# Работа над ошибками

№1(2)

1) $-\frac{9}{64} \Rightarrow$ представим $\frac{9}{64}$

$\frac{9}{64} - \frac{1}{2} < 0 \Rightarrow 2^{-2}\left(\frac{1}{2} + 0 + 0\right) = \frac{1}{8}$ — отбрасываем $-\frac{16}{64}$ — дополнение

4) $-15 \Rightarrow 2^3\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) = 7 \Rightarrow |7| < |-15| \Rightarrow fl(-15) = \infty$

3) $\frac{23}{256} < \frac{1}{8} \Rightarrow fl\left(\frac{23}{256}\right) = 0$

№1(3)

$\eta_{\frac{9}{64}} = \left[ \left|\frac{9}{8} - \frac{9}{64}\right| = \left|\frac{16}{64} - \frac{9}{64}\right| = \frac{1}{64} \right.$

$\eta_{\frac{23}{256}} = \frac{23}{256} \quad \forall \left|UFL - \frac{23}{256}\right| = \frac{9}{256}$

$\eta_{-15} = \infty \quad \forall \left|OFL - 15\right| = 8$

$\eta_{\frac{9}{16}} = \left[ \begin{array}{l} \left|\frac{1}{2} - \frac{9}{16}\right| = \frac{1}{8} \\ \left|\frac{5}{6} - \frac{9}{16}\right| = \frac{1}{8} \end{array} \right.$

№ 2 (3)

Выполним 3 итерации

ПВР:

1) $\begin{pmatrix} -0,74 \\ -0,79 \end{pmatrix}$ 2) $\begin{pmatrix} 0,074 \\ -1,11 \end{pmatrix}$ 3) $\begin{pmatrix} 0,06 \\ -1,01 \end{pmatrix}$

Зейдель:

1) $\begin{pmatrix} -0,56 \\ -0,70 \end{pmatrix}$ 2) $\begin{pmatrix} -0,17 \\ -0,91 \end{pmatrix}$ 3) $\begin{pmatrix} -0,05 \\ -0,97 \end{pmatrix}$

Якоби:

1) $\begin{pmatrix} -0,57 \\ -1 \end{pmatrix}$ 2) $\begin{pmatrix} -0,[illegible] \\ -0,7 \end{pmatrix}$ 3) $\begin{pmatrix} -0,17 \\ -1 \end{pmatrix}$

№ 2 (2)

$A\begin{pmatrix} x \\ y \end{pmatrix} = b \qquad A = \begin{pmatrix} 44 & 25 \\ 25 & 48 \end{pmatrix} \quad b = \begin{pmatrix} -25 \\ -48 \end{pmatrix}$

Итерация Зейделя

$$x_i^{(s+1)} = \sum_{j=1}^{i-1} c_{ij} x_j^{s+1} + \sum_{j=1}^{n} c_{ij} x_j^{s} + d_i \qquad d_i = \frac{b_i}{a_{ii}} \quad c_{ij} = -\frac{a_{ij}}{a_{ii}}$$

$c_{ii} = 0$

$$\begin{cases} x^{s+1} = -\frac{a_{12}}{a_{11}} y^{s} + \left(-\frac{25}{44}\right) + c_{11} x^{s} \\ y^{s+1} = -\frac{a_{21}}{a_{22}} x^{s} + \left(-\frac{48}{48}\right) \end{cases} \qquad \begin{cases} x^{s+1} = \\ y^{s+1} = \end{cases}$$

$y^{s+1} = \frac{a_{21}}{a_{22}} x^{s+1}$

$$\begin{cases} x^{s+1} = c_{11} x_1^s + c_{12} y^s + d_1 \\ y^{s+1} = c_{21} x^{s+1} + c_{22} y^s + d_2 \end{cases}$$

$$c_{11} = 0$$

$$c_{12} = -\frac{a_{12}}{a_{11}} = -\frac{25}{44} \qquad d_1 = \frac{b_1}{a_{11}}$$

$$c_{21} = -\frac{a_{21}}{a_{22}} = -\frac{25}{48} \qquad d_2 = \frac{b_2}{a_{22}} =$$

$$\begin{cases} x^{s+1} = -\frac{25}{44} y^s - \frac{25}{44} \\ y^{s+1} = -\frac{25}{48} x^{s+1} - \frac{48}{48} \end{cases}$$

$$\begin{cases} x^{s+1} = -\frac{25}{44}(1 + y^s) \\ y^{s+1} = -\frac{25}{48} x^{s+1} - \frac{48}{48} \end{cases}$$

ПВР: $\omega = 1,3$

$$\begin{cases} x^{s+1} = (1 - 1,3) x^s - \frac{1,3 \cdot 25}{44}(1 - y^s) \\ y^{s+1} = (1 - 1,3) y^s + \omega\left(-\frac{25}{48} x^{s+1} - 1\right) \end{cases}$$

$$\begin{cases} x^{s+1} = -0,3 x^s - 0,73(1 - y^s) \\ y^{s+1} = -0,3 y^s + 1,3\left(-\frac{25}{48} x^{s+1} - 1\right) \end{cases}$$

№5 (1)

$\|x\| = \max(2|x_1|, |2x_1 + 2x_2|)$

$C = \begin{pmatrix} 2 & 0 \\ 2 & 2 \end{pmatrix}$

1) $\|\lambda x\| = |\lambda| \|x\|$
2) $\|x + y\| \le \|x\| + \|y\|$ } доказано в кр
3) $\|x\| = 0 \Leftrightarrow x = 0$

$$\begin{cases} x_1 = 0 \\ 2x_1 + 2x_2 = 0 \end{cases} \Rightarrow x_1 = x_2 = 0 \Rightarrow x_1 = x_2 = 0$$

$$x_1 = x_2 = 0 \Leftrightarrow \|x\| = 0$$

№5 (2)

$A = \begin{pmatrix} 0 & 2 \\ -2 & -4 \end{pmatrix}$

$$\|A\|_* = \sup_{x \neq 0} \frac{\|Ax\|_*}{\|x\|_*} = \sup_{\|x\|_* = 1} \|Ax\|_*$$

$$\sup_{x \neq 0} \frac{\|Cx\|_1}{\|Cx\|_1} = \sup_{x \neq 0} \frac{\|CAx\|_1}{\|Cx\|_1} = \left| y = Cx \right| = \sup_{y \neq 0} \frac{\|CAx\|_1}{\|y\|_1} \Leftarrow$$

Значит, что

$$\|A\|_1 = \sup_{x \neq 0} \frac{\|Ax\|_1}{\|x\|_1} \qquad y = Cx$$

$$\Leftarrow \sup_{y \neq 0} \frac{\|CAC^{-1}y\|_1}{\|y\|_1} = \left| \|B\| = \|CAC^{-1}\| \right| = \sup_{y \neq 0} \frac{\|By\|_1}{\|y\|_1}$$

$$\|B\|_1 = \begin{pmatrix} -2 & 2 \\ 0 & -2 \end{pmatrix} = 4$$

№ 4 (1)

$$f'_{i+3} = \frac{f_{i+4} - f_{i+4}}{8h}$$

1) Порядок аппроксимации

$$R = f'(x_i + 3h) - \frac{f(x_i + 4h) - f(x_i - 4h)}{8h} =$$

$$= \begin{cases} f(x_i + 4h) = f(\underbrace{x_i + 3h}_{z_0} + h) = f(z_0) + f'(z_0)h + \frac{1}{2}f''(z_0)h^2 + O(h^3) \\ f(x_i - 4h) = f(z_0) + f'(z_0)(-7h) + \frac{1}{2}f''(z_0)(-7h)^2 + O(h^3) \end{cases}$$

$$= f'(z_0) \approx \frac{1}{8h}\left(+8h f'(z_0) + 24h^2 f''(z_0) + O(h^3)\right) =$$

$$= 3f'(z_0)h + O(h^2) \Rightarrow \text{1ый порядок аппроксимации}$$

2) $f'_{i+3} \approx \frac{f(x_{i+4}) - f(x_{i-4})}{8h} \Rightarrow \left| \tilde{f}(x_{i+4}) \Rightarrow f(x_{i+4}) + \Delta f_{i+4} \right.$
$\left. |\tilde{f}_{i+4} - f_{i+4}| = |\Delta f_{i+4}| \right.$

$$err = f'(x_{i+3}) - \frac{\tilde{f}(x_{i+4}) - \tilde{f}(x_{i-4})}{8h} = \overbrace{f'(x_{i+3}) - \frac{f(x_{i+4}) - f(x_{i-4})}{8h}}^{3f''(x_{i+4})\cdot h} +$$

$$+ \underbrace{\frac{f(x_{i+4}) - \tilde{f}(x_{i+4})}{8h} + \frac{\tilde{f}(x_{i-4}) - f(x_{i-4})}{8h}}$$

$$\leq \frac{|\Delta f_{i+4}|}{8h} + \frac{|\Delta f_{i-4}|}{8h} \leq \frac{2M_0\varepsilon}{8h} = \frac{M_0\varepsilon}{4h}$$

$$|err| \leq 3|f''(x_{i+3})|h + \frac{\varepsilon M_0}{4h} \to \min$$

$$E(h) = 3M_2 h + \frac{\varepsilon M_0}{4h} \qquad E' = 3M_2 - \frac{M_0\varepsilon}{4h^2} = 0 \qquad \boxed{h_{opt} = \sqrt{\frac{M_0\varepsilon}{12M_2}}}$$

3) $f(x) = \sin\left(8x + \frac{\pi}{16}\right)$

$f''(x) = -64 \sin\left(8x + \frac{\pi}{16}\right) \Rightarrow M_2 = 64$

$M_0 = 1$

float: $h_{opt} = \sqrt{\frac{1 \cdot 6 \cdot 10^{-8}}{12 \cdot 64}} = 8,8 \cdot 10^{-6}$

double: $h_{opt} = \sqrt{\frac{1 \cdot 60 \div 16}{12 \cdot 64}} = 3,6 \cdot 10^{-10}$