Orthogonal Frequency Division Multiplexing

a.k.a <u>OFDM</u>

<u>Motivating principles</u>:

- Suppose that we are given a large amount of spectrum over which we can transmit information over. Our aim is to achieve both <u>reliable</u> and <u>high data rate</u> communications.

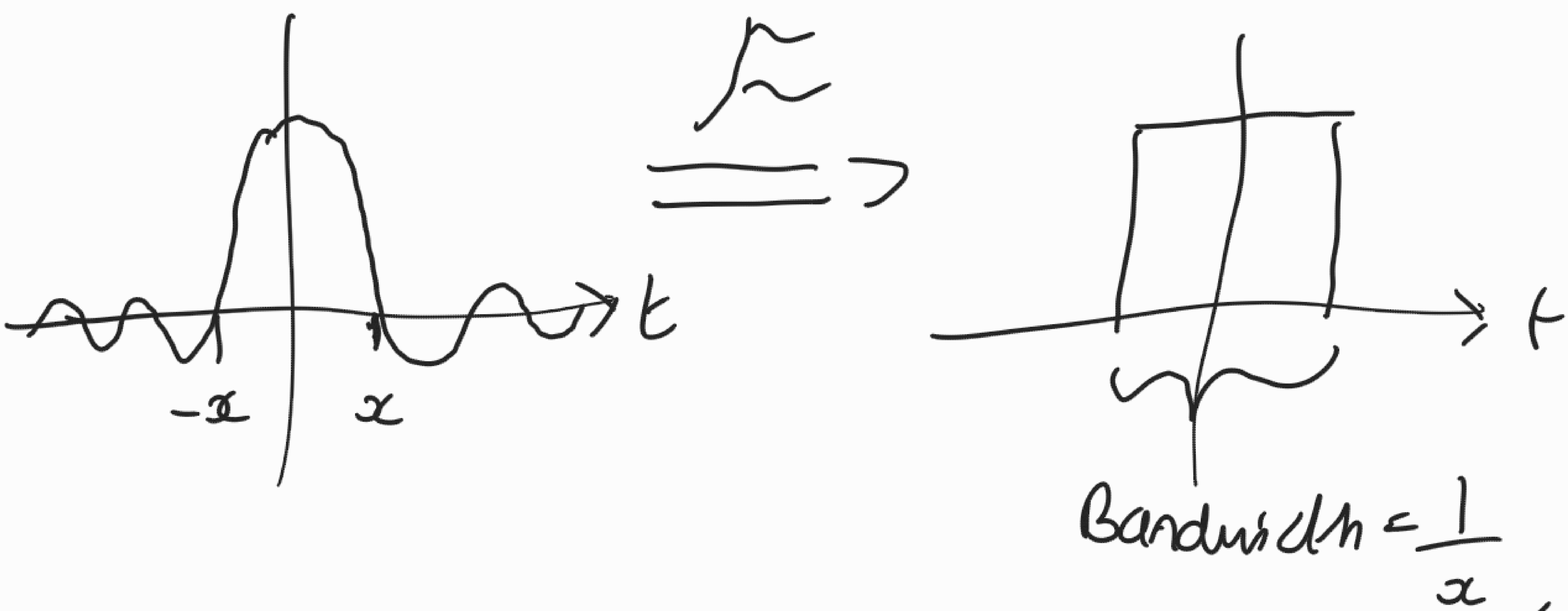
E.g. GSM or Global Standard for Mobile Communications.

in GSM-850 (Used in US/Canada), you have 25 MHz of spectrum for uplink and downlink separately.

- Let us examine a single link for now.

Recall:

- Using sinc pulse shaping with $T = x$, we occupy a bandwidth of $\frac{1}{x}$ in freq. domain



$\therefore$ In theory, given 25 MHz, of spectrum, I can choose a symbol duration of $\frac{1}{25M}$ s.

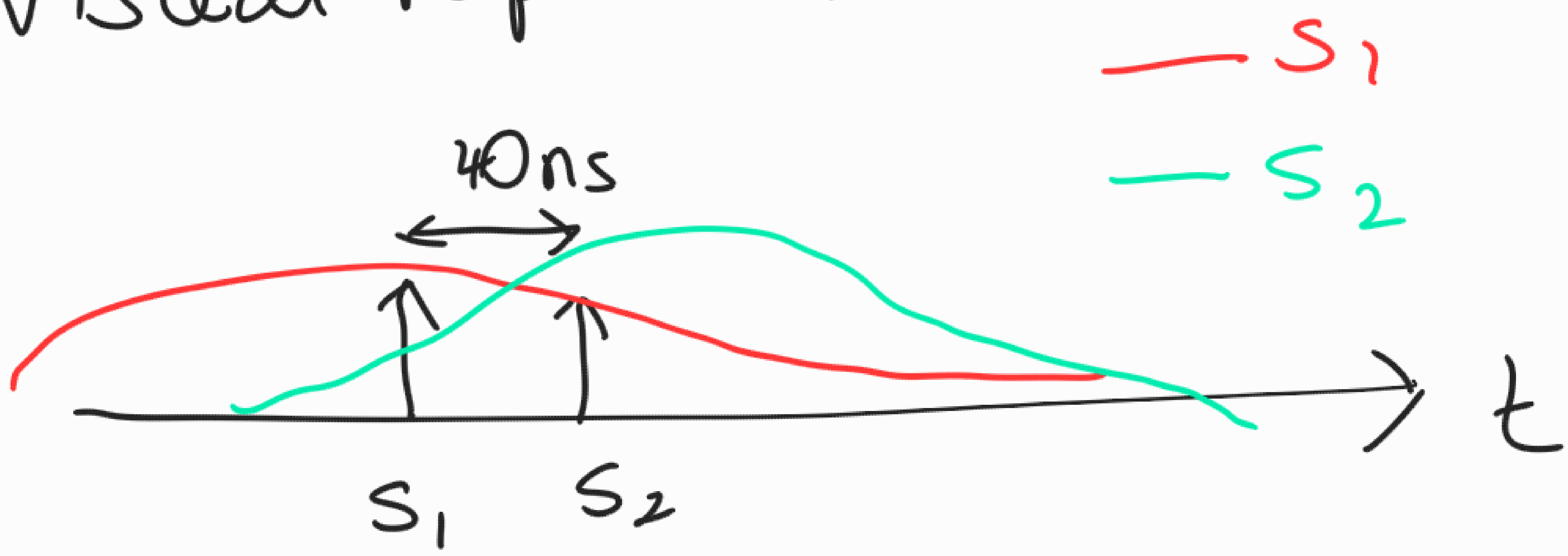
$= 0.04\,\mu s$ or 40 ns.

- This is however is very impractical.

- Wireless communication happens in multipath channels, so what appears at the receiver is the sum of delayed and reflected components of the same signal.

- A transmitted symbol duration is effectively <u>extended</u> due to reflected copies of the signal arriving at the receiver.
- A measure that quantifies the difference in time of arrival between the first and last copy of the signal is known as the <u>delay spread</u>.
- The delay spread is typically $O(1\mu s)$.

Visual representation.



- Delay spread $>> T$ $\therefore$ Inter-symbol interference (ISI) occurs. //

- We therefore can conclude from this that the larger T, the lesser the effect of the delay spread on the symbol and thus less ISI occurs.

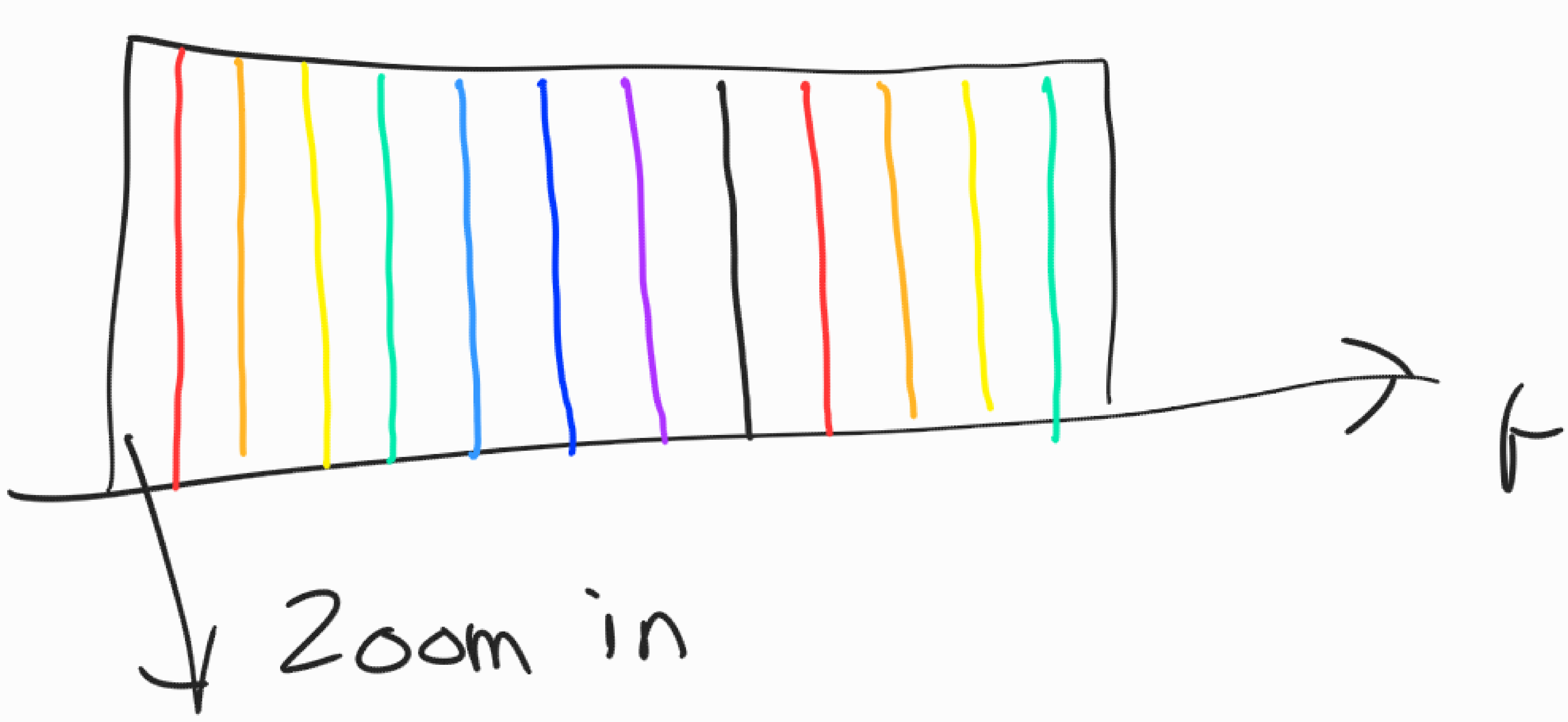
$\therefore$ The larger our T the more RELIABLE our communications are.

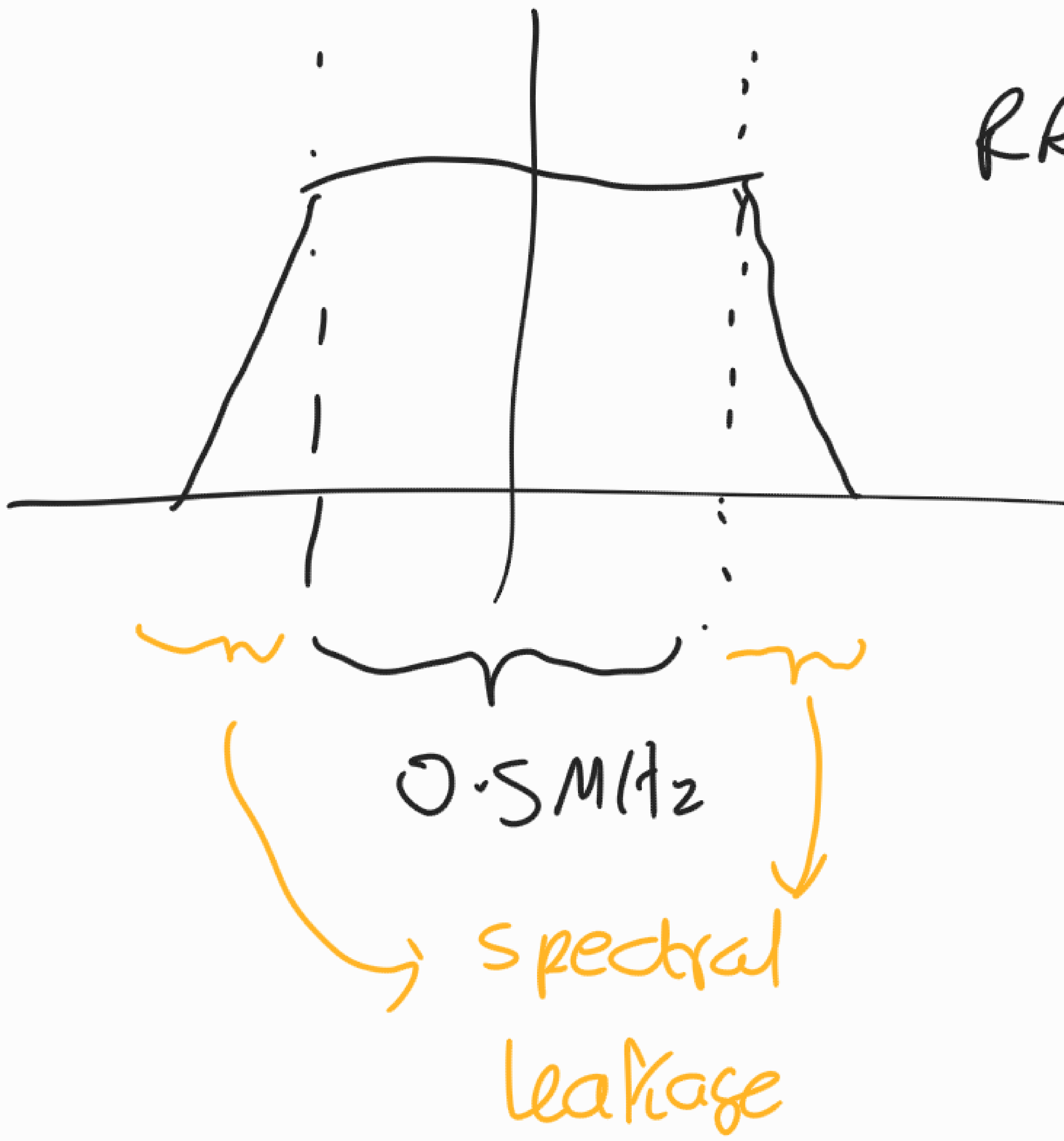
Reliability >> Data rate

- A solution? Use multiple low bandwidth channels, i.e. split spectrum up into multiple channels.

  e.g. suppose I know I can perform communications reliably with a single channel with 0.5MHz bandwidth.

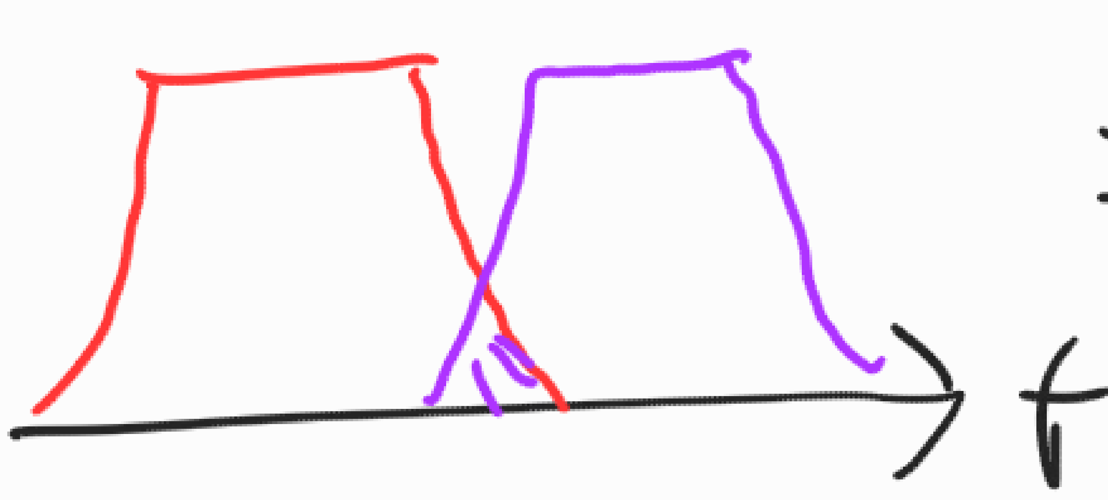
  - Split up 25MHz into 50 0.5MHz channels



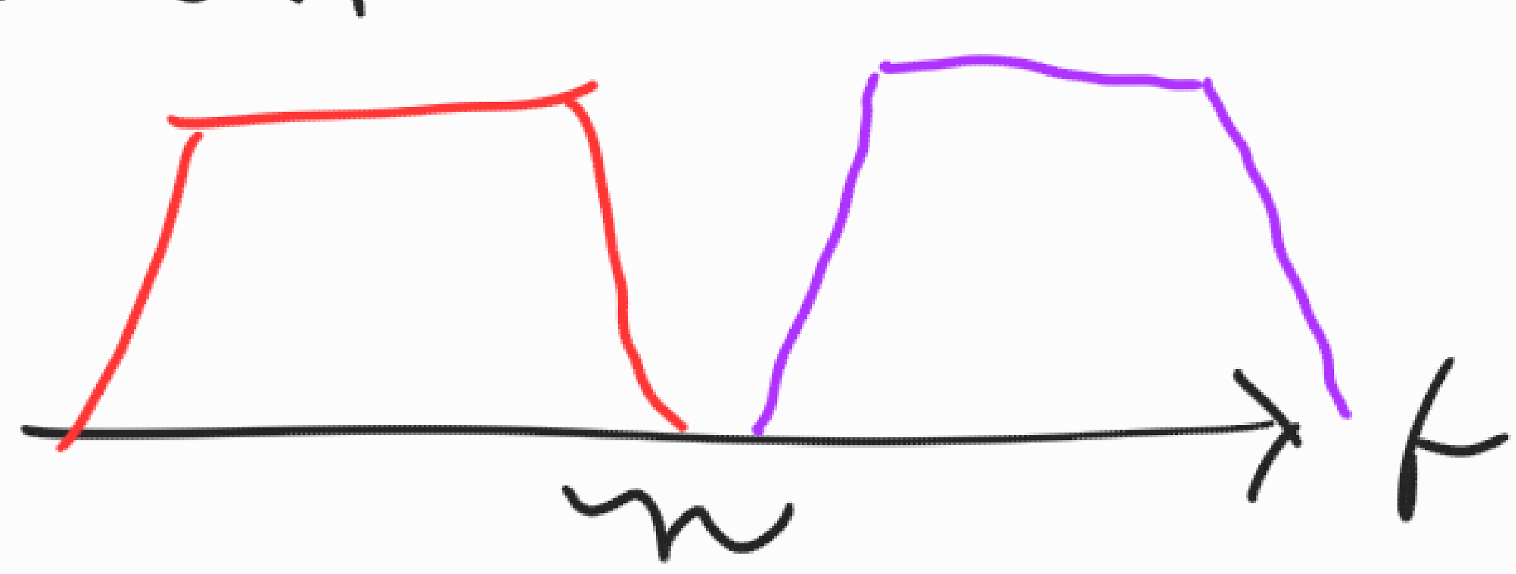


RRC pulse shaping

- Splitting up bandwidth into multiple frequency channels is known as Frequency Division Multiplexing (FDM)
- To make sure neighbouring channels do not overlap in freq. domain, we often define guard bands between channels.

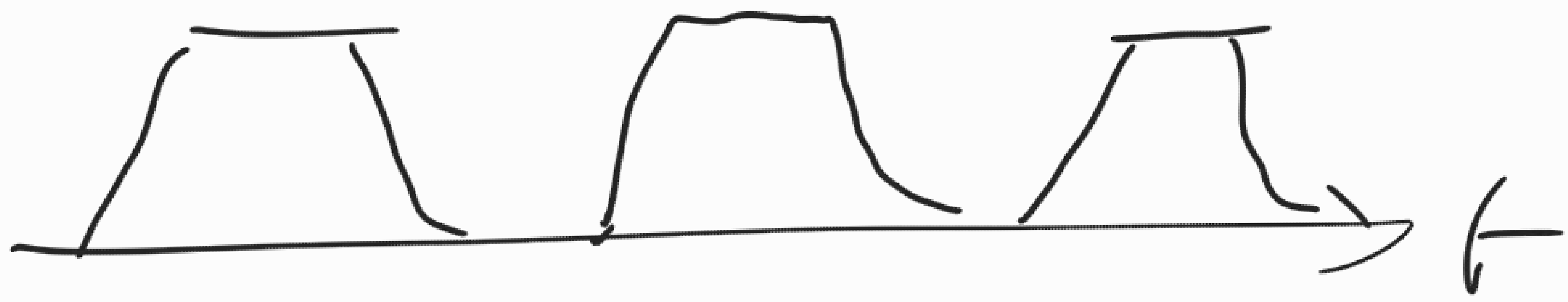


$\Longrightarrow$



Effectively:

Note: GSM uses 124 channels with 200kHz spacing

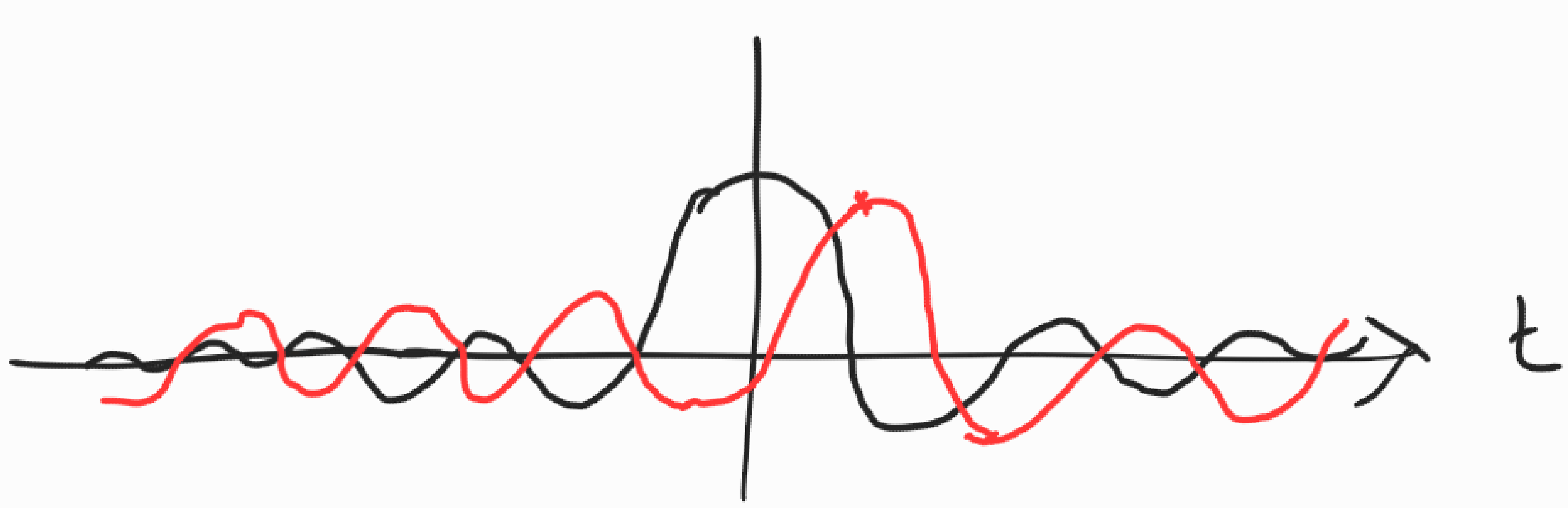


- Using such a scheme we achieve reliable comms with reasonable data rates. However, we have spectral inefficiency because of this spacing between chanels.

$\therefore$ Need to come up with a scheme that allows for better <u>spectral efficiency</u>.

<u>Recalling basic principles</u>:

- Earlier this semester, we were able to see that when we transmit shifted versions of sinc(t/T), we were able to distinguish every symbol from another.



∴ Mathematically, for N symbol transmissions, we can express the resulting signal as:

$$x(t) = \sum_{n=0}^{N-1} \operatorname{sinc}\left(\frac{t-nT}{T}\right)$$

- Note that we can distinguish each transmitted signal from the other even with the fact that <u>they overlap in time</u>.
- We also know in theory that if we keep decreasing T, we can still be able to distinguish each symbol even if we pack a lot of them per unit time.
- So what allows us to do this,

The class of signals $\operatorname{sinc}\left(\frac{t-nT}{T}\right)$ are <u>orthogonal</u> $\forall n \in \mathbb{Z}$.

Defining orthogonality of functions:

$$\frac{1}{L}\int_{L}^{L} f(x)\,g(x) = 0$$

or inner product of functions = 0 //

- Other pulse shaping functions we used such as RRC also have this property allowing us to pack many symbols per unit time.
- Now, what if we extend this concept of orthogonality to the frequency domain?

Orthogonal Frequency Division Multiplexing (OFDM)

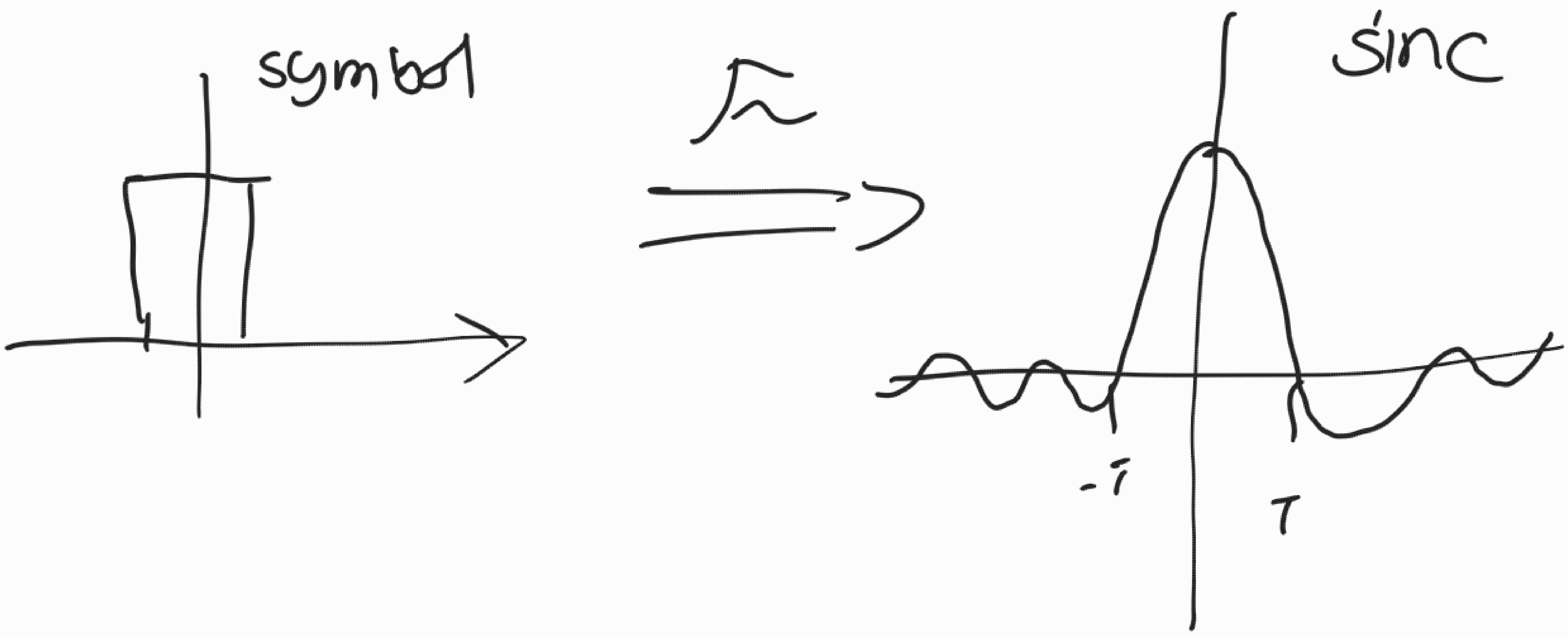
- We first need to find an <u>orthonormal basis set</u> in the frequency domain.

Recall from signals theory (301/438):

- The class of complex discrete complex exponentials $e^{\frac{2\pi}{N}k}$, $k = 0 \ldots N-1$ form an orthonormal basis set.
- This is the basis of the discrete fourier series / discrete fourier transform (DFT).

Also:

- A symbol has a defined interval T in time



Now suppose we compute the following:

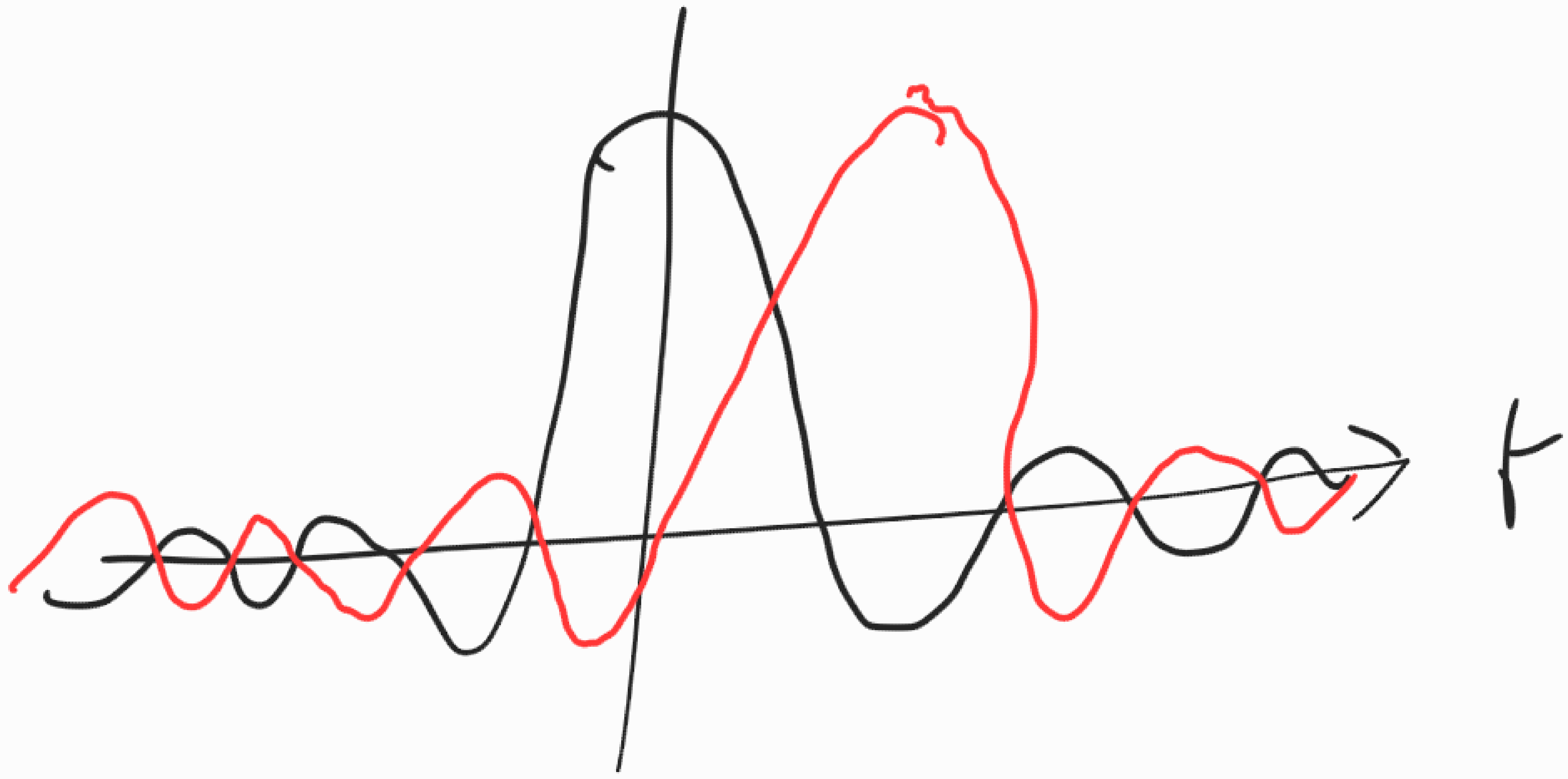
For K symbols where K = N:

$$X[K] = \frac{1}{N} \sum_{K=0}^{N-1} x[k] e^{\frac{2\pi}{N} k n}$$

what is this equivalent to in freq domain:

- <u>shifted sinc pulses in freq domain</u>

For N carriers:



∴ We have overlapping frequency channels that are orthogonal.

- Operation above is known as <u>inverse discrete fourier transform</u> (IDFT).

Channel spacing $= \frac{B}{N} = \frac{1}{T}$ for each channel.

- We can pack a lot of these channels in a given bandwidth.

- Given symbols for which we compute the IDFT, we can retrieve the symbols by retrieve the symbols by inverse operation, the DFT.

- Lastly, need guard band in time.

<u>Now: Schematic Representation</u>