A Matrix Primer

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A matrix of size $m \times n$ is a 2D grid of values, typically decimal numbers, with m rows and n columns. Element $E[i, j]$ of a matrix, refers to the value in row i and column j. Element $E[1, 1]$ is in the top left corner of the matrix and element $E[m, n]$ is the bottom right corner of the matrix. A matrix can be added with another matrix, multiplied by a scalar, or multiplied by another matrix.

Two matrices can be added only if they have the same size. Addition is performed element-wise, meaning that if $C = A + B$, where C, B, and A, are matrices, then element $C[i, j] = A[i, j] + B[i, j]$ for all elements of C. E.g.,

$$\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Any matrix can be multiplied by a scalar, which is typically a decimal value. Scalar multiplication is performed element-wise, meaning that if $C = sA$, where s is a scalar value and A is a matrix, then $C[i, j] = s \times A[i, j]$. E.g.,

$$\begin{bmatrix} -3 & -6 \\ -9 & -12 \end{bmatrix} = -3 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Two matrices can be multiplied only if the width of the first matrix is equal to the height of the second matrix. If $C = AB$, where A is a matrix of size $m \times n$, B is a matrix of size $n \times p$, and C is a matrix of size $m \times p$, then

$$C[i, j] = A[i, 1] \times B[1, j] + A[i, 2] \times B[2, j] + A[i, 3] \times B[3, j] + \ldots + A[i, n] \times B[n, j]$$

E.g.,

$$\begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 29 & 40 & 51 \\ 39 & 54 & 69 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$