Lecture 1 / Homework 1 for The Software Defined Radio (SDR) of Purdue VIP

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The main difficulty of learning how to use the USRP is due to the following two challenges.

Challenge 1: We need to learn the basic principle of digital communication.
Challenge 2: We need to learn how to implement a communication system in a digital way.

As can be seen, the two challenges have very similar wording: Both involves the keywords "digital" and "communication". However, as we will see that the difference is substantial. Students of the SDR team need to address both challenges before starting to develop a working system. For the following, we will discuss both challenges separately.

Along our discussion, you will be assigned several "tasks". Please complete the tasks in a different notebook and submit them as homework assignment. Your final grade will be partially based on your performance on individual tasks.

Challenge 1: Learning the basic principle of digital communications.

By physics, any communication must be carried in an analog form (EM waves). However, the goal of digital communication is to send digital bits through analog waveforms.

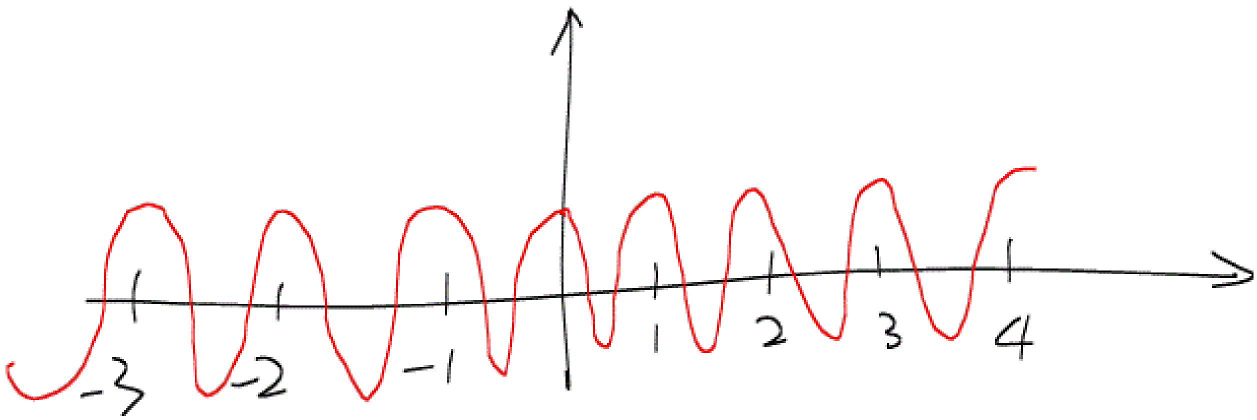
Take the following example, suppose we would like to send a single bit b=0 or b=1. One easy way is the following. Consider two continuous time (CT) waveforms: $f_0(t)=\cos(2\pi t)$ and $f_1(t)=\cos(4\pi t)$.

Task 1: Plot $f_0(t)$ and $f_1(t)$ for the range of t=-4 to 4.

One can see that the two analog waveforms are obviously different. One way of sending a single bit from the transmitter Tx to the receiver Rx is by sending either $f_0(t)$ or $f_1(t)$ depending on whether the message bit b is 0 or 1.

Task 2: If I would like to send a digital bit b=0, plot the <u>analog waveform</u> sent by Tx. If I would like to send a digital bit b=1, plot the *analog waveform* sent by Tx.

Task 3: What if Rx receives the following waveform, can you deduce what is the *b* value transmitted by the transmitter Tx?



As you can see, the above constitutes a basic digital communication system (sending digital bit *b* through analog waveforms).

You should ask yourself: What is the problem of the above system? The main problem is that it can only send 1 bit. For a practical system, we want to send as many bits as fast as possible. For the following, we will modify the above system to send more digital bits but still uses only analog waveforms.

Consider two waveforms $f_0(t)$ and $f_1(t)$ but this time we choose a different pair of waveforms. We choose

Equation 1

$f_0(t)$ by $f_0(t)=1$ if $0<t<1$, and $f_0(t)=0$ otherwise. We choose $f_1(t)=\cos(2\pi t)$ if $0<t<1$ and $f_1(t)=0$ otherwise.

Task 4: Plot $f_0(t)$ and $f_1(t)$ for the range of t=-4 to 4.

If we are only interested in sending 1 bit, then we can repeat the process as described previously.

Task 5: If I would like to send a digital bit b=0, plot the <u>analog waveform</u> sent by Tx. If I would like to send a digital bit b=1, plot the *analog waveform* sent by Tx.

However, as discussed, our goal is to send multiple bits. For example, we can send 3 bits b_1, b_2, and b_3 by the following way. That is, we let Tx send

Equation 2

$x(t)=(1-b_1)f_0(t)+b_1f_1(t) + (1-b_2)f_0(t-3)+b_2f_1(t-3) + (1-b_3)f_0(t-6)+b_3f_1(t-6)$.

Task 6: If I would like to send three digital bits $b_1=0$, $b_2=1$, $b_3=1$, plot the analog waveform sent by Tx for the range of t=-3 to 9. If I would like to send three digital bit $b_1=1$, $b_2=0$, $b_3=1$, plot the *analog waveform* sent by Tx for the range of t=-3 to 9.

Task 7: Is the bit-to-waveform mapping unique? Or equivalently, can you find the inverse map from analog waveforms to digital bit strings? That is, if you can only observe the analog waveform, can you deduce the three bit values sent by Tx?

The above principle can be used to send infinitely many bits b_1, b_2, b_3, …… by shifting $f_0(t)$ and $f_1(t)$ for $3k$ seconds when sending the k-th bit b_k.

Task 8: Are you confident that for any bit string values b_1, b_2, b_3, …… you can plot the corresponding waveform? Are you confident that for any observed waveform, you can deduce the original b_1, b_2, b_3, …… values?

Task 9: In average, how many *bits per second* can we achieve through the above scheme?

Task 10: How to improve the transmission rate so that we can transmit a higher number of bits per second using the above scheme?

One can also transmit 2 bits per time instant. That is, we have 4 analog waveforms $f_{00}(t)$, $f_{01}(t)$, $f_{10}(t)$, and $f_{11}(t)$ to consider. When Tx would like to communicate 2 bit values being 10, Tx simply transmits one waveform $f_{10}(t)$. The Tx can repeat the above time-shift scheme to send multiple ``2-bit values” in a similar way as in Equation 2.

Task 11: Design your own $f_{00}(t)$, $f_{01}(t)$, $f_{10}(t)$, and $f_{11}(t)$ and then use them to send two 2-bit values, 01 and 10 with the time-shift spacing being T=2 second.

Task 12: What is the rate "bits per second" of the above scheme (in the long run)? Can you increase the rate as you did in Task 10?

For illustration purpose, we only consider two waveforms $f_0(t)$ and $f_1(t)$ in this document instead of the setting of four waveforms $f_{00}(t)$, $f_{01}(t)$, $f_{10}(t)$, and $f_{11}(t)$. One can see that there are two major design parameters of the system. First, what are the $f_0(t)$ and $f_1(t)$ of interest; and secondly, what is the time-shift spacing T used for the system. For any time-shift spacing T, the transmission rate is 1/T bits per second. Given any time-shift spacing T (or equivalently, given any desired throughput 1/T bps), a popular choice of $f_0(t)$ and $f_1(t)$ are

Equation 3

$$f_0(t) = \mathrm{sinc}(t/T) \text{ and } f_1(t) = -\mathrm{sinc}(t/T)$$

[For students who have not taken 301, you can google it to learn what the function "sinc(x)" is.]

Task 13: Suppose T=2 second and suppose the above choices of $f_0(t)$ and $f_1(t)$ are used. Similar to Task 4, if we want to send a single bit b=1, plot the final analog waveform x(t) that will be sent by the transmitter Tx. For this task, you have to plot the waveform by MATLAB, instead of by paper and pencil. Please plot $x(t)$ for the range of t=-5 to 15. (In this task, we only send 1 bit)

One thing to notice is that the above choice of $f_0(t)$ and $f_1(t)$ in Equation 3 is quite different from the previous choice in Equation 1. In particular, in Equation 1, the duration for which $f_0(t)$ is non-zero is finite. However, in Equation 3 the duration for which $f_0(t)$ is non-zero is infinite. Whether the duration of "$f_0(t)$ and $f_1(t)$ being non-zero" being finite or infinite *is not* the most critical factor when designing $f_0(t)$ and $f_1(t)$. There are some other reasons why Equation 3 is a good / popular choice of $f_0(t)$ and $f_1(t)$.

Task 14: Suppose T=2 second and suppose the above choices of $f_0(t)$ and $f_1(t)$ are used. If we want to send 10 bits 0010101110 in a similar way as in Task 6, plot the final analog waveform x(t) that will be sent by the transmitter Tx. For this task, you have to plot the waveform by MATLAB, instead of by paper and pencil. Please plot $x(t)$ for the range of t=-5 to 25.

Task 15: Plot another *x*(t) waveform for the case of the 10 bits being 1011010111. Are you able to take a look at the final analog waveform *x*(t) to "guess" what are the 10 bits being transmitted? There is an easy way to do that. Can you find it?

This concludes our brief discussion about Challenge 1. Next time, we will talk about the second challenge.