# Decision Methods and Models

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# Chapter 1

# Introduction

## 1.1 Terminology

A **system** is the portion of the world affected by the decision, it should allow different configurations, it should not be given once and for all, otherwise no decision would be possible.

A **solution** or **alternative** is the combination of all controllable aspects of the system, and **outcome** or **scenario** the combination of all its uncontrollable aspects.

A system combines controllable and uncontrollable aspects into a **configuration**. Each configuration is associated to an **impact**, that describes all aspects relevant for the decision.

**Decision-maker** or **stakeholder** refers to everybody who contributes to the choice of the alternative. The former indicates who takes part to the choice, while the latter also includes who does not participate but has interests at stake and could react to a disagreeable choice, exerting an indirect influence on the choice.

With **preference**, we denote the relative satisfaction between impacts.

A decision problem requires to *choose an alternative*:

- so as to *move the system into a configuration*
- such that *the decision-makers prefers its associated impact* to those of other configurations

- keeping into account that *the actual configuration depends on the alternative, but also on the scenario*

A **decision problem** implies two fundamental conditions:

- **Freedom**, i.e., availability of different choices (otherwise there is no decision)
- **Rationality**, i.e., the existence of preference criteria (otherwise the choice cannot be motivated)

This is a concept different from the definition of "decision problem" typically given in computer science (problem which admits only two possible solutions, *yes* or *no*). The decision problems considered here can be considered special cases of optimization/search problems, whose solution is an object with the maximum value (or minimum cost).

The focus is on practical decisions where a large amount of data must be taken into account, many choices are possible and the cost of a wrong choice is high.

We want to discuss *what make a decision complicated*, present the mathematical models to describe complicated situations and present the methods to deal with such situations, while recognizing limits and errors of this approach.

## 1.2 Modeling approach to decision

The modeling approach to decision requires a series of intermediate passages:

- Building a model of the problem
- Solving the model with algorithms, i.e., formal methods
- Interpreting the solution, with suitable methods

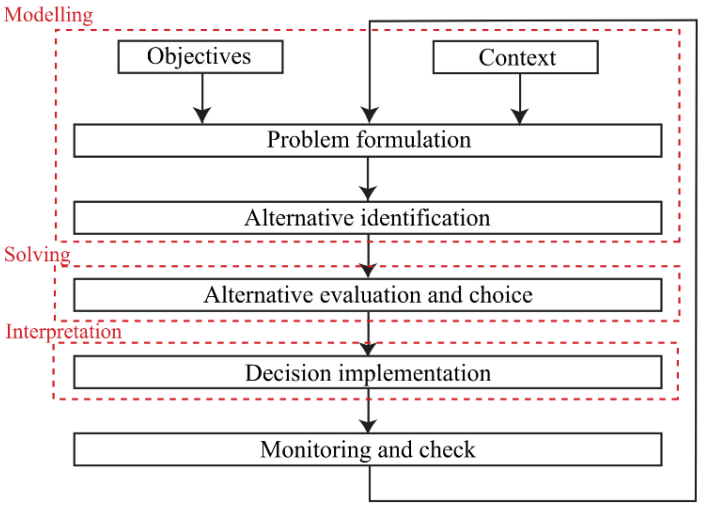
The strategy is to first **make a model**, then **compute**, and **finally decide**. From a problem to a model, solved by an algorithm, then actions are taken based on the solution.

The **decision process** occurs in an **iterative correction approach**:

1. *Problem formulation:* delineate the system, identifying impacts and preferences (Objectives) on one hand, decision-makers and scenarios

(Context) on the other

2. *Alternative identification:* define the set of feasible alternatives
3. *Alternative evaluation and choice:* evaluate the impact associated to each configuration (alternative and scenario) and choose an alternative based on the preferences of the decision-makers
4. *Decision implementation:* apply or simulate the alternative selected
5. *Monitoring and check:* observe the consequences of the decision; if unsatisfactory, make correction and repeat the process, introducing new scenarios, objectives, alternatives and evaluation methods




## 1.3 Why a formal approach?

A formal approach allows to:

- Predict in a more certain and precise way the impact of a decision, using descriptive models instead of intuition and experience
- Accelerate the decision process, using algorithms and information technology
- Consider a much larger number of possible alternatives
- Clarifying and certifying the decision process

  - making explicit the assumptions made on alternatives, scenarios, preferences and decision-makers
  - guaranteeing repeatability of the process
  - allowing specific changes to the process, without starting from scratch

The formal approach is based on

- **Models**, to take decisions and predict their outcomes
- **Methods**, to build models, solve them, and interpret their results

## 1.4 Prescriptive and descriptive models

A decision model includes a number of submodels, which can be classified based on the data they require and the result they produce. Decision models usually combine:

1. **Prescriptive models** that
   - receive impacts and preferences in input
   - return a suggested alternative in output

   *if this is the case, you should do that*
2. **Descriptive/predictive** models
   - receive the system, an alternative and a scenario in input
   - return an impact in output

   *If you do this and if this happens, you will obtain that*

The two families of models can have subtle and complex interactions. For example, any prescriptive model uses a set of descriptive models to obtain the impacts of possible alternatives and scenarios; on the other hand, some descriptive models can include prescriptive ones.

Example: a model prescribes a decision (*close or open streets*) based on models that describe a system (*amount of traffic*), including decisions prescribed by models (*satellite navigators*)

## 1.5 What makes a decision problem complicated?

A decision problem can be *complicated* due to:

- An **insufficient model** of the system
- **Complicating features** of the model, such as
  - complex preference structure, insufficient to define an optimum
  - uncertain environment, impact depends also on unknown scenario
  - multiple decision-makers, with potentially conflicting preferences
- A **computationally complex model**, everything is clearly defined, but no efficient algorithm is known to solve the problem

The three main complexity sources for decision problems are:

1. **Preference structure**: simple or complex
2. **Uncertainty**: a single scenario or many
3. **Decision-makers**: single or many

giving rise to $2^3 = 8$ families of decision problems.

We'll consider four basic families of prescriptive models:

1. Simple preference, a single scenario and a single decision maker:
   - mathematical programming
   - multi-attribute utility theory
2. Complex preference, a single scenario and a single decision maker:
   - paretian preferences
   - weak rationality models (AHP and ELECTRE methods)
3. Simple preference, multiple scenarios and a single decision-maker:
   - decisions in condition of ignorance (robust programming)
   - decisions in conditions of risk (stochastic programming)
4. Simple preference, a single scenario and multiple decision-makers:
   - independent decision-makers (game theory)
   - cooperating decision-makers (group decisions)