Hybrid Optimization in Photoacoustic Image Reconstruction: Integrating NSGA-III and Tikhonov Regularization

by

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Author Qualifications

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Master of Science

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Abstract

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> Finishing a PhD is like moving a mountain. Luckily, you only have to move one rock at a time.
>
> (Reddit somewhere: pg. ??)

Acknowledgements

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Contribution statement

Declaration by author

(this section is presently incomplete)

Supervisors

Title. Supervisor One[1], Title. Supervisor Two[2], Title. Supervisor Three[1], and Title. Supervisor Four[3,4]

[1]A *long* affiliation (e.g., School)
allows for three (e.g., Faculty)
lines to be specified (e.g., Institution)

[2]A *short* affiliation (e.g., Department)
only has two (e.g., Company)

[3]A supervisor may have
multiple affiliations

[4]An affiliation may be used by
multiple supervisors

List of publications

(this section is presently incomplete)

Publications included in this thesis

(to be completed)

Submitted manuscripts included in this thesis

(to be completed)

Other publications during candidature

(to be completed)

Contents

List of Figures

List of Tables

Chapter 1

Introduction

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1.1 Photoacoustic Imaging

The generation of images using the photoacoustic effect has been established as a valuable non-invasive technique for investigating the composition and structure of various materials. This phenomenon occurs when a photo-absorbing material interacts with light pulses, generating increases in temperature and mechanical pressure that propagate as photoacoustic signals. These signals are detected by ultrasonic transducers, enabling the creation of photoacoustic images, which are particularly relevant in biomedical applications such as the assessment of arterial structures, tissue oxygenation monitoring, gastrointestinal endoscopies, molecular imaging, and in vivo brain studies in experimental animals.

The reconstruction of photoacoustic images is carried out by capturing and analyzing the photoacoustic waves recorded by sensors placed on the surface of the investigated material. Among the most widely used reconstruction methods are Time Reversal, Delay and Sum, and Regularization techniques.

This study focuses on an innovative approach that combines an exploratory and multi-objective perspective with evolutionary algorithms. This methodology allows for the exploration of diverse solutions without the need to determine an optimal regularization term, and by carefully selecting objectives, it facilitates the reconciliation of different constraints for image reconstruction.

1.1.1 Research Questions

This research will investigate Lorem ipsum dolor sit amet, consectetuer adipiscing elit.. Specifically, this research will provide a foundation for Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis.. From this foundation, Curabitur dictum gravida mauris..

The research question is as follows:

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RQ3: *Curabitur dictum gravida mauris.*

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1.1.2 Contributions

This research provides the following contributions to knowledge:

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1.2 Image Reconstruction

1.3 Multi-Objective Optimization

1.4 Overview

The format of this thesis is by publication, so I will be also attempting to publish this research as the following papers:

P1: This is the full title of my first paper (RQ1; C1)

P2: This is where I would put my second paper ***if I had any*** (RQ2; C2, C3)

P3: You get the idea (RQ3; C4, C5)

P4: ... (RQ4; C6)

Chapter 2

Literature review

2.1 Photoacoustic Imaging

2.1.1 Fundamentals of Photoacoustic Imaging Reconstruction

2.1.2 Linear State Space Model for Photoacoustic Imaging Reconstruction

2.2 Regression Methods

2.2.1 Least Squares Regression

2.2.2 Ridge Regression

2.2.3 Regularization

Lasso

Tikhonov Regularization

2.3 Multi-Objective Optimization for Image Reconstruction

2.3.1 Introduction to Multi-Objective Optimization

2.3.2 NSGA-II

2.4 Hybrid Optimization

2.5 Limitations and Challenges in Photoacoustic Imaging Reconstruction

Chapter 3

Methodology

3.1 Simulation and Experimental Setup

The experiments conducted in this study are based on synthetic data, allowing precise control over the noise levels and the number of samples. This approach provides valuable insights into the accuracy and stability of the proposed methods for estimating $\mathbf{d}$ and the absorption profile μ. The simulations were generated using the Linear State Space Model described in Section 2.1.2 and the forward model outlined in Section 2.1.1, which relies on the photoacoustic effect—a phenomenon where light absorption by a material generates sound waves.

As discussed in Section 2.1.2, the linear state space model allows the estimation of the absorption profile μ by solving the following linear matrix equation:

$$\mathbf{y} = \mathbf{H}\mathbf{d} + \mathbf{w}, \tag{3.1}$$

where $\mathbf{y}$ represents the measurements, $\mathbf{H}$ is the system matrix derived from experimental data, $\mathbf{d}$ is the data vector, and $\mathbf{w}$ is the noise term with distribution $\mathcal{N}(0, \sigma^2)$. From the linear state space model, μ can be computed using the relationship:

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{N_z} \end{bmatrix} = \begin{bmatrix} \mu_0 \\ \mu_1 a_0 \\ \mu_2 a_1 a_0 \\ \vdots \\ \mu_{N_z-1} a_{N_z-2} a_{N_z-3} \cdots a_1 a_0 \end{bmatrix}. \tag{3.2}$$

The equation 3.1 can theoretically be solved for $\mathbf{d}$ using least squares regression. However, due to the ill-conditioned nature of the problem, this approach often results in unstable solutions. To stabilize the solution, regularization techniques are applied, leading to the following optimization problem:

$$\hat{\mathbf{d}} = \arg\min_{\mathbf{d}} \left\{ \|\mathbf{y} - \mathbf{H}\mathbf{d}\|_2^2 + \lambda \|\mathbf{d}\|_2^2 \right\}, \tag{3.3}$$

where λ is the regularization parameter that controls the trade-off between the data fidelity term and the regularization term. The solution to this problem is given by:

$$\hat{\mathbf{d}} = \left(\mathbf{H}^T\mathbf{H} + \lambda\mathbf{I}\right)^{-1}\mathbf{H}^T\mathbf{y}. \tag{3.4}$$

3.2 Tikhonov Regularization for Image Reconstruction

Tikhonov regularization is a widely used technique for stabilizing ill-conditioned problems. By introducing a regularization term that penalizes large coefficients, it prevents overfitting and enhances the stability of the solution. In the context of image reconstruction, this method solves equation 3.3 by balancing data fidelity and regularization.

The choice of the regularization parameter λ is crucial. The L-curve method, which plots the residual norm $\left\|\mathbf{y} - \mathbf{H}\hat{\mathbf{d}}\right\|_2$ against the regularization norm $\left\|\hat{\mathbf{d}}\right\|_2$, helps identify the optimal λ. The corner of the L-curve represents the best trade-off between accuracy and stability.

However, Tikhonov regularization typically provides a single solution, which may not fully capture the range of possible trade-offs between objectives. This limitation motivates the use of multi-objective optimization.

3.3 Multi-Objective Optimization for Image Reconstruction

Multi-objective optimization enhances the reconstruction process by providing a diverse set of solutions, each capturing a unique trade-off between competing objectives. In the context of ill-posed regularization problems, these objectives are typically the accuracy of the data fit, quantified by $\|\mathbf{y} - \mathbf{H}\mathbf{d}\|_2$, and the stability of the solution, measured by $\|\mathbf{d}\|_2$.

We use NSGA-II to estimate the data vector $\mathbf{d}$ and subsequently derive the absorption profile μ. The algorithm generates a Pareto front of solutions, offering a diverse set of trade-offs between accuracy and stability. Unlike traditional approaches that yield a single solution, the Pareto front allows users to select the solution that best aligns with specific application requirements or constraints.

The choice of objectives is inspired by the L-curve method, where the optimal regularization parameter λ balances data fidelity and regularization. Minimizing $\|\mathbf{d}\|_2$ aligns with Tikhonov regularization, ensuring solution stability while mitigating numerical instabilities and overfitting. Each solution in the Pareto front corresponds to a unique balance between these objectives, enabling a comprehensive exploration of the trade-offs involved.

3.4 Hybrid Optimization for Image Reconstruction

The hybrid optimization approach combines Tikhonov regularization and NSGA-II to leverage the strengths of both techniques. Tikhonov regularization provides an initial estimate of $\mathbf{d}$, offering a stable starting point for the optimization process. NSGA-II then refines this estimate, exploring a broader solution space and generating a diverse set of solutions in the Pareto front.

By using the Tikhonov solution as the initial population for NSGA-II, the algorithm achieves faster convergence and produces a more diverse Pareto front. This approach not only improves the quality of the reconstruction but also enables the selection of solutions tailored to specific trade-offs between accuracy and stability.

The Pareto front generated by this hybrid method offers a comprehensive range of solutions. Users can evaluate and choose the most suitable solution based on application-specific requirements, such as prioritizing higher accuracy or enhanced stability. This flexibility represents a significant advantage over single-solution methods, which may not adequately address the diverse needs of real-world applications.

Chapter 4

Experiments and Results

This section presents the results of the experimentes. Experiments in one dimension are destineted to compere regularized optimization with Tikhonov regularization, multi-objective optimization with NSGA-II and hybrid optimization. Experiments in two dimensions are destineted to compare regularized optimization with Tikhonov regularization and hybrid optimization.

4.1 Experiments in one dimension

To compare the performance of the three optimization methods, we conducted experiments using synthetic data generated from the linear state space model described in Section 2.1.2. The forward model outlined in Section 2.1.1 was used to simulate the photoacoustic effect, generating measurements that were subsequently reconstructed using the three optimization methods. The experiments were designed to evaluate the accuracy and stability of the solutions obtained by each method, providing insights into their performance under varying noise levels and sample sizes.

4.1.1 Regularized vs Hybrid optimization

4.2 Experiments in two dimensions

4.3 Something meaningful

Chapter 5

Analysis

Note: Here I could write something to provide some meta-information for an annual review, if I wanted to.

5.1 Advantages and Limitations of the Regression and Regularization Approach

5.2 Advantages and Limitations of the Multi-objective Optimization Approach

5.3 Advantages and Limitations of the Hybrid Optimization Approach

Chapter 6

Conclusion

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References

Bibliography