Overview

Final material: Chapter 1 (1.1 - 1.5), Chapter 2($2.1-2.5$), Chapter 3($3.1-3.4$), Chapter 4 (4.1 - 4.5 (so without Poisson processes)), Chapter 5 (5.1 - 5.2), Chapter 6 (6.1 - 6.3), Chapter 7 (7.1 - 7.2), Chapter 8 (8.1 - 8.4), Chapter 9 (9.1 - 9.3), Chapter 10 (10.1 - 10.3).

Here is the preliminary list of covered topics

Chapter 1.

- Probability space (sample space, collection of events, probability measure and its axioms)
- Counting techniques (Appendix C)
- Equally likely outcomes $\left(P(A)=\frac{\#A}{\#\Omega}\right)$
- Sampling (balls chosen from urns: with replacement with order, without replacement with/without order)
- Decomposing an event as the disjoint union of simpler events.
- Probability of event and its compliment
- Inclusion-exclusion formula
- Definition of random variables; discrete random variable, probability mass function

Chapter 2.

- Conditional probability of A given B
- The multiplication rule for conditional probabilities
- Decomposition $P(A)=\sum P(A|B_i)P(B_i)$ for a partition $\Omega=\bigcup B_i$
- Bayes' formula
- Independence of two events A and B, connection with independent of compliments
- Mutual and pairwise independence
- Independence of random variables, simplified definition for the discrete case
- Independence of functions of random variables
- Independent trials and distributions constructed from them (Bernoulli, Binomial, Geometric)
- Hypergeometric distribution
- Conditional independence of events

Chapter 3 + Part of Chapters 8.1-8.2.

- Continuous distributions: the definition of the probability density function, computing of $P(B)$ using p.d.f
- Uniform distribution, exponential distribution
- The cumulative distribution function of a random variable, definition, basic properties
- How does the c.d.f. of a discrete random variable look like.
- How to identify the probability mass function from the c.d.f. and vice versa.
- How to compute the p.d.f. of a continuous random variable from the c.d.f.
- Expected value: discrete and continuous case
- Computing $E[g(X)]$, basic properties of expectation
- Variance (two formulas: $E\big[(X-E[X])^2\big]$ and $E[X^2]-(E[X])^2$) and its properties
- Linearity of expectation; expectation of product of independent random variables; variance of the sum of independent random variables;
- The normal (Gaussion) distribution $N(\mu,\sigma^2)$; definition φ and Φ; if $X\sim N(\mu,\sigma^2)$, then $(X-\mu)/\sigma\sim N(0,1)$;

Chapter 4.

- The Central Limit Theorem for binomial random variables (the normal approximation); continuity correction;
- The weak law of large numbers for binomial random variables;
- Let $S_n\sim \text{Bin}(n,p)$ denote the number of successes in a sequence of independent trials with an unknown success probability p. Then a natural estimate for p is $\hat{p}_n=\frac{S_n}{n}$ (the frequency of successes) and we can estimate the error using the normal approximation:

$$P(\hat{p}_n-p|\le\varepsilon)\ge 2\Phi\big(2\varepsilon\sqrt{n}\big)-1$$

- The definition of a confidence interval corresponding to a certain percentage; application to polling;
- Poisson random variable;
- The Poisson approximation of the binomial distribution; Poisson distribution as a model for counting rare events;
- Exponential distribution and its properties, including memoryless property;

Chapter 5 + Chapter 8.3.

- The moment generating function of the random variable and its properties; moment generating function for discrete, Poisson, exponential, and normal random variables;
- Computations of moments via moment generating function;
- Moment generating function of the sum of independent random variables;
- The moment generating function identifies the distribution of the random variable; Identifying the pmf of a random variable from the moment generating function;
- The sum of independent Poisson random variables is a Poisson rv;
- Linear combinations of independent normal random variables is normal;
- Computing p.m.f of $g(X)$ for a discrete random variable X;
- Computing p.d.f of $g(X)$ for a continuous random variable X via cdf techniques (you do not need general formulas from the book!);

Chapter 6.

- Joint distribution of random variables (discrete and continuous cases); How to use the joint pmf or pdf to compute various probabilities about random variables (probability of some region; marginal pmf or pdf; expectations, etc);
- Joint distribution of random variables and independence;
- The multinomial distribution.

Chapter 7.

- Distribution (pmf or pdf) of the sum of two independent discrete or continuous random variables; definition of convolution;
- Negative binomial distribution;
- Exchangeability of random variables and main examples: independent identically distributed rv and sampling without replacement;

Chapter 8 (remaining part).

- The indicator method: if the random variable X is non-negative integer valued then often it can be represented as the sum of indicator random variables (i.e. random variables that are 0 or 1). Then the expected value of X is just the sum of the expectations of the indicators. Since the expectation of an indicator is just the probability that it is equal to 1 , this method can lead to simpler computation then going through the original definition of the expectation (using the pmf).
- The definition of covariance, properties
- General formula for the variance of the sum of random variables

$$\text{Var}(X_1+\ldots+X_n)=\sum_{i=1}^{n}\text{Var}(X_i)+2\sum_{1\le i<j\le n}\text{Cov}\big(X_i,X_j\big)$$

Chapter 9.

- Estimating probabilities of the form $P(X\ge c)$ using Markov's and Chebyshev's inequality.
- The weak law of large numbers for i.i.d. random variables with a finite variance.
- The Central Limit Theorem for i.i.d. random variables with a finite mean and variance; normal approximation;

Chapter 10.

- Conditional probability mass function and conditional expectation of a discrete random variable with respect to an event B with $P(B)>0$
- The averaging principle: how to get the unconditional pmf or expectation from the conditional ones if we have a partition.
- The conditional pmf and expectation of X given $Y=y$. The averaging identity in this case.
- Conditional distribution of jointly continuous random variables. The conditional pdf and expectation of X given $Y=y$; averaging identity
- The conditional expectation of X given Y: the random variable $E[X|Y]$ for discrete and continuous case;