CAFFÈ BELTRAMI

UNIVERSITÀ DEGLI STUDI DI PAVIA

Caffè Beltrami presenta…

Massimiliano Ghiotto

Massimiliano Ghiotto è dottorando del XXXIX ciclo presso il Dipartimento di Matematica dell'Università di Pavia. La sua ricerca si concentra nell'approssimazione di PDEs tramite operatori neurali, con particolare attenzione allo sviluppo di operatori neurali applicabili a problemi definiti su geometrie multipatch.

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Operatori neurali

Deep Learning per matematici

Deep Learning

Applicazioni interessanti

Operatori neurali

Approsimazione di equazioni differenziali

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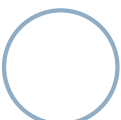
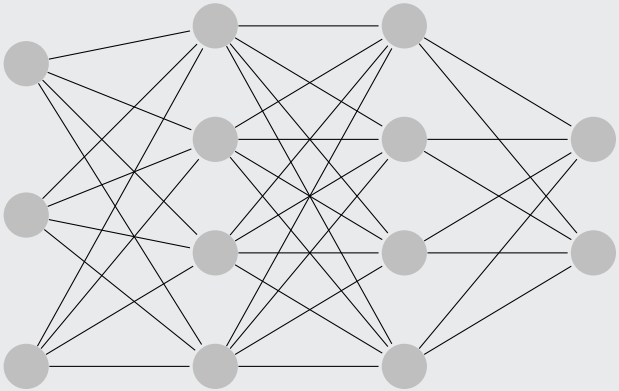
Deep Learning

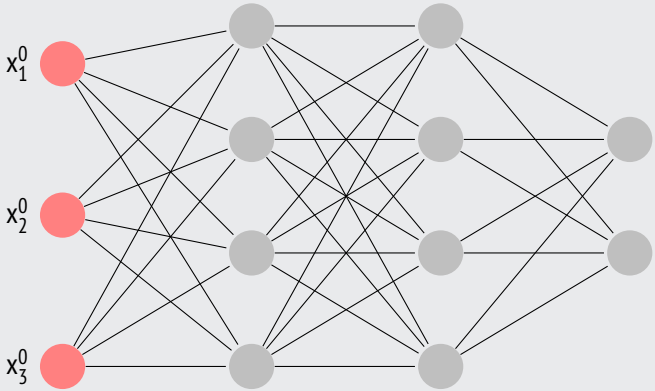


Multi-layer Perceptron



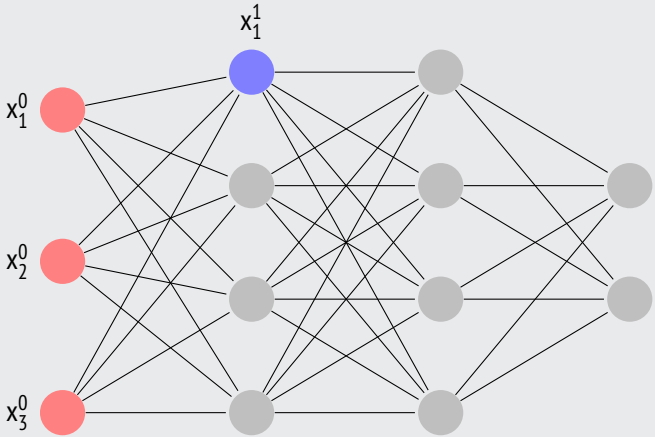
Esempio di architettura di un Multi-layer Perceptron con 3 neuroni di input, 4 neuroni nel primo hidden layer, 4 neuroni nel secondo hidden layer e 2 neuroni di output.

Multi-layer Perceptron



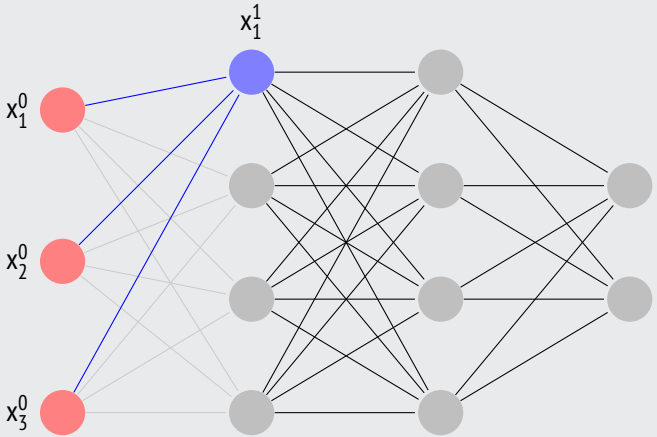
Layer di input con 3 neuroni.

Multi-layer Perceptron



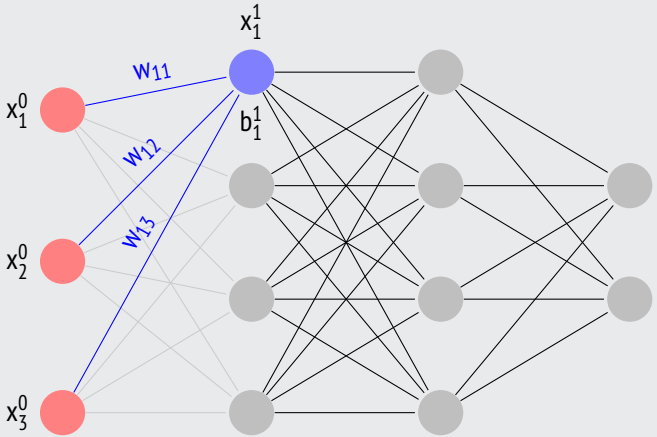
Vogliamo definire il valore del neurone x_1^1.

Multi-layer Perceptron



Evidenziamo le connesioni con i neuroni nel layer precedente.

Multi-layer Perceptron



$$x_1^1 = \sigma\left(b_1^1 + \sum_{i=1}^{3} w_{1i} x_i^0\right)$$

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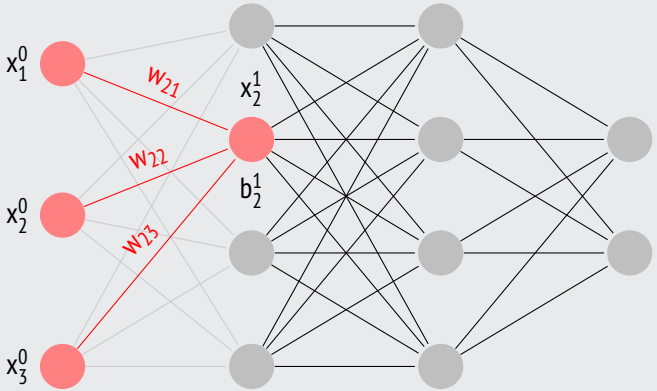
Multi-layer Perceptron



$$x_1^1 = \sigma\left(b_1^1 + \sum_{i=1}^{3} w_{1i} x_i^0\right)$$

$$x_2^1 = \sigma\left(b_2^1 + \sum_{i=1}^{4} w_{2i} x_i^0\right)$$

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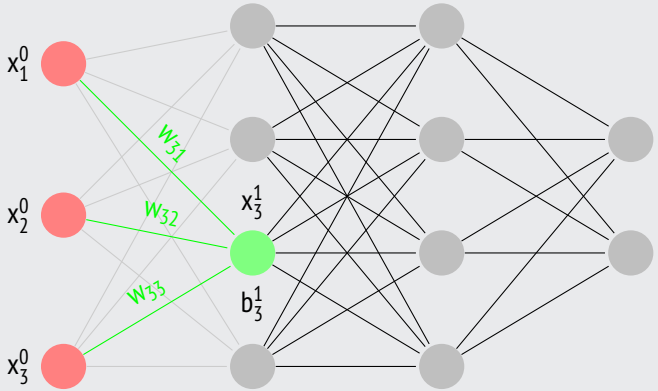
Multi-layer Perceptron

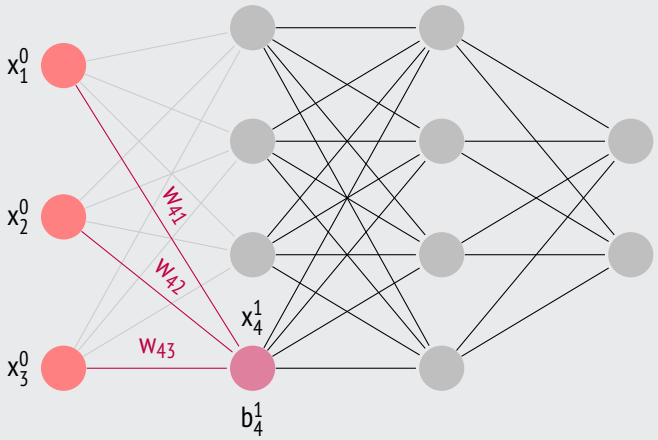


$$x_1^1 = \sigma\left(b_1^1 + \sum_{i=1}^{3} w_{1i}x_i^0\right)$$

$$x_2^1 = \sigma\left(b_2^1 + \sum_{i=1}^{4} w_{2i}x_i^0\right)$$

$$x_3^1 = \sigma\left(b_3^1 + \sum_{i=1}^{4} w_{3i}x_i^0\right)$$

Multi-layer Perceptron



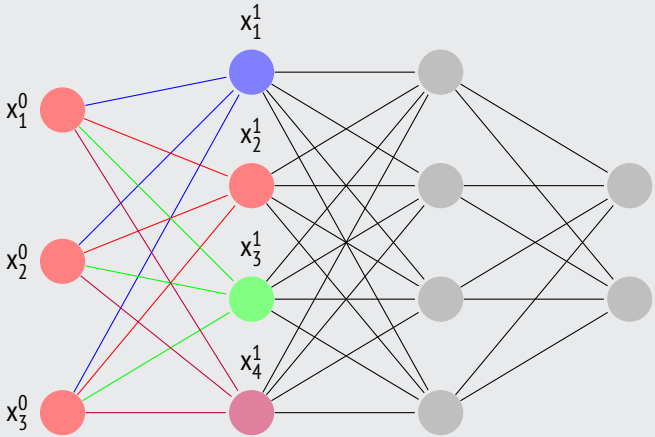
$$x_1^1 = \sigma\left(b_1^1 + \sum_{i=1}^{3} w_{1i}x_i^0\right)$$

$$x_2^1 = \sigma\left(b_2^1 + \sum_{i=1}^{4} w_{2i}x_i^0\right)$$

$$x_3^1 = \sigma\left(b_3^1 + \sum_{i=1}^{4} w_{3i}x_i^0\right)$$

$$x_4^1 = \sigma\left(b_4^1 + \sum_{i=1}^{2} w_{4i}x_i^0\right)$$

Multi-layer Perceptron

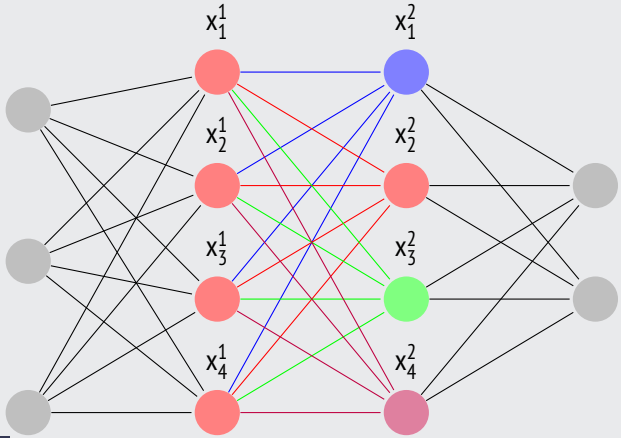


Dato $\mathbf{x}^0 \in \mathbb{R}^3$

$$\mathbf{x}^1 := \sigma\left(\mathbf{W}_1\mathbf{x}^0 + \mathbf{b}_1\right) \in \mathbb{R}^4$$

con $\mathbf{W} \in \mathbb{R}^{4\times 3}$, $\mathbf{b} \in \mathbb{R}^4$ e σ funzione di attivazione non lineare applicata componente per componente.

Multi-layer Perceptron



Dato $\mathbf{x}^0 \in \mathbb{R}^3$

$$\mathbf{x}^1 := \sigma\left(\mathbf{W}_1\mathbf{x}^0 + \mathbf{b}_1\right) \in \mathbb{R}^4$$

$$\mathbf{x}^2 := \sigma\left(\mathbf{W}_2\mathbf{x}^1 + \mathbf{b}_2\right) \in \mathbb{R}^4$$

con $\mathbf{W}_2 \in \mathbb{R}^{4\times 4}$ e $\mathbf{b} \in \mathbb{R}^4$.

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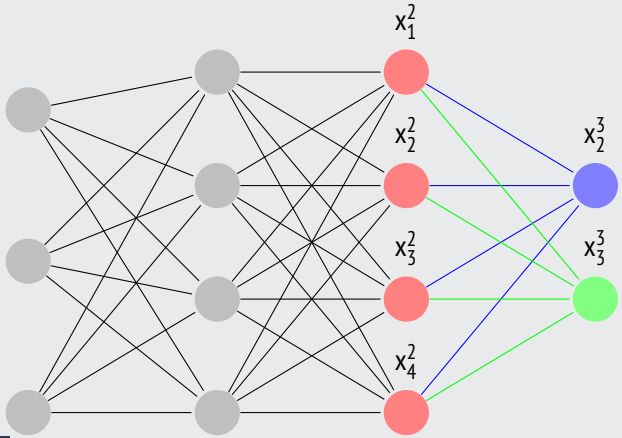
Multi-layer Perceptron



Dato $\mathbf{x}^0 \in \mathbb{R}^3$

$$\mathbf{x}^1 := \sigma\left(\mathbf{W}_1\mathbf{x}^0 + \mathbf{b}_1\right) \in \mathbb{R}^4$$

$$\mathbf{x}^2 := \sigma\left(\mathbf{W}_2\mathbf{x}^1 + \mathbf{b}_2\right) \in \mathbb{R}^4$$

$$\mathbf{x}^3 := \sigma\left(\mathbf{W}_3\mathbf{x}^2 + \mathbf{b}_3\right) \in \mathbb{R}^2$$

con $\mathbf{W}_3 \in \mathbb{R}^{2\times 4}$ e $\mathbf{b} \in \mathbb{R}^2$.

Multi-layer Perceptron

Definizione

Multi-layer Perceptron Un Multi-layer Perceptron (MLP) con input $\mathbf{x}^0 \in \mathbb{R}^{d_0}$, L hidden layers con $d_1, \ldots, d_L$ neuroni e output $\mathbf{x}^L \in \mathbb{R}^{d_L}$ è definito come

$$\mathbf{x}^1 := \sigma\left(\mathbf{W}_1\mathbf{x}^0 + \mathbf{b}_1\right) \in \mathbb{R}^{d_1}$$

$$\vdots$$

$$\mathbf{x}^L := \sigma\left(\mathbf{W}_L\mathbf{x}^{L-1} + \mathbf{b}_L\right) \in \mathbb{R}^{d_L}$$

dove $\mathbf{W}_\ell \in \mathbb{R}^{d_\ell \times d_{\ell-1}}$ e $\mathbf{b}_\ell \in \mathbb{R}^{d_\ell}$ per $\ell = 1, \ldots, L$ e σ è una funzione di attivazione non lineare applicata componente per componente.

Architetture diverse per problemi diversi

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Transformer

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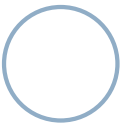
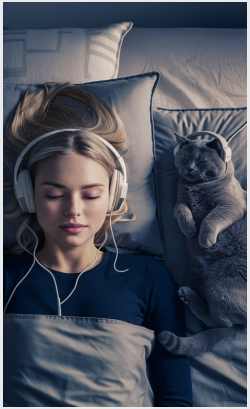
Applicazioni interessanti



Applicazioni alle immagini

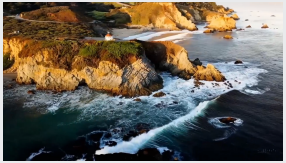
Esistono molte applicazioni per la generazione di immagini come DALL-E3, Midjourney e Ideogram



Applicazioni ai video

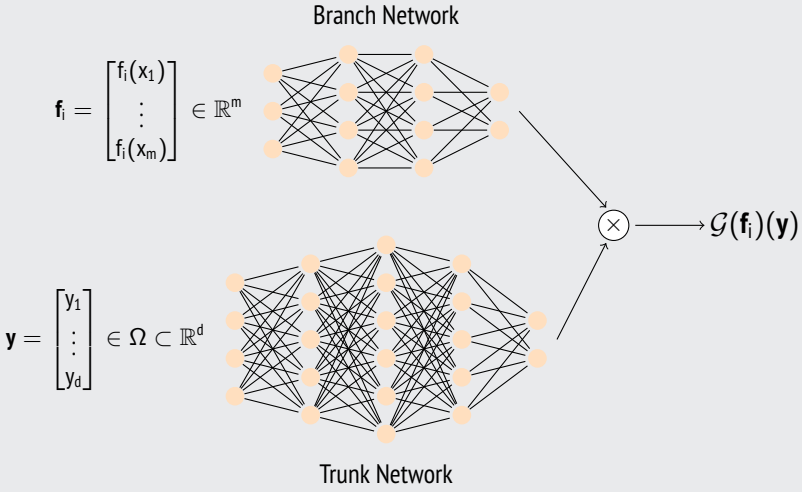
Deep learning per la generazione di video SORA o per modificare video anche in tempo reale NVIDIA Broadcast.



Operatori neurali

Deep Operator Network



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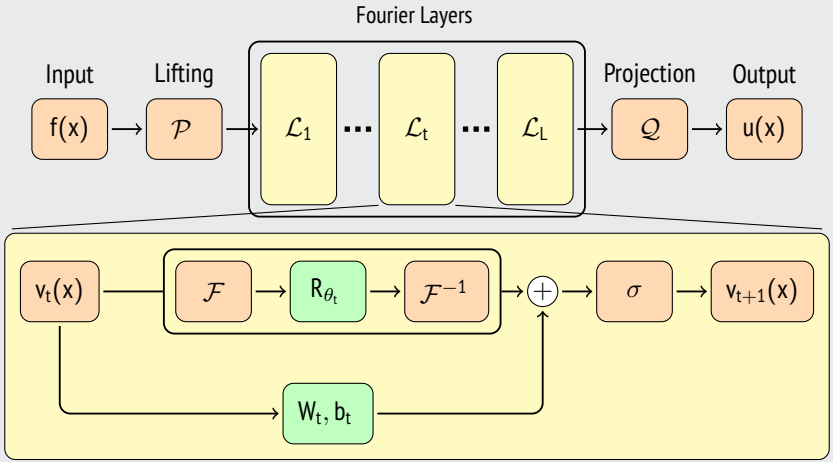
Fourier Neural Operator



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Approsimazione di equazioni differenziali