A Library of ADMM for Sparse and Low-rank Optimization

version 1.0

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https://github.com/canyilu/LibADMM

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1 Introduction

The LibADMM toolbox solves many popular compressive sensing problems (see Table 1) by M-ADMM proposed in [14]. Please refer to the readme.txt for more information. Some more details will come soon.

Citing. In citing this toolbox in your papers, please use the following references [10] [14]:

Canyi Lu. A Library of ADMM for Sparse and Low-rank Optimization. National University of Singapore, June 2016. https://github.com/canyilu/LibADMM.

Canyi Lu, Jiashi Feng, Shuicheng Yan and Zhouchen Lin. A Unified Alternating Direction Method of Multipliers by Majorization Minimization. IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 40, pp. 527-541, 2018

The corresponding BiBTeX citations are given below:

```
@manual{lu2016libadmm,
    author        =   {Lu, Canyi},
    title         =   {A Library of {ADMM} for Sparse and Low-rank Optimization},
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@article{lu2018unified,
    author      =   {Lu, Canyi and Feng, Jiashi and Yan, Shuicheng and Lin, Zhouchen},
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    year        =   {2018},
    volume      =   {40},
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```

TABLE 1: Applicability of the LibADMM package

Model	Problem		Function	Description and Reference
Sparse models	$\min_{\mathbf{x}}\ r(\mathbf{x})$ s.t. $\mathbf{A}\mathbf{x}=\mathbf{b}$	$r(\mathbf{x})=\\|\mathbf{x}\\|_1$	`l1`	ℓ_1 [16]
		$r(\mathbf{x})=\sum_{g\in\mathcal{G}}\\|\mathbf{x}_g\\|_2$	`groupl1`	Group Lasso [19]
		$r(\mathbf{x})=\\|\mathbf{x}\\|_1+\lambda_2\\|\mathbf{x}\\|_2^2$	`elasticnet`	Elastic net [21]
		$r(\mathbf{x})=\\|\mathbf{x}\\|_1+\lambda_2\sum_{i=2}^{p}\|x_i-x_{i-1}\|$	`fusedl1`	Fused Lasso [17]
		$r(\mathbf{x})=\\|\mathbf{A}\mathrm{Diag}(\mathbf{x})\\|_*$	`tracelasso`	Trace Lasso [12]
		$r(\mathbf{x})=\frac{1}{2}\\|\mathbf{x}\\|_{\text{ksp}}^2$	`ksupport`	k support norm [6]
	$\min_{\mathbf{x},\mathbf{e}}\ l(\mathbf{e})+\lambda r(\mathbf{x})$ s.t. $\mathbf{A}\mathbf{x}+\mathbf{e}=\mathbf{b}$	$l(\mathbf{e})=\\|\mathbf{e}\\|_1$ $l(\mathbf{e})=\frac{1}{2}\\|\mathbf{e}\\|_2^2$	`l1R`	Reg. ℓ_1
			`groupl1R`	Reg. Group Lasso
			`elasticnetR`	Reg. Elastic net
			`fusedl1R`	Reg. Fused Lasso
			`tracelassoR`	Reg. Trace Lasso
			`ksupportR`	Reg. k support norm
Low-rank matrix models	$\min_{\mathbf{L},\mathbf{S}}\ \\|\mathbf{L}\\|_*+\lambda l(\mathbf{S}),\ \text{s.t. } \mathbf{X}=\mathbf{L}+\mathbf{S}$		`rpca`	Robust PCA [2]
	$\min_{\mathbf{X}}\ \\|\mathbf{X}\\|_*,\ \text{s.t. } \mathcal{P}_\Omega(\mathbf{X})=\mathcal{P}_\Omega(\mathbf{M})$		`lrmc`	Low-rank matrix completion [1]
	$\min_{\mathbf{X},\mathbf{E}}\ \\|\mathbf{X}\\|_*+\lambda l(\mathbf{E}),\ \text{s.t. } \mathcal{P}_\Omega(\mathbf{X})+\mathbf{E}=\mathbf{M}$		`lrmcR`	Reg. Low-rank matrix completion
	$\min_{\mathbf{X},\mathbf{E}}\ \\|\mathbf{X}\\|_*+\lambda l(\mathbf{E}),\ \text{s.t. } \mathbf{A}=\mathbf{B}\mathbf{X}+\mathbf{E}$		`lrr`	Low-rank representation [7]
	$\min_{\mathbf{Z},\mathbf{L},\mathbf{E}}\ \\|\mathbf{Z}\\|_*+\\|\mathbf{L}\\|_*+\lambda l(\mathbf{E})$ s.t. $\mathbf{X}\mathbf{Z}+\mathbf{L}\mathbf{X}-\mathbf{X}=\mathbf{E}$		`latlrr`	Latent low-rank representation [8]
	$\min_{\mathbf{X},\mathbf{E}}\ \\|\mathbf{X}\\|_*+\lambda_1\\|\mathbf{X}\\|_1+\lambda_2 l(\mathbf{E})$ s.t. $\mathbf{A}=\mathbf{B}\mathbf{X}+\mathbf{E}$		`lrsr`	Low-rank and sparse representation [20]
	$\min_{\mathbf{L}_i,\mathbf{S}_i}\ \\|\mathbf{L}\\|_*+\lambda\sum_{i=1}^{m}\\|\mathbf{S}_i\\|_1,$ s.t. $\mathbf{X}_i=\mathbf{L}+\mathbf{S}_i,\ i=1,\cdots,m,\ \mathbf{L}\geq 0,\ \mathbf{L}\mathbf{1}=\mathbf{1}$		`rmsc`	Robust multi-view spectral clustering [18]
	$\min_{\mathbf{Z}_i,\mathbf{E}_i}\ \sum_{i=1}^{K}(\\|\mathbf{Z}_i\\|_*+\lambda l(\mathbf{E}_i))+\alpha\\|\mathbf{Z}\\|_{2,1}$ s.t. $\mathbf{X}_i=\mathbf{X}_i\mathbf{Z}_i+\mathbf{E}_i,\ i=1,\cdots,K$		`mlap`	Multi-task low-rank affinity pursuit [4]
	$\min_{\mathbf{L},\mathbf{S}}\ \\|\mathbf{L}\\|_*+\lambda\\|\mathbf{C}\circ\mathbf{S}\\|_1,\ \text{s.t. } \mathbf{A}=\mathbf{L}+\mathbf{S},0\leq\mathbf{L}\leq 1$		`igc`	Improved graph clustering [3]
	$\min_{\mathbf{P}}\ \langle\mathbf{P},\mathbf{L}\rangle+\lambda\\|\mathbf{P}\\|_1,\ \text{s.t. } 0\preceq\mathbf{P}\preceq\mathbf{I},\mathrm{Tr}(\mathbf{P})=k$		`sparsesc`	Sparse spectral clustering [15]
Low-rank tensor models	$\min_{\boldsymbol{\mathcal{L}},\boldsymbol{\mathcal{S}}}\ \sum_{i=1}^{k}\alpha_i\\|\boldsymbol{\mathcal{L}}_{i(i)}\\|_*+\\|\boldsymbol{\mathcal{S}}\\|_1,\ \text{s.t. } \boldsymbol{\mathcal{X}}=\boldsymbol{\mathcal{L}}+\boldsymbol{\mathcal{S}}$		`trpca_snn`	Tensor robust PCA based on sum of nuclear norm [5]
	$\min_{\boldsymbol{\mathcal{X}}}\ \sum_{i=1}^{k}\alpha_i\\|\boldsymbol{\mathcal{X}}_{i(i)}\\|_*,\ \text{s.t. } \mathcal{P}_\Omega(\boldsymbol{\mathcal{X}})=\mathcal{P}_\Omega(\boldsymbol{\mathcal{M}})$		`lrtc_snn`	Low-rank tensor completion based on sum of nuclear norm [9]
	$\min_{\boldsymbol{\mathcal{X}},\boldsymbol{\mathcal{E}}}\ \sum_{i=1}^{k}\alpha_i\\|\boldsymbol{\mathcal{X}}_{i(i)}\\|_*+\lambda l(\boldsymbol{\mathcal{E}})$ s.t. $\mathcal{P}_\Omega(\boldsymbol{\mathcal{X}})+\boldsymbol{\mathcal{E}}=\boldsymbol{\mathcal{M}}$		`lrtcR_snn`	Reg. low-tank tensor completion based on sum of nuclear norm
	$\min_{\boldsymbol{\mathcal{L}},\boldsymbol{\mathcal{S}}}\ \\|\boldsymbol{\mathcal{L}}\\|_*+\lambda\\|\boldsymbol{\mathcal{S}}\\|_1,\ \text{s.t. } \boldsymbol{\mathcal{X}}=\boldsymbol{\mathcal{L}}+\boldsymbol{\mathcal{S}}$		`trpca_tnn`	Tensor Robust PCA based on tensor nuclear norm [11]
	$\min_{\boldsymbol{\mathcal{X}}}\ \\|\boldsymbol{\mathcal{X}}\\|_*,\ \text{s.t. } \mathcal{P}_\Omega(\boldsymbol{\mathcal{X}})=\mathcal{P}_\Omega(\boldsymbol{\mathcal{M}})$		`lrtc_tnn`	Low-rank tensor completion based on tensor nuclear norm [13]
	$\min_{\boldsymbol{\mathcal{X}},\boldsymbol{\mathcal{E}}}\ \\|\boldsymbol{\mathcal{X}}\\|_*+\lambda l(\boldsymbol{\mathcal{E}}),\ \text{s.t. } \mathcal{P}_\Omega(\boldsymbol{\mathcal{X}})+\boldsymbol{\mathcal{E}}=\boldsymbol{\mathcal{M}}$		`lrtcR_tnn`	Reg. low-rank tensor completion based on tensor nuclear norm [13]
	$\min_{\boldsymbol{\mathcal{X}}}\ \\|\boldsymbol{\mathcal{X}}\\|_*,\ \text{s.t. } \mathbf{y}=\Phi(\boldsymbol{\mathcal{X}})$		`lrtr_Gaussian_tnn`	Low-rank tensor recovery from Gaussian measurements based on tensor nuclear norm [13]

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