Chapter 1

Implementation of Clearing Model for the Day-Ahead Market

The simplified version of the EUPHEMIA algorithm is based on the model presented by Chatzigiannis et al. [1] that incorporates the use of block orders in exclusive groups. A reduction of complexity is achieved by limiting the algorithm and subsequent tests to a single bidding zone. Accordingly all constraints related to the existence of multiple bidding zones are removed, as no flow between neighboring zones has to be modeled. Further, the order types that are included are limited to solely hourly orders and the different types of block orders. Hourly orders that are considered, each consist of a single pair of values for quantity and price. Linear piecewise hourly orders are not part of the model. In the case of block orders, profile block orders are excluded, as they are neither acknowledged in the official EUPHEMIA public description [2], nor in the underlying implementation by Chatzigiannis et al. [1]. Linked block orders, exclusive group of block orders and flexible hourly orders prevail. The remaining order types, including complex orders, scalable complex orders, as well as PUN and merit orders are not available in the majority of bidding zones. Therefore, the added complexity of including them would not be justified by a meaningful improvement of accuracy of the test results.

1.1 Description of Solving Process

As was presented in the EUPHEMIA public description [2], the solution for the European day-ahead market can be found by solving the social welfare maximization problem. Here, it is modeled as a mixed integer linear program (MILP). As linear piecewise orders are not considered in this clearing model, the quadratic component of the original EUPHEMIA

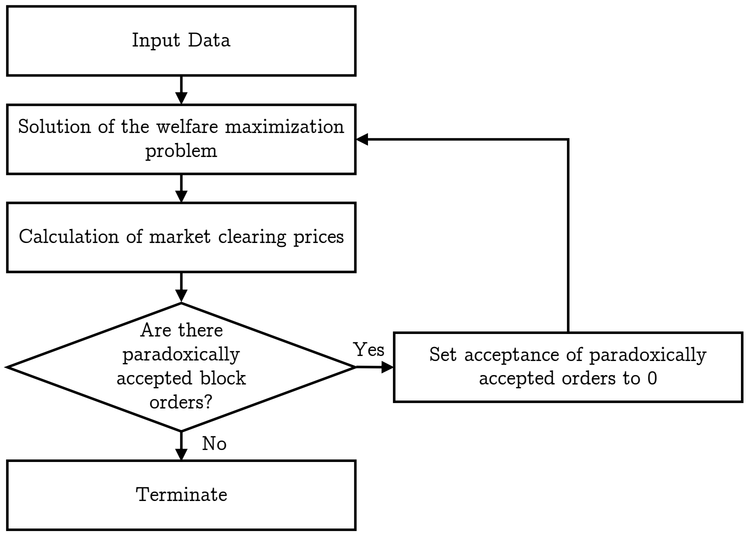

Figure 1.1: Schematic illustration of solving process.

mechanism can be omitted, and the problem formulated as a linear program. The exact mathematical formulation will be presented in the following section 1.3. The complete solving process is depicted in Figure 1.1. After solving the MILP, prices for each period are determined and subsequently, paradoxically accepted block orders are searched for, as described in section 1.4. If none can be identified, the algorithm terminates and returns a set of accepted orders, as well as clearing prices. Else, the paradoxically accepted block orders are set as rejected and the MILP is solved again. This procedure is repeated until the termination criterion of no paradoxically accepted orders existing is fulfilled.

1.2 Input Data

The solving process accepts input data that represents a list of orders. Included order types are simple hourly orders and block orders. Hourly orders are divided into supply and demand orders. Each is defined by three parameters, a price, a volume, and the specific hour. Block orders are defined with a price and a volume for any of the 24 hours of the day. Supply orders have negative volumes specified. A further parameter characterizes the type of the block order. Options are a simple block order, a linked block order, an exclusive group of block orders and a flexible hourly order. If a block order is linked or part of an exclusive group, the block code parameter includes additional information on

Parameter	Type	Unit	Description
side	String		Demand or supply order
hour	number		Hour that the bid is placed for
price	number	€/MWh	Maximum price for demand order, minimum for supply order
volume	number	MWh	Demanded or offered volume

(1.1.A) Hourly Orders

Parameter	Type	Unit	Description
blockType	String		Differentiation between simple block, linked block, block that is part of an exclusive group and flexible hourly order
blockCode	String		Block id of block to which this block is linked to or exclusive group id
price	number	€/MWh	Maximum price for demand order, minimum for supply order
volumes	list	[MWh]	Volumes for each hour, negative values for supply orders, flexible hourly orders only have volume specified for first hour

(1.1.B) Block Orders

Table 1.1: Parameters of order types that are considered by implementation of clearing mechanism.

that. In the case of a linked order, it takes the value of the ID of the block that the specific block is linked to. All blocks that are part of an exclusive group take the same value here, that corresponds to the ID of the exclusive group. For flexible hourly orders, only the volume of the first hour is specified. That volume later can be covered at any hour. The parameters of the input data are listed in Table 1.1.

1.3 Mathematical Formulation

The formulation of the MILP that represents the social welfare maximization problem is presented in this section. After presenting the nomenclature, first, the objective function is formulated, and subsequently, the constraints are constructed.

1.3.1 Nomenclature

Before formulating the problem, in this section the nomenclature for sets, parameters, and variables are introduced, that are part of the MILP, as well as the abbreviations for the result values.

Indices and Sets

$d(D)$	Index (set) of hourly demand orders
$s(S)$	Index (set) of hourly supply orders
$b(B)$	Index (set) of block orders
$t(T)$	Index (set) of trading periods within a day (typically an hour)
$lb(LB)$	Index (set) of linked block orders
$eg(EG)$	Index (set) of exclusive groups of block orders
$fh(FH)$	Index (set) of flexible hourly orders
$pa(PA)$	Index (set) of paradoxically accepted block orders

Parameters

P_d	Price of demand order d in €
Q_d	Quantity of demand order d in MWh
P_s	Price of supply order s in €
Q_s	Quantity of supply order s in MWh
P_b	Price of block order b in €
Q_b^t	Quantity of block order b in period t in MWh
R_b	Minimum acceptance ratio of block order b
A_b^{lb}	1, if block order lb is linked child of block b, else 0
A_b^{eg}	1, if block b is part of exclusive group eg, else 0
Y_d^t	1, if demand order d is placed for period t, else 0
Y_s^t	1, if demand order s is placed for period t, else 0
U_b	1, if block order b was accepted, else 0
U_{fh}^t	1, if flexible hourly order fh was accepted for period t, else 0

Variables

Positive Variables

x_d	Acceptance Ratio of demand order d
x_s	Acceptance Ratio of supply order s
x_b	Acceptance Ratio of block order b
u_b^*	Continuous acceptance Status of block order b
u_{fh}^{t*}	Continuous acceptance Status of flexible hourly order fh in period t

Binary Variables

u_b	Acceptance Status of block order b
u_{fh}^t	Acceptance Status of flexible hourly order fh in period t

Results

w^{tot}	Social welfare
ut^{dem}	Utility of demand side, excluding block orders
c^{sup}	Suppliers' cost, excluding block orders
w^{block}	Contribution of block orders to social welfare
v_{demand}^t	Accepted volume of hourly demand orders for period t
v_{supply}^t	Accepted volume of hourly supply orders for period t
v_{block}^t	Accepted volume of block orders for period t
π_t	Clearing price for period t
p_b	Profit generated by block order b

1.3.2 Objective Function

The total social welfare can be represented as the sum of the supplier and consumer surplus. The congestion rent that is also considered in the actual implementation of EUPHEMIA can be neglected, as in this model only one bidding zone exists.

$$\max w^{tot} = ut^{dem} - c^{sup} + w^{block} \tag{1.1}$$

The objective function of maximizing the total social welfare w^{tot} is given in equation 1.1. The sum of the consumer and supplier surplus can be attained by computing it individually for each order type. That includes the generated welfare through block orders w^{block}, as well as the welfare gained by hourly orders, which results from the subtraction of the suppliers' cost c^{sup} from the utility of the demand side ut^{dem}. For block orders, demand and supply do not have to be considered separately, as supply orders are denoted by specifying a negative demand. How the different components are calculated can be taken from equations 1.2, 1.3 and 1.4.

$$ut^{dem} = \sum_{d \in D} x_d * P_d * Q_d \tag{1.2}$$

$$c^{sup} = \sum_{s \in S} x_s * P_s * Q_s \tag{1.3}$$

$$w^{block} = \sum_{b \in B} \sum_{t \in T} x_b * P_b * Q_b^t \tag{1.4}$$

Single orders only contribute to the total social welfare if their acceptance x_d, x_s or x_b respectively is larger than 0. The quantitative effect of such orders is derived by multiplication of the accepted volume and the order price, with demand orders adding to the welfare and supply orders taking away from it. Thus, the price of supply orders is considered their cost that have to be covered and the utility of a matched demand order is their price.

1.3.3 Constraints

The constraints that are added to the optimization problem are defined by the various order types, as well as ensuring supply and demand volumes are matching.

Order Clearing Constraints

Firstly, an upper limit is set to the acceptance values x_d and x_s of hourly demand and supply orders. They can not exceed 1.

$$x_d \leq 1 \quad \forall d \in D \tag{1.5}$$

$$x_s \leq 1 \quad \forall s \in S \tag{1.6}$$

Further, the same constraint has to be made for any block order in the input data set. Additionally, the acceptance value of block orders has a lower bound that is determined by the minimum acceptance ratio. If a block order is fully rejected, thus, binary u_b equaling 0, the lower and upper bound will also turn into 0, as can be taken from equations 1.7 and 1.8.

$$x_b \leq u_b \quad \forall b \in B \tag{1.7}$$

$$x_b \geq R_b * u_b \quad \forall b \in B \tag{1.8}$$

Additional constraints have to be set connected with the special block order types. In linked block orders, the acceptance ratio of a child block can not be larger than the one of its parent block.

$$x_{lb} \leq \sum_{b \in B} A_b^{lb} * x_b \quad \forall lb \in LB \tag{1.9}$$

The combined acceptance ratio of all blocks contained in an exclusive group may not exceed 1.

$$\sum_{b \in B} A_b^{eg} * x_b \leq 1 \quad \forall eg \in EG \tag{1.10}$$

Regarding any flexible hourly order, the binary acceptance u^t_{fh} can only take 1 for exactly one t.

$$\sum_{t \in T} u^t_{fh} \leq 1 \quad \forall fh \in FH \tag{1.11}$$

Power Balance Constraints

For every period, a power balance constraint is created to ensure that the cleared energy on the demand side is matched by the cleared supply.

$$v^t_{demand} + v^t_{block} = v^t_{supply} \quad \forall t \in T \tag{1.12}$$

The volume for a period of supply and demand of the hourly orders equals the sum of all accepted orders of the respective side in that period. The binary variable Y^t_o denotes whether order o was entered for period t.

$$v^t_{demand} = \sum_{d \in D} x_d * Y^t_d * Q_d \tag{1.13}$$

$$v^t_{supply} = \sum_{s \in S} x_s * Y^t_s * Q_s \tag{1.14}$$

Due to supply block orders being modeled with negative quantities, the volume of block orders in a period already covers demand, as well as supply that is cleared in the specific hour of all accepted block orders.

$$v^t_{block} = \sum_{b \in B} x_b * Q^t_b \tag{1.15}$$

1.4 Price Determination

Determining the price follows a method that was introduced by O'Neill et al. [3]. After solving for the optimal variables in the previously presented optimization model, the binary decision variables are fixed at their determined values and the problem is resolved as a continuous one. In the fixed model equations 1.1 - 1.15 remain unchanged with the only adaption being binary variables u_b and u^t_{fh} being replaced by continuous variables

u_b^* and u_{fh}^{t*} respectively. Added constraints set the new variables at the value that was determined by solving the MILP before, which are denoted by U_b and U_{fh}^t respectively.

$$u_b^* = U_b \quad \forall b \in B \tag{1.16}$$

$$u_{fh}^{t*} = U_{fh}^t \quad \forall fh \in FH, \quad \forall t \in T \tag{1.17}$$

This way, the problem is now a linear program without any integer variables and a dual can be derived. After solving, the dual variables of the power balance constraint in equation 1.12 form the clearing prices for each hour.

Determination of Paradoxically Accepted Block Orders

A paradoxically accepted block order is a cleared order through which acceptance the market participant is receiving a negative profit. After having determined the clearing prices π_t, identifying such orders is possible. The contribution of a block order b to the social welfare p_b corresponds to the individual profit for the market participant submitting that order. It is given in equation 1.18.

$$p_b = \sum_{t \in T} x_b * [Q_b^t * (P_b - \pi_t)] \tag{1.18}$$

Any block order with a negative p_b is considered to be paradoxically accepted. For the following iteration of solving the MILP, the acceptance of any such order pa will be set to zero.

$$u_{pa} = 0 \quad \forall pa \in PA \tag{1.19}$$

If there are no more paradoxically accepted orders after solving, the algorithm terminates.

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