DMM Summary

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Disclaimer This "summary" it's not supposed to be a standalone tool, it contains more *pragmatic* definitions, aimed more at the exercises/exam itself. It's not complete *and it's not supposed to be.*

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1 Decision Problems

1.1 Preference Relation

1.1.1 Properties

Considering impact set F and a binary relation Π, the main properties of said relation can be:

- **Reflexivity:** $\forall a \in F$, $(a, a) \in \Pi$; to check for it, on the graph representation every impact has to have a self-loop
- **Transitivity:** $\forall a, b, c \in F$, if $(a, b) \in \Pi$ and $(b, c) \in \Pi$ then $(a, c) \in \Pi$; to check for it, on the graph representation follow every arc and check that every possible "triangle" is complete
- **Antisymmetry:** $\forall a, b \in F$, if $(a, b) \in \Pi$ and $(b, a) \in \Pi$ then $a = b$; to check for it, on the graph representation there must be no impacts that point at each other
- **Completeness:** $\forall a, b \in F$, if $(a, b) \notin \Pi$ then $(b, a) \in \Pi$; to check for it, on the graph representation, from each node there must be an arc (either outgoing or incoming) connecting it to every other

1.1.2 Types of relations (orders)

Combining the properties, we can get different kinds of preferences:

- **Preorder:** reflexivity and transitivity. Guarantees that the set of nondominated alternatives is nonempty, even if not all alternatives are comparable, we can find at least one nondominated solution
- **Partial order:** reflexivity, transitivity and antisymmetry. This limits indifference, but still allows for incomparability, and thus not always leading to a definitive choice
- **Weak order:** reflexivity, transitivity and completeness. Guarantees that nondominated solutions exists and are all mutually indifferent, any of them can be chosen. Such orders admit representation by a value function (with ties), turning the decision problem into an optimization problem
- **Total order:** reflexivity, transitivity, antisymmetry and completeness. Provides a unique linear ranking of alternatives, there is always a unique best alternative

1.1.3 Derived relations

From the weak preference relation, one can derive:

- **Indifference relation:** Ind_Π

  $$(a, b), (b, a) \in \mathrm{Ind}_\Pi \Leftrightarrow (a, b) \in \Pi \wedge (b, a) \in \Pi$$

  To build it from the graph representation, add all self loops and each pair of arcs that point at each other

- **Strict preference relation:** Str_Π

  $$(a, b) \in \mathrm{Str}_\Pi \Leftrightarrow (a, b) \in \Pi \wedge (b, a) \notin \Pi$$

  To build it from the graph representation, add all arcs which do not have an equal one in the opposite direction (the set difference of the last one w.r.t. Π)

- **Incomparability relation:** Inc_Π

  $$(a, b), (b, a) \in \mathrm{Inc}_\Pi \Leftrightarrow (a, b) \notin \Pi \wedge (b, a) \notin \Pi$$

  To build it from the graph representation, add all arcs which are not present in either direction in the graph

2 Basic Decision Models

2.1 Structured preferences

2.1.1 Dominance relation

Some ways to sort *the stuff*.

Lexicographic order Order the alternatives w.r.t. the value of the first indicator (for some ordering of the indicators) and break ties with the subsequent one. This yields a total order.

The variant with aspiration levels introduces a “minimum requirement” ϵ_i, rejecting all alternatives with indicator f_i worse than ϵ_i (higher or lower depending on whether it’s a benefit or cost).

Utopia point Identify an ideal impact with the best possible value for each indicator (optimize them independently) and evaluate all alternatives with the distance from the ideal impact.

Different definitions of distance yield different results. Some definitions:

- L_1 Manhattan distance

$$d(f, f') = \sum_{l \in P} |f_l - f'_l|$$

- L_2 Euclidean distance

$$d(f, f') = \sqrt{\sum_{l \in P} \left(f_l - f'_l\right)^2}$$

- L_∞ Chebyshev distance/maximum norm

$$d(f, f') = \max_{l \in P} |f_l - f'_l|$$

Borda count In the case of finite alternatives, they can be sorted by counting how many alternatives are worse than each one

$$B(f) = \left|\left\{f' \in F \mid f \preceq f'\right\}\right|$$

2.2 MAUT

2.2.1 Indifference curves

An *indifference curve* is a subset of the impact space $I \subseteq F$ of reciprocally indifferent impacts. By definition:

- The curves cover F
- Any two curves have empty intersection
- Weak order on impacts maps to total order on curves

Usually, continuity is assumed (they are mathematical objects and not a general set of points), and each indifference curve is expressed in the implicit form $u(f) = c$, each c identifies a curve.