_{GPS}$$ ，跟式(1)一样，这里重写下，方便推导

两边同时对t求导，

$$\frac{d\, {}^G\hat{p}_{GPS}}{dt} = \frac{d\, {}^G\hat{p}_I}{dt} + \frac{d(\hat{R}_I^G\, {}^I p_{GPS})}{dt}$$

$${}^G\hat{v}_{GPS} = {}^G\hat{v}_I + \frac{d(\hat{R}_I^G)}{dt} \cdot {}^I p_{GPS} + \hat{R}_I^G \cdot \frac{d({}^I p_{GPS})}{dt}$$

又∵ ${}^I p_{GPS}$ 在IMU系下是常量，对t不变. $\therefore \frac{d({}^I p_{GPS})}{dt} = 0$　　$\dot{R} = R[w_L]_\times$

$$\therefore \text{上式} \Rightarrow {}^G\hat{v}_{GPS} = {}^G\hat{v}_I + \hat{R}_I^G [w_L]_\times\, {}^I p_{GPS} \qquad (5)$$

2.1 因此GPS速度残差为：

$$\begin{aligned} {}^G\tilde{v}_{GPS} &= {}^G v_{GPS} - {}^G\hat{v}_{GPS} \\ &= {}^G v_{GPS} - \left({}^G\hat{v}_I + \hat{R}_I^G [w_L]_\times\, {}^I p_{GPS}\right) \qquad (6) \end{aligned}$$

$$w_L = w_I - w_b \qquad (7)$$

（w_I：测量，w_b：零偏）

都是对估计加扰动

2.2 GPS速度残差关于误差状态的雅各比

(a) 残差 ${}^G\tilde{v}_{GPS}$ 关于 ${}^G\tilde{v}_I$ 的雅各比，对式(1)中 ${}^G\hat{p}_{GPS}$ 加扰动 ${}^G\tilde{p}_{GPS}$：

$${}^G\hat{p}_{GPS} + {}^G\tilde{p}_{GPS} = {}^G\hat{p}_I + {}^G\tilde{p}_I + \hat{R}_I^G\, {}^I p_{GPS}$$

$$ {}^G\hat{p}_{GPS} - {}^G\hat{p}_I + {}^G\tilde{p}_{GPS} = {}^G\tilde{p}_I + \hat{R}_I^G {}^I p_{GPS} $$

相等.

$$ \therefore {}^G\tilde{p}_{GPS} = {}^G\tilde{p}_I $$

两边同时对t求导

$$ \frac{d({}^G\tilde{p}_{GPS})}{dt} = \frac{d({}^G\tilde{p}_I)}{dt} $$

$$ \Rightarrow {}^G\tilde{v}_{GPS} = {}^G\tilde{v}_I $$

因此

$$ \frac{\partial\, {}^G\tilde{v}_{GPS}}{\partial\, {}^G\tilde{v}_I} = I_{3\times3} \quad (8) $$

或者直接对观测值式(5)中 ${}^G\hat{v}_I$ 加扰动 ${}^G\tilde{v}_I$:

$$ {}^G\hat{v}_{GPS} + {}^G\tilde{v}_{GPS} = {}^G\hat{v}_I + {}^G\tilde{v}_I + \hat{R}_I^G [w_L]_\times {}^I p_{GPS} $$

，去掉无关项 推导

(b) 残差 ${}^G\hat{v}_{GPS}$ 关于 ${}^G\tilde{\theta}_I$ 的雅各比，对式(5)中的 $\hat{R}_I^G$ 加扰动 ${}^G\tilde{\theta}_I$:

$$ {}^G\hat{v}_{GPS} + {}^G\tilde{v}_{GPS} = {}^G\hat{v}_I + \hat{R}_I^G (I + [{}^G\tilde{\theta}_I]_\times) [w_L]_\times {}^I p_{GPS} $$

$$ = {}^G\hat{v}_I + \hat{R}_I^G [w_L]_\times {}^I p_{GPS} + \hat{R}_I^G [{}^G\tilde{\theta}_I]_\times [w_L]_\times {}^I p_{GPS} $$

无关项

省略无关项，

$$ {}^G\tilde{v}_{GPS} = -\hat{R}_I^G \left[[w_L]_\times {}^I p_{GPS}\right]_\times {}^G\tilde{\theta}_I $$

$w - w_b$

$$ \frac{\partial\, {}^G\tilde{v}_{GPS}}{\partial\, {}^G\tilde{\theta}_I} = -\hat{R}_I^G \left[[w_L]_\times {}^I p_{GPS}\right]_\times \quad (9) $$

(c) 残差 ${}^G\hat{v}_{GPS}$ 关于 $\tilde{w}_b$ 的雅各比，将式(7)代入式(5)中，

根据《捷联惯导》公式(4.1-2)

$$ {}^G\hat{v}_{GPS} = {}^G\hat{v}_I + \hat{R}_I^G [w_I - w_b]_\times {}^I p_{GPS} $$

$(w_b + \tilde{w}_b)$

$$ = {}^G\hat{v}_I + (\hat{R}_I^G [w_I]_\times - \hat{R}_I^G [w_b]_\times) {}^I p_{GPS} $$

$$ = {}^G\hat{v}_I + \hat{R}_I^G [w_I]_\times {}^I p_{GPS} - \hat{R}_I^G [w_b]_\times {}^I p_{GPS} \quad (10) $$

无关项

对式(10)中 w_b 加扰动 $\tilde{w}_b$，

$$ {}^G\hat{v}_{GPS} + {}^G\tilde{v}_{GPS} = -\hat{R}_I^G [w_b + \tilde{w}_b]_\times {}^I p_{GPS} $$

$$^G\hat{V}_{GPS} + {}^G\tilde{V}_{GPS} = -\hat{R}_I^G[w_L]_\times {}^I p_{GPS} - \hat{R}_I^G[\tilde{w}_b]_\times {}^I p_{GPS}$$

天线项

省略天线项. $${}^G\tilde{V}_{GPS} = +\hat{R}_I^G[{}^I p_{GPS}]_\times \tilde{w}_b$$

$$\therefore \frac{\partial {}^G\tilde{V}_{GPS}}{\partial \tilde{w}_b} = \hat{R}_I^G[{}^I p_{GPS}]_\times \quad (11)$$

$$\text{同理 } H_{3\times 15} = \left[0_{3\times 3}, \frac{\partial {}^G\tilde{V}_{GPS}}{\partial {}^G\tilde{V}_I}, \frac{\partial {}^G\tilde{V}_{GPS}}{\partial {}^G\tilde{\theta}_I}, 0_{3\times 3}, \frac{\partial {}^G\tilde{V}_{GPS}}{\partial \tilde{w}_b}\right]$$