Глава 7

№ 6.4

1) Ф-ла трапеции

$$I_{тр} = h\sum_{i=0}^{N-1} \frac{1}{2}\left(f(x_i) + f(x_{i+1})\right)$$

$$\mathcal{E} \leq \frac{1}{12} \max_{[a;b]} |f''(\xi)| h^2 (b-a)$$

2) Ф-ла Симпсона

$$I_C = \frac{h}{6}\sum_{i=0}^{N-1}\left(f(x_i) + 4f(x_{i+\frac{1}{2}}) + f(x_{i+1})\right)$$

$$\mathcal{E} \leq \frac{1}{2880} \max_{[a,b]} |f^4(\xi)| h^2 (b-a)$$

П-ло Рунге

$$I - I\left(\frac{h}{2}\right) = \frac{I\left(\frac{h}{2}\right) - I(h)}{3}$$

$$I = \frac{4}{3} I\left(\frac{h}{2}\right) - \frac{1}{3} I(h) = \frac{1}{3}\left(4\sum_{i=0}^{N-1} \frac{f(x_i) + f(x_{i+\frac{1}{2}})}{2} \cdot \frac{h}{2} + \right.$$

$$\left. + 4\sum_{i=0}^{N-1} \frac{f(x_{i+\frac{1}{2}}) + f(x_{i+1})}{2} \cdot \frac{h}{2} - \sum_{i=1}^{N-1} \frac{f(x_i) + f(x_{i+1})}{2} h\right) =$$

$$= \frac{h}{6}\sum_{i=0}^{N-1}\left(f(x_i) + 4f(x_{i+\frac{1}{2}}) + f(x_{i+1})\right)$$

Порядок возрос до 4 со 2 => $\Delta = 2$

№ 8.6

$P(x) = ax^2 + bx + c$ — интерполяция на отрезке $[nh, (n+1)h]$

$$\int_{a+nh=l}^{a+(n+1)h=r} (ax^2+bx+c)\,dx = \frac{1}{3}ah(l^2+lr+r^2) + \frac{1}{2}bh(l+r) + ch =$$

$$= \frac{h}{6}\left((al^2+bl+c) + (ar^2+br+c) + al^2 + 2arl + ar^2 + 2bl + 2br + 4c\right) =$$

$$= \frac{h}{6}\left(P(l) + P(r) + 4\left(a\frac{(l+r)^2}{4} + b\frac{(l+r)}{2} + c\right)\right) = \frac{h}{6}\left(P(r) + 4P\left(\frac{r+l}{2}\right) + P(l)\right)$$

$$I = \sum_{i=0}^{N-1} \frac{h}{6}\left(P(x_i) + 4P\left(x_{i+\frac{1}{2}}\right) + P(x_{i+1})\right)$$ ч.т.д.

№ 8.14 (а)

$$I = \int_0^1 \sin(x^2)\,dx = \frac{1}{2}\int_{-1}^{1} \sin(x^2)\,dx \approx y$$

т.к. $n = 2$

$$= y = \sum_{i=1}^{2} c_i f(x_i) = \frac{1}{2}\left(1 \cdot \sin\left(\frac{\sqrt{3}}{3}\right)^2 + 1 \cdot \sin\left(-\frac{\sqrt{3}}{3}\right)^2\right) = 2 \cdot 0{,}327$$

$\Downarrow$

$I = 0{,}327$

Оценим погрешность

$$\Delta = \frac{(b-a)^{2n+1}(n!)^4}{(2n+1)((2n)!)^3} f^{(2n)}(\xi) = \frac{2 \cdot 16}{5 \cdot 24^3} \max_{[-1;1]} f^{4} = 0{,}036$$

Ответ: $I = 0{,}327$; $\Delta = 0{,}036$

№ 8.19

$$I = \int_0^1 \frac{\ln x}{\sqrt{1-x}}\,dx, \ \varepsilon_0 = 10^{-3}$$

1) $t = \sqrt{1-x}, \ x = 1-t^2, \ dx = -2t\,dt$

$$I = 2\underbrace{\int_0^1 \ln(1-t)\,dt}_{I_1} + 2\underbrace{\int_0^1 \ln(1+t)\,dt}_{I_2}$$

~~$\varepsilon(I_1) \le \frac{\varepsilon_0}{4}$~~

~~$\varepsilon(I_2) \le \frac{\varepsilon_0}{4}$~~

2) ~~ε~~ $I_1 = -\int_0^1 \ln(1-t)\,d(1-t) = -1$

3) $\varepsilon(I_2) \le \frac{\varepsilon_0}{2}$ Будем считать по ф-ле Трапеции

$$\frac{1}{12}\max_{[0,1]}\left(\frac{1}{(1+t)^2}\right)h^2 \le \frac{\varepsilon_0}{2}$$

$$h \le \sqrt{6\varepsilon_0} = 0{,}08$$

$$N = \left[\frac{1}{0{,}08}\right] = 13$$

$$I_n = 2(I_1 + I_2) = -1{,}228$$

Ответ: $-1{,}228$

№ 8.25в

$$\int_0^4 \frac{e^{-\sqrt{x}}-1}{\sqrt{4x^2-x^3}}\,dx, \ \varepsilon \qquad , \ t=\sqrt{4-x}$$

$$I = 2\int_0^2 \frac{e^{-\sqrt{4-t^2}}-1}{4-t^2}\,dt = 2\int_0^2 \frac{e^{-\sqrt{4-t^2}}-1+\sqrt{4-t^2}}{4-t^2}\,dt -$$

$$-2\int_0^2 \frac{dt}{\sqrt{4-t^2}} \qquad \text{по методу средних}$$

$$\frac{\pi}{2}$$