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Lab session # 5
ALTEGRAD 2024

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# 1 Question 1

For directed graphs, deepwalk should only follow the direction of the edges. This means that during the random walk generation, the next node could be chosen from the outgoing neighbors of the current node.

As for the weighted graphs, the probability of choosing the next node should be proportional to the edge weights. Therefore, nodes connected by higher-weight edges should be more likely to be visited during the walk.

# 2 Question 2

We consider the rotation matrix for a 90-degree counterclockwise rotation:

$$R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

We have that:

$$X_2 = X_1 \cdot R$$

Therefore, $X_2$ is a 90-degree counterclockwise rotation of $X_1$.

# 3 Question 3

In the GCN architecture implemented in Task 10, there are two message-passing layers. Therefore, the receptive field of each node includes nodes that are up to 2 edges away.

In the general case, with $k$ message-passing layers, the receptive field of a node includes nodes that are up to $k$ edges away. This means that the features of nodes up to $k$ edges away from the given node are taken into account in the prediction $\hat{Y}_i$.

# 4 Question 4

- Let's compute $Z_1$ for $K_4$, we have:

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

We have $\hat{A}$ :

$$\hat{A} = D^{-1}A = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{bmatrix}$$

We want $Z_1$, which is:

$$Z_1 = \text{ReLU}(\hat{A}XW_0)$$

Calculating $\hat{A}XW_0$:

$$\hat{A}XW_0 = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -0.8 & 0.5 \end{bmatrix} = \begin{bmatrix} -0.3 & 0.5 \\ -0.3 & 0.5 \\ -0.3 & 0.5 \\ -0.3 & 0.5 \end{bmatrix}$$

With ReLU:

$$Z_1 = \text{ReLU}\left(\begin{bmatrix} -0.3 & 0.5 \\ -0.3 & 0.5 \\ -0.3 & 0.5 \\ -0.3 & 0.5 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0.5 \\ 0 & 0.5 \\ 0 & 0.5 \\ 0 & 0.5 \end{bmatrix}$$

- Let's compute $Z_1$ for $S_4$. The method is the same:

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

The degree matrix $D$ is:

$$D = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We have $\hat{A}$ :

$$\hat{A} = D^{-1}A = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Calculating $\hat{A}XW_0$:

$$\hat{A}XW_0 = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -0.8 & 0.5 \end{bmatrix} = \begin{bmatrix} -0.3 & 0.5 \\ -0.3 & 0.5 \\ -0.3 & 0.5 \\ -0.3 & 0.5 \end{bmatrix}$$

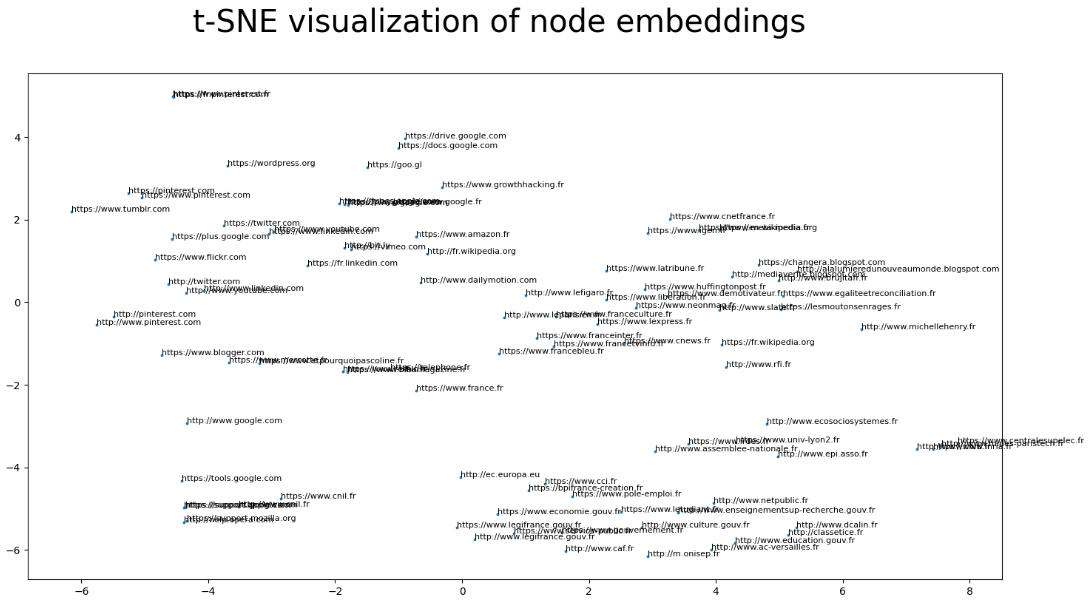
Finally:

$$Z_1 = \text{ReLU}\left(\begin{bmatrix} -0.3 & 0.5 \\ -0.3 & 0.5 \\ -0.3 & 0.5 \\ -0.3 & 0.5 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0.5 \\ 0 & 0.5 \\ 0 & 0.5 \\ 0 & 0.5 \end{bmatrix}$$

For both $K_4$ and $S_4$, the $Z_1$ are identical, which means that the GCN architecture with the given weights produces the same output for these graphs when the input features are all ones.

With randomly sampled node features $X$ from a random uniform distribution, the observed structure would be different as the features would introduce variability, which would lead to different representations for each node.

# 5 Diagrams



t-SNE visualization of node embeddings
https://drive.google.com
https://docs.google.com
https://wordpress.org
https://goo.gl
https://www.growthhacking.fr
https://pinterest.com
https://www.pinterest.com
https://www.tumblr.com
https://twitter.com
https://plus.google.com
https://www.amazon.fr
http://fr.wikipedia.org
https://www.cnetfrance.fr
https://www.flickr.com
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https://www.latribune.fr
http://twitter.com
http://www.dailymotion.com
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http://ec.europa.eu
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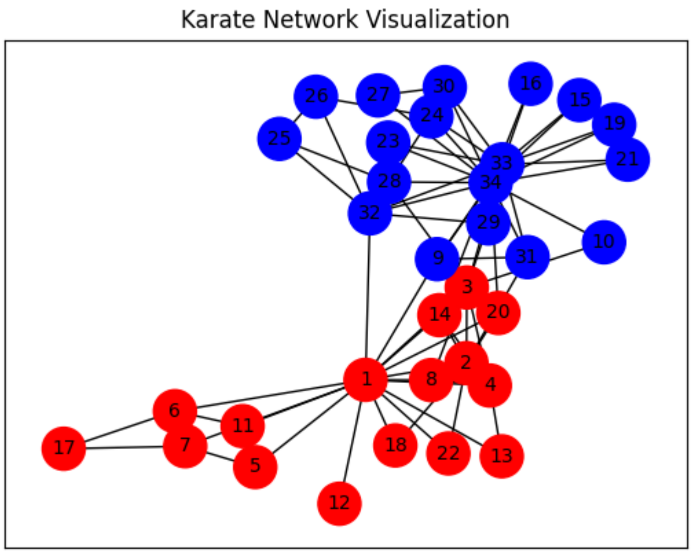
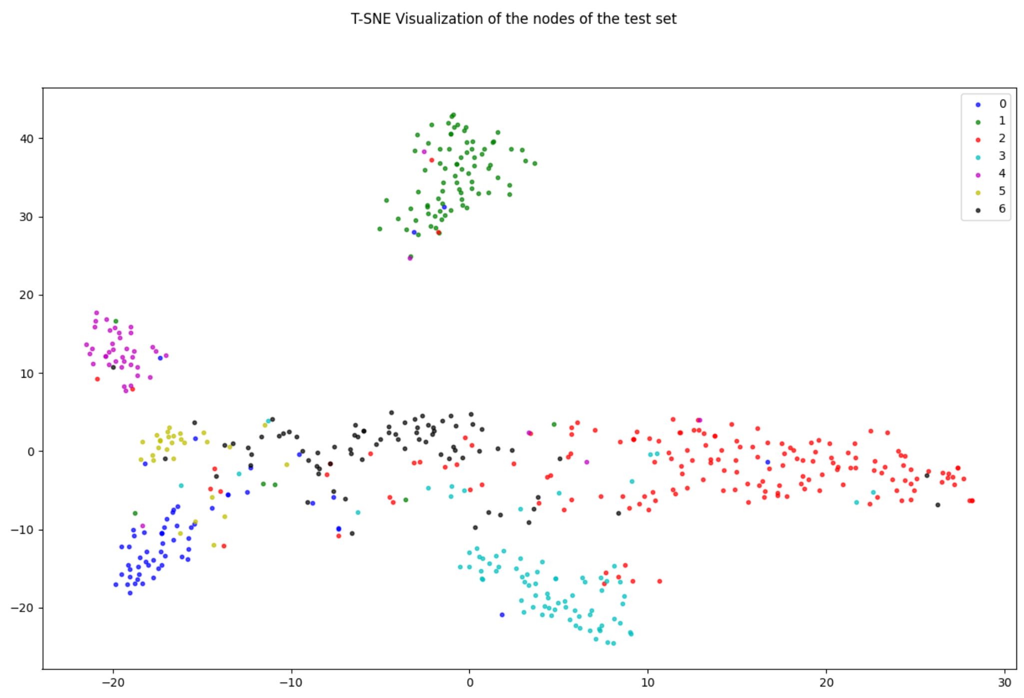
http://www.culture.gouv.fr
http://www.dcalin.fr
http://classetice.fr
http://www.legifrance.gouv.fr
http://www.education.gouv.fr
http://www.caf.fr
http://www.ac-versailles.fr
http://m.onisep.fr

Karate Network Visualization


T-SNE Visualization of the nodes of the test set
0
1
2
3
4
5
6
40
30
20
10
0
−10
−20
−20
−10
0
10
20
30