Title of Paper:

A Simple Transmit Diversity Technique for Wireless Communications

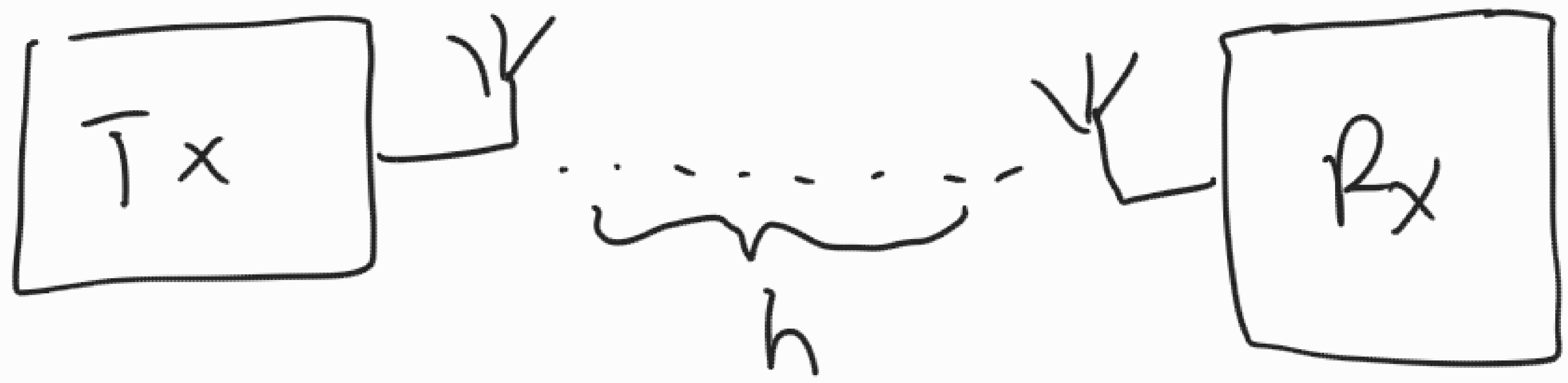
By: Siavash M. Alamouti

- Two fundamental concepts we need to understand before delving into details of scheme.

1 Concept of a fading channel:

- Between a transmitter and receiver, we do not a have a direct link.
- Reflections due to objects between Tx and Rx, result in signals reaching Tx and Rx in different ways. Relative movement between Tx and Rx causes this too.
- This is known as multipath fading.
- Difficult to deterministically predict this phenomenon, so fading channels are usually modelled as time-varying random variables.

Considering a single 1×1 ($Tx \rightarrow Rx$) link:



- For a given transmitted symbol s, the received symbol r is given as:

$$r = h \cdot s$$

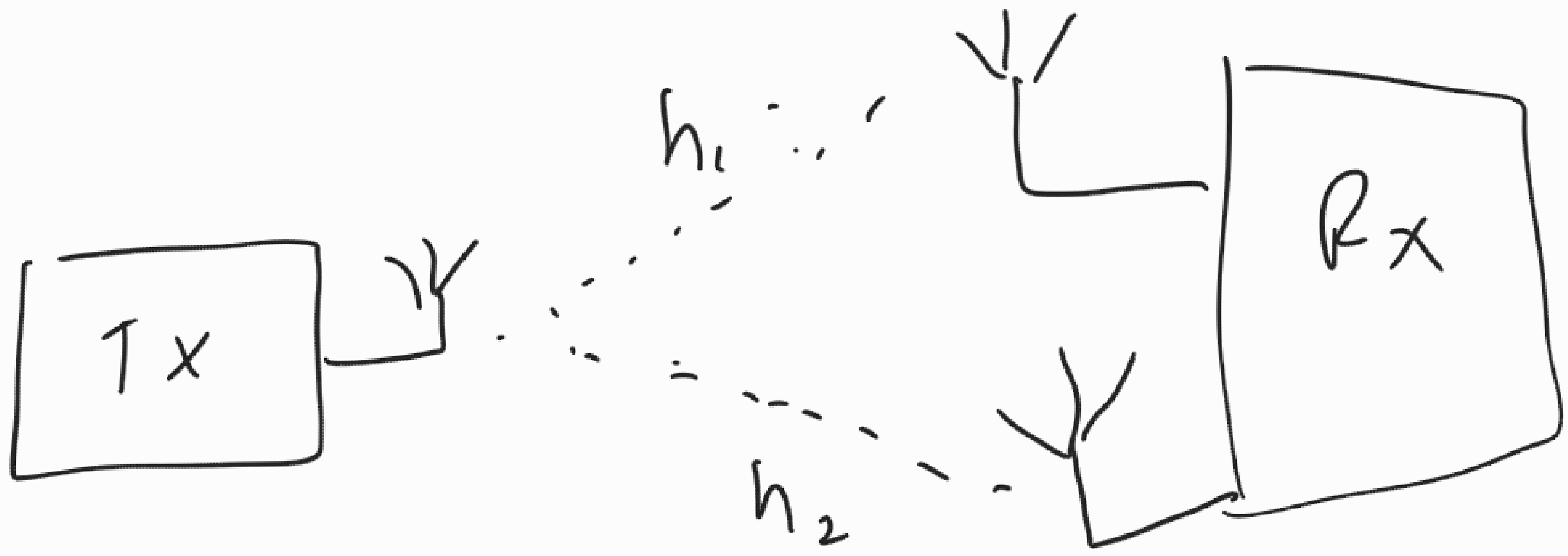
where $h = \alpha e^{j\theta}$, i.e. a multiplicative complex distortion

② Concept of Antenna Diversity

Considering a single 1×1 (SISO) link:

- If there is an event which causes us to not be able to detect at signal at Rx due to fading, we effectively lose the single link we have between Tx and Rx.

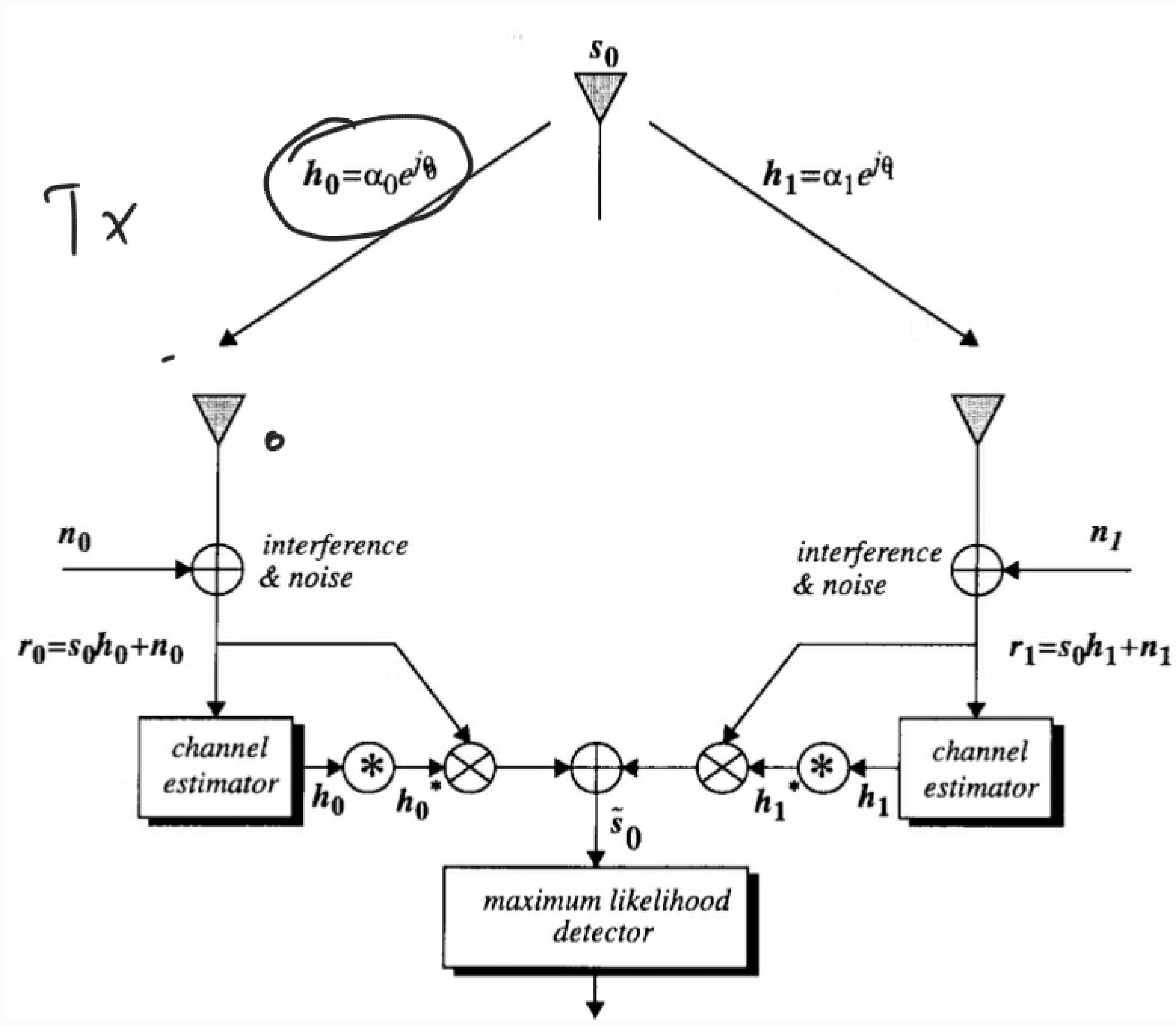
Considering a 1×2 (SIMO) link:



- Two channels of communication between Tx and Rx.
- Increasing number of receivers is known as <u>receive diversity</u>.
- Increasing number of transmitters is known as <u>transmit diversity</u>.

Let us examine a scheme for a 1×2 system known as the maximal-ratio receive combining (MRRC) scheme.

- Fig. 1: MRRC scheme illustration (from Alamouti paper)



- A signal s_0 is sent at a given time.
- Two channels, h_0 and h_1, between Tx and Rx where:

$$h_0 = \alpha_0 e^{j\theta_0} \qquad (1)$$
$$h_1 = \alpha_1 e^{j\theta_1}$$

- At receiver, noise and interference is added, e.g. due to AWGN (Additive White Gaussian Noise) such that received signals can be expressed by

$$r_0 = h_0 s_0 + n_0 \qquad (2)$$
$$r_1 = h_1 s_0 + n_1$$

n_0, n_1 are uncorrelated noise sources.

Assumptions:

1. We have good knowledge of channel coefficients.

- Given that we know h_0, h_1 and r_0 and r_1, we need to come up with a method to allow us to make the <u>optimal</u> decision of what symbol does that signal represent.

Decision making scheme consists of:

1. Combiner
2. Maximum likelihood detector

- It can be shown that optimum combining scheme is as follows:

$$\tilde{s}_0 = h_0^* r_0 + h_1^* r_1 \qquad (3)$$

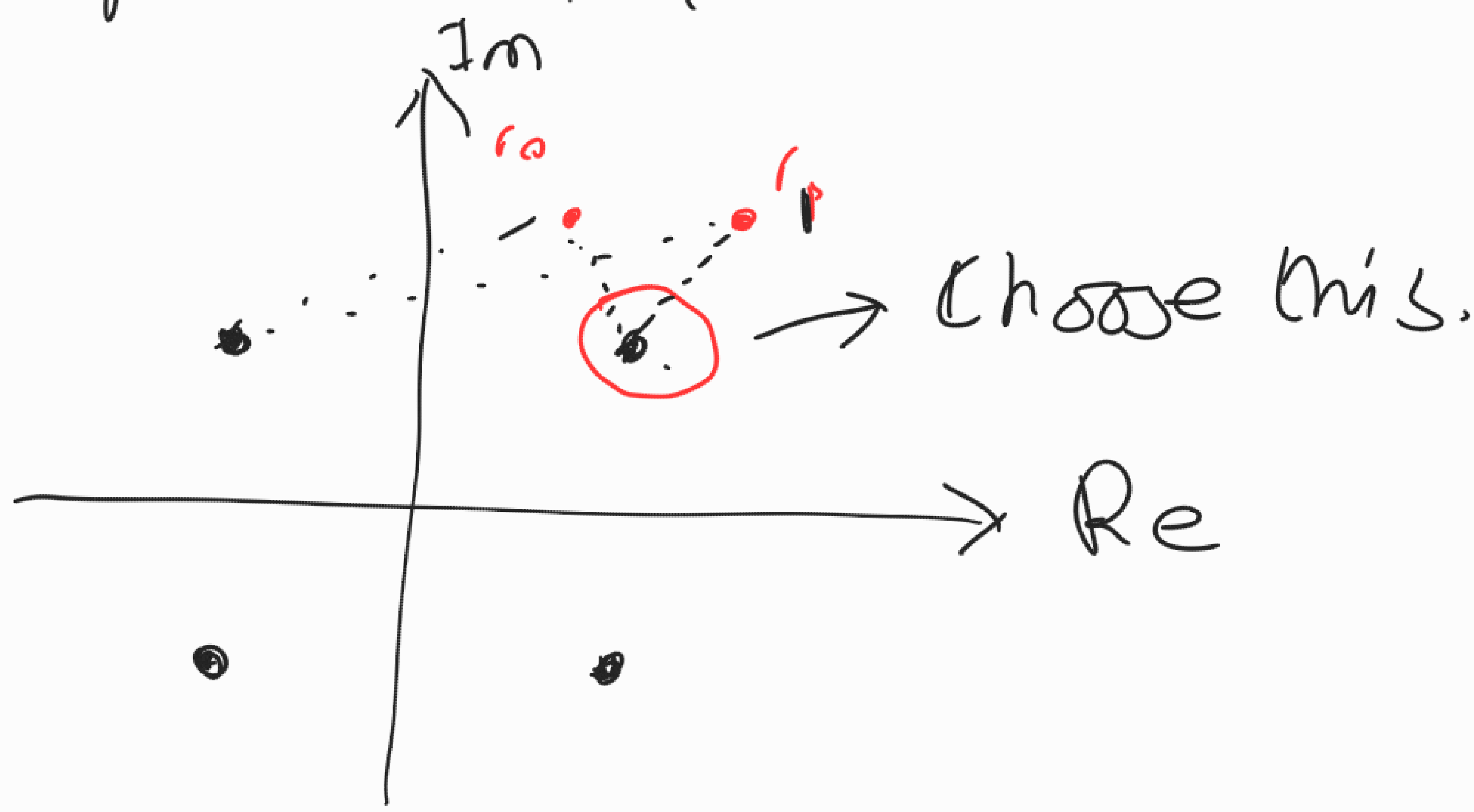
$$= h_0^* (h_0 s_0 + n_0) + h_1^* (h_1 s_0 + n_1)$$

$$= (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1^* n_1$$

Given this combining scheme, we know that our decision should be made by:

$$d^2(r_0, h_0 s_i) + d^2(r_1, h_1 s_1) \le d^2(r_0, h_0 s_k) + d^2(r_1, h_1 s_k) \quad (4) \qquad \forall i \neq k$$

For example for 4-QAM system:



$$d^2(x, y) = (x - y)(x^* - y^*) \quad \text{(Euclidean distance)} \quad (5)$$

Using (3), (4), (5) maximum likelihood detector can be defined as follows:

$$(\alpha_0^2 + \alpha_1^2 - 1)|s_i|^2 + d^2(\tilde{s}_0, s_i) \le (\alpha_0^2 + \alpha_1^2 - 1)|s_k|^2 + d^2(\tilde{s}_0, s_k) \quad \forall i \neq k \quad (6)$$

- This scheme allows us to obtain max SNR at receiver.

$$SNR = \frac{\text{Signal power}}{\text{Noise power}} = \frac{\|h\|^2 \, p}{\sigma^2}$$

$p \rightarrow$ Transmit power

$\sigma^2 \rightarrow$ noise variance

$h = \begin{bmatrix} h_0 \\ h_1 \end{bmatrix}$, vector of channels

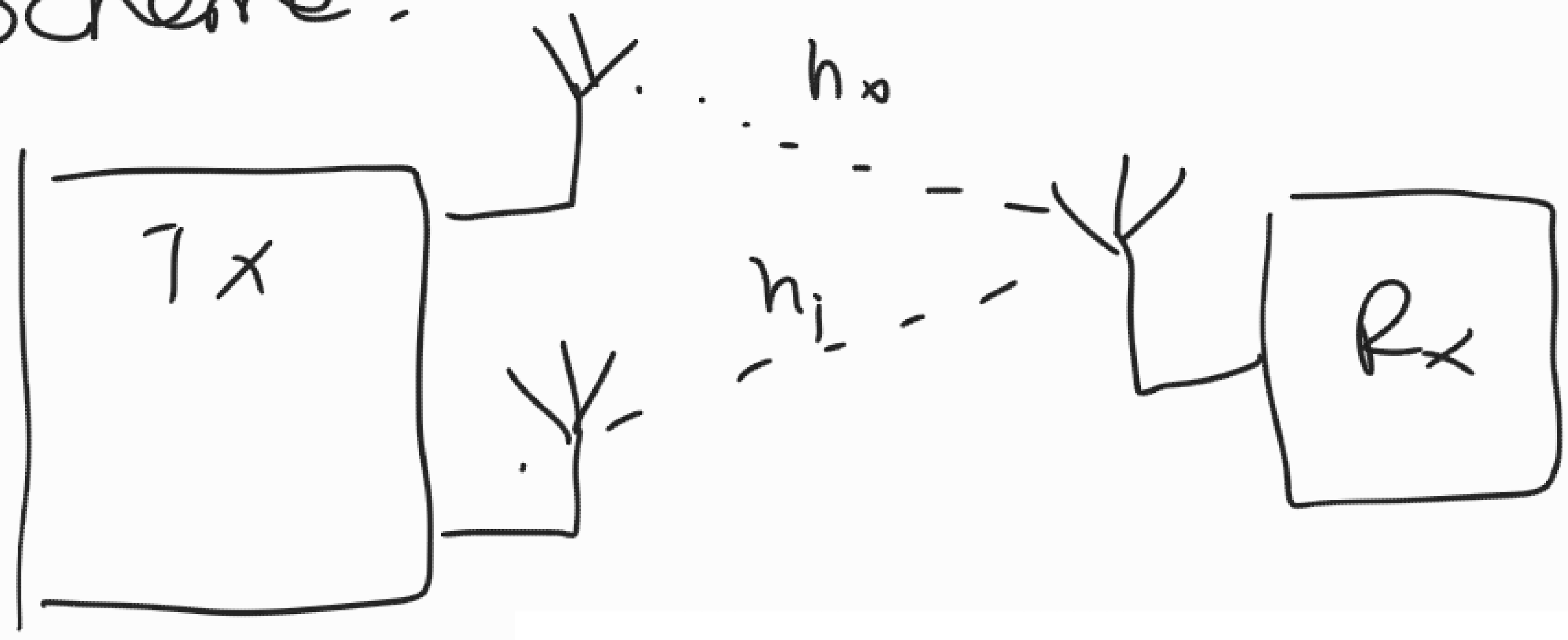
and $\|h\|^2 = \alpha_0^2 + \alpha_1^2$

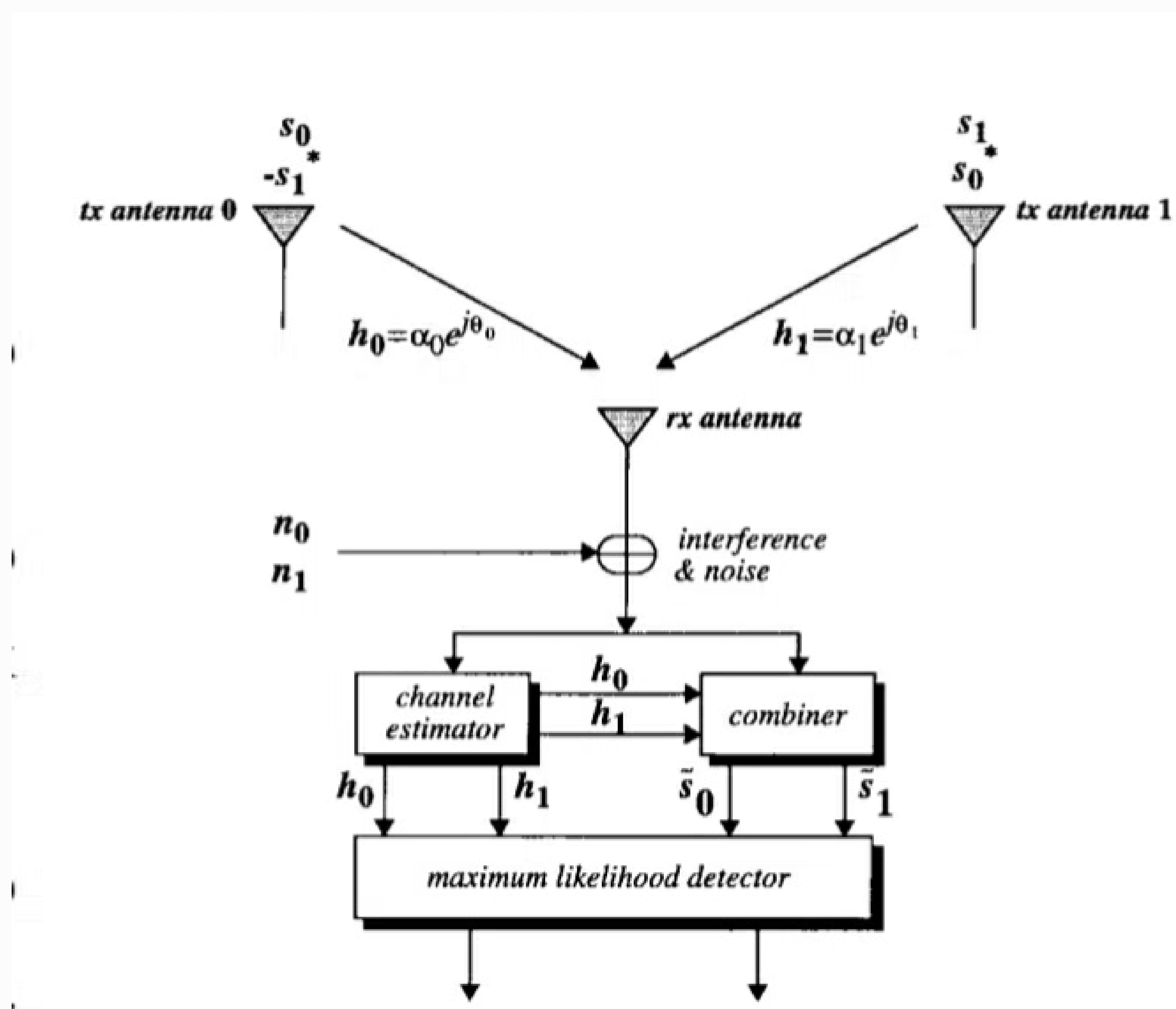
Note: Key is in Combining scheme, $\tilde{s}_0$ allows for optimal decision.

Limitations of method above:

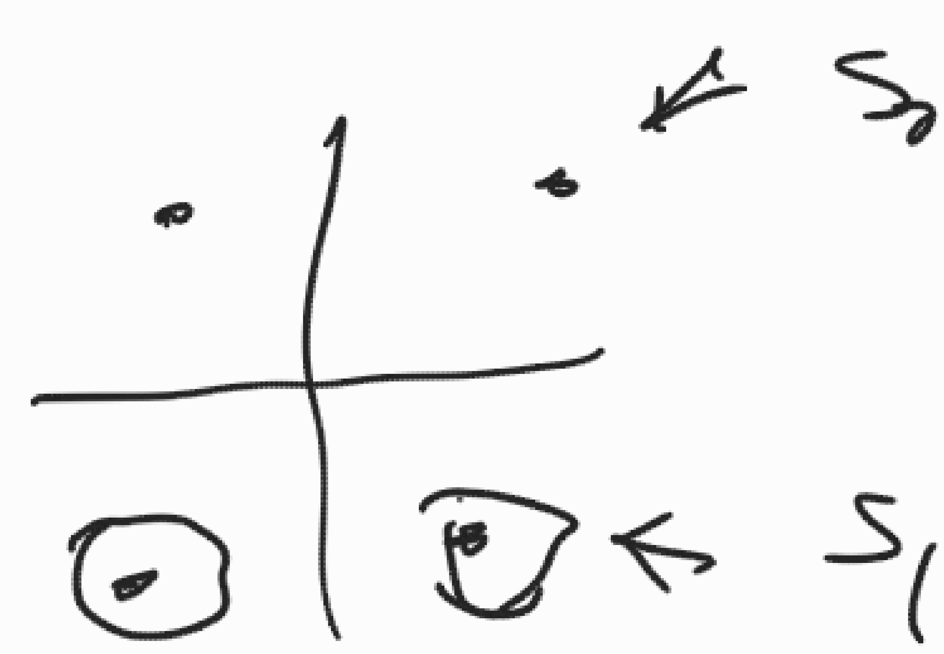
- In practical implementations, we are limited on number of receive antennas due to power and size constraints of receive devices.
- More practical techniques such as <u>transmit diversity</u> need to be explored.

Considering Alamouti's Transmit Diversity Scheme: (2 x 1) MISO





- Suppose I want to transmit a TWO symbol message $\{s_0, s_1\}$.
- At a given instant, let antenna 0 transmit s_0 and antenna 1 transmit s_1. $(n = t)$
- At the next transmit instant, i.e. $n = t+T$ antenna 0 transmits $-s_1^*$ and antenna 1 transmits s_0^*.



	antenna 0	antenna 1
time t	s_0	s_1
time $t+T$	$-s_1^*$	s_0^*

- For a sufficiently small T, a reasonable assumption is that:

$$h_0(t) = h_0(t+T) = \alpha_0 e^{j\theta_0}$$
$$h_1(t) = h_1(t+T) = \alpha_1 e^{j\theta_1} \qquad (7)$$

Therefore, we can define:

$$r_0 = r(t) = h_0 s_0 + h_1 s_1 + n_0$$
$$r_1 = r(t+T) = -h_0 s_1^* + h_1 s_0^* + n_1 \qquad (8)$$

<u>Combining Scheme:</u>

- We want to decode two symbols:

$$\tilde{s}_0 = h_0^* r_0 + h_1 r_1^* \qquad (9)$$
$$\tilde{s}_1 = h_1^* r_0 - h_0 r_1^*$$

Substituting (7), (8) into (9):

$\tilde{s}_0$:

$$\tilde{s}_0 = h_0^*(h_0 s_0 + h_1 s_1 + n_0) + h_1(-h_0^* s_1 + h_1^* s_0 + n_1^*)$$

$$= \alpha_0^2 s_0 + \cancel{h_0^* h_1 s_1} + h_0^* n_0 - \cancel{h_0 h_1} s_1^* + \alpha_1^2 s_0 + h_1 n_1^*$$

$$\therefore \tilde{s}_0 = (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1 n_1^* \quad (10)$$

$$\tilde{s}_1 = h_1^*(h_0 s_0 + h_1 s_1 + n_0) - h_0(-h_0^* s_1 + h_1^* s_0 + n_1^*)$$

$$\tilde{s}_1 = \cancel{h_0 h_1^*} s_0 + \alpha_1^2 s_1 + h_1^* n_0 + \alpha_0^2 s_1 - \cancel{h_0 h_1^*} s_0 - h_0 n_1^*$$

$$\tilde{s}_1 = (\alpha_0^2 + \alpha_1^2) s_1 - h_0 n_1^* + h_1^* n_0 \quad (11)$$

Notice:
(10), (11) are exactly as (3) meaning that this scheme allows us to achieve same order of diversity as MRRC!

- We then use decision making scheme in (6).

For given transmit power, P, power is evenly distributed among Tx antennas.

$$\therefore \text{Effective SNR / symbol} = \frac{P/2 \, \|h\|^2}{\sigma^2}$$

i.e. −3dB loss, not much really

What we achieved:

(1) Same diversity order as (1×2) system,

(2) No loss in data rate, decreased sensitivity to fading, no bandwidth expansion.

(3) Scheme can be expanded for 2M Tx antennas, M receive antennas.

(4) Allows us to achieve equivalent diversity order in a 2×2 MIMO system compared to 1×4 SIMO system, as shown in paper.

Implementation Issues:

- Several, one key one:

  Soft failure: ←

  - For $h_1 = 0$

  $$\tilde{s}_0 = \alpha_0^2 s_0 + h_0^* s_0$$

  $$\tilde{s}_1 = \alpha_0^2 s_1 - h_0 n_1^*$$

- No diversity order gain!

  ∴ Use 2×2 MIMO for additional redundancy.

Fun Fact: Iphones are 2×2 or 4×4 MIMO.

BER Simulation.