

Basics

Maxwell's Mixture Model

(Under Untruthful Responding)

Direct Question → Did you cheat on the exam?

Indirect Question → Were you honest on the exam?

Unrelated Question → Were you born in April?

You choose p,q,a

Untruthfulness

1. Direct Question:

Yes → No

No → No

2. Indirect Question:

Yes → Yes

No → Yes

3. Unrelated Question:

Yes → Yes

No → No

Calculator of Estimator

$$P(yes) = p\pi_x a + q(1 - \pi_x) + q\pi_x(1 - a) + (1 - p - q)\pi_y$$

$$P(yes) = \pi_x[pa - q + q(1 - a)] + q + (1 - p - q)\pi_y$$

$$P(yes) = \pi_x(pa - qa) + q + (1 - p - q)\pi_y$$

$$P(yes) = \pi_x a(p - q) + q + (1 - p - q)\pi_y \quad (1)$$

$$\hat{\pi}_x = \frac{\hat{P}_y - q - (1 - p - q)\pi_y}{a(p - q)} \quad (2)$$

If you are aware that untruthfulness was going on, then

$$E(\hat{\pi}_x) = \frac{E(\hat{P}_y) - q - (1 - p - q)\pi_y}{a(p - q)}$$

$$E(\hat{\pi}_x) = \frac{[\pi_x a(p - q) + q + (1 - p - q)\pi_y] - q - (1 - p - q)\pi_y}{a(p - q)}$$

$$E(\hat{\pi}_x) = \pi_x$$

But if you do not know what untruthfulness is occurring

$$\hat{\pi}_x = \frac{\hat{P}_y - q - (1 - p - q)\pi_y}{p - q} \quad (3)$$

With $E(\hat{P}_y)$ given by (1)

$$E(\hat{\pi}_x) = \frac{\pi_x a(p - q)}{p - q}$$

$$Var(\hat{\pi}_x) = \frac{P_y^*(1 - P_y^*)}{(n - 1)(p - q)^2}$$

$$text{Bias}(\hat{\pi}_x) = (\pi_x a) - \pi_x = \pi_x(a - 1)$$

Correct Estimator is given by (2) with unknown untruthfulness
 Incorrect Estimator is given by (3)