Hybrid Optimization in Photoacoustic Image Reconstruction: Integrating NSGA-III and Tikhonov Regularization

by

Maximiliano Galindo
Author Qualifications

Submitted in fulfilment of the requirements for the degree of
Master of Science

Universidad Nacional Autónoma de México
November 2021

Abstract

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Finishing a PhD is like moving a mountain. Luckily, you only have to move one rock at a time.

(Reddit somewhere: pg. ??)

Acknowledgements

(this section is presently blank)

Contribution statement

Declaration by author

(this section is presently incomplete)

Supervisors

Title. Supervisor One[1], Title. Supervisor Two[2], Title. Supervisor Three[1], and Title. Supervisor Four[3,4]

[1]A *long* affiliation (e.g., School)
allows for three (e.g., Faculty)
lines to be specified (e.g., Institution)

[2]A *short* affiliation (e.g., Department)
only has two (e.g., Company)

[3]A supervisor may have
multiple affiliations

[4]An affiliation may be used by
multiple supervisors

List of publications

(this section is presently incomplete)

Publications included in this thesis

(to be completed)

Submitted manuscripts included in this thesis

(to be completed)

Other publications during candidature

(to be completed)

Contents

1 Introduction **2**

1.1 Photoacoustic Imaging 3

1.1.1 Research Questions 3

1.1.2 Contributions 3

1.2 Image Reconstruction 4

1.3 Multi-Objective Optimization 4

1.4 Overview 4

2 Literature review **5**

2.1 Photoacoustic Imaging 5

2.1.1 Fundamentals of Photoacoustic Imaging Reconstruction 5

2.1.2 Linear State Space Model for Photoacoustic Imaging Reconstruction . . . 5

2.2 Regression Methods 5

2.2.1 Least Squares Regression 5

2.2.2 Ridge Regression 5

2.2.3 Regularization 5

2.3 Multi-Objective Optimization for Image Reconstruction 5

2.3.1 Introduction to Multi-Objective Optimization 5

2.3.2 NSGA-III 5

2.4 Hybrid Optimization 5

2.5 Limitations and Challenges in Photoacoustic Imaging Reconstruction 5

3 Methodology **6**

3.1 Simulation and Experimental Setup 6

3.2 Tikhonov Regularization for Image Reconstruction 7

3.3 Multi-Objective Optimization for Image Reconstruction 7

3.4 Hybrid Optimization for Image Reconstruction 7

4 Experiments and Results **8**

4.1 Experiments in one dimension 8
4.1.1 Regularized vs Multi-objective optimization 8
4.1.2 Regularized vs Hybrid optimization 8
4.2 Experiments in two dimensions 8
4.3 Something meaningful 8

5 Analysis **9**
5.1 Advantages and Limitations of the Regression and Regularization Approach 9
5.2 Advantages and Limitations of the Multi-objective Optimization Approach 9
5.3 Advantages and Limitations of the Hybrid Optimization Approach 9

6 Conclusion **10**

List of Figures

List of Tables

Chapter 1

Introduction

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula. Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

1.1 Photoacoustic Imaging

1.1.1 Research Questions

This research will investigate Lorem ipsum dolor sit amet, consectetuer adipiscing elit.. Specifically, this research will provide a foundation for Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis.. From this foundation, Curabitur dictum gravida mauris..

The research question is as follows:

> *Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna.*

Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. As such, this research has been discretised as to be addressed in the following sub-questions:

RQ1: *Lorem ipsum dolor sit amet, consectetuer adipiscing elit.*

RQ2: *Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis.*

RQ3: *Curabitur dictum gravida mauris.*

RQ4: *Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna.*

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

1.1.2 Contributions

This research provides the following contributions to knowledge:

C1: Lorem ipsum dolor sit amet, consectetuer adipiscing elit..

C2: Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis..

C3: Curabitur dictum gravida mauris..

C4: Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna..

C5: Donec vehicula augue eu neque..

C6: Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas..

1.2 Image Reconstruction

1.3 Multi-Objective Optimization

1.4 Overview

The format of this thesis is by publication, so I will be also attempting to publish this research as the following papers:

P1: This is the full title of my first paper (RQ1; C1)

P2: This is where I would put my second paper ***if I had any*** (RQ2; C2, C3)

P3: You get the idea (RQ3; C4, C5)

P4: ... (RQ4; C6)

Chapter 2

Literature review

2.1 Photoacoustic Imaging

2.1.1 Fundamentals of Photoacoustic Imaging Reconstruction

2.1.2 Linear State Space Model for Photoacoustic Imaging Reconstruction

2.2 Regression Methods

2.2.1 Least Squares Regression

2.2.2 Ridge Regression

2.2.3 Regularization

Lasso

Tikhonov Regularization

2.3 Multi-Objective Optimization for Image Reconstruction

2.3.1 Introduction to Multi-Objective Optimization

2.3.2 NSGA-III

2.4 Hybrid Optimization

2.5 Limitations and Challenges in Photoacoustic Imaging Reconstruction

Chapter 3

Methodology

In this section I will present the methodology used to evaluate and compare the proposed approaches.

3.1 Simulation and Experimental Setup

The experiments I conducted are based on synthetic data, enabling precise control over the noise level and the number of samples. This setup provides valuable insights into the accuracy of the proposed methods for estimating $\mathbf{d}$ and the absorption profile μ. The simulations were generated using the Linear State Space Model described in Section 2.1.2 and the forward model outlined in Section 2.1.1. The forward model relies on the photoacoustic effect—a physical phenomenon in which a material absorbs light and subsequently generates sound waves.

As we see in Section 2.1.2, given the linear state space model we can estimate the absorption profile μ by solving the following linear equation matrix:

$$\mathbf{y} = \mathbf{H}\mathbf{d} + \mathbf{w} \tag{3.1}$$

After estimating $\mathbf{d}$, we can estimate μ since from the linear state space model we have that:

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{N_z} \end{bmatrix} = \begin{bmatrix} \mu_0 \\ \mu_1 a_0 \\ \mu_2 a_1 a_0 \\ \vdots \\ \mu_{N_z-1} a_{N_z-2} a_{N_z-3} \cdots a_1 a_0 \end{bmatrix} \tag{3.2}$$

The equation 3.1 could be solved for $\mathbf{d}$ using least squares regression, but typically the problem is ill-conditioned and the solution is unstable. To stabilize the solution, we can use regularization techniques. The problem can be formulated as follows:

$$\hat{\mathbf{d}} = \arg\min_{\mathbf{d}} \left\{ \|\mathbf{y} - \mathbf{H}\mathbf{d}\|_2^2 + \lambda \|\mathbf{d}\|_2^2 \right\} \tag{3.3}$$

where λ is the regularization parameter that controls the trade-off between the data fidelity term and the regularization term. The solution to this problem is given by:

$$\hat{\mathbf{d}} = \left(\mathbf{H}^T\mathbf{H} + \lambda\mathbf{I}\right)^{-1}\mathbf{H}^T\mathbf{y} \tag{3.4}$$

3.2 Tikhonov Regularization for Image Reconstruction

The Tikhonov regularization technique is a popular method for stabilizing ill-conditioned reconstruction problems. It introduces a regularization term that penalizes large coefficients in the solution, thereby preventing overfitting and improving the stability of the reconstruction. The Tikhonov regularization problem solves the by adding a regularization term to the least squares objective function.

To find the optimal value of λ, we use the L-curve method, which plots the residual norm $\left\|\mathbf{y} - \mathbf{H}\hat{\mathbf{d}}\right\|_2$ against the regularization norm $\left\|\hat{\mathbf{d}}\right\|_2$. The optimal value of λ corresponds to the corner of the L-curve, where the trade-off between data fidelity and regularization is balanced.

3.3 Multi-Objective Optimization for Image Reconstruction

Multi-objective optimization aims to find a set of solutions that optimize multiple objectives simultaneously. In the context of photoacoustic image reconstruction, the objectives are typically accuracy and efficiency. The NSGA-III algorithm is a popular method for solving multi-objective optimization problems. It uses a genetic algorithm to evolve a population of candidate solutions and maintain a diverse set of non-dominated solutions in the Pareto front.

We use NSGA-III to estimate the data vector $\mathbf{d}$ and from it find the absorption profile μ. The algorithm generates a set of solutions that represent the trade-off between minimizing the data residual $\|\mathbf{y} - \mathbf{H}\mathbf{d}\|_2$ and minimizing the regularization term $\|\mathbf{d}\|_2$. The solutions in the Pareto front provide a range of possible solutions that balance accuracy and stability in the reconstruction process.

3.4 Hybrid Optimization for Image Reconstruction

The hybrid optimization approach integrates NSGA-III and Tikhonov regularization to improve the accuracy and stability of the reconstruction process. The algorithm combines the benefits of multi-objective optimization and regularization techniques to find a set of solutions that balance accuracy and stability. The problem is the same as in 3.3, but first we find an approximated solution using Tikhovon regularization and then we use NSGA-III to find a set of solutions that optimize the objectives. Using the first solutin provided by Tikhonov regularization as the initial population of NSGA-III, we can improve the convergence of the algorithm and obtain a more diverse set of solutions in the Pareto front. The best solution in the Pareto front could be more accurate and stable than the solution provided by Tikhonov regularization alone.

Chapter 4

Experiments and Results

This chapter presents the experiments conducted to evaluate the proposed approach. The experiments are divided into three parts. The first part is a comparation of the regularizaded and multi-objective optimization approaches in one dimension. The second part is a comparation of the regularizaded and the hybrid optimization approaches in one dimension. The third part is a comparation of the regularizaded and the hybrid optimization approaches in two dimensions. The results are presented in the form of tables and figures. The tables show the results of the experiments in terms of the number of evaluations, the number of generations, the number of solutions in the Pareto front, and the hypervolume. The figures show the Pareto fronts of the solutions obtained by the algorithms.

4.1 Experiments in one dimension

4.1.1 Regularized vs Multi-objective optimization

4.1.2 Regularized vs Hybrid optimization

4.2 Experiments in two dimensions

4.3 Something meaningful

Chapter 5

Analysis

Note: Here I could write something to provide some meta-information for an annual review, if I wanted to.

5.1 Advantages and Limitations of the Regression and Regularization Approach

5.2 Advantages and Limitations of the Multi-objective Optimization Approach

5.3 Advantages and Limitations of the Hybrid Optimization Approach

Chapter 6

Conclusion

(this section is presently blank)

References

Bibliography