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An improved moving average technical trading rule[☆]

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HIGHLIGHTS

- The suggested approach uses a threshold which acts as a dynamic trailing stop.
- This modification increases the cumulative return and Sharpe ratio of the investor.
- It results in smaller maximum drawdown and drawdown duration.

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ABSTRACT

This paper proposes a modified version of the widely used price and moving average cross-over trading strategies. The suggested approach (presented in its 'long only' version) is a combination of cross-over 'buy' signals and a dynamic threshold value which acts as a dynamic trailing stop. The trading behaviour and performance from this modified strategy are different from the standard approach with results showing that, on average, the proposed modification increases the cumulative return and the Sharpe ratio of the investor while exhibiting smaller maximum drawdown and smaller drawdown duration than the standard strategy.

1. Introduction

The use of averages underlies all attempts of empirical modelling and the use of moving averages, in particular, has a long and distinguished history in smoothing and forecasting at least from the time of the publication of the book of Brown [1]. Moving averages form the simplest statistical construct that is widely used in trading the financial markets of all types, foreign exchange and equities more than others, in a variety of different interpretations of trading strategies (or rules). The purpose of this paper is to propose a modification to the standard cross-over strategy, based on prices & moving averages, that enhances its performance along all evaluation measures, providing (on average) higher cumulative returns, higher Sharpe ratios and lower drawdowns.

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Moving averages are a staple in the arsenal of tools in technical analysis trading and their properties and efficacy have been considered in many previous academic studies[2] some of which we discuss below. Brown and Jennings [2] is an early reference from economists on technical analysis. Brock et al. [3] examine some simple technical trading rules and associate them with the properties of stock returns while Neely [4] provides a review of technical analysis (with emphasis on moving average rules) in foreign exchange markets and LeBaron [5] examines the profitability of technical trading rules and foreign exchange intervention. Neely and Weller [6] provide further discussion on Neely's earlier paper. Lo et al. [7] have a comprehensive review of technical analysis, that includes the use of moving averages, where they try to provide some underlying statistical foundations to technical analysis trading rules. Vandewally et al. [8] analyse the use of moving averages from a physicist perspective and more recently, Okunev and White [9], Nicolau [10], Faber [11], Friesen et al. [12], Harris and Yilmaz [13], and Zhu and Zhou [14] have interesting theory and applications that are based on moving average technical trading rules. Okunev and White [9] examine the profitability of moving average-type rules, and the reasons behind it, in currency markets. Nicolau [10] and Zhu and Zhou [14] develop continuous time models that are used to explain various aspects of behaviour of moving averages; the latter paper is particularly interesting since it shows how to optimize a moving average approach for asset allocation. The same underlying intuition, with the application but without the theory, underlies the work of Faber [11] which is concerned with the use of moving averages as 'market timing' instruments. His main concern, from a practitioner's perspective, is whether a simple, 200-day moving average, price cross-over strategy can be used to avoid the pitfalls and large drawdowns of the buy & hold strategy—and subsequently be used in an asset allocation framework. Friesen et al. [12] discuss reasons and explanations behind trading rule profitability, including 'confirmation bias' and show how certain price patterns arise and lead to certain autocorrelation structure. Finally, Harris and Yilmaz [13] examine whether a smoothing approach can be used profitably in foreign exchange trading, by comparing moving average rules with the use of the Hodrick–Prescott [15] filter and kernel smoothing. There are many more academic references on the use and profitability of technical trading rules, beyond moving averages, whereas the above short list is mainly aimed on some papers that used smoothing methods for trading.

The modification that we propose in this paper is simple, intuitive, has a probabilistic explanation (based on the notion of 'return to the origin' in random walk parlance) and can easily be implemented for actual applications. It consists of a rule that relates the current price of an asset with the price of the last 'buy' signal issued by a moving average strategy (making this latter price a dynamic threshold) and it works as a dynamic trailing stop. We present a 'long only' version of the strategy but the adaptation to both long-and-short trading is immediate. We further discuss this modification in the next section. We present comparative results on the performance of the modified strategy on the Dow Jones index, the S&P500 index and the EUR/USD exchange rate. We use the latter currency in order to show the wide applicability of the strategy. Our results support the proposed modified strategy and show that considerable performance improvements can be effected to the standard cross-over rules.

The rest of the paper is organized as follows: in Section 2 we present our methodology; in Section 3 we discuss our data; in Section 4 we have the main discussion of our empirical results while in Sections 4.1 and 4.2 we comment on a variety of secondary series; in Section 5 we have a brief discussion on the choice of moving average type, length of the moving average and other implementation issues; Section 6 has some concluding remarks and prospects for further work.

2. Methodology

2.1. Trading strategies

Consider the (closing) price $\{P_t\}_{t\in\mathbb{N}_+}$ of an asset and let $M_t(k)$ denote the kth period[3] backward moving average, that is:

$$M_t(k) \stackrel{\text{def}}{=} \frac{1}{k}\sum_{j=0}^{k-1} P_{t-j}. \tag{1}$$

The moving average is one of the most frequently used indicators in trading strategies. Two of the easiest and most popular such strategies are based on a price cross-over and on moving averages cross-over. The first strategy issues a 'buy' signal when the price of the asset crosses above the moving average while the second strategy issues a 'buy' signal when a faster moving average crosses above a slower moving average; 'sell' signals are defined in the opposite direction. If the strategies are 'long only' ones then an 'exit' signal (usually reverting to a risk-free asset) is issued. We are going to be concerned with such 'long only' strategies so that the signals are binary.[4] The signal variable based on a price cross-over is defined as follows:

$$S^P_{t+\tau}(k) \stackrel{\text{def}}{=} \begin{Bmatrix} 1 & \text{while} & P_{t-1+\tau} \geq M_{t-1+\tau}(k) \\ 0 & \text{while} & P_{t-1+\tau} < M_{t-1+\tau}(k) \end{Bmatrix} \tag{2}$$

2 The literature on technical analysis from the practitioners' perspective is huge and cannot possibly be reviewed here.

3 Sometimes called the 'look-back' period.

4 It is straightforward to use all material that follows with sell signals as well but, as in Ref. [11], we assume that the investor exits the market and stays with a risk-free asset; in the present analysis we focus on the differential performance among strategies and we assume that the risk-free rate is zero.

for $\tau = 0, 1, \ldots$, where we note the one period transaction-delay in buying the asset—this is what will actually happen if one was implementing the strategy in real time.

Suppose that the first buy (or entry) signal is issued at time t_1 and the first exit signal is issued after s periods at time $t_1 + s$. The total (cumulative) return of the strategy over this holding period is then given by:

$$TR^P_{t_1+s+1} \stackrel{\text{def}}{=} \left\{ \prod_{\tau=t_1+1}^{t_1+s+1} (1 + R_\tau) \right\} - 1 \tag{3}$$

where $R_\tau \stackrel{\text{def}}{=} P_\tau / P_{\tau-1} - 1$ is the percentage return for the τth period. The total return of the strategy over a sequence of holding periods, for a sample of size n, is given by:

$$TR^P_n \stackrel{\text{def}}{=} \left\{ \prod_{\tau=t_1+1}^{n} (1 + R^P_\tau) \right\} - 1 \tag{4}$$

where $R^P_\tau \stackrel{\text{def}}{=} S_{\tau-1}(k) R_\tau$ is the sequence of the strategy's returns.

Similarly, we may define the signal variable for the moving averages cross-over as follows:

$$S^M_{t+\tau}(k_1, k_2) \stackrel{\text{def}}{=} \begin{Bmatrix} 1 & \text{while} & M_{t-1+\tau}(k_1) \geq M_{t-1+\tau}(k_2) \\ 0 & \text{while} & M_{t-1+\tau}(k_1) < M_{t-1+\tau}(k_2) \end{Bmatrix} \tag{5}$$

where $\tau = 0, 1, \ldots$ and $k_1 < k_2$. The strategy's returns and total return are defined in an analogous fashion to the price cross-over case and we denote them by R^M_τ and TR^M_n respectively.

Our modification[5] to the above strategies is very simple: in order to stay in the market (the initial 'buy' signal always being provided by a moving average strategy) we require that the current price is greater or equal than the convex combination of the entry price and the current price, which is equivalent to having the current price greater or equal than the entry price. While this appears exceedingly simplistic it does have an underlying intuition, a probabilistic justification and, as we will see, it works quite well in practice. This modification allows for improved entry and exit periods, compared to the plain moving average strategies, because it provides a well-defined local 'trendline' and 'confirmation' on market direction; in addition, as it will be seen, it acts as a dynamic stop loss.

To see the workings of this modification consider the following example. A moving average strategy, say $S^P_t(k)$, provides an entry signal at period t_i and we mark the entry price P_{t_i} and track the current price $P_{t_i+\tau}$, for $\tau > 0$. Now, at each point in time there is a probability of staying in the market $\mathsf{P}\left[S^P_{t_i+\tau}(k) = 1\right]$ and a corresponding probability of exiting the market $\mathsf{P}\left[S^P_{t_i+\tau}(k) = 0\right] = 1 - \mathsf{P}\left[S^P_{t_i+\tau}(k) = 1\right]$. Think of the "expected" price $P^*_{t_i+\tau}$ at each period $t_i + \tau$ as the convex combination, the straight line, that passes through the two price levels, that is:

$$P^*_{t_i+\tau} \stackrel{\text{def}}{=} \mathsf{P}\left[S^P_{t_i+\tau}(k) = 1\right] P_{t_i} + \left(1 - \mathsf{P}\left[S^P_{t_i+\tau}(k) = 1\right]\right) P_{t_i+\tau}. \tag{6}$$

It is rather natural to require that the current price is at least as large as the "expected" price to stay into the market, i.e. $P_{t_i+\tau} \geq P^*_{t_i+\tau}$ which is easily seen to boil down to a rule the requires $P_{t_i+\tau} \geq P_{t_i}$. Note that the use of probabilities is not really required, although they are more intuitive than an arbitrary convex combination of the current and the entry price. We immediately observe that the modified strategy will not necessarily use all the moving average signals but only those that will conform to the price inequality we just noted. Furthermore, it becomes a function of the different entry prices at times t_i, i.e., while being into a trade with our modified strategy the reference entry time and reference entry price may change. To formally state our approach we provide a definition of the entry times and the new signal variable. Using again the price cross-over strategy for illustration, we have:

$$t_i(k) \equiv t_i \stackrel{\text{def}}{=} \left\{ t \in \mathbb{N}_+ : S^P_t(k) > S^P_{t-1}(k) \right\} \tag{7}$$

for the definition of the moving average-based entry times and let $t_\ell \stackrel{\text{def}}{=} \max_i t_i$ denote the latest entry time for all $t_i \leq t$. Then, the signal variable is defined as:

$$C^P_{t+\tau}(k, t_\ell) \stackrel{\text{def}}{=} \begin{Bmatrix} 1 & \text{while} & P_{t-1+\tau} \geq P_{t_\ell} \\ 0 & \text{while} & P_{t-1+\tau} < P_{t_\ell} \end{Bmatrix} \tag{8}$$

for $\tau = 0, 1, \ldots$, and note that this modified signal becomes a function of the cross-over entry time t_ℓ and entry price P_{t_ℓ}. A similar expression applies to the case where instead of a price cross-over we have moving averages cross-over $C^M_{t+\tau}(k, t_\ell)$. As with the plain cross-over signals a one-period delay applies for the modified signals as well. For future reference we denote

[5] In what follows we will call the price and moving average cross-overs 'standard' strategies while we will call them 'modified' strategies when they incorporate they changes that we propose below.

the modified strategies' returns by $R_\tau^{P,C} \stackrel{\text{def}}{=} C_{\tau-1}^P(k, \tau_\ell)R_\tau$ and by $R_\tau^{M,C} \stackrel{\text{def}}{=} C_{\tau-1}^M(k_1, k_2, \tau_\ell)R_\tau$ and the total returns by $TR_n^{P,C}$ and by $TR_n^{M,C}$ respectively. We can now summarize the main aspects of our modified strategy, again using the price cross-over for illustration, as follows:

1. The initial entry time t_1 is determined by the cross-over signal variable $S_t^P(k)$.
2. Once we enter into a trade the exit condition is determined by the modified signal variable $C_t^P(k, t_\ell)$ and not the cross-over signal variable $S_t^P(k)$.
3. During the duration of a trade the reference entry time and reference entry price will change if the cross-over signal variable issues an exit signal and later an entry signal *while the modified signal variable does note change*. This makes the latest entry price P_{t_ℓ} to act as a dynamic trailing stop.
4. The modified strategy's entry and exit times do not coincide with the cross-over strategy's entry and exit times.

Why would one expect, *a priori*, this modified strategy to work? As the new signal variable depends on a price distance, we can actually provide a probabilistic explanation under the assumption that prices follow a (symmetric) random walk. Although the assumptions of a random walk, particularly the one of independent increments and constant volatility, are known not to hold it is still instructive to use the random walk model since we have available results on the probability of exiting from the modified strategy, i.e. on $p_t(\tau) \stackrel{\text{def}}{=} \mathsf{P}\left[C_{t+\tau}^P(k, t_\ell) = 0 \wedge C_t^P(k, t_\ell) = 1\right]$ for $\tau > 0$ and for fixed t_ℓ. This probability corresponds to the event of a 'return to the origin' in random walk parlance and its probabilistic behaviour is well known. In fact, we are particularly interested in the probability of the 'first passage to the origin' after τ-periods we are in a trade (thus the fixed t_ℓ—this is so since the random walk's origin does not matter insofar it is fixed).

Under these assumptions for the random walk it is known (for details see Refs. [16,17, vol. 1, Chapters 3,13 and 14]) that the probability of a 'first passage to the origin' declines exponential as τ increases.[6] The probability of an immediate first passage is $p_t(2) = 50\%$ (because of the symmetry assumption) which declines to about $p_t(10) = 2.8\%$ in 10 periods and to about $p_t(20) = 0.94\%$ in 20 periods. If the random walk is not symmetric then these probabilities change. However, it is interesting to note that even when the odds are against a price increase the probabilities still decline exponentially albeit they start from higher levels: that is, if the trade is not terminated soon then it will probably continue. For example, if the odds of a negative return each period are 30% then the probability of an immediate first passage is $p_t(2) = 70\%$ which declines to $p_t(10) = 2.70\%$ in 10 periods and to about $p_t(20) = 0.60\%$ in 20 periods. Therefore, irrespective of the odds structure, the probability of exiting a successful trade declines as τ increases but for fixed t_ℓ only; when the reference entry time and price change the 'origin' changes again and the probabilities 'reset'. It is in this sense that the proposed strategy has P_{t_ℓ} acting as a dynamic trailing stop.

2.2. Strategy evaluation

To evaluate our proposed modification on the moving average trading rules we use a variety of averages, as used by practitioners and trading platforms, as well as a number of practical trading evaluation measures. Besides the plain moving average we also employ the exponential moving average and the weighted moving average.[7] These rules have been studied in the academic literature from a different perspective; see, among others, Ohnishi et al. [18] for an alternative way to decide the weights in a weighted moving average rule and Grebenkov and Serror [19] who analyse the trend in stock price series using an exponential moving average. For all of these averages we used a number of combinations for k and (k_1, k_2) conforming to the most popular choices for daily data: 5, 20, 50, 100 and 200-period averages were used. Specifically, the following pairs (k_1, k_2) were considered: $(5, 20), (10, 20), (20, 50), (20, 100)$ and $(50, 200)$—the more relevant of those being the last three pairs which we discuss more extensively. To perform our exercise in a real-time fashion we split the sample into two parts $n_0 + n_1 = n$, where n_1 is the evaluation period—we use a variety of evaluation periods (see discussion of data and results) to account for different market periods. For each of the averages and for each of the four strategies (price cross-over, modified price cross-over, moving averages cross-over and modified moving averages cross-over) we compute the following evaluation measures (R_t^s denotes the returns of any of the four strategies):

- The total return, as in Eq. (4),

$$TR^s \stackrel{\text{def}}{=} \left\{ \prod_{\tau=t_1^s+1}^{n} (1 + R_\tau^s) \right\} - 1.$$

- The average return $AR^s \stackrel{\text{def}}{=} \frac{1}{N_s}\sum_{t=n_s}^n R_t^s$, where t_1^s denotes the first trading period for the sth strategy, $n_s \stackrel{\text{def}}{=} n_0 + t_1^s + 1$ denotes the first evaluation period and $N_s \stackrel{\text{def}}{=} n - n_s + 1$ denotes the evaluation observations. The average return is reported annualized.

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6 This probability is the same as the probability of a 'first return to the origin' but the latter does not require a positive price distance for all τ prior to the return.

7 Arithmetic weighting is conventionally used in technical analysis.

Table 1
Data sample splits as strategy evaluation periods.

	DJIA		SP500		EUR/USD	
	Date	n_1	Date	n_1	Date	n_1
S1	08/01/1929	20 618	11/01/1950	15 310	06/21/2001	2558
S2	01/02/1970	10 519	01/02/1970	10 519	11/25/2002	2187
S3	01/02/1990	5 465	01/02/1990	5 645	02/03/2006	1353
S4	01/03/2000	2 937	01/03/2000	2 937	03/19/2009	539

- The standard deviation of the return $SD^s \stackrel{\text{def}}{=} \sqrt{\frac{1}{N_s}\sum_{t=n_s}^{n}\left(R_t^s - AR^s\right)^2}$, annualized.
- The Sharpe ratio $SR^s \stackrel{\text{def}}{=} AR^s/SD^s$, annualized.
- The maximum drawdown MD^s. Let TR_t^s denote the running total return of a strategy up to time $t > n_s$ and let $\mathcal{M}_t^s \stackrel{\text{def}}{=} \max_{t' \leq t} TR_{t'}^s$ denote the running maximum return. Then the maximum drawdown is defined as $MD^s \stackrel{\text{def}}{=} \frac{1+\mathcal{M}_t^s}{1+TR_t^s} - 1$.
- The maximum drawdown duration, denoted MDD^s.

We choose as our benchmark the standard moving average strategies as detailed above and we report the above measures as differences with respect to that benchmark. So, again using the price cross-over strategy, $s = P$, as an illustration, the final statistics are given in a form like:

1. The difference in total returns $TR \stackrel{\text{def}}{=} TR^{P,C} - TR^P$.
2. The difference in average returns $AR \stackrel{\text{def}}{=} AR^{P,C} - AR^P$.
3. The difference in standard deviations $SD \stackrel{\text{def}}{=} SD^{P,C} - SD^P$.
4. The difference in the Sharpe ratios $SR \stackrel{\text{def}}{=} SR^{P,C} - SR^P$.
5. The difference in maximum drawdowns $MD \stackrel{\text{def}}{=} MD^{P,C} - MD^P$.
6. The difference in maximum drawdown durations $MDD \stackrel{\text{def}}{=} MDD^{P,C} - MDD^P$

and similarly for $s = M$. Detailed results are also available on the comparative performance of these strategies with respect to the buy & hold strategy and we comment on their differences in the coming discussion. However, our main focus is to compare two active strategies and not an active versus a passive strategy.

3. Data

We apply the methodology described in the previous section to representative series from two asset classes. First, for equities, we use two long data sets for the Dow Jones (DJIA) and the S&P500 (SP500) indices. Second, we use the EUR/USD foreign exchange rate to illustrate the applicability of the strategy across different asset classes.[8] Our choice of data series is based on data availability, 'popularity' and a combination of high volume and liquidity and low transaction costs in their trading. For the DJIA and the SP500, which are not directly tradable, the analysis can be thought of in terms of 'market timing' as in Ref. [11] or their ETFs counterparts can be used (i.e. DIA and SPY). The EUR/USD exchange rate is of prime interest to currency traders worldwide and its modelling is especially relevant during these turbulent times.

The data for the DJIA and the SP500 was downloaded from Yahoo Finance! website. We use the longest records available, from 1928 and 1950 respectively—the corresponding sample observations are 20 826 days for the DJIA (ending in 02/09/2011) and 15 519 days for the SP500 (ending in 02/09/2011 as well). The data for the EUR/USD exchange rate were publicly available from the FRED database of the Federal Reserve Bank of St Louis, from 01/03/2000 until 04/13/2011 for a total of 2943 observations.

As mentioned in the methodology section, in evaluating our trading strategies we split our sample into training and evaluation periods and let the sample roll forward based on the length of the largest moving average. We have selected two different splitting dates so as to provide results that are (as much as possible) free from bias due to the starting date of the evaluation period. We summarize them in Table 1.

4. Discussion of results

We focus on the following combination of parameters (k_1, k_2) of (20, 50), (20, 100) and (50, 200) for the indices and (5, 20), (10, 20) and (20, 50) for the EUR/USD exchange rate. We will also discuss the performance of (a) the largest evaluation period (S1), (b) the smallest evaluation period (S4) and (c) the average performance across all evaluation periods (not just those in (a) and (b)).[9] The selection of these sample splits is based on sample size considerations (as in S1) and on having a period that exhibits at least part of cycle (trough & peak as in S3 and S4).

[8] More results using the SPY, QQQ, XLF, XLE, EWJ and IYR ETFs are available on request.

[9] In the next subsections we are interested on average performance across strategies and evaluation periods; see Section 5 on additional discussion on results for strategy usage and comparisons among price cross-overs and moving average cross-overs.

4.1. Results on DJIA and S&P500

We begin our discussion with the results on the longest series of DJIA which are given in Table 2. The table, as all the ones that follow, has three panels as in (a), (b) and (c) mentioned above. Starting with the results for the longest evaluation period (S1) we see that, in terms of the total return difference *TR*, the proposed modified strategy is better 89% of the time, across all cross-over strategies and (k_1, k_2) combinations, with an average gain[10] over the standard strategies of 2900% (while the average total return among all strategies, and not just those that our modified strategies are better, is 2400%). These numbers are neither unreasonable nor 'alarming': they simply reflect the fact that, over the long run of 80 years that we examine, the index has been steadily rising until 2000 and the current price would almost always be greater than the updated entry price. This is precisely the effect associated to the 'return to the origin' and the probability of long leads in a random walk context. As we will see immediately below for shorter evaluation periods the numbers are correspondingly smaller.

Among the price cross-over strategies the best performers are the modified 50-day weighted moving average with a gain of 4100% and the modified 50-day moving average with a gain of 3200%, while among the moving average cross-overs the best performers are the modified (20, 50)-days weighted moving average and the modified (20, 100)-days simple moving average with gains of 9100% and 9000% respectively. Here, and in many cases for other series, we find that the moving average cross-over strategies are better than the price cross-over ones. It is interesting that while the difference in total return is quite substantial we do not find any difference in terms of the average return: the average annualized return *AR* gain is the same across winning strategies and across all strategies and equals to 1%. On the other hand, the risk–reward trade-off is much better with the use of the modified strategies: 74% of the time the modified strategies have larger Sharpe ratios, with an average gain of 12% for the winning strategies and of 8% or all strategies. Based on these criteria the average performance of the proposed modified strategy is better than that of the standard cross-over rules. However, even more important is the fact that the modified strategy exhibits lower maximum drawdown and lower drawdown duration: (35%) of the time the modified strategies have lower maximum drawdown with an average gain of −20% although the maximum drawdown is larger (at 42%) across all strategies. For the maximum drawdown duration we have that 60% of the time the modified strategies have lower duration with corresponding averages of −578 and −136 days: with the modified strategies an investor will emerge from a price slump more than a year earlier, on average, than by using the standard cross-over strategies. These results are, of course, conditioned to the choice of moving average and the choice of the look-back parameters (k_1, k_2). They do not imply that the modified strategies will *always* be better but *on average* an investor will be much better off using the modified strategies rather than the standard ones.

We next turn to the results from the smallest evaluation sample (S4), the one that includes the last 20 years that contain a full cycle (trough to trough) of two bull and bear markets. This is an important evaluation period for momentum-based strategies such as the ones we are considering. The results, in the second panel of Table 2, are encouraging: for the difference in total return we find that 81% of the time the modified strategies are better than the standard ones with an average gain of 19% across these winning strategies (and 14% across all strategies). So we again find that the cumulative worth for an investor is on average higher when using the modified strategies, even during a crisis-and-recovery period. Among the price cross-over strategies the best performers are the modified 200-day exponential moving average with a gain of 43% and the modified 50-day exponential moving average with a gain of 32%, while among the moving average cross-over strategies the best performers are the modified (20, 50)-days and (20, 100)-days simple moving averages with gains of 35% and 22% respectively. For the Sharpe ratio we find that the modified strategies are also better 78% of the time with an average gain to risk–reward trade-off of 19% (among the winning strategies) and 13% (among all strategies); these averages are actually better than the ones for the largest evaluation period discussed above and this could be interpreted as a sign of certain 'robustness' for the proposed modification. Furthermore, the performance based on maximum drawdown and its duration is also better than before: based on maximum drawdown the modified strategies were better 57% of the time with an average gain of −14% across the winning strategies while the average gain was 1% across all strategies. The results are even more encouraging for the maximum drawdown duration, where 85% of the time the modified strategies have smaller duration with an average of −352 days, while the overall average duration is again better at −313 days. We see that the performance of the new approach is indeed robust and shows to be more profitable than the standard cross-over strategies in a period where there were many 'breaks' in the main market trend.

A similar picture emerges if we look at the average performance across evaluation periods, in the third panel of Table 2. Here, we again have that 89% of the time the modified strategies outperform the standard ones in terms of the difference in total return, with an average gain of 637% and 795%, across the winning and all strategies respectively. The Sharpe ratio, maximum drawdown and drawdown duration exhibit equally good performance as in the previously two examined evaluation periods.

It is quite interesting to compare the above results with those on S&P500, which are presented in Table 3. The reader will immediately notice the smaller numbers due to the smaller evaluation period, compared to that of the DJIA. In the first panel

[10] These, and the other average differences that are discussed below are computed as follows: for each of the panels in Table 1 let s_{ij} denote the cell value for strategy i and evaluation measure j for example $i = MA_1$ the simple price cross-over based on k_1 and $j = TR$ be the total return. For each evaluation measure there are 3 (k_1, k_2) combinations and 9 average types for a total of 27 cell entries. Then, the average difference among the winning strategies is $\frac{1}{27}\sum_{\forall(k_1,k_2)}\sum_{\forall i} s_{ij} \cdot I_{ij}$ where $I_{ij} \stackrel{\text{def}}{=} I(s_{ij} > 0)$ for $j = TR, AR, SD, SR$ and $I_{ij} \stackrel{\text{def}}{=} I(s_{ij} < 0)$ for $j = MD, MDD$. The average difference among all strategies is $\frac{1}{27}\sum_{\forall(k_1,k_2)}\sum_{\forall i} s_{ij}$. The same applies to all tables in the sequel.

Table 2
Strategy evaluation statistics for DJIA index.

		MA_1	MA_2	WMA_1	WMA_2	EMA_1	EMA_2	$MACO$	$WMACO$	$EMACO$
						$S_1, n_1 = 20\,618$				
$k_1 = 20$	TR	15.46	32.38	11.99	40.68	25.84	27.22	45.37	91.95	11.54
$k_2 = 50$	AR	0.01	0.01	0.01	0.02	0.02	0.01	0.05	0.05	0.00
	SD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
	SR	0.07	0.08	0.09	0.14	0.14	0.06	0.36	0.31	−0.01
	MD	0.31	0.10	−0.07	0.51	0.20	0.04	−0.07	0.05	0.69
	MDD	−1690	28	−1149	−437	−365	−260	−1244	4	446
$k_1 = 20$	TR	15.46	3.76	11.99	−3.54	25.84	19.54	90.49	22.60	44.39
$k_2 = 100$	AR	0.01	0.00	0.01	0.00	0.02	0.01	0.04	0.01	0.02
	SD	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	0.00	0.00
	SR	0.07	0.00	0.09	−0.03	0.14	0.04	0.29	0.09	0.11
	MD	0.31	0.12	−0.07	0.10	0.20	−0.19	0.31	0.11	−0.10
	MDD	−1690	−34	−1149	−12	−365	−133	195	474	−199
$k_1 = 50$	TR	32.38	4.97	40.68	4.65	9.04	−63.52	52.13	41.57	−19.73
$k_2 = 200$	AR	0.01	0.00	0.02	0.00	0.00	−0.02	0.01	0.01	−0.01
	SD	0.00	0.01	0.00	0.00	0.01	0.03	−0.01	−0.01	0.03
	SR	0.08	−0.02	0.14	0.00	−0.01	−0.26	0.11	0.13	−0.16
	MD	0.10	−0.54	0.51	−0.07	0.33	4.85	−0.74	−0.29	4.66
	MDD	28	62	−437	−127	−260	2398	846	33	1346
						$S_4, n_1 = 2937$				
$k_1 = 20$	TR	0.24	0.05	0.10	0.21	0.32	0.28	0.35	0.09	−0.13
$k_2 = 50$	AR	0.03	0.01	0.01	0.02	0.04	0.04	0.05	0.01	−0.02
	SD	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	−0.01	0.00
	SR	0.18	0.05	0.08	0.16	0.26	0.28	0.38	0.09	−0.11
	MD	−0.19	0.09	−0.04	−0.02	−0.27	0.04	0.07	0.11	0.42
	MDD	−112	−60	−726	67	−781	−329	−423	−78	845
$k_1 = 20$	TR	0.24	−0.07	0.10	−0.06	0.32	0.18	0.22	−0.09	0.29
$k_2 = 100$	AR	0.03	−0.01	0.01	−0.01	0.04	0.03	0.03	−0.01	0.04
	SD	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	−0.01	0.00
	SR	0.18	−0.10	0.08	−0.07	0.26	0.21	0.23	−0.09	0.30
	MD	−0.19	0.11	−0.04	0.25	−0.27	−0.17	0.28	0.55	−0.15
	MDD	−112	−35	−726	−17	−781	−395	−124	416	−385
$k_1 = 50$	TR	0.05	−0.07	0.21	0.11	0.32	0.43	0.05	0.06	0.00
$k_2 = 200$	AR	0.01	−0.01	0.02	0.02	0.04	0.06	0.01	0.01	0.00
	SD	0.00	0.01	0.00	0.00	0.00	0.01	0.01	0.00	0.02
	SR	0.05	−0.07	0.16	0.12	0.32	0.42	0.04	0.06	−0.04
	MD	0.09	−0.02	−0.02	−0.08	0.03	−0.16	−0.16	−0.28	0.21
	MDD	−60	−23	67	−64	−568	−840	−728	−558	−153
						Average across samples given k_1, k_2				
$k_1 = 20$	TR	4.56	8.74	3.66	11.53	7.47	7.25	12.54	23.79	2.45
$k_2 = 50$	AR	0.02	0.01	0.02	0.03	0.04	0.02	0.05	0.03	0.00
	SD	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	0.00	0.00
	SR	0.14	0.08	0.14	0.17	0.24	0.13	0.33	0.21	−0.05
	MD	−0.06	0.12	−0.16	0.10	−0.14	0.06	−0.01	0.16	0.52
	MDD	−786	6	−1160	−271	−534	−277	−643	−62	772
$k_1 = 20$	TR	4.56	0.36	3.66	−1.35	7.47	5.53	23.95	5.53	12.33
$k_2 = 100$	AR	0.02	−0.01	0.02	−0.01	0.04	0.02	0.03	0.00	0.03
	SD	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	0.00	0.00
	SR	0.14	−0.07	0.14	−0.05	0.24	0.11	0.24	0.02	0.19
	MD	−0.06	0.12	−0.16	0.22	−0.14	−0.19	0.28	0.40	−0.20
	MDD	−786	−34	−1160	−13	−534	−199	8	417	−398
$k_1 = 50$	TR	8.74	0.06	11.53	0.82	2.77	−15.35	13.04	10.70	−4.34
$k_2 = 200$	AR	0.01	−0.01	0.03	0.01	0.02	0.02	0.01	0.01	0.00
	SD	0.00	0.01	0.00	0.00	0.00	0.02	0.00	0.00	0.02
	SR	0.08	−0.09	0.17	0.02	0.12	0.07	0.04	0.07	−0.03
	MD	0.12	−0.14	0.10	−0.07	0.13	1.03	−0.25	−0.25	1.12
	MDD	6	41	−271	−111	−337	205	59	−378	−90

Notes: (1) Table entries have the differences in strategy evaluation statistics between the proposed modified rule and the standard cross-over rules as in Section 2.2 of the main text; (2) MA_i denotes the price cross-over strategy based on k_i with WMA_i and EMA_i denoting weighted and exponential moving averages respectively; (3) *MACO*, *WMACO* and *EMACO* denote the moving averages cross-over strategies; (4) *TR* is the difference in total return, *AR* is the difference in annualized return, *SD* is the difference in annualized std. deviation, *SR* is the difference in annualized Sharpe ratios, *MD* is the maximum drawdown and *MDD* is the maximum drawdown duration (in days).

Table 3
Strategy evaluation statistics for S&P500 index.

		MA_1	MA_2	WMA_1	WMA_2	EMA_1	EMA_2	*MACO*	*WMACO*	*EMACO*
					$S_1, n_1 = 15\,310$					
$k_1 = 20$	TR	6.26	−6.35	8.24	7.17	10.44	3.95	42.37	5.51	7.55
$k_2 = 50$	AR	0.02	−0.01	0.02	0.01	0.02	0.00	0.06	0.01	0.01
	SD	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	0.00	0.01
	SR	0.12	−0.10	0.17	0.11	0.19	0.03	0.48	0.05	0.03
	MD	−0.02	0.14	−0.13	−0.59	−0.61	−0.28	−0.22	0.18	0.08
	MDD	−294	293	−556	−406	−96	−417	−1195	247	297
$k_1 = 20$	TR	6.26	8.08	8.24	10.55	10.44	−13.27	48.08	27.60	39.61
$k_2 = 100$	AR	0.02	0.01	0.02	0.01	0.02	−0.01	0.04	0.02	0.02
	SD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	SR	0.12	0.03	0.17	0.07	0.19	−0.09	0.35	0.15	0.15
	MD	−0.02	−0.21	−0.13	−0.23	−0.61	0.12	0.06	0.00	0.22
	MDD	−294	−38	−556	−270	−96	179	−185	126	138
$k_1 = 50$	TR	−6.35	−13.59	7.17	−11.71	4.39	−13.11	−32.25	39.70	−28.35
$k_2 = 200$	AR	−0.01	−0.01	0.01	−0.01	0.00	−0.01	−0.03	0.02	−0.03
	SD	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.00	0.01
	SR	−0.10	−0.11	0.11	−0.09	0.03	−0.12	−0.18	0.13	−0.25
	MD	0.14	0.39	−0.59	0.18	−0.28	0.37	0.23	0.19	0.57
	MDD	293	1159	−406	225	−417	711	1027	64	1516
					$S_4, n_1 = 2937$					
$k_1 = 20$	TR	−0.09	−0.20	0.21	0.35	0.28	0.16	0.72	0.12	0.22
$k_2 = 50$	AR	−0.01	−0.03	0.04	0.05	0.05	0.02	0.10	0.01	0.02
	SD	0.00	0.00	−0.01	0.00	0.00	0.00	−0.02	−0.01	0.00
	SR	−0.09	−0.19	0.22	0.36	0.32	0.13	0.67	0.10	0.18
	MD	−0.01	0.21	−0.21	−0.41	−0.48	−0.08	−0.39	0.01	−0.05
	MDD	24	594	−71	−968	−238	−280	−1327	−240	−148
$k_1 = 20$	TR	−0.09	0.08	0.21	0.01	0.28	0.06	0.72	0.26	0.67
$k_2 = 100$	AR	−0.01	0.01	0.04	0.00	0.05	0.01	0.08	0.03	0.07
	SD	0.00	0.00	−0.01	0.00	0.00	0.00	−0.01	−0.01	0.00
	SR	−0.09	0.08	0.22	0.01	0.32	0.04	0.59	0.22	0.55
	MD	−0.01	−0.11	−0.21	0.02	−0.48	−0.02	−0.26	−0.05	−0.14
	MDD	24	−103	−71	−39	−238	−32	−1042	−148	−696
$k_1 = 50$	TR	−0.20	0.06	0.35	0.01	0.14	−0.09	−0.65	0.00	−0.86
$k_2 = 200$	AR	−0.03	0.01	0.05	0.00	0.02	−0.01	−0.07	0.00	−0.10
	SD	0.00	0.01	0.00	0.00	0.00	0.02	0.00	0.00	0.05
	SR	−0.19	0.03	0.36	0.00	0.11	−0.14	−0.46	−0.01	−0.65
	MD	0.21	0.04	−0.41	0.02	−0.05	0.02	0.13	0.00	0.82
	MDD	594	19	−968	4	−260	9	729	72	1264
					Average across samples given k_1, k_2					
$k_1 = 20$	TR	1.81	−2.12	2.59	2.43	3.33	1.34	13.53	1.39	2.11
$k_2 = 50$	AR	0.00	−0.02	0.03	0.03	0.04	0.01	0.07	0.01	0.01
	SD	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	0.00	0.00
	SR	0.03	−0.13	0.19	0.23	0.26	0.08	0.54	0.06	0.06
	MD	−0.01	0.15	−0.15	−0.55	−0.58	−0.23	−0.31	0.06	0.01
	MDD	−139	368	−265	−637	−132	−383	−1279	−23	29
$k_1 = 20$	TR	1.81	2.28	2.59	3.21	3.33	−4.12	14.26	7.57	11.94
$k_2 = 100$	AR	0.00	0.01	0.03	0.01	0.04	−0.01	0.05	0.02	0.04
	SD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	SR	0.03	0.05	0.19	0.06	0.26	−0.07	0.40	0.13	0.26
	MD	−0.01	−0.18	−0.15	−0.17	−0.58	0.09	−0.14	−0.03	0.03
	MDD	−139	−54	−265	−212	−132	126	−665	−31	−313
$k_1 = 50$	TR	−2.12	−4.27	2.43	−3.65	1.45	−4.20	−11.17	11.70	−10.08
$k_2 = 200$	AR	−0.02	0.00	0.03	0.00	0.01	−0.01	−0.04	0.01	−0.06
	SD	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.02
	SR	−0.13	−0.06	0.23	−0.04	0.08	−0.11	−0.27	0.11	−0.39
	MD	0.15	0.24	−0.55	0.10	−0.22	0.19	0.17	0.06	0.70
	MDD	368	712	−637	112	−378	410	798	−139	1576

Notes: (1) Table entries have the differences in strategy evaluation statistics between the proposed modified rule and the standard cross-over rules as in Section 2.2 of the main text; (2) MA_i denotes the price cross-over strategy based on k_i with WMA_i and EMA_i denoting weighted and exponential moving averages respectively; (3) *MACO*, *WMACO* and *EMACO* denote the moving averages cross-over strategies; (4) *TR* is the difference in total return, *AR* is the difference in annualized return, *SD* is the difference in annualized std. deviation, *SR* is the difference in annualized Sharpe ratios, *MD* is the maximum drawdown and *MDD* is the maximum drawdown duration (in days).

of Table 3 we see that, in terms of the total return difference *TR*, the proposed modified strategy is better 70% of the time with an average gain of 1600% (while the average total return among all strategies is 650%). Among the price cross-over strategies the best performers are the modified 20-day weighted moving average with a gain of 1000% and the modified 20-day moving average with a gain of 1000% as well, while among the moving average cross-overs the best performers are the modified (20, 50) and (20, 100)-days simple moving average (as in the case of the DJIA) with gains of 4800% and 4200% respectively. The modified strategies are also better in terms of their Sharpe ratios: 70% of the time they are better with average gains of 13% (across the winning strategies) and 6% (across all strategies) respectively. The modified strategies exhibits consistently lower maximum drawdown and lower drawdown duration: the average drawdown gain is −33% for the winning strategies, with duration gains of −382 days, while the corresponding gains across all strategies are −4% and −39% days, still quite substantial improvements over the standard strategies.

Turning next to the results on the smallest evaluation period (S4), which is directly comparable to the DJIA, we see improved performance as well. Regarding the difference in total return (second panel of Table 3) we find that 70% of the time the modified strategies are better than the standard ones with an average gain of 24% across these winning strategies (and 10% across all strategies). Among the price cross-over strategies the best performers are the modified 50-day weighted moving average with a gain of 35% and the modified 20-day exponential moving average with a gain of 28%, while among the moving average cross-over strategies the best performers are the modified (20, 100)-days exponential moving average and (20, 100)-days weighted moving average with gains of 72% and 64% respectively. For the Sharpe ratio we find that the modified strategies are better 70% of the time with an average gain to risk–reward trade-off of 22% (among the winning strategies) and 10% (among all strategies); these averages are again better than the ones for the largest evaluation period. Furthermore, the performance based on maximum drawdown and its duration is also better than before: based on maximum drawdown the modified strategies were better 67% of the time with an average gain of −18% across the winning strategies while the average gain was −7% across all strategies. The results for the maximum drawdown duration, where 63% of the time the modified strategies had smaller duration, are also very good with an average gain in duration of −460 days, while the overall average duration is again better at −131 days. All in all, the results on these major US indices over two different time spans show that the proposed modification can produce substantial gains in terms of both higher return and lower risk for an active investor. The robustness of these findings is further examined in the discussion on the ETFs that follows. As in the case of the DJIA, the performance results for the average across evaluation periods in the third panel of Table 3 continue to support the modified strategy.

4.2. Results on EUR/USD exchange rate

For the results on the EUR/USD exchange rate we concentrate on faster look-back periods of (k_1, k_2) equal to (5, 20), (10, 20) and (20, 50) days (with all other cases available as well). The nature of the foreign exchange market, with trading taking place around the clock and more 'aggressive' investors, is such that it allows for higher profitability in shorter horizons. To provide a flavour of the method in a different set of moving average parameters we have in Table 4 the results from these shorter look-back periods. Looking at the first panel of Table 4 we see that, in terms of the total return difference *TR*, modified strategy is better 78% of the time with an average gain of 25% (across all winning strategies) and of 18% (across all strategies) respectively. Among the price cross-over strategies the best performer is the modified 10-day weighted moving average with a gain of 69% (the 20-day moving average is second best with a gain of 35%) while among the moving average cross-overs the best performers are the modified (5, 20) moving average and the (10, 20) weighted moving average with gains of 55% and 37% respectively. In terms of the risk–reward the modified strategies are better 70% of the time with average Sharpe ratio gains over the standard ones of 25% (across the winning strategies) and 14% (across all strategies) respectively. Turning to the maximum drawdown and its duration we see something quite interesting: while in terms of drawdown the modified and standard strategies are basically on par in terms of drawdown duration the modified strategies easily outperform the standard ones buy over −100 days.

Finally, when we look at the results on the second panel of the table for the period starting from March 2009 we see some interesting results as well. Here, 67% of the time the modified strategies have better total return and Sharpe ratio compared to the standard ones. However, the gains are small for total return and large for Sharpe ratio (in fact, the risk–reward gains are the highest among those presented in Table 4). Across all strategies the gain in total return is just 2% but the gain in the Sharpe ratio is 25%, the latter rising to 63% among the winning strategies. Note that the average maximum drawdown duration among all strategies is essentially 'destroyed' by a single strategy (exponential moving average cross-over) since in 70% of the time the modified strategies have smaller duration than the standard ones.

The results across all evaluation periods are qualitatively similar to what we just discussed, as can be seen from the third panel of Table 4.

5. Discussion on strategy usage

Of interest is to examine a number of additional issues with the use of the proposed methodology. First, which one of the two types of cross-overs – price or moving averages – performs best on average? Focusing on the set of results for the two

Table 4
Strategy evaluation statistics for EUR/USD.

		MA_1	MA_2	WMA_1	WMA_2	EMA_1	EMA_2	$MACO$	$WMACO$	$EMACO$
						$S_1, n_1 = 2558$				
$k_1 = 5$	TR	0.30	0.35	0.02	−0.07	0.26	0.17	0.55	0.29	0.17
$k_2 = 20$	AR	0.03	0.03	−0.01	−0.01	0.01	0.01	0.04	0.03	0.01
	SD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	SR	0.29	0.34	−0.05	−0.14	0.19	0.16	0.44	0.29	0.10
	MD	−0.03	−0.01	−0.01	0.05	0.02	0.00	0.01	0.02	0.00
	MDD	−164	−423	−64	−7	106	−162	−123	−105	−58
$k_1 = 10$	TR	0.38	0.35	0.69	−0.07	0.17	0.17	0.21	0.37	−0.01
$k_2 = 20$	AR	0.04	0.03	0.07	−0.01	0.01	0.01	0.01	0.03	−0.02
	SD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	SR	0.39	0.34	0.76	−0.14	0.17	0.16	0.10	0.38	−0.11
	MD	−0.02	−0.01	−0.10	0.05	0.01	0.00	0.02	0.01	0.07
	MDD	−198	−423	−243	−7	−96	−162	45	−116	97
$k_1 = 20$	TR	0.35	0.13	−0.07	0.05	0.17	−0.11	0.19	−0.10	0.02
$k_2 = 50$	AR	0.03	0.01	−0.01	−0.01	0.01	−0.02	0.01	−0.02	0.00
	SD	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	−0.01	0.00
	SR	0.34	0.10	−0.14	−0.09	0.17	−0.19	0.15	−0.19	0.01
	MD	−0.01	0.02	0.05	0.05	0.00	−0.02	−0.13	−0.04	−0.02
	MDD	−423	1	−7	−24	−162	−100	−186	129	−53
						$S_4, n_1 = 539$				
$k_1 = 5$	TR	−0.02	0.04	0.01	0.02	0.08	0.00	0.11	0.04	0.05
$k_2 = 20$	AR	−0.02	0.05	0.01	0.04	0.11	−0.02	0.15	0.04	0.04
	SD	0.00	−0.01	0.01	0.01	0.00	−0.01	−0.01	−0.01	0.00
	SR	−0.18	0.49	0.14	0.35	1.03	−0.09	1.51	0.48	0.41
	MD	0.00	−0.02	−0.02	−0.01	−0.03	−0.01	−0.06	0.01	0.01
	MDD	−2	−9	−8	−3	−107	−2	−46	45	32
$k_1 = 10$	TR	0.05	0.04	0.10	0.02	0.05	0.00	0.07	0.09	−0.01
$k_2 = 20$	AR	0.06	0.05	0.14	0.04	0.07	−0.02	0.09	0.12	−0.06
	SD	0.00	−0.01	0.00	0.01	0.00	−0.01	−0.01	−0.01	0.01
	SR	0.55	0.49	1.30	0.35	0.65	−0.09	0.88	1.18	−0.58
	MD	−0.02	−0.02	−0.05	−0.01	−0.02	−0.01	−0.05	−0.06	0.06
	MDD	−46	−9	−131	−3	−16	−2	−38	−44	62
$k_1 = 20$	TR	0.04	−0.05	0.02	−0.07	0.00	0.03	0.05	−0.06	−0.13
$k_2 = 50$	AR	0.05	−0.08	0.04	−0.10	−0.01	0.03	0.08	−0.08	−0.14
	SD	−0.01	0.00	0.01	0.00	−0.01	0.00	0.00	0.01	0.00
	SR	0.49	−0.76	0.35	−0.97	−0.09	0.36	0.70	−0.83	−1.32
	MD	−0.02	0.05	−0.01	0.03	−0.01	−0.03	−0.01	0.09	0.11
	MDD	−9	57	−3	58	14	−4	−15	40	212
						Average across samples given k_1, k_2				
$k_1 = 5$	TR	0.10	0.16	0.00	−0.02	0.15	0.11	0.28	0.11	0.02
$k_2 = 20$	AR	0.01	0.04	0.00	0.01	0.06	0.02	0.09	0.03	−0.01
	SD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	SR	0.09	0.38	0.01	0.04	0.54	0.19	0.81	0.27	−0.03
	MD	−0.01	0.01	−0.02	0.04	−0.01	−0.02	0.00	0.03	0.03
	MDD	−92	−149	−39	−25	−40	−111	−21	3	32
$k_1 = 10$	TR	0.21	0.16	0.36	−0.02	0.11	0.11	0.08	0.19	−0.09
$k_2 = 20$	AR	0.07	0.04	0.13	0.01	0.04	0.02	0.02	0.06	−0.06
	SD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	SR	0.61	0.38	1.13	0.04	0.36	0.21	0.22	0.61	−0.50
	MD	−0.03	0.01	−0.08	0.04	0.01	−0.02	0.01	0.00	0.06
	MDD	−133	−149	−234	−25	−89	−112	97	−40	192
$k_1 = 20$	TR	0.16	−0.02	−0.02	−0.03	0.11	−0.14	0.12	−0.10	−0.09
$k_2 = 50$	AR	0.04	−0.04	0.01	−0.05	0.02	−0.05	0.05	−0.05	−0.06
	SD	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	−0.01	0.01
	SR	0.38	−0.34	0.04	−0.40	0.20	−0.43	0.44	−0.49	−0.55
	MD	0.01	0.03	0.04	0.05	−0.02	0.04	−0.09	−0.01	0.05
	MDD	−149	53	−25	55	−93	57	−137	53	85

Notes: (1) Table entries have the differences in strategy evaluation statistics between the proposed modified rule and the standard cross-over rules as in Section 2.2 of the main text; (2) MA_i denotes the price cross-over strategy based on k_i with WMA_i and EMA_i denoting weighted and exponential moving averages respectively; (3) *MACO*, *WMACO* and *EMACO* denote the moving averages cross-over strategies; (4) *TR* is the difference in total return, *AR* is the difference in annualized return, *SD* is the difference in annualized std. deviation, *SR* is the difference in annualized Sharpe ratios, *MD* is the maximum drawdown and *MDD* is the maximum drawdown duration (in days).

Table 5
Number of additional trades of the modified strategy.

	n_1	(k_1, k_2)	MA_1	MA_2	WMA_1	WMA_2	EMA_1	EMA_2	MACO	WMACO	EMACO
DJIA	20 618	20, 50	49	107	−51	55	−39	44	308	273	216
		20, 100	49	112	−51	72	−39	79	216	243	203
		50, 200	107	104	55	97	44	85	137	197	114
	2 937	20, 50	11	22	−8	2	−18	13	57	55	41
		20, 100	11	31	−8	37	−18	22	62	46	54
		50, 200	22	22	2	11	14	25	28	52	20
SP500	20 618	20, 50	44	101	−33	67	−38	27	212	208	174
		20, 100	44	104	−33	86	−38	30	179	187	140
		50, 200	101	48	67	76	27	26	113	106	77
	2 937	20, 50	28	29	−7	19	−1	9	48	50	37
		20, 100	28	38	−7	20	−1	15	44	52	35
		50, 200	101	48	67	76	27	26	113	106	77
EUR/USD	2 558	5, 20	−61	−2	−93	−10	−86	−20	27	29	28
		10, 20	−20	−2	−36	−10	−25	−20	35	41	36
		20, 50	−2	22	−10	−4	−20	7	26	33	27
	5 39	5, 20	−3	−2	−12	−1	−16	−5	7	5	1
		10, 20	1	−2	−6	−1	−2	−6	3	10	4
		20, 50	−2	2	−1	−1	−6	3	8	9	13

Notes: (1) Table entries have the differences in strategy evaluation statistics between the proposed modified rule and the standard cross-over rules as in Section 2.2 of the main text; (2) MA_i denotes the price cross-over strategy based on k_i with WMA_i and EMA_i denoting weighted and exponential moving averages respectively; (3) *MACO*, *WMACO* and *EMACO* denote the moving averages cross-over strategies; (4) *TR* is the difference in total return, *AR* is the difference in annualized return, *SD* is the difference in annualized std. deviation, *SR* is the difference in annualized Sharpe ratios, *MD* is the maximum drawdown and *MDD* is the maximum drawdown duration (in days).

indices we find the moving average cross-overs are better performers (in terms of difference in total return) than the price cross-overs 54% of the time. In particular, the percentage of outperformance rises to 78%.

Second, which of the types of moving averages used (plain, weighted and exponential) appears as a top performer most of the time? Again focusing on the difference in total return, we find that for the price cross-over strategies the plain moving average is top performer 26% of the time, the weighted moving average 34% of the time and the exponential moving average 40% of the time; the corresponding percentages for the moving average cross-over strategy are 32%, 40% and 28%. If we look at DJIA and S&P500 we find that for the price cross-over strategy the weighted moving average is best 56% of the time and the exponential moving average is best 44% of the time; for the moving average cross-over strategy the plain moving average is best 16% of the time, the weighted moving average 28% of the time and the exponential moving average 56% of the time. One cannot easily draw a generic conclusion as to which type of moving average works best with the modified strategy but the weighted and exponential moving averages appear to be safer bets to use than the plain moving average. For the two indices, where the moving average cross-over strategy is better 78% of the time, we get a clear indication that the exponential moving average works best most of the time.

Third, for the price cross-over strategy, what is the average and median look-back period for the top performers? We find that the average (median) length of the moving average is 36 (20) days for the two indices. Since we have concentrated on fixed look-back periods the median values are here more appropriate and the results do support the use of the 20-day look-back period in the price cross-over strategy.

An important practical issue on any strategy relates to the number of trades, as these affect the transaction costs. Since the proposed modification acts as a dynamic trailing stop we expect a possibly increased number of trades compared to the standard strategy, although it turns out that this is highly data specific. We present our results in Table 5, in the same form as in previous tables, i.e. as differences with respect to the trades of the standard strategy—and we discuss the same types of averages across the tables cells as before. We start off by discussing the results for the largest evaluation period. For the EUR/USD exchange rate we actually have 4 less trades than the standard strategy, on average, with 55% of the time having less rather than more trades. For the two indices we find that the average number of extra trades is 103 for the DJIA and 77 for the S&P500, that correspond to less than 0.5% of the days of their evaluation samples.

If we next look at the number of trades for the smaller evaluation periods we find that, on average, there are no more trades for EUR/USD compared to the standard strategy. For this exchange rate series (and for the chosen look-back periods) the strategy appears that can be used safely and successfully. For the other series we have results similar to the larger evaluation period: the average number of extra trades is 22 for the DJIA and 40 for the S&P500, that correspond to less than 1% of the days of their evaluation samples. These results are in line with our previous findings: it appears that the smaller drawdowns and the smaller duration may be attributable (in part) to the timing of these extra trades (for the equity series) or the decreased trades (for the exchange rate series).

The effect of these extra trades on total return is, of course, negative but it should not affect our results considerably—the final effect depends on the strategy and its performance and rests with the investor's trade-off with respect to increased gains & lower drawdowns vs. increased number of trades.

6. Concluding remarks

In this paper we present a modification to, the widely used, price and moving average cross-over trading strategies. The modification is based on an updated threshold value which is defined by the 'buy' signal of the standard cross-over strategy and acts as a dynamic trailing stop. This implies a different behaviour and performance for the modified strategy compared to the standard one and we find that, on average, the modification improves trading performance by a wide margin across a number of evaluation measures. More importantly, besides increasing the cumulative return of an investor it does so without increasing the risk–reward ratio: the modified strategy exhibits, on average, smaller maximum drawdown and smaller drawdown duration.

Our analysis is evaluated in three series: the DJIA and S&P500 for a long-run period of 80 and 60 years respectively and the EUR/USD exchange rate for over 10 years. Our results show that, across moving average types, look-back periods and cross-over types, the modified strategy works well and, on average, outperforms both the standard strategy and the buy & hold strategy, sometimes very substantially.

An important aspect of our on-going work is to examine in more detail the performance of the proposed modification, particularly across yet different evaluation periods, and to understand further and better the underlying reasons for which it appears to work. In particular, further study is required on the properties of the returns generated by the modified strategy, on additional results in foreign exchange markets and on the timing and quality of its trading signals. We are pursuing them in current work.

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