DIFFERENTIALS, TODA BRACKETS, AND ALGEBRAICITY

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1. SUMMARY

In this post I am going to write down the details of the claim that "every" differential is secretly a Toda bracket and discuss a notion of "algebraicity" inherent in this point of view. In doing this, I will review the theory of Toda Brackets as well as explain what exactly I mean when I say "every" differential. Along the way we will discuss the theory of coherent chain complexes and decalage. Much of this blog post follows Ariotta's paper on coherent chain complexes https://arxiv.org/abs/2109.01017.

2. TODA BRACKETS

When things are trivial, they usually provide little to no information. In homotopy theory, however, triviality is *witnessed* by nullhomotopies. These witnesses may be treated as extra data themselves which can be used to produce extra structure from triviality. The threefold bracket is easiest to explain. Fix a stable ∞-category $\mathcal{C}$, th

3. SPECTRAL SEQUENCES OF STABLE HOMOTOPY TYPES

4. DIFFERENTIALS AS TODA BRACKETS

5. ALGEBRAICITY