NATIONAL AGRARIAN UNIVERSITY

LA MOLINA

FACULTY OF SCIENCES

ACADEMIC DEPARTMENT OF PHYSICS AND METEOROLOGY

COURSE: APPLIED GEOMATICS

REPORT 02

Validation of interpolated maps with Spline and Kriging methods

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LIMA - 2024

I. INTRODUCTION

The objective of this study is to determine the level of error of the Spline and Kriging interpolation methods for the meteorological variables of precipitation, temperature and relative humidity in the department of Puno - Peru and its surrounding areas in order to estimate the data in places where there is no data. Puno, a region located in the southeastern region of the country, is known for its varied topography that includes everything from highland plains to high Andean mountains. This geographic diversity significantly influences climate patterns, making Puno an ideal region to study the precision of different meteorological data interpolation methods.

Data from SENAMHI stations over a 10-year period and ERA-5 reanalysis data will be used to determine the 30-year climatology. As a previous step, in the data from meteorological stations, homogenization of the data is carried out and it is determined which date intervals have the least data gap, a monthly average of the variables of interest is made in the period from 2001 to 2010. , and from this average monthly interpolations of the maximum temperature, minimum temperature and precipitation are made and the data are exported in a multi-temporal grid format or NCDF. For these interpolations, two types of interpolation are carried out, which are *Warfare* and *Spline nearest*.

Statistical measurements of scalar pressure such as MSE, RMSE, MAE and MAD are carried out to compare the precision of both interpolation methods during the 12 months of the year. This analysis will help identify patterns, generation of a high spatial resolution grid database and possible improvements in the application of statistics in the Puno region, this by identifying of the interpolation technique with greater precision compared to the others. (Wilks, 2011).

II. GOALS

General Objective:

- Analyze scalar precision statistics comparing the kriging interpolation method and the Spline method.

Specific Objectives:

- Homogenize the data obtained from the SENAMHI stations for more precise calculations
- Perform Spline and Kriging interpolation methods for precipitation, temperature and relative humidity in Puno and surrounding areas, using data from SENAMHI and ERA-5 for 2001-2010.
- Calculate MSE, RMSE, MAE and MAD for each method and variable, generating maps to visualize the spatial distribution of the error.

III. THEORETICAL FRAMEWORK

Homogenization

The homogenization Data collection is a process based on the analysis of climate trends and variability due to the inhomogeneities that may exist in long-term climate data series. These inhomogeneities can arise from changes in station locations, site exposure, instrumentation, observers, and observing procedures. The homogenization seeks to identify and adjust for these non-climatic variations to ensure that the data reflect accurately the actual weather conditions. This process is essential to validate and improve the quality of climate change analyses, ensuring that data is consistent and reliable over time. Tools such as the "CLIMATOL" software and R packages are used for this purpose, facilitating the correction and filling of missing data, and allowing rigorous control of the quality and homogeneity of the data.

Interpolation Warfare

Kriging is an advanced geostatistical interpolation method used to estimate values at unsampled locations from a sparse set of points with known (z) values. Unlike other interpolation methods that simply calculate values based on physical proximity or mathematical relationships, Kriging incorporates a statistical model that considers spatial autocorrelation between sample points. This autocorrelation reflects how z values vary with distance and direction between points, which is essential for capturing complex spatial patterns in natural and environmental phenomena.

The Kriging method involves several key steps. First, an exploratory statistical analysis of the data is performed to understand the spatial autocorrelation structure. This involves creating variograms, which are graphs that show how the semivariance between points changes with distance. From these variograms, a mathematical model is fitted that describes the spatial relationship of the data. This variogram modeling is crucial because it provides information about the spatial dependence of z values, thus allowing more accurate predictions to be made at unsampled locations.

Once the variogram model is adjusted, predictions are made using the weights derived from the adjusted variogram. These weights are more sophisticated than other interpolation methods, as they not only consider the distance between points, but also the overall spatial structure of the data. This ensures that the predictions are more accurate and that the interpolated surface better captures the spatial variability of the studied phenomenon. In summary, Kriging is a powerful method that not only generates interpolated surfaces, but also provides estimates of the uncertainty associated with those predictions, which is essential in scientific and environmental applications where precision and confidence in the results are crucial.

Spline Interpolation

Spline interpolation is a method that uses mathematical functions to estimate values between input points, minimizing the overall curvature of the interpolated surface. This approach produces smooth surfaces that pass exactly through the data points and are continuous in first derivatives. The technique is based on the minimization of the total curvature of the surface, ensuring that the second derivative of the surface is minimal.

There are two main types of spline: regularized and tension. The regularized spline incorporates third derivative terms in the minimization to obtain smoother surfaces. The weight parameter controls the influence of these terms, with higher values being appropriate to obtain very smooth surfaces. On the other hand, the stress spline uses first derivative terms in the minimization, allowing the stiffness of the interpolated surface to be adjusted.

Additionally, other parameters such as the number of points used in the calculation of each interpolated cell can be adjusted, which affects the smoothness and processing time of the output surface. In summary, spline interpolation is a powerful tool for generating smooth surfaces from discrete data, being useful in applications such as modeling elevations, water levels, and contaminant concentrations.

Mean Square Error MSE

MSE is defined as the mean of and_t^2, that is, the average of the errors between the estimator and what is estimated squared, its formula is:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2$$

Where n is the number of samples and $\widehat{and}_t$ is the estimate of and_t. From the formula it is deduced that the loss function of the measurement is the quadratic or mean squared error. This is a measure that relates the variance and bias of an estimator; a lower MSE indicates a better fit to the real data. By squaring differences, negative and positive data do not cancel out and larger errors are given more weight, which is useful in some contexts.

However, the MSE does not share units with the data or the estimator and varies between 0 and infinity. Its disadvantages include its sensitivity to outliers, which can make it unreliable, and its difficulty in interpretation due to different units. Furthermore, if the MSE is greater than the variance of the explanatory variable, including it in a forecasting model can worsen the results, as noted by Ashley (1983, 1988).

Root Mean Square Error RMSE

RMSE is defined as the square root of the mean squared errors, its formula is:

$$RMSE = \sqrt[2]{MSE} = \sqrt[2]{\frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2}$$

Where n is the number of samples, $\widehat{and}_t$ corresponds to the observed values and_t and this is modeled in values at time or place t. The loss function of the mean is the quadratic or mean squared error.

This measure measures the difference between the values predicted by a model and those observed, being useful to evaluate the predictive capacity of the model. RMSE is preferred over MAE when a Gaussian error distribution is expected, according to Chai and Draxler (2014), and differs from MSE in that the result is in the original units of the data. Although it is useful to impress due to its complexity and in contexts with a quadratic loss function, its use should be avoided in the presence of outliers, as these strongly influence the measurement. Armstrong (2001) and Willmott and Matsuura (2005) criticize its sensitivity to outliers and its potential to be a misleading indicator of average model performance. Furthermore, error variance can vary over time due to nonlinearities and variation in exogenous variables (Woschnagg & Cipan, 2004).

Mean Absolute Error MAE

MAE is defined as the average magnitude of a forecasting exercise without taking into account its sign, that is, the average of the absolute values of the calculated errors, its formula is:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|y_t - \hat{y}_t|$$

Where n is the number of samples and $\widehat{and}_t$ is the estimate of and_t. The loss function of the measurement is that of the absolute error.

It is a measure that provides a directly interpretable value, since the loss is in the same units of the output variable and assigns the same weight to all errors, being suitable for uniformly distributed errors. Willmott and Matsuura (2005) consider it a natural measure of error, and other metrics such as RMSE are derived from it. Its absolute nature makes it less sensitive to outliers and varies between 0 and infinity. However, most models assume normally distributed errors, which makes RMSE more suitable in such cases (Chai and Draxler, 2014). Additionally, the MAE may not discriminate well between model outputs and the inclusion of absolute values may complicate the assessment of model sensitivity.

Median Absolute Error MAD

MAD is defined as the measure of the errors of a forecasting exercise without taking into account their sign, that is, the median of the absolute values of the calculated errors. Its formula is:

$$MdAE = Mediana\ (|y_t - \hat{y}_t|)\ para\ t = 1 \dots n$$

Where n is the number of samples and $\widehat{and}_t$ is the estimate of and_t. The loss function of the measurement is that of the absolute error.

It is a measure of precision that depends on the scale of the data, being useful for comparing methods within the same data set, but not for those with different scales. Although the MAD is not affected by extreme values, this characteristic is also a limitation, as it does not fully use the available information about the errors, which, according to Swanson, Tayman, and Bryan (2011), is essential for any "robust" measure. ".

IV. DATA AND METHODS

Study area

The study area is the department of Puno, located in the southeastern region of Peru, covering an area of approximately 71,999 km². Puno presents a varied topography that includes highlands, mountains and a part of the Amazon rainforest, resulting in a diversity of microclimates. The climate varies between cold and warm, with average maximum temperatures of 22°C and minimum temperatures of 1.4°C. Rainfall is concentrated in four months (December to March), although it can vary, causing floods, droughts, frosts and hailstorms.

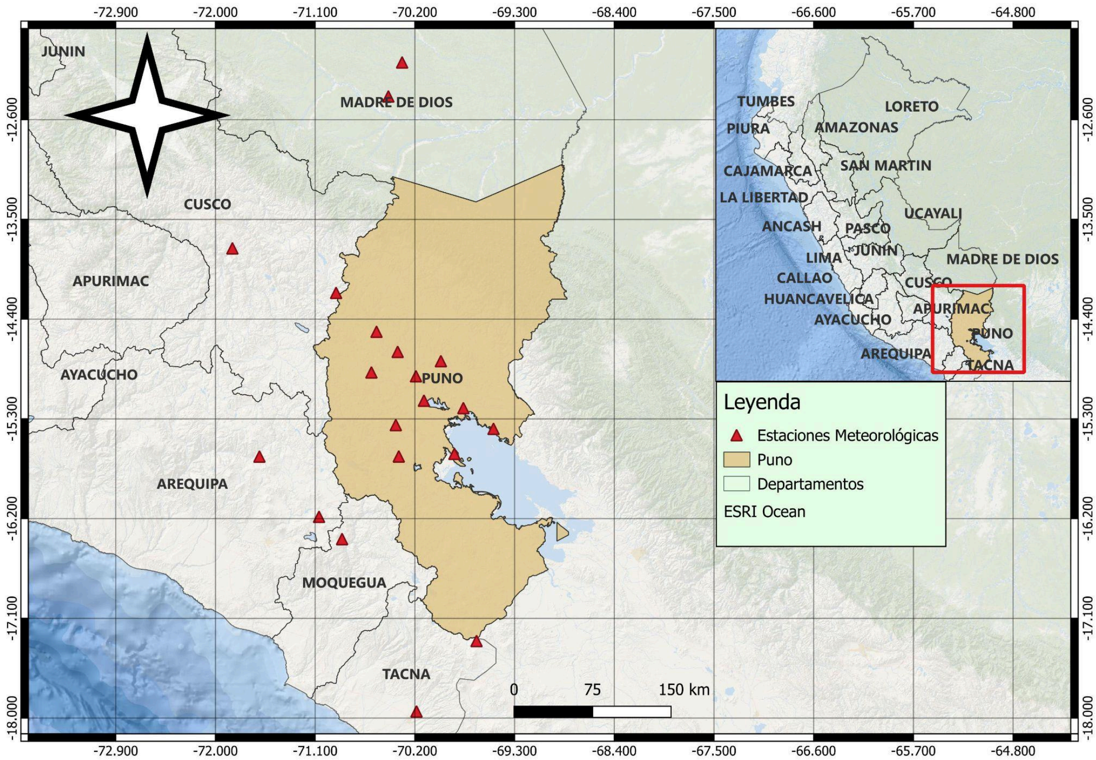


Figure 1. Location map of Puno

Source: Own elaboration

Data

Data from the following weather stations were used for the interpolation methods. The study period is 10 years, this interval is from 2000 to 2010.

Name	Length (°)	Latitude (°)	Altitude (masl)
AYAVIRI	-70.593	-14.881	3920
PROGRESS	-70.356	-14.695	3905
LAMP	-70.373	-15.356	3900
CABANILLAS	-70.346	-15.639	3890
AZANGARO	-70.191	-14.915	3863
ARAB	-70.119	-15.136	3920
EIGHT	-69.966	-14.779	4119
HUANCANE	-69.763	-15.203	3860
HUARAYAMOHO	-69.491	-15.39	3890
CAPACHICA	-69.844	-15.616	3819
ACOMAYO	-70.316	-12.083	3227
CCATCCA	-70.44	-12.39	3693
WE ARE BEYOND	-70.547	-14.514	3827
THEY HAVE	-70.912	-14.163	4445
PANTS	-70.186	-17.941	871
UBINAS	-70.858	-16.385	3491
CHUAPALCA	-68.356	-16.695	4177
PARURO	-71.848	-13.761	3047
CHIVAY	-71.604	-15.64	3661
THE FRIAR	-71.066	-16.184	4119

Methods

Homogenization Methodology

For homogenization, the R programming language was used through the RStudio IDE. First, the data files of the meteorological stations were imported, which include information on their location and data on meteorological variables such as maximum temperature, minimum temperature and precipitation from 2001 to 2010. Then, the stations corresponding to the area of study, in this case, stations in Puno and surrounding areas. Using the 'Climatol' libraries, the meteorological variables were homogenized.

Kriging and Spline Interpolation Methodology

To perform the Kriging and Spline interpolation, the data of the monthly averages were obtained for the period of years, two one-dimensional arrays were created with the data of the longitude and latitude coordinates and a two-dimensional array that presents the scalar field of the variables of interest. The variables of this scalar field are minimum temperature, maximum temperature and precipitation.

In order to do an interpolation quickly, for all the months and for the 3 variables, an iteration is done that allows us to perform the interpolation omitting the empty values or the months where there is no data, for these months a two-dimensional arrangement is made. with empty values. On the contrary, for the months in which there are values, interpolation is performed where the output data is a scalar field with the interpolated Kriging and Spline values.

It should be noted that for both methodologies the same spatial resolution has been used in the same spatial region; these coincide with the resolution of the data extracted from ERA5 and therefore it is not necessary to rescale the data.

Finally, in a *dataset* Separately, the average temperature is calculated using the average of the scalar fields of the maximum and minimum temperature, and the scalar field of relative humidity is calculated, where, for this, the Tetens equation is used to determine the water vapor pressure. and saturation pressure. For the saturation pressure, the average temperature is used and for the water vapor pressure, the minimum temperature is used.

Scalar precision measurement methodology

When both climatological data and interpolated data are available, it is crucial that they are in the same arrangement format. This implies that they must have the same spatial and temporal resolution. This uniformity is essential because these data sets are handled as matrices, allowing direct and comparative mathematical calculations to be carried out between them. This ensures coherence and precision in the analysis, facilitating the correct interpretation of climatic variations and their interpolated representations.

When arrays follow this format, it is easier to perform mathematical calculations to evaluate the errors described in the theoretical framework. The matrices represent a structured organization of the data, thus allowing the precise application of tests such as MSE (Mean Squared Error), RMSE (Root Mean Squared Error), MAE (Mean Absolute Error) and MAD (Median Absolute Deviation). This ordered structure not only simplifies the comparison between climatological data and their interpolations, but also ensures consistency in the evaluation of the accuracy and precision of the models used.

It is important to note that the distinction between observed data and predicted data is emphasized in the error descriptions. In this context, the interpolated data are considered as the observed values, while the climatological data are interpreted as the predicted values. However, this ordering does not significantly alter the calculations, since the absolute value of the difference between both sets of data is used. It is crucial to note that these differences are calculated between arrays containing corresponding values on the respective dates.

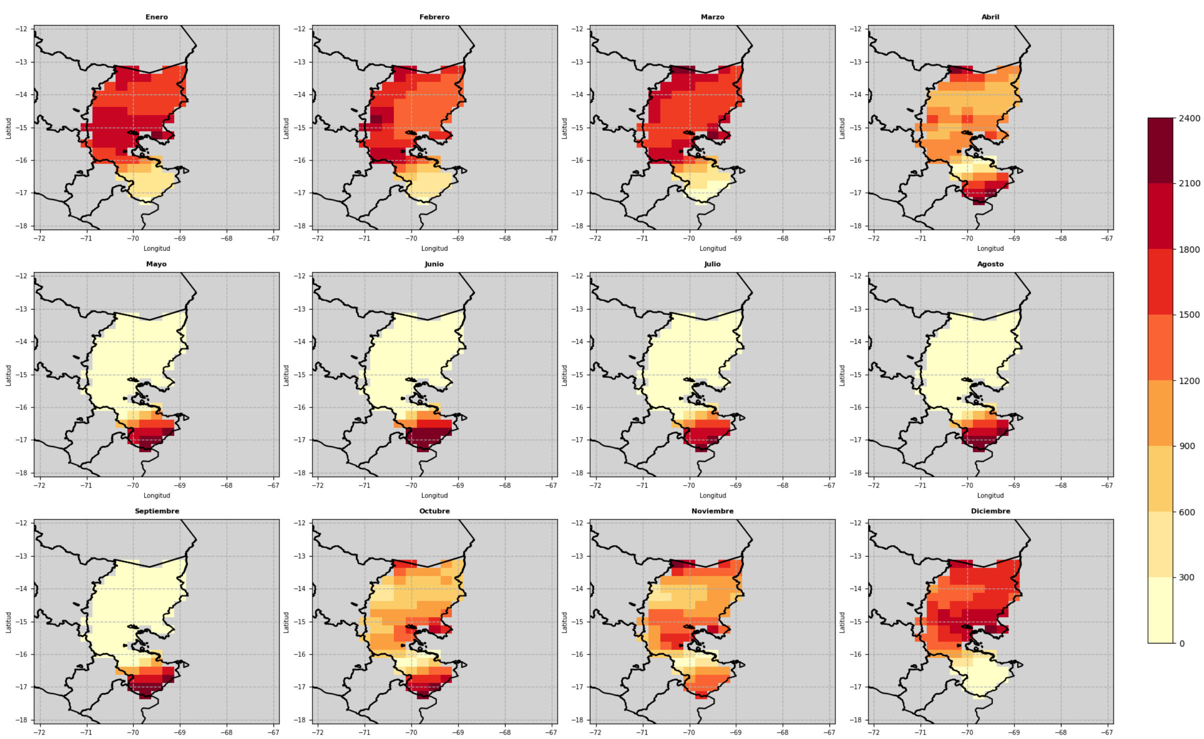
V. RESULTS AND DISCUSSIONS

A. Errors between Kriging interpolation and climatology.

Mean square error (MSE)

Figure 1

Mean square error (MSE) between climatology and kriging interpolation for mean temperature.

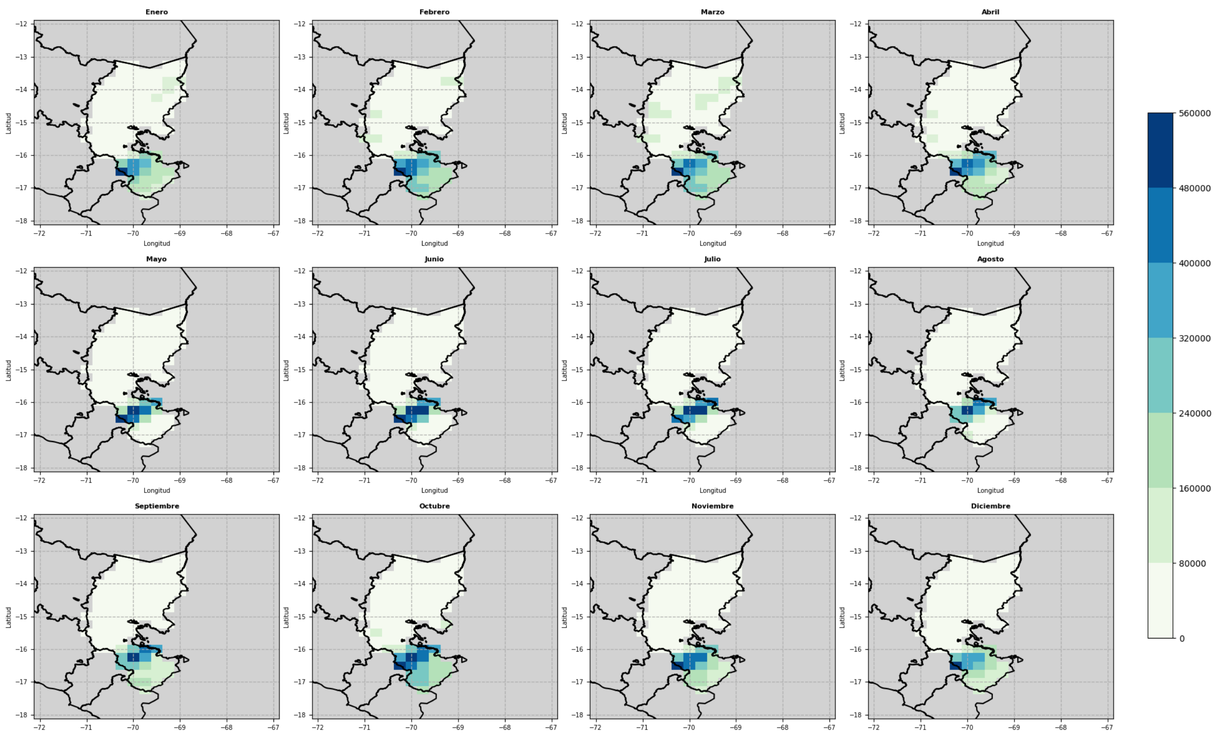


Fountain: Own elaboration

Two periods can be distinguished in the year: the dry season and the wet season. According to Figure 1, the areas with the highest degree of error are found in the north, and to a lesser extent, in the south. However, this fact only applies to the wet season months, since the opposite occurs during the dry season.

Figure 2

Mean square error (MSE) between climatology and kriging interpolation for precipitation.

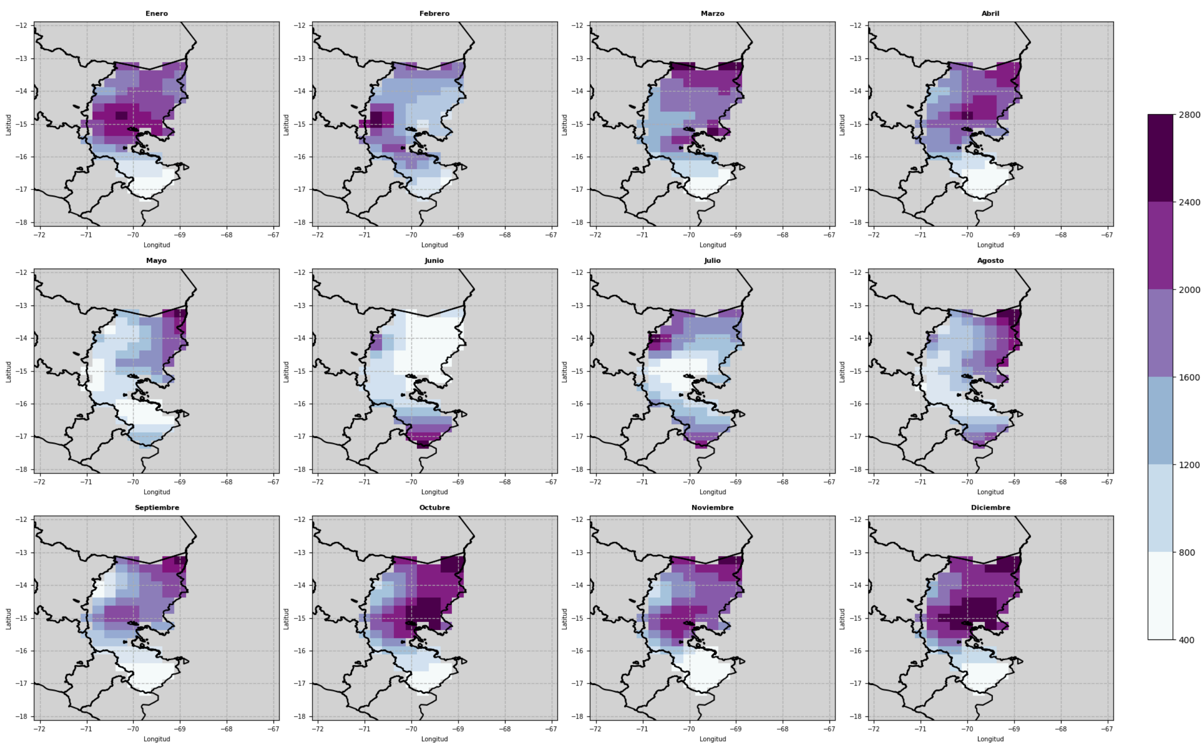


Fountain: Own elaboration

As can be seen in Figure 2, the mean squared errors can be seen in all stations, where at the level of their spatial distribution, it can be identified that a greater error appears in the wet season before the dry season. However, in the dry season there is an excessive increase in the MSE error, this value is highly accentuated in the southwestern area of Lake Titicaca, a value that is too high suggests that it is not possible to rely on this in the use of a model, since it can worsen the prognosis rather than help. A possible cause is that for said area there is high variability or occurrence of extremes as well as high geographic or climatic variability. This assumption should be corroborated with alternative field data.

Figure 3

Mean square error (MSE) between climatology and kriging interpolation for relative humidity.



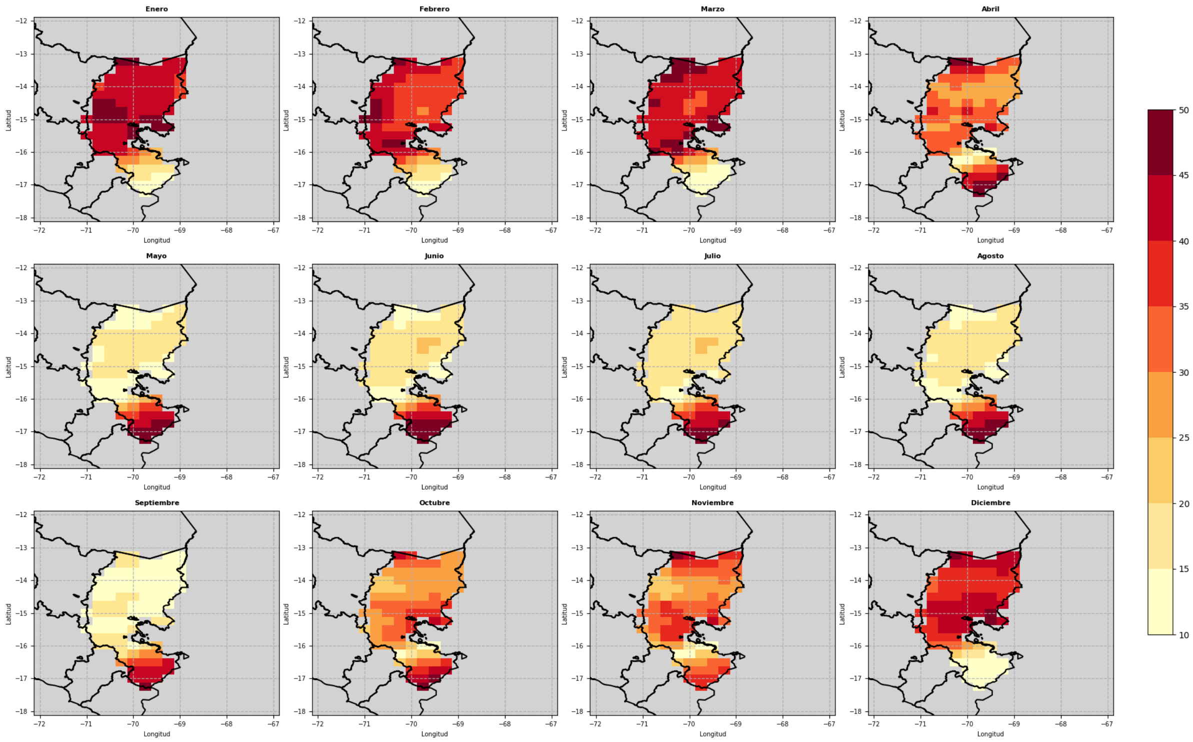
Fountain: Own elaboration

According to Figure 3, the highest concentration and the highest values of errors are found in the northern part, while the southern regions present a lower degree of error. This trend remains constant during most months. However, in the month of June, this observation is not true.

Square root mean square error (RMSE)

Figure 4

Root mean square error (RMSE) between climatology and kriging interpolation for mean temperature.



Fountain: Own elaboration

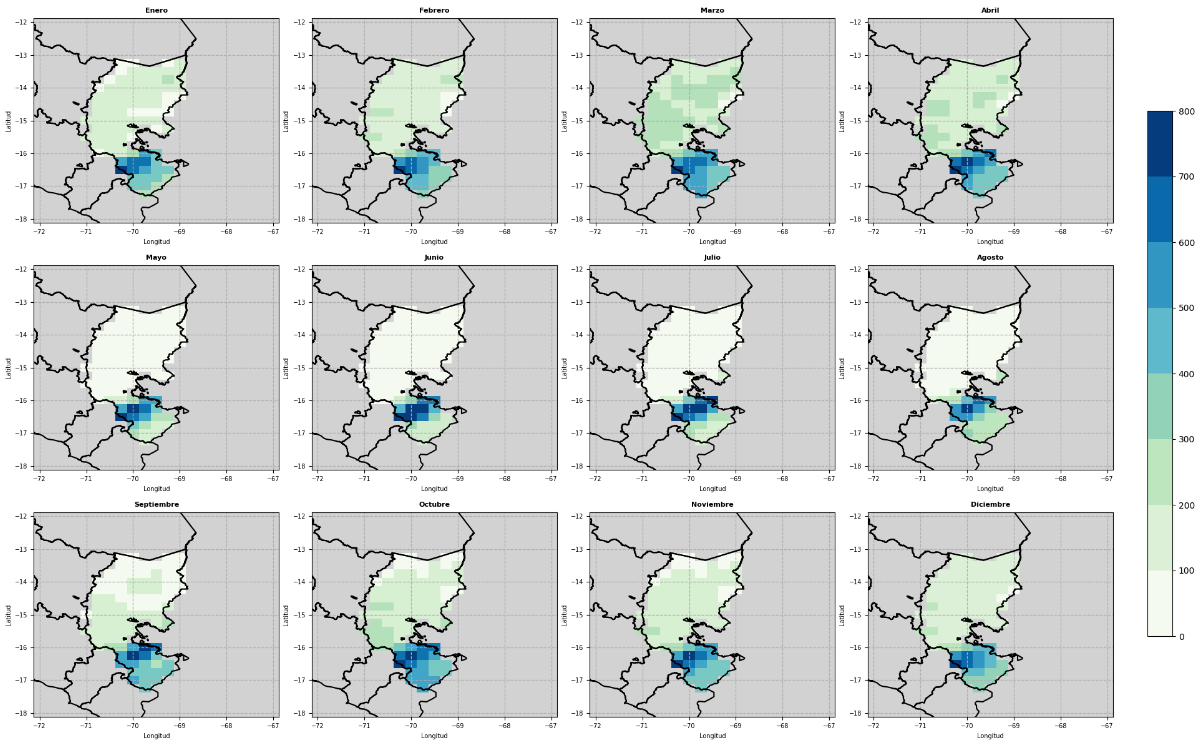
As in Figure 1, which shows the MSE error, the RMSE reflects the same quadratic relationship of this parameter. Therefore, the same conclusion is reached about the relationship of error levels between the northern and southern zones.

As can be seen in Figure 5, it can be noted that there is a low error value for the dry or winter season, while there are high values for a summer or humid season. An increase in the error can be seen southwest of Lake Titicaca, this suggests that there is high variability or presence of outliers. This spatial configuration is also repeated for the MAE and the MAD.

The RMSE measures the difference between the predicted and observed values in the model. However, this indicator is mostly used for Gaussian distributions. Since a precipitation distribution is not considered Gaussian or normal, but rather a positive symmetric distribution, this can influence a larger amount of error.

Figure 5

Root mean square error (RMSE) between climatology and kriging interpolation for precipitation.

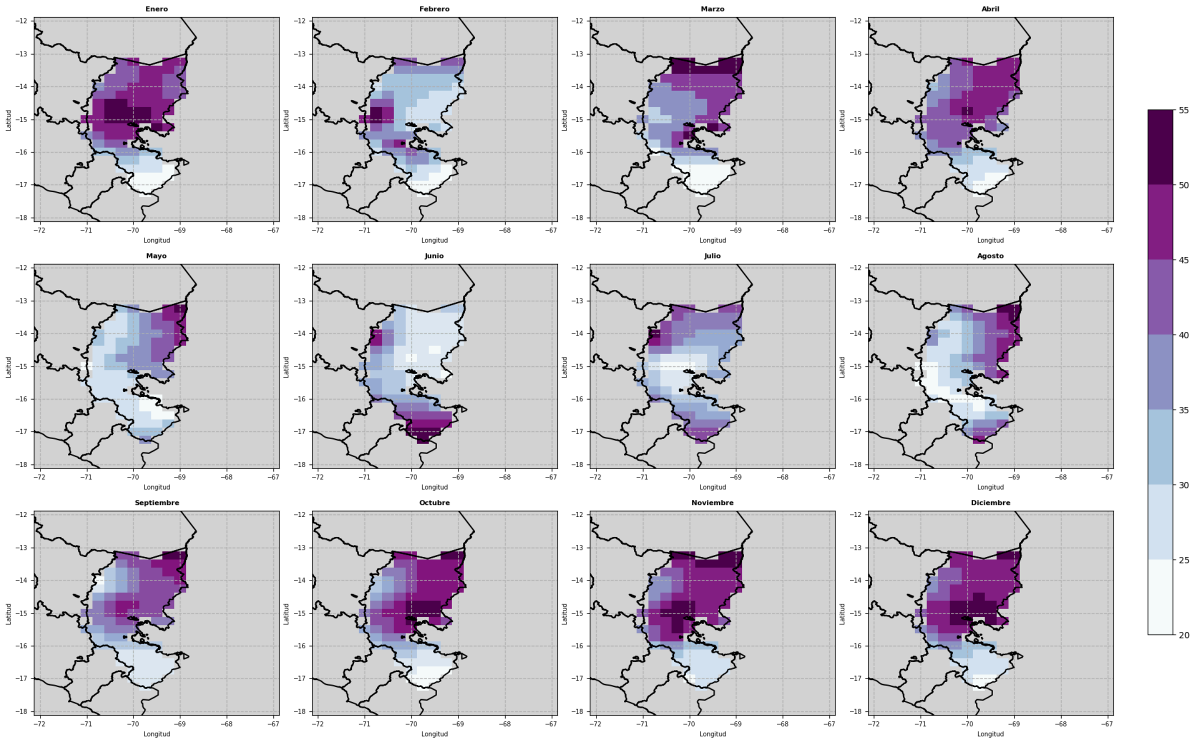


Fountain: Own elaboration

The highest error values are found in the southern area. While, the lowest values are found in the northern part.

Figure 6

Root mean square error (RMSE) between climatology and kriging interpolation for relative humidity.



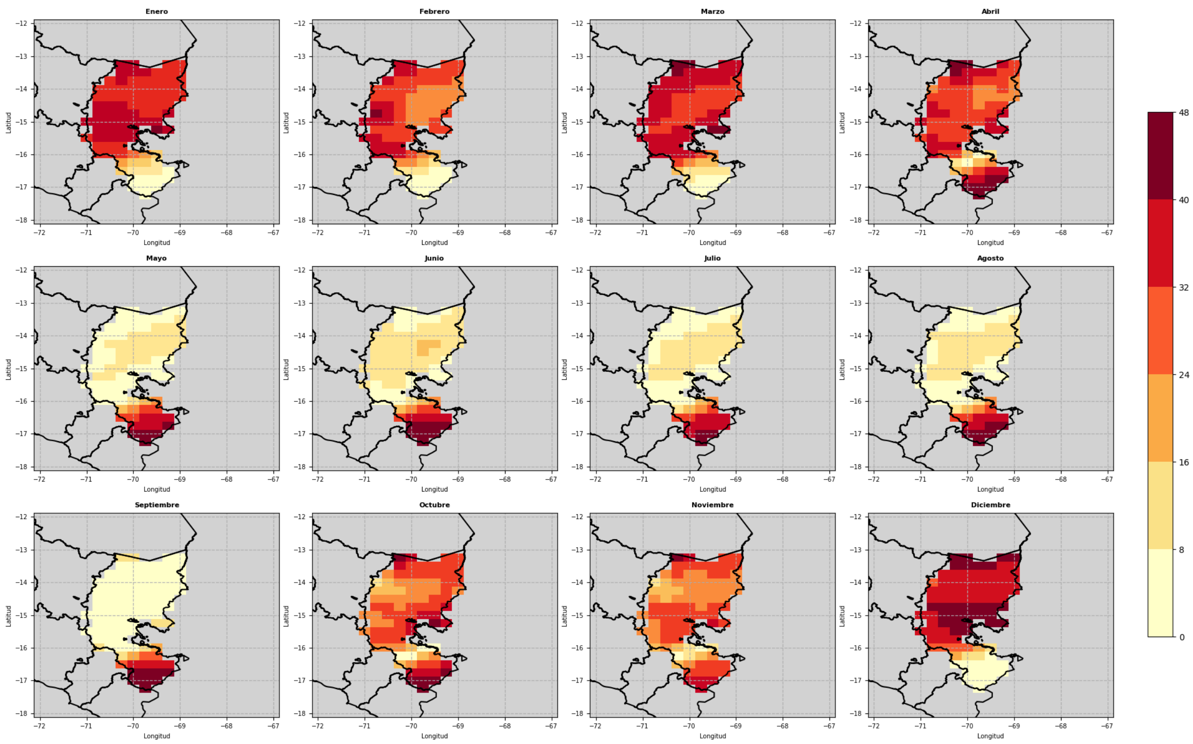
Fountain: Own elaboration

The highest error values are found in the northern part of the department; this fact remains constant in all months, except in June when the opposite occurs.

Mean absolute error (MAE)

Figure 7

Mean absolute error (MAE) between climatology and kriging interpolation for mean temperature.



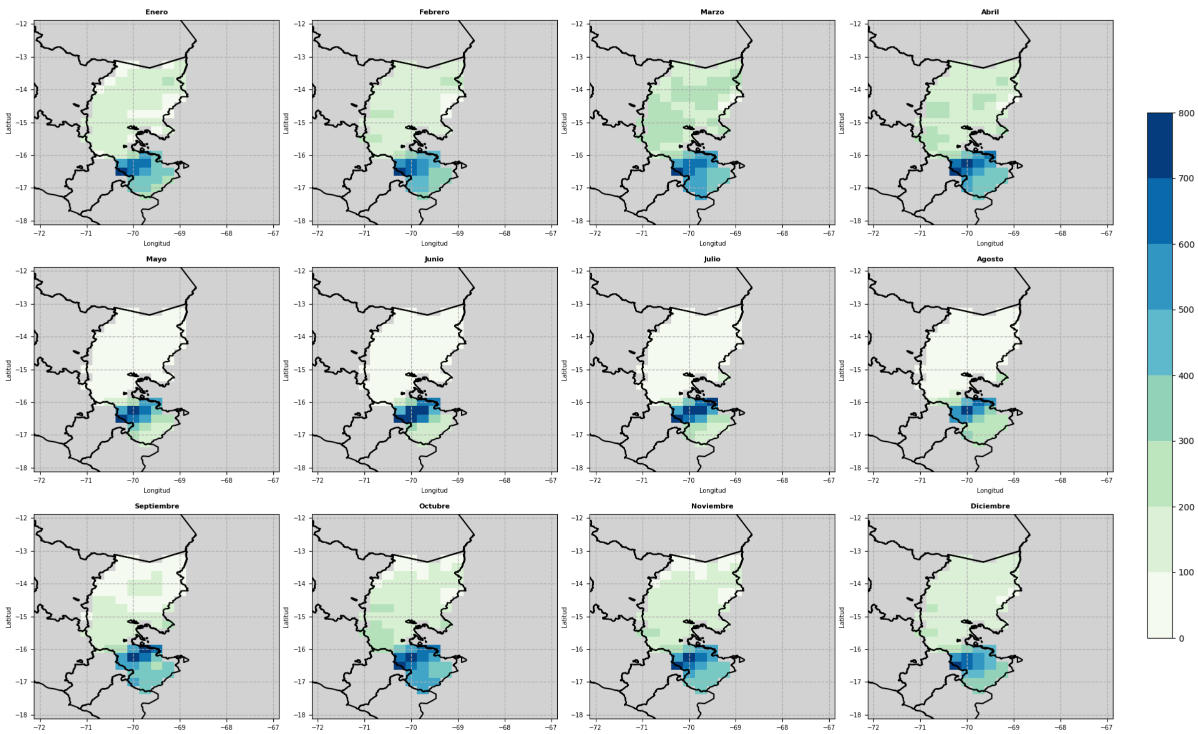
Fountain: Own elaboration

The error level in the temperature error figures (MSE and RMSE) shows a very similar monthly distribution. During the wet season, higher error values are observed in the northern region compared to the south. In contrast, during the dry season, this trend is reversed.

As can be seen in Figure 8, the MAE varies according to the seasonality of each month, with the wet season months being the months in which a higher MAE value is present; while, in the dry season months, a lower MAE value may be shown. This means that for the months of the dry season it can be represented much better. However, in every month an increase in the scalar measurement of precision can be seen in the southeastern part of Puno and the southwestern part of Lake Titicaca. According to the percentage of data that these stations located in the southeast have, they have uninterrupted data from 2000 to 2010. It is suggested that this increase may be possible because these stations did not follow correct quality control when processing the information. or because the Calana station is at a lower altitude compared to the others, this could have caused differences in precipitation.

Figure 8

Mean absolute error (MAE) between climatology and kriging interpolation for precipitation.

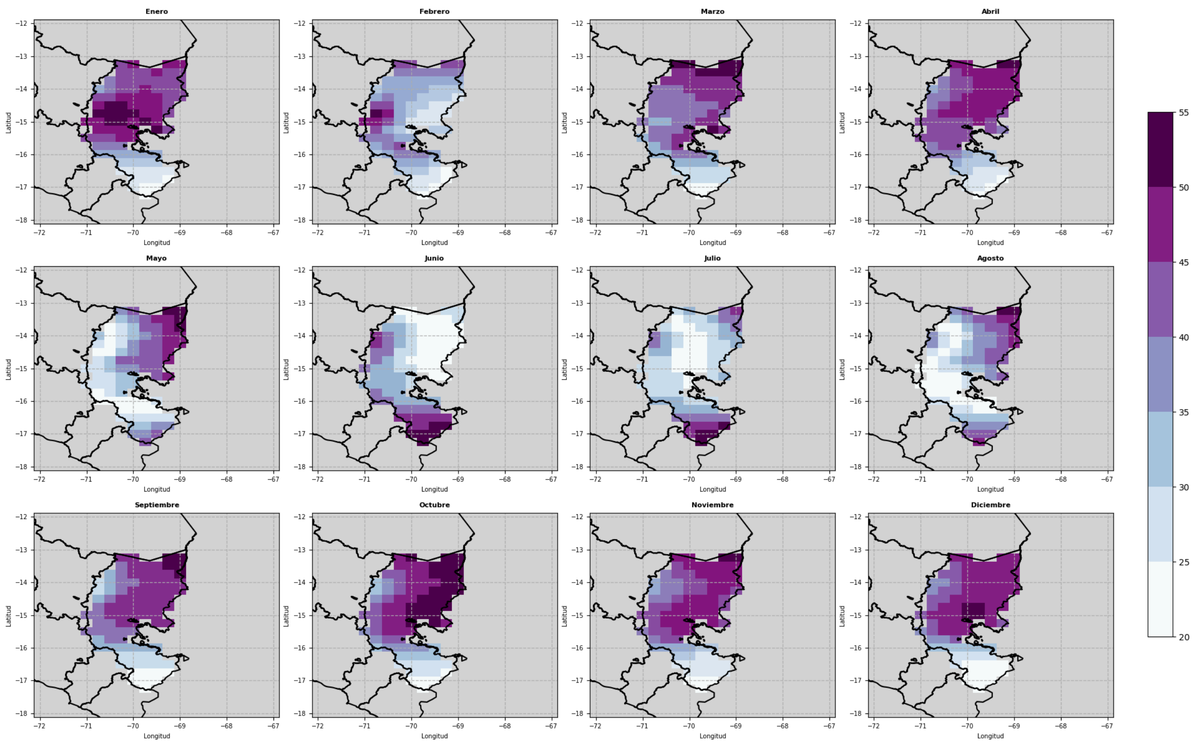


Fountain: Own elaboration

The error level is high in the southern region, while in the northern regions the error level is low.

Figure 9

Mean absolute error (MAE) between climatology and kriging interpolation for relative humidity.



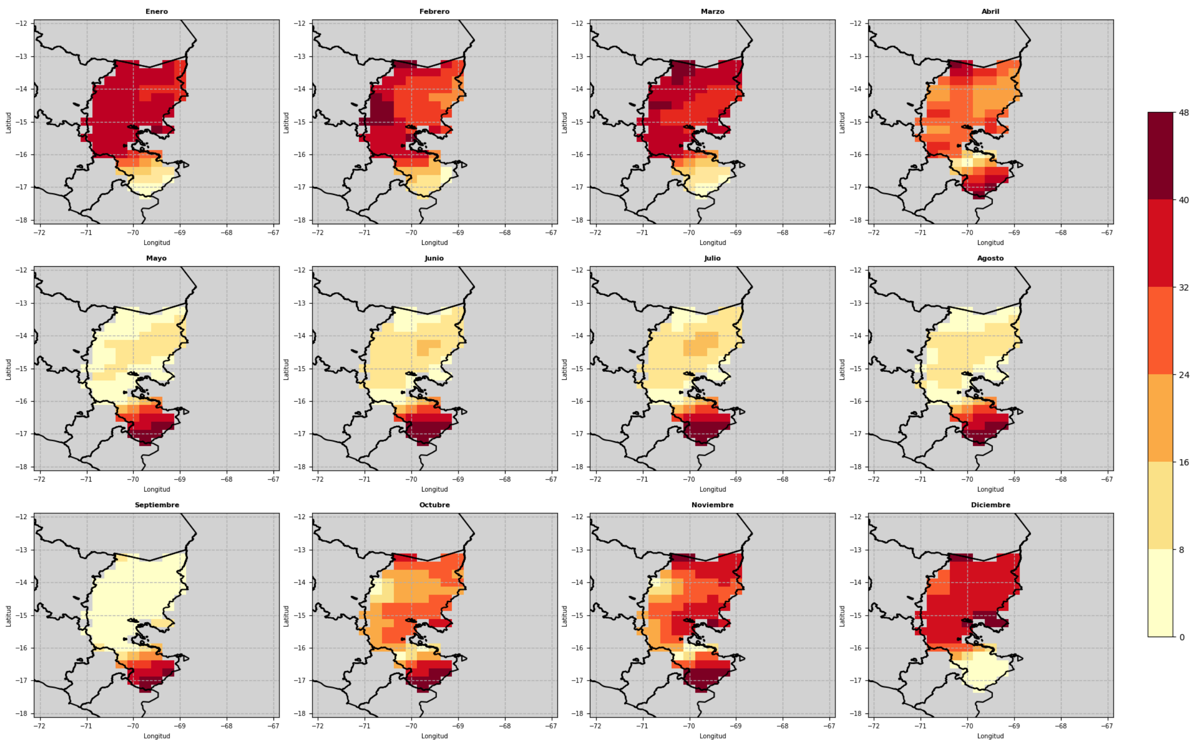
Fountain: Own elaboration

The pattern of errors remains similar to the previous sections. However, it is notable how this fact becomes more evident in some months, especially in July and August.

Median absolute error (MAD).

Figure 10

Median absolute error (MAD) between climatology and kriging interpolation for mean temperature.

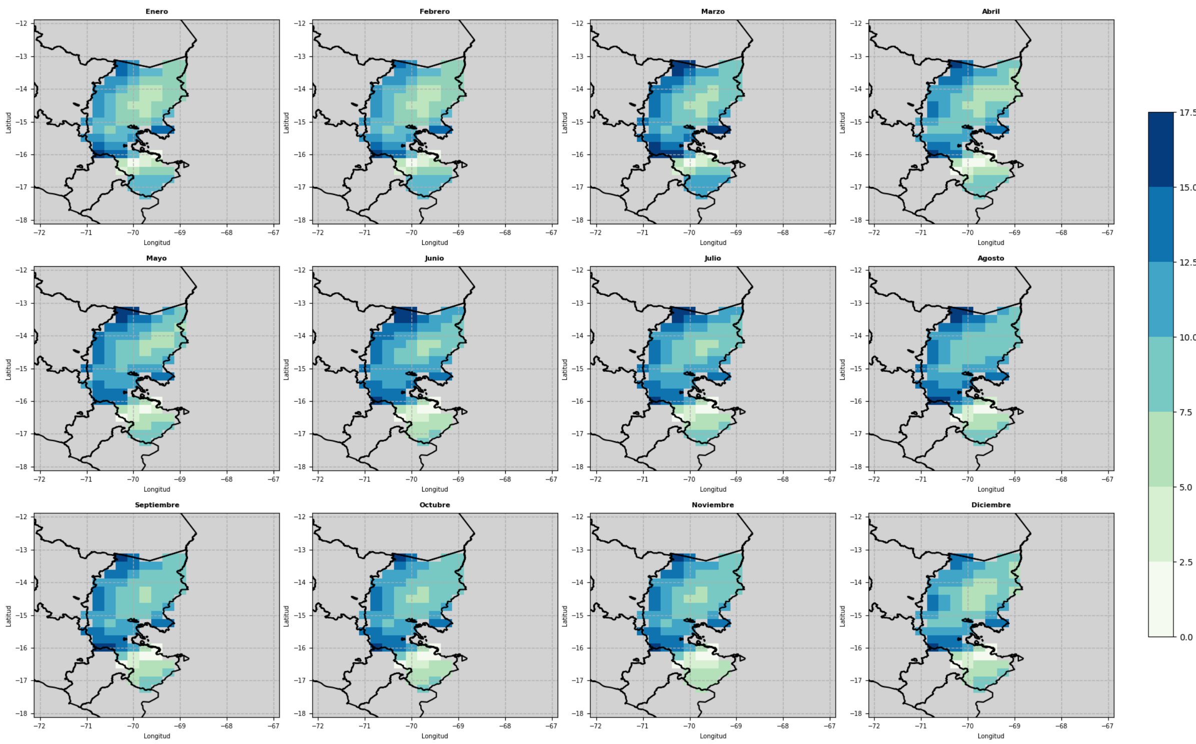


Fountain: Own elaboration

The value of the median errors is maintained consistently in both the wet and dry seasons.

Figure 11

Median absolute error (MAD) between climatology and kriging interpolation for precipitation.



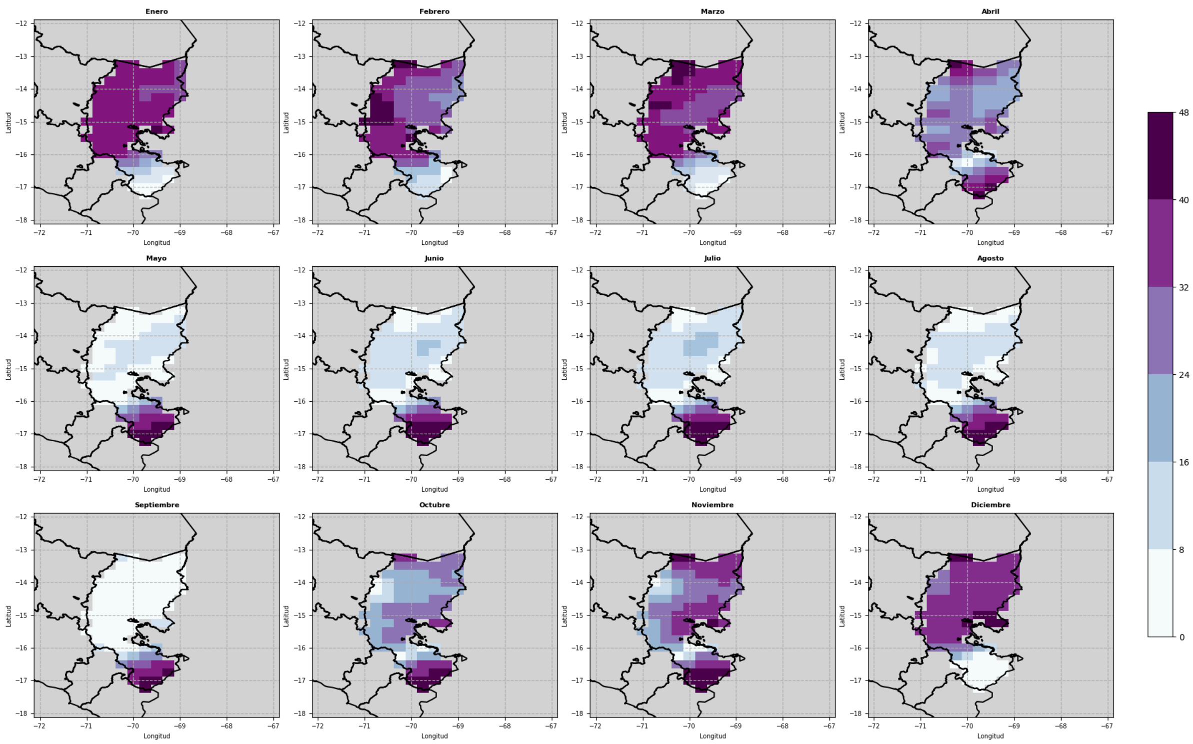
Fountain: Own elaboration

As shown in Figure 11, the MAD error measure is presented, as can be seen it has lower errors for the wet season months, while the highest values are shown for the dry season. According to the definition of mad, it only uses the median of the absolute values, however, by only considering the median within the dispersion measure, extreme values are excluded. These extreme values are characteristic throughout Puno, since being a mountain area, the high orographic and intraseasonal variability can cause extreme precipitation in some areas.

The error levels for precipitation in Figure 11 are maintained almost uniformly for all areas of Puno.

Figure 12

Median absolute error (MAD) between climatology and kriging interpolation for relative humidity.



Fountain: Own elaboration

During the wet season months, the highest error values are concentrated in the northern part, while for the dry season the opposite occurs: the highest error values are concentrated in the southern regions.

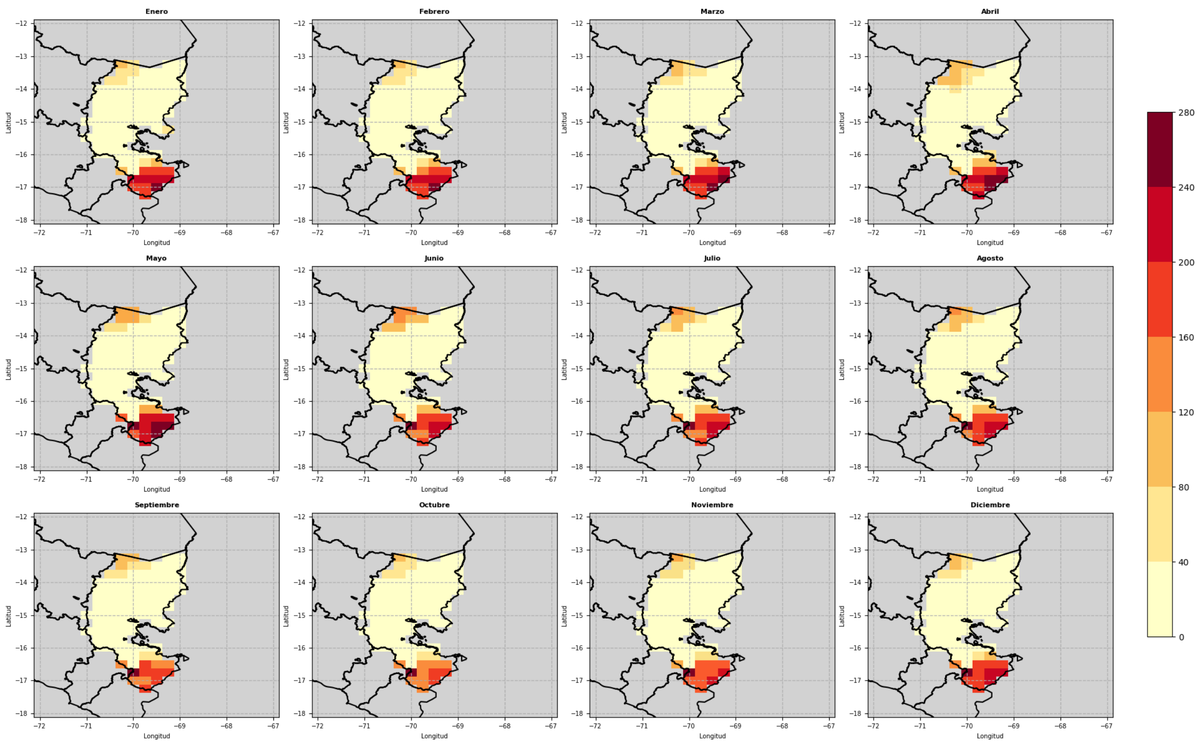
B. Errors between Spline interpolation and climatology.

Mean square error (MSE)

According to figure 13 and making a comparison with figure 1. It is possible to distinguish that the level of error in the figures are very different. Since the error presented in the MSE with the spline method is much smaller. So you show that the spline methodology ends up being better.

Figure 13

Mean square error (MSE) between climatology and spline interpolation for mean temperature.

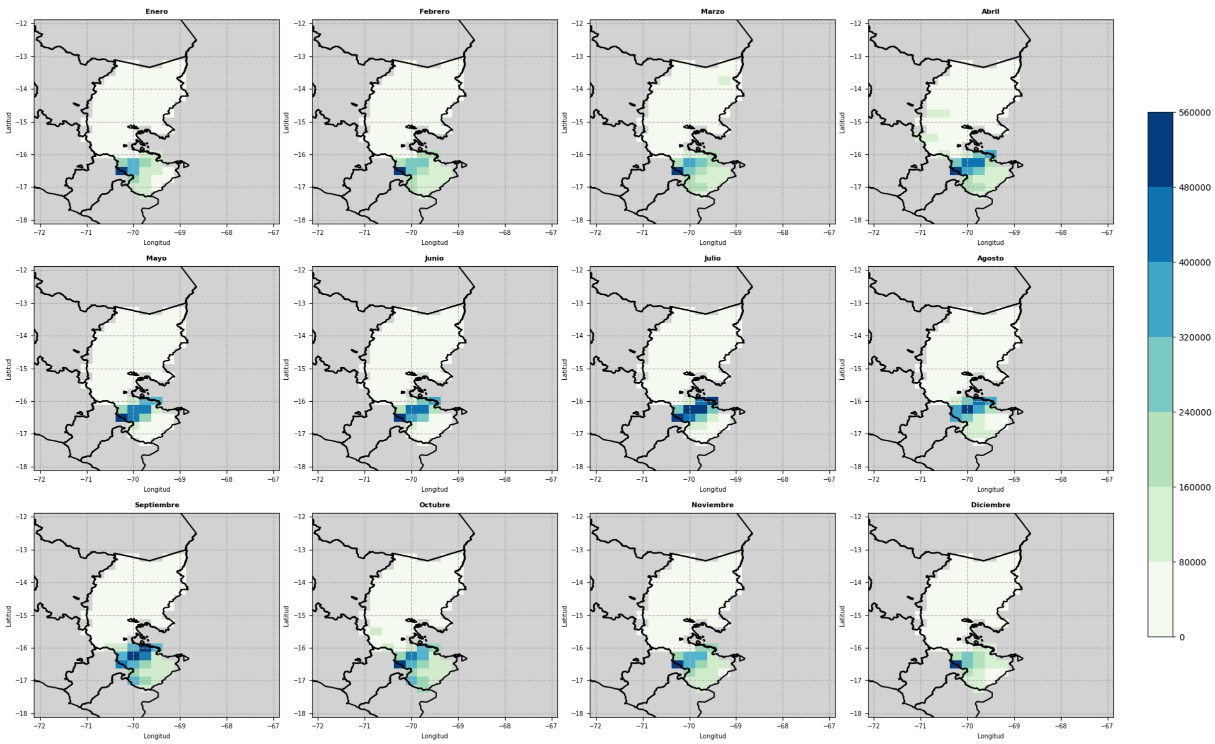


Fountain: Own elaboration

As can be seen in Figure 14, for the MSE error for spline interpolation, when compared with kriging interpolation, it follows a similar monthly pattern. Where the error statistic is accentuated again in the summer months or wet season and an increase in the value in the southwest area of Titicaca, as has already been shown with the spatial distribution of the RSME, this may be due to the fact that there is a high variability or presence of extremes in which it is necessary to verify the orographic and climatic condition with a greater amount of in situ data.

Figure 14

Mean square error (MSE) between climatology and spline interpolation for precipitation.



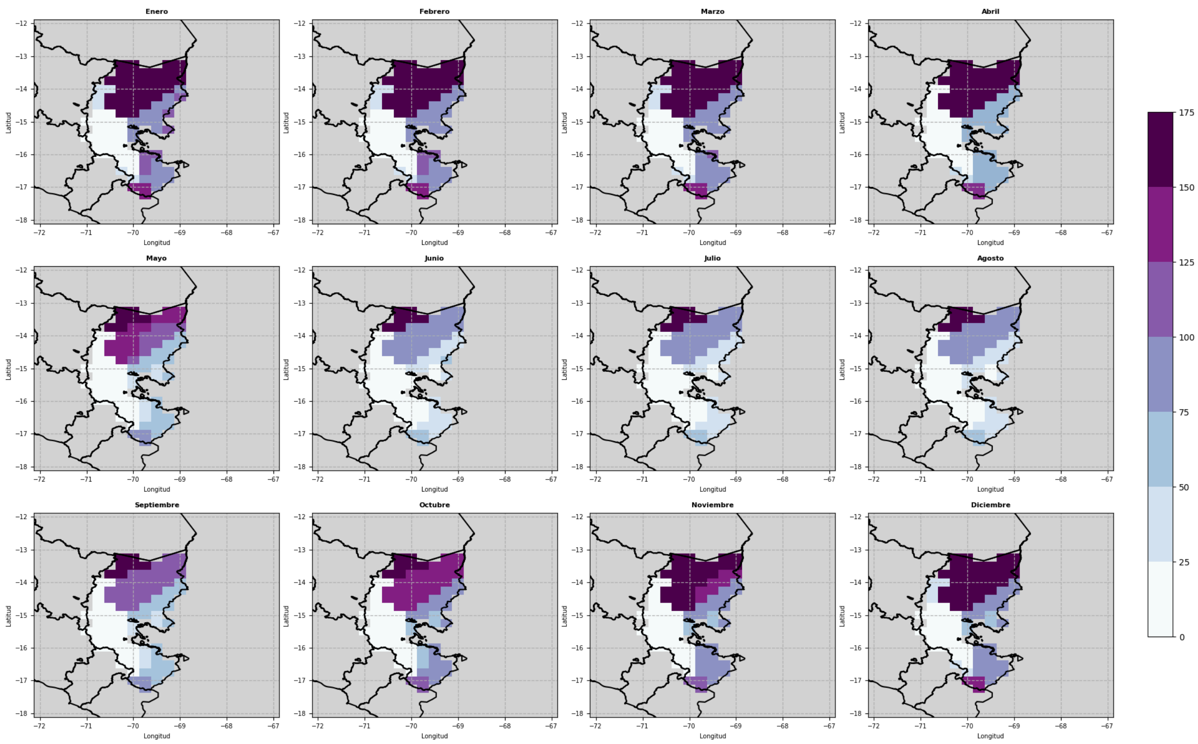
Fountain: Own elaboration

The error level in Figure 15, which represents the MSE obtained with the spline method, shows a much smaller bias than that of the kriging method. However, the distribution of these errors in different areas is completely different.

While with the kriging method the errors occur more uniformly throughout the entire region, in the case of the spline, the highest errors are concentrated in the northern area of the department, while in the southern areas almost none. errors occur.

Figure 15

Mean square error (MSE) between climatology and spline interpolation for relative humidity.



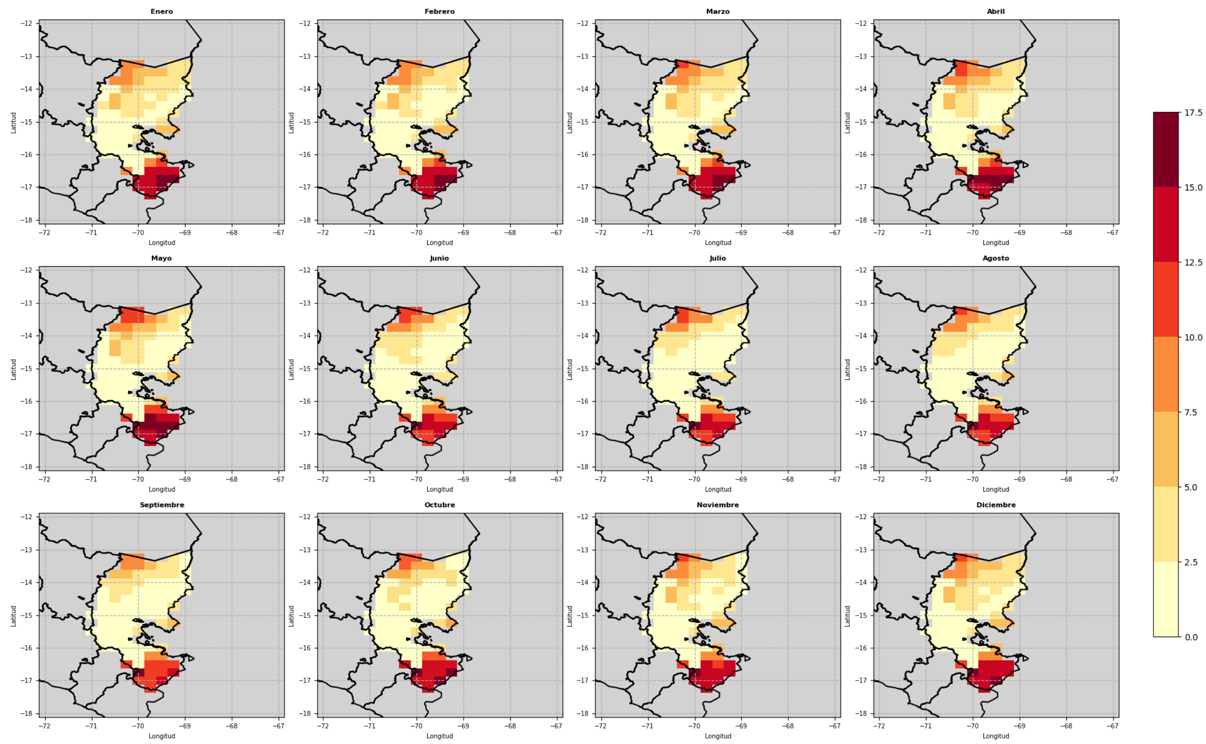
Fountain: Own elaboration

Square root mean square error (RMSE)

The RMSE, representing the square root of the MSE, shows a distribution of the data and a level of proportion in the difference between the errors of the MSE and the RMSE that are similar. This leads to the conclusion that the level of error with the spline method is much lower than with the kriging method. This can be concluded from what is shown in figure 16.

Figure 16

Root mean square error (RMSE) between climatology and spline interpolation for mean temperature.

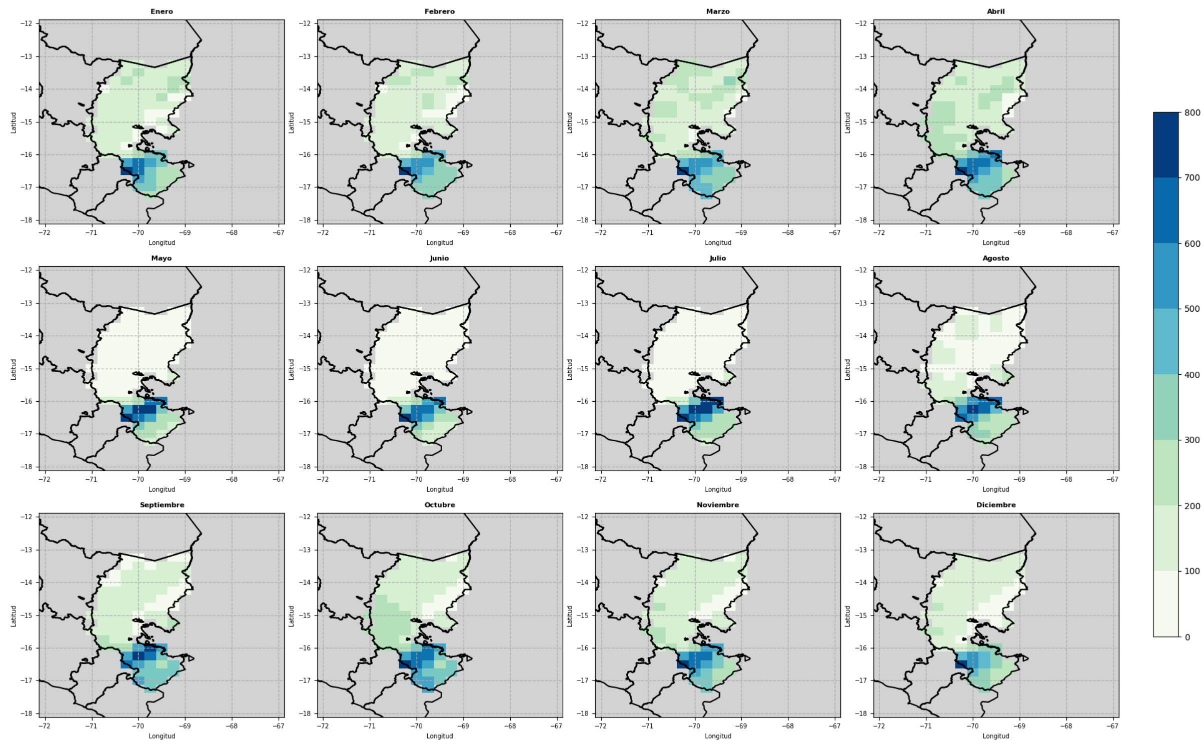


Fountain: Own elaboration

As can be seen in Figure 17, the mean square error obtains higher values in the months of the wet season than in the months of the dry season; it follows a similar pattern when compared to the error of kriging interpolation. Also, a high error appears in the southwest area of Lake Titicaca, this could be due to the fact that in this area there is a high sensitivity to atypical values or extreme values of precipitation. This may be characteristic of the area due to the great orographic and climatic difference throughout the study area.

Figure 17

Root mean square error (RMSE) between climatology and spline interpolation for precipitation.

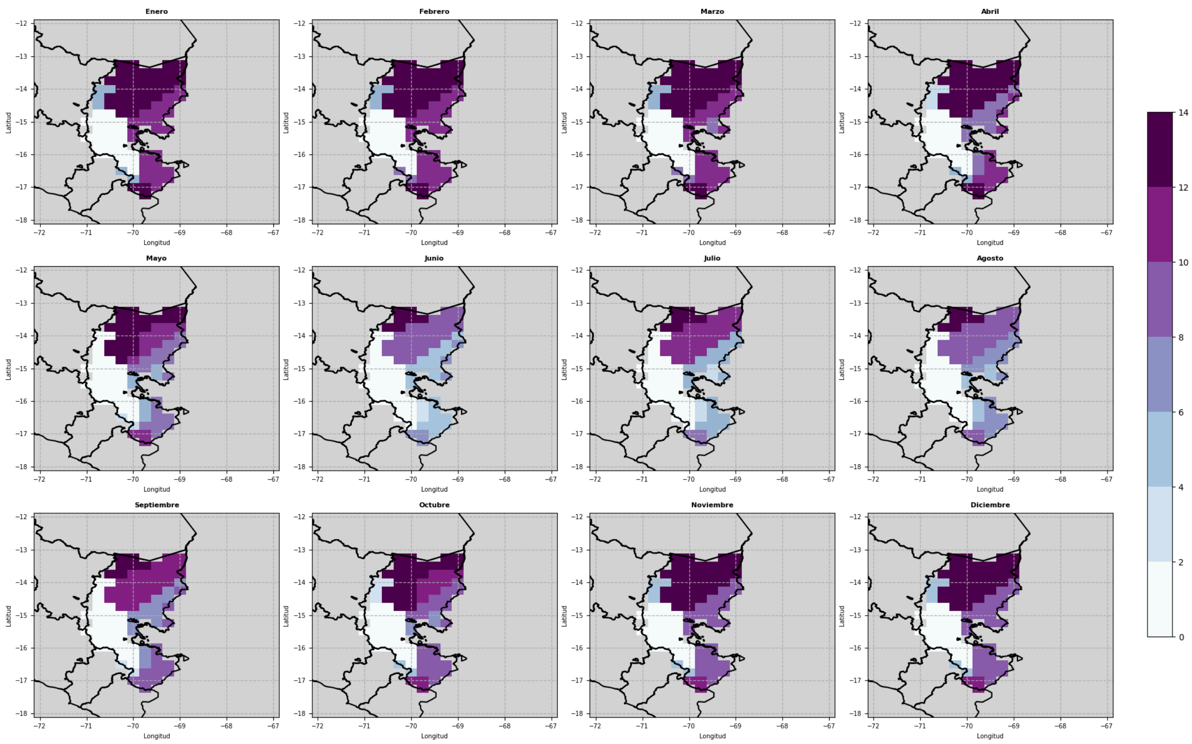


Fountain: Own elaboration

As in the case of temperature, Figure 18 shows the root of the MSE, which allows us to observe both the proportion in the distribution of errors in the department of Puno and the level of error in comparison with the kriging method.

Figure 18

Root mean square error (RMSE) between climatology and spline interpolation for relative humidity.



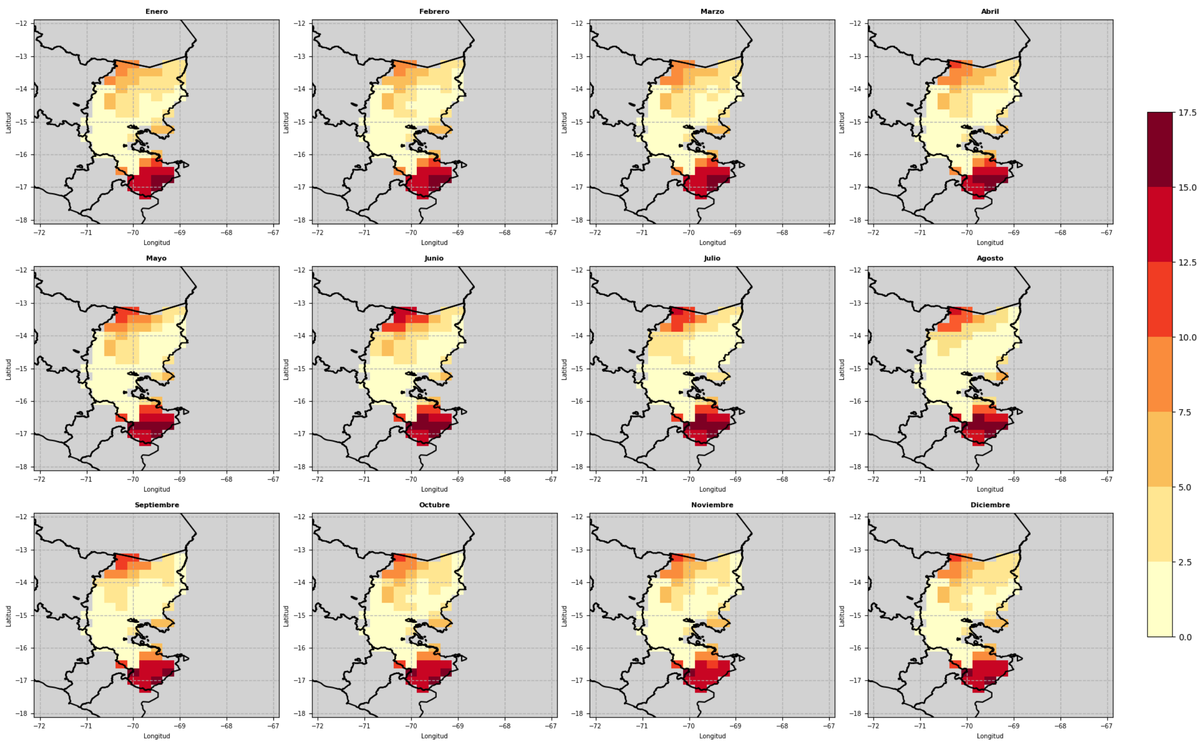
Fountain: Own elaboration

Mean absolute error (MAE)

The level of error is comparable to that shown in the previous figures, reflecting consistency in the results obtained. And, from what is presented and comparing the errors, the spline ends up having a better margin of error. Shown in figure 19.

Figure 19

Mean absolute error (MAE) between climatology and spline interpolation for mean temperature.

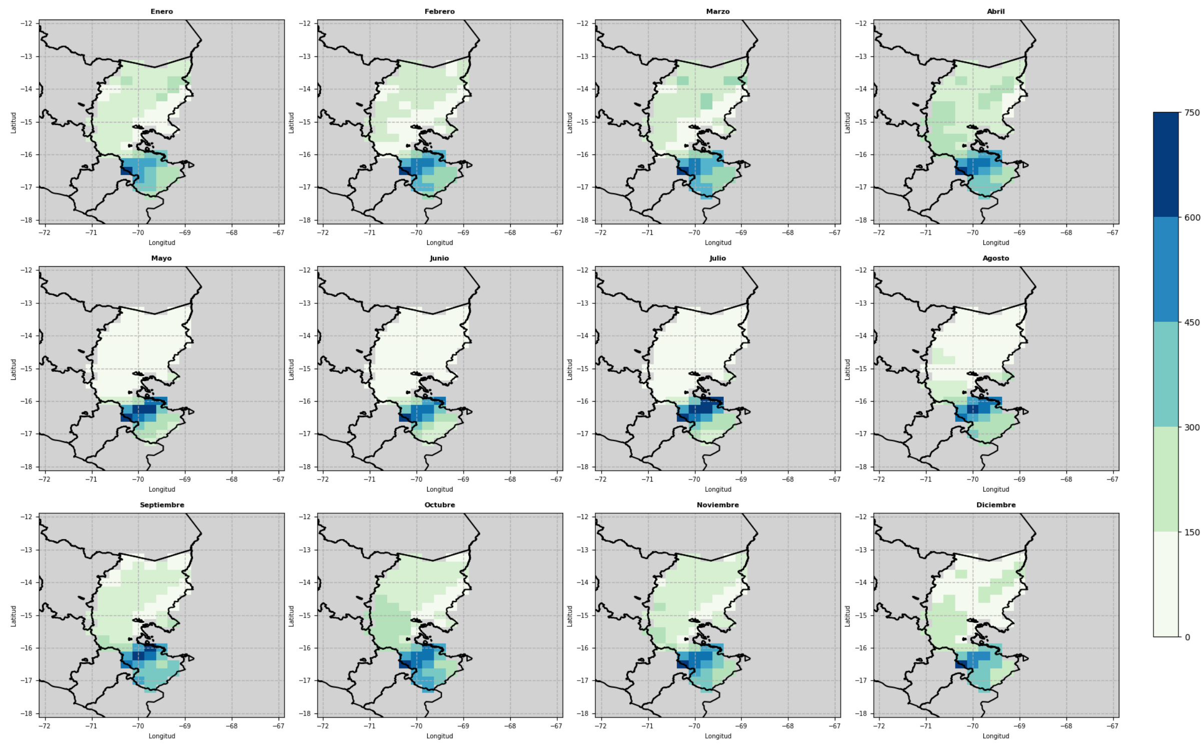


Fountain: Own elaboration

As can be seen in Figure 20, this figure follows the same pattern of the MAE for the kriging interpolation method. The highest values are found southeast of Puno and northwest of Lake Titicaca. The lowest values are found in the dry season and the highest values are found in the wet season, this is because in wet seasons there is a less normalized distribution than in a dry season which can influence the statistical measurement of precision.

Figure 20

Mean absolute error (MAE) between climatology and spline interpolation for precipitation.

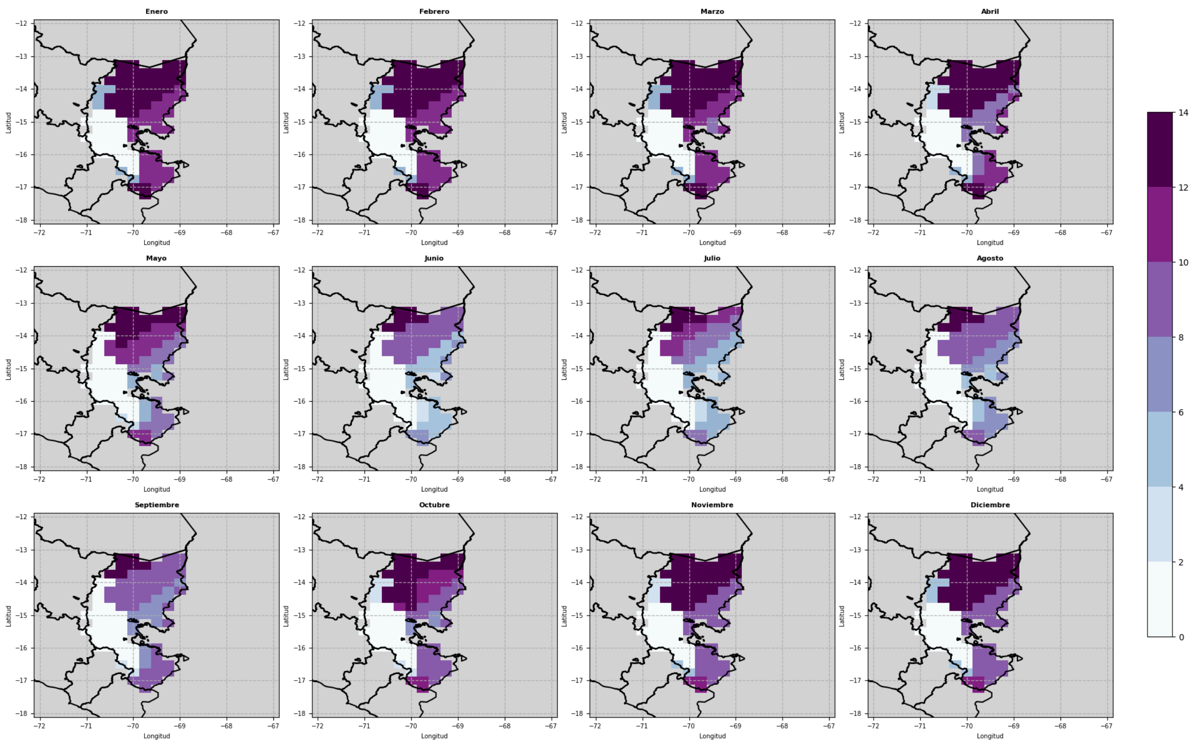


Fountain: Own elaboration

As illustrated in Figure 21, most of the errors are concentrated in the north and south, particularly on the eastern side of the department of Puno. In contrast, the western zone has a significantly smaller margin of error. This observation is consistent with other methodologies used, but further highlights the effectiveness of the spline method, which shows a much lower magnitude of errors compared to the kriging method. This reduction in error levels underscores the precision and reliability of the spline method, positioning it as a superior tool for data analysis and interpretation in this region.

Figure 21

Mean absolute error (MAE) between climatology and spline interpolation for relative humidity.



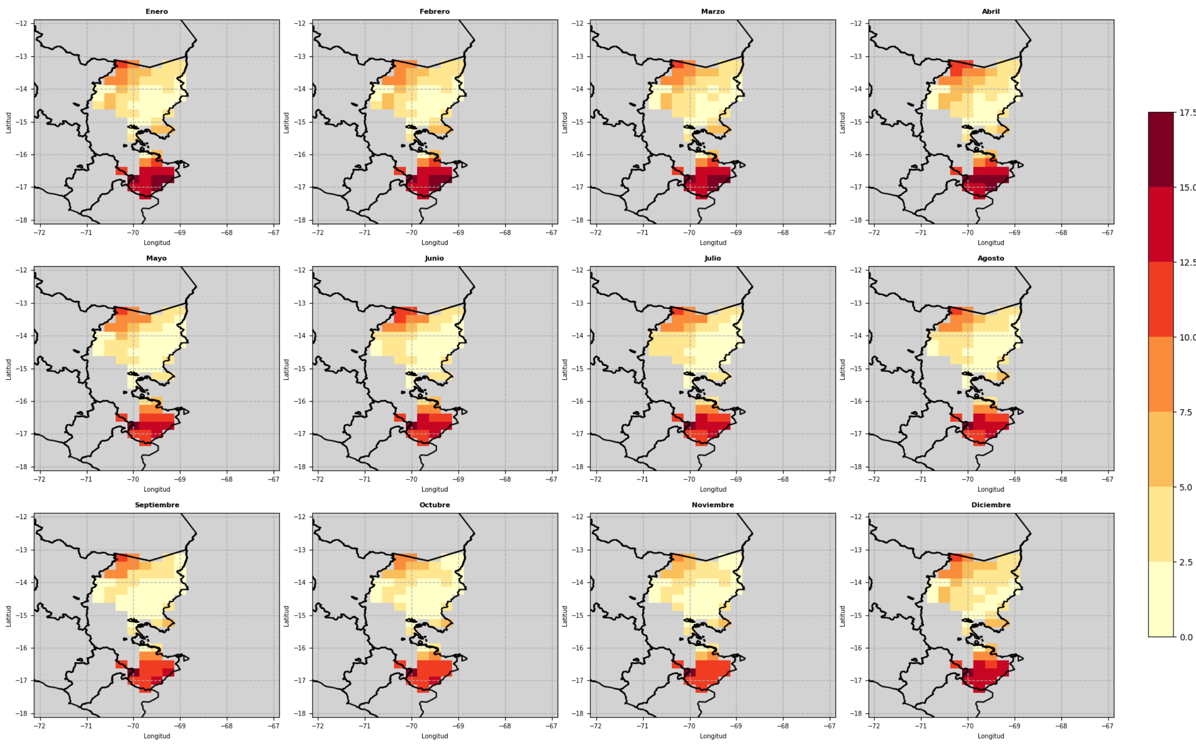
Fountain: Own elaboration

Median absolute error (MAD).

The average value of the mean in Figures 22, 23 and 24 reveals a notable consistency with the results obtained using the MAE, underlining a significant consistency in the measurements. This agreement not only reinforces the validity of the methods used, but also provides a robust basis for the evaluation and comparison of the models. This level of precision is crucial to more deeply understand the variations and trends present in the data, thus facilitating informed decision making in future studies and practical applications.

Figure 22

Median absolute error (MAD) between climatology and spline interpolation for mean temperature.

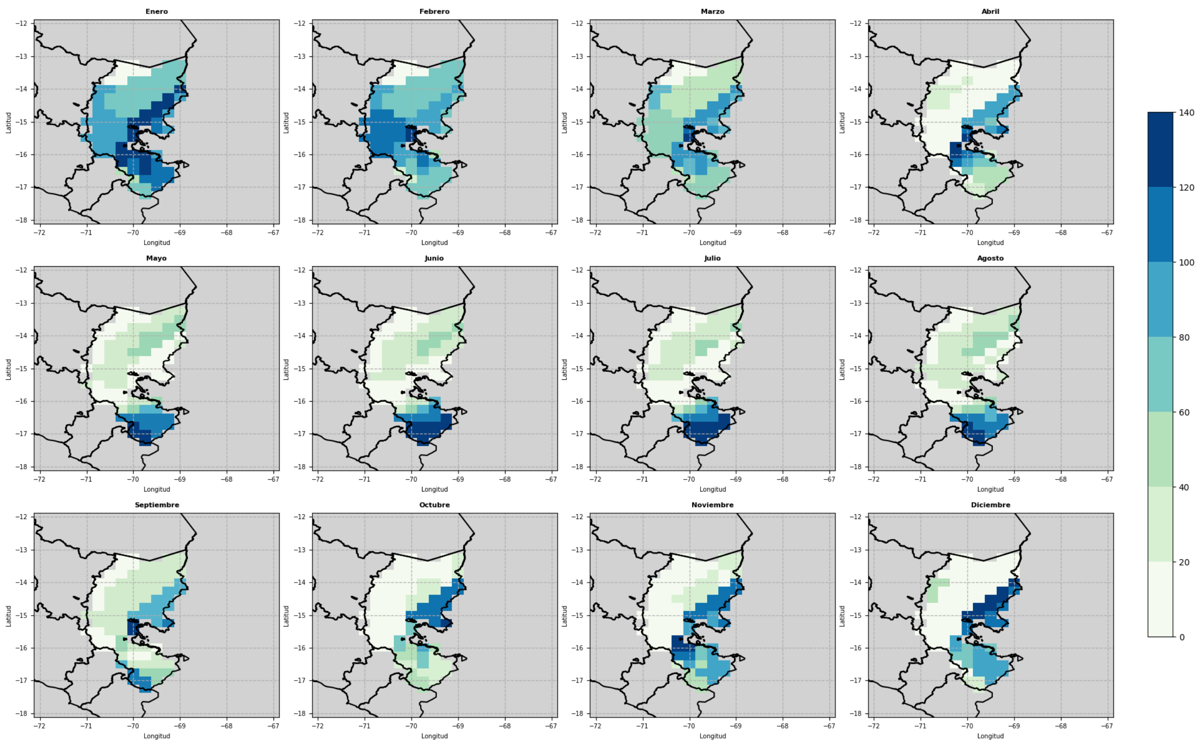


Fountain: Own elaboration

In Figure 23, if compared with the results proposed by the Kriging interpolation method, a decrease in the absolute median error can be verified for the months of the dry season, however, the MAD values are still high during the months of the dry season. months of the wet season. In almost every month of the year an increase in MAD can be identified in the southwestern part of Titicaca and in the eastern part of Puno. These places suggest high variability due to their intrinsic geographical and climatological characteristics.

Figure 23

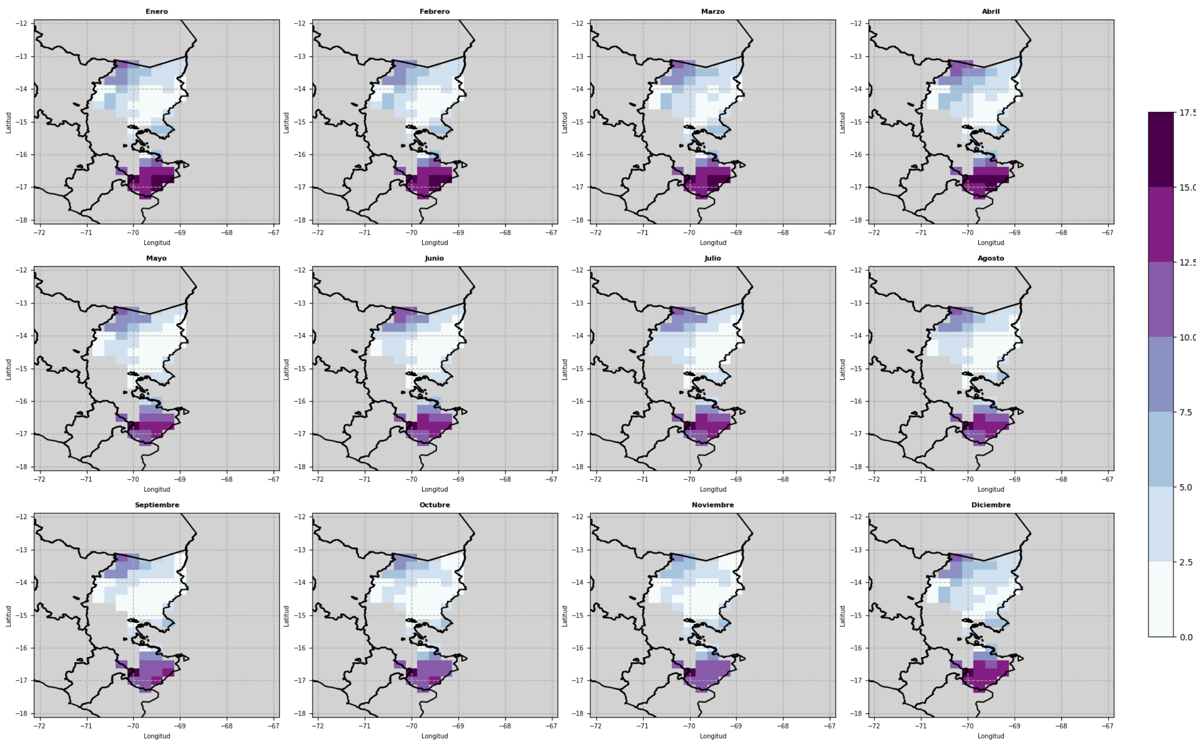
Median absolute error (MAD) between climatology and spline interpolation for precipitation.



Fountain: Own elaboration

Figure 24

Median absolute error (MAD) between climatology and spline interpolation for relative humidity.



Fountain: Own elaboration

VI. CONCLUSIONS

- It was possible to analyze the scalar precision statistics and it was found that in most of these the kriging interpolation method presented higher values compared to the spline method.
- It was possible to homogenize using free packages such as Climatol which completed the missing data and eliminate the *outliers* Of these, this allowed an improvement in terms of data quality as well as in the performance of interpolations.
- From the data cleaning, it was possible to calculate monthly averages during the period from 2000 to 2010 and the monthly climatology of the multidimensional ERA5 reanalysis product.

- Spline and Kriging interpolation methods were carried out using precipitation, maximum temperature and minimum temperature as scalar fields, using open libraries such as Parkridge and Scipy.
- The MSE, RMSE, MAE and MAD could be calculated for each method and variable, generating maps to visualize the spatial distribution of the error, different spatial configurations were obtained, positive values can be explained by the presence of high variability or occurrence of extremes. within the study area.

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VIII. ATTACHMENTS

The codes used can be found in the following link, along with the information obtained from me.using different methods such as those downloaded from other sources: https://drive.google.com/drive/folders/1icWaTP1VAvuQhhbOlreaYpfn-akA4bGL?usp=sharing