Maxence Lasbordes
maxence.lasbordes@dauphine.eu

Lab session # 4
ALTEGRAD 2024

11/19/24

1 Question 1

The graph G consists of:

- Let N be the number of edges of a complete graph with 100 vertices:

$$N = \binom{100}{2} = 4950$$

- Let M be the number of edges of a complete bipartite graph with 50 vertices in each partition:

$$M = 50 \cdot 50 = 2500$$

- So the total number of edges in G is 7450

Let's compute the number of Triangles in G.

- In the complete graph with 100 vertices, the number of triangles N' is:

$$N' = \binom{100}{3} = 161700$$

- In complete bipartite graph with 50 vertices in each partition, no triangles exist because it is a bipartite graph.

- Total triangles in G is 161700

2 Question 2

- Graph (a):

$$\begin{aligned} Q &= \left[\frac{6}{13} - \left(\frac{12}{26}\right)^2\right] + \left[\frac{6}{13} - \left(\frac{13}{26}\right)^2\right] \\ &= 0.46 \end{aligned}$$

- Graph (b):

$$\begin{aligned} Q &= \left[\frac{2}{13} - \left(\frac{11}{26}\right)^2\right] + \left[\frac{4}{13} - \left(\frac{15}{26}\right)^2\right] \\ &= -0.05 \end{aligned}$$

3 Question 3

We have :

$$\phi(C_4) = [4, 2, 0, \ldots, 0]$$
$$\phi(P_4) = [3, 2, 1, 0, \ldots, 0]$$

Therefore :

$$K(C_4, C_4) = \phi(C_4) \cdot \phi(C_4) = 20$$

$$K(C_4, P_4) = \phi(C_4) \cdot \phi(P_4) = 16$$

$$K(P_4, P_4) = \phi(P_4) \cdot \phi(P_4) = 14$$

4 Question 4

A kernel value of 0, means that the two graphs G and G' have no common graphlets of size 3. Basically, the feature vectors representing the counts of graphlets of size 3 in G and G' are orthogonal.

Here is an example. Let's consider the following two graphs:

- G: A triangle (3-cycle)
- G': A path of length 3 (3-path)

The graphlet kernel of size 3 will count the occurrences of 3-node graphlets in each graph. A triangle has one 3-cycle graphlet, while a 3-path has no 3-cycle graphlets but has other 3-node graphlets. So they don't share any common graphlets of size 3, and the kernel value is 0.