A Library of ADMM for Sparse and Low-rank Optimization

Version 1.0

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https://github.com/canyilu/LibADMM

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The LibADMM toolbox solves many popular compressive sensing problems (see Table 1) by M-ADMM proposed in [14]. Please refer to the readme.txt for more information. Some more details will come soon.

Citing. In citing this toolbox in your papers, please use the following references: [10] [14]

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TABLE 1: Applicability of the LibADMM package

Model	Problem		Function	Description and Reference
Sparse models	$\min_{\mathbf{x}}\ r(\mathbf{x})$ s.t. $\mathbf{A}\mathbf{x}=\mathbf{b}$	$r(\mathbf{x})=\lVert\mathbf{x}\rVert_1$	`l1`	ℓ_1 [16]
		$r(\mathbf{x})=\sum_{g\in\mathcal{G}}\lVert\mathbf{x}_g\rVert_2$	`groupl1`	Group Lasso [19]
		$r(\mathbf{x})=\lVert\mathbf{x}\rVert_1+\lambda_2\lVert\mathbf{x}\rVert_2^2$	`elasticnet`	Elastic net [21]
		$r(\mathbf{x})=\lVert\mathbf{x}\rVert_1+\lambda_2\sum_{i=2}^{p}\lvert x_i-x_{i-1}\rvert$	`fusedl1`	Fused Lasso [17]
		$r(\mathbf{x})=\lVert\mathbf{A}\text{Diag}(\mathbf{x})\rVert_*$	`tracelasso`	Trace Lasso [12]
		$r(\mathbf{x})=\frac{1}{2}\lVert\mathbf{x}\rVert_{\text{ksp}}^2$	`ksupport`	k support norm [6]
	$\min_{\mathbf{x},\mathbf{e}}\ l(\mathbf{e})+\lambda r(\mathbf{x})$ s.t. $\mathbf{A}\mathbf{x}+\mathbf{e}=\mathbf{b}$	$l(\mathbf{e})=\lVert\mathbf{e}\rVert_1$ $l(\mathbf{e})=\frac{1}{2}\lVert\mathbf{e}\rVert_2^2$	`l1R`	Reg. ℓ_1
			`groupl1R`	Reg. Group Lasso
			`elasticnetR`	Reg. Elastic net
			`fusedl1R`	Reg. Fused Lasso
			`tracelassoR`	Reg. Trace Lasso
			`ksupportR`	Reg. k support norm
Low-rank matrix models	$\min_{\mathbf{L},\mathbf{S}}\ \lVert\mathbf{L}\rVert_*+\lambda l(\mathbf{S}),$ s.t. $\mathbf{X}=\mathbf{L}+\mathbf{S}$		`rpca`	Robust PCA [2]
	$\min_{\mathbf{X}}\ \lVert\mathbf{X}\rVert_*,$ s.t. $\mathcal{P}_\Omega(\mathbf{X})=\mathcal{P}_\Omega(\mathbf{M})$		`lrmc`	Low-rank matrix completion [1]
	$\min_{\mathbf{X},\mathbf{E}}\ \lVert\mathbf{X}\rVert_*+\lambda l(\mathbf{E}),$ s.t. $\mathcal{P}_\Omega(\mathbf{X})+\mathbf{E}=\mathbf{M}$		`lrmcR`	Reg. Low-rank matrix completion
	$\min_{\mathbf{X},\mathbf{E}}\ \lVert\mathbf{X}\rVert_*+\lambda l(\mathbf{E}),$ s.t. $\mathbf{A}=\mathbf{B}\mathbf{X}+\mathbf{E}$		`lrr`	Low-rank representation [7]
	$\min_{\mathbf{Z},\mathbf{L},\mathbf{E}}\ \lVert\mathbf{Z}\rVert_*+\lVert\mathbf{L}\rVert_*+\lambda l(\mathbf{E})$ s.t. $\mathbf{X}\mathbf{Z}+\mathbf{L}\mathbf{X}-\mathbf{X}=\mathbf{E}$		`latlrr`	Latent low-rank representation [8]
	$\min_{\mathbf{X},\mathbf{E}}\ \lVert\mathbf{X}\rVert_*+\lambda_1\lVert\mathbf{X}\rVert_1+\lambda_2 l(\mathbf{E})$ s.t. $\mathbf{A}=\mathbf{B}\mathbf{X}+\mathbf{E}$		`lrsr`	Low-rank and sparse representation [20]
	$\min_{\mathbf{L}_i,\mathbf{S}_i}\ \lVert\mathbf{L}\rVert_*+\lambda\sum_{i=1}^{m}\lVert\mathbf{S}_i\rVert_1,$ s.t. $\mathbf{X}_i=\mathbf{L}+\mathbf{S}_i,\ i=1,\cdots,m,\ \mathbf{L}\geq 0,\ \mathbf{L}\mathbf{1}=\mathbf{1}$		`rmsc`	Robust multi-view spectral clustering [18]
	$\min_{\mathbf{Z}_i,\mathbf{E}_i}\ \sum_{i=1}^{K}(\lVert\mathbf{Z}_i\rVert_*+\lambda l(\mathbf{E}_i))+\alpha\lVert\mathbf{Z}\rVert_{2,1}$ s.t. $\mathbf{X}_i=\mathbf{X}_i\mathbf{Z}_i+\mathbf{E}_i,\ i=1,\cdots,K$		`mlap`	Multi-task low-rank affinity pursuit [4]
	$\min_{\mathbf{L},\mathbf{S}}\ \lVert\mathbf{L}\rVert_*+\lambda\lVert\mathbf{C}\circ\mathbf{S}\rVert_1,$ s.t. $\mathbf{A}=\mathbf{L}+\mathbf{S},0\leq\mathbf{L}\leq 1$		`igc`	Improved graph clustering [3]
	$\min_{\mathbf{P}}\ \langle\mathbf{P},\mathbf{L}\rangle+\lambda\lVert\mathbf{P}\rVert_1,$ s.t. $0\preceq\mathbf{P}\preceq\mathbf{I},\text{Tr}(\mathbf{P})=k$		`sparsesc`	Sparse spectral clustering [15]
Low-rank tensor models	$\min_{\mathcal{L},\mathcal{S}}\ \sum_{i=1}^{k}\alpha_i\lVert\mathcal{L}_{i(i)}\rVert_*+\lVert\mathcal{S}\rVert_1,$ s.t. $\mathcal{X}=\mathcal{L}+\mathcal{S}$		`trpca_snn`	Tensor robust PCA based on sum of nuclear norm [5]
	$\min_{\mathcal{X}}\ \sum_{i=1}^{k}\alpha_i\lVert\mathcal{X}_{i(i)}\rVert_*,$ s.t. $\mathcal{P}_\Omega(\mathcal{X})=\mathcal{P}_\Omega(\mathcal{M})$		`lrtc_snn`	Low-rank tensor completion based on sum of nuclear norm [9]
	$\min_{\mathcal{X},\mathcal{E}}\ \sum_{i=1}^{k}\alpha_i\lVert\mathcal{X}_{i(i)}\rVert_*+\lambda l(\mathcal{E})$ s.t. $\mathcal{P}_\Omega(\mathcal{X})+\mathcal{E}=\mathcal{M}$		`lrtcR_snn`	Reg. low-tank tensor completion based on sum of nuclear norm
	$\min_{\mathcal{L},\mathcal{S}}\ \lVert\mathcal{L}\rVert_*+\lambda\lVert\mathcal{S}\rVert_1,$ s.t. $\mathcal{X}=\mathcal{L}+\mathcal{S}$		`trpca_tnn`	Tensor Robust PCA based on tensor nuclear norm [11]
	$\min_{\mathcal{X}}\ \lVert\mathcal{X}\rVert_*,$ s.t. $\mathcal{P}_\Omega(\mathcal{X})=\mathcal{P}_\Omega(\mathcal{M})$		`lrtc_tnn`	Low-rank tensor completion based on tensor nuclear norm [13]
	$\min_{\mathcal{X},\mathcal{E}}\ \lVert\mathcal{X}\rVert_*+\lambda l(\mathcal{E}),$ s.t. $\mathcal{P}_\Omega(\mathcal{X})+\mathcal{E}=\mathcal{M}$		`lrtcR_tnn`	Reg. low-rank tensor completion based on tensor nuclear norm [13]
	$\min_{\mathcal{X}}\ \lVert\mathcal{X}\rVert_*,$ s.t. $\mathbf{y}=\Phi(\mathcal{X})$		`lrtr_Gaussian_tnn`	Low-rank tensor recovery from Gaussian measurements based on tensor nuclear norm [13]

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