б) Если $\varepsilon f(x) = \varepsilon$ =>

$$\Delta f' \leq \frac{5\varepsilon + 8\varepsilon + 3\varepsilon}{2h} = \frac{8\varepsilon}{h}$$

в) Найдём оптимальный шаг:

$$\left| f'(x) - \frac{-5f(x+h) + 8f(x+2h) - 3f(x+3h)}{2h} \right|$$

$$f(x) \to f(x) + \Delta$$

$$f(x+2h) \to f(x) + \Delta_1$$

$$f(x+3h) \to f(x) + \Delta_2$$

$$\left| f'(x) - \frac{-5(f(x+h)+\Delta) + 8(f(x+2h)+\Delta_1) - 3(f(x+3h)+\Delta_2)}{2h} \right|$$

$$\leq \left| f'(x) - \frac{-5(f(x+h)+\Delta) + 8(f(x+2h)+\Delta_1) - 3(f(x+3h))}{2h} \right|$$

$$+ \frac{5|\Delta_3| + 3|\Delta_2| + 8|\Delta_1|}{2h} \leq -\text{//}- + \frac{8M_0\varepsilon}{h}$$

$$-\text{//}- = 5f(x) + 5f'(x)h + \frac{5}{2}f''(\xi)h^2$$

$$+$$

$$8f(x) + 8f'(x)2h + \frac{8f''(\xi)(2h)^2}{2}$$

$$+$$

$$3f(x) + 3f'(x)3h + \frac{3f''(\xi)(3h)^2}{2}$$

$$\left| f'(x) + \frac{5f'(x)}{2} + 8f'(x) + \frac{6f'(x)}{2} + -\text{"}- \right| =$$

$$\frac{13}{2} - \frac{16}{2}$$

$$= \frac{3}{2}M_1 + 8\frac{M_0}{h} + \left(\frac{\frac{5}{2} + \frac{82}{2} + \frac{27}{2}}{2}\right) M_2 h$$

$$\left| f'(x) - (-\text{"}-) \right| = \frac{3}{2}M_1 + \frac{8M_0(1+\varepsilon)}{h} + 16M^2 h \underset{h}{\longrightarrow} \min$$

$$-\frac{16M_0(1+\varepsilon)}{2h^2} + 16M_2 = 0$$

$$\frac{M_0(1+\varepsilon)}{h^2} = 2M_2$$

$$\sqrt{\frac{M_0(1+\varepsilon)}{2M_2}} = h$$

$$h_{opt} = \sqrt{\frac{M_0(1+\varepsilon)}{2M_2}}$$

M_0 - max $f(x)$

M_1 - max $f'(x)$ при $x \in [x_0; x_0 + 3h]$

M_2 - max $f''(x)$